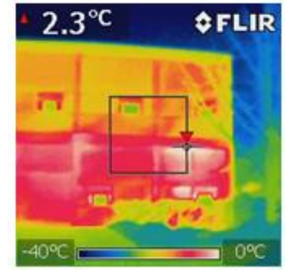
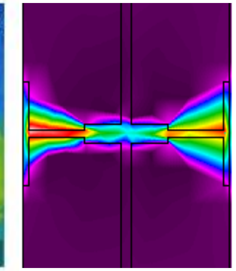
CAE 331/513
Building Science
Fall 2019





October 1, 2019
Psychrometrics (equations)

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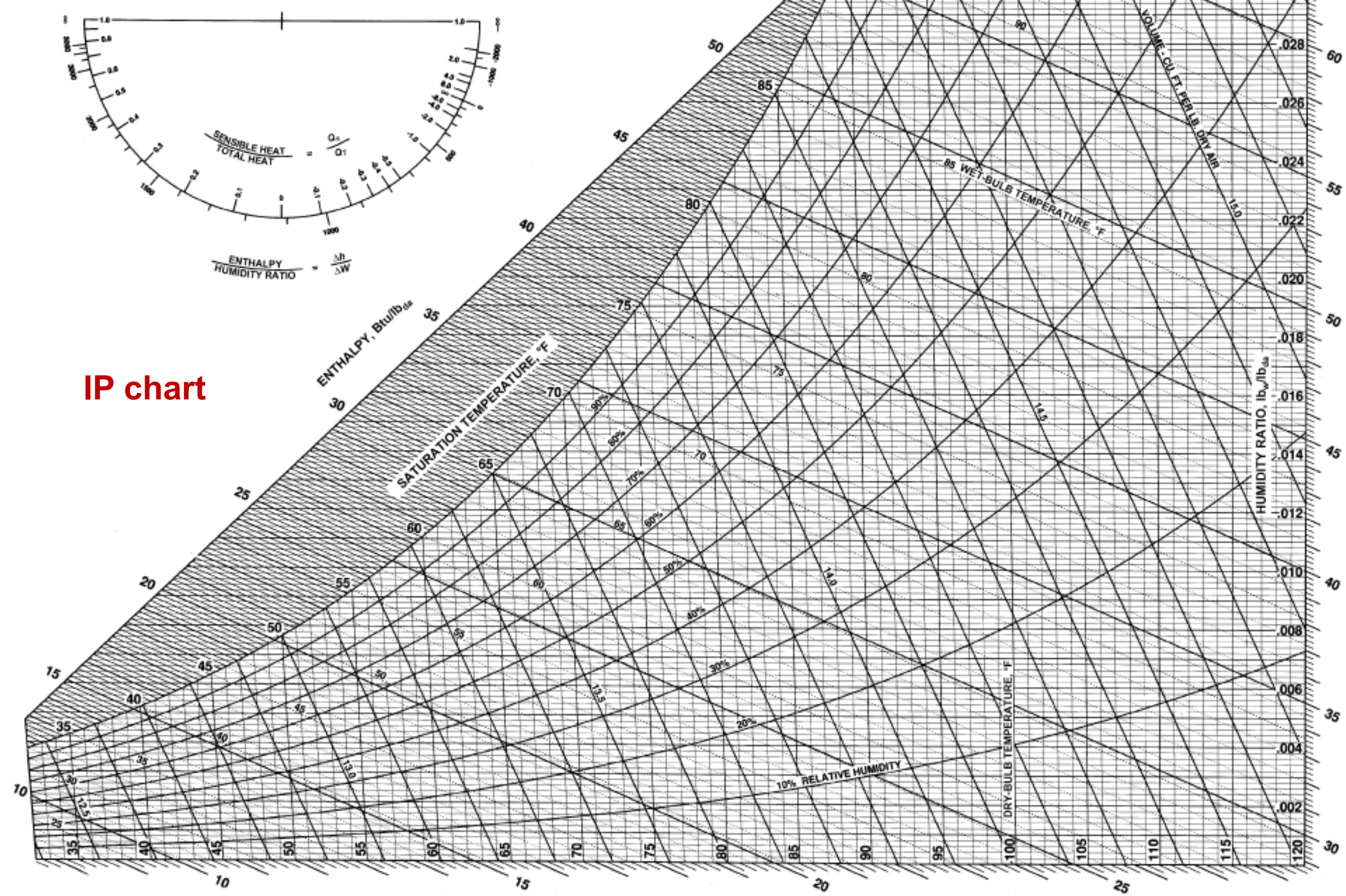
Last time

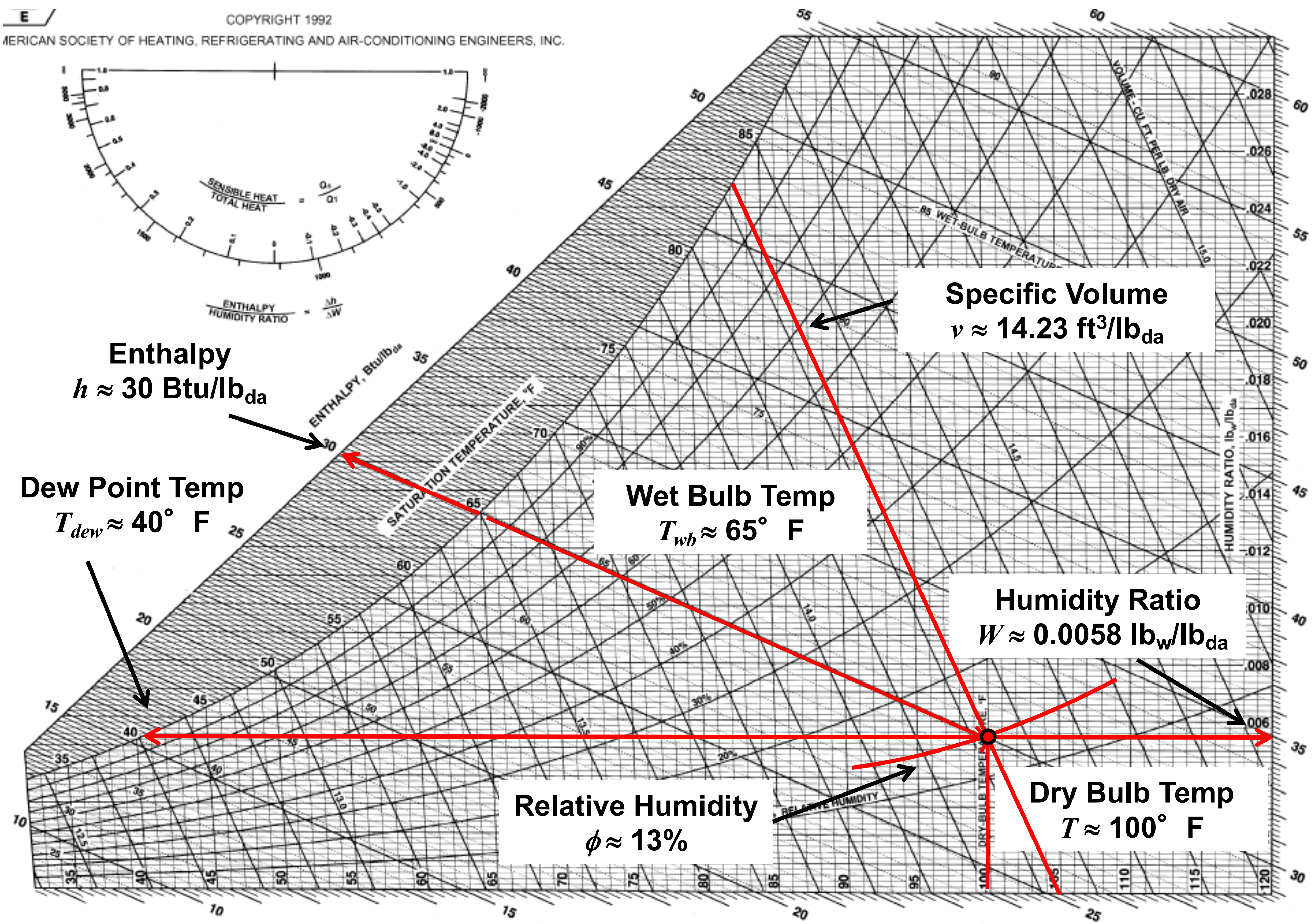
Introduced Psychrometrics and several key terms:

1. Dry bulb temperature
2. Vapor pressure
3. Saturation
4. Relative humidity
5. Absolute humidity (or humidity ratio)
6. Dew point temperature
7. Wet bulb temperature
8. Enthalpy
9. Density
10. Specific volume

E

IP chart


SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY, Btu/lbda
SATURATION TEMPERATURE, °F
85 WET-BULB TEMPERATURE, °F
VOLUME - CU. FT. PER LB. DRY AIR
HUMIDITY RATIO, lbw/lbda
DRY-BULB TEMPERATURE, °F
10% RELATIVE HUMIDITY



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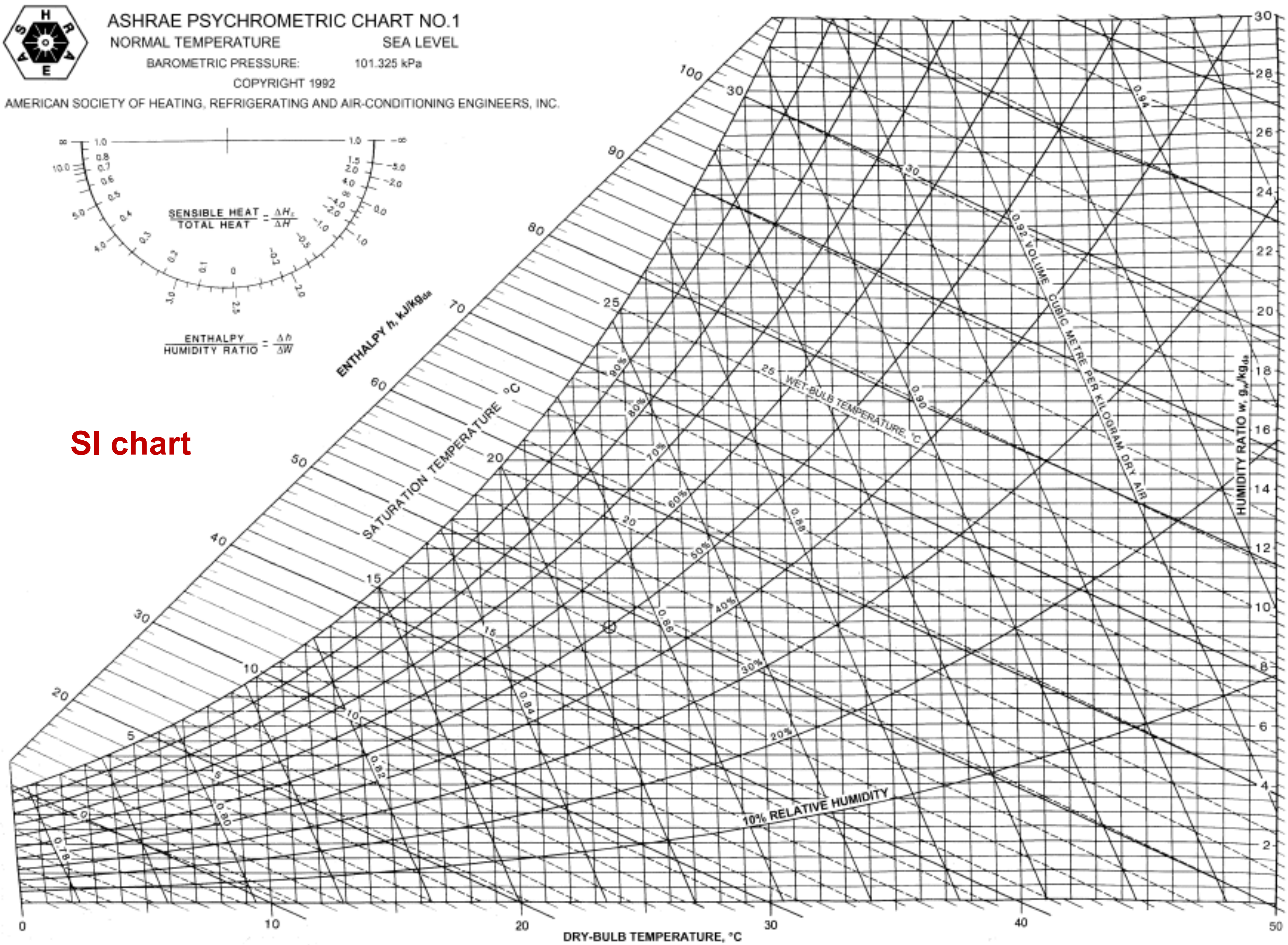
AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.
SENSIBLE HEAT / TOTAL HEAT = Q_s / Q_t
ENTHALPY / HUMIDITY RATIO = $\Delta h / \Delta W$
ENTHALPY, Btu/lb da
SATURATION TEMPERATURE, °F
VOLUME, CU FT PER LB DRY AIR
WET-BULB TEMPERATURE
HUMIDITY RATIO, lbw/lbda
RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F
Specific Volume
$v \approx 14.23\ ft^3/lb_{da}$
Enthalpy
$h \approx 30\ Btu/lb_{da}$
Dew Point Temp
$T_{dew} \approx 40°\ F$
Wet Bulb Temp
$T_{wb} \approx 65°\ F$
Humidity Ratio
$W \approx 0.0058\ lb_w/lb_{da}$
Relative Humidity
$\phi \approx 13\%$
Dry Bulb Temp
$T \approx 100°\ F$


ASHRAE PSYCHROMETRIC CHART NO.1
NORMAL TEMPERATURE
SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
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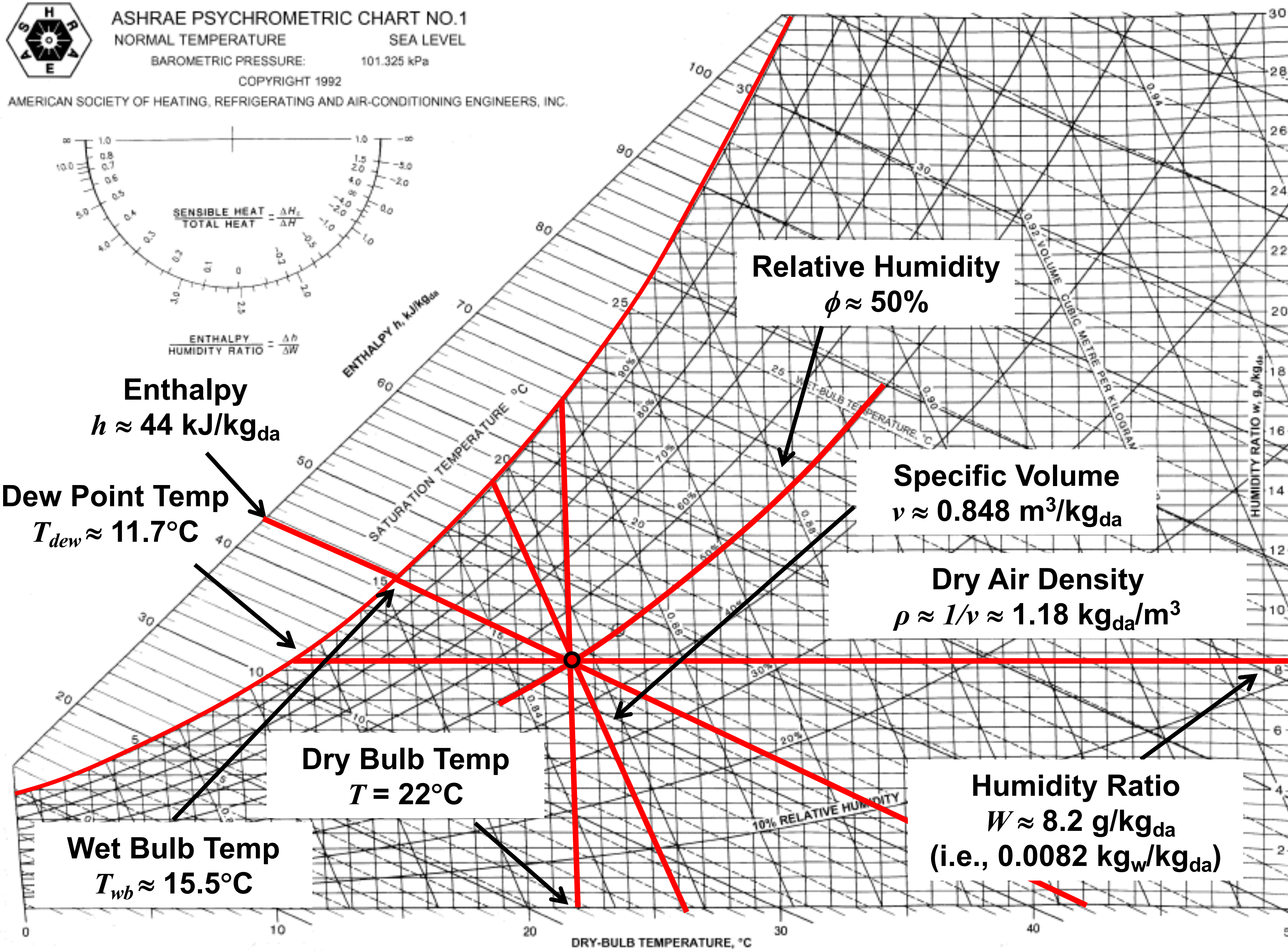
SENSIBLE HEAT / TOTAL HEAT = ΔHs / ΔH
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
SI chart
ENTHALPY h, kJ/kg_da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM DRY AIR
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, g_w/kg_da
DRY-BULB TEMPERATURE, °C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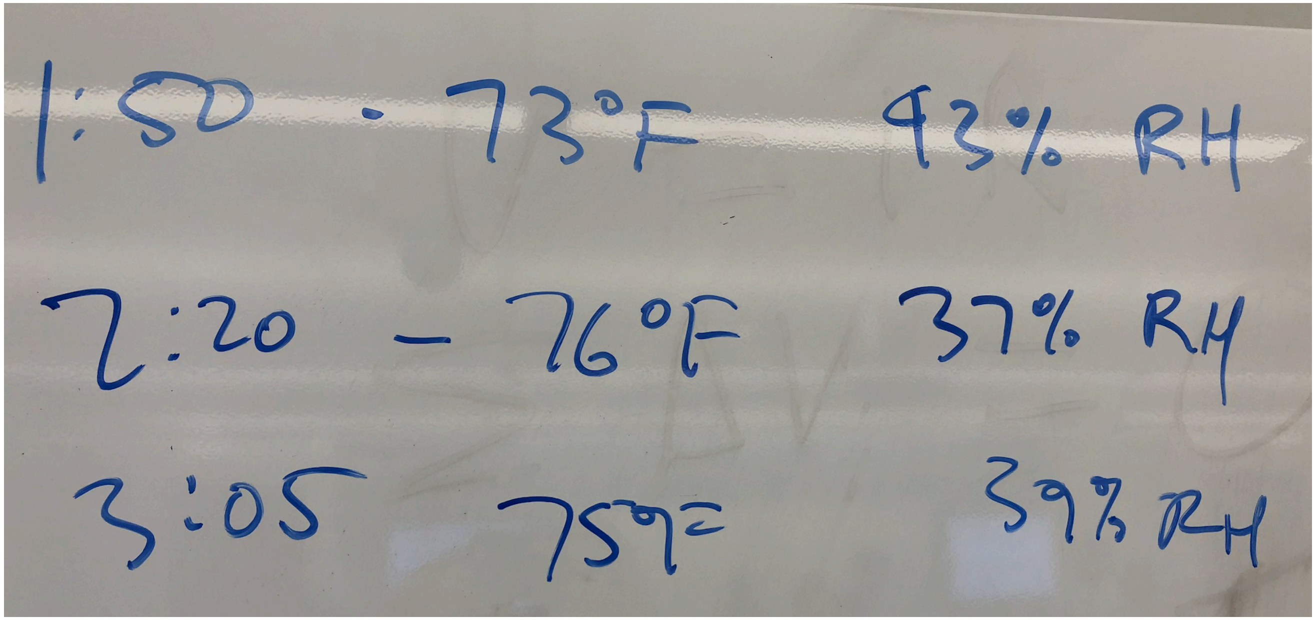
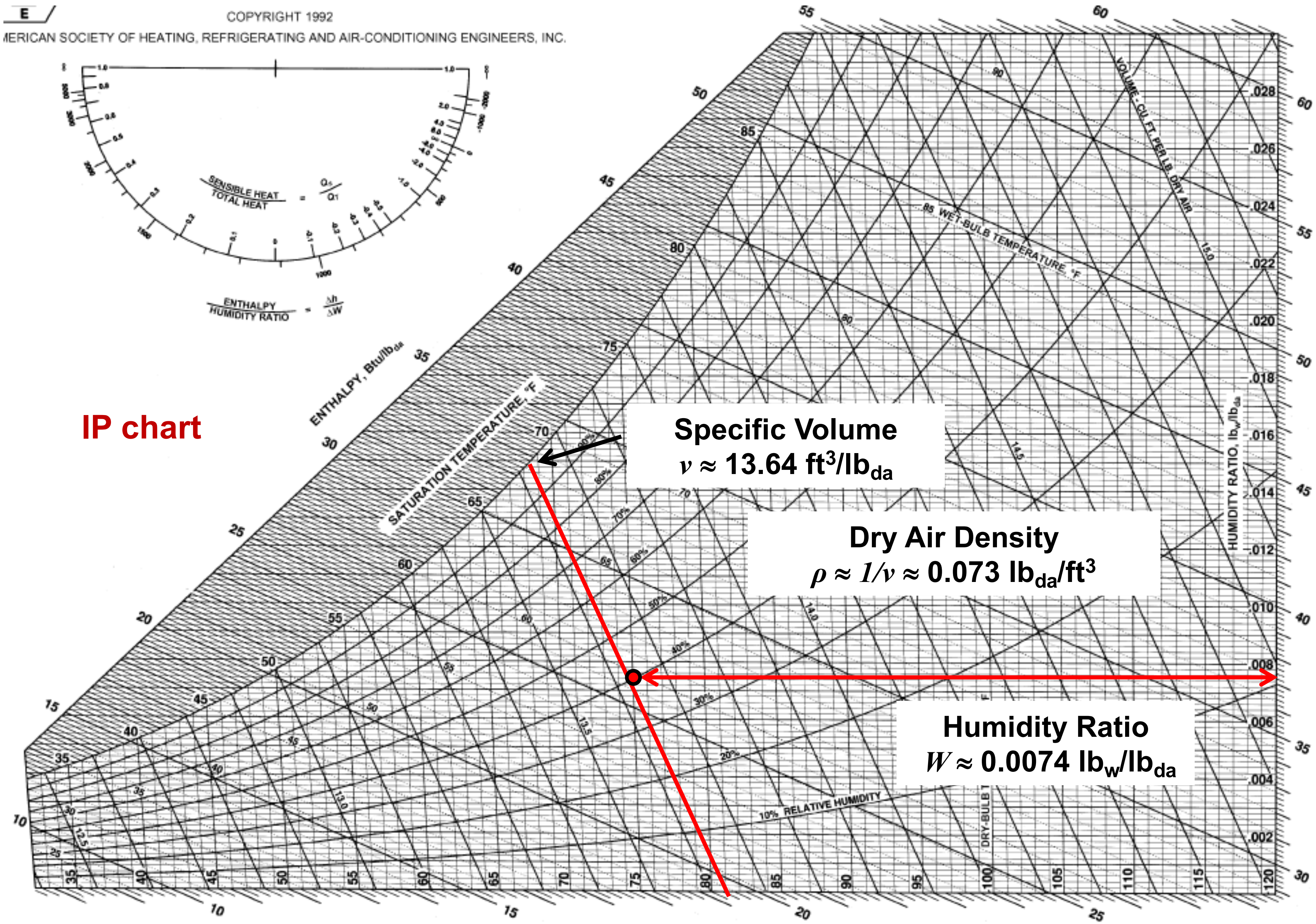
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, gw/kgda
DRY-BULB TEMPERATURE, °C
Relative Humidity
$\phi \approx 50\%$
Enthalpy
$h \approx 44$ kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx 11.7$°C
Specific Volume
$v \approx 0.848$ m^3/kg$_{da}$
Dry Air Density
$\rho \approx 1/v \approx 1.18$ kg$_{da}$/m^3
Dry Bulb Temp
$T = 22$°C
Wet Bulb Temp
$T_{wb} \approx 15.5$°C
Humidity Ratio
$W \approx 8.2$ g/kg$_{da}$
(i.e., 0.0082 kg$_w$/kg$_{da}$)

Using these parameters

- Question:
  - What was the mass of water vapor in the classroom last time?
  - Dry bulb temperature ≈ 75°F
  - RH ≈ 40%

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ENTHALPY, Btu/lb_da
SATURATION TEMPERATURE, °F
VOLUME, CU.FT. PER LB. DRY AIR
WET-BULB TEMPERATURE, °F
HUMIDITY RATIO, lb_w/lb_da
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °F
Specific Volume
$v \approx 13.64$ ft^3/lb$_{da}$
Dry Air Density
$\rho \approx 1/v \approx 0.073$ lb$_{da}$/ft^3
Humidity Ratio
$W \approx 0.0074$ lb$_w$/lb$_{da}$

Using these parameters

- Question:
  - What was the mass of water vapor in the classroom last time?
  - Dry bulb temperature ≈ 75°F
  - RH ≈ 40%

- Answer:
  - $W \approx$ 0.0074 lb_w/lb_{da}
  - $v \approx$ 13.64 ft^3/lb_{da}
  - $\rho \approx 1/v \approx$ 0.073 lb_{da}/ft^3
  - Volume of room, $V \approx$ ______ ft^3
  - Mass of dry air, $m_{da} = \rho V$
  - Mass of water vapor, $m_w = W m_{da}$

PSYCHROMETRIC EQUATIONS

Psychrometric equations

- When we need more precise answers, or when we need to automate engineering calculations, we must:
  - Use the underlying **equations** that govern moist air properties and processes and make up the psychrometric chart

- This begins by treating air as an **ideal gas**

Treating air as an ideal gas

- At typical temperatures and pressures within buildings, air and its constituents act approximately as ideal gases
- Each gas i in the mixture, as well as the entire mixture, will follow the ideal gas law:

Ideal Gas Law (Boyle's law + Charles's law)

$$pV = nRT$$

p = pressure (lb/ft^2)
V = volume (ft^3)
n = number of moles (#)
R = gas constant ($lb_f \cdot ft/(lb_{mol}\, R)$)*
T = absolute temperature (R)

*Units on R vary with units of pressure

Air as an ideal gas

- We can treat air as a composition of ideal gases
  - A bunch of ideal gases acting as an ideal gas

- For individual gases (e.g., N_2, O_2, H_2O, CO_2, constituent i):

$$P_i V = n_i RT$$

P_i = partial pressure exerted by gas i
n_i = # of moles of gas i
R, V, T = gas constant, volume, temperature

$$P_i = \frac{n_i}{V} RT$$

Rearrange so that n_i/V is the molar concentration

$$P_i = y_i P_{tot}$$

P_{tot} = total pressure of air (atm, Pa, psia, etc.)
y_i = mole fraction of gas i in air (moles i / moles air)

Air as an ideal gas

- Air as a composite mixture

$$P_i = y_i P_{tot}$$

$$P_{tot} = \sum P_i = \sum \frac{n_i}{V} RT = \frac{RT}{V} \sum n_i = \frac{RT}{V} n_{tot}$$

$$PV = nRT$$

Universal gas constant

- The universal gas constant relates energy and temperature
  - It takes many forms depending on units

Universal gas constant

$$PV = nRT$$

Value of R	**Units ($V\ P\ T^{-1}\ n^{-1}$)**
8.314	$J/(K \cdot mol)$
8.314	$m^3 \cdot Pa/(K \cdot mol)$
0.08206	$L \cdot atm/(K \cdot mol)$
8.205×10^{-5}	$m^3 \cdot atm/(K \cdot mol)$
1545.349	$ft \cdot lb_f/(R \cdot lb_{mol})$
1.986	$Btu/(lb_{mol} \cdot R)$

Dalton's law of partial pressures for psychrometrics

- In an ideal gas, the total pressure can be considered to be the sum of the partial pressures of the constituent gases

$$p = p_{N_2} + p_{O_2} + p_{H_2O} + p_{CO_2} + p_{Ar} + \ldots$$

- We can consider moist air as dry air combined with water vapor and break the pressure into only two partial pressures:
  - Dry air (da)
  - Water vapor (w)

$$p = p_{da} + p_w$$

Dalton's law of partial pressures for psychrometrics

- We can analyze the dry air, the water vapor, and the mixture of each gas using the ideal gas law and assuming they are all at the same temperature

$$p_{da}v_{da} = R_{da}T \quad \& \quad p_w v_w = R_w T \quad \& \quad pv = RT$$

- For each individual gas, a mole fraction (Y_i) can be defined as the ratio of the partial pressure of gas i to the total pressure

$$\frac{n_i}{n} = \frac{p_i}{p} = Y_i$$

Specific gas constants

- To work with air and water vapor we can also work with specific gas constants (which are functions of molecular weight)
- Dry air (no water vapor): MW_{da} = 28.966 g/mol

$$R_{da} = \frac{R}{MW_{da}} = \frac{1545.349 \dfrac{\text{ft} \cdot \text{lb}_\text{f}}{^\circ\text{R} \cdot \text{lb}_\text{mol}}}{28.965 \dfrac{\text{lb}_\text{m,da}}{\text{lb}_\text{mol}}} = 53.35 \frac{\text{ft} \cdot \text{lb}_\text{f}}{^\circ\text{R} \cdot \text{lb}_\text{da}}$$

- Water vapor alone: MW_w = 18.015 g/mol

$$R_{w} = \frac{R}{MW_{w}} = \frac{1545.349 \dfrac{\text{ft} \cdot \text{lb}_\text{f}}{^\circ\text{R} \cdot \text{lb}_\text{mol}}}{18.015 \dfrac{\text{lb}_\text{m,w}}{\text{lb}_\text{mol}}} = 85.78 \frac{\text{ft} \cdot \text{lb}_\text{f}}{^\circ\text{R} \cdot \text{lb}_\text{w}}$$

$$pv = \frac{p}{\rho} = R_i T$$

Specific gas constant:

$$R_i = \frac{R}{MW_i}$$

Note IP units:
1 g/mol = 1 $\text{lb}_\text{m}/\text{lb}_\text{mol}$

Air pressure variations

- The barometric (atmospheric) pressure and temperature of air vary with both altitude and local weather conditions
  - But there are standard values for pressure as a function of altitude that are normally used
- At sea level, the standard temperature is 59°F and the standard pressure is 14.696 psia (i.e. 101.325 kPa or 1 atm)
  - Temperature is assumed to decrease linearly with altitude
    - Pressure is more complicated

$$p = 14.696(1 - 6.8754 \times 10^{-6} Z)^{5.2559}$$

The equation for temperature as a function of altitude is

$$t = 59 - 0.00356620Z$$

where

Z = altitude, ft
p = barometric pressure, psia
t = temperature, °F

$$pv = \frac{p}{\rho} = RT$$

Air pressure variations

Table 1 Standard Atmospheric Data for Altitudes to 30,000 ft

Altitude, ft	Temperature, °F	Pressure, psia	
–1000	62.6	15.236	
–500	60.8	14.966	Chicago, IL
0	59.0	14.696	
500	57.2	14.430	
1,000	55.4	14.175	
2,000	51.9	13.664	Denver, CO
3,000	48.3	13.173	Big Sky, MT
4,000	44.7	12.682	
5,000	41.2	12.230	
6,000	37.6	11.778	Breckenridge, CO
7,000	34.0	11.341	
8,000	30.5	10.914	
9,000	26.9	10.506	
10,000	23.4	10.108	
15,000	5.5	8.296	
20,000	–12.3	6.758	
30,000	–47.8	4.371	

Source: Adapted from NASA (1976).

Specifying the state of moist air

In order to specify the state of moist air, we need total atmospheric pressure, p, the air temperature, T, and at least one other property

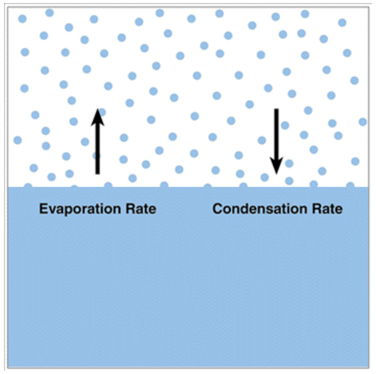
- W, ϕ, h, p_w, or T_{dew}

- We can use the psychrometric chart
- We can also use the **underlying equations** for greater accuracy and automation

Remember: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is the pressure exerted by a vapor in thermodynamic equilibrium with its condensed phases

p_w *Units of pressure, psia (Pa or kPa) (aka "partial pressure")

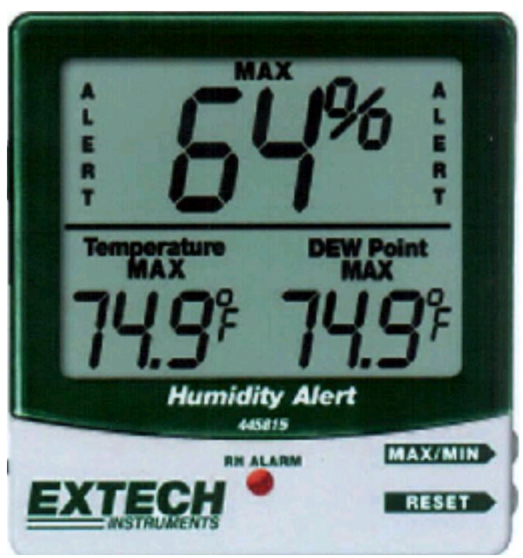


- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
  - We call the limit ***saturation***

p_{ws} *Units of pressure, psia (Pa or kPa) (aka "saturation vapor pressure")

Relative humidity, ϕ (RH)

- The relative humidity ratio, ϕ, is the mole fraction of water vapor (x_w) relative to the water vapor that would be in the mixture if it were saturated at the given T and P (x_{ws})
  - We can also describe RH by partial pressures (ideal gas)
- Relative humidity is a common measure that relates well to how we perceive moisture in air



$$\phi = \left[\frac{x_w}{x_{ws}}\right]_{T,P} = \frac{p_w / p_{tot}}{p_{ws} / p_{tot}} = \frac{p_w}{p_{ws}}$$

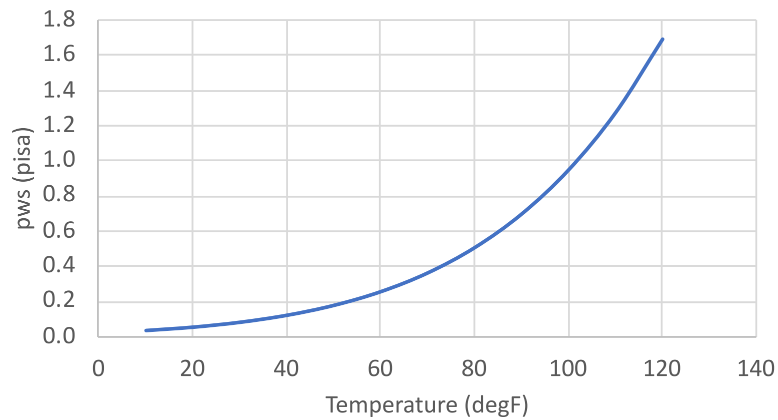
p_{ws} for 32°F < T < 392°F (IP units)

For p_{ws}, the saturation pressure over **liquid water**:

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13}\ln T$$

where

$$\begin{aligned}
C_8 &= -1.044\,039\,7 \text{ E+04} \\
C_9 &= -1.129\,465\,0 \text{ E+01} \\
C_{10} &= -2.702\,235\,5 \text{ E-02} \\
C_{11} &= 1.289\,036\,0 \text{ E-05} \\
C_{12} &= -2.478\,068\,1 \text{ E-09} \\
C_{13} &= 6.545\,967\,3 \text{ E+00}
\end{aligned}$$



Units:

p_{ws} = saturation pressure, psia
T = absolute temperature, °R = °F + 459.67

Note:
These constants are only for IP units
<u>SI units are different</u>

p_{ws} for -148°F < T < 32°F (IP units)

For p_{ws}, the saturation pressure over **ice**:

$$\ln p_{ws} = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T$$

where

$C_1 = -1.021\,416\,5$ E+04
$C_2 = -4.893\,242\,8$ E+00
$C_3 = -5.376\,579\,4$ E−03
$C_4 = 1.920\,237\,7$ E−07
$C_5 = 3.557\,583\,2$ E−10
$C_6 = -9.034\,468\,8$ E−14
$C_7 = 4.163\,501\,9$ E+00

Note:
These constants are only for IP units
SI units are different

Units:

p_{ws} = saturation pressure, psia
T = absolute temperature, °R = °F + 459.67

Humidity ratio, W (IP units)

- The humidity ratio, W, is ratio of the mass of water vapor to mass of dry air in a given volume
  - We use W when finding other mixture properties
  - Note 1: W is small (W < 0.03 for most real building conditions)
  - Note 2: W is also expressed in grains/lb where 1 lb = 7000 grains

Units:

$$W = \frac{m_w}{m_{da}} = \frac{MW_w x_w}{MW_{da} x_{da}} = 0.622 \frac{x_w}{x_{da}} \qquad \left[\frac{\mathrm{lb_w}}{\mathrm{lb_{da}}}\right]$$

$$x_{da} = \frac{p_{da}}{p_{da} + p_w} = \frac{p_{da}}{p_{tot}} \qquad x_w = \frac{p_w}{p_{da} + p_w} = \frac{p_w}{p_{tot}}$$

Humidity ratio, W (IP units)

- The humidity ratio, W, is ratio of the mass of water vapor to mass of dry air in a given volume
  - We use W when finding other mixture properties
  - Note 1: W is small (W < 0.03 for most real building conditions)
  - Note 2: W is also expressed in grains/lb where 1 lb = 7000 grains

$$W = 0.622\frac{x_w}{x_{da}} = 0.622\frac{p_w / p_{tot}}{p_{da} / p_{tot}} = 0.622\frac{p_w}{p_{da}} = 0.622\frac{p_w}{p_{tot} - p_w}$$

where: $p_{tot} = p_{da} + p_w = 14.696$ psia @ sea level

Saturation humidity ratio, W_s (IP units)

- At a given temperature T and pressure P there is a maximum W that can be obtained
- If we try to add any more moisture, it will just condense out
  - It is when the partial pressure of vapor has reached the saturation pressure
- This maximum humidity ratio is called the saturation humidity ratio, W_s
  - From our previous equation we can write:

UNITS

$$W_s = 0.622 \frac{p_{ws}}{p_{da}} = 0.622 \frac{p_{ws}}{p_{tot} - p_{ws}} \qquad \left[\frac{\mathrm{lb_w}}{\mathrm{lb_{da}}}\right]$$

Degree of saturation, μ (IP units)

- The degree of saturation, μ (dimensionless), is the ratio of the humidity ratio W to that of a saturated mixture W_s at the same T and P
  - Note that μ and ϕ are not quite the same
  - Their values are very similar at lower temperatures but may differ a lot at higher temperatures

$$\mu = \left[\frac{W}{W_s}\right]_{T,P}$$

$$\mu = \frac{\phi}{1 + (1-\phi)W_s / (0.622)}$$

$$\phi = \frac{\mu}{1 - (1-\mu)\, p_{ws} / p_{tot}}$$

Specific volume, v, and density, ρ (IP units)

- The **specific volume** of moist air (or the volume per unit mass of air, ft^3/lb_{da}) can be expressed as:

$$v = \frac{R_{da}T}{p_{da}} = \frac{R_{da}T}{p_{tot} - p_w} = \frac{R_{da}T(1 + 1.6078W)}{p_{tot}}$$

$$v \approx 0.370486(T + 459.67)(1 + 1.6078W)\ /\ p_{tot}$$

where

v = specific volume, ft^3/lb_{da}
t = dry-bulb temperature, °F
W = humidity ratio, lb_w/lb_{da}
p = total pressure, psia

- If we have v we can also find **moist air density**, ρ (lb/ft^3):

$$\rho = \frac{m_{da} + m_w}{V} = \frac{1}{v}\left(1 + W\right)$$

Enthalpy, h (IP units)

- The enthalpy of a mixture of perfect gases equals the sum of the individual partial enthalpies of the components
- Therefore, the enthalpy (h) for moist air is: $$h = h_{da} + W h_g$$

  h = enthalpy for moist air [BTU/lb_{da}]
  h_g = specific enthalpy for saturated water vapor (i.e., h_{ws}) [BTU/lb_w]
  h_{da} = specific enthalpy for dry air [BTU/lb_{da}]

- Some approximations: $h_{da} \approx 0.24T$ $\quad h_g \approx 1061 + 0.444T$

$$h \approx 0.24T + W(1061 + 0.444T)$$

*where T is in °F and h is in BTU/lb

A note on h_g, h_f, and h_{fg} (i.e., h_w)

100% liquid water

100% water vapor (i.e., steam)

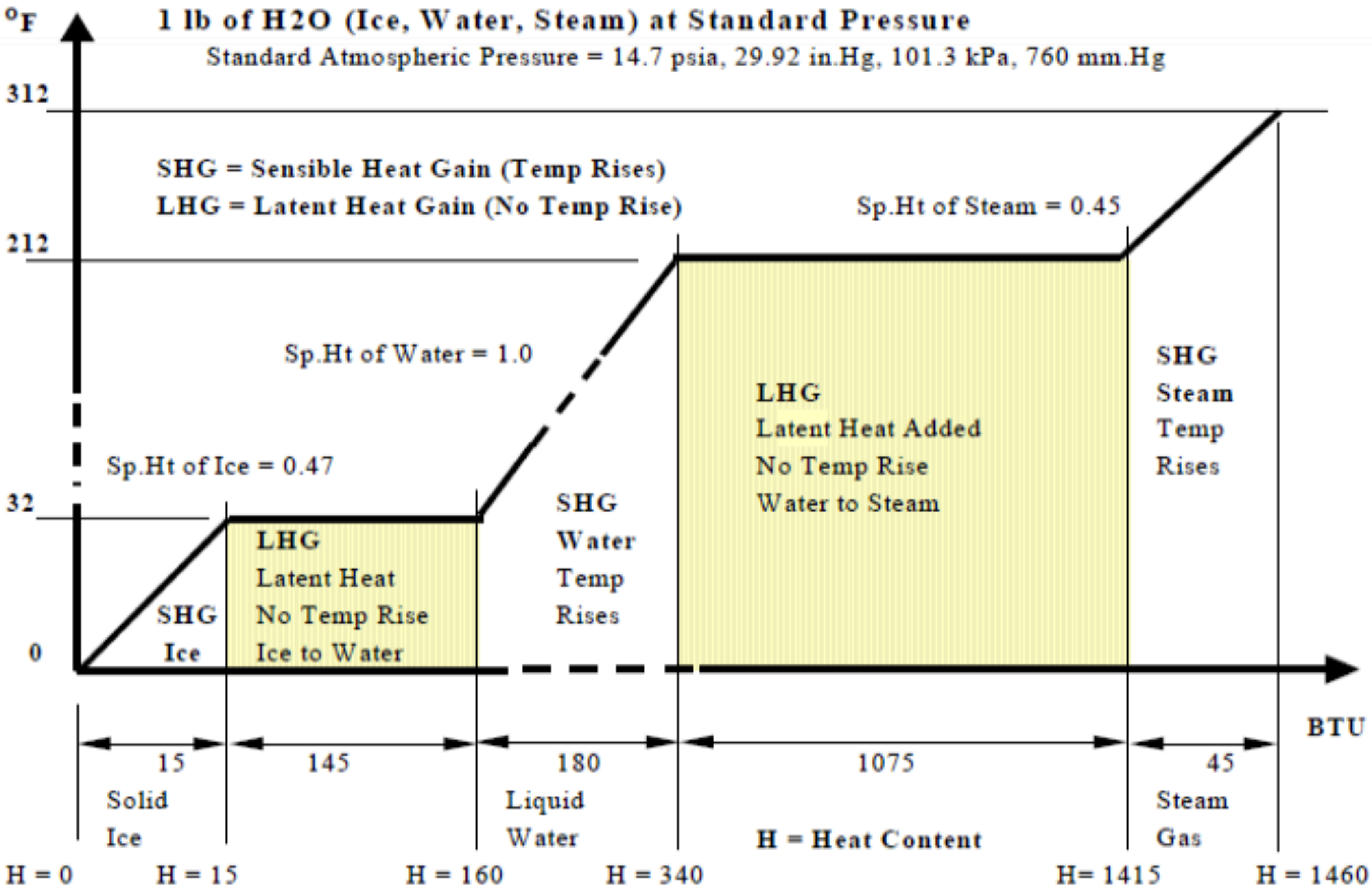
Table 3 Thermodynamic Properties of Water at Saturation (*Continued*)

Temp., °F t	Absolute Pressure p_{ws}, psia	Specific Volume, ft³/lb$_w$ Sat. Liquid v_i/v_f	Specific Volume Evap. v_{ig}/v_{fg}	Specific Volume Sat. Vapor v_g	Specific Enthalpy, Btu/lb$_w$ Sat. Liquid h_i/h_f	Specific Enthalpy Evap. h_{ig}/h_{fg}	Specific Enthalpy Sat. Vapor h_g	Specific Entropy, Btu/lb$_w$·°F Sat. Liquid s_i/s_f	Specific Entropy Evap. s_{ig}/s_{fg}	Specific Entropy Sat. Vapor s_g	Temp., °F t
53	0.19903	0.01603	1532.94	1532.96	21.07	1063.31	1084.38	0.0420	2.0741	2.1160	53
54	0.20646	0.01603	1480.62	1480.64	22.07	1062.75	1084.82	0.0439	2.0689	2.1129	54
55	0.21414	0.01603	1430.31	1430.32	23.07	1062.18	1085.26	0.0459	2.0638	2.1097	55
56	0.22206	0.01603	1381.92	1381.94	24.08	1061.62	1085.69	0.0478	2.0587	2.1065	56
57	0.23024	0.01603	1335.38	1335.39	25.08	1061.05	1086.13	0.0497	2.0536	2.1034	57
58	0.23868	0.01603	1290.60	1290.61	26.08	1060.49	1086.56	0.0517	2.0486	2.1003	58
59	0.24740	0.01603	1247.51	1247.53	27.08	1059.92	1087.00	0.0536	2.0435	2.0972	59

As liquid water transitions to vapor (steam), it will be in a mixed state:

- Part liquid, part vapor

A note on h_g, h_f, and h_{fg} (i.e., h_w)



http://energy-models.com/gases-liquids-fluids-and-solids

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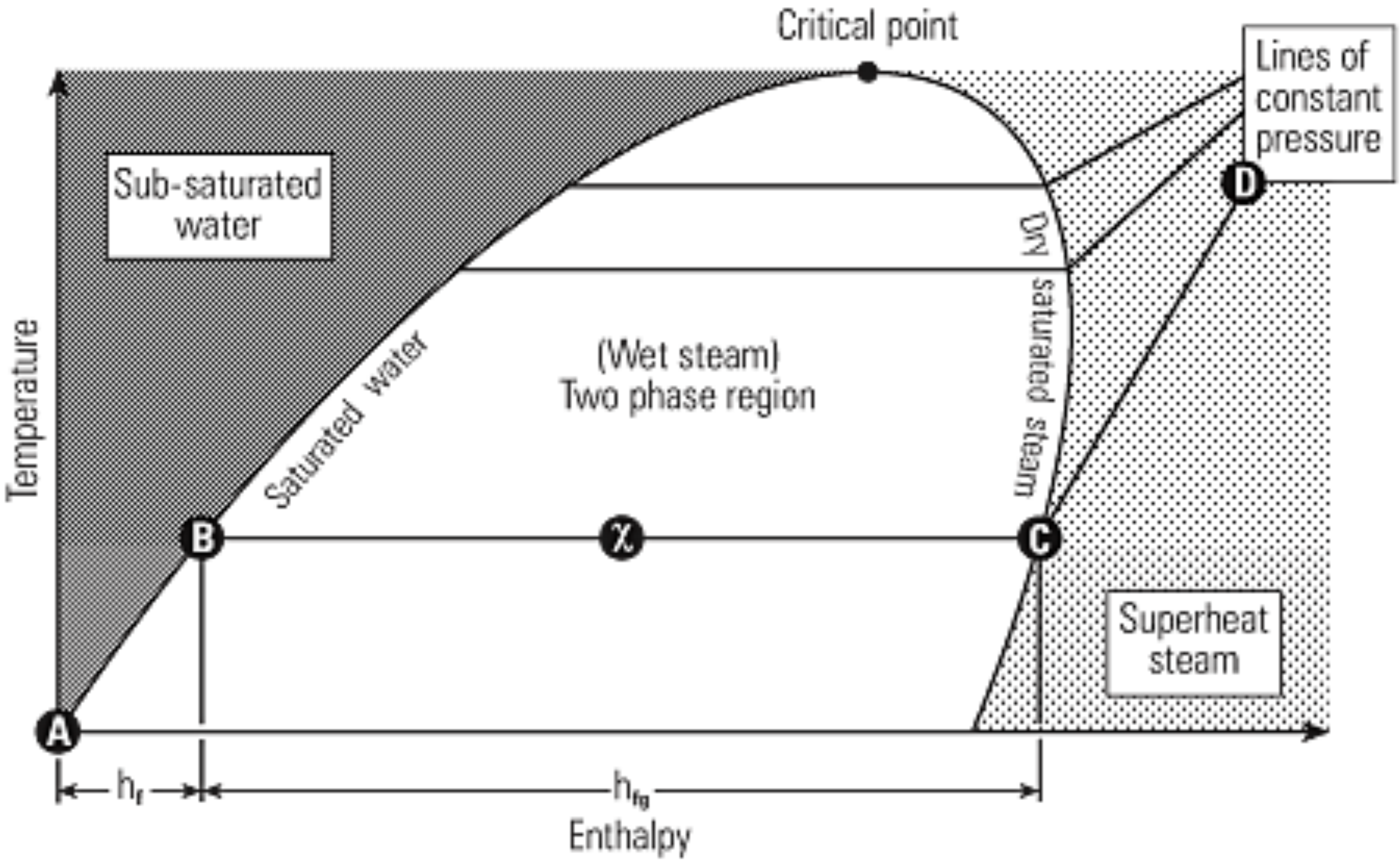


http://pointing.spiraxsarco.com/images/resources/steam-engineering-tutorials/2/2/fig_2_2_3.gif

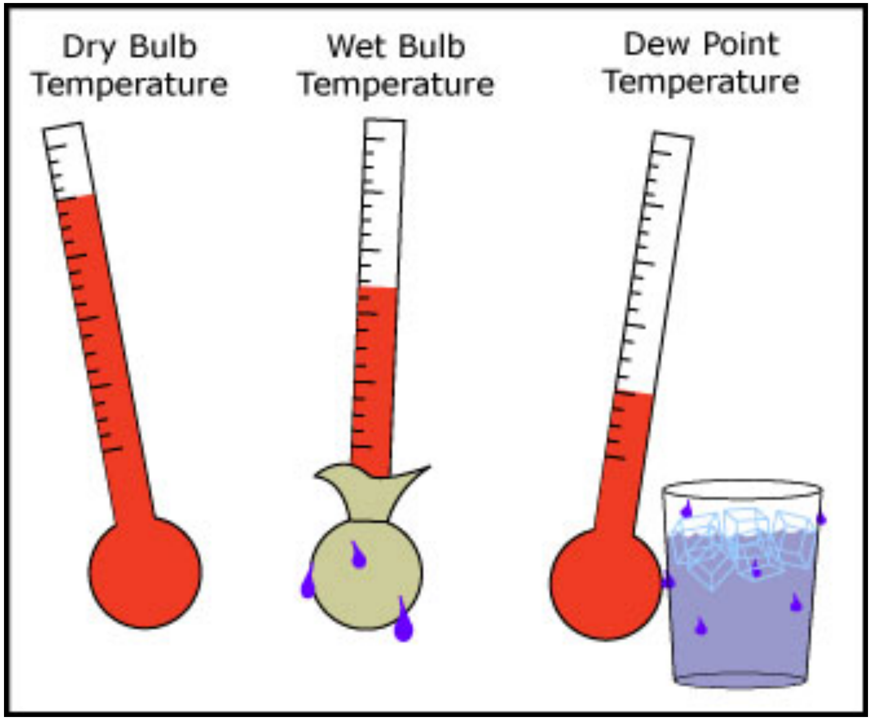
Remember: 3 different temperatures T, T_{dew}, and T_{wb}

The standard temperature, T, we are all familiar with is called the **dry-bulb** temperature, or T_d

- It is a measure of internal energy

We can also define:

- **Dew-point** temperature, T_{dew}
  - Temperature at which water vapor changes into liquid (condensation)
  - Air is maximally saturated with water vapor
- **Wet-bulb** temperature, T_{wb}
  - The temperature that a parcel of air would have if it were cooled to saturation (100% relative humidity) by the evaporation of water into it



Units of Celsius, Fahrenheit, or Kelvin

✓ The energy needed to evaporate liquid water (heat of vaporization) is taken from the air in the form of sensible heat and converted to latent heat, which lowers the temperature at constant enthalpy

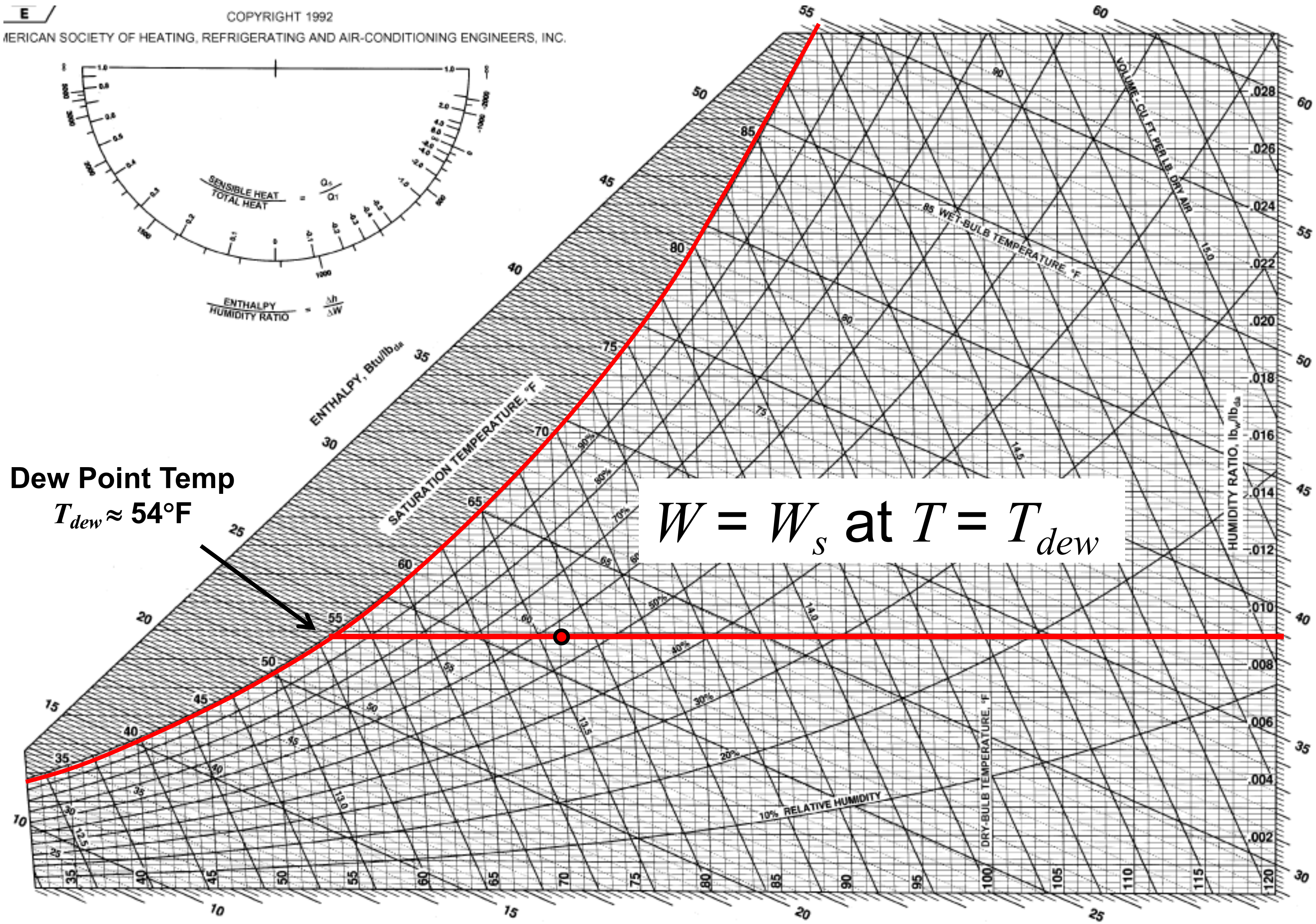
Dew-point temperature, T_{dew}

The dew point temperature, T_{dew}, is the air temperature at which the current humidity ratio (W) is equal to the saturation humidity ratio (W_s) at the same temperature

i.e., $W_s(p, T_{dew}) = W$

When the air temperature is lowered to the dew-point at constant pressure, the relative humidity rises to 100% and condensation occurs

T_{dew} is a direct measure of the humidity ratio W since $W = W_s$ at $T = T_{dew}$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SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY, Btu/lb_da
SATURATION TEMPERATURE, °F
WET-BULB TEMPERATURE, °F
VOLUME, CU.FT. PER LB. DRY AIR
HUMIDITY RATIO, lb_w/lb_da
DRY-BULB TEMPERATURE, °F
10% RELATIVE HUMIDITY
Dew Point Temp
$T_{dew} \approx 54$°F
$W = W_s$ at $T = T_{dew}$

Dew-point temperature, T_{dew} (IP units)

- Dew-point temperature, T_{dew}

Between dew points of 32 to 200°F,

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}(p_w)^{0.1984}$$

Below 32°F,

$$t_d = 90.12 + 26.142\alpha + 0.8927\alpha^2$$

where

t_d = dew-point temperature, °F
$\alpha = \ln p_w$
p_w = water vapor partial pressure, psia
$C_{14} = 100.45$
$C_{15} = 33.193$
$C_{16} = 2.319$
$C_{17} = 0.17074$
$C_{18} = 1.2063$

Note:
These constants are only for IP units
[SI units are different]

Wet-bulb temperature, T_{wb} (IP units)

- Wet-bulb temperature, T_{wb}
- Requires iterative solving… find the T_{wb} that satisfies the following equation (above freezing):

$$W = \frac{(1093 - 0.556T_{wb})W_{s@T_{wb}} - 0.240(T - T_{wb})}{1093 + 0.444T - T_{wb}} \quad \text{= actual } W$$

- And for T below freezing:

$$W = \frac{(1220 - 0.04T_{wb})W_{s@T_{wb}} - 0.240(T - T_{wb})}{1220 + 0.444T - 0.48T_{wb}} \quad \text{= actual } W$$

*Where T_{wb} and T are in Fahrenheit

Obtaining these data from ASHRAE Tables

ASHRAE HoF Chapter 1 Table 2 gives us W_s, v_{da} , v_s , h_{da}, and h_s directly at different temperatures:

Table 2 Thermodynamic Properties of Moist Air at Standard Atmospheric Pressure, 14.696 psia (*Continued*)

Temp., °F	Humidity Ratio	Specific Volume, ft^3/lb_{da}			Specific Enthalpy, Btu/lb_{da}			Specific Entropy, $Btu/lb_{da} \cdot °F$		Temp., °F
t	W_s, lb_w/lb_{da}	v_{da}	v_{as}	v_s	h_{da}	h_{as}	h_s	s_{da}	s_s	t
60	0.011089	13.0955	0.2328	13.3283	14.412	12.055	26.467	0.02947	0.05389	60
61	0.011498	13.1208	0.2418	13.3626	14.653	12.504	27.157	0.02993	0.05522	61
62	0.011921	13.1461	0.2512	13.3973	14.893	12.968	27.861	0.03039	0.05657	62
63	0.012357	13.1713	0.2609	13.4322	15.133	13.448	28.581	0.03085	0.05795	63
64	0.012807	13.1966	0.2709	13.4675	15.373	13.944	29.318	0.03131	0.05936	64
65	0.013272	13.2219	0.2813	13.5032	15.614	14.456	30.070	0.03177	0.06080	65
66	0.013753	13.2472	0.2920	13.5392	15.854	14.986	30.840	0.03223	0.06226	66
67	0.014249	13.2724	0.3031	13.5755	16.094	15.532	31.626	0.03268	0.06376	67
68	0.014761	13.2977	0.3146	13.6123	16.335	16.097	32.431	0.03314	0.06529	68
69	0.015289	13.3230	0.3265	13.6494	16.575	16.680	33.255	0.03360	0.06685	69
70	0.015835	13.3482	0.3388	13.6870	16.815	17.282	34.097	0.03405	0.06844	70
71	0.016398	13.3735	0.3515	13.7250	17.056	17.903	34.959	0.03450	0.07007	71
72	0.016979	13.3988	0.3646	13.7634	17.296	18.545	35.841	0.03496	0.07173	72
73	0.017578	13.4241	0.3782	13.8022	17.536	19.208	36.744	0.03541	0.07343	73
74	0.018197	13.4493	0.3922	13.8415	17.776	19.892	37.668	0.03586	0.07516	74
75	0.018835	13.4746	0.4067	13.8813	18.017	20.598	38.615	0.03631	0.07694	75

Obtaining these data from ASHRAE Tables

ASHRAE HoF Chapter 1 Table 3 gives us p_{ws} at different temperatures:

Table 3 Thermodynamic Properties of Water at Saturation (*Continued*)

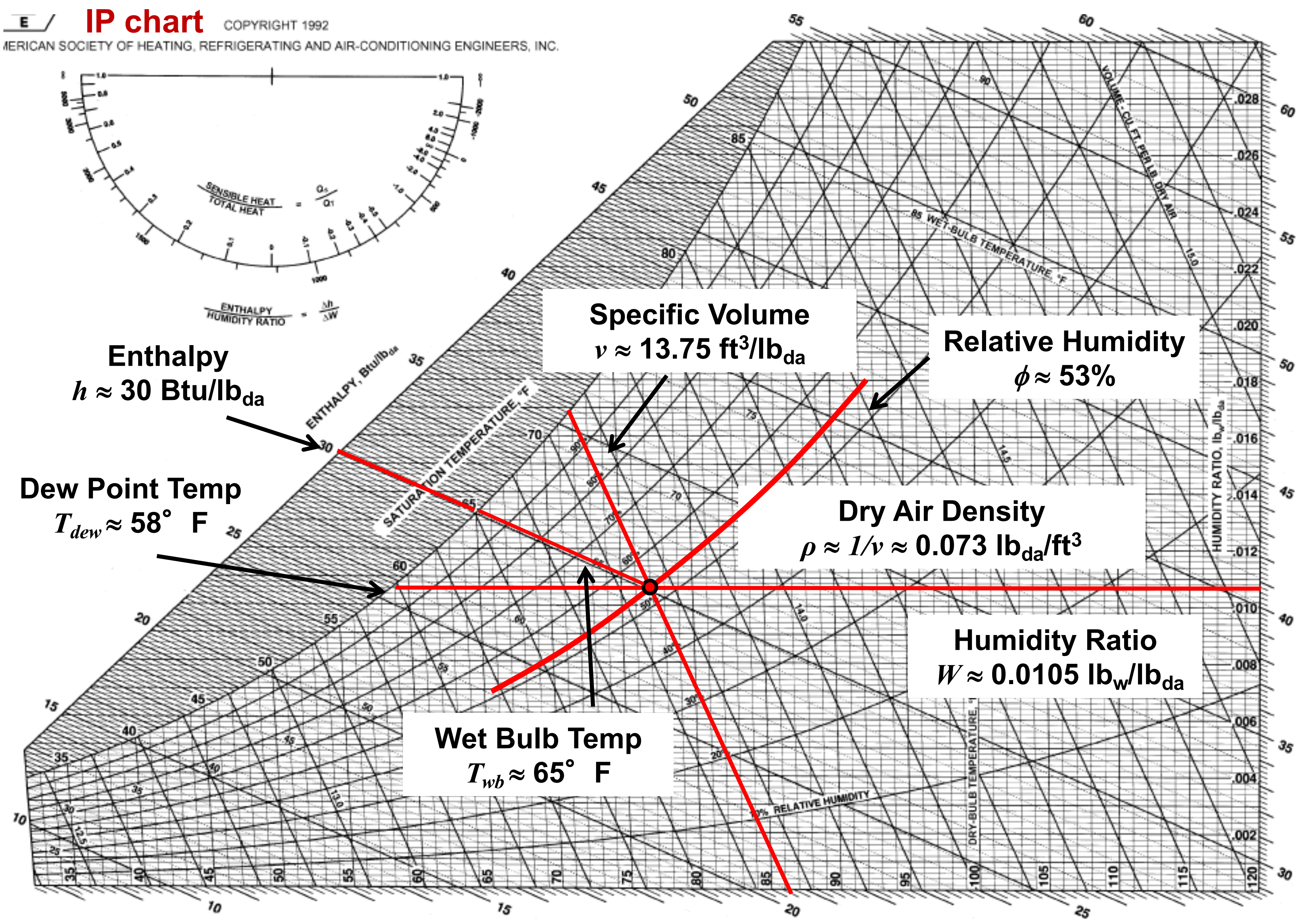
Temp., °F t	Absolute Pressure p_{ws}, psia	Specific Volume, ft³/lb$_w$ Sat. Liquid v_i/v_f	Specific Volume Evap. v_{ig}/v_{fg}	Specific Volume Sat. Vapor v_g	Specific Enthalpy, Btu/lb$_w$ Sat. Liquid h_i/h_f	Specific Enthalpy Evap. h_{ig}/h_{fg}	Specific Enthalpy Sat. Vapor h_g	Specific Entropy, Btu/lb$_w$·°F Sat. Liquid s_i/s_f	Specific Entropy Evap. s_{ig}/s_{fg}	Specific Entropy Sat. Vapor s_g	Temp., °F t
53	0.19903	0.01603	1532.94	1532.96	21.07	1063.31	1084.38	0.0420	2.0741	2.1160	53
54	0.20646	0.01603	1480.62	1480.64	22.07	1062.75	1084.82	0.0439	2.0689	2.1129	54
55	0.21414	0.01603	1430.31	1430.32	23.07	1062.18	1085.26	0.0459	2.0638	2.1097	55
56	0.22206	0.01603	1381.92	1381.94	24.08	1061.62	1085.69	0.0478	2.0587	2.1065	56
57	0.23024	0.01603	1335.38	1335.39	25.08	1061.05	1086.13	0.0497	2.0536	2.1034	57
58	0.23868	0.01603	1290.60	1290.61	26.08	1060.49	1086.56	0.0517	2.0486	2.1003	58
59	0.24740	0.01603	1247.51	1247.53	27.08	1059.92	1087.00	0.0536	2.0435	2.0972	59
60	0.25639	0.01603	1206.05	1206.07	28.08	1059.36	1087.44	0.0555	2.0385	2.0941	60
61	0.26567	0.01604	1166.14	1166.16	29.08	1058.79	1087.87	0.0575	2.0335	2.0910	61
62	0.27524	0.01604	1127.72	1127.74	30.08	1058.23	1088.31	0.0594	2.0285	2.0879	62
63	0.28511	0.01604	1090.73	1090.74	31.08	1057.66	1088.74	0.0613	2.0236	2.0849	63
64	0.29529	0.01604	1055.11	1055.12	32.08	1057.10	1089.18	0.0632	2.0186	2.0818	64

IP unit example

Assume:

- Dry bulb temperature = 25°C = 77°F
- RH = 53%
- Sea level (14.696 psia)

Find all other relevant parameters using equations



IP chart
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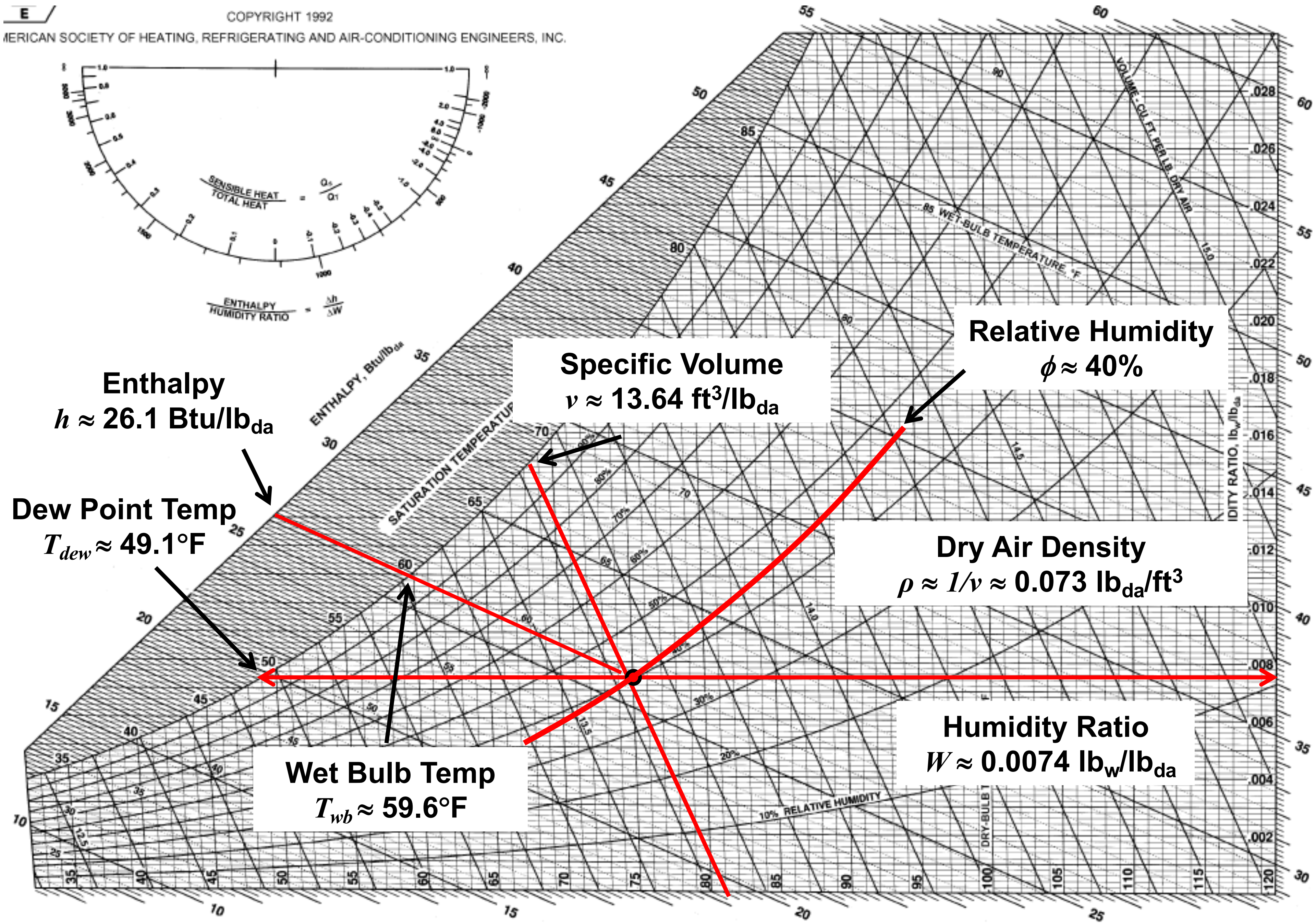
SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
Enthalpy
h ≈ 30 Btu/lb_da
Specific Volume
v ≈ 13.75 ft³/lb_da
Relative Humidity
φ ≈ 53%
Dew Point Temp
T_dew ≈ 58° F
Dry Air Density
ρ ≈ 1/v ≈ 0.073 lb_da/ft³
Humidity Ratio
W ≈ 0.0105 lb_w/lb_da
Wet Bulb Temp
T_wb ≈ 65° F
ENTHALPY, Btu/lb_da
SATURATION TEMPERATURE, °F
85 WET-BULB TEMPERATURE, °F
VOLUME - CU. FT. PER LB. DRY AIR
HUMIDITY RATIO, lb_w/lb_da
DRY-BULB TEMPERATURE, °F
RELATIVE HUMIDITY

Revisit classroom example

- Dry bulb temperature ≈ 75°F
- RH ≈ 40%
- Sea level (14.696 psia)

Find all other relevant parameters using equations



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SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY, Btu/lbda
SATURATION TEMPERATURE
85 WET-BULB TEMPERATURE, °F
VOLUME, CU.FT. PER LB. DRY AIR
10% RELATIVE HUMIDITY
DRY-BULB T
HUMIDITY RATIO, lbw/lbda
Enthalpy
h ≈ 26.1 Btu/lbda
Specific Volume
v ≈ 13.64 ft3/lbda
Relative Humidity
ϕ ≈ 40%
Dew Point Temp
Tdew ≈ 49.1°F
Dry Air Density
ρ ≈ 1/v ≈ 0.073 lbda/ft3
Humidity Ratio
W ≈ 0.0074 lbw/lbda
Wet Bulb Temp
Twb ≈ 59.6°F

Revisit an SI example from last class

Moist air exists at 30°C dry-bulb temperature with a 15°C dew point temperature

Find the following:

(a) the humidity ratio, W

(b) degree of saturation, μ

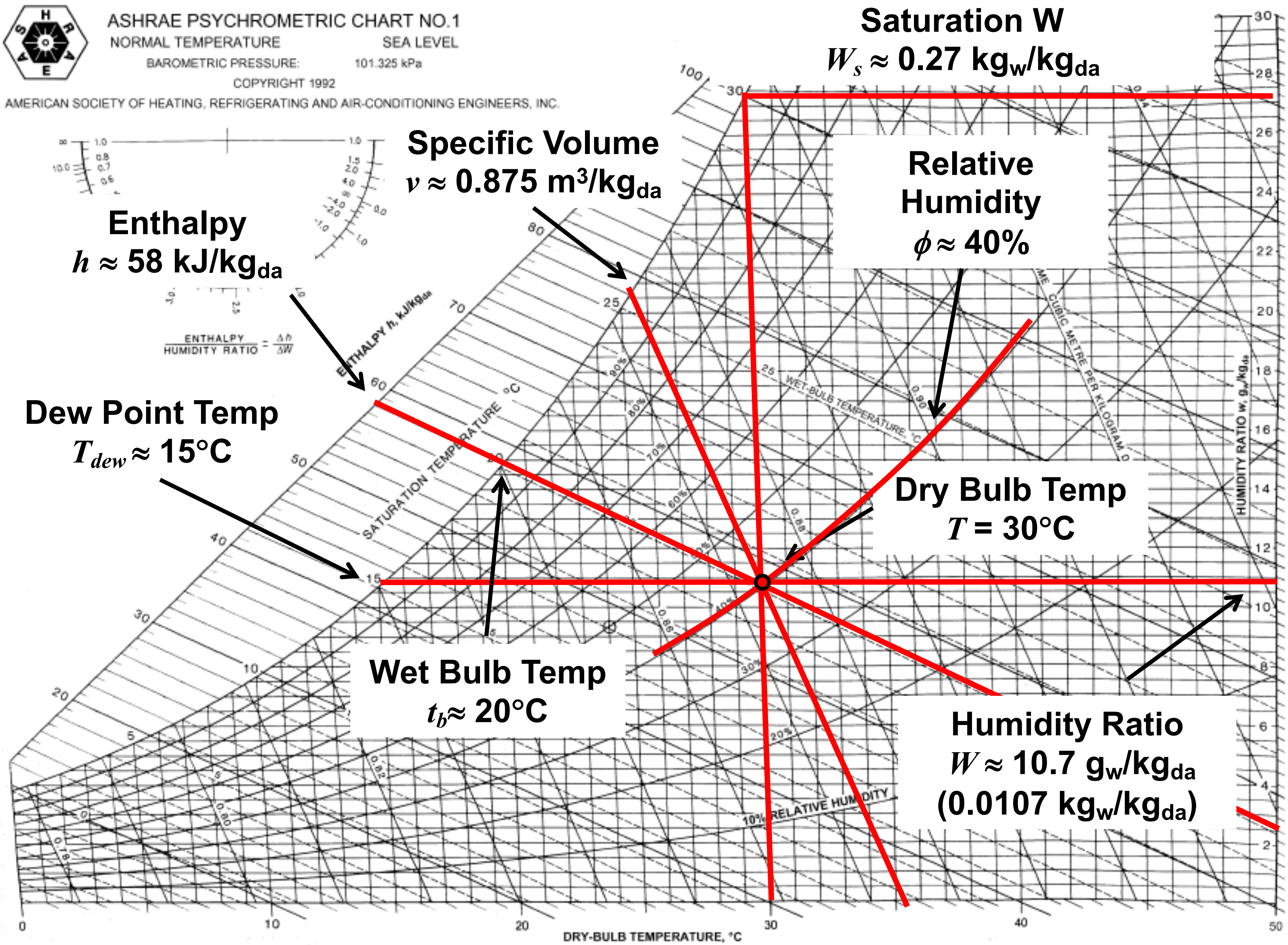
(c) relative humidity, ϕ

(d) enthalpy, h

(e) specific volume, v

(f) density, ρ

(g) wet bulb temperature, T_{wb}


ASHRAE PSYCHROMETRIC CHART NO.1
NORMAL TEMPERATURE
SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
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Saturation W
$W_s \approx 0.27\ kg_w/kg_{da}$
Specific Volume
$v \approx 0.875\ m^3/kg_{da}$
Relative Humidity
$\phi \approx 40\%$
Enthalpy
$h \approx 58\ kJ/kg_{da}$
Dew Point Temp
$T_{dew} \approx 15°C$
Dry Bulb Temp
$T = 30°C$
Wet Bulb Temp
$t_b \approx 20°C$
Humidity Ratio
$W \approx 10.7\ g_w/kg_{da}$
$(0.0107\ kg_w/kg_{da})$
ENTHALPY h, kJ/kg
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
CUBIC METRE PER KILOGRAM
HUMIDITY RATIO W, g_w/kg_{da}
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C

Humidity ratio

$$W = 0.622 \left. \frac{p_w}{p - p_w} \right|_{@T=30°C}$$

Assume p = 101.325 kPa (sea level)

- For a known T_{dew} = 15°C, we know that the actual humidity ratio in the air, W, is by definition the same as the saturation humidity ratio, W_s, at an air temperature of 15°C

$$W_{@T=30°C} = W_{s@T=15°C} = 0.622 \left. \frac{p_{ws}}{p - p_{ws}} \right|_{@T=15°C}$$

Temp., °C t	Absolute Pressure p_{ws}, kPa
14	1.5989
15	1.7057

$p_{ws@15C}$ = 1.7057 kPa

$$\boxed{W_{@T=30°C} = W_{s@T=15°C} = 0.622 \frac{1.7057}{101.325 - 1.7057} = 0.01065 \frac{\text{kg}_\text{w}}{\text{kg}_\text{da}}}$$

Degree of saturation

- Need the saturation humidity ratio @ T = 30°C:

$$\mu = \left[\frac{W}{W_s}\right]_{@T=30°C}$$

$$W_{s@T=30°C} = 0.622 \left.\frac{p_{ws}}{p - p_{ws}}\right|_{@T=30°C}$$

Temp., °C t	Absolute Pressure p_{ws}, kPa
30	4.2467
31	4.4966

$p_{ws@15C}$ = 4.2467 kPa

$$W_{s@T=30°C} = 0.622 \frac{4.2467}{101.325 - 4.2467} = 0.02720 \frac{\text{kg}_\text{w}}{\text{kg}_\text{da}}$$

$$\boxed{\mu = \frac{W}{W_s} = \frac{0.01065}{0.02720} = 0.39}$$

Relative humidity

- From previous: $$\phi = \frac{p_w}{p_{ws}}$$

$$p_{w@T=30°C} = p_{ws@T=15°C} = 1.7057\,kPa$$

$$p_{ws@T=30°C} = 4.2467\,kPa$$

$$\boxed{\phi = \frac{1.7057}{4.2467} = 0.40 = 40\%}$$

Enthalpy

$$h \approx 1.006T + W(2501 + 1.86T)$$

*where T is in °C

$$\boxed{h \approx 1.006(30) + (0.01065)(2501 + 1.86(30)) = 57.4\,\frac{\text{kJ}}{\text{kg}}}$$

Specific volume and density

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

$$v \approx 0.287042(30 + 273.15)(1 + 1.6078(0.01065)) / (101.325)$$

$$\boxed{v \approx 0.873 \frac{\mathrm{m}^3}{\mathrm{kg}_{\mathrm{da}}}}$$

$$\boxed{\rho = \frac{1}{v}\left(1 + W\right) = \frac{1}{0.873}\left(1 + 0.01065\right) = 1.157 \frac{\mathrm{kg}}{\mathrm{m}^3}}$$

Wet-bulb temperature

- Wet-bulb temperature is the T_{wb} that fits this equation:

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = 0.01065$$

where: $T = 30°C$ $\quad T_{wb} = ?°C$

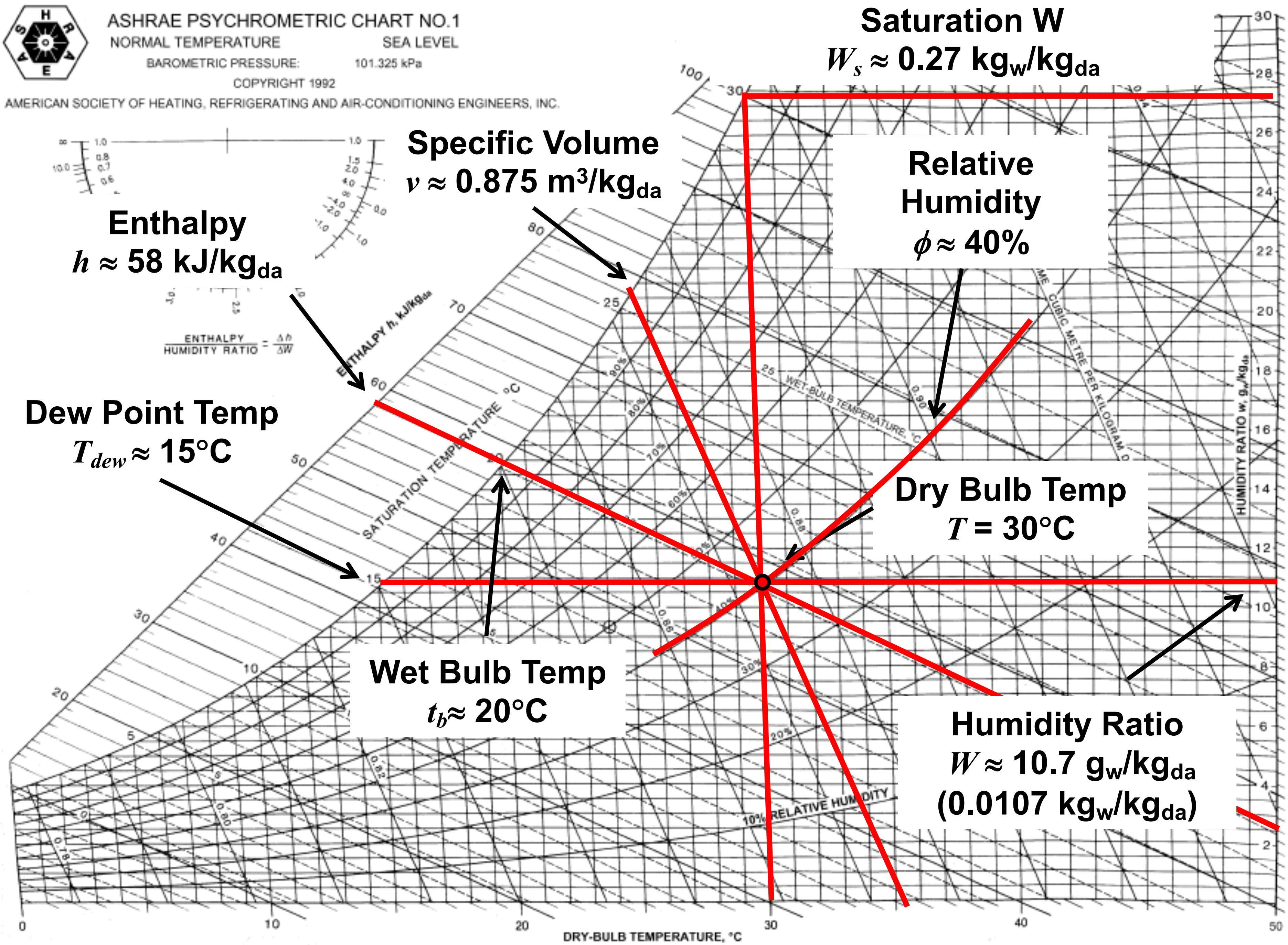
$$W_{s@T_{wb}=?} = 0.622 \left.\frac{p_{ws}}{p - p_{ws}}\right|_{@T_{wb}=?}$$

Procedure:

- Guess T_{wb}, calculate pws for that T, calculate W_s for that T
  - Repeat until W calculated based on those values (and original T) in equation above is equal to actual W (0.01065 in our case)

$$\boxed{T_{wb} = 20.1°C}$$

*Where T_{wb} and T are in <u>Celsius</u>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$(0.0107\ kg_w/kg_{da})$
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
HUMIDITY RATIO W, g_w/kg_{da}
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C

HW 3 assigned

- HW 3 assigned on Blackboard
  - Building (and using) an Excel-based psychrometric calculator
  - Due Tuesday October 8