CAE 463/524
Building Enclosure Design
Spring 2016

Week 6: February 16, 2016
Complex conduction in building enclosures
Introduce THERM

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Campus enclosure assessment projects

- Need to do thermal assessments by early/mid March
  - Don’t wait too long!
  - All teams have now been formed

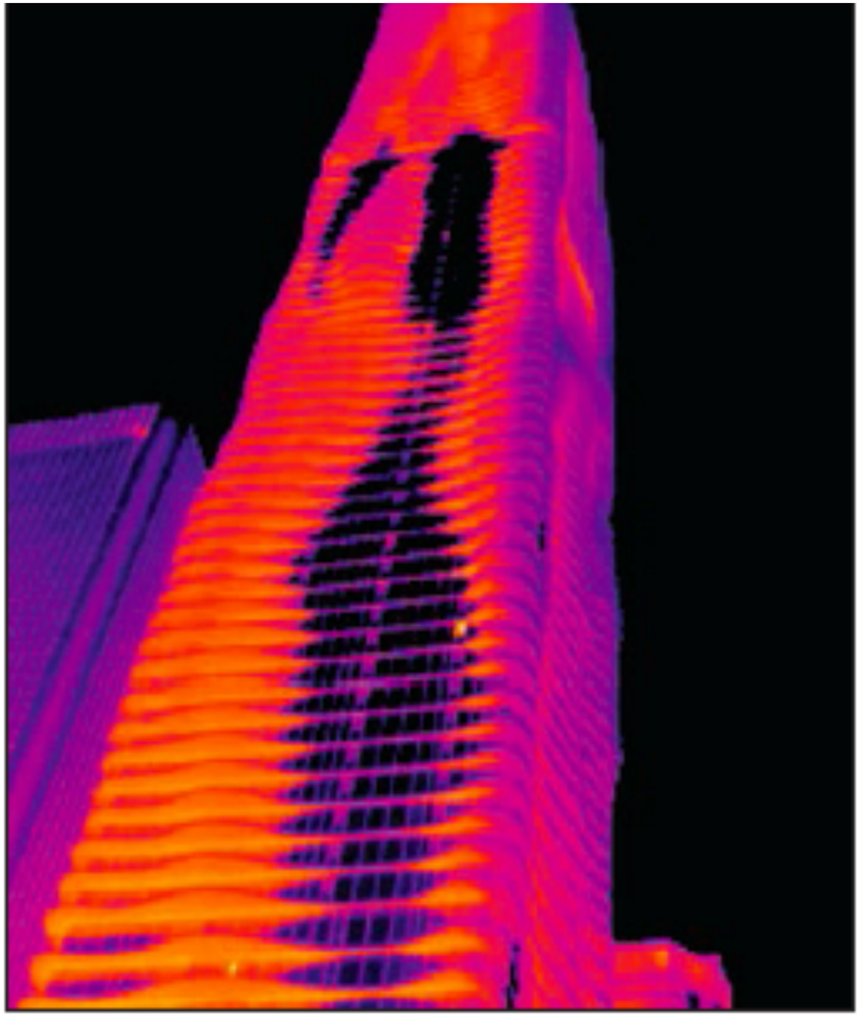
Team #	Members	Building
1	Naveen, Julia, Xu, Luanzhizi, Steve	Alumni
2	Bianca, Al, Taylor, David	SSV
3	Nina, Dina, Lindsey, Salvatore, JiWan	Vandercook
4	Andrea, Ben, Keonho, Kevin	Crown
5	Afshin, Ali, Mehdi, Jose, Kamal	Siegel

- Email me (brent@iit.edu) and our TA Akram (aali21@hawk.iit.edu) when you’re ready to check out the IR camera, IR thermometer, and air T/RH sensor

Last time

- Conduction in building enclosures
  - Common insulation materials and properties
  - Identifying and eliminating/reducing “thermal bridges”

"Thermal Bridge Redux" by J. Lstiburek, ASHRAE Journal July 2012



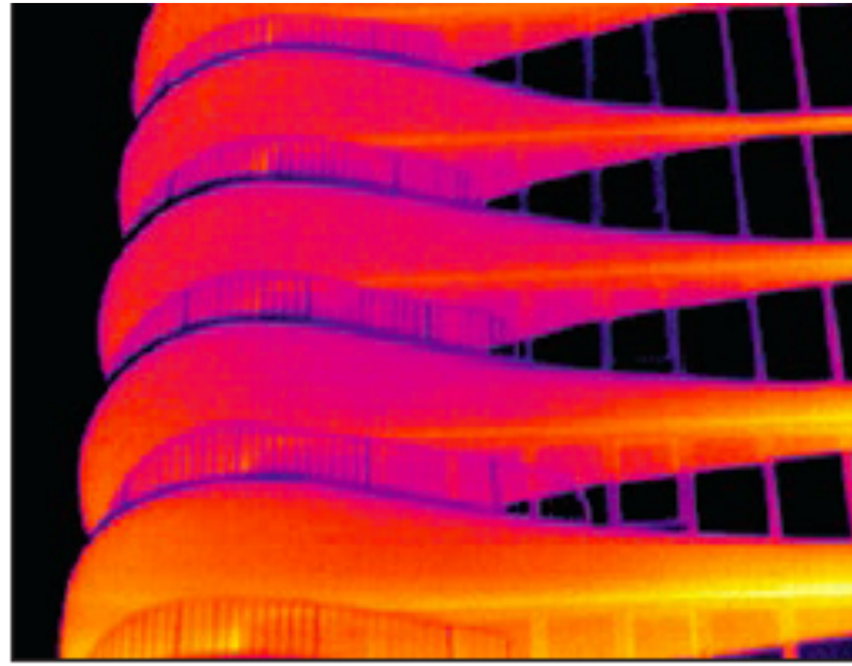


Photo 1f (left): Infrared of Aqua Tower.
Photo 1g (right): Infrared of Aqua Tower Balcony.

Thermographic images courtesy of Dave Robley, Thermographer, Fluke

Aqua Tower design process

https://www.youtube.com/watch?v=cProXspD2S0

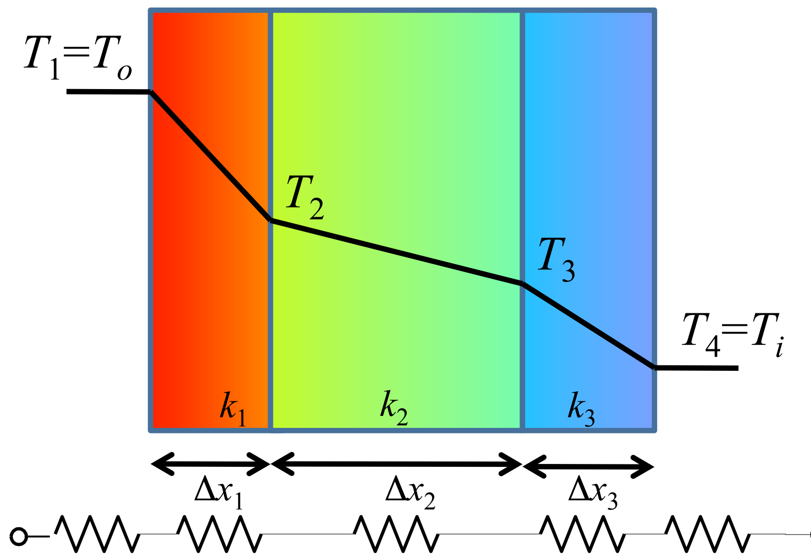
Today's objectives

- More complex conduction in building enclosures
  - Multiple layers
  - Heat transfer in parallel
  - More complex thermal networks (isothermal and parallel path)
  - Numerical solvers (THERM)
  - Below- and on-grade heat transfer

CONDUCTION THROUGH MULTIPLE LAYERS

Conduction through multiple layers

- Just as in electrical circuits, the overall thermal resistance of a series of elements (layers) can be expressed as the sum of the resistances of each layer
  - Don't forget the interior and exterior convective resistances



$$R_o = 0.044 \quad R_1 = \frac{\Delta x_1}{k_1} \quad R_2 = \frac{\Delta x_2}{k_2} \quad R_3 = \frac{\Delta x_3}{k_3} \quad R_i = 0.11$$

Typical "film" values

- By continuity of energy we can write

$$q = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3}$$

so

$$q = \frac{T_1 - T_4}{R_{total}} \quad \text{where} \quad R_{total} = R_o + R_1 + R_2 + R_3 + R_i$$

Can only add resistances (R) in series, not conductances (U)

Simple conduction through multiple layers

- Calculate the R-value of an enclosure assembly

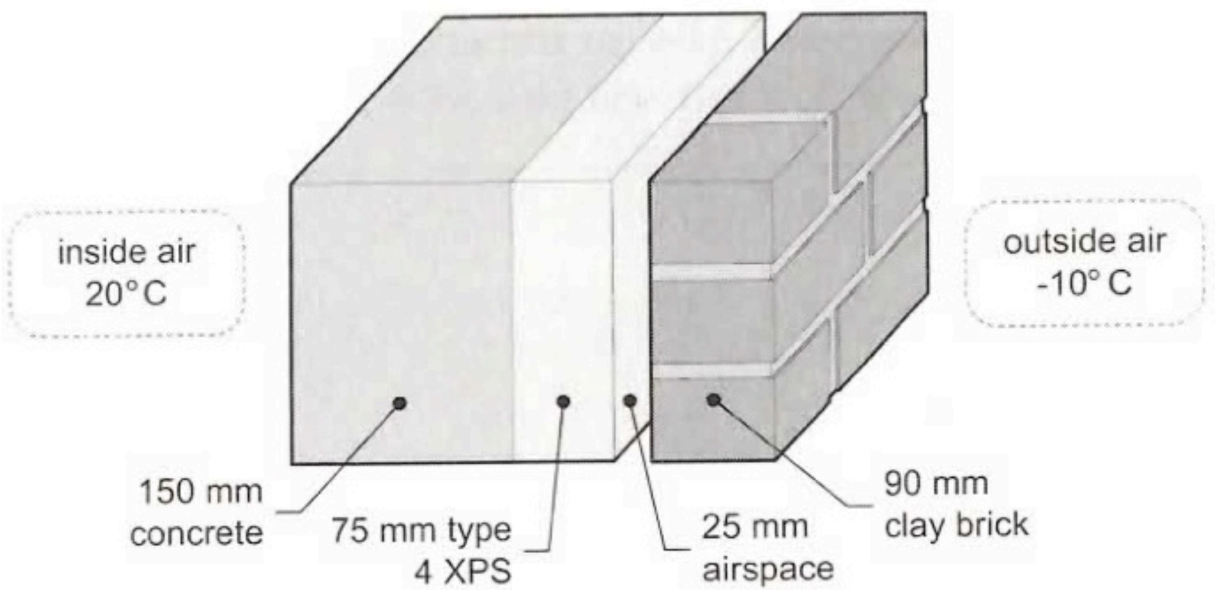
Steps:

1. List each material in the assembly
   - And its conductivity and thickness
2. Calculate conductance of each layer
   - $U = k/L$
3. Calculate thermal resistance of each layer
   - $R = 1/U$
4. Sum the individual thermal resistances to get R_{total}

Conduction through multiple layers

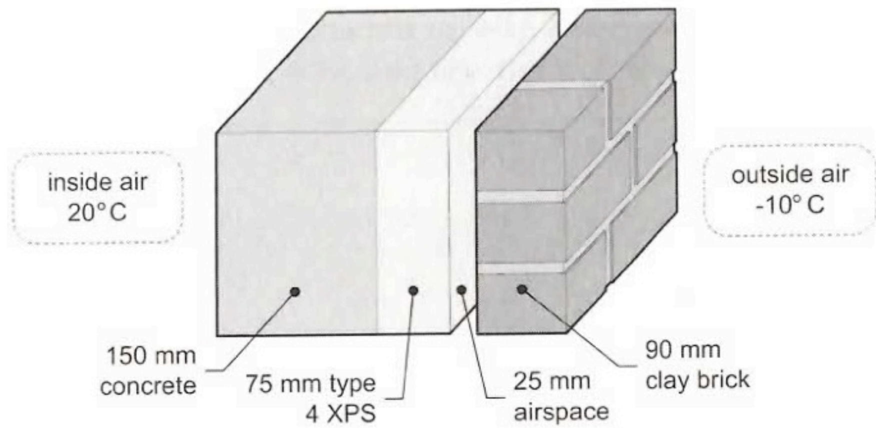
Example problem:

- Calculate the total thermal resistance, R_{total}, and the temperature distribution through the wall shown below



Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for material data



Layer material	Conductivity k = [W/mK]	Thickness L = [m]	Conductance U = [W/m^{2}K]	Resistance R = [m^{2}K/W]

A note on R-values of air cavities

- ASHRAE has measured the combined convective + radiative R-values for thin planar cavities of various orientations and depths with various "ε_{eff}"

- These are the best data to use for air spaces in assemblies
  – If you do not know that the material in the cavity is reflective or "low e", just assume that both walls of the cavity have ε=0.9 for each surface, so that when combined, ε_{eff}= 0.82

$$\varepsilon_{eff} = \varepsilon_1 \varepsilon_2$$

2013 ASHRAE Handbook, Chapter 26 (small cavities)

R-values for different air gap characteristics

Table 3 Thermal Resistances of Plane Air Spaces[a,b,c], $(m^2 \cdot K)/W$

Position of Air Space	Direction of Heat Flow	Air Space: Mean Temp.[d], °C	Air Space: Temp. Diff.[d], °C	13 mm Air Space[c]: Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82	20 mm Air Space[c]: Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82
Horiz.	Up ↑	32.2	5.6	0.37	0.36	0.27	0.17	0.13	0.41	0.39	0.28	0.18	0.13
		10.0	16.7	0.29	0.28	0.23	0.17	0.13	0.30	0.29	0.24	0.17	0.14
		10.0	5.6	0.37	0.36	0.28	0.20	0.15	0.40	0.39	0.30	0.20	0.15
		–17.8	11.1	0.30	0.30	0.26	0.20	0.16	0.32	0.32	0.27	0.20	0.16
		–17.8	5.6	0.37	0.36	0.30	0.22	0.18	0.39	0.38	0.31	0.23	0.18
		–45.6	11.1	0.30	0.29	0.26	0.22	0.18	0.31	0.31	0.27	0.22	0.19
		–45.6	5.6	0.36	0.35	0.31	0.25	0.20	0.38	0.37	0.32	0.26	0.21
45° Slope	Up ↗	32.2	5.6	0.43	0.41	0.29	0.19	0.13	0.52	0.49	0.33	0.20	0.14
		10.0	16.7	0.36	0.35	0.27	0.19	0.15	0.35	0.34	0.27	0.19	0.14
		10.0	5.6	0.45	0.43	0.32	0.21	0.16	0.51	0.48	0.35	0.23	0.17
		–17.8	11.1	0.39	0.38	0.31	0.23	0.18	0.37	0.36	0.30	0.23	0.18
		–17.8	5.6	0.46	0.45	0.36	0.25	0.19	0.48	0.46	0.37	0.26	0.20
		–45.6	11.1	0.37	0.36	0.31	0.25	0.21	0.36	0.35	0.31	0.25	0.20
		–45.6	5.6	0.46	0.45	0.38	0.29	0.23	0.45	0.43	0.37	0.29	0.23
Vertical	Horiz. →	32.2	5.6	0.43	0.41	0.29	0.19	0.14	0.62	0.57	0.37	0.21	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.51	0.49	0.35	0.23	0.17
		10.0	5.6	0.47	0.45	0.33	0.22	0.16	0.65	0.61	0.41	0.25	0.18
		–17.8	11.1	0.50	0.48	0.38	0.26	0.20	0.55	0.53	0.41	0.28	0.21
		–17.8	5.6	0.52	0.50	0.39	0.27	0.20	0.66	0.63	0.46	0.30	0.22
		–45.6	11.1	0.51	0.50	0.41	0.31	0.24	0.51	0.50	0.42	0.31	0.24
		–45.6	5.6	0.56	0.55	0.45	0.33	0.26	0.65	0.63	0.51	0.36	0.27
		32.2	5.6	0.44	0.41	0.29	0.19	0.14	0.62	0.58	0.37	0.21	0.15

Usually we use values from the $\varepsilon_{eff} = 0.82$ column unless one material is low-e

2013 ASHRAE Handbook, Chapter 26 (small cavities)

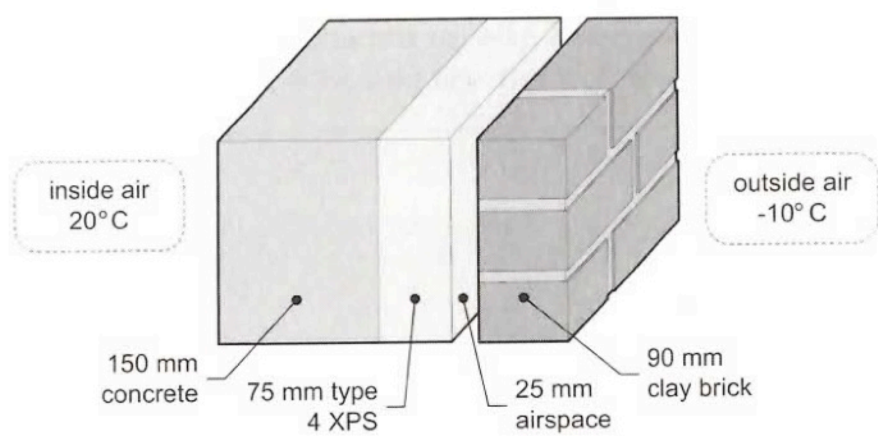
R-values for different air gap characteristics

Position of Air Space	Direction of Heat Flow	Mean Temp.[d], °C	Temp. Diff.[d], °C	40 mm Air Space[c] Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82	90 mm Air Space[c] Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82
		32.2	5.6	0.45	0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
		10.0	16.7	0.33	0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
		10.0	5.6	0.44	0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
Horiz.	Up	–17.8	11.1	0.35	0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
		–17.8	5.6	0.43	0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
		–45.6	11.1	0.34	0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
		–45.6	5.6	0.42	0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
		32.2	5.6	0.51	0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
		10.0	16.7	0.38	0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
		10.0	5.6	0.51	0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
45° Slope	Up	–17.8	11.1	0.40	0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
		–17.8	5.6	0.49	0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
		–45.6	11.1	0.39	0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
		–45.6	5.6	0.48	0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
		32.2	5.6	0.70	0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
		10.0	5.6	0.67	0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
Vertical	Horiz.	–17.8	11.1	0.49	0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
		–17.8	5.6	0.62	0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
		–45.6	11.1	0.46	0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
		–45.6	5.6	0.58	0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26
		32.2	5.6	0.89	0.80	0.45	0.24	0.16	0.85	0.76	0.44	0.24	0.16

Usually we use values from the $\varepsilon_{eff} = 0.82$ column

Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for material data



Layer material	Conductivity k = [W/mK]	Thickness L = [m]	Conductance U = [W/m^{2}K]	Resistance R = [m^{2}K/W]
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17

R_{total} (IP) = 17.3 hr·ft^2·°F/Btu

R-values of deeper cavities

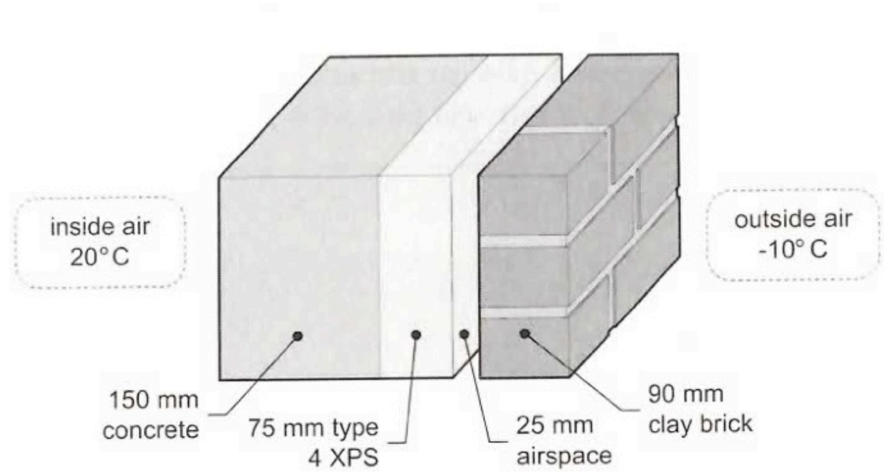
- The R-value of cavities stops increasing much at 3 inches (75 mm) depth
  - Beyond 3 inches (75 mm), convection and radiation dominate
  - For a deep cavity, either compute R-values with more advanced methods or use the 3 inch (75 mm) value

- Do **NOT** take the R value of a 1 inch (25 mm) cavity and multiply by the thickness of the cavity for thick cavities
  - If you did that, you would guess that an 8 foot attic would have an R value of about 100 $hr \cdot ft^2 \cdot °F/Btu$, which is a factor of 20 too high!

Conduction through multiple layers

- U_{total} = 0.33 W/m^{2}K

- Calculate steady-state heat flux through the enclosure



- q = UΔT
- q = (0.33 W/m^{2}K)*(T_{inside} - $T_{outside}$)
- q = (0.33 W/m^{2}K)*(30 K) = 10 W/m^2
  - From inside to outside

Conduction through multiple layers

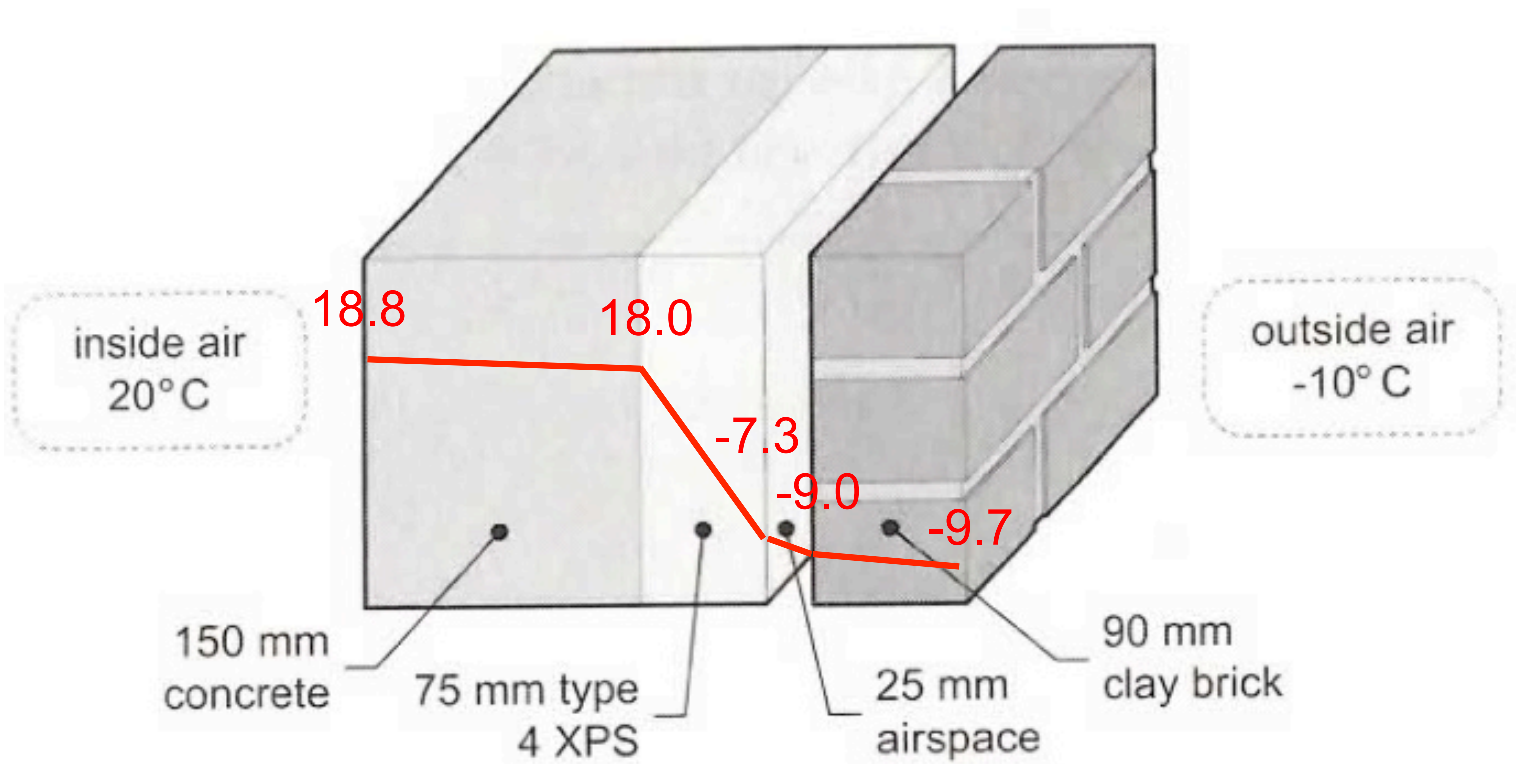
- Calculating the **temperature gradient** through an enclosure of i materials

$$\Delta T_i = \frac{T_{internal} - T_{external}}{\sum_{i=0}^{n} R_i} R_i$$

Layer	Conductivity W/mK	Thickness m	Conductance W/m²K	Resistance m²K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			R_{total} (m²K/W)	3.04
			U_{total} (W/m²K)	0.33

Conduction through multiple layers

- Calculating the **temperature gradient** through an enclosure of i materials

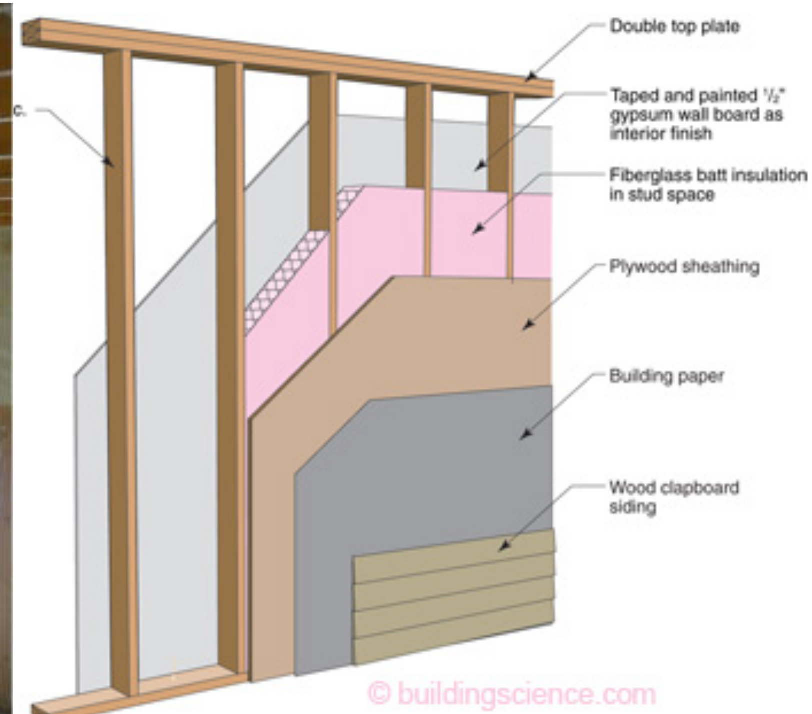


Why would temperature distributions be helpful to know?

http://www.energyvanguard.com/blog-building-science-hers-bpi/a-tale-of-two-roofs-frost-snow-and-attic-heat-loss

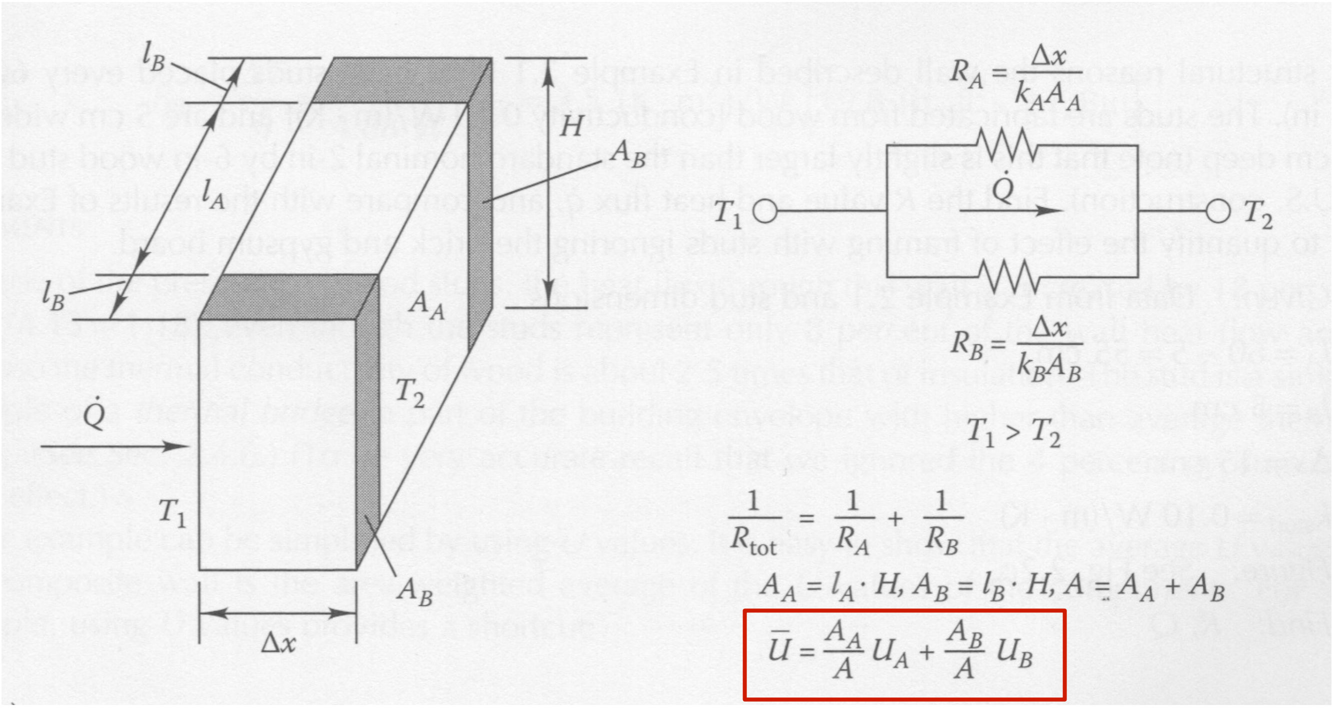
What about more realistic constructions?

- Building walls rarely exist in complete, homogenous layers
- Structural elements – studs – are usually located within the envelope matrix at regular intervals



Accounting for structural elements (studs)

- Parallel-resistance heat flow



Treat resistances like resistors in parallel

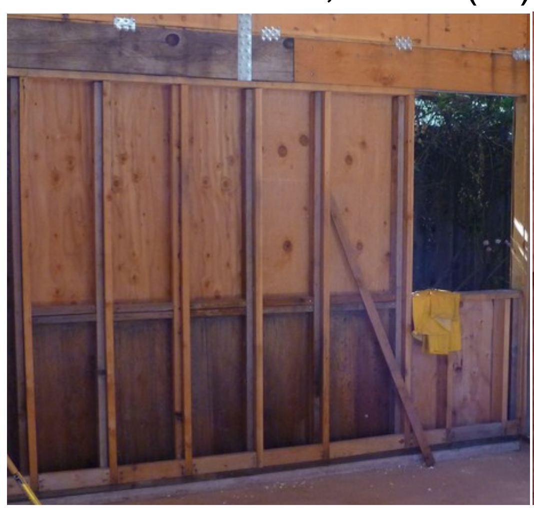
- Simply use **<u>weighted average U values</u>**:

$$U_{total} = \frac{A_1}{A_{total}} U_1 + \frac{A_2}{A_{total}} U_2 + \ldots$$

Example: Accounting for structural elements (studs)

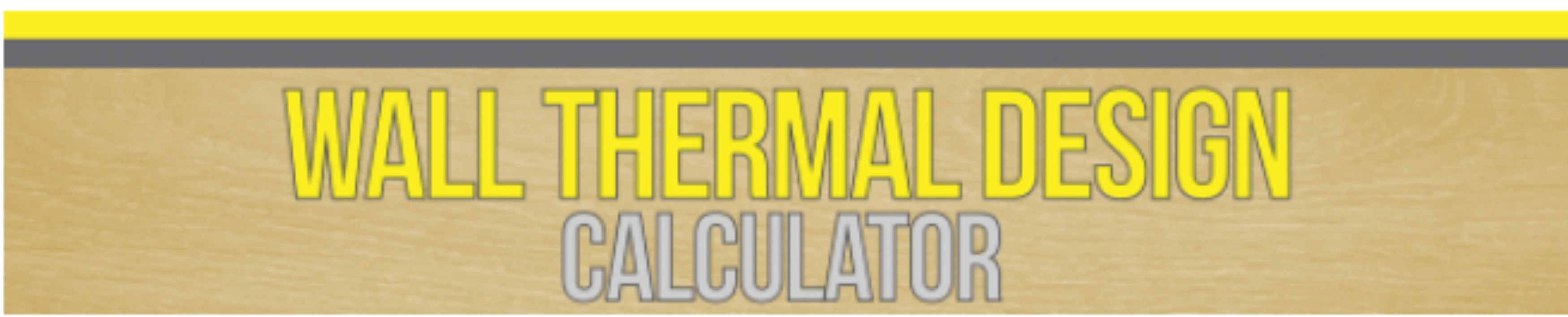
- For structural reasons the wall below must have studs placed every 24”
  - “24 in o.c.” = 24 inches on center
- The studs are wood and are 2” wide and 6” deep
- **Problem:** Find the R value of this assembly

Brick layer: 4” thick, R-0.6 (IP)
Fiberglass insulation and studs: 6” thick, R-21.3 (IP)
Wood studs: 6” thick, R-6.5 (IP)
Gypsum board: 0.5” thick, R-0.4 (IP)

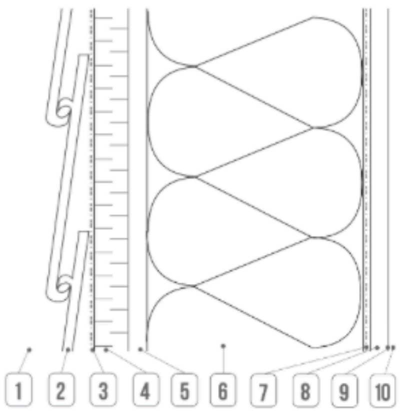


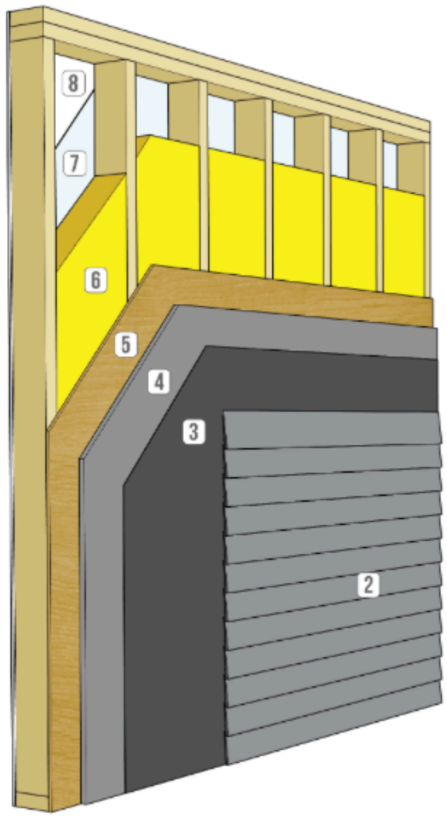
Online tools to help us out with thermal bridges

- There are some online "modified zone" calculators developed by Oakridge National Laboratory
  - http://www.ornl.gov/sci/roofs+walls/calculators/modzone/index.html
  - http://www.ornl.gov/sci/roofs+walls/calculators/modzone/modzone2.html
- Building Science Corp:
  - http://www.buildingscience.com/documents/information-sheets/high-r-value-wall-assemblies/?topic=/doctypes/information-sheets
- Canadian Wood Council:
  - http://www.cwc.ca/index.php/en/resources/wallthermaldesignmainpage



Wall thermal design calculator





	WALL ASSEMBLY COMPONENTS [1]	RSI	R
1	exterior air film	0.03	0.17
2	vinyl siding (no air space)	0.11	0.62
3	asphalt impregnated paper [2]	0.00	0.00
4	1" (25.4 mm) unfaced expanded polystyrene	0.66	3.75
5	7/16" (11.1 mm) OSB sheathing	0.11	0.62
6	2x4 framing filled with R14 batt @ 16" o.c.	1.62	9.20
7	polyethylene	0.00	0.00
8	1/2" (12.7 mm) gypsum board	0.08	0.45
9	finish: 1 coat primer	0.00	0.00
10	interior air film	0.12	0.68
	Effective RSI / R Value of Entire Assembly	**2.73**	**15.50**
	Centre of Cavity RSI / R Value	3.58	20.33
	Installed Insulation RSI / R Value (nominal)	3.13	17.75
	Effective RSI / R Value of Assembly with Advanced Framing (advanced framing as defined by NBC 9.36.2.4.(1))	2.84	16.10

Note: [1]Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. [2] Sheathing membrane material must comply with CAN/CGSB-51.32, "Sheathing Membrane, Breather Type."

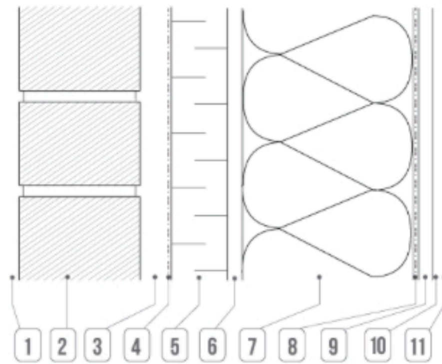
LEGEND: Pass; Proceed with caution

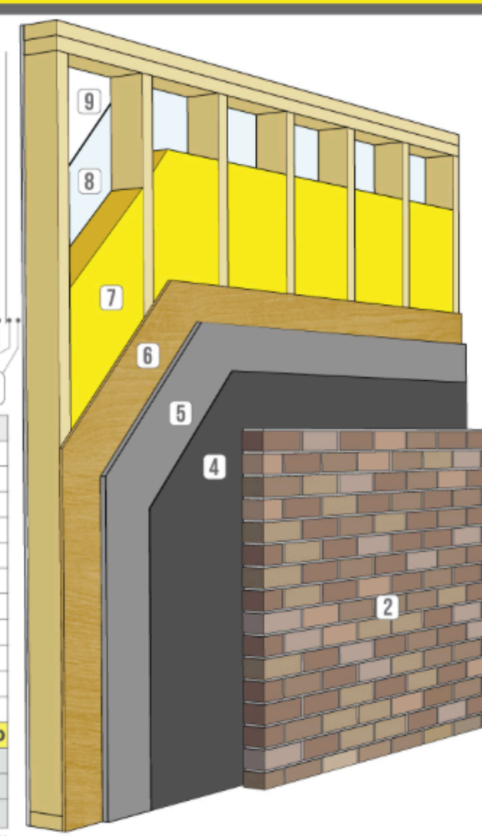
SIMULATED DURABILITY ANALYSIS

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WUFI HYDROTHERMAL MODELING	Pass	Pass	Pass	Pass	Pass

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 60 ng/(Pa•s•m2) must comply to NBC 9.25.5.2.

R_{eff} 15.5





	WALL ASSEMBLY COMPONENTS [1]	RSI	R
1	exterior air film	0.03	0.17
2	fired clay brick 4" (102 mm) [2]	0.07	0.40
3	1" (25.4 mm) air space w/ weep holes at base of wall	0.18	1.02
4	asphalt impregnated paper [3]	0.00	0.00
5	2" (50.8 mm) extruded polystyrene type 3 / 4	1.78	10.00
6	7/16" (11.1 mm) OSB sheathing	0.11	0.62
7	2x8 framing filled with R28 batt @ 24" o.c.	3.44	19.55
8	polyethylene	0.00	0.00
9	1/2" (12.7 mm) gypsum board	0.08	0.45
10	finish: 1 coat primer	0.00	0.00
11	interior air film	0.12	0.68
	Effective RSI / R Value of Entire Assembly	**5.81**	**33.00**
	Centre of Cavity RSI / R Value	7.30	41.44
	Installed Insulation RSI / R Value (nominal)	6.69	38.00
	Effective RSI / R Value of Assembly with Advanced Framing (advanced framing as defined by NBC 9.36.2.4.(1))	5.96	33.82

Note: [1]Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. [2] The thermal resistance of mortar was not considered. [3]Sheathing membrane material must comply with CAN/CGSB-51.32, "Sheathing Membrane, Breather Type."

OUTBOARD TO INBOARD RATIO 0.39

LEGEND: Pass; Proceed with caution; Check permeance of material

SIMULATED DURABILITY ANALYSIS

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WUFI HYDROTHERMAL MODELING	Pass	Pass	Pass	Pass	Pass
OUTBOARD TO INBOARD RATIO COMPLIANCE	Pass 0.2	Pass 0.3	Pass 0.2	Pass 0.2	Pass 0.2

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 60 ng/(Pa•s•m2) must comply to NBC 9.25.5.2.

R_{eff} 33.0

Limitations to the summation rule

The summation rule for finding R_{total} has several limitations:

- Only works for **layers**
- Layers must be **same area**
- Layers must be **uniform thickness**
- Layers must have **constant material properties**
  - This is the **biggest** limitation

What do we do with more realistic constructions?

- Parallel path or ISO thermal equivalents
- Computer modeling

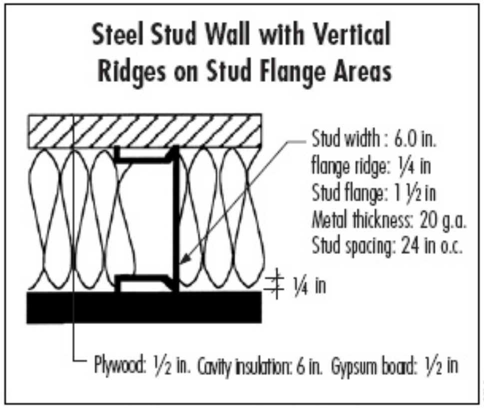


Figure 1. Vertical ridges on a steel stud reduce the contact area between the stud and the sheathing material and improve the whole wall R-value.

CONDUCTION IN MORE COMPLICATED ENCLOSURE ASSEMBLIES

Thermal networks

- We have already learned how to combine layers to get an overall thermal resistance (or U value) for an assembly made of homogenous layers
  - But we often have to find R (or U) of a *more complicated* assembly

- The first level of more complicated analysis is best done using more complicated thermal networks

- References:
  - ISO 6946 Building components and building elements thermal resistance and thermal transmittance calculation method
  - ASHRAE Handbook of Fundamentals

Developing a thermal network

1. Identify the layers of the assembly
2. Identify all elements in the layers with differing thermal conductivity
3. Find the R-value for each element
4. Draw a resistor for each element

    Don't forget the internal and external convection resistances

5. Set the resistance to the R-value ***divided by*** the fractional cross sectional area of the element
6. Connect resistors assuming isothermal or parallel path conditions

Isothermal vs. Parallel Path?

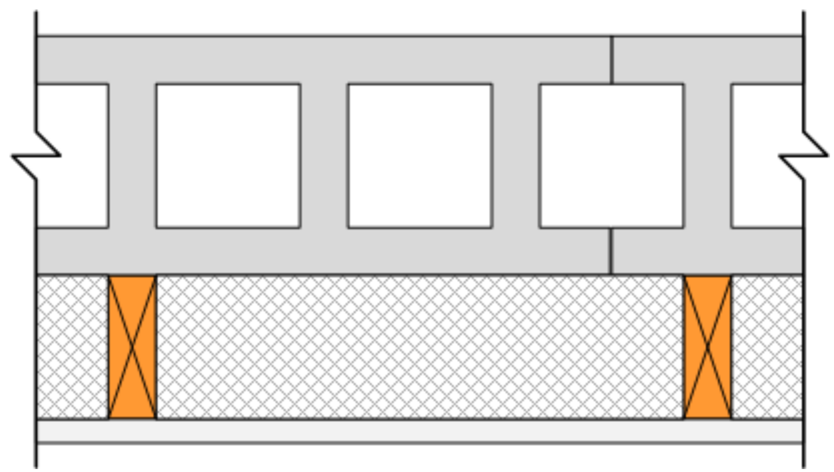
- In the **isothermal** assumption, the temperature at any layer interface is assumed to be constant, even if the layer is made of more than one material
  - This means that there is a network node that corresponds to the interface of each layer

- In the **parallel path** assumption, the heat transfer is assumed to be only normal to planes
  - This means that the network is several parallel branches

- This is probably best illustrated by example

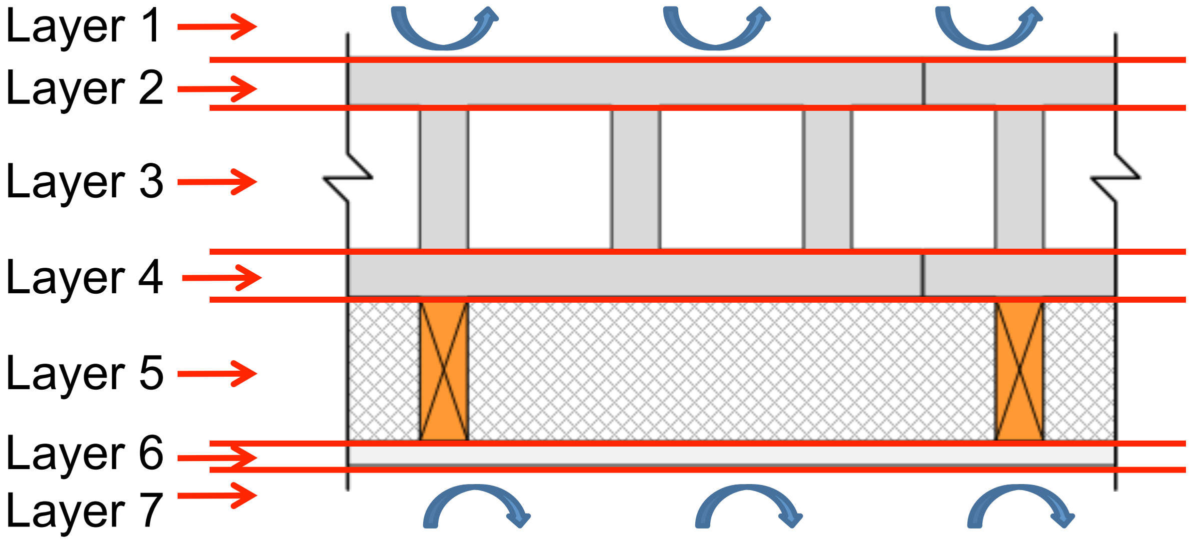
Isothermal/parallel path example

- Consider a CMU Block wall with an attached stud wall
  - A section of the wall is shown below in plan view
  - Draw the isothermal and parallel path thermal networks



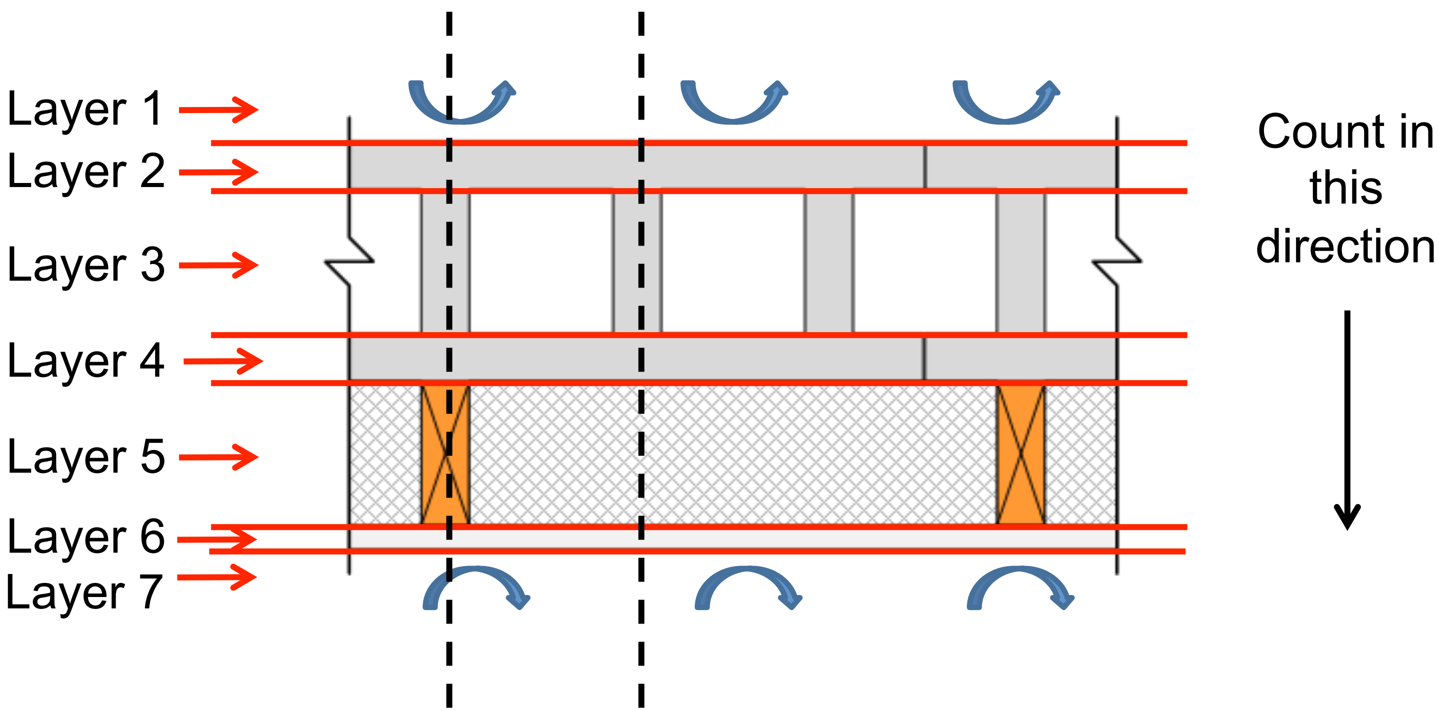
Isothermal/parallel path example

1. **Identify the layers**
   - This example has seven layers
   - Five are within the assembly
     - The other two are internal and external convection resistances



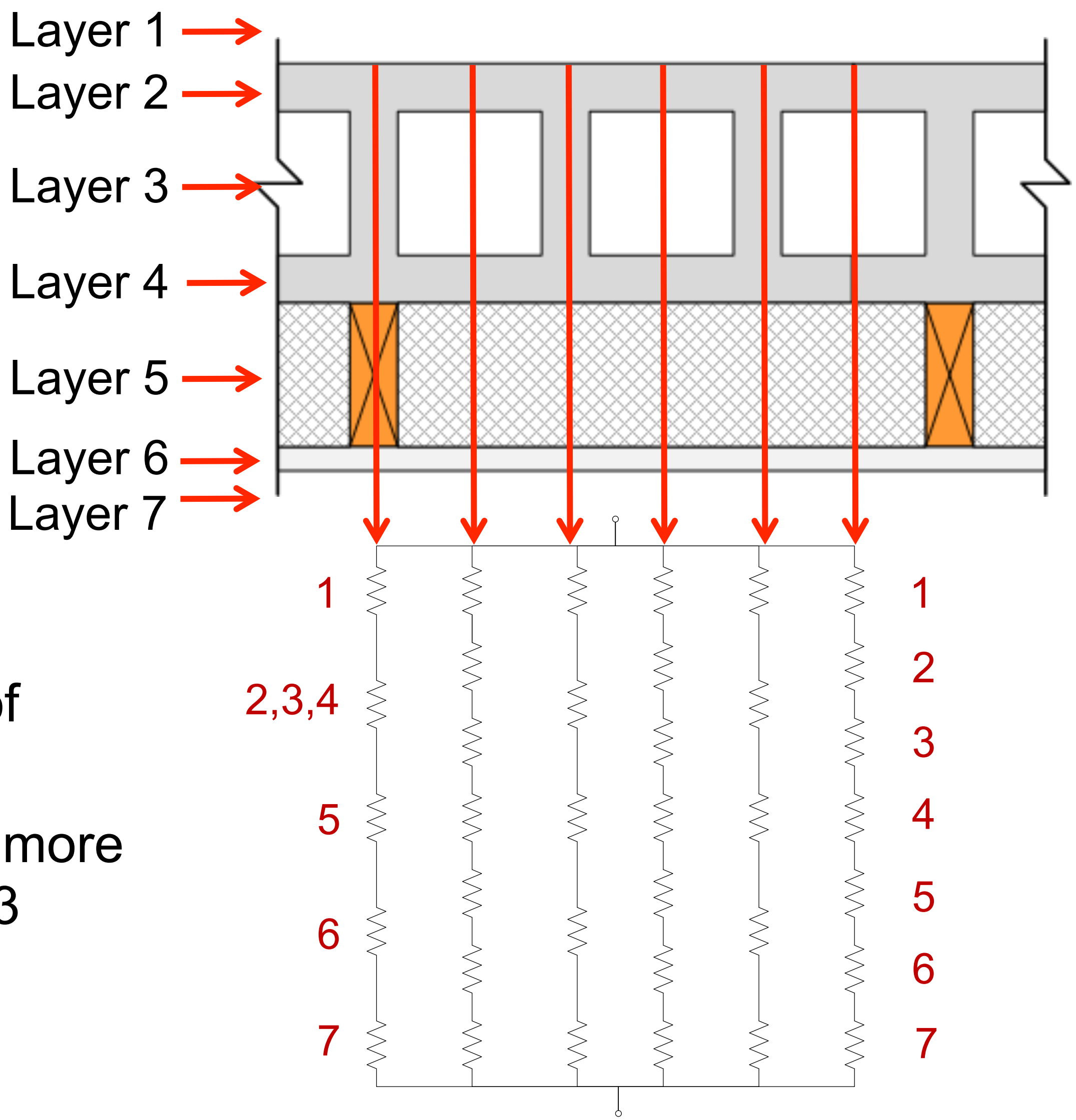
2. Identify the elements and 3. Find R-value for each element

- Layer 1 and 7 have 1 element (convection)
- Layer 2 and 4 have 1 element (1.5 inches of concrete)
- Layer 6 has 1 element (0.5 inches of gypsum wallboard)
- Layer 3 has 2 elements (3.5 inches air cavity and 3.5 inches concrete)
- Layer 5 has 2 elements (3.5 inches insulation and 3.5 inches wood stud)



Parallel Path method

- Identify the different paths and draw them in parallel



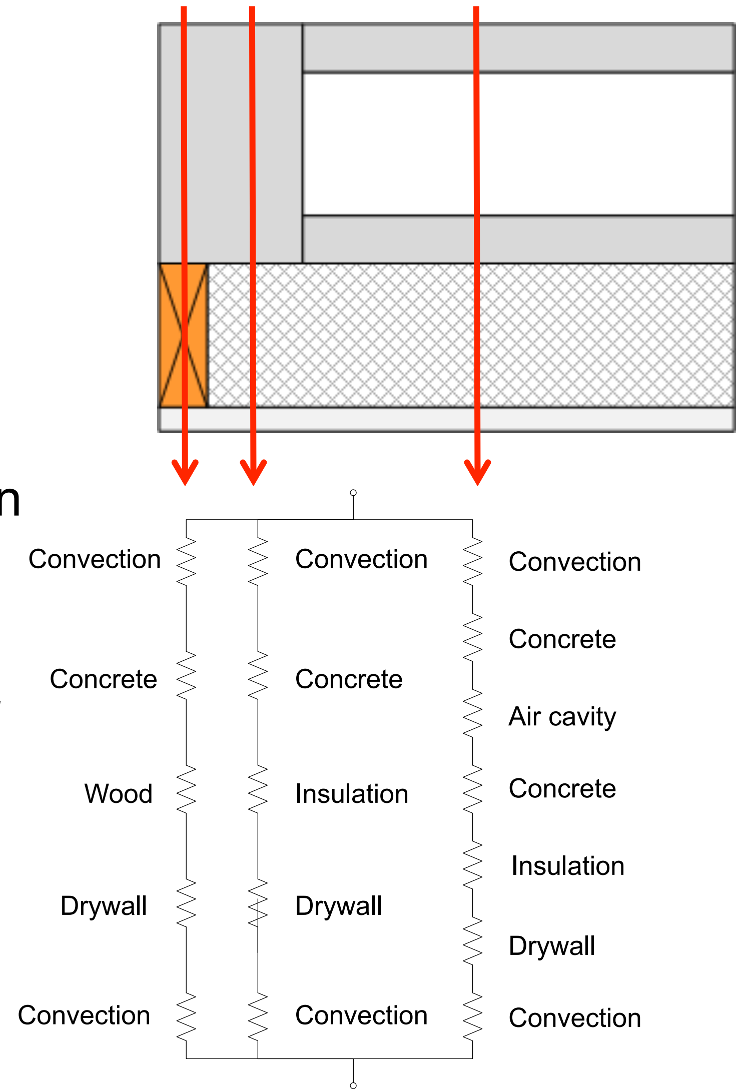
Note: In this example, several of these paths are identical

- So although there are many more components, there are only 3 unique ones

Alternate/Shortened Parallel Path

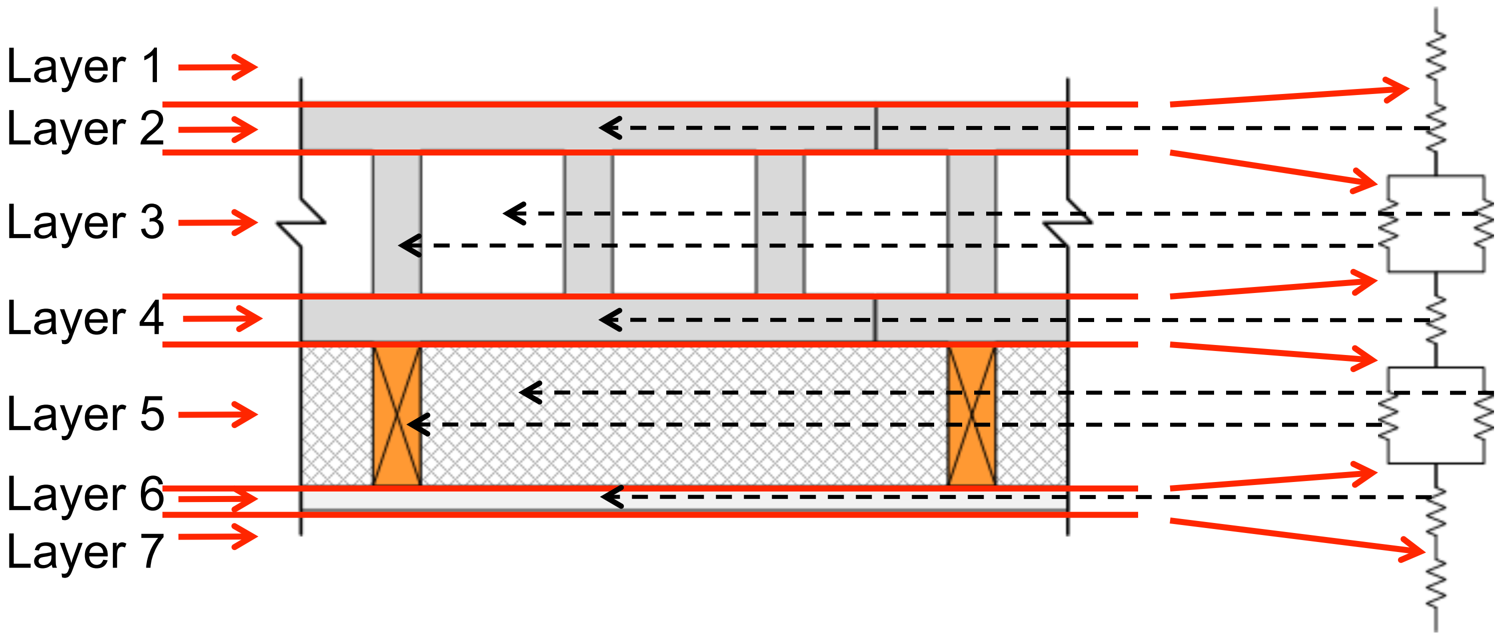
- We could combine like elements in each layer to reduce the number of paths to analyze without changing the answer for either isothermal or parallel path
  - This parallel network only has three paths, but will have the same temperatures at each interface location as the previous network

- Note the difference in the number of resistances between each path – varies according to # of elements/ layers involved in each path



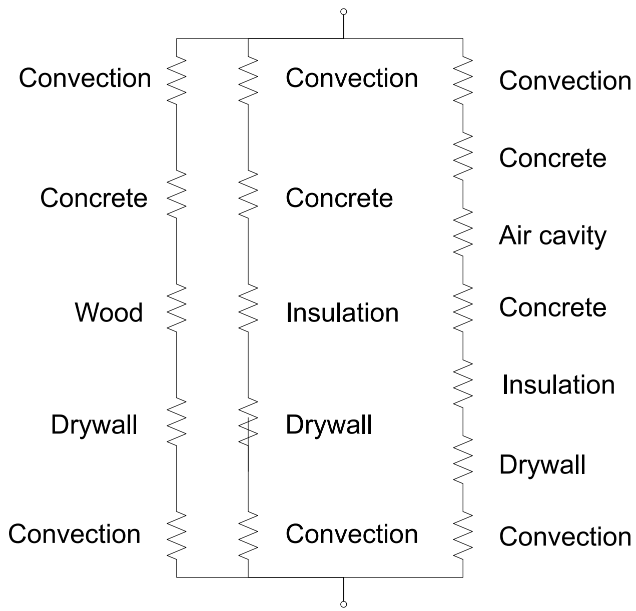
Isothermal method

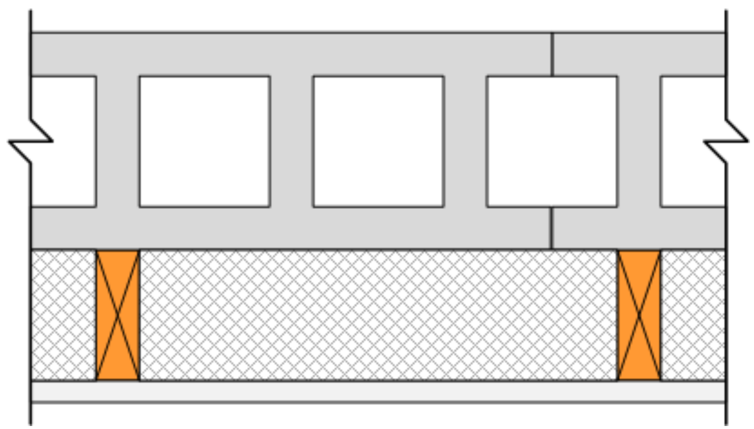
- To apply the isothermal method, we put a node at each layer interface and add a resistor for each element in the layer:



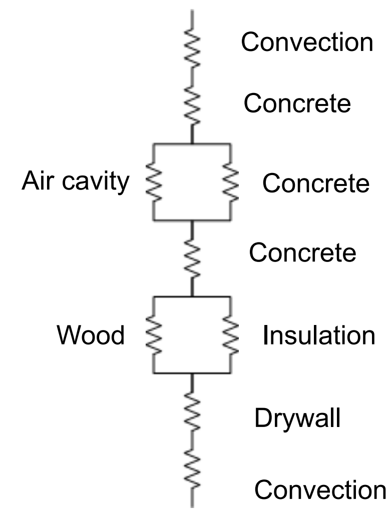
Isothermal vs. Parallel Path setup

Parallel Path





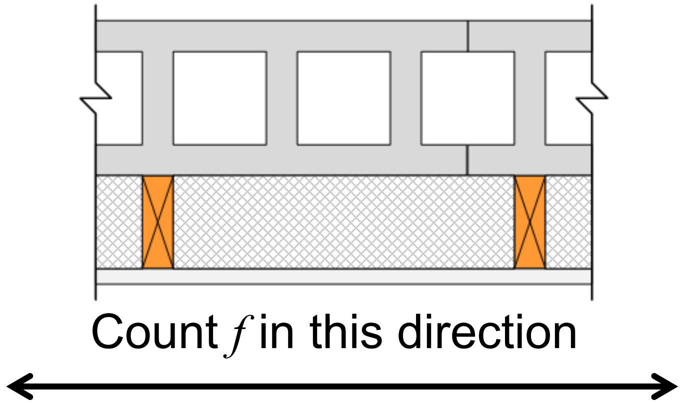
Isothermal



Assigning resistance values

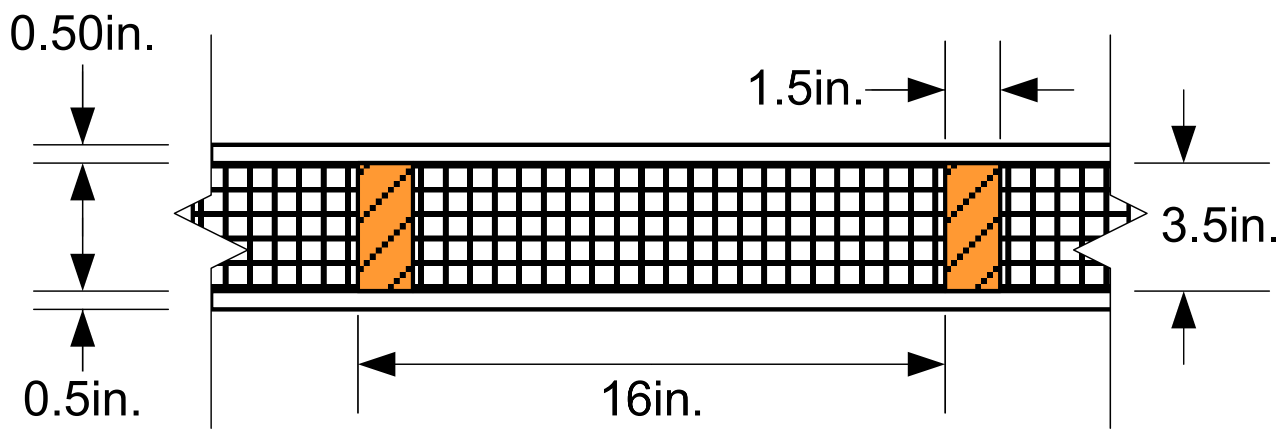
- The resistance value for the network elements will be the R-value of the element represented **divided by the fractional area (*f*)** of each element
  - Fractional area is the fraction of the entire cross section that the element takes up
  - This must be a number between 0 and 1

$$R_{element,network} = \frac{R_{element}}{f}$$



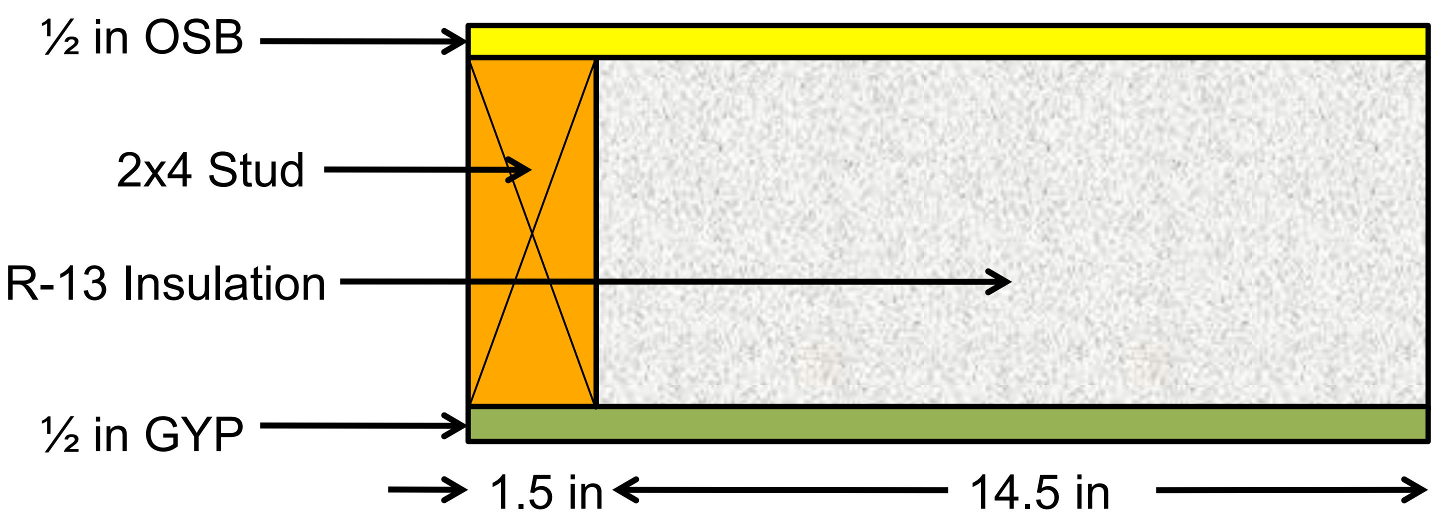
Example problem

- A wood frame wall has 2x4 studs spaced 16 inches OC ("on center") with R-13 (IP) insulation in the cavity
- The interior wall is 0.5 inches of gypsum wallboard and the outside wall is 0.5 inches of OSB sheathing
- Draw the isothermal and parallel path networks for this wall
  - Assume winter outdoor conditions



Example problem continued

- Let's redraw the wall construction to make it easier to identify the layers
  - We can show only the unique part of the wall if we prefer
    - Note: This drawing is not to scale



Example problem continued

Identify the layers and elements:

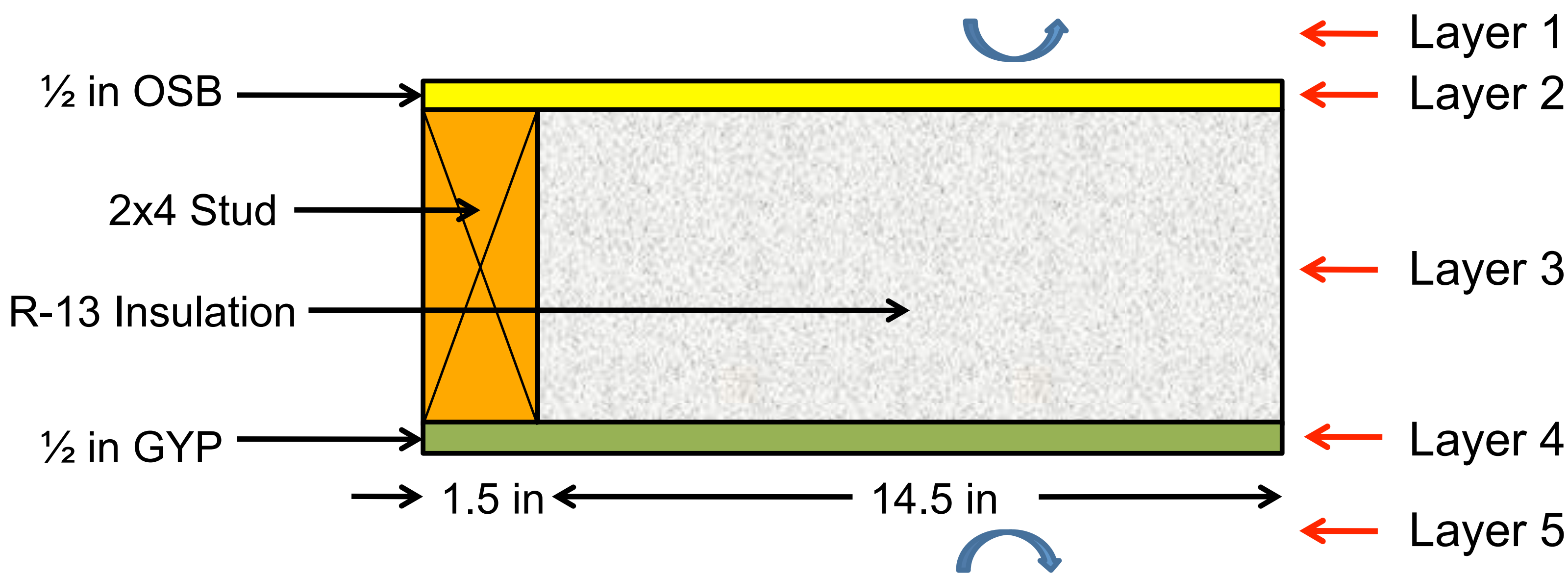
Layer 1: Exterior convection (winter conditions)

Layer 2: 0.5 inches of OSB

Layer 3: 3.5 inches of wood stud and 3.5 inches R-13 insulation

Layer 4: 0.5 inches of gypsum wallboard

Layer 5: Interior convection



Example problem continued

Look up R values for each element:

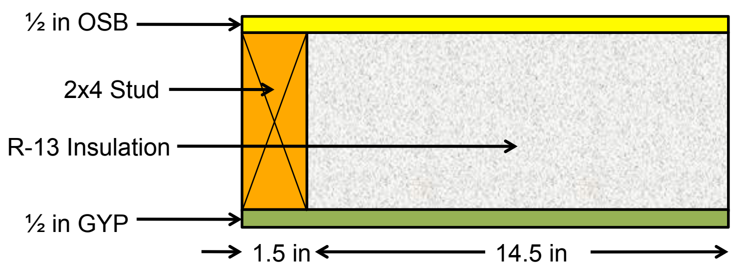
- Outdoor winter convection: R_{ext} = 0.03 (m^2K)/W
- ½ in. OSB, R_{osb} = 0.12 (m^2K)/W
- R-13 insulation: R_{ins} = 2.29 (m^2K)/W
- 2x4 (3.5 in. thick) wood stud: R_{2x4} = 0.96 (m^2K)/W
- ½ in. gypsum: R_{gyp} = 0.079 (m^2K)/W
- Indoor convection: R_{int} = 0.12 (m^2K)/W

Example problem continued

Find fractional areas of each element

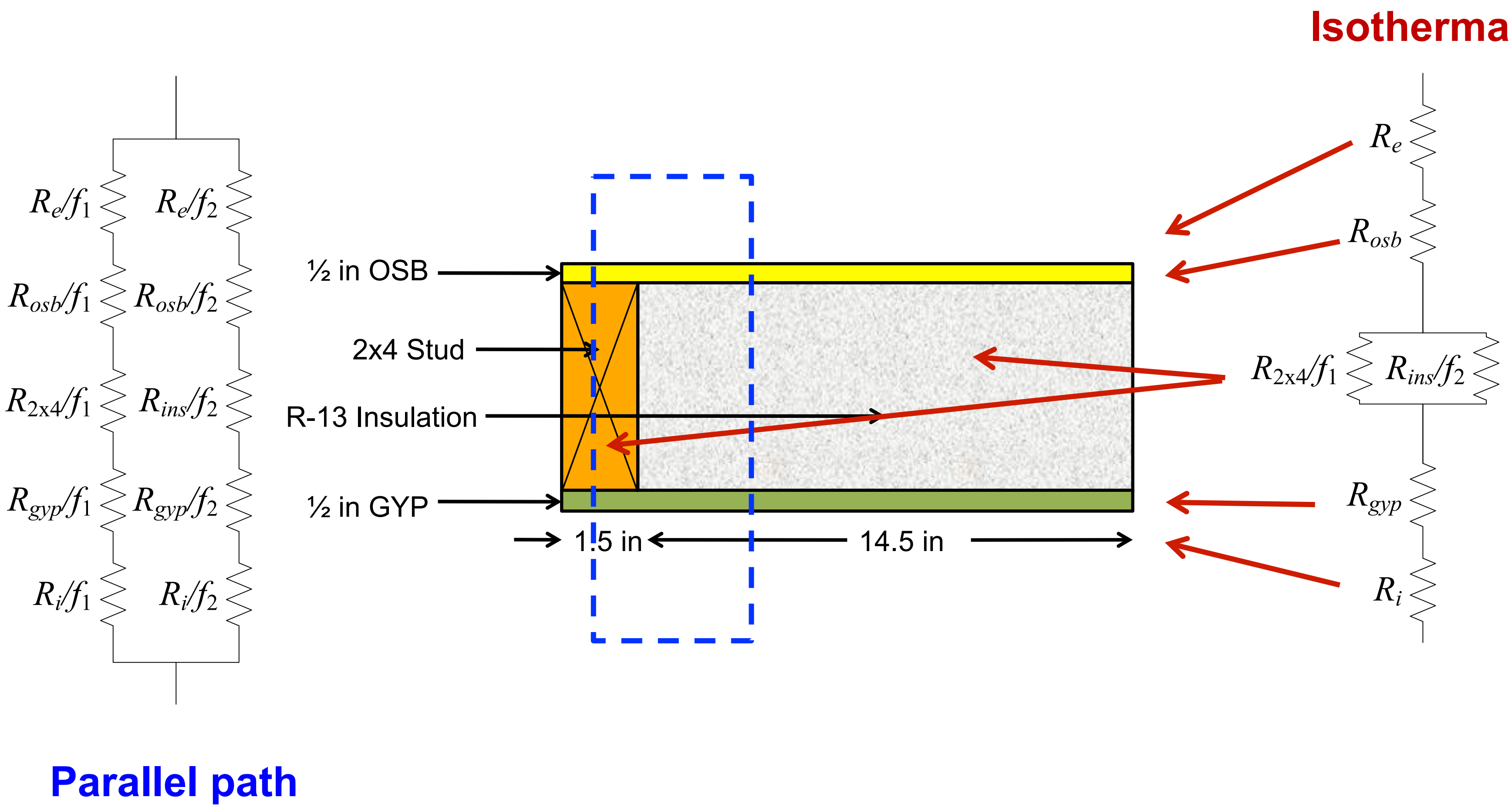
- The full width of the assembly is 16 inches
- All elements are full height of the wall, so the fractional width = fractional area

- Layers 1, 2, 4 and 5 are all 16 inches
  - $f = 1.0$
- 2x4 stud is 1.5 inches
  - $f_1 = 1.5/16 = 0.094$
- R-13 insulation is 14.5 inches
  - $f_2 = 14.5/16 = 0.906$



Example problem continued

- Draw the thermal networks



Example problem continued

Parallel path network resistor values:

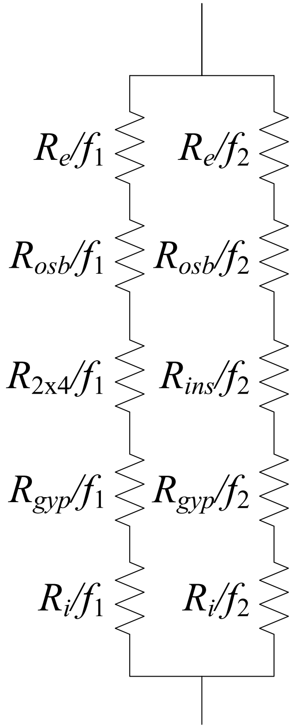
$$\frac{R_{ext}}{f_1} = \frac{0.03}{0.094} = 0.32\,\tfrac{m^2K}{W}, \qquad \frac{R_{ext}}{f_2} = \frac{0.03}{0.906} = 0.03\,\tfrac{m^2K}{W}$$

$$\frac{R_{int}}{f_1} = \frac{0.12}{0.094} = 1.27\,\tfrac{m^2K}{W}, \qquad \frac{R_{int}}{f_2} = \frac{0.12}{0.906} = 0.13\,\tfrac{m^2K}{W}$$

$$\frac{R_{osb}}{f_1} = \frac{0.12}{0.094} = 1.27\,\tfrac{m^2K}{W}, \qquad \frac{R_{osb}}{f_2} = \frac{0.12}{0.906} = 0.13\,\tfrac{m^2K}{W}$$

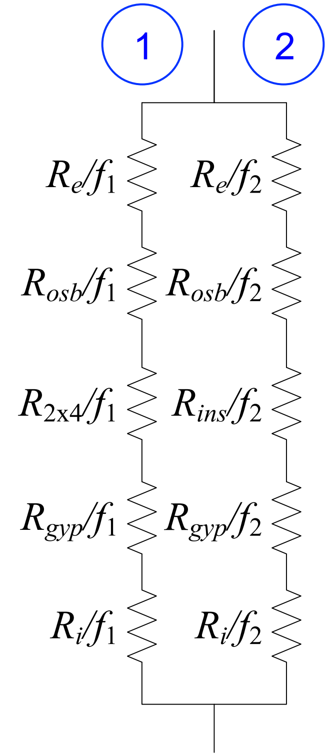
$$\frac{R_{gyp}}{f_1} = \frac{0.08}{0.094} = 0.84\,\tfrac{m^2K}{W}, \qquad \frac{R_{gyp}}{f_2} = \frac{0.08}{0.906} = 0.09\,\tfrac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25\,\tfrac{m^2K}{W}, \qquad \frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53\,\tfrac{m^2K}{W}$$



Example problem continued

Parallel path network resistor values



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 = R_{ext}/f_1 + R_{osb}/f_1 + R_{gyp}/f_1 + R_{int}/f_1 + R_{2x4}/f_1$$

$$R_2 = R_{ext}/f_2 + R_{osb}/f_2 + R_{gyp}/f_2 + R_{int}/f_2 + R_{insulation}/f_2$$

$$R_1 = 0.32 + 1.27 + 10.25 + 1.27 + 0.84 = 13.96$$

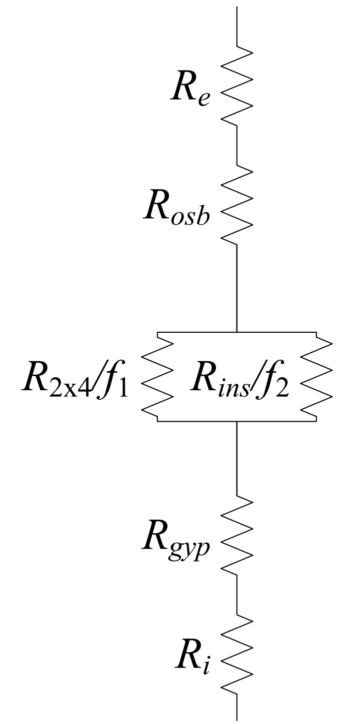
$$R_2 = 0.03 + 0.13 + 2.53 + 0.13 + 0.09 = 2.91$$

$$\frac{1}{R_{total}} = \frac{1}{13.96} + \frac{1}{2.91} = 0.415 \frac{\text{W}}{\text{m}^2\text{K}}$$

$$R_{total} = \frac{1}{0.415 \frac{\text{W}}{\text{m}^2\text{K}}} = 2.41 \frac{\text{m}^2\text{K}}{\text{W}} = \text{R-}13.68\ (\text{IP})$$

Example problem continued

We can now find the network resistor values for the **isothermal** network:



$$R_{ext} = \frac{0.03}{1.0} = 0.03 \frac{m^2K}{W} \qquad R_{int} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W}$$

$$R_{osb} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W} \qquad R_{gyp} = \frac{0.08}{1.0} = 0.08 \frac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25 \frac{m^2K}{W} \qquad \frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53 \frac{m^2K}{W}$$

$$R_{total} = R_e + R_{osb} + \frac{1}{\dfrac{1}{R_{2x4}/f_1} + \dfrac{1}{R_{insulation}/f_2}} + R_{gyp} + R_i$$

$$R_{total} = 0.03 + 0.12 + \frac{1}{1/10.25 + 1/2.53} + 0.08 + 0.12 = 2.38 \frac{\text{m}^2\text{K}}{\text{W}} = \text{R-13.49 (IP)}$$

Utility of thermal networks

By developing the full thermal network and then combining elements, we can better see the heat transfer paths

In particular:

- We can identify thermal bridges more easily
  - Areas of particularly low resistance
- We can identify the relative contribution of the elements to heat transfer more easily
- We can use nodal analysis techniques to find the temperature everywhere quickly and easily

Limitations to these methods

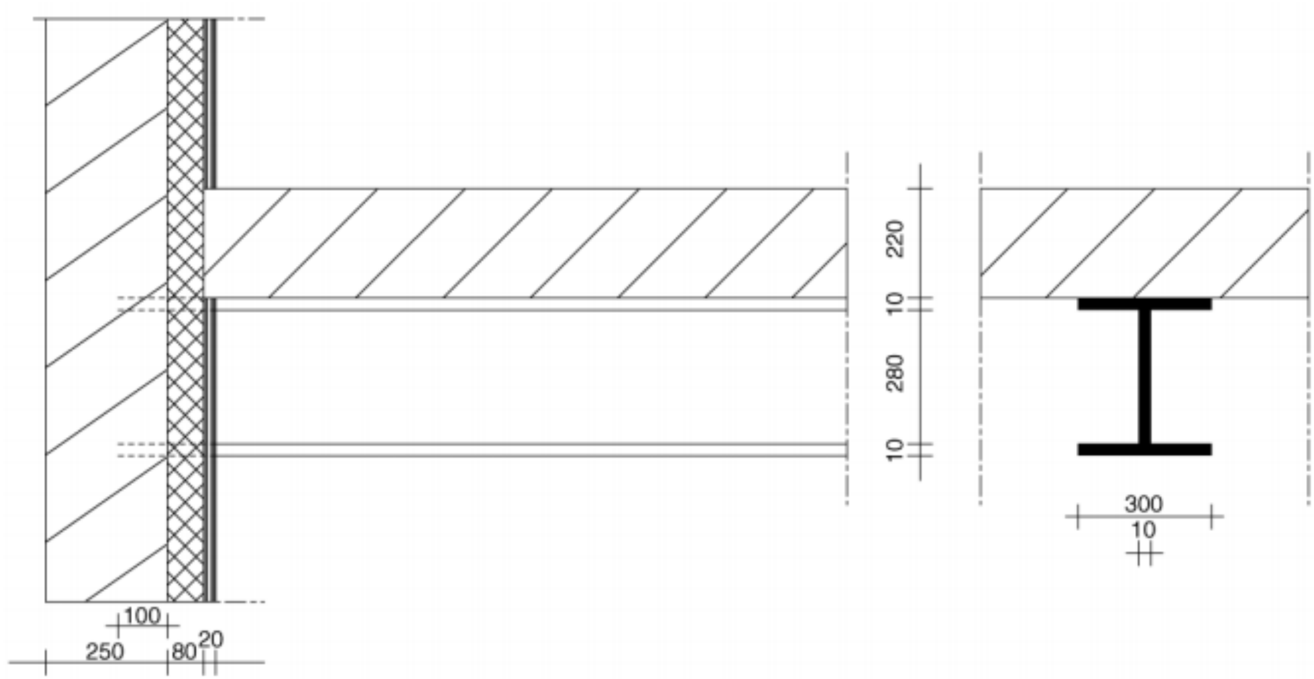
- Previous methods do not work as well when the assembly has **metal** thermal bridges
  - For example: Embedded steel beams can create thermal bridges

- Any flanges on metal beams draw heat into metal areas that cross through layers
  - Increases heat transfer
  - Often odd geometries and narrow dimensions

- We need more accurate methods of estimating U-values

Thermal analysis of these even more complex geometries

- How do we estimate U and R values for complex geometries and combinations of materials?



- Simple 1-D calculations can have significant errors
  - Hard (or impossible) to capture all phenomena
- Need to model 2-D or 3-D heat transfer using computer simulations

2-D AND 3-D HEAT TRANSFER USING NUMERICAL SOLVERS

Numerical solutions

- In addition to our analytical solution methods, there is another way to solve these problems:
  - Numerical analysis using computer solvers

- For assemblies with thermal bridges, it is probably a better solution to utilize finite-element or similar heat transfer software to estimate the U-value, R-value, and temperatures in and around the assembly

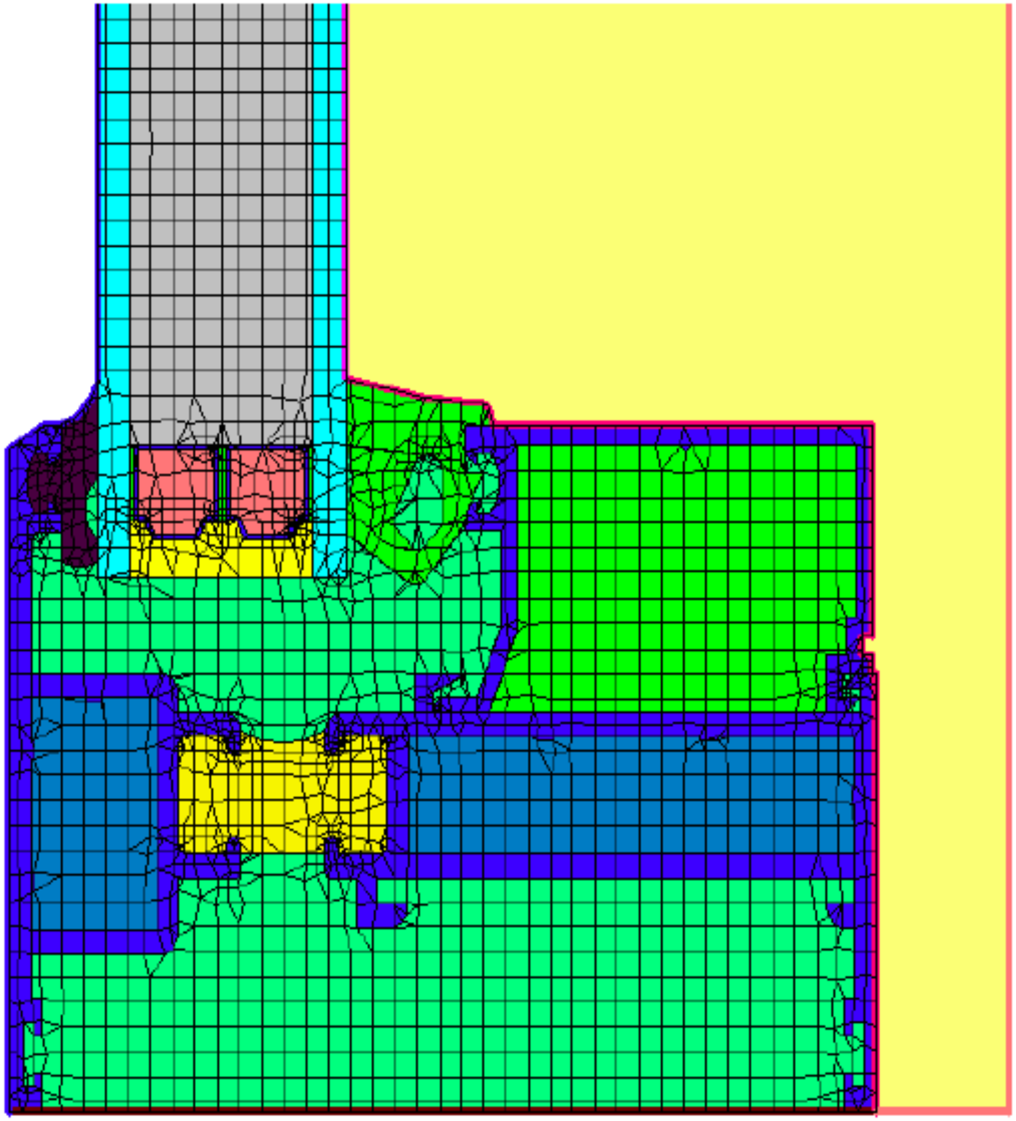
Basic idea of 2-D or 3-D numerical analysis

- Break assembly into a hundreds or thousands of homogenous elements

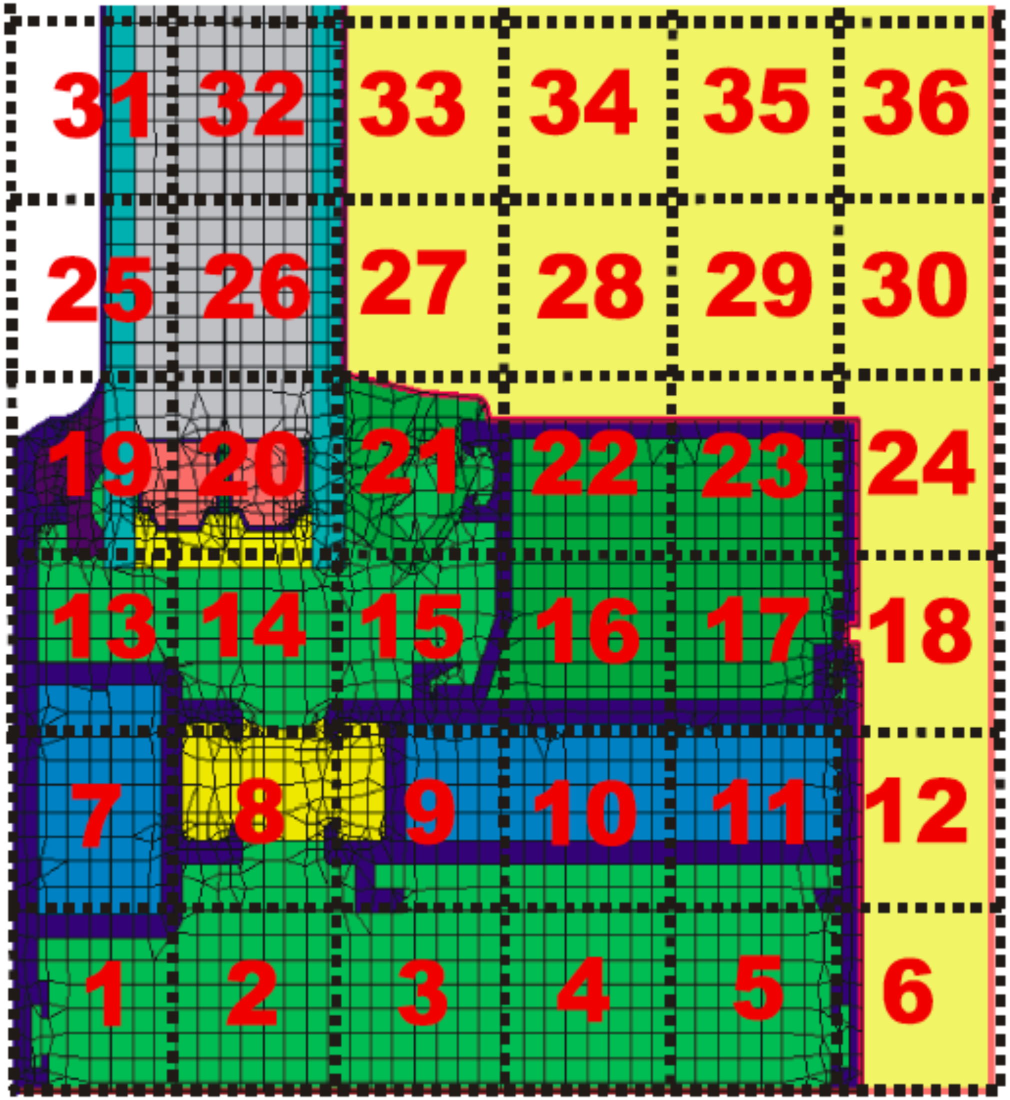
- Use the basic equations of heat transfer and write heat balances on each element to create a huge set of simultaneous equations

- Solve the simultaneous equations numerically to find heat flow and temperatures throughout the system

Grid for numerical solver

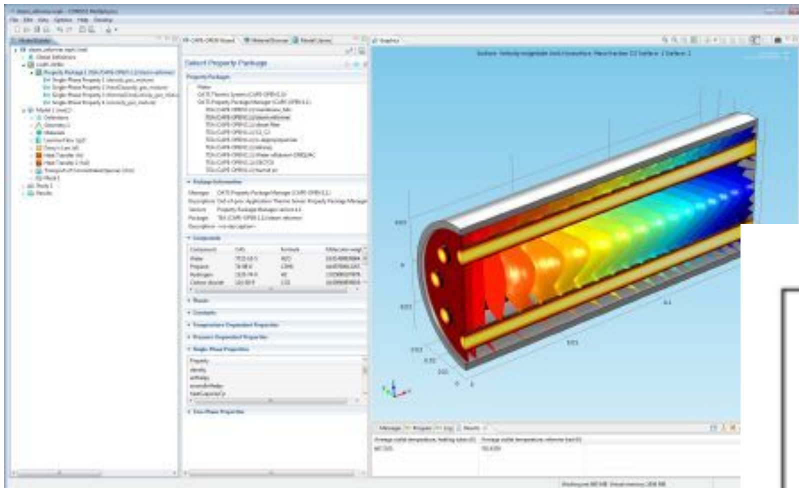


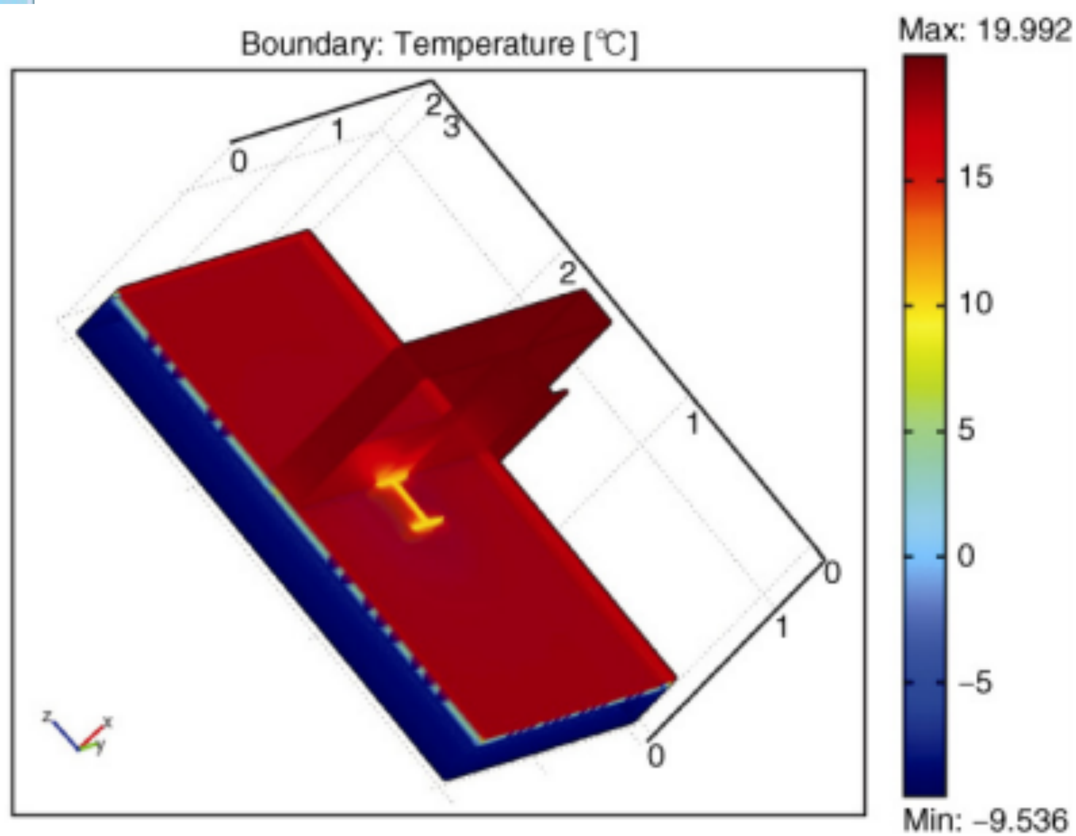
Grid for numerical solver



3-D solvers

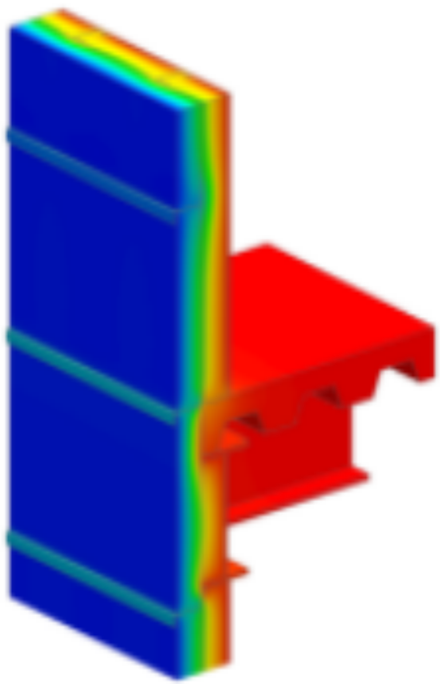
- COMSOL finite element solver

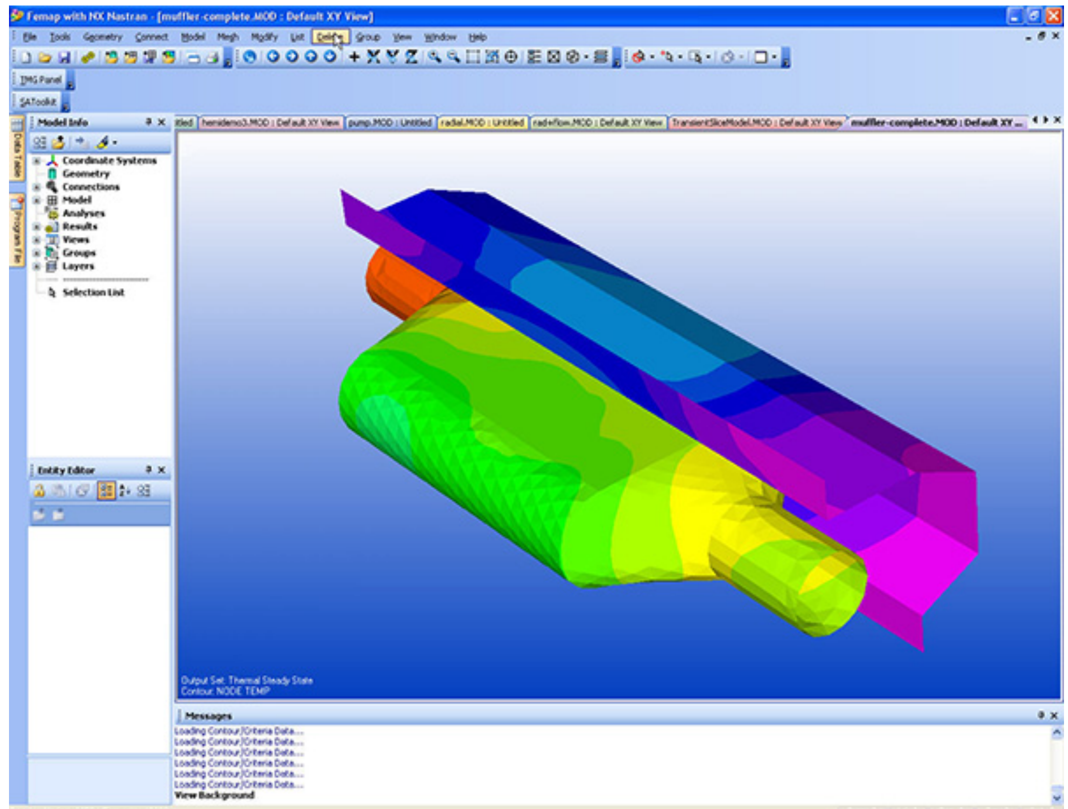




3-D solvers

- Femap and Nx finite element analysis





SIEMENS

THERM – popular 2-D solver

- A very popular 2-D heat transfer program used in the US – especially for window analysis - is called THERM
  - URL to download: http://windows.lbl.gov/software/therm/therm.html
  - Requires registration but is FREE

- THERM is used with WINDOW for designing windows
  - But can also be used as a stand alone program for any assembly
  - Very similar 2-D and 3-D programs often used in Europe are called heat2 and heat3

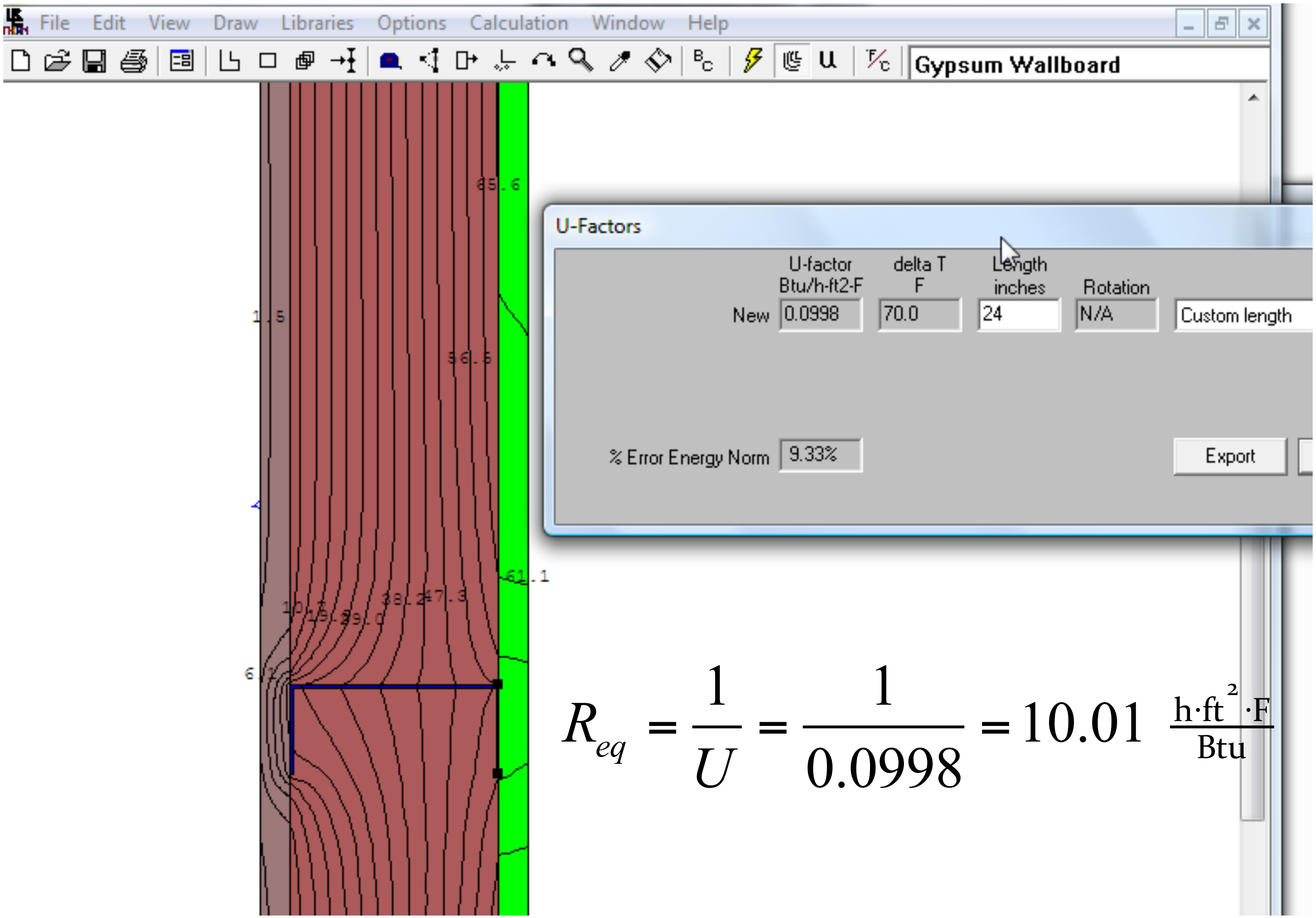
- You will use THERM on a HW
  - You can download from my website: http://built-envi.com/wp-content/uploads/2012/09/THERM63Setup.exe
  - Only runs on Windows ☹
  - It is also installed in the AM 218 computer lab

Using THERM

- In THERM you “draw” out an assembly
  - Assign materials
  - Assign boundary conditions
  - And let the program solve for the temperatures and heat flow throughout the assembly

- The program then can analyze that output to calculate a U value for the entire 2-D assembly
  - You can then calculate R = 1/U for the assembly

Example screenshot: THERM analysis of steel stud



$$R_{eq} = \frac{1}{U} = \frac{1}{0.0998} = 10.01 \ \frac{\mathrm{h \cdot ft^2 \cdot F}}{\mathrm{Btu}}$$

Governing equations for THERM

1-D Conduction

$$q_{cond} = -k\frac{dT}{dx} = -\frac{k}{L}dT$$

2-D Conduction

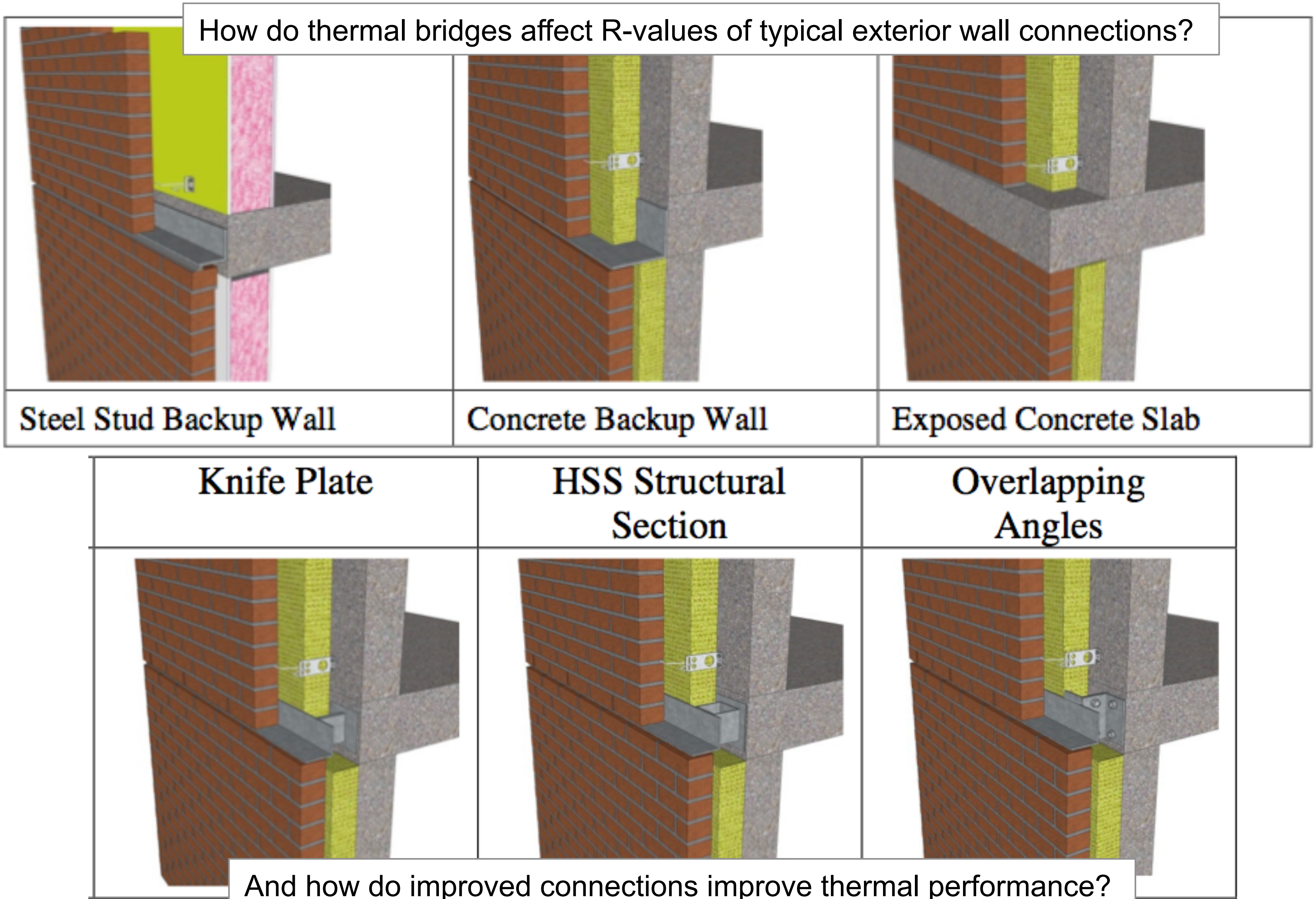
$$q_{cond} = -k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

- Energy balance at a surface of each discretized element is the same as previously, except the conduction term contains X and Y components

- Set boundary conditions on interior and exterior and the solver will compute temperature throughout assembly
  - Then it can calculate a U-value for the whole assembly

What you can do with THERM

How do thermal bridges affect R-values of typical exterior wall connections?

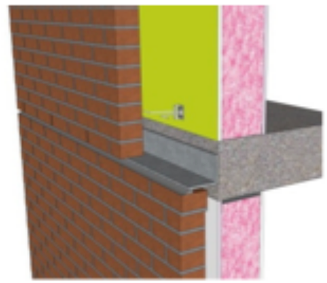
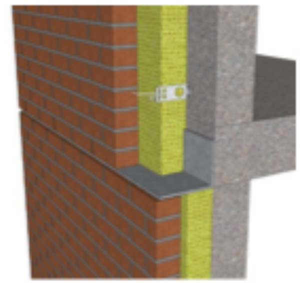
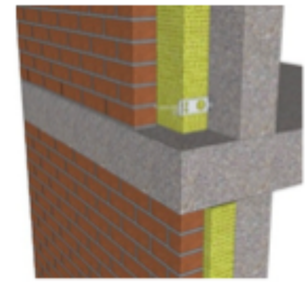
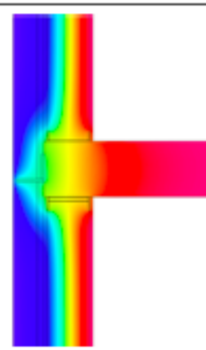
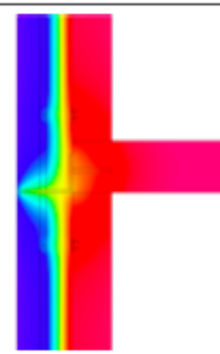
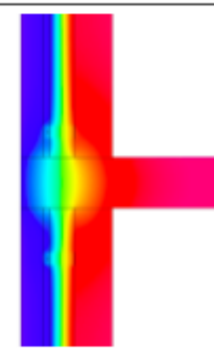


And how do improved connections improve thermal performance?

Finch et al. 2013, 12th Canadian Masonry Symposium

What you can do with THERM

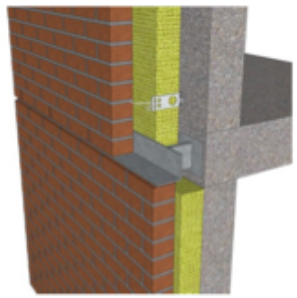
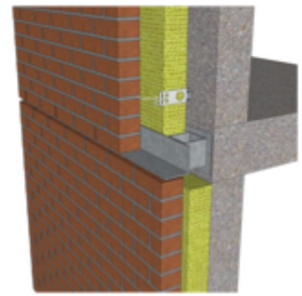
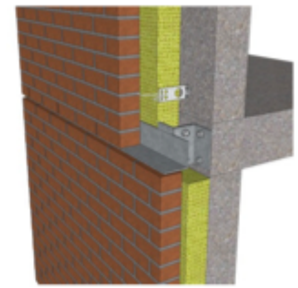
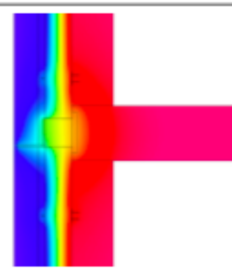
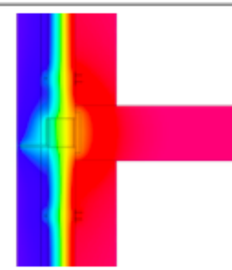
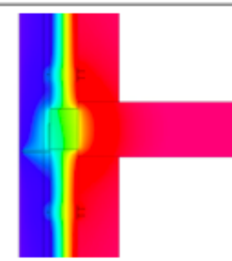
Table 1: Summary of Nominal and Effective R-Values and U-Values for Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Steel Stud Backup	Poured Concrete Backup	Exposed Slab Edge
			
			
Nominal Insulation R-Value/U-Value	R-20 (RSI 3.52) U-0.05 (USI 0.284)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-7.3 (RSI 1.29) U-0.137 (USI 0.777)	R-10.5 (RSI 1.84) U-0.096 (USI 0.543)	R-9 (RSI 1.58) U-0.112 (USI 0.634)
Effective Reduction	63.5%	37.5%	46.4%
Linear Transmission	-	ψ = 0.339 IP (0.586 SI)	ψ = 0.478 IP (0.827 SI)

Finch et al. 2013, 12th Canadian Masonry Symposium

What you can do with THERM

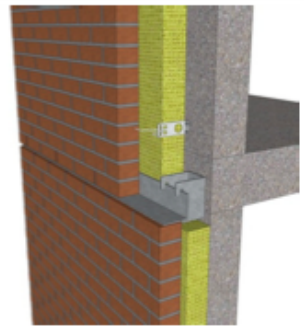
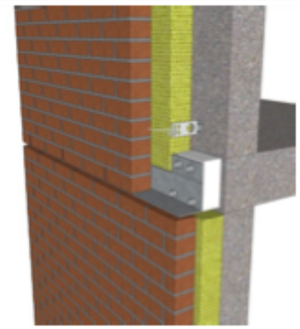
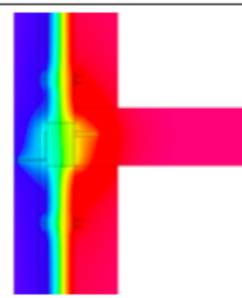
Table 2: Summary of Nominal and Effective R-Values and U-Values for Typical Stand-Off Modifications to the Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Knife Plate	HSS Structural Section	Overlapping Angles
			
			
	shelf angle: 4"x4"x1/4" outside of insulation. 4"x4"x3/4" stand-off knife plates welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 4"x4"x1/4" HSS tube welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 2-6"x4"x5/16" angles bolted to slab edge at 48" o.c.
Nominal Insulation R-Value/U-Value	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-14.8 (RSI 2.6) U-0.068 (USI 0.384)	R-14.8 (RSI 2.6) U-0.068 (USI 0.385)	R-15.0 (RSI 2.64) U-0.067 (USI 0.379)
Effective Reduction	16.4%	16.5%	15.3%
Linear Transmission	ψ = 0.096 IP (0.166 SI)	ψ = 0.097 IP (0.168 SI)	ψ = 0.089 IP (0.153 SI)

Finch et al. 2013, 12th Canadian Masonry Symposium

What you can do with THERM

Table 3: Summary of Nominal and Effective R-Values and U-values for Propriety Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Standoff Bracket	4-Bolt Cast-In
		
		
	shelf angle 4"x4"x1/4" outside insulation. Proprietary clip is ¼" thick steel, 4"x4"x1/4" 6 "lg C-section. Non-welded connection. .	Shelf angle 4"x4"x1/4" outside insulation. Pre-manufactured cast-in place thermal break connection with 4 stainless steel bolts attached to 7"x7"x 3/8" plate.
Nominal Insulation R-Value/U-Value	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-14.9 (RSI 2.62) U-0.067 (USI 0.381)	R-16.4 (RSI 2.9) U-0.061 (USI 0.345)
Effective Reduction	16.4%	7%
Linear Transmission	ψ = 0.091 IP (0.158 SI)	ψ = 0.037 IP (0.064 SI)

Finch et al. 2013, 12th Canadian Masonry Symposium

THERM demonstration

WRAPPING UP COMPLEX CONDUCTION

(1) Below-grade walls and floors
(2) On-grade heat transfer

Below-grade heat flow

- Where does heat flow?
  - Depends on surface and ground temperature distributions

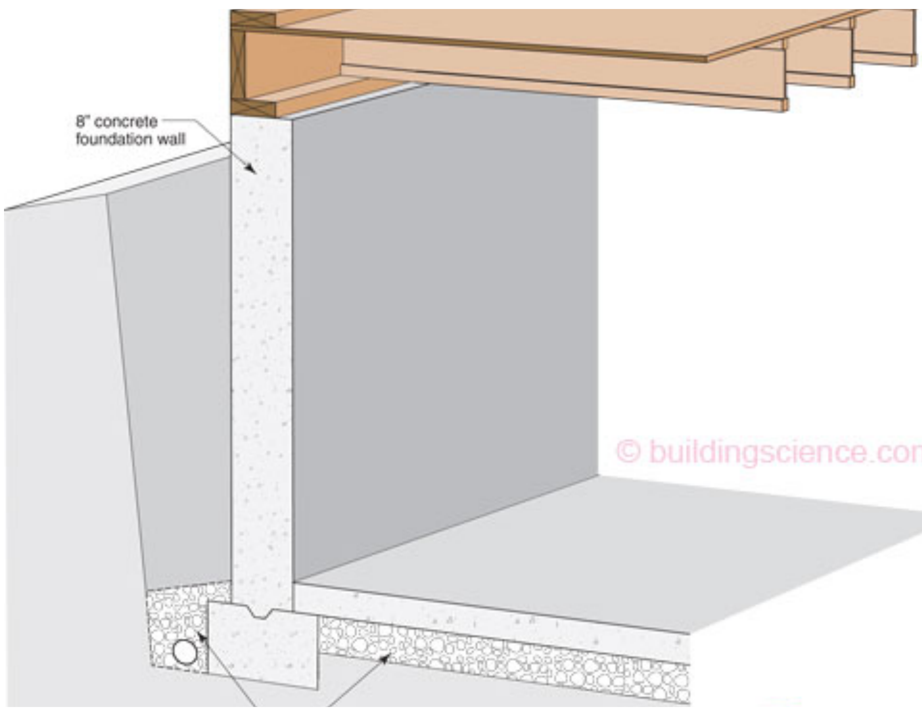
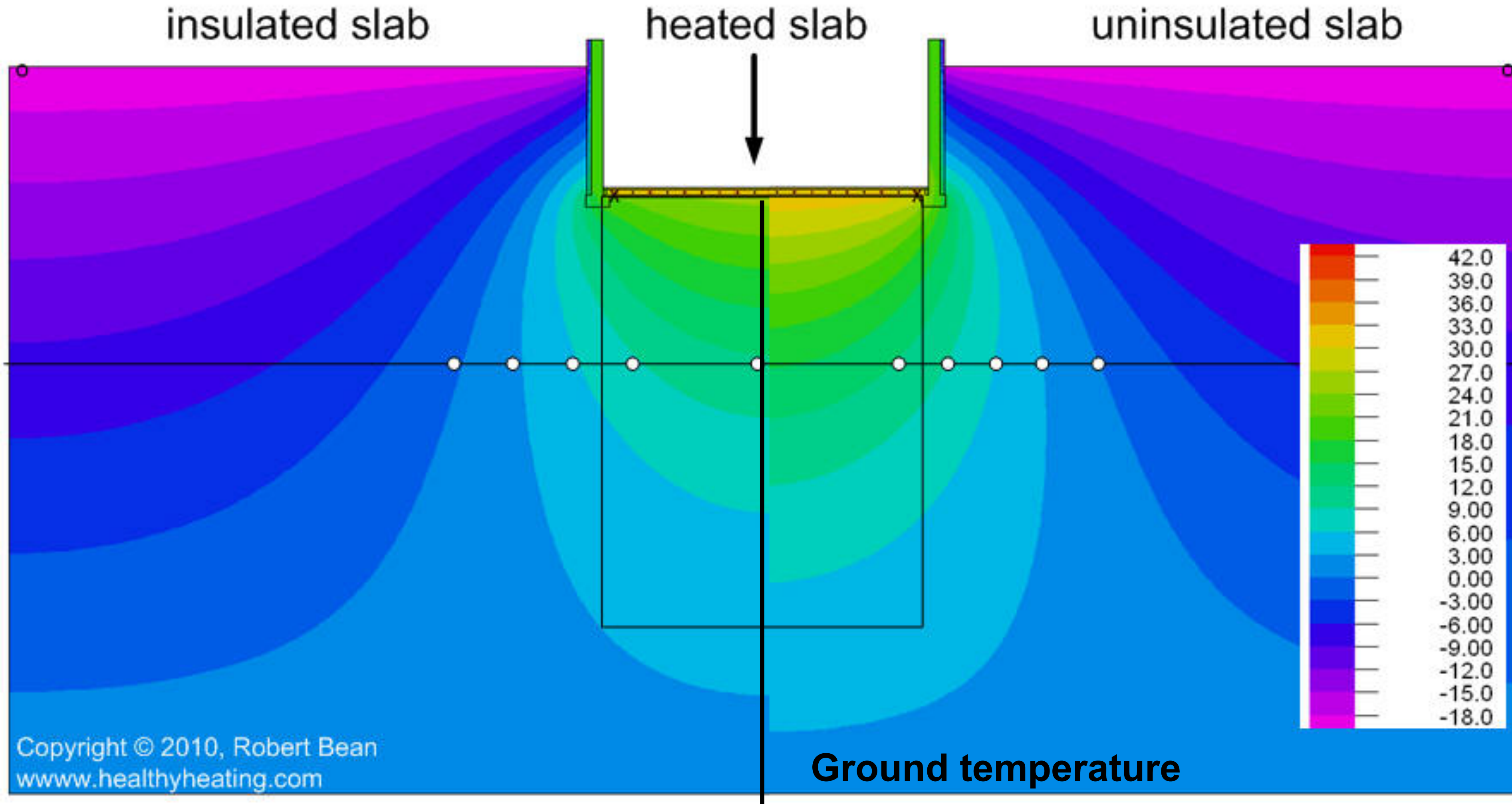


Fig. 4 Heat Flow from Basement

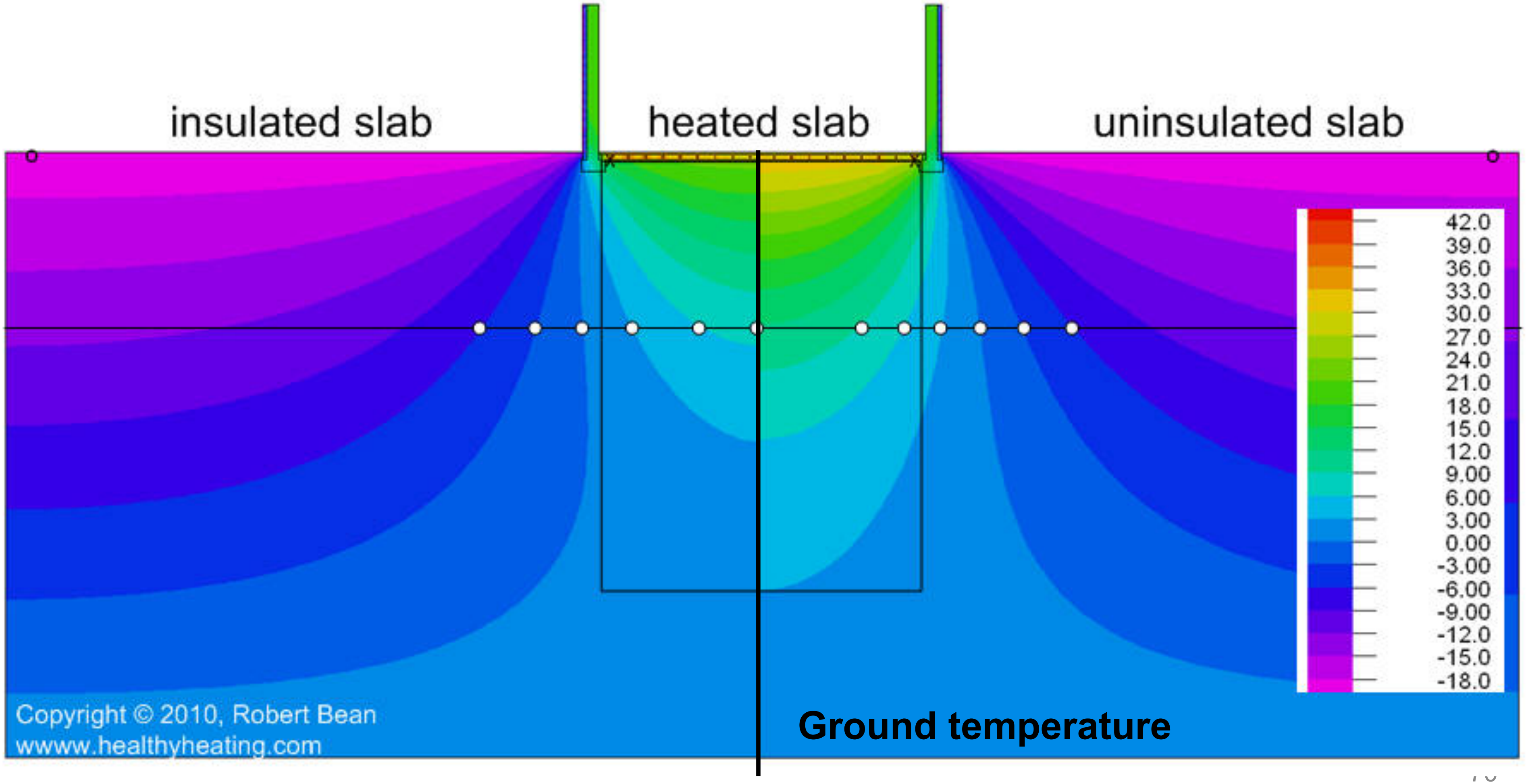
Below-grade heat flow

- Often we have walls and floors built below-grade, or "submerged" within the soil



On-grade heat flow

- Often we have floors built directly on grade, in contact with the ground



Average annual ground temperatures

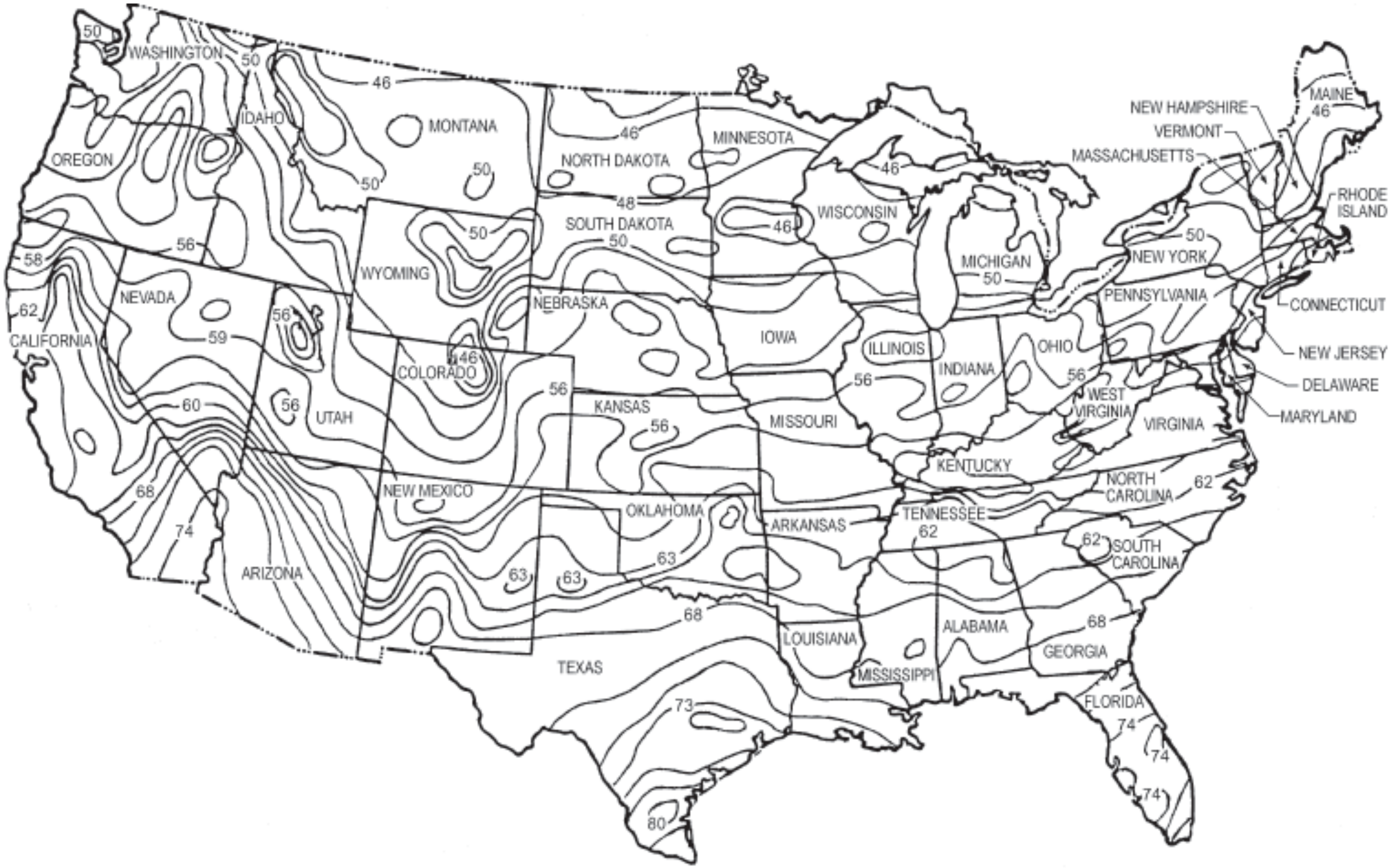
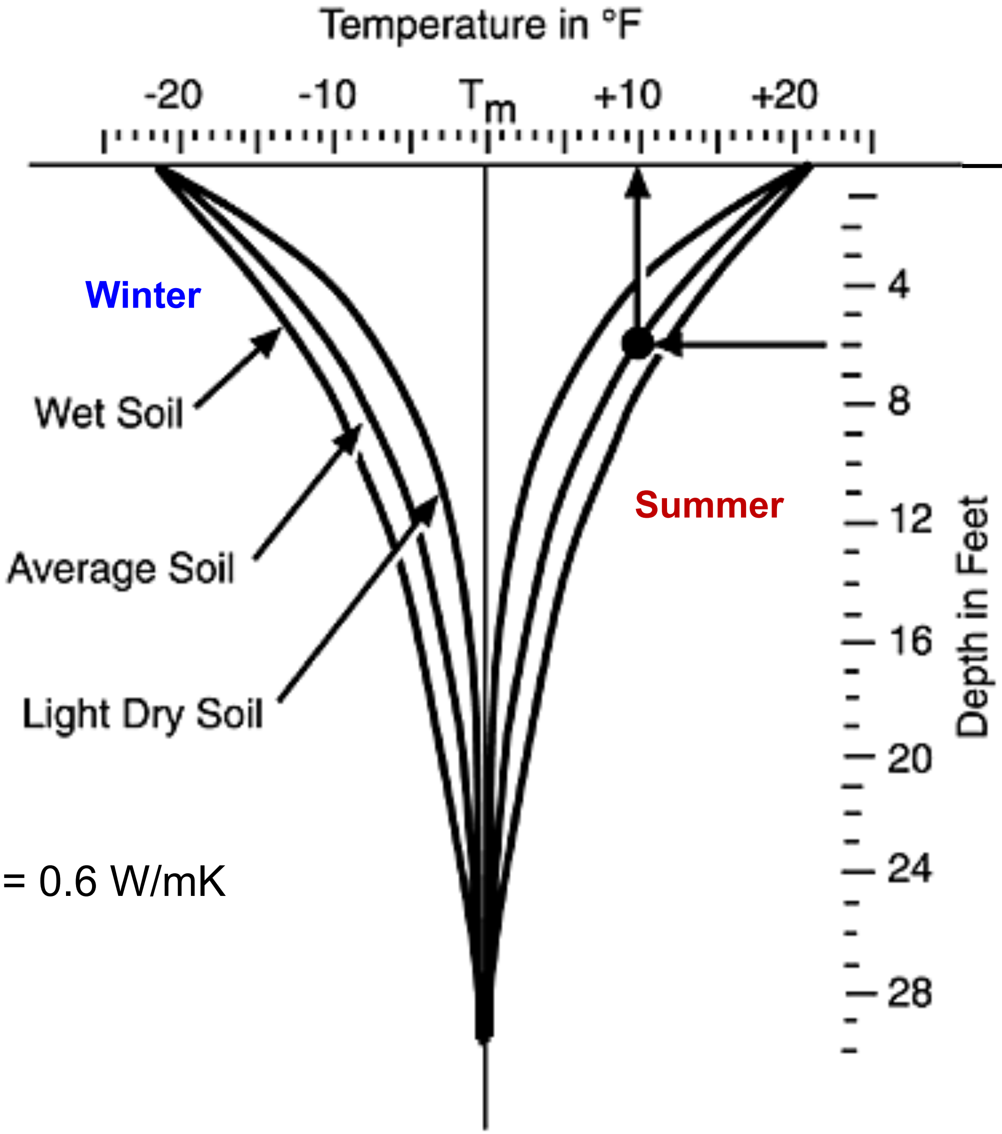
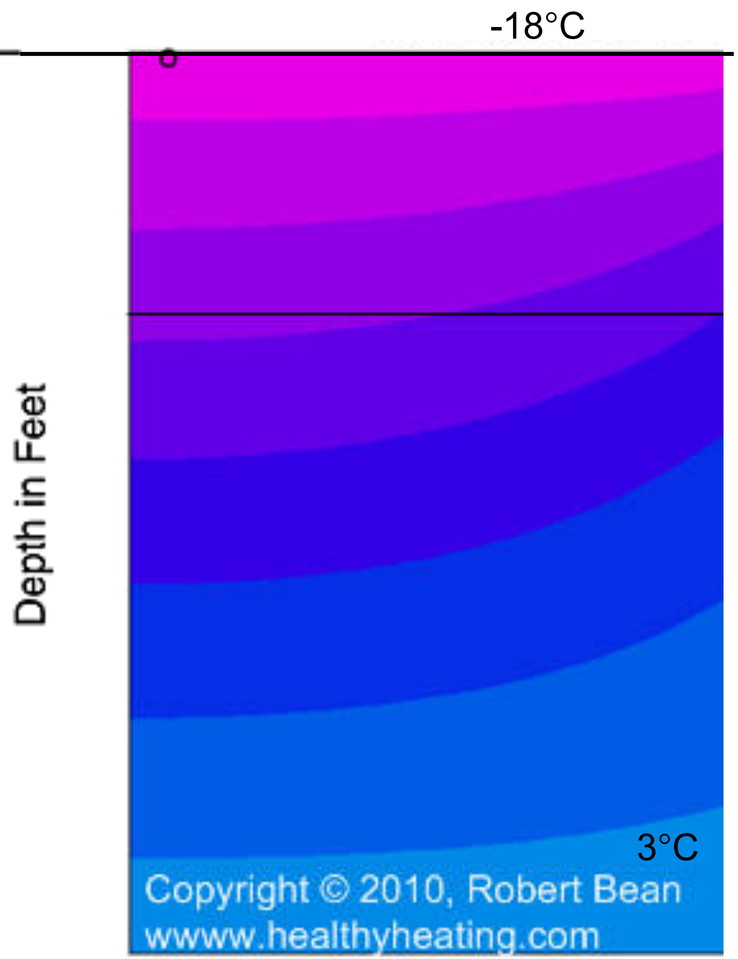


Fig. 17 Approximate Groundwater Temperatures (°F) in the Continental United States

Source: 2011 HVAC Apps Chapter 34, Figure 17

Ground temperatures vary with depth and soil moisture



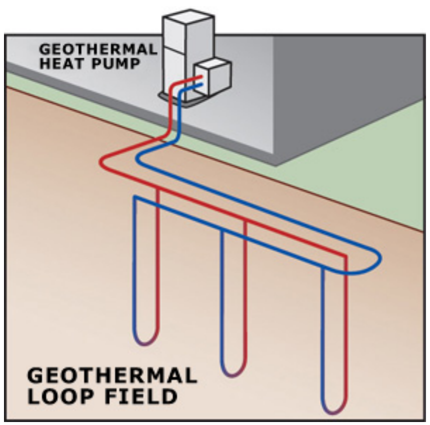


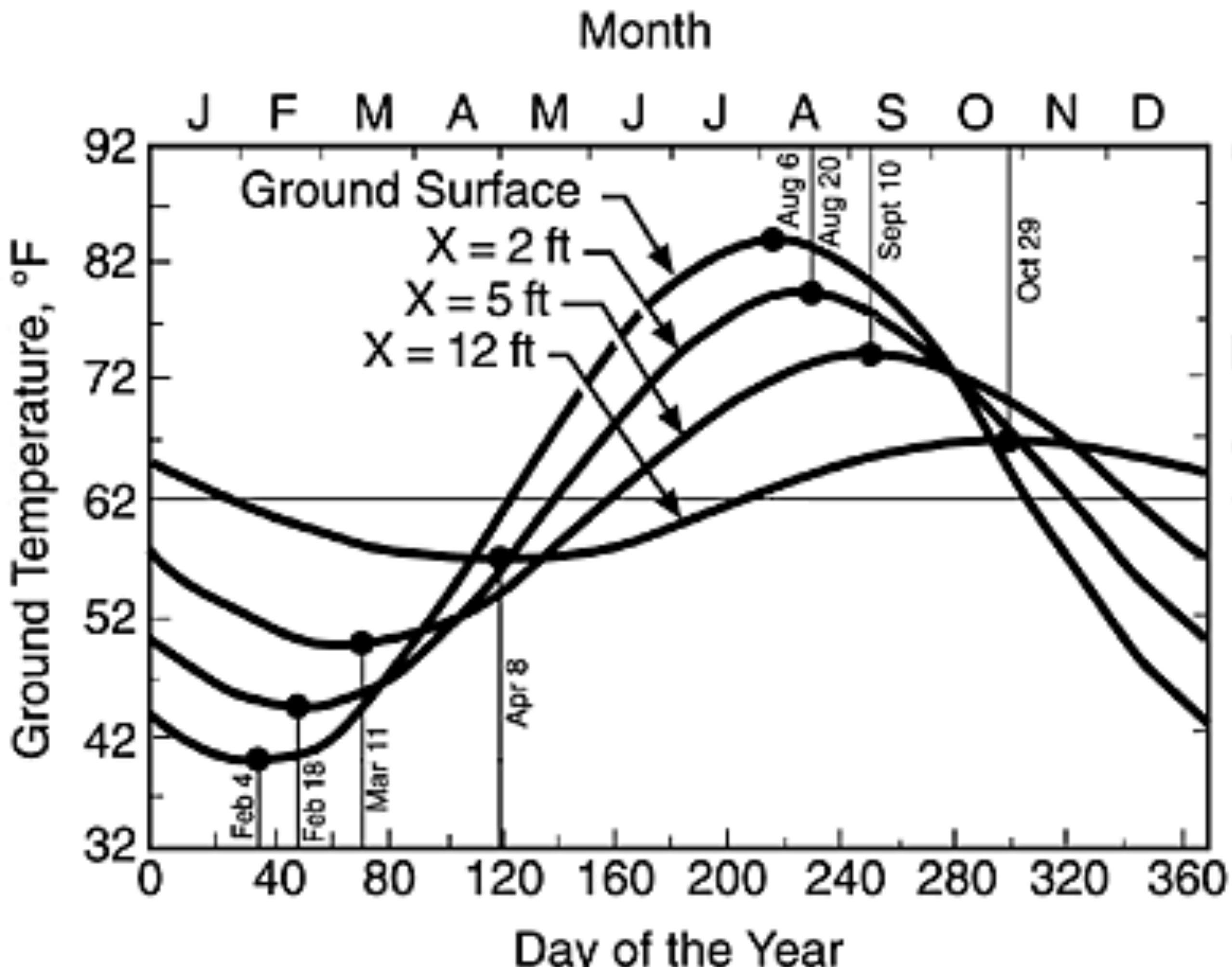
k_{H2O} = 0.6 W/mK

Source: http://www.geo4va.vt.edu/A1/A1.htm

Ground temperatures also vary with season

- Ground surface approximately equal to outdoor air
- The deeper you go, the more constant your temperature remains





Source: http://www.geo4va.vt.edu/A1/A1.htm

Design ground temperatures

- Design (worst-case) ground temperatures
  - Adjust mean ground temperature by a peak seasonal amplitude:

$$T_{gr} = T_{gm} - T_A$$

where

T_A = the ground temperature variation amplitude (right)

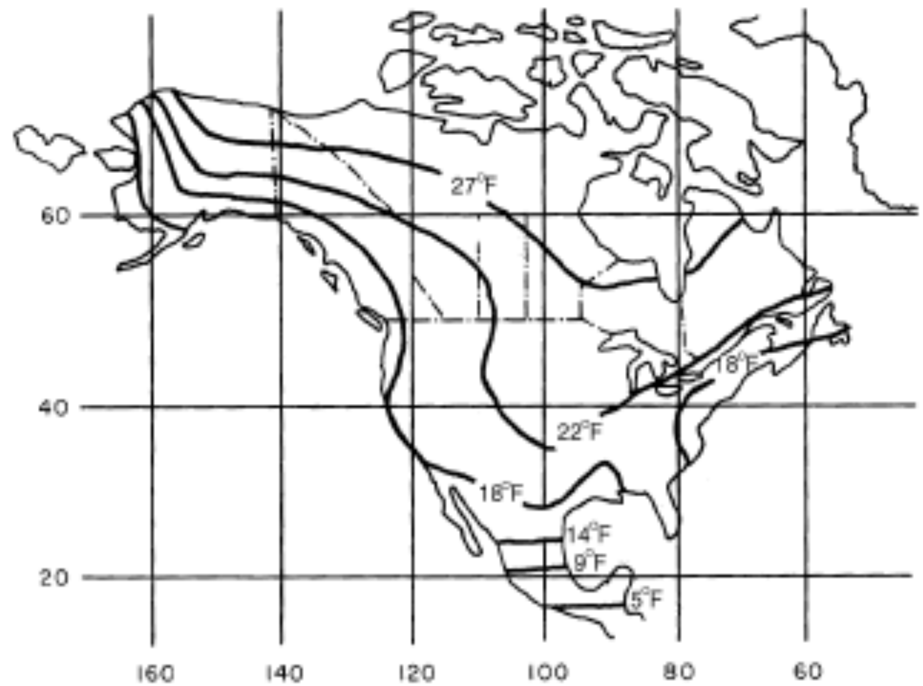
T_{gm} = mean ground temperature

*Note for Chicago:

$T_A \approx 22°F$ $T_{gm} \approx 54°F$

$T_{gr} \approx 54-22 = 32°F$

Alternatively, T_{gr} can be estimated as the mean air temperature in the coldest month (often done)

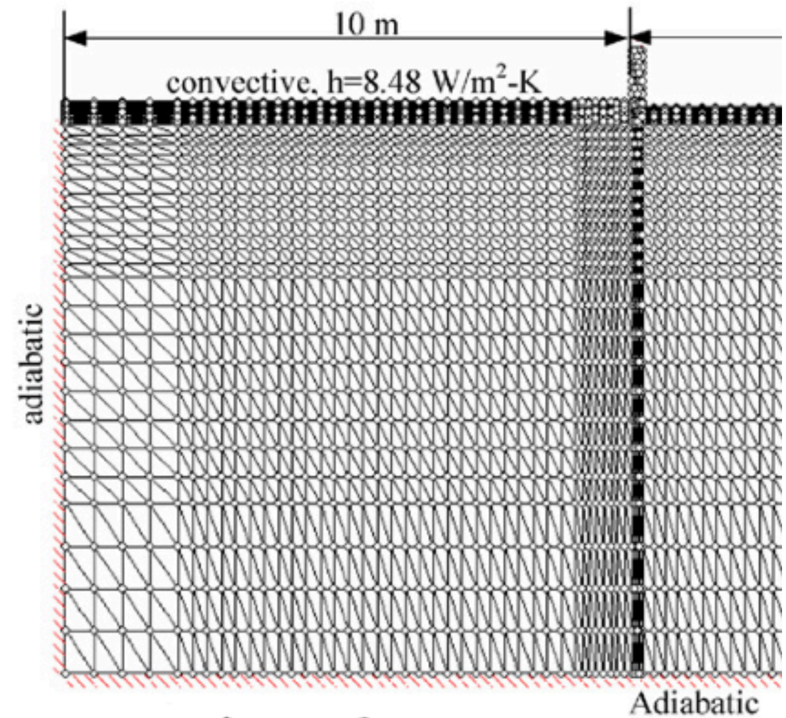


Ground Temp **Amplitude**, T_A

Source: 2005 ASHRAE Handbook Chapter, 29, Fig 2

Below-grade heat transfer

- Heat transfer through below-grade walls and floors
  - Conduction is truly 2-D or 3-D
  - 1-D modeling is not appropriate



- Heat transfer through walls
  - Between inside and surrounding soil
    - (not exterior air)
  - Depends on the wall area

- Heat transfer through the floor
  - Between inside and the soil below
  - Depends on the floor area

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

- ASHRAE HOF has some guidelines for transforming 2-D into 1-D

Simplified below-grade heat transfer

$$Q = AU_{avg}\,(T_i - T_{gr}) \quad \text{[W]}$$

$$q = U_{avg}\,(T_i - T_{gr}) \quad [\text{W/m}^2]$$

where

A is the wall or floor area below grade [m^2] (analyze any wall portion above-grade in the normal way)

T_i is the below grade inside temp [K]

T_{gr} is the design ground surface temp [K]

U_{avg} is the average U factor for the below grade surface [$W/(m^2K)$] (see following slides)

Below grade depth parameters for estimating U value

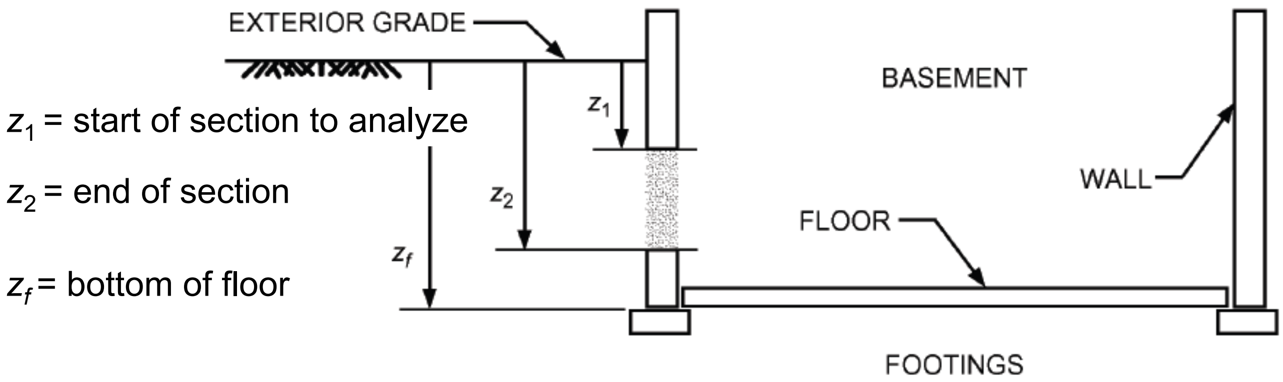


Fig. 14 Below-Grade Parameters

- For **average below-grade floor** value with a floor depth of height z_f from ground ("grade")

$$U_{avg,bf} = \frac{2k_{soil}}{\pi w_b} \times \left[\ln\left(\frac{w_b}{2} + \frac{z_f}{2} + \frac{k_{soil}R_{other}}{\pi}\right) - \ln\left(\frac{k_{soil}R_{other}}{\pi}\right)\right] \quad (40)$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK

R_{other} = R value of floor + insulation + convection [m^2K/W]

w_b = shortest dimension of basement width [m]

z_f = floor depth below grade [m]

Pre-computed tables for $U_{avg,bf}$

- Assuming **un-insulated concrete** floor

Table 17 Average U-Factor for Basement Floors

z_f (depth of floor below grade), m	$U_{avg,bf}$, W/(m^2·K)			
	w_b (shortest width of basement), m			
	6	7	8	9
0.3	0.370	0.335	0.307	0.283
0.6	0.310	0.283	0.261	0.242
0.9	0.271	0.249	0.230	0.215
1.2	0.242	0.224	0.208	0.195
1.5	0.220	0.204	0.190	0.179
1.8	0.202	0.188	0.176	0.166
2.1	0.187	0.175	0.164	0.155

Soil conductivity is 1.4 W/(m·K); floor is uninsulated. For other soil conductivities and insulation, use Equation (38).

$U_{avg,bw}$ for below-grade **walls**

$$U_{avg,bw} = \frac{2k_{soil}}{\pi(z_1 - z_2)} \times \left[\ln\left(z_2 + \frac{2k_{soil}R_{other}}{\pi}\right) - \ln\left(z_1 + \frac{2k_{soil}R_{other}}{\pi}\right)\right] \quad (39)$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
R_{other} = R value of wall, insulation and inside surface resistance [m²K/W]
z_1, z_2 = depths of top and bottom of wall segment under consideration [m]

Table 16 Average U-Factor for Basement Walls with Uniform Insulation

Depth, m	$U_{avg,bw}$ from grade to depth, W/(m²·K)			
	Uninsulated	R-0.88	R-1.76	R-2.64
0.3	2.468	0.769	0.458	0.326
0.6	1.898	0.689	0.427	0.310
0.9	1.571	0.628	0.401	0.296
1.2	1.353	0.579	0.379	0.283
1.5	1.195	0.539	0.360	0.272
1.8	1.075	0.505	0.343	0.262
2.1	0.980	0.476	0.328	0.252
2.4	0.902	0.450	0.315	0.244

Soil conductivity = 1.4 W/(m·K); insulation is over entire depth. For other soil conductivities and partial insulation, use Equation (37).

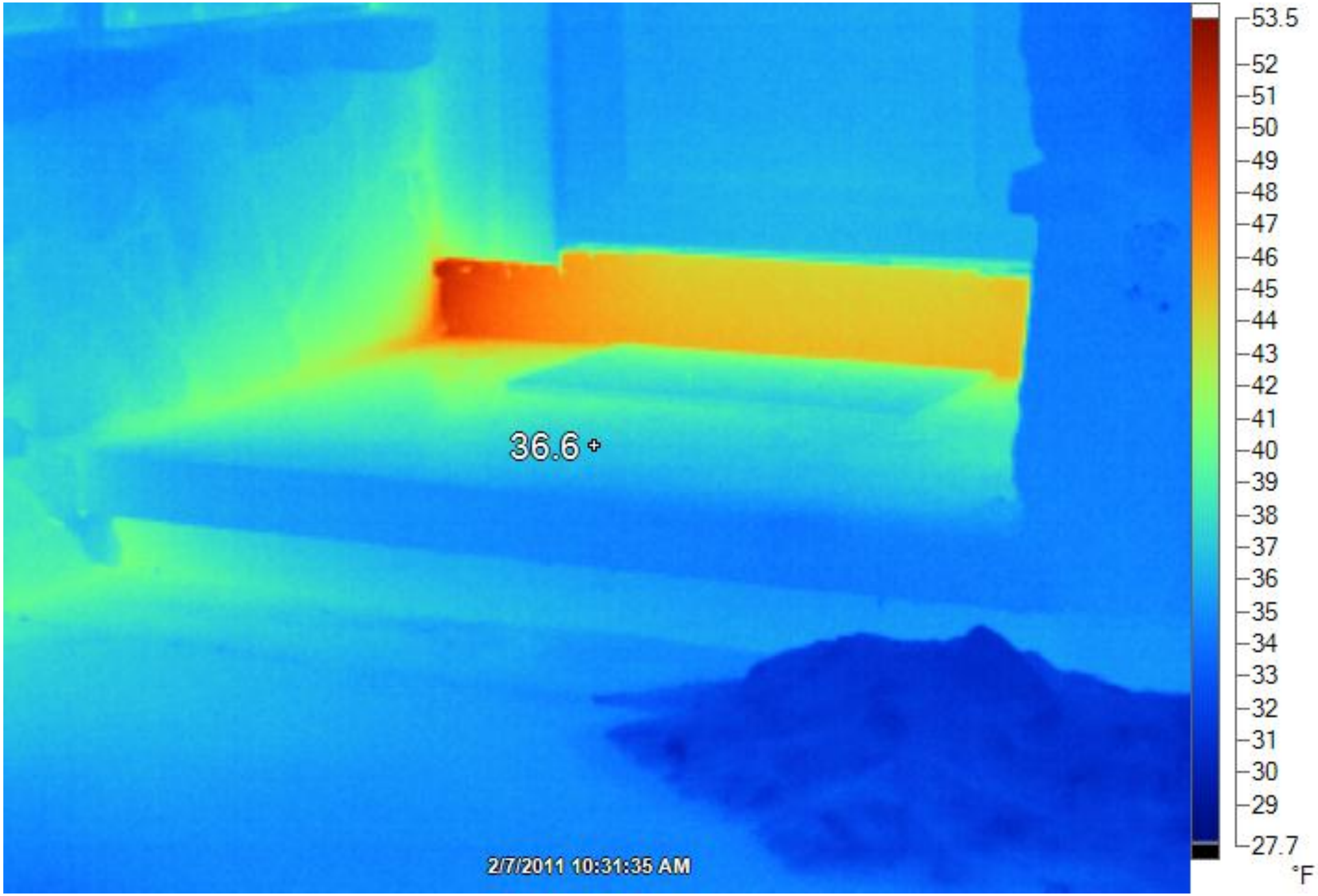
Assuming **concrete** walls with **uniform insulation**

On-grade heat transfer

On-grade heat transfer

- Heat transfer for slab-on-grade floors
  - Concrete slabs can be heated or unheated
  - In either case:
    - The ground is often at a lower temperature than indoor air
      - Soil and concrete are fairly conductive
    - Perimeter can be exposed directly to outdoor air
  - It turns out that the perimeter is often most important for both energy and comfort
    - Need to insulate the perimeter

Slab-on-grade floors



Slab-on-grade floors

- Simplified heat transfer through slab-on-grade floors
  - Function of perimeter of slab (not area)

$$Q = pF_p\left(T_i - T_o\right)$$

where T_i and T_o are the inside and outside temps [K]
p is the perimeter of the exposed floor surface [m]
F_p is the heat loss coefficient per unit length of perimeter [W/mK]

Design considerations

- To reduce heat transfer through slab on grade floors, we obviously need to:
  - Reduce the perimeter length, and/or
  - Decrease the heat loss coefficient, F_p
- Decreasing F_p is as simple as adding insulation to the foundation exterior
  - Typically no need to exceed R-8 (IP)

Figure 3. Insulated Form Board Field Installation

Heat loss coefficient: F_p

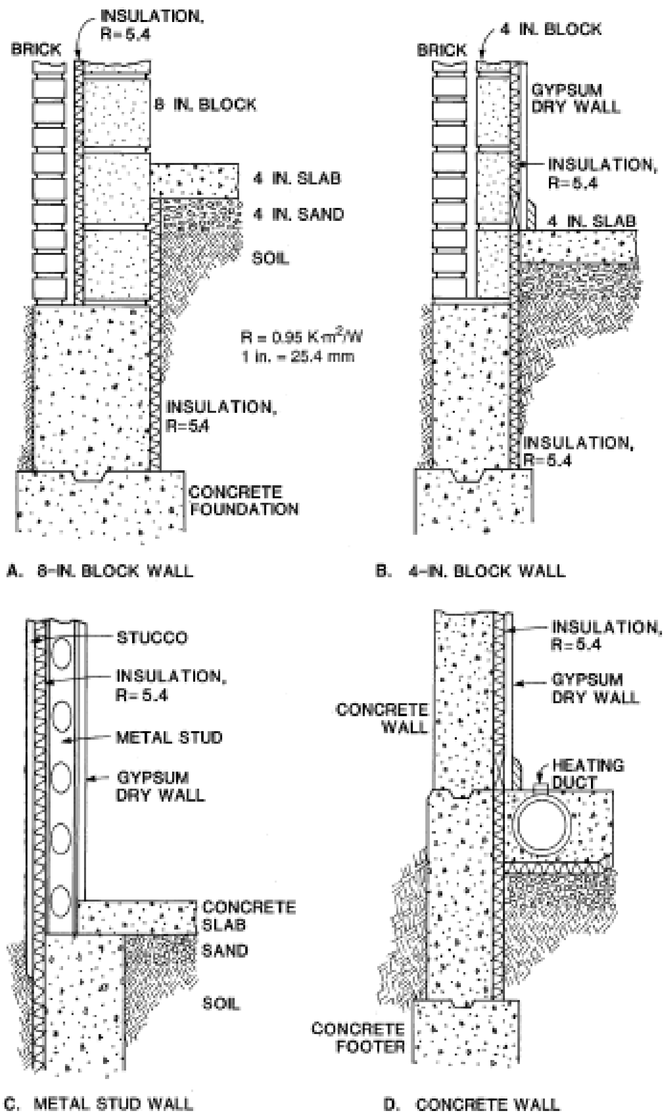


Fig. 8 Slab-on-Grade Foundation Insulation

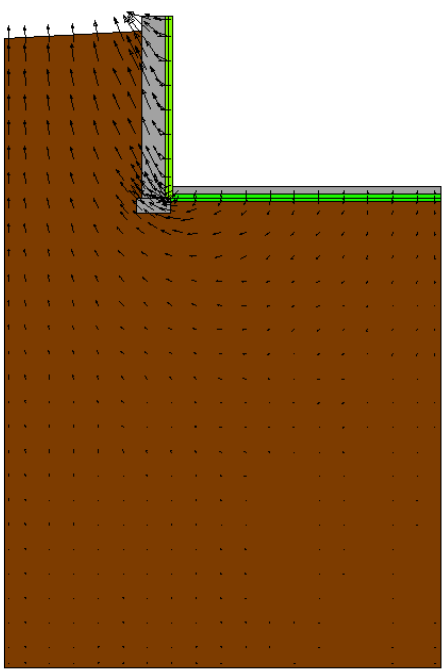
Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

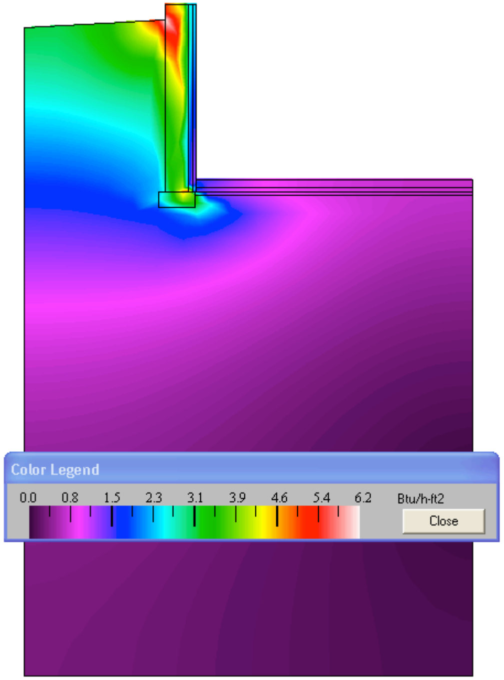
Construction	Insulation	F_p, W/(m·K)
200 mm block wall, brick facing	Uninsulated	1.17
	R-0.95 (m^2·K)/W from edge to footer	0.86
4 in. block wall, brick facing	Uninsulated	1.45
	R-0.95 (m^2·K)/W from edge to footer	0.85
Metal stud wall, stucco	Uninsulated	2.07
	R-0.95 (m^2·K)/W from edge to footer	0.92
Poured concrete wall with duct near perimeter*	Uninsulated	3.67
	R-0.95 (m^2·K)/W from edge to footer	1.24

*Weighted average temperature of the heating duct was assumed at 43°C during heating season (outdoor air temperature less than 18°C).

Using THERM for finding U_{avg}

- You can use THERM to model 2-D heat transfer but be careful …
  - Must create new convection boundary conditions for interior convection to/from floor
  - Must model a large area of soil around the foundation as a solid with adiabatic boundary conditions
  - Must model outside soil/air interface with new exterior convection
- It’s really a 3-D problem, so THERM has limitations



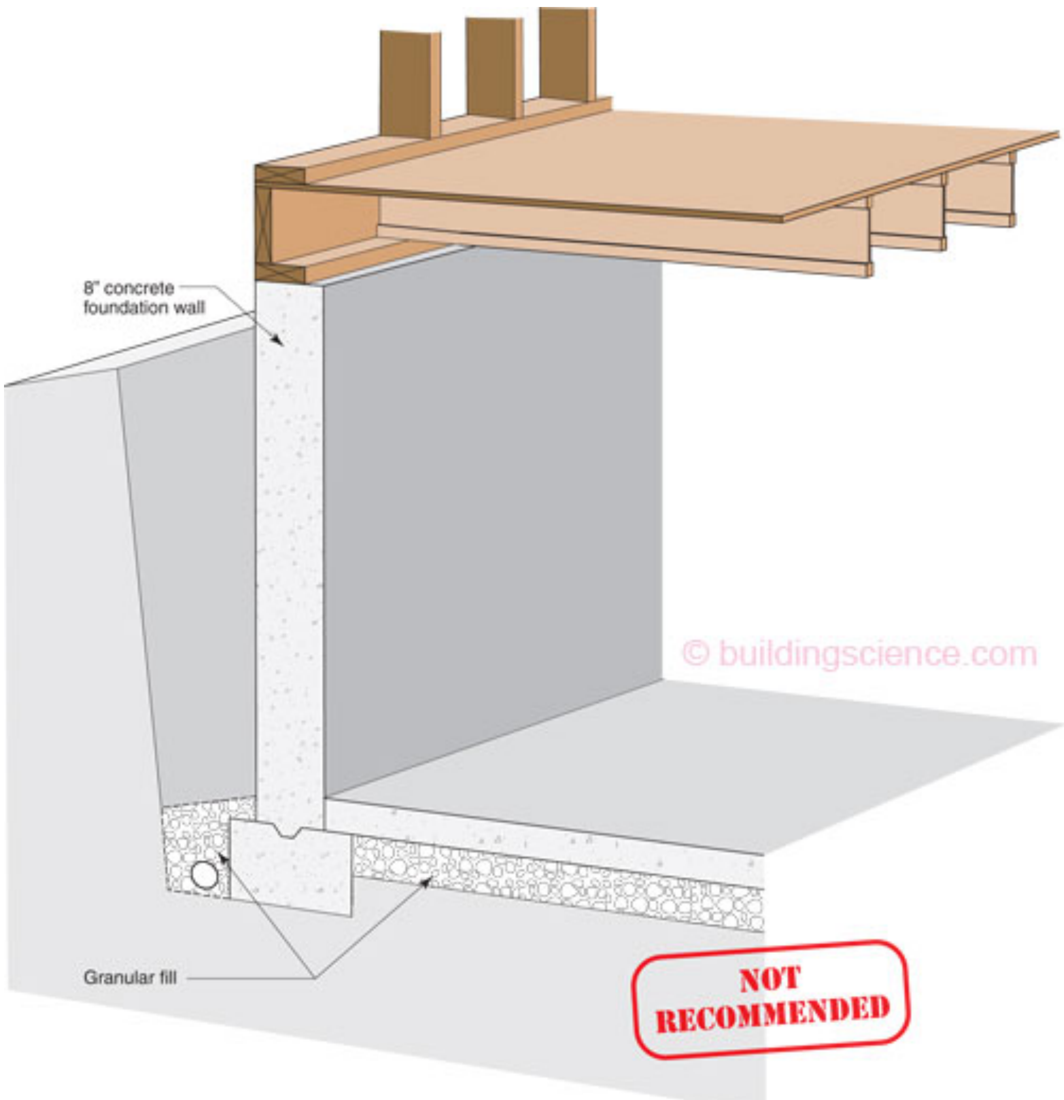


http://nesea.org/conversation/masters-blog/heat-loss-ground-part-1

EXAMPLE DETAILS FOR COMMON BELOW-GRADE ENCLOSURES (RESIDENTIAL)

Below-grade enclosures (residential)

Uninsulated concrete foundation wall and slab

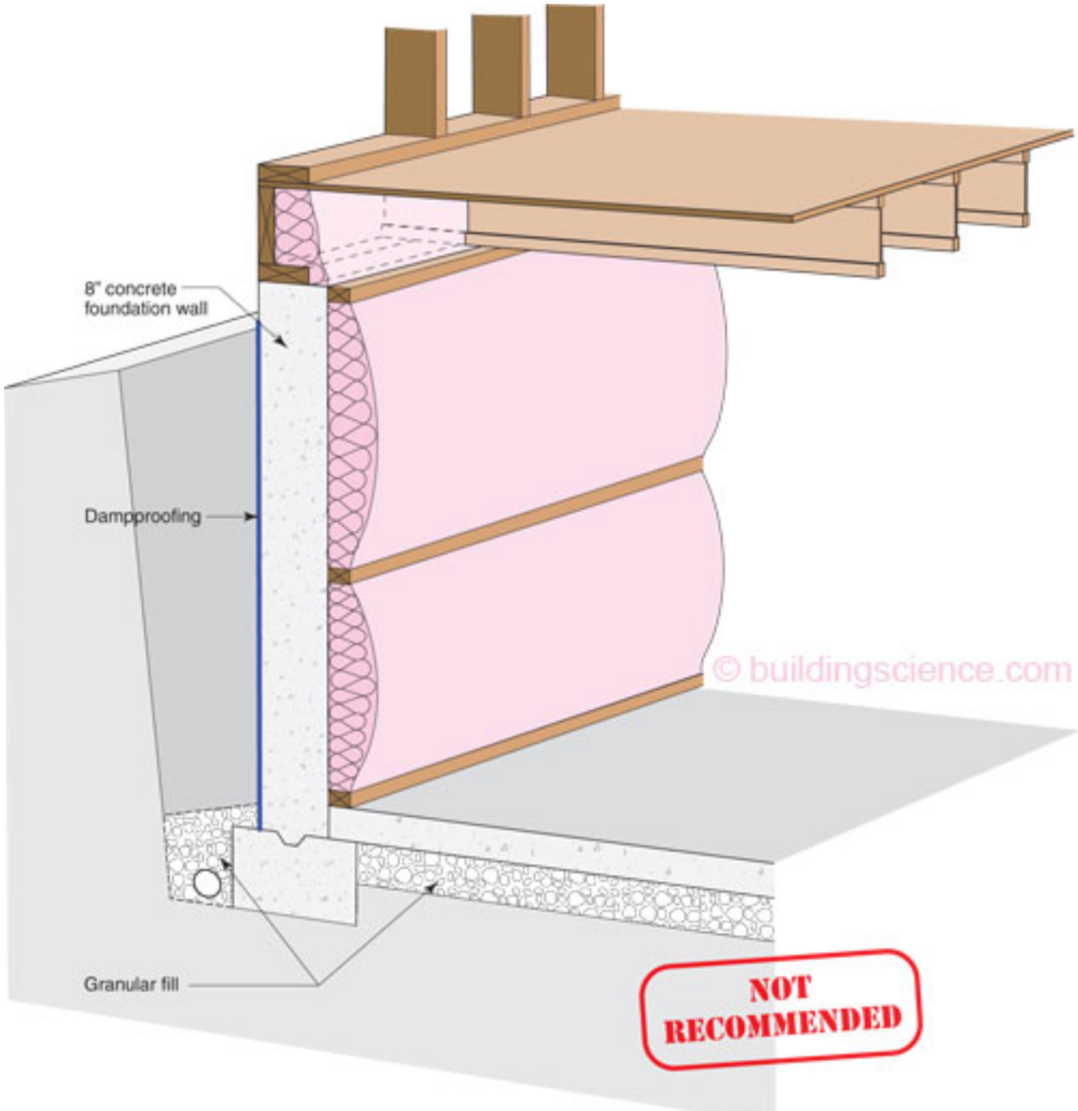


- No thermal control
- Not even allowed by code if basement is conditioned
- No moisture control
- Water vapor diffusion and capillary action are near-constant moisture sources

http://www.buildingscience.com/

Below-grade enclosures (residential)

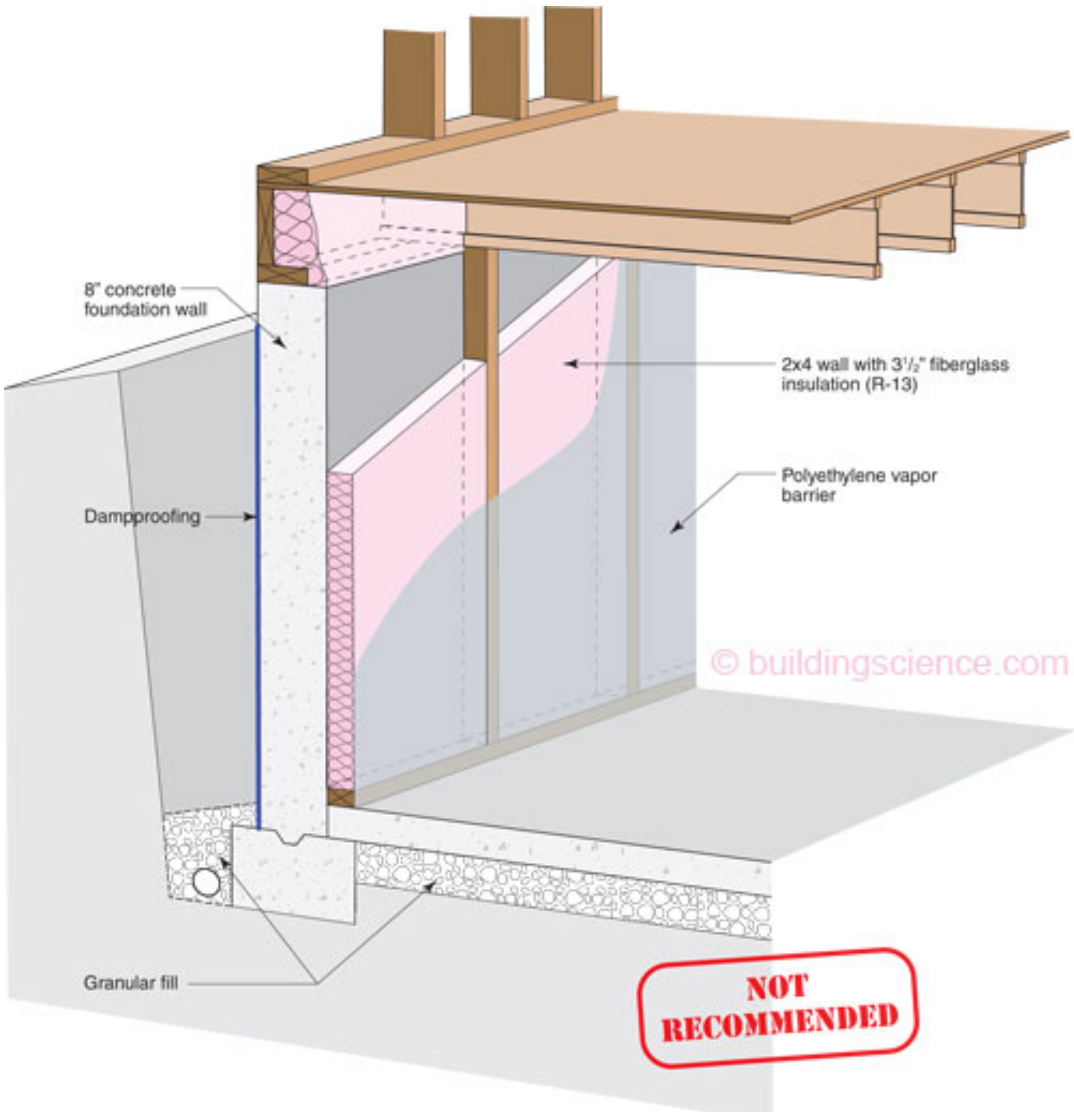
Code minimum R-10 (IP) continuous insulation in a framed wall



- Slab not insulated
- Better thermal control
- Inexpensive
- Sometimes wall insulation batt is covered with vapor barrier
- Moisture issues (batt is air and vapor permeable)
- High RH at concrete wall most of the year

Below-grade enclosures (residential)

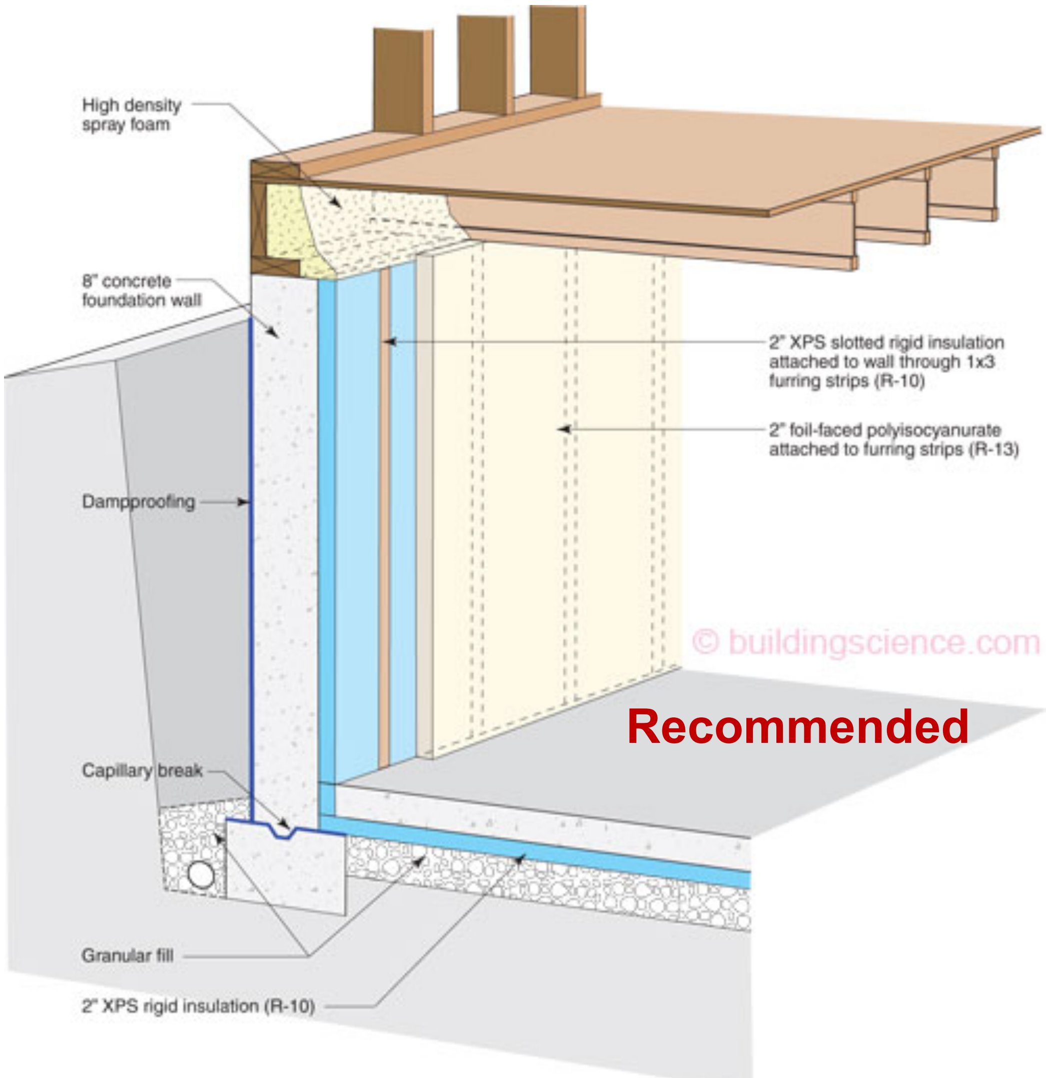
R-13 (IP) insulation in a 2x4 framed wall

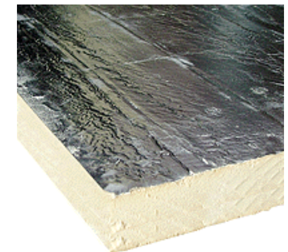


- Similar to last construction
- Moisture issues
- High RH at concrete wall most of the year
- Particularly a problem if there is any air leakage

Below-grade enclosures (residential)

2” XPS rigid insulation + 2” foil-face polyisocyanurate foam board

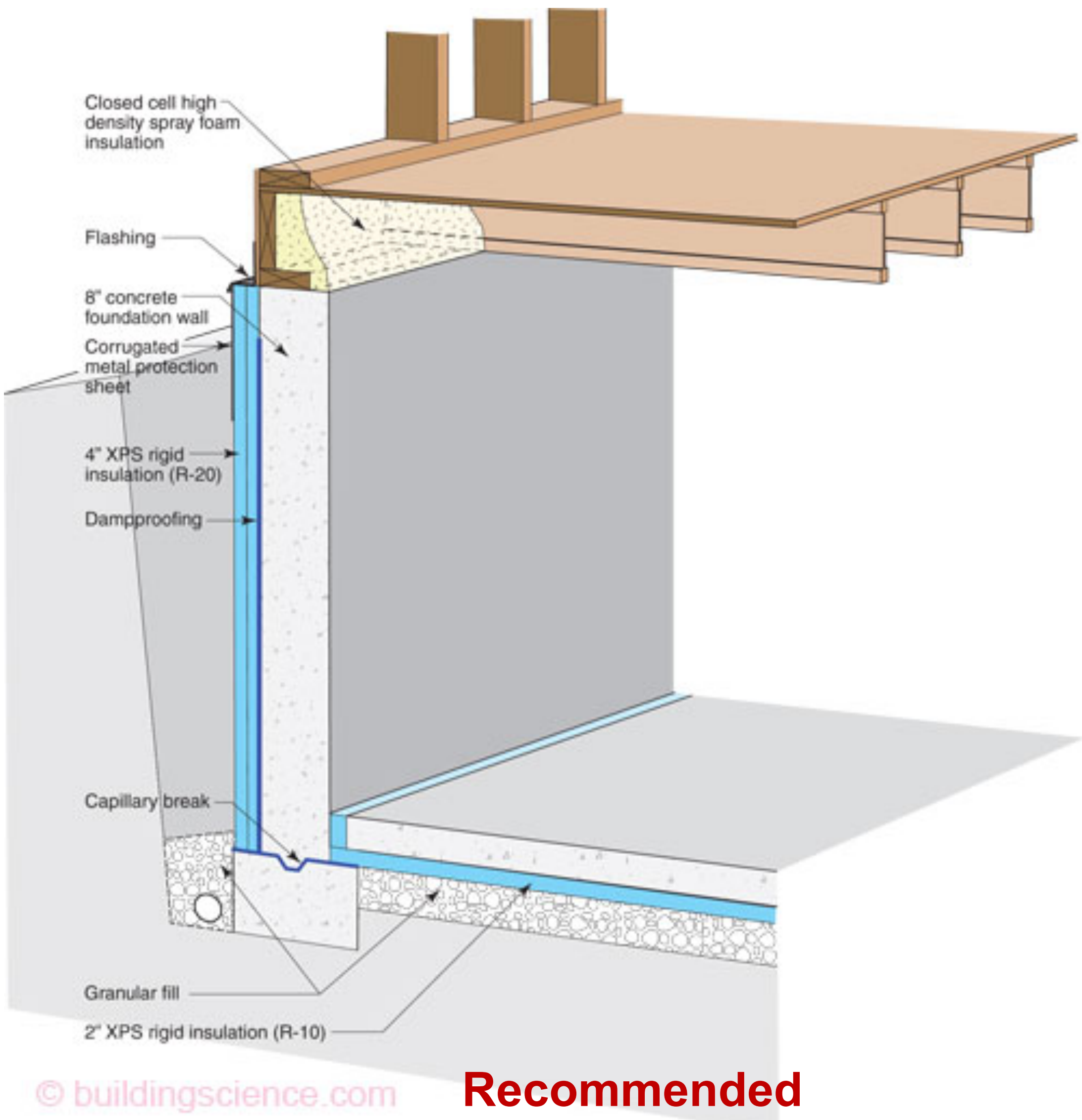




- Very good thermal control (R-23 walls)
- Water vapor diffusion is prevented
- Capillary action is prevented by the thermal/capillary break at the edge of the slab and top of footing

Below-grade enclosures (residential)

Rigid XPS exterior insulation



Recommended

- Very good thermal control (R-20 walls)
- Exterior insulation can be joined with first floor insulation
- Excellent resistance to vapor diffusion
- Capillary action is a potential problem (through the footing)
  - Need a break
- Exposed concrete provides moisture buffer after it dries
- May be hard to construct

HW 2

- HW 2 is assigned today, due next week in class
  - Uses THERM

- Next lecture (Feb 23): Moisture flows in building encosures

- You should also be starting your campus enclosure assessments by now