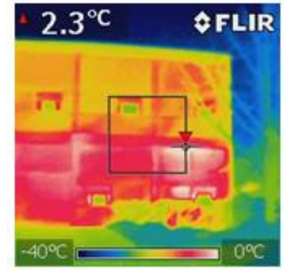
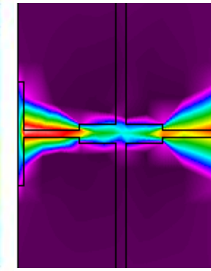
CAE 331/513
Building Science
Fall 2019



November 12, 2019
Building energy balances

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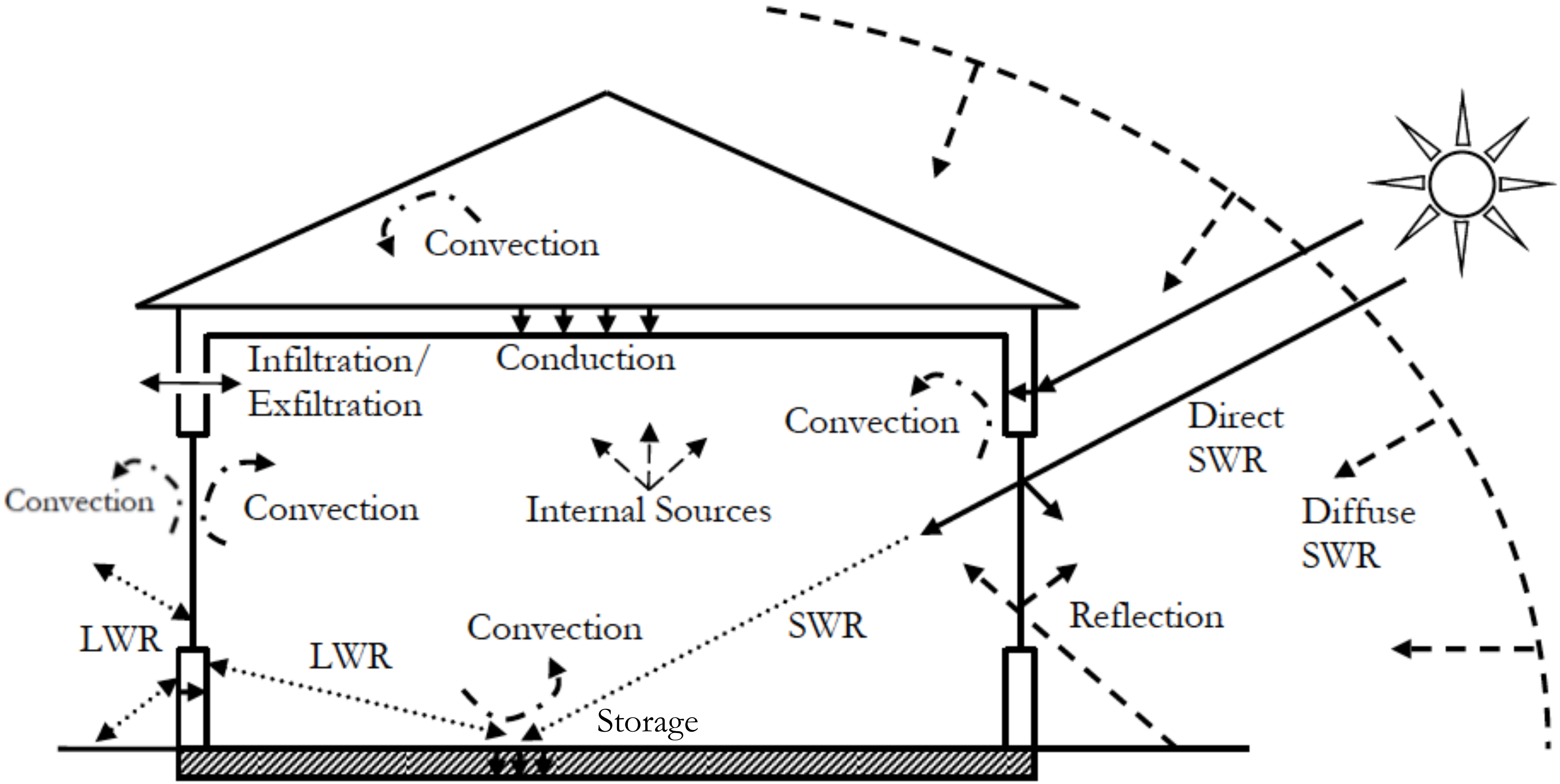
Last time

- Air infiltration and natural ventilation

BUILDING ENERGY BALANCES

Building energy balances: Back to heat transfer



What are energy balances useful for?

1. Heating and cooling load calculations
   – i.e., equipment sizing to meet peak heating/cooling loads

2. Annual energy estimation

Surface energy balances: Revisited

- We know that multiple modes of heat transfer are typically acting at the same time at multiple points point within a building…

  … So we can also write expressions to quantify heat flow/flux to/from these various points simultaneously by accounting for all relevant modes of heat transfer

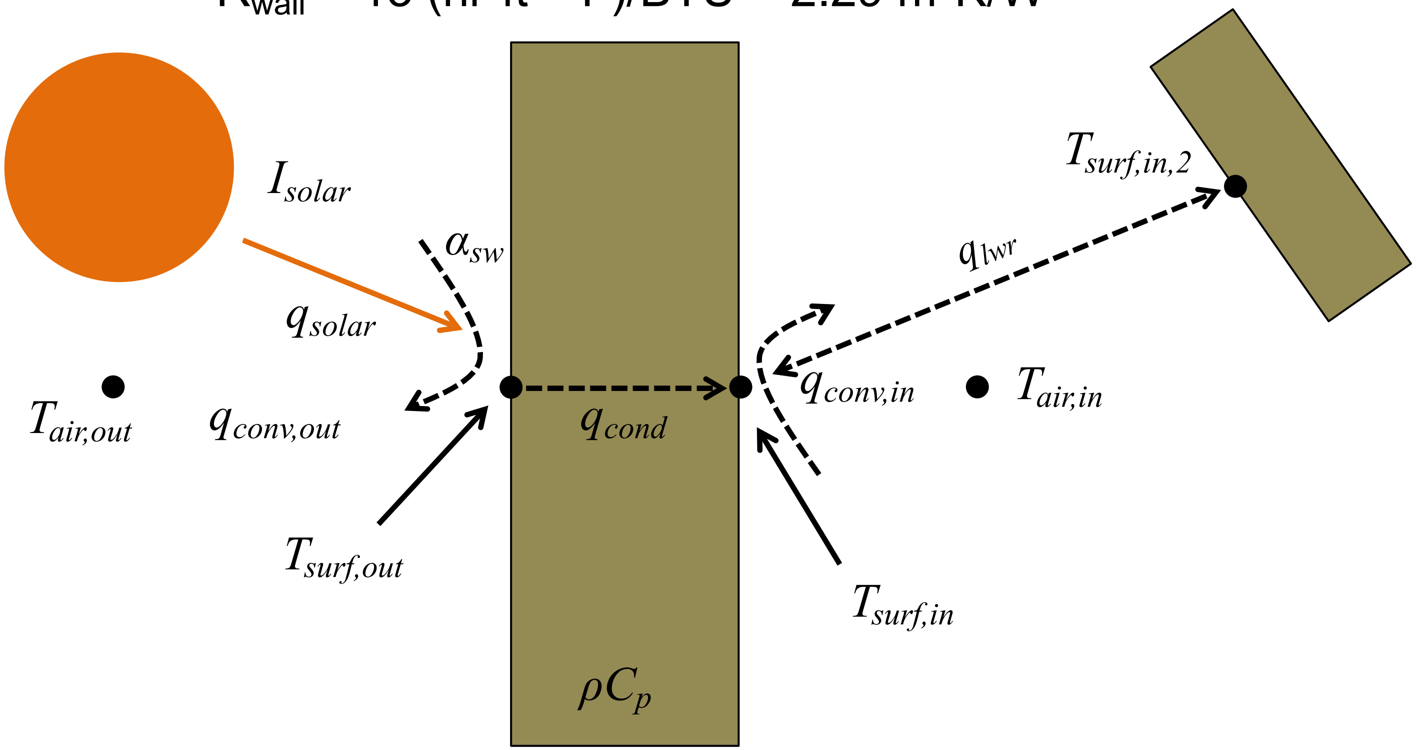
  – **Writing “building energy balances”**

  – **Solving systems of equations**

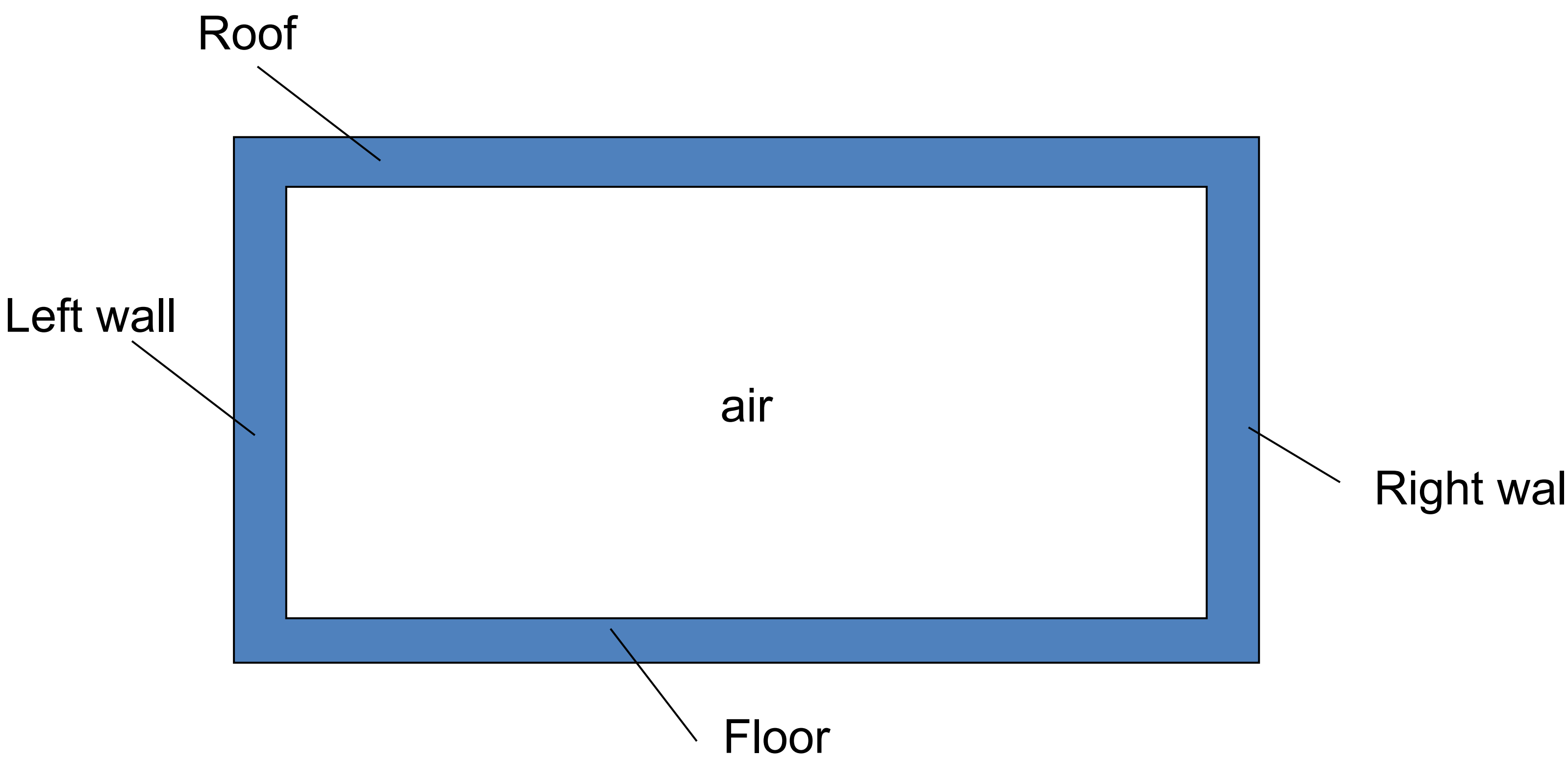
Surface energy balances: Revisited

(Re)imagine an external wall of a building:

R_{wall} = 13 (hr-ft^2-°F)/BTU = 2.29 m^{2}K/W



Building energy balance: Simple box model

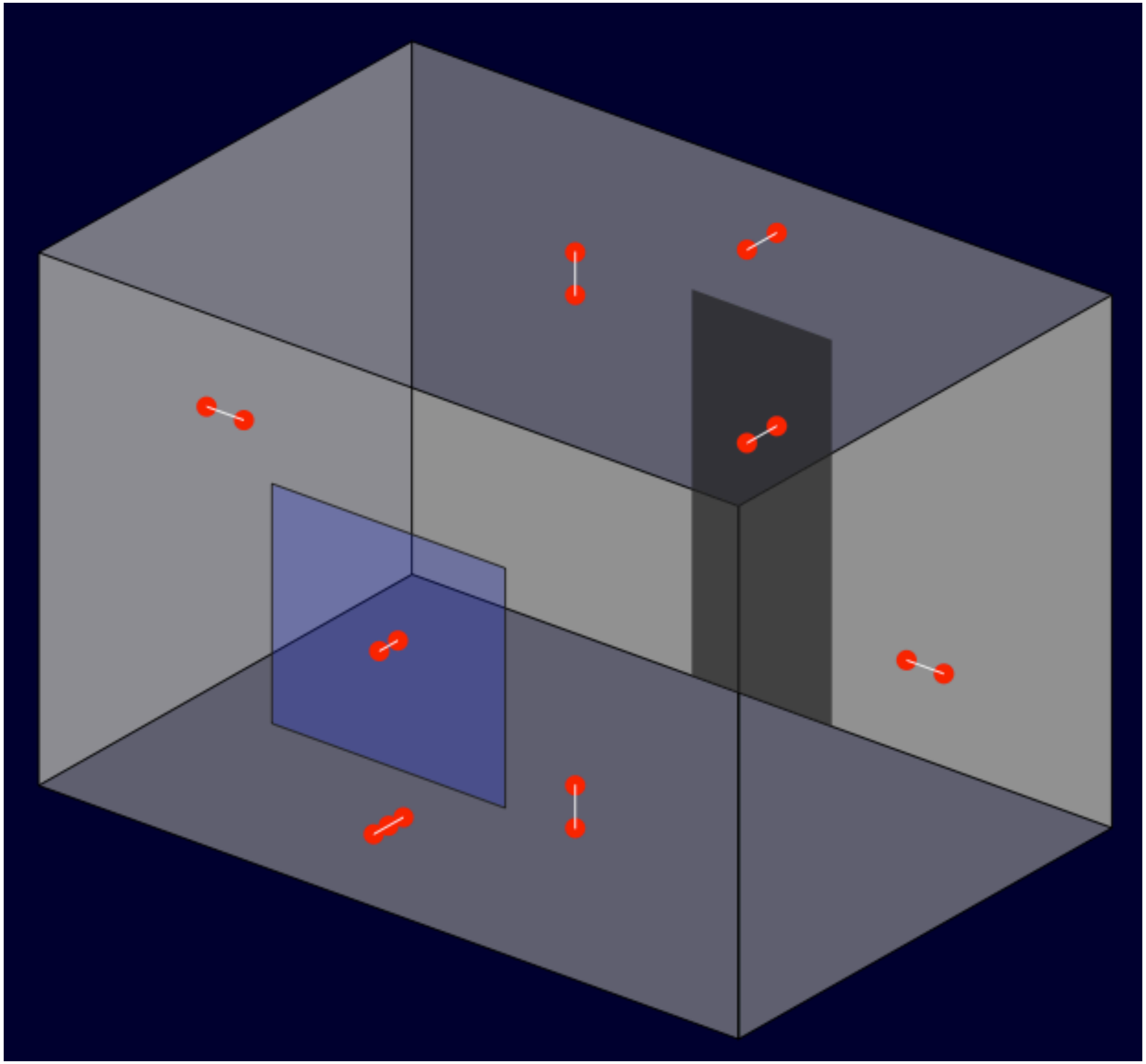


Elements are connected by:

1) Convection – air node
2) Radiation – surface nodes

Slide courtesy Atila Novoselac, University of Texas at Austin

Simple box model: Conduction elements



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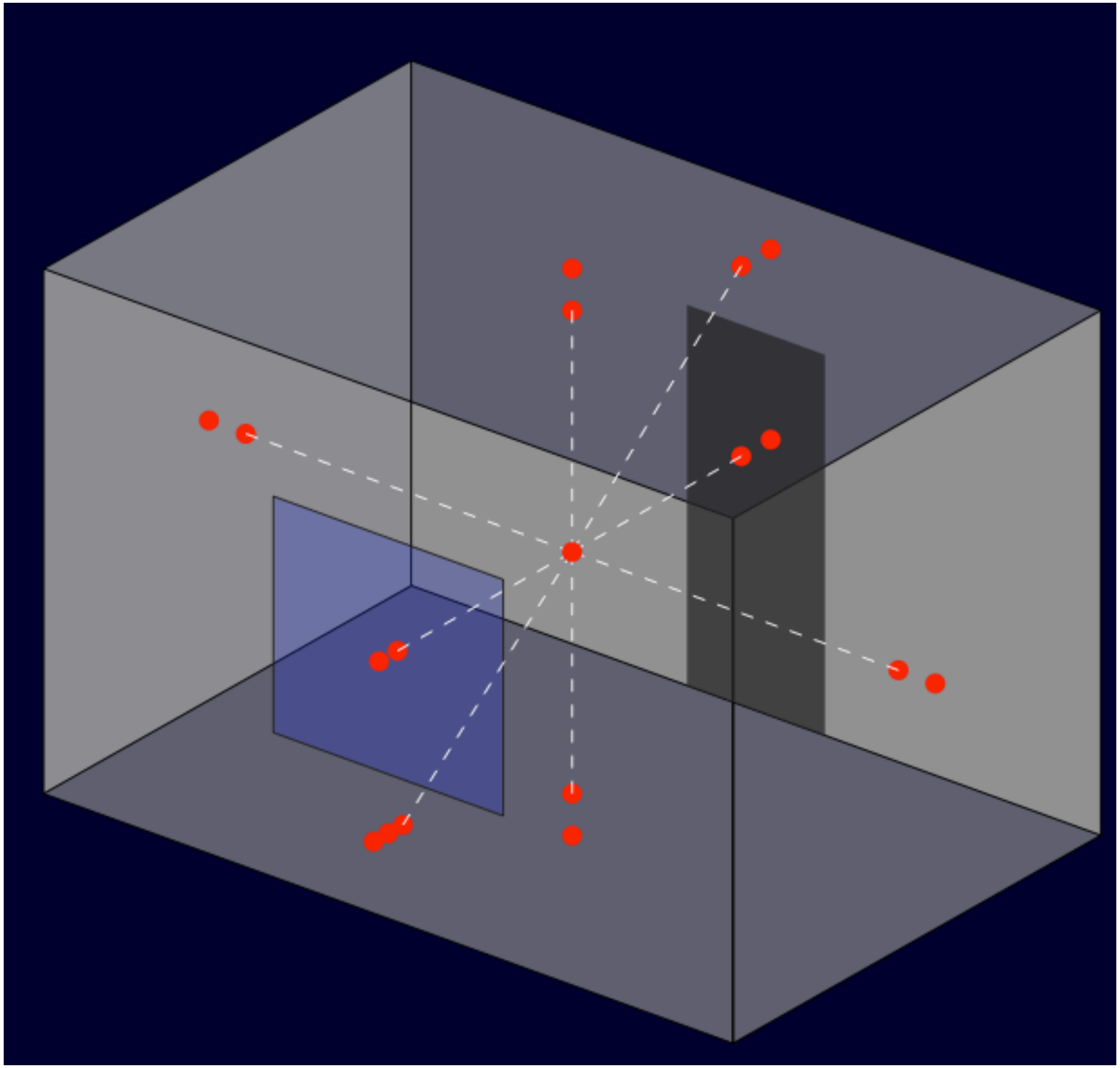
Simple box model: Interior convection elements



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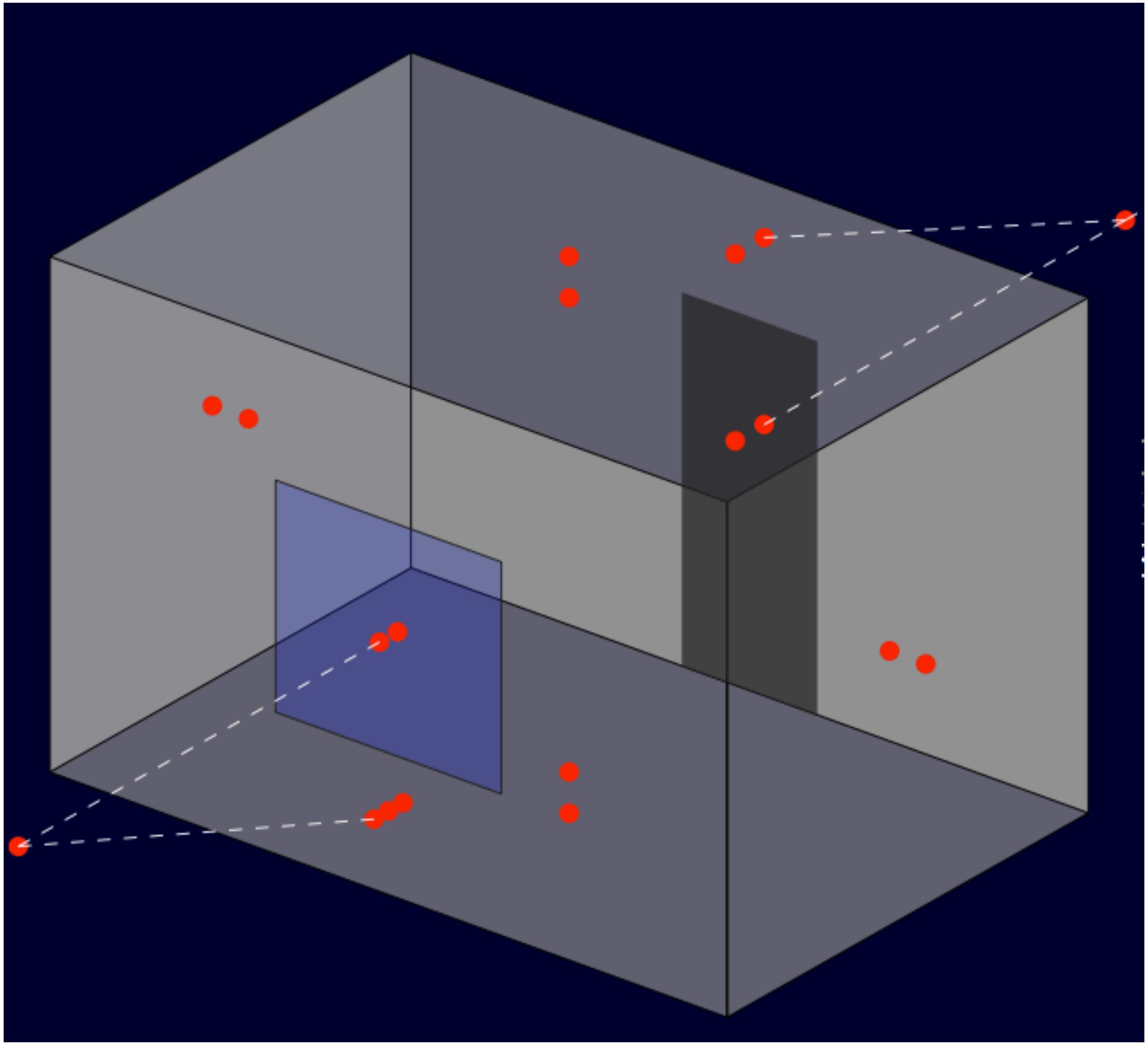
Simple box model: Exterior convection elements



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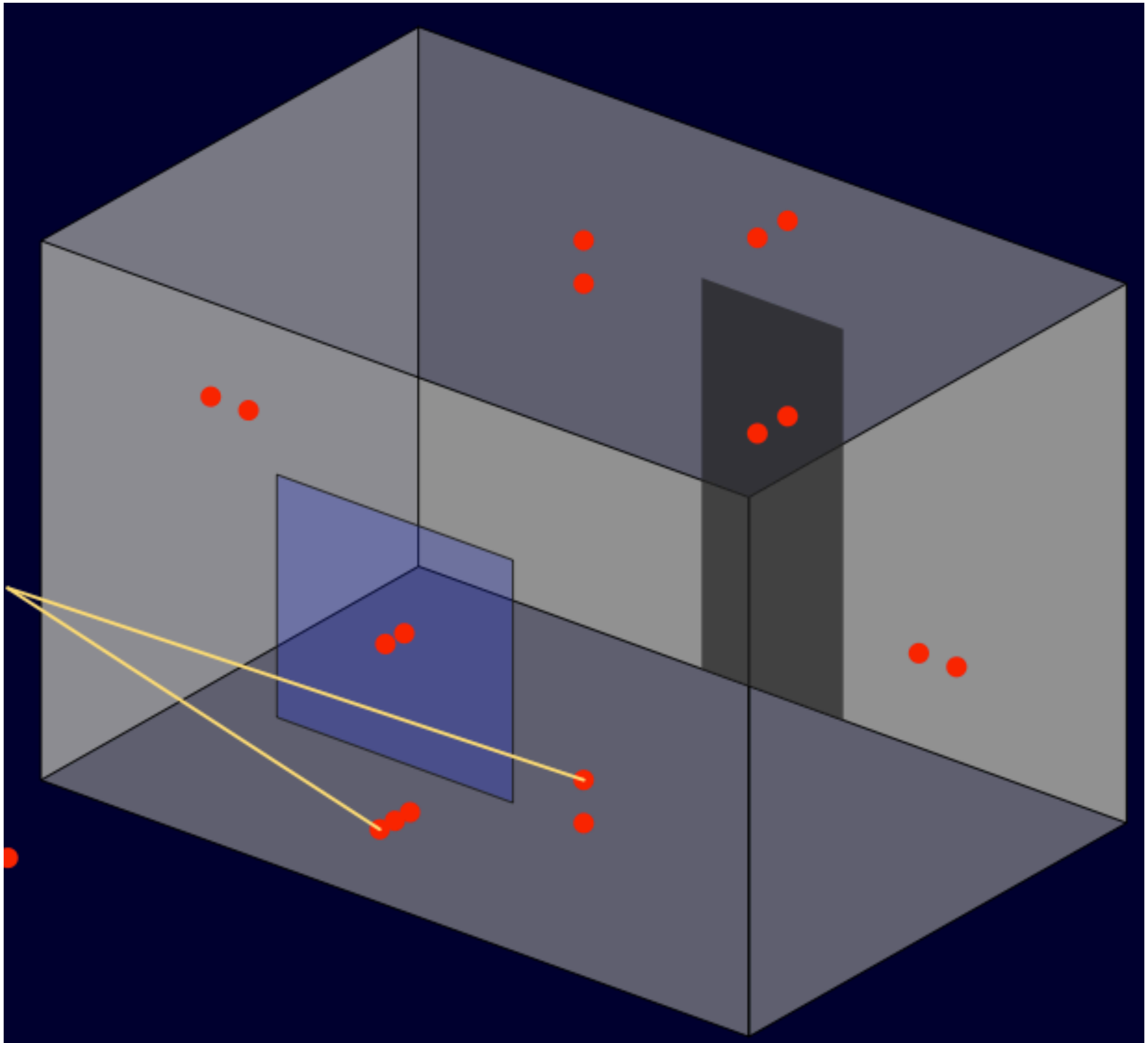
Simple 3-D box model: Solar (direct + diffuse)



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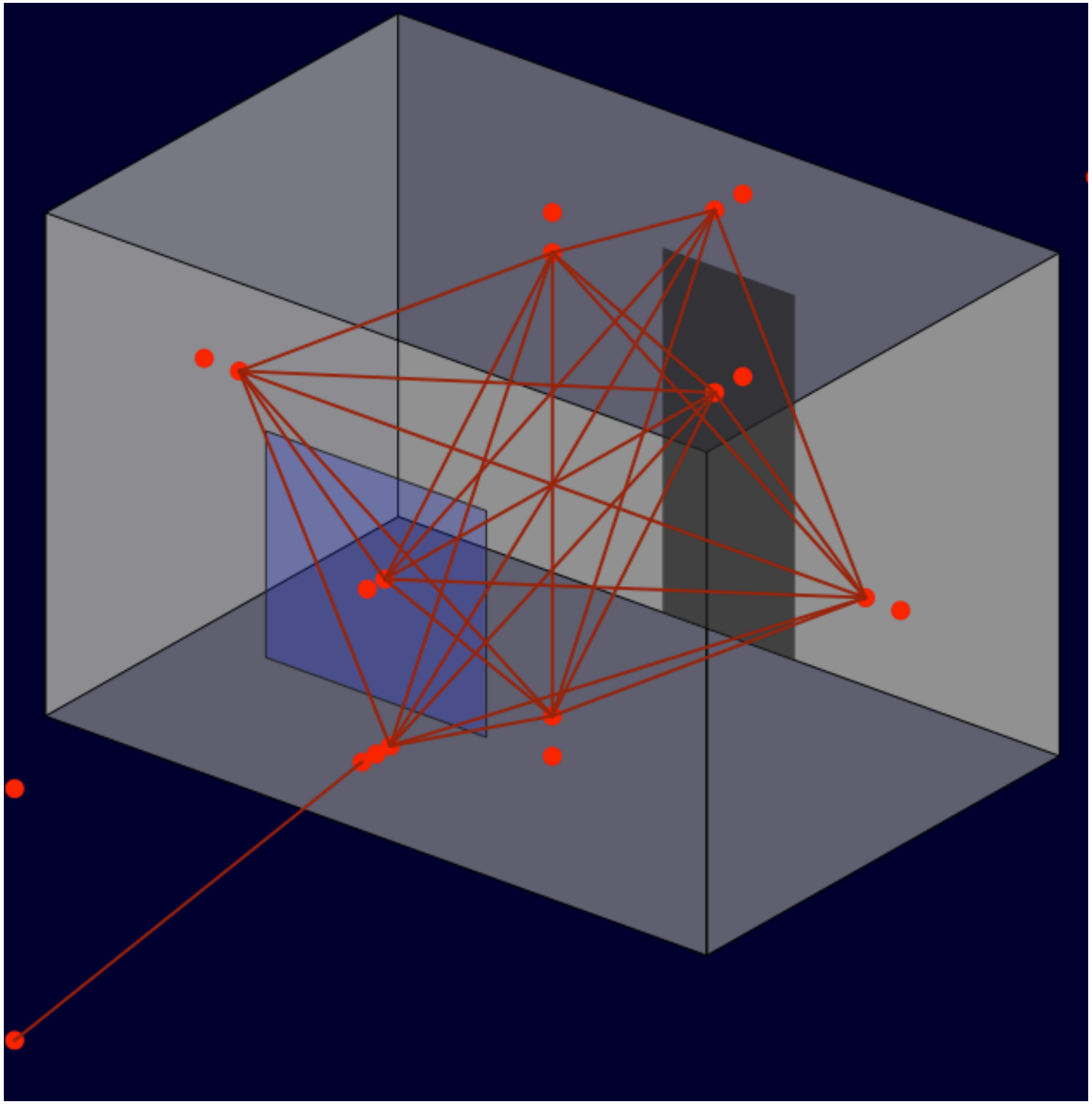
Simple box model: Long wave radiation elements



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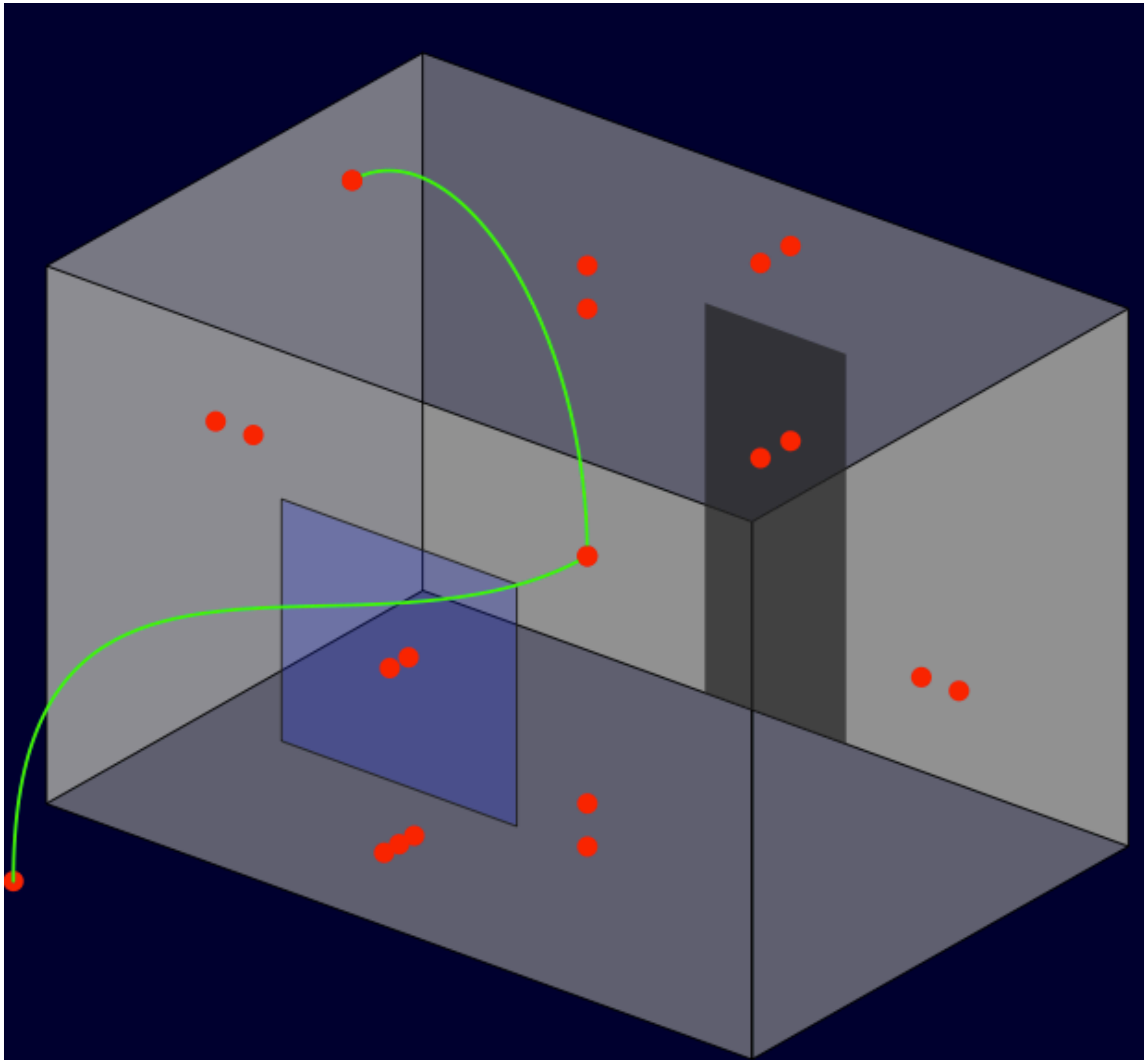
Simple box model: Ventilation/infiltration elements



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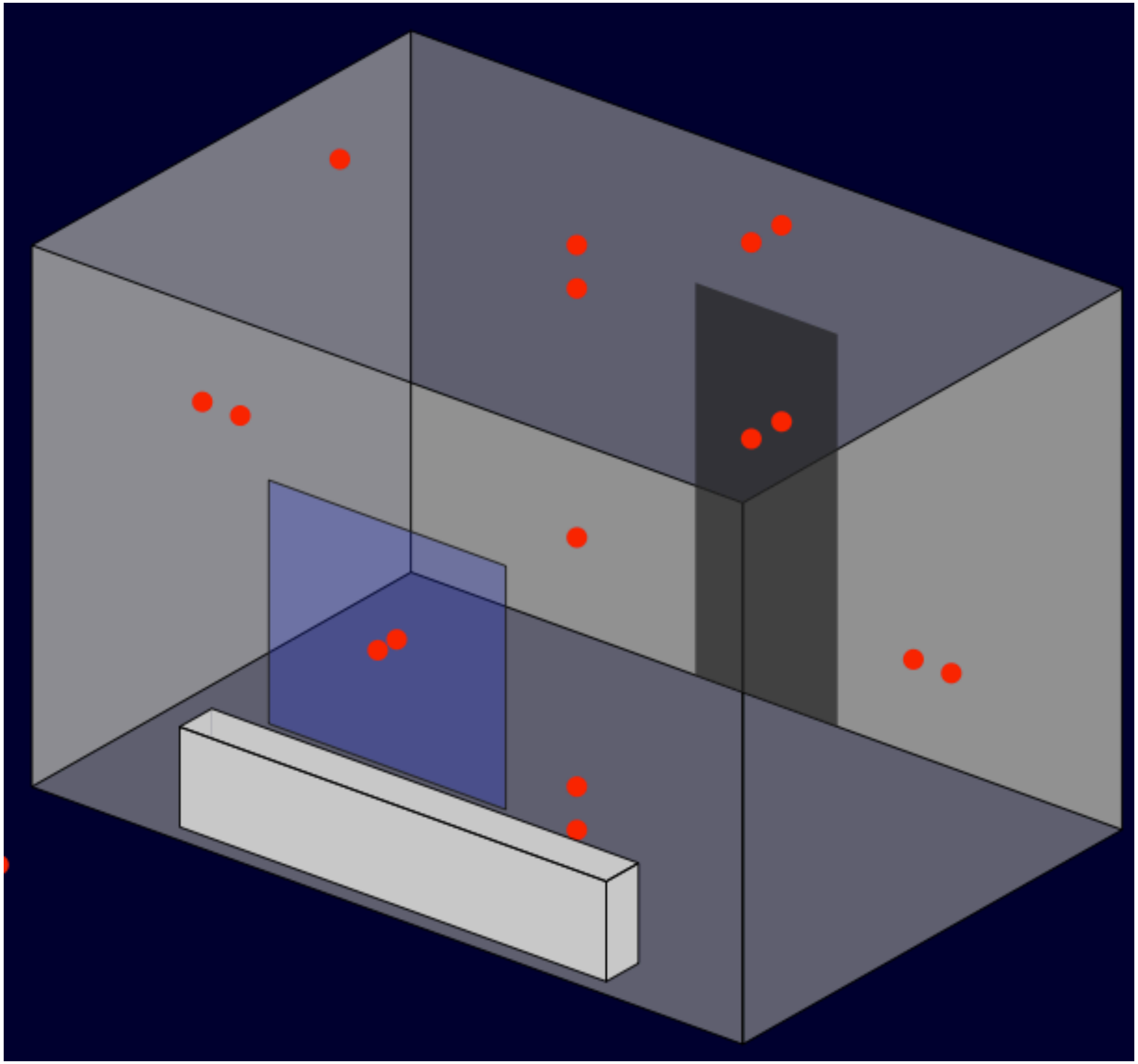
Simple box model: Internal mass (e.g., furniture)



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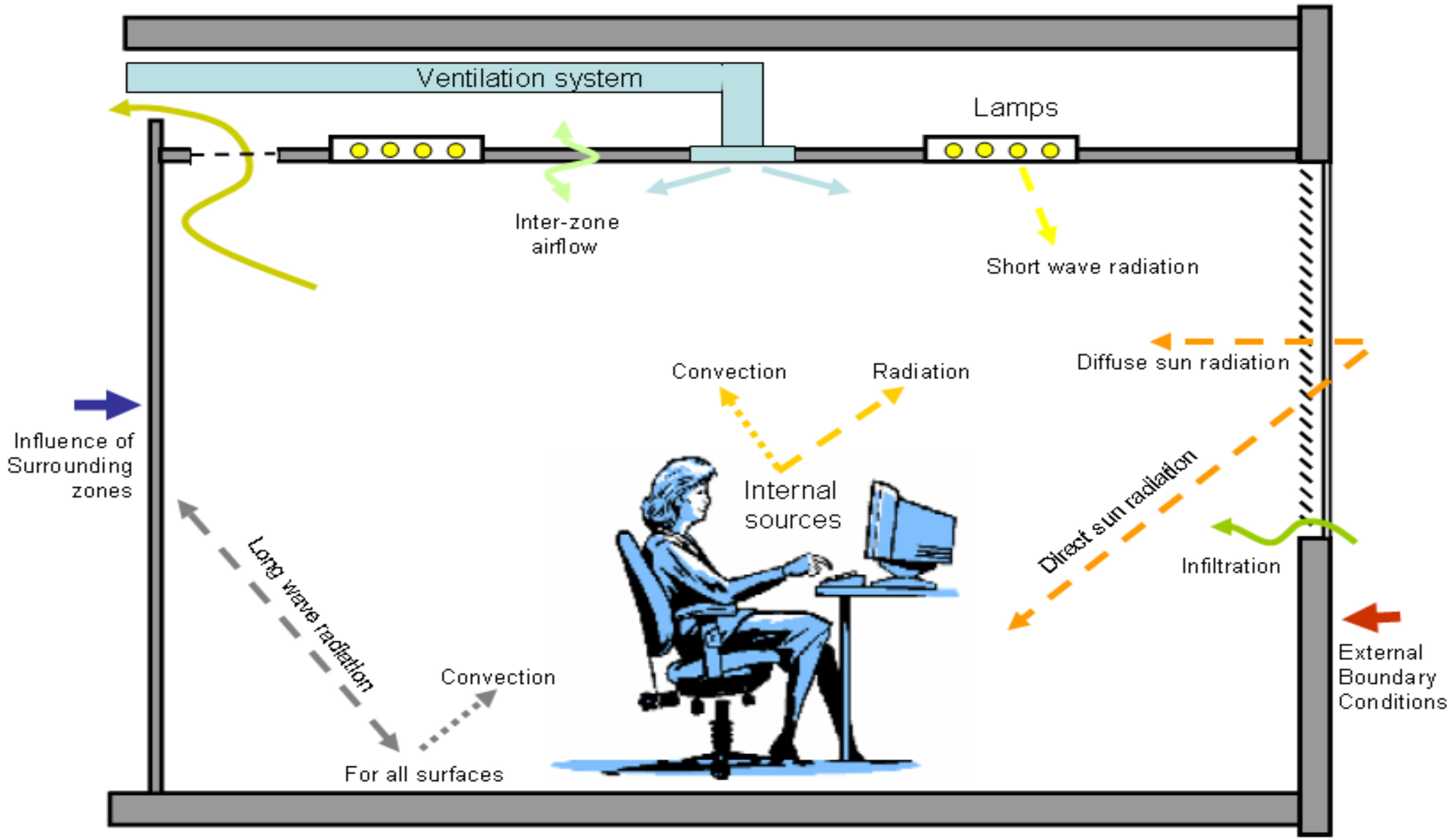
Adding occupants, equipment, and lights



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Convective/radiative components of internal gains

- Need to account for heat capacity of building materials and the fraction of radiative versus convective heat given off by <u>systems</u> and <u>equipment</u>

	Radiative, percent	Convective, percent
Fluorescent lights	50	50
People	33	67
External walls and roofs	60	40
Appliance and machines	20–80	80–20

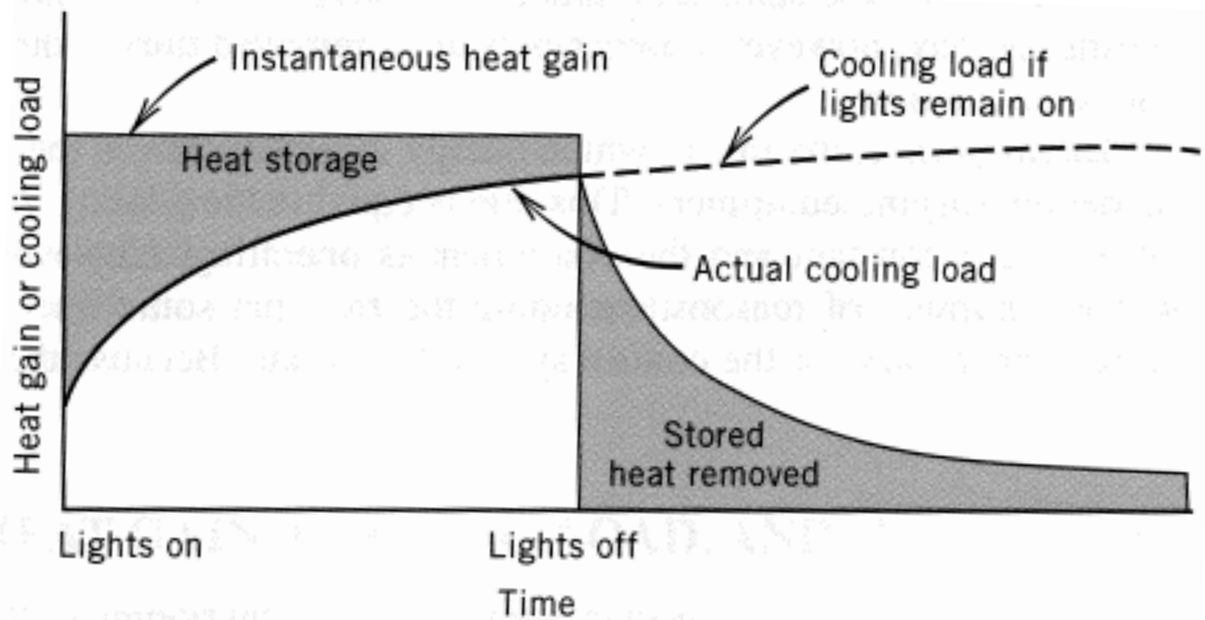
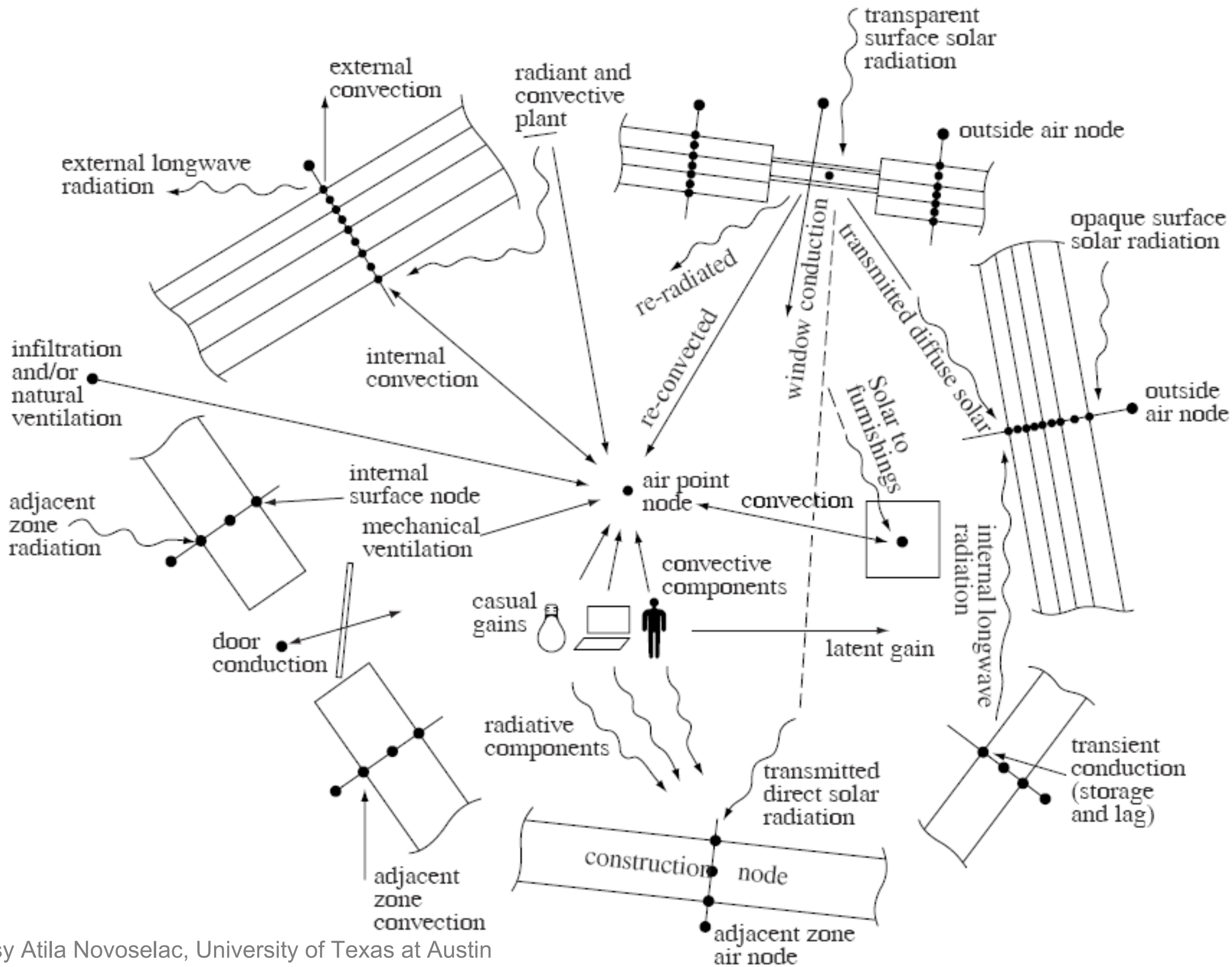


Figure 8-3 Actual cooling load from fluorescent lights.

Whole building energy balances



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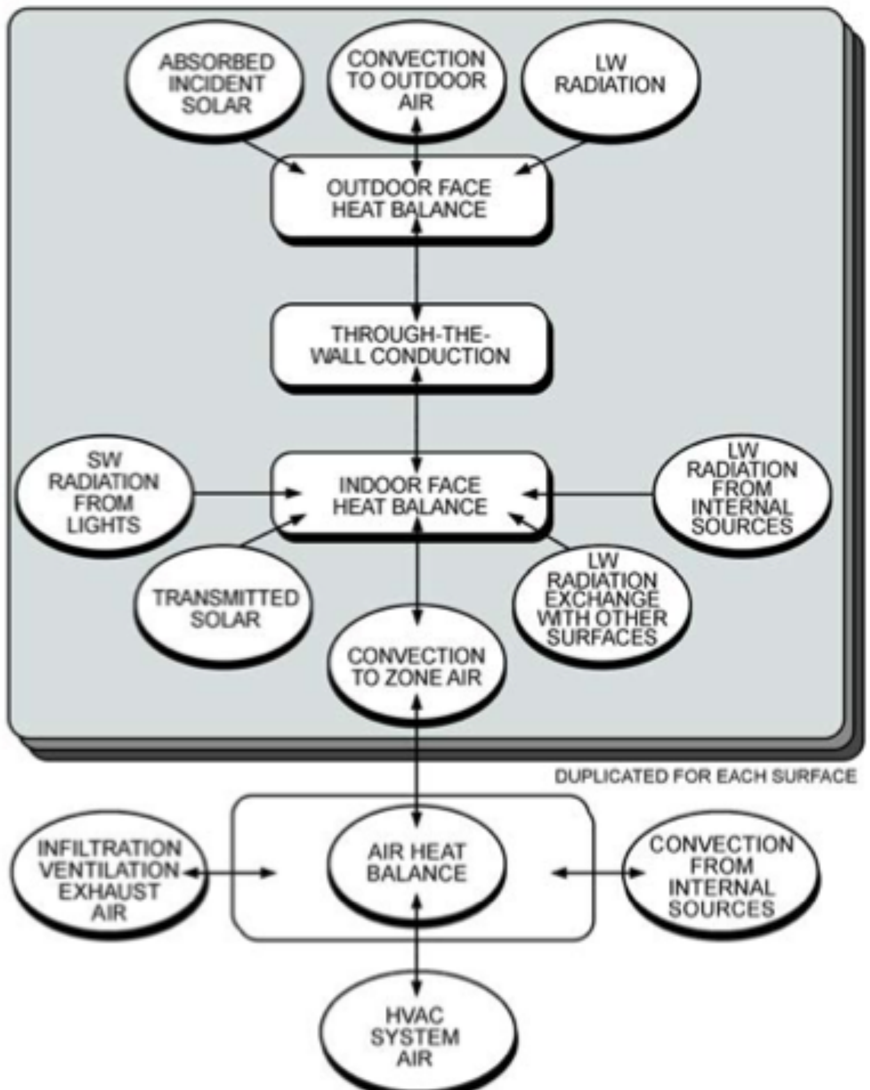
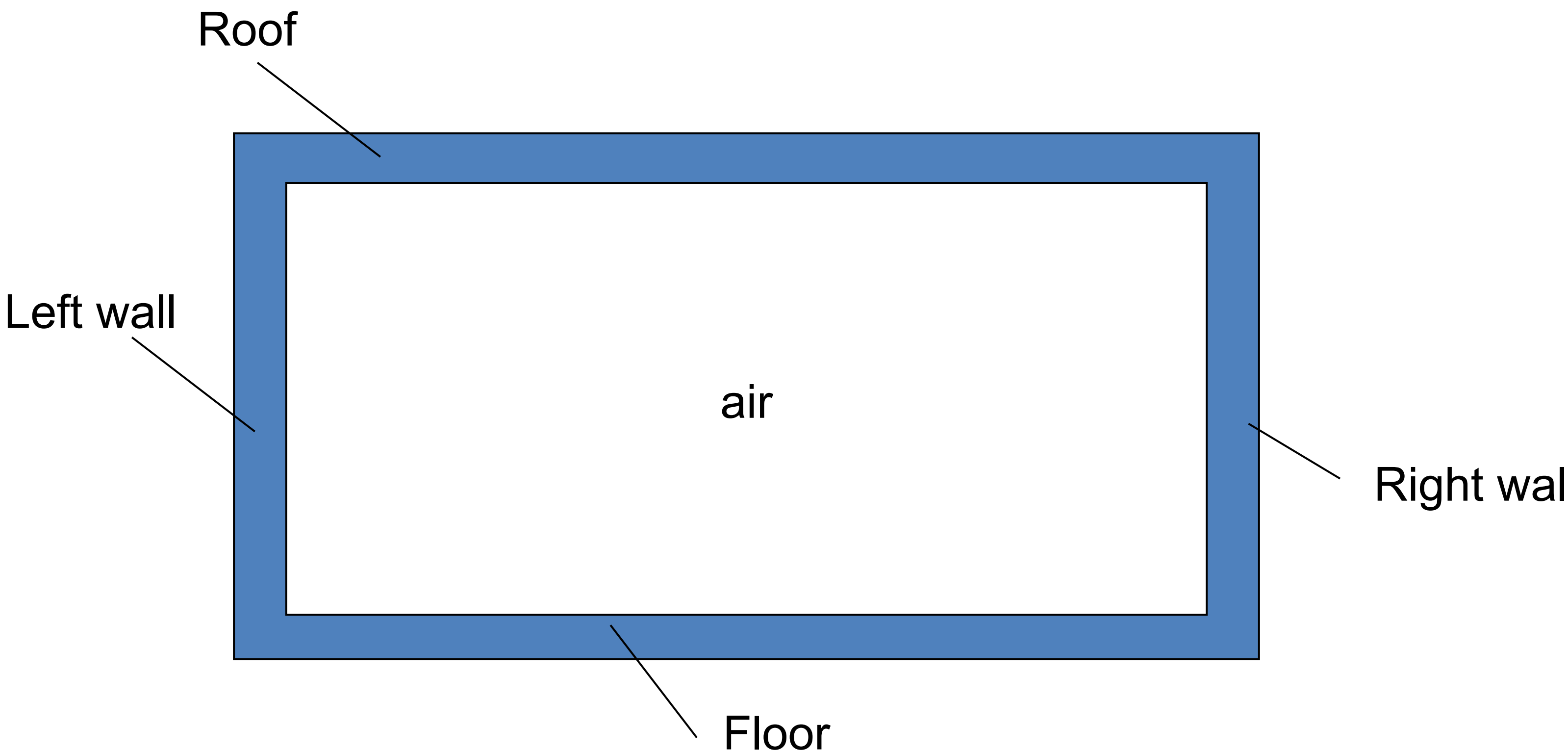
Energy balances: Applications of heat balance method



***Also used for cooling load calculations (later)**

Building energy balance: Simple box model



Elements are connected by:

1) Convection – indoor air node
2) Radiation – surface nodes

Building energy balance: Simple box model

- For an example 2-D room like this, you would setup a system of equations where the temperature at each node (either a surface or within a material) is unknown
  - 12 material nodes + 1 indoor air node

Heat Xfer @ external surfaces:
Radiation and convection

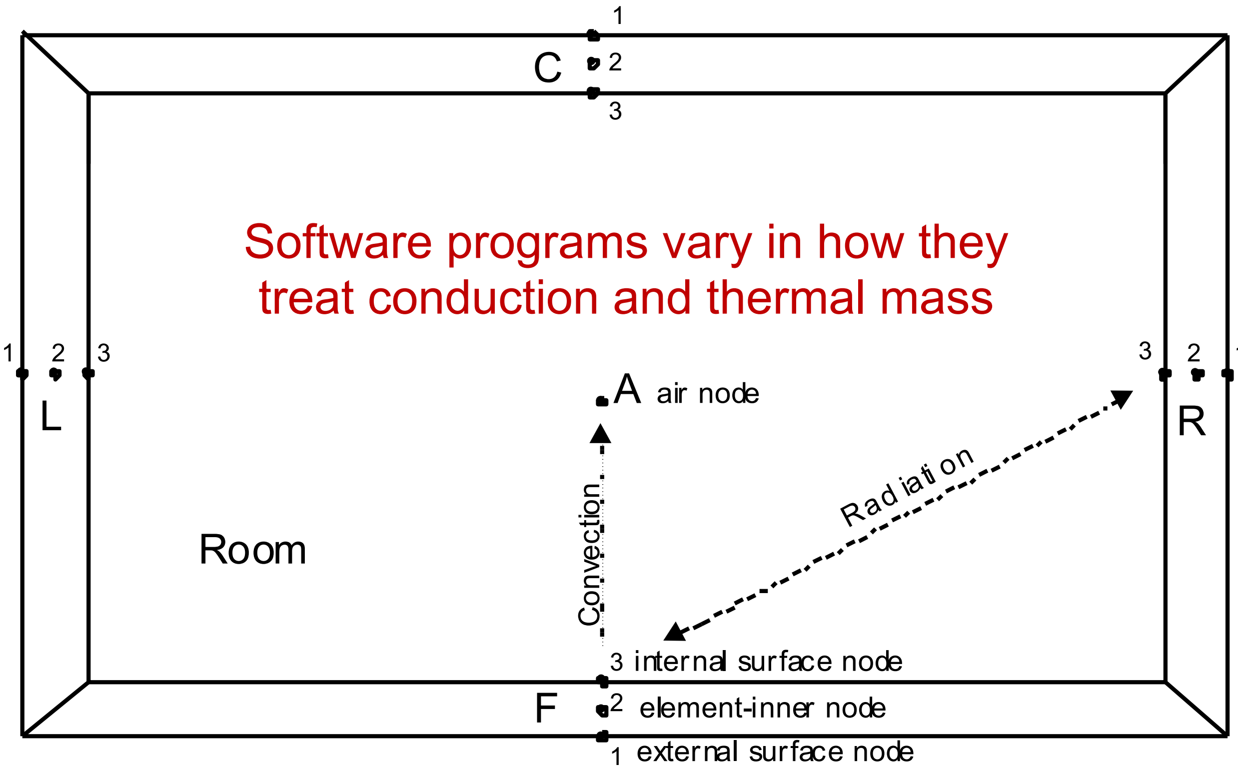
At surface nodes:

$$\sum q = 0$$

At nodes inside materials:

$$mc_p \frac{dT}{dt} = \sum q_{at\ boundaries}$$

Unsteady conduction (storage) is based on density and heat capacity of material

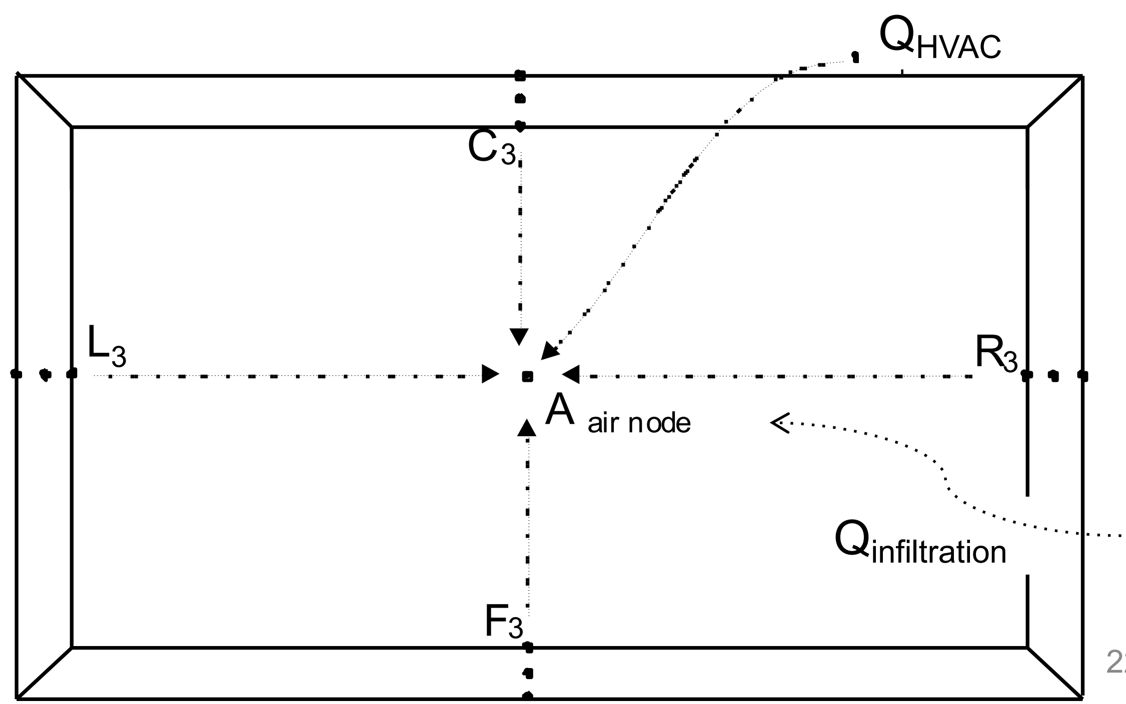


Unsteady energy balance for indoor air node

- To get the impact on indoor air temperature (and close the system of equations):
  - Write an energy balance on the indoor air node
  - Air is impacted directly only by convection (bulk and surface)

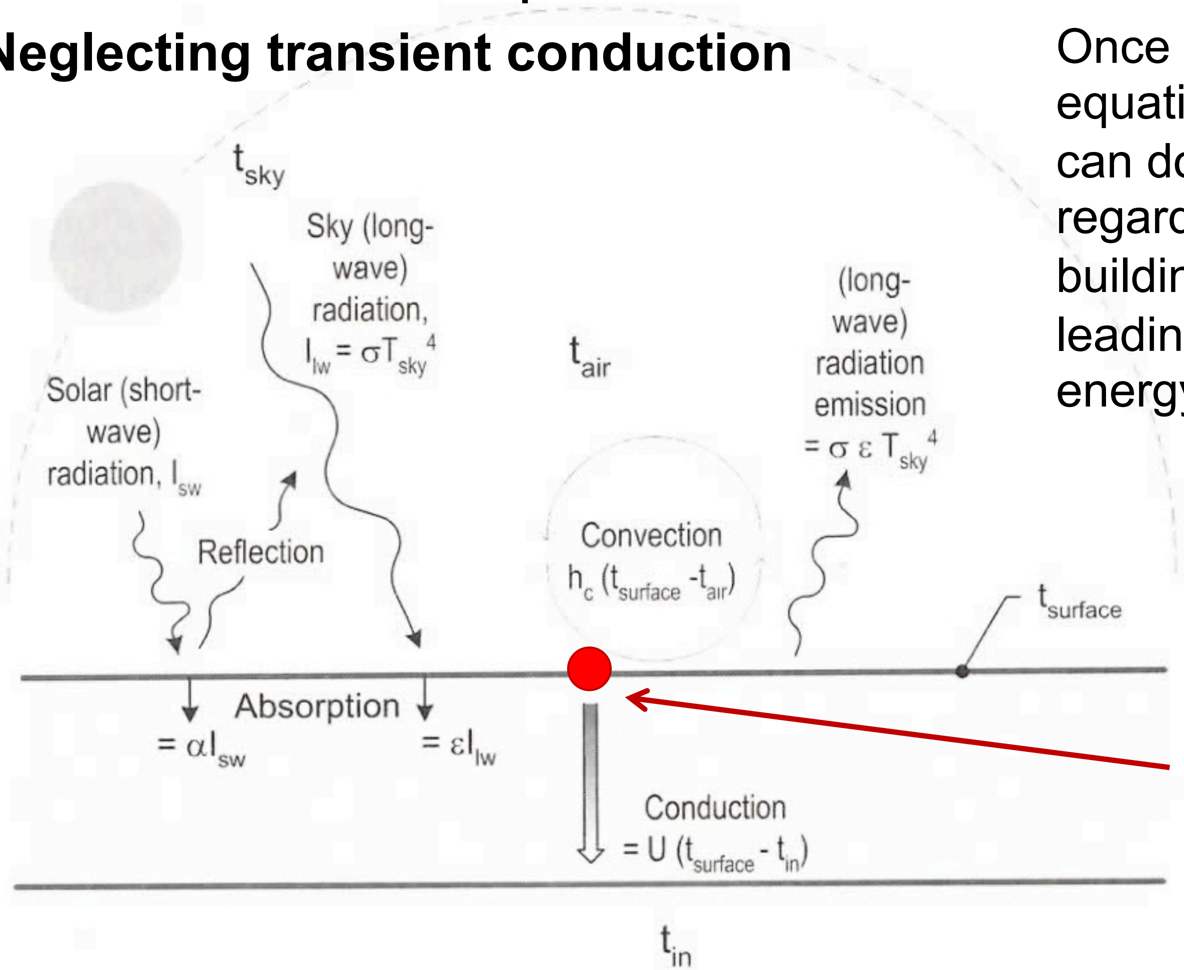
$$(V_{room}\rho_{air}c_{p,air})\frac{dT_{air,in}}{dt} = \sum_{i=1}^{n} h_i A_i \left(T_{i,surf} - T_{air,in}\right) + \dot{m}c_p\left(T_{out} - T_{air,in}\right) + Q_{HVAC}$$

In plain English:
The change in indoor air temperature is equal to the sum of convection from each interior surface plus outdoor air delivery (by infiltration or dedicated outdoor air supply), plus the bulk convective heat transfer delivered by the HVAC system



Surface energy balance example

- Exterior surface example: **Roof**
  - **Neglecting transient conduction**



Once you have this equation described, you can do just about anything regarding heat transfer in building science/analysis, leading into full-scale energy modeling

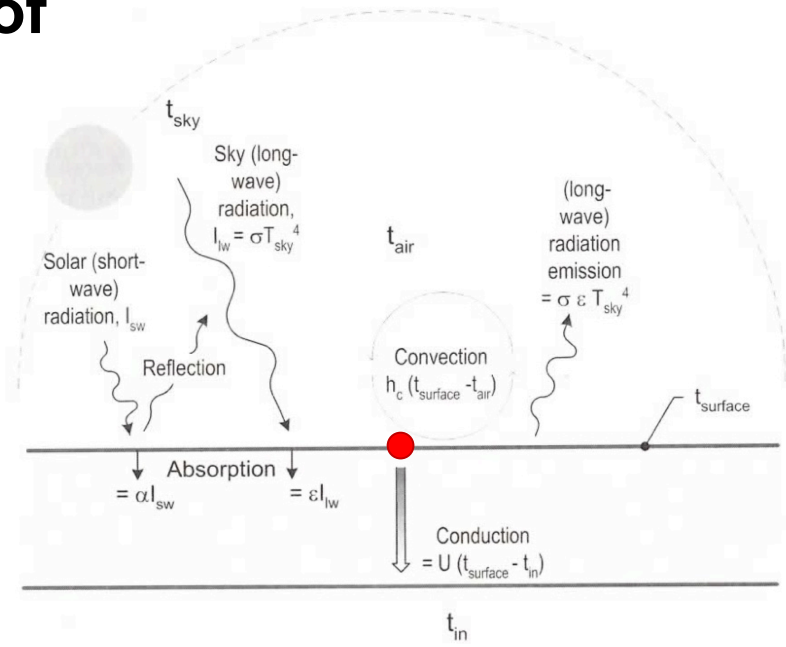
Steady-state energy balance at this exterior surface:
What enters must also leave (no storage)

$$\boxed{q_{solar} + q_{longwaveradiation} + q_{convection} - q_{conduction} = 0}$$

Surface energy balance example

- Exterior surface example: **Roof**

$$\boxed{\sum q = 0}$$



We can use this equation to estimate indoor and outdoor surface temperatures

At steady state, net energy balance is zero

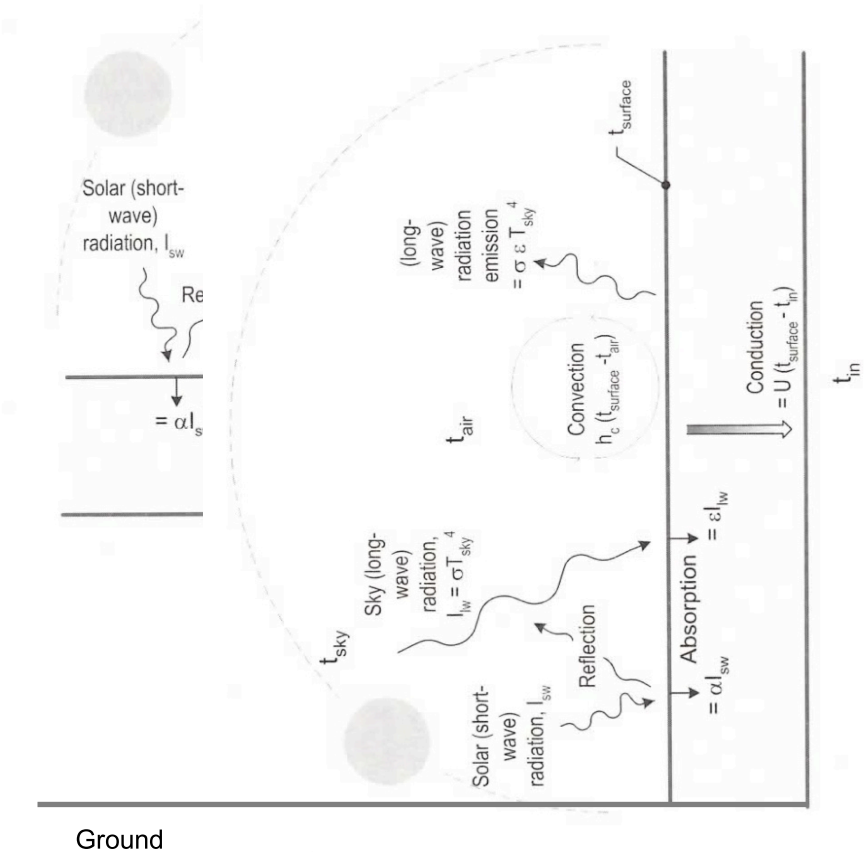
- Because of T^4 term, often requires iteration

Solar gain	αI_{solar}	$q_{sw,solar}$
Surface-sky radiation	$+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surface}^4)$	$+q_{lw,surface-sky}$
Convection on external wall	$+h_{conv}(T_{air} - T_{surface})$	$+q_{convection}$
Conduction through wall	$-U(T_{surface} - T_{surface,interior}) = 0$	$-q_{conduction} = 0$

Surface energy balance example

- Similarly, for a vertical surface:

$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$



$$\alpha I_{solar}$$

$$+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surface}^4)$$

$$+\varepsilon_{surface}\sigma F_{ground}(T_{ground}^4 - T_{surface}^4)$$

$$+h_{conv}(T_{air} - T_{surface})$$

$$-U(T_{surface} - T_{surface,interior}) = 0$$

A note on sign conventions

- Move from left to right (or top to bottom)
- Assume that the temperature to the left (or upstream) is higher than the temperature to the right (or downstream)
  - The signs will work themselves out and let you know if you assumed the wrong direction
  - Just be consistent!

Sky temperatures

Sky temperatures

- There are many ways to get "apparent sky temperatures"
  - Varying levels of detail and accuracy

- For a partly cloudy night sky: $T_{sky} = T_{air}\left[0.8 + \frac{(T_{dewpoint} - 273)}{250}\right]^{1/4}$
  - For 50% cloud cover

- For daytime: $T_{sky} = \left(\varepsilon_{sky} T_{air}^4\right)^{0.25}$

$$\varepsilon_{sky} = \left[0.787 + 0.764 \ln\left(\frac{T_{dewpoint}}{273}\right)\right]\left(1 + 0.0224N - 0.0035N^2 + 0.00028N^3\right)$$

  - For a clear sky: $N = 0$
  - For 50% cloud cover, $N = 0.5$

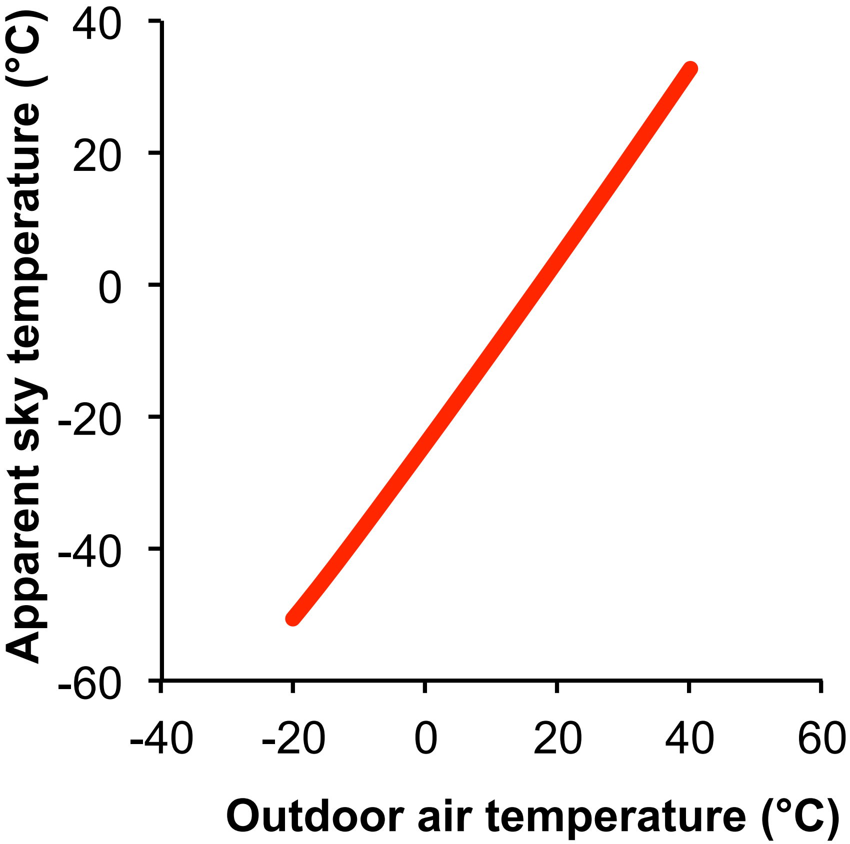
Where N = cloud cover (tenths)

Simpler sky temperatures

- Other models estimate apparent sky temperatures ignoring differences in water vapor:

$$T_{\mathrm{sky}} = 0.0552 T_{\mathrm{at}}^{1.5}$$

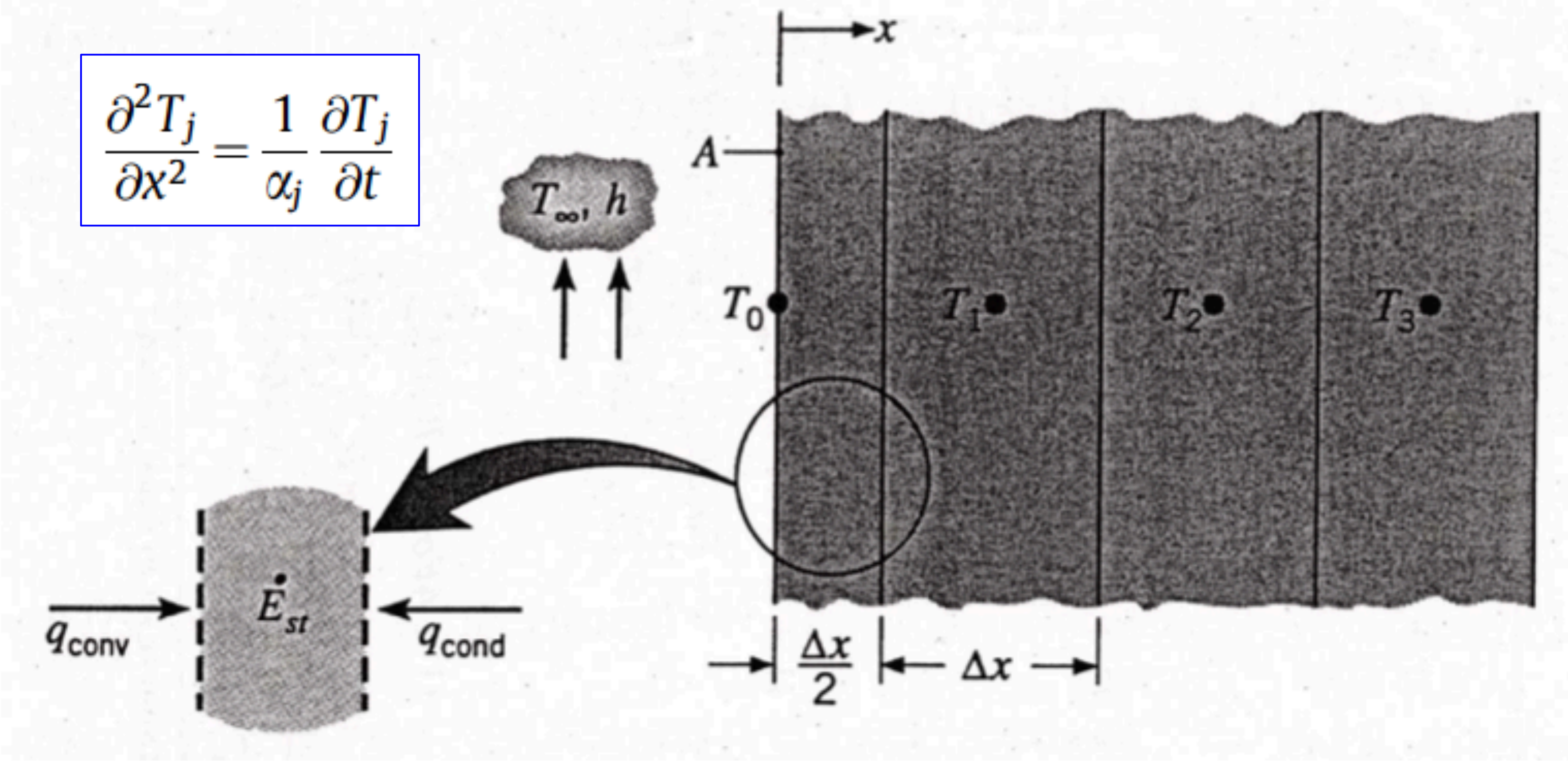
Where T_{sky} is in K and T_{at} is ambient air temperature $[K]$



Modeling thermal mass: Transient (unsteady) conduction

- If you need to model unsteady conduction through materials:
  - Recall transient heat conduction equation

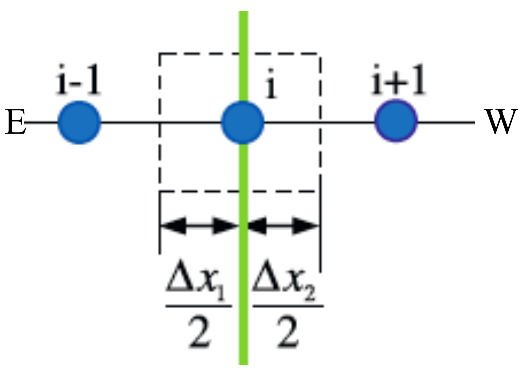
$$\frac{\partial^2 T_j}{\partial x^2} = \frac{1}{\alpha_j}\frac{\partial T_j}{\partial t}$$



Transient conduction: Example numerical approach

- Conduction finite difference solution (**implicit**)

$$C_p \rho \Delta x \frac{T_i^{j+1} - T_i^{j}}{\Delta t} = \frac{1}{2}\left[\left(k_W \frac{\left(T_{i+1}^{j+1} - T_i^{j+1}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j+1} - T_i^{j+1}\right)}{\Delta x}\right) + \left(k_W \frac{\left(T_{i+1}^{j} - T_i^{j}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j} - T_i^{j}\right)}{\Delta x}\right)\right] \quad (36)$$



Where:

T = node temperature

Subscripts:

i = node being modeled

i+1 = adjacent node to interior of construction

i-1 = adjacent node to exterior of construction

j+1 = new time step

j = previous time step

Δt = calculation time step

Δx = finite difference layer thickness (always less than construction layer thickness)

C_p = specific heat of material

k_W = thermal conductivity for interface between i node and i+1 node

k_E = thermal conductivity for interface between i node and i-1 node

ρ = density of material

Selecting grid size:

$$\left(Fo = \alpha \Delta t / \Delta x^2\right) < 0.5$$

Implicit = temperatures are evaluated at time j+1 as a function of temperatures at time j

Transient conduction: Lumped capacitance model

- Conduction and thermal mass together can also be modeled using a **lumped capacitance** approach in 1-dimension:

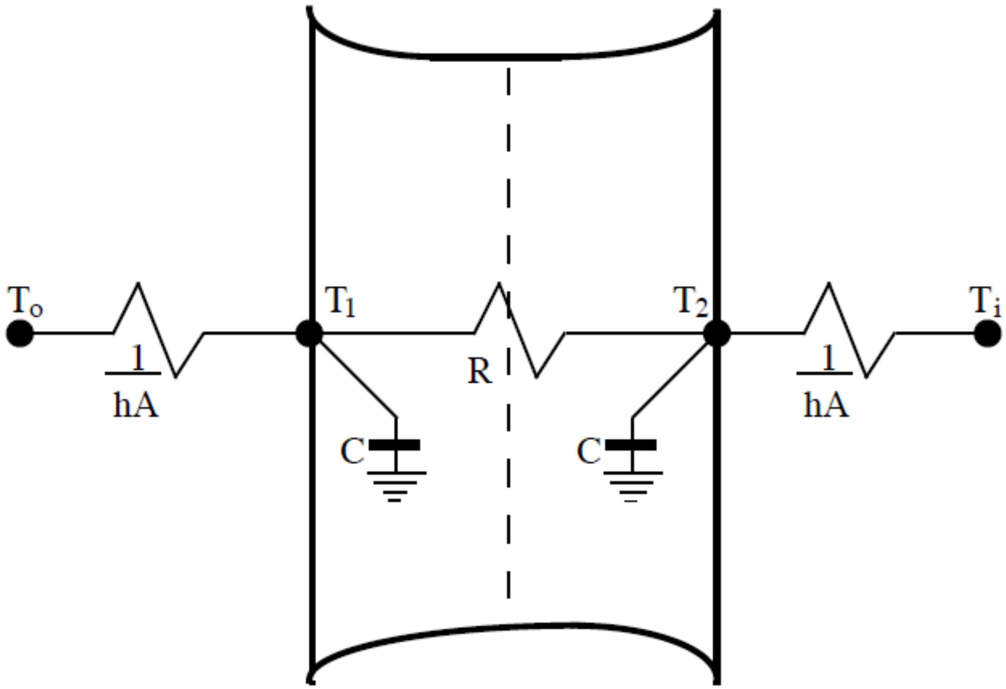


Figure 9. Two Node State Space Example.

$$C\frac{dT_1}{dt} = hA(T_o - T_1) + \frac{T_2 - T_1}{R}$$

$$C\frac{dT_2}{dt} = hA(T_i - T_2) + \frac{T_1 - T_2}{R}$$

where:

$$R = \frac{\ell}{kA},$$

$$C = \frac{\rho c_p \ell A}{2}$$

Transient conduction: Lumped capacitance model

- Wall example: Exterior surface balance at T_1 changes

*Easier, simpler

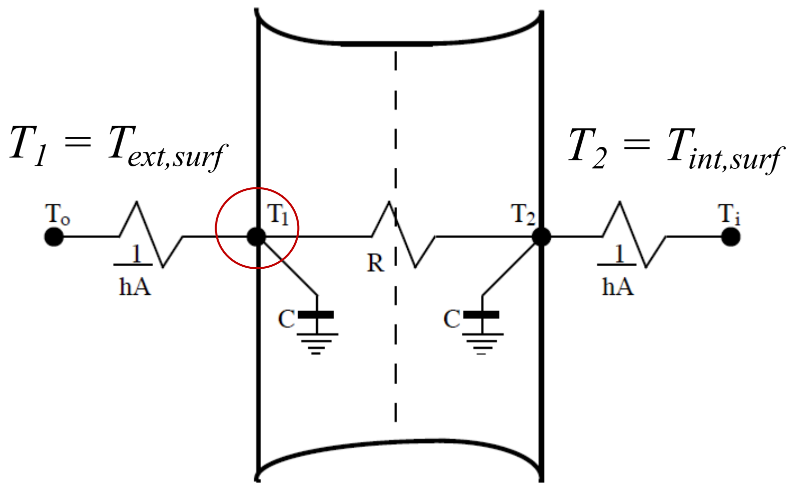


Figure 9. Two Node State Space Example.

From:

$$q_{sw,solar} + q_{lw,surface-sky} + q_{lw,surface-ground} + q_{convection} - q_{conduction} = 0$$

To:

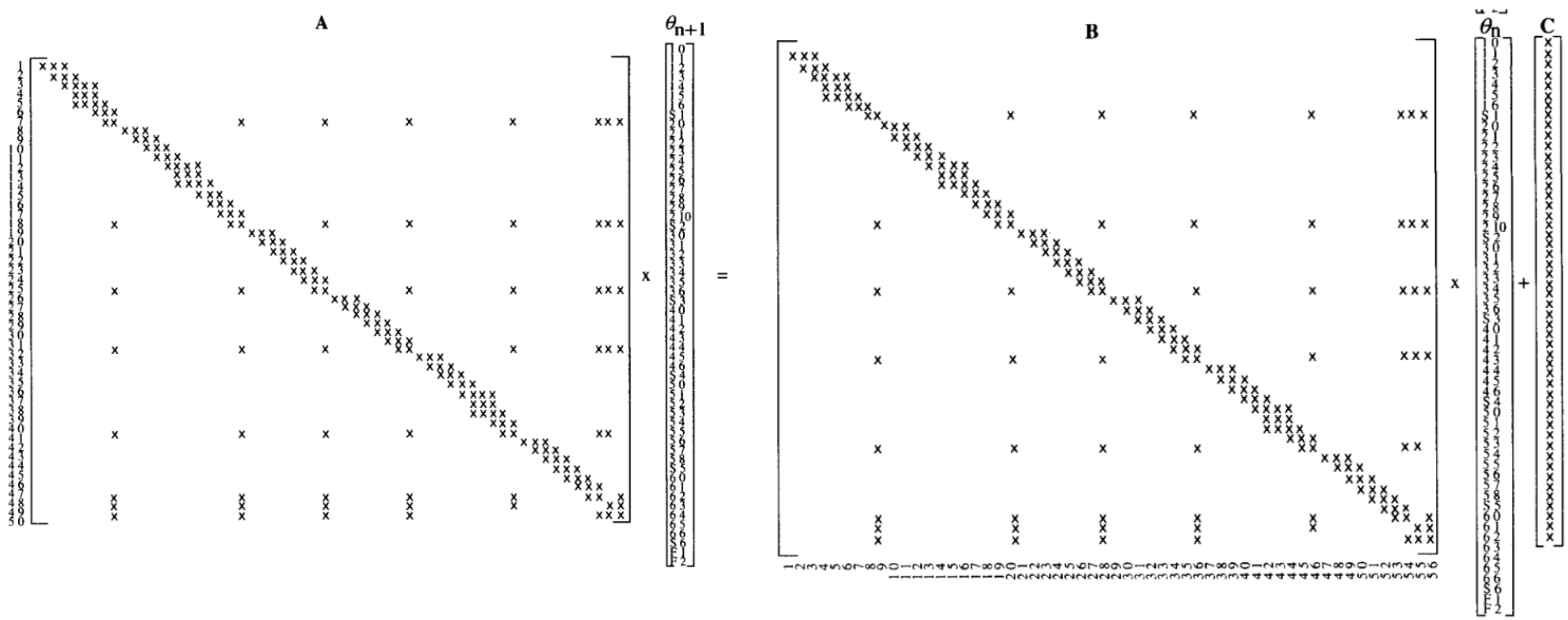
$$q_{sw,solar} + q_{lw,surface-sky} + q_{lw,surface-ground} + q_{convection} - q_{conduction} = \rho C_p \frac{L}{2} \frac{dT}{dt}$$

Solve the system of equations

In matrix notation, the system of equations can be expressed as

$$\mathbf{A}\theta_{n+1} = \mathbf{B}\theta_n + \mathbf{C} = \mathbf{Z} \tag{4.1}$$

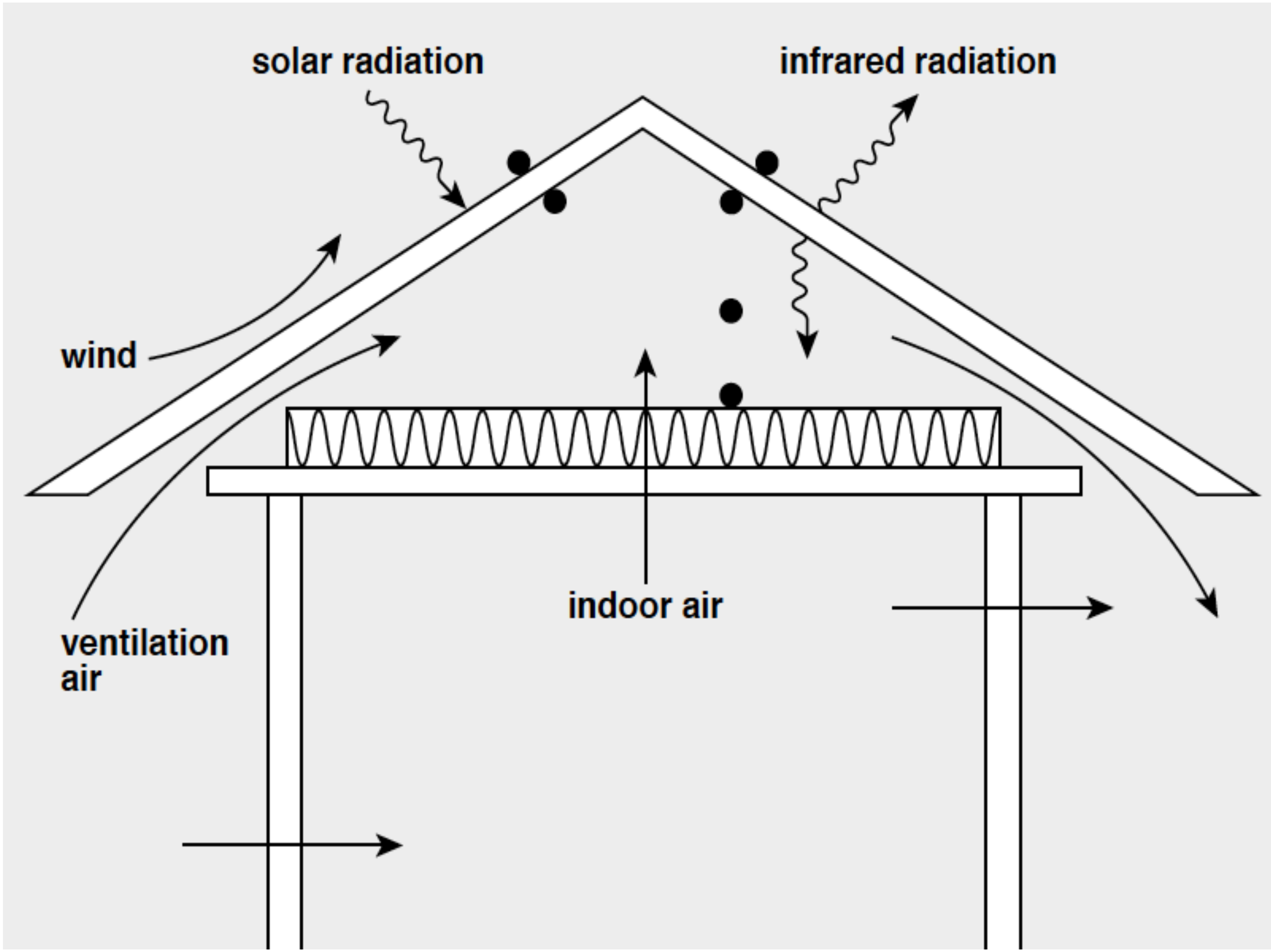
where $\mathbf{A}$ is a sparse matrix of future time-row coefficients of the nodal temperature or heat injection terms of the conservation equations, $\mathbf{B}$ the corresponding matrix established at the present time-row, $\mathbf{C}$ a column matrix of known boundary excitations relating to the present and future time-rows, θ a column matrix of nodal temperatures and heat injections, $n+1$ refers to the future time-row, n the present time-row, and $\mathbf{Z}$ is a column matrix. Initial conditions are given by $\theta(0) = \theta_0$.



Clarke 2001 *Energy Simulation in Building Design*

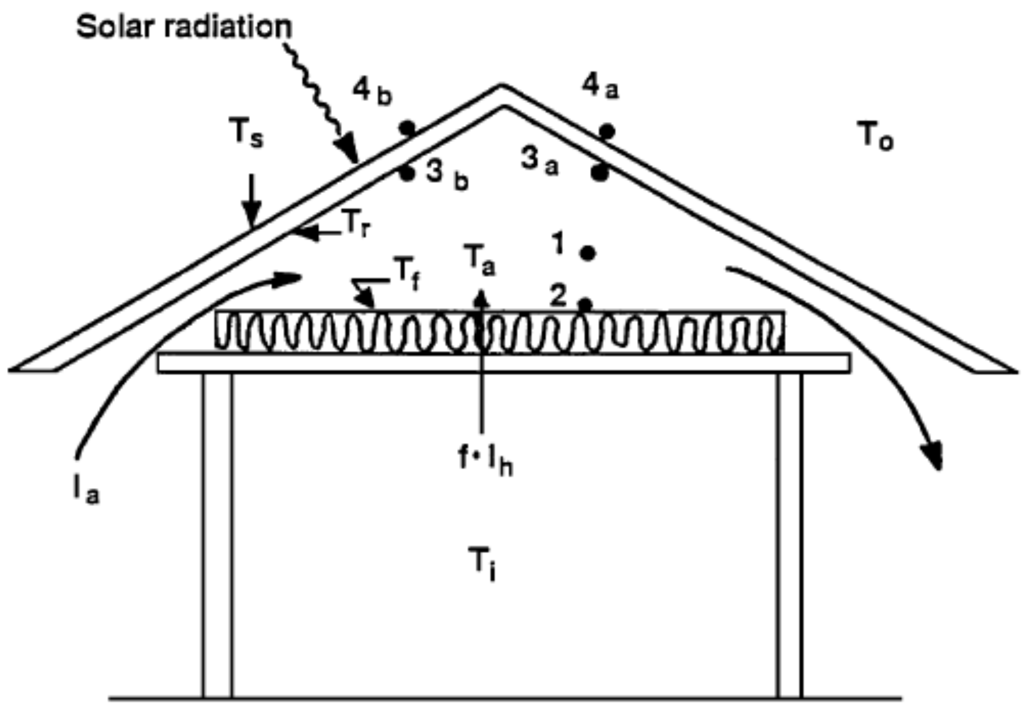
BUILDING ENERGY BALANCE EXAMPLE: ATTIC HEAT TRANSFER

Attic simulation example



TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer



● Nodes for heat balance

T_a, T_i, T_f, Temperatures of attic, indoor air, attic floor
T_o, T_r, T_s outside air, underside of roof, surface of roof

f Fraction of air entering attic

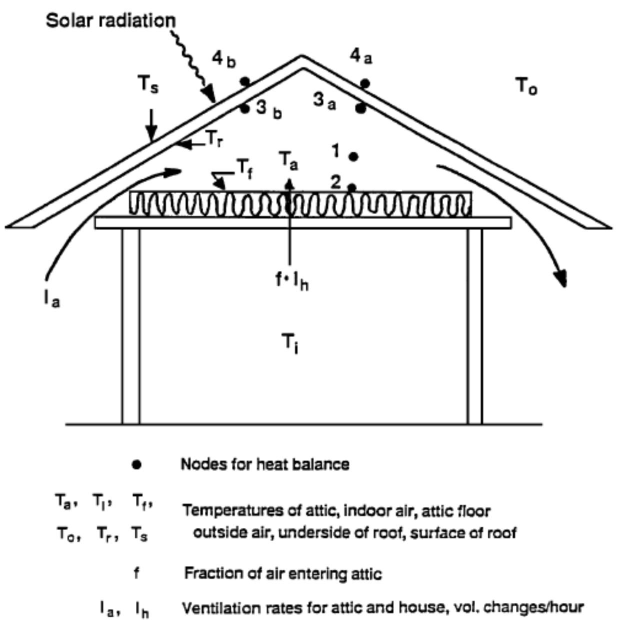
I_a, I_h Ventilation rates for attic and house, vol. changes/hour

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

Attic Air—The heat balance for the attic air (node 1) is given by

$$A_c h_f (T_f - T_a) + f I_h V_h \rho C_p (T_i - T_a)$$
$$= \frac{A_r}{2} h_{r,1}(T_a - T_{r,1}) + \frac{A_r}{2} h_{r,2}(T_a - T_{r,2}) + \frac{A_{es}}{R_{es}}(T_a - T_o)$$
$$+ I_a \rho C_p V_a (T_a - T_o)$$



where

A_c, A_r, A_{es} = surface areas of ceiling, total roof area, and combined area of soffit and end walls (ft^2)
C_p = specific heat of air (Btu/lb·°F)
f = fraction of house exfiltration that transfers into attic
h_f, $h_{r,n}$ = convective heat transfer coefficients at attic floor and underside of roof (surface n), respectively (Btu/h·ft^2·°F)
I_h = house exfiltration rate (h^{-1})
I_a = attic ventilation rate, i.e., outdoor air entering attic (h^{-1})
$R_{e,s}$ = average thermal resistance of end walls and eaves (h·ft^2·°F/Btu)
T_a = attic air temperature (°R)
T_f = temperature of attic floor surface (°R)
T_i = indoor air temperature (°R)
T_o = outdoor air temperature (°R)
$T_{r,1}$ = temperature of roof sheathing underside a (°R)
$T_{r,2}$ = temperature of roof sheathing underside b (°R)
V_a = volume of attic space (ft^3)
V_h = volume of house (ft^3)
ρ = density of air (lb/ft^3)

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

Attic Floor—The heat balance for the attic floor (node 2) is

$$\frac{T_i - T_f}{R_c} = h_f(T_f - T_a) + \frac{F_1}{2}(T_f - T_{r,1}) + \frac{F_2}{2}(T_f - T_{r,2}) \qquad (3)$$

where

R_c = thermal resistance of attic floor (ceiling) ($h \cdot ft^2 \cdot °F/Btu$)

F_1, F_2 = radiative heat transfer coefficients between attic floor and undersides of roof ($Btu/h \cdot ft^2 \cdot °F$), with the following definition:

$$F_n = \frac{\sigma}{(1/\varepsilon_f) + (1/\varepsilon_r) - 1}(T_f^2 + T_{r,n}^2)(T_f + T_{r,n}) \quad n = 1,2$$

where

ε_f = emissivity of attic floor surface (assumed to be 0.9)

ε_r = emissivity of roof sheathing surface (underside, assumed to be 0.9)

σ = Stefan–Boltzman constant ($Btu/h \cdot ft^2 \cdot °R^4$)

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

Sheathing—The heat balance at the underside of sheathing (nodes 3a and 3b) is

$$\frac{T_{\mathrm{r},n} - T_{\mathrm{s},n}}{R_{\mathrm{r}}} = h_{\mathrm{r}}(T_{\mathrm{a}} - T_{\mathrm{r},n}) + \frac{A_{\mathrm{c}}}{A_{\mathrm{r}}} F_n (T_{\mathrm{f}} - T_{\mathrm{r},n}) + \frac{2}{A_{\mathrm{r}}} L_{\mathrm{h}} W_{\mathrm{r},n} \quad n = 1,2 \qquad (5)$$

where

L_{h} = latent heat of vaporization (1,050 Btu/lb)
R_{r} = thermal resistance of roof (h·ft^2·°F/Btu)
$T_{\mathrm{s},n}$ = temperature of outside roof surface (°R)
$W_{\mathrm{r},n}$ = rate of moisture adsorption into sheathing (lb/h)

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

The heat balance at the top surface of sheathing (nodes 4a and 4b) is

$$\frac{T_{s,n} - T_{r,n}}{R_r} = \alpha I_n + (h_{o,n} + h_{IR})(T_o - T_{s,n}) - L_{IR} \qquad n = 1,2 \qquad (6)$$

where

h_{IR}, L_{IR} = adjustments for infrared radiation exchange with sky (see Eqs. (7), (8), and (9))

$h_{o,n}$ = convective heat transfer coefficient at exterior roof surface n (Btu/h·ft^2·°F)

I_n = total solar radiation incident on roof surface (Btu/h·ft^2)

α = solar absorptance

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

$$h_{IR} = 4\varepsilon_s \sigma T_o^3 \quad (7)$$

$$L_{IR} = \varepsilon_s \sigma T_o^4 (1 - \varepsilon_{IR}) \quad (8)$$

where

ε_s = emissivity of roof shingles (assumed to be 0.9)

ε_{IR} = sky emissivity with clouds

The sky emissivity is calculated with equations from Martin and Berdahl (1984). The emissivity for a clear sky is

$$\varepsilon_0 = 0.711 + 0.56\frac{T_d}{100} + 0.73\left(\frac{T_d}{100}\right)^2 + 0.013\cos\left(\frac{2\pi t}{24}\right) \quad (9)$$

where

ε_0 = emissivity of clear sky

T_d = outdoor dew point temperature (°C)

t = time (h)

and the emissivity of the sky with clouds is

$$\varepsilon_{IR} = \varepsilon_0 + 0.784C(1 - \varepsilon_0) \quad (10)$$

where C = total cloud cover as recorded by National Climatic Center (Asheville, NC). Values of C range from 0 to 10.

TenWolde, Anton. 1997. FPL roof temperature and moisture model: Description and verification. Res. Pap. FPL–RP–561. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory. 48 p.

Attic heat transfer

Sheathing surface temperature: Measured vs. modeled

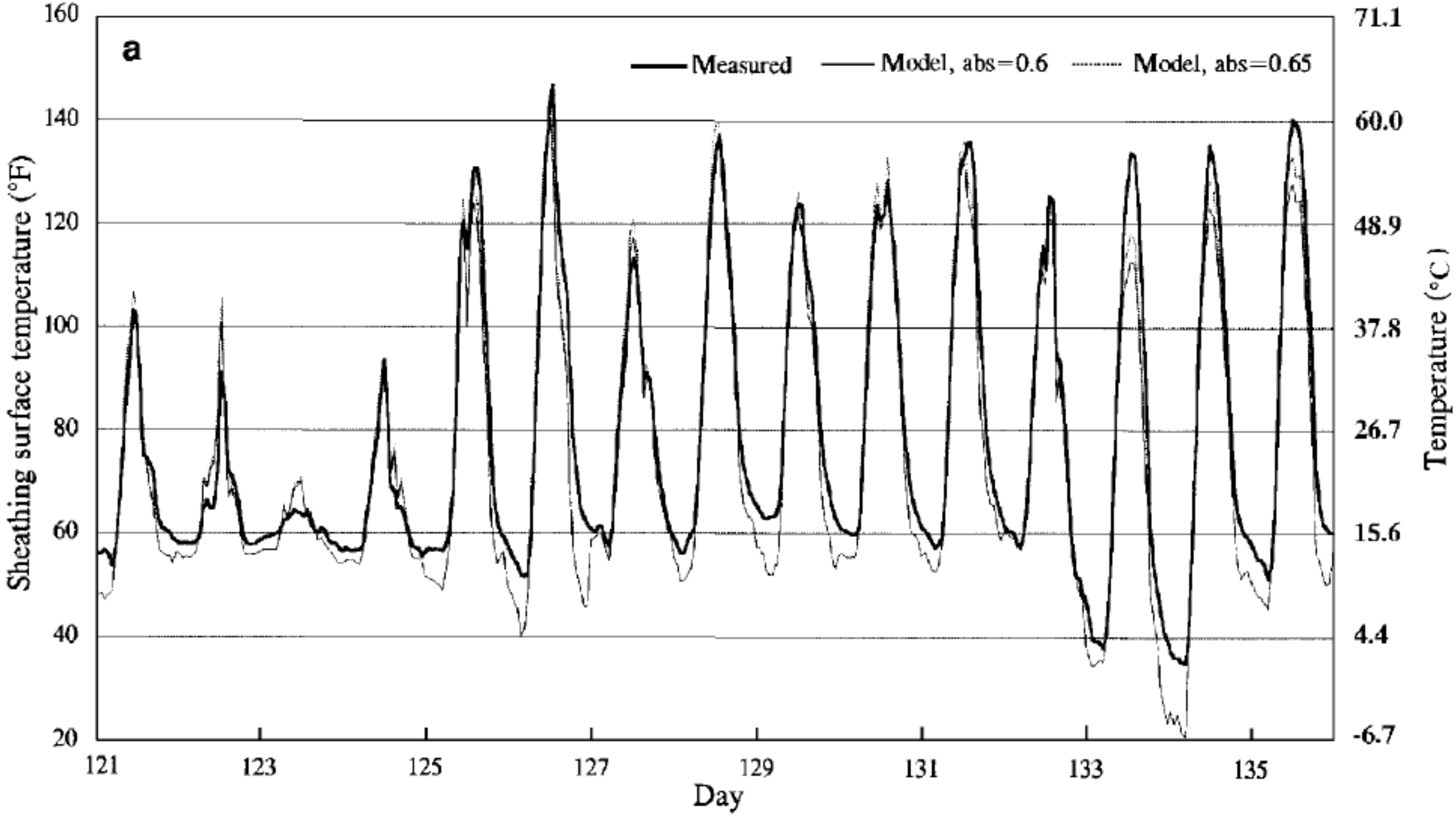


Figure 4—Exterior sheathing surface temperature, bay 2, May 1–15, 1993.

Attic heat transfer

Attic air temperature: Measured vs. modeled

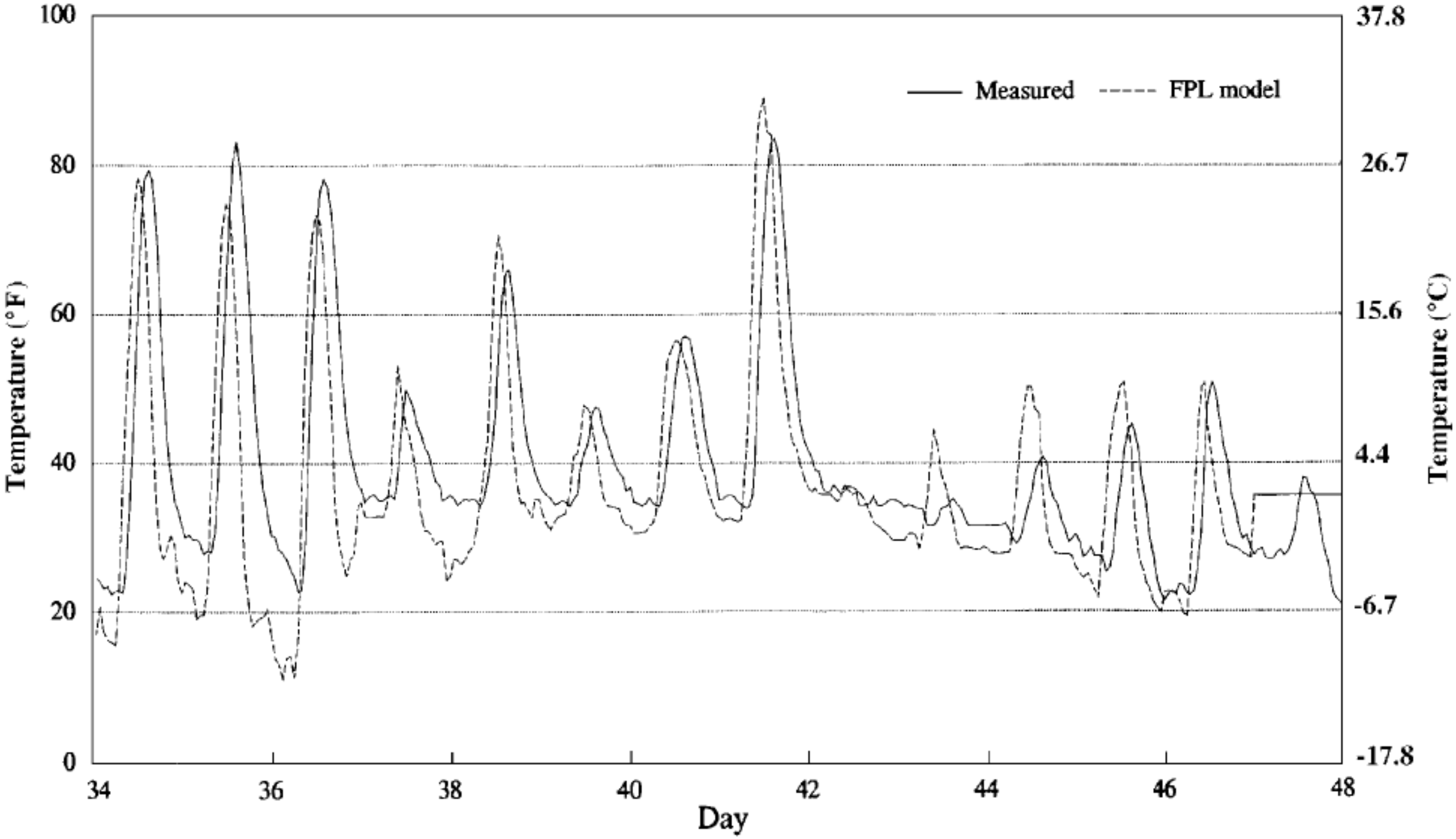
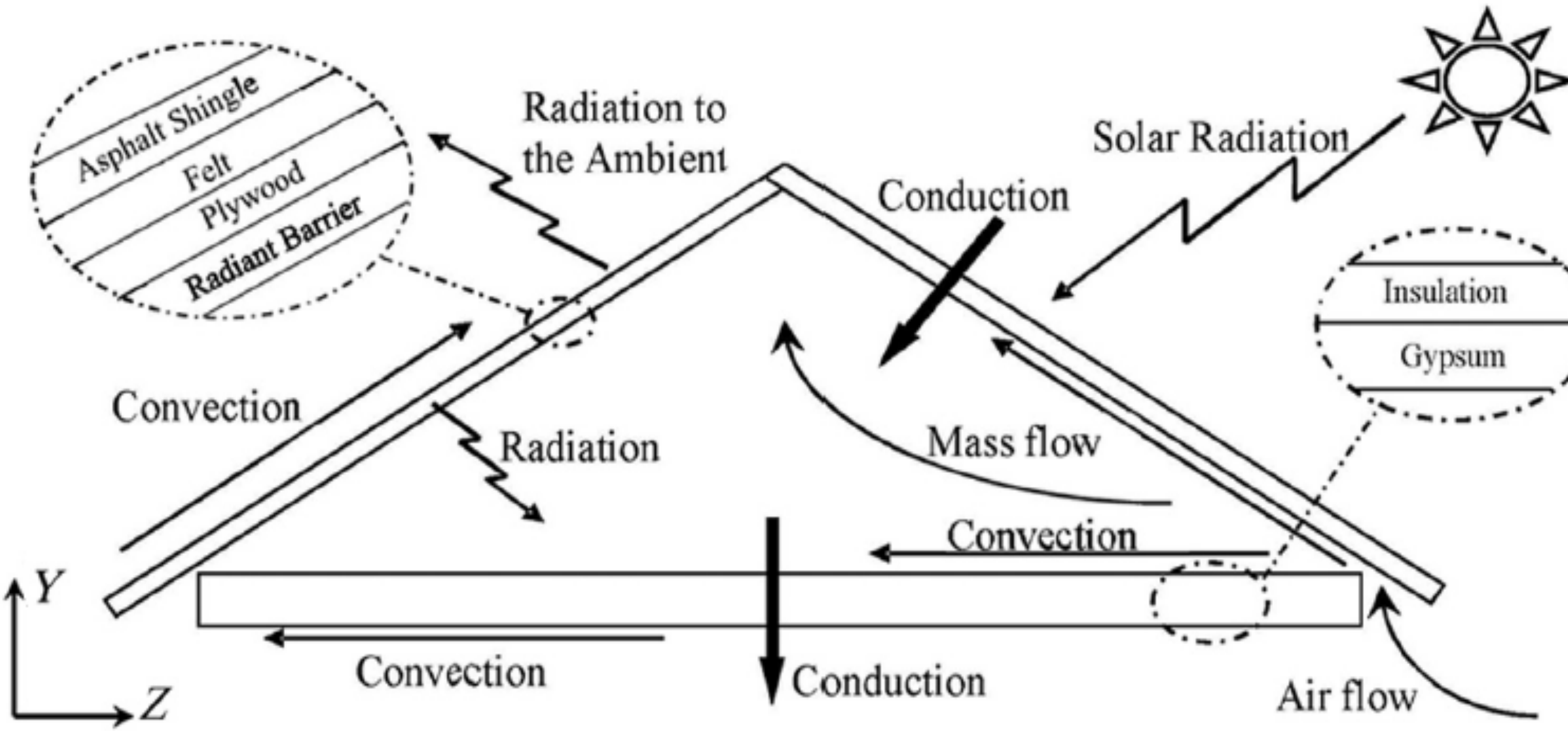


Figure 5—Attic air temperature, bay 2, February 3–16, 1993; absorptance = 0.65.

Attic heat transfer: Radiant barriers

- Radiant barriers typically have LW emissivity less than ~0.1
  - Approximately 90% of materials have emissivity of 0.9
- Inhibits heat flow through radiation only
  - Doesn't directly impact convection or conduction

Modeling the impacts of radiant barriers



Asadi et al., 2012. Performance evaluation of an attic radiant barrier system using three dimensional transient finite element method. *Journal of Building Physics*.

Modeling the impacts of radiant barriers

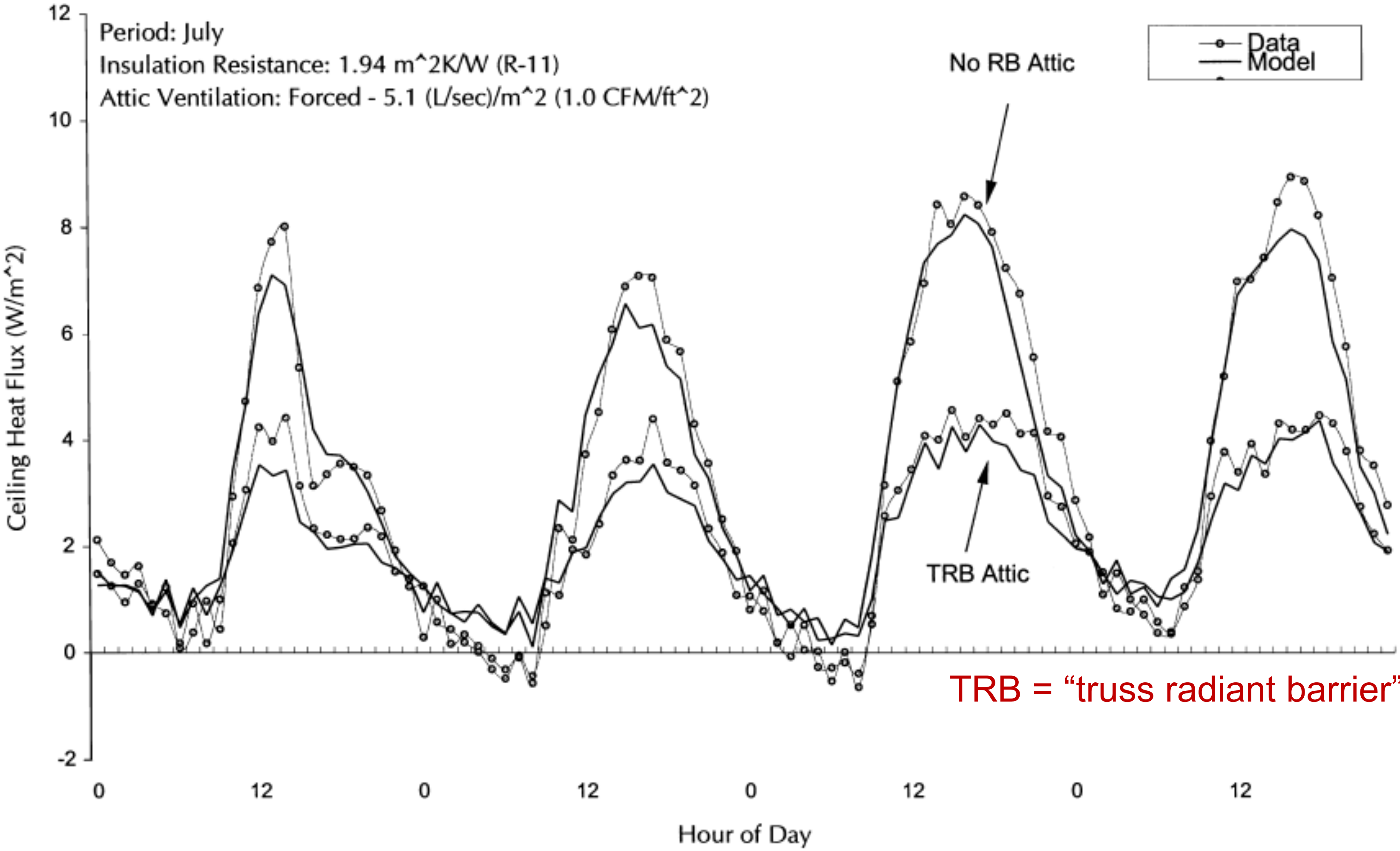


Fig. 6. Ceiling heat fluxes (TRB case, insulation resistance: 1.94 m^2 K/W, R-11; with attic airflow rate: 5.1 l/s/m^2, 1.0 CFM/ft^2).

Medina, M. 2000. On the performance of radiant barriers in combination with different attic insulation levels. *Energy and Buildings* 33:31-40.

Modeling the impacts of radiant barriers

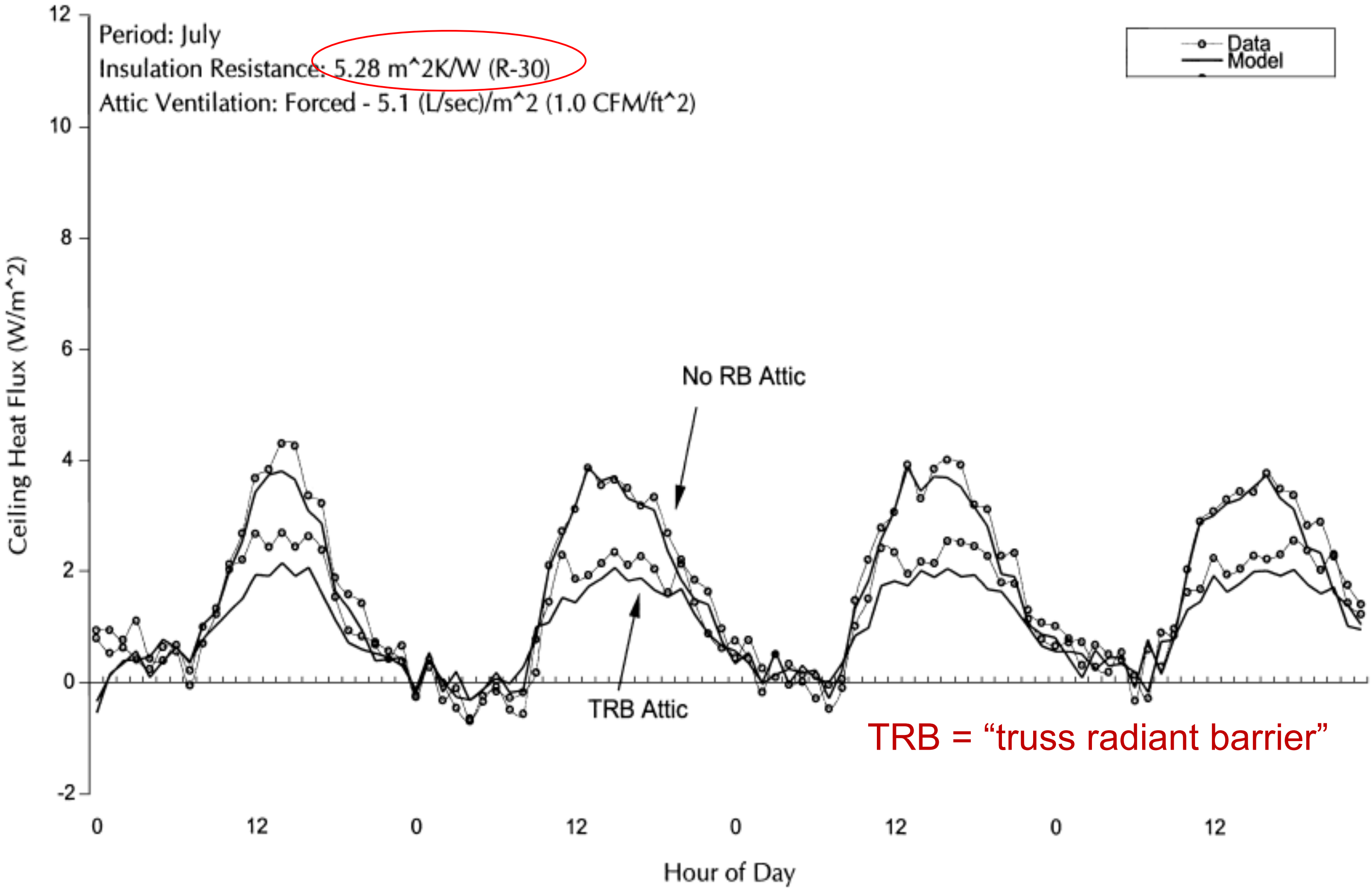
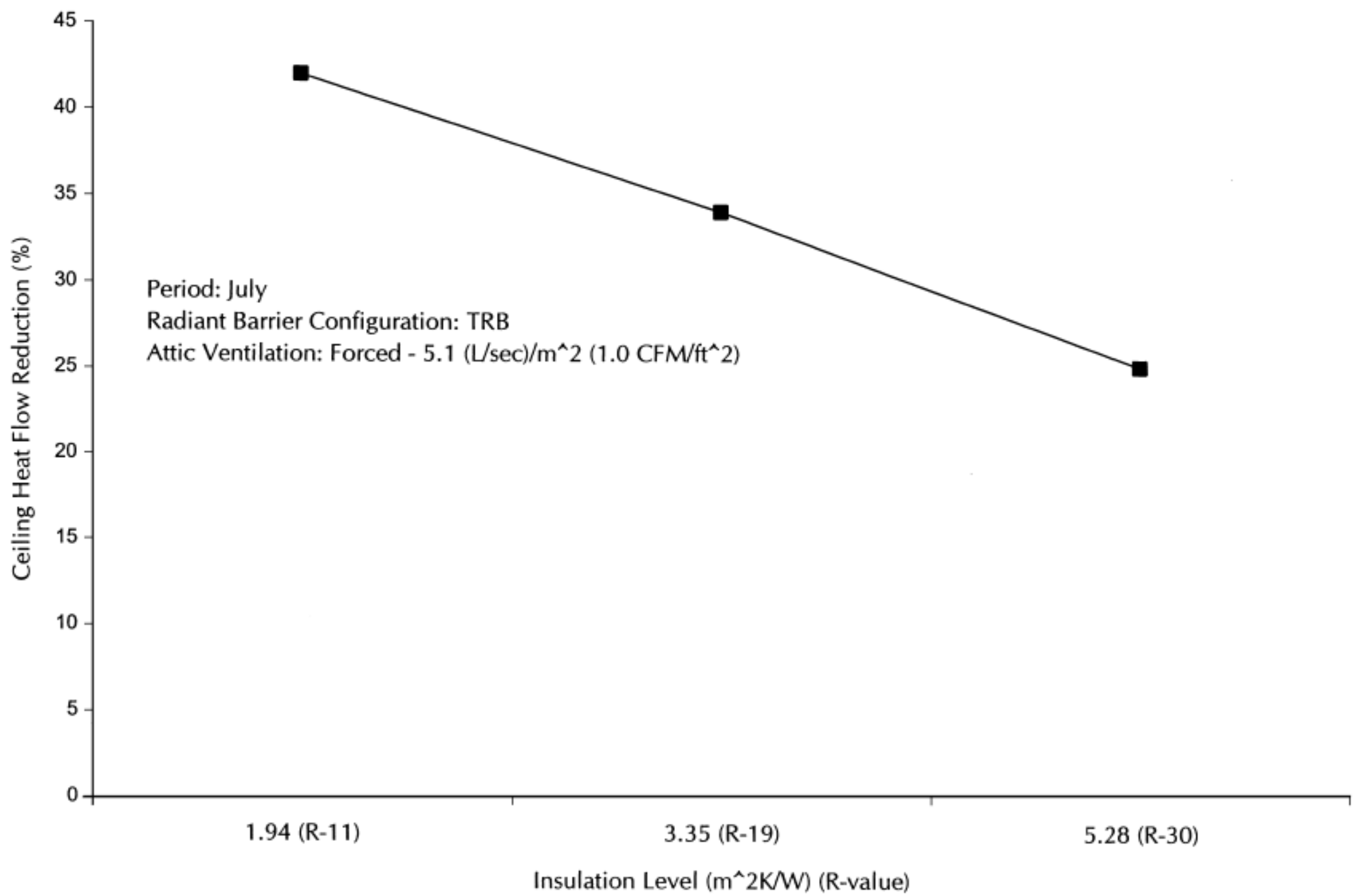


Fig. 7. Ceiling heat fluxes (TRB case, insulation resistance: 5.28 m^2 K/W, *R*-30; with attic airflow rate: 5.1 l/s/m^2, 1.0 CFM/ft^2).

Medina, M. 2000. On the performance of radiant barriers in combination with different attic insulation levels. *Energy and Buildings* 33:31-40.

Modeling the impacts of radiant barriers

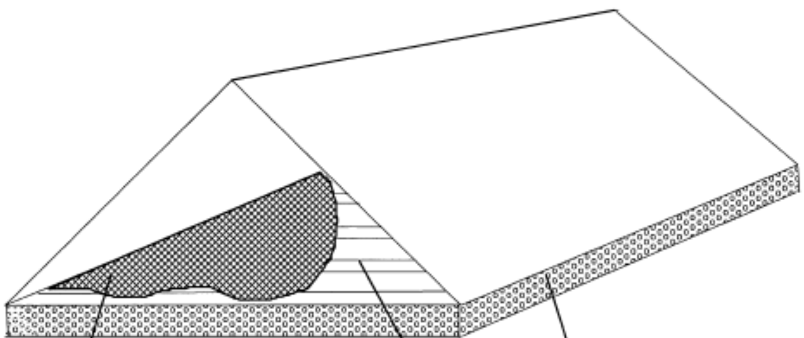
- Radiant barriers have less of an impact on well-insulated roofs

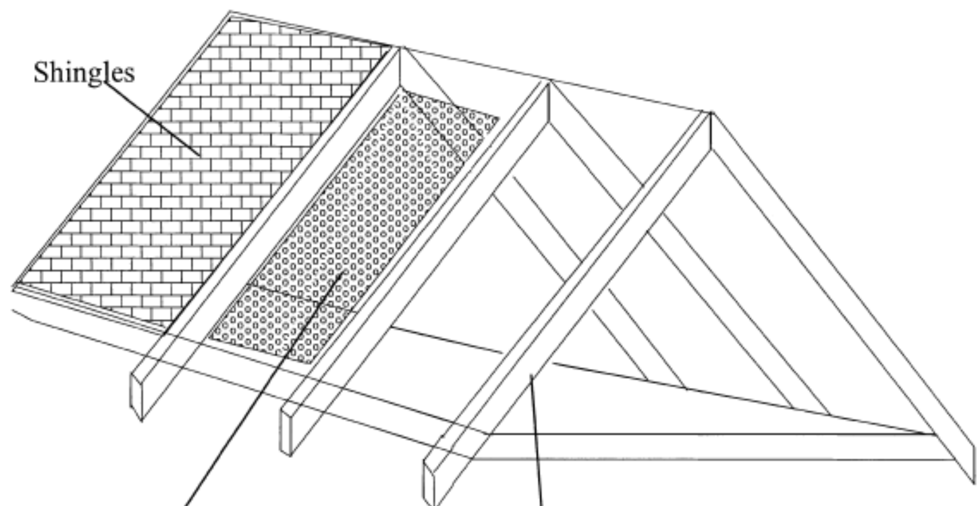


Medina, M. 2000. On the performance of radiant barriers in combination with different attic insulation levels. *Energy and Buildings* 33:31-40.

Modeling the impacts of radiant barriers

- Should we install radiant barriers on attic floors or underside of roof sheathing?

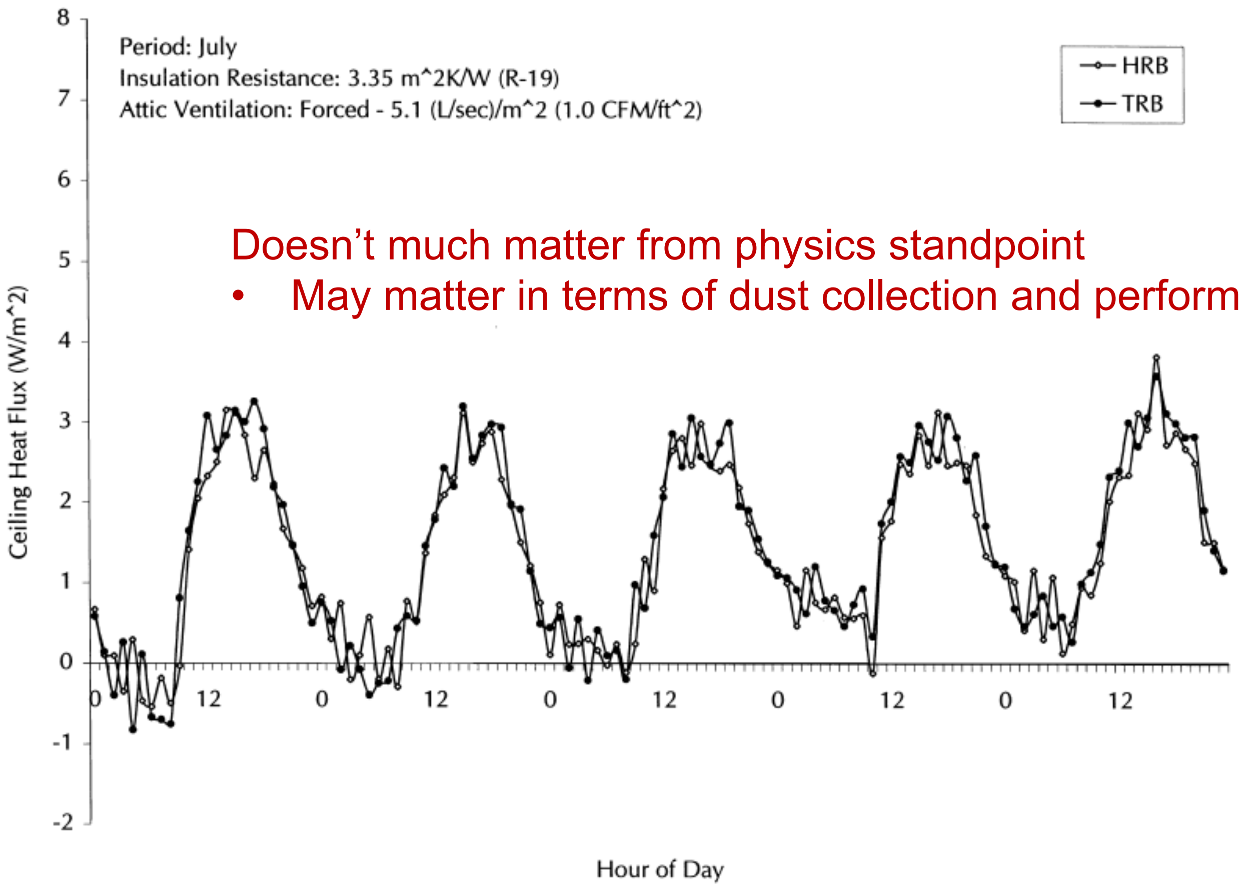




Medina, M. 2000. On the performance of radiant barriers in combination with different attic insulation levels. *Energy and Buildings* 33:31-40.

Modeling the impacts of radiant barriers

- Should we install radiant barriers on attic floors or underside of roof sheathing?



Doesn't much matter from physics standpoint

- May matter in terms of dust collection and performance over time

Fig. 9. Comparison between HRB and TRB configurations — experimental (insulation resistance: 3.35 m^2 K/W, *R*-19; with airflow rate: 5.1 l/s/m^2, 1.0 CFM/ft^2).

Medina, M. 2000. On the performance of radiant barriers in combination with different attic insulation levels. *Energy and Buildings* 33:31-40.

BUILDING ENERGY BALANCES: HW 6

HW 6 Part 1 – Little boxes

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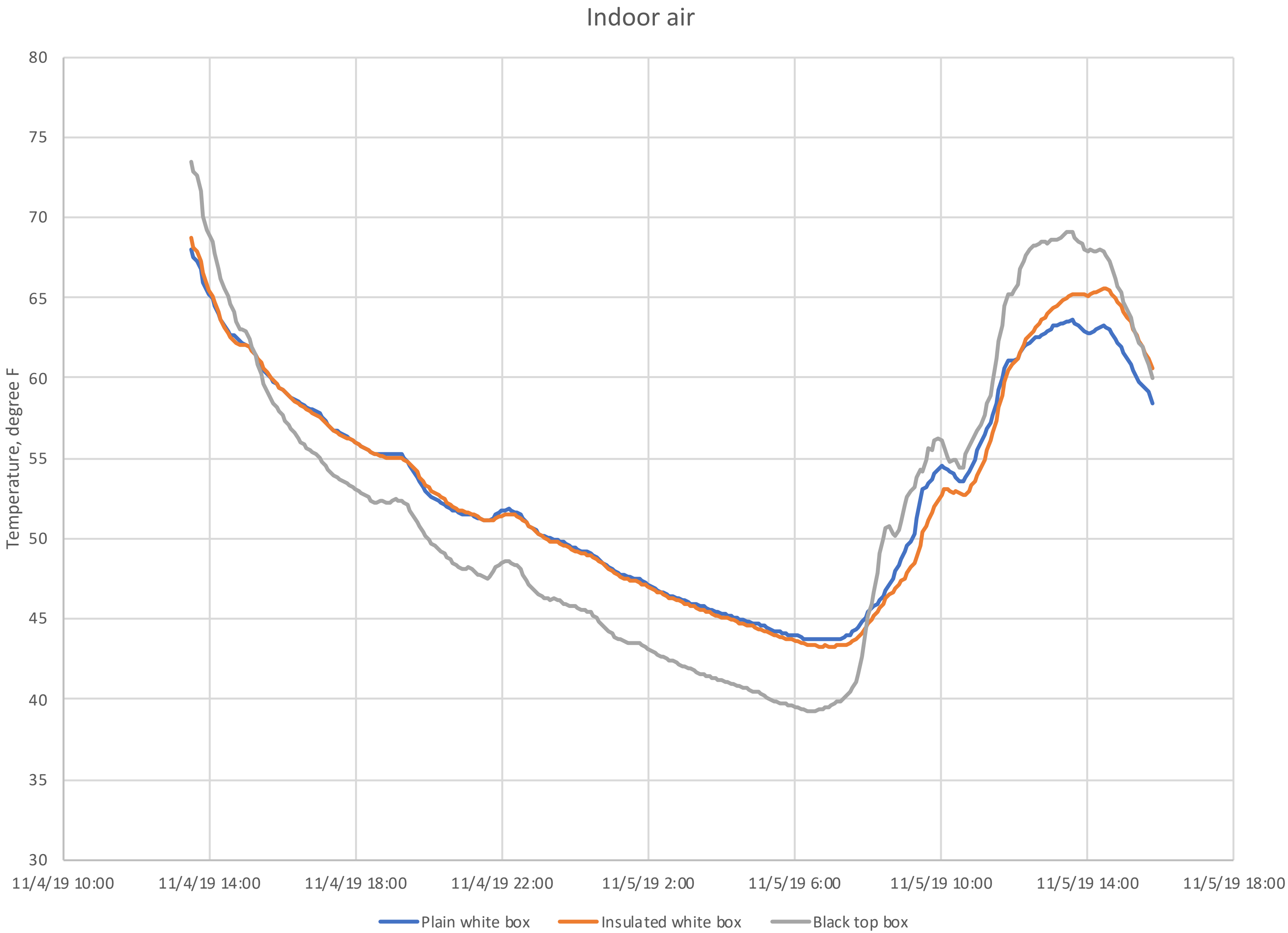
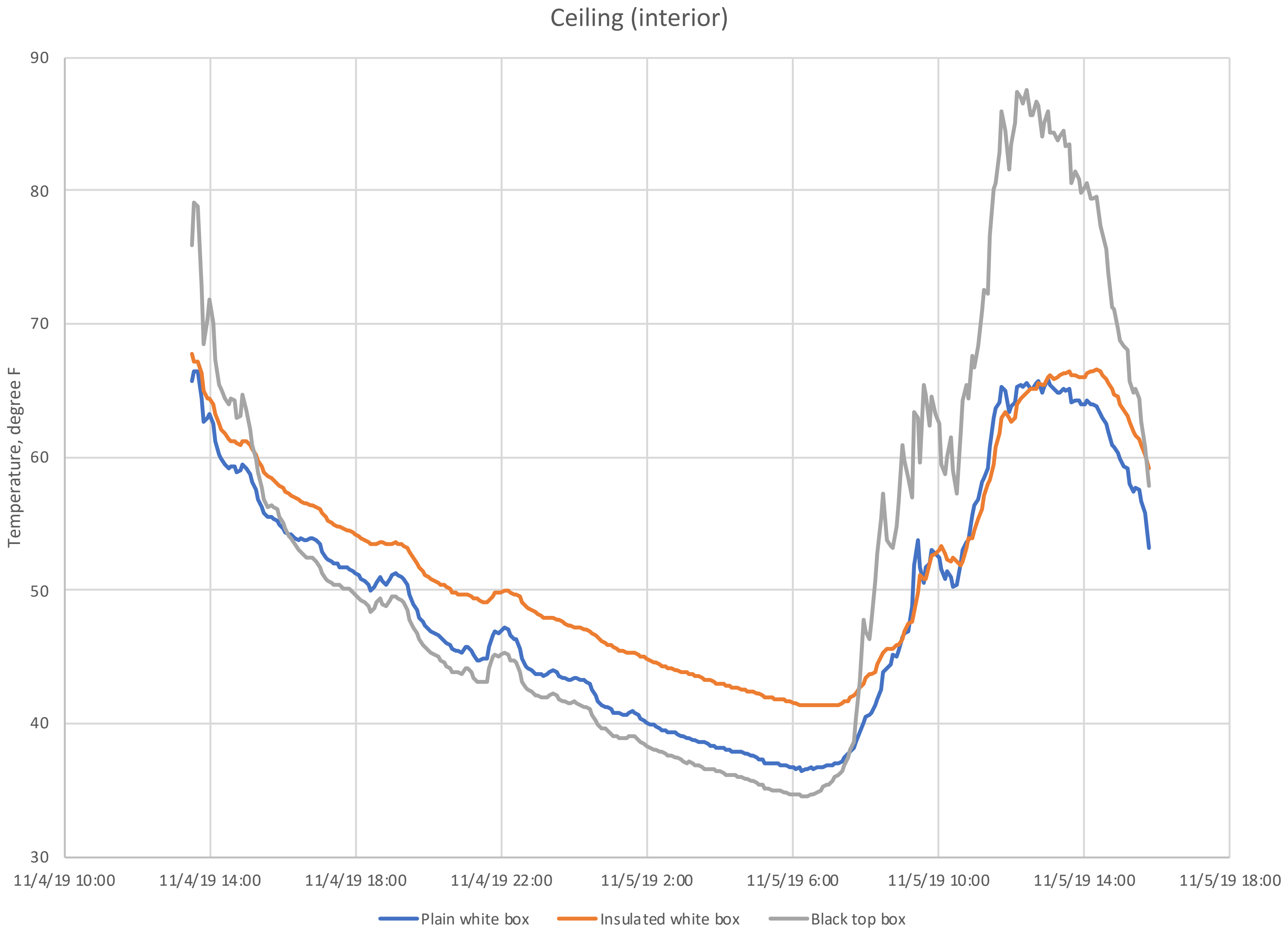
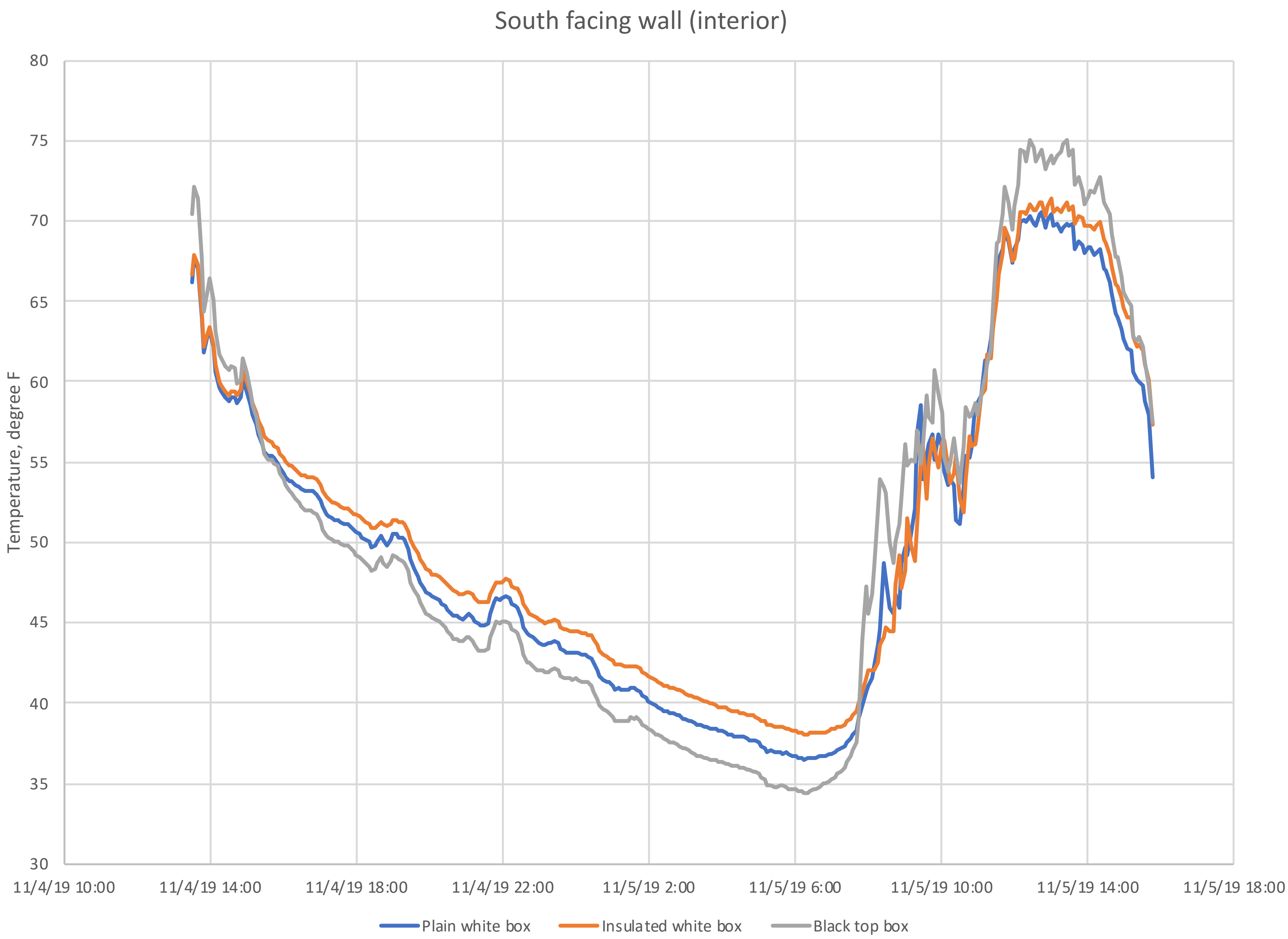


HW 6 Part 1 – Little boxes



HW 6 Part 1 – Little boxes



HW 6 Part 1 – Little boxes

November 5, 2019 https://www.wunderground.com/dashboard/pws/KILCHICA692/graph/2019-11-5/2019-11-5/daily