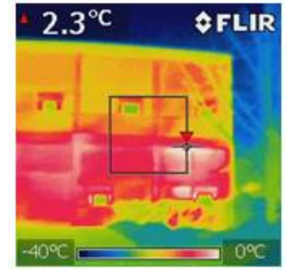
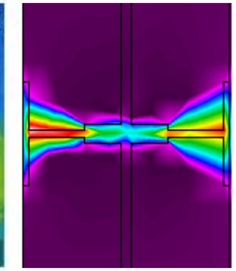
CAE 331/513
Building Science
Fall 2017



October 3, 2017
Psychrometrics (chart and definitions)

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Last time and this time

- Introduced HVAC systems and processes

- Today:
  - Psychrometrics (chart and some equations)
  - Assign HW #3 (due Tuesday Oct 10)

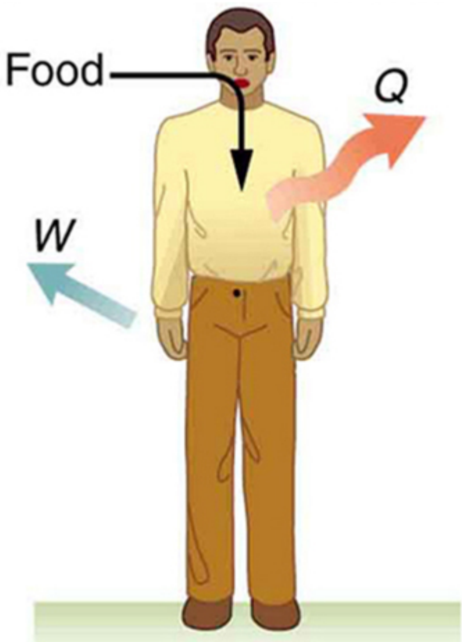
Remember our human thermal comfort lecture?

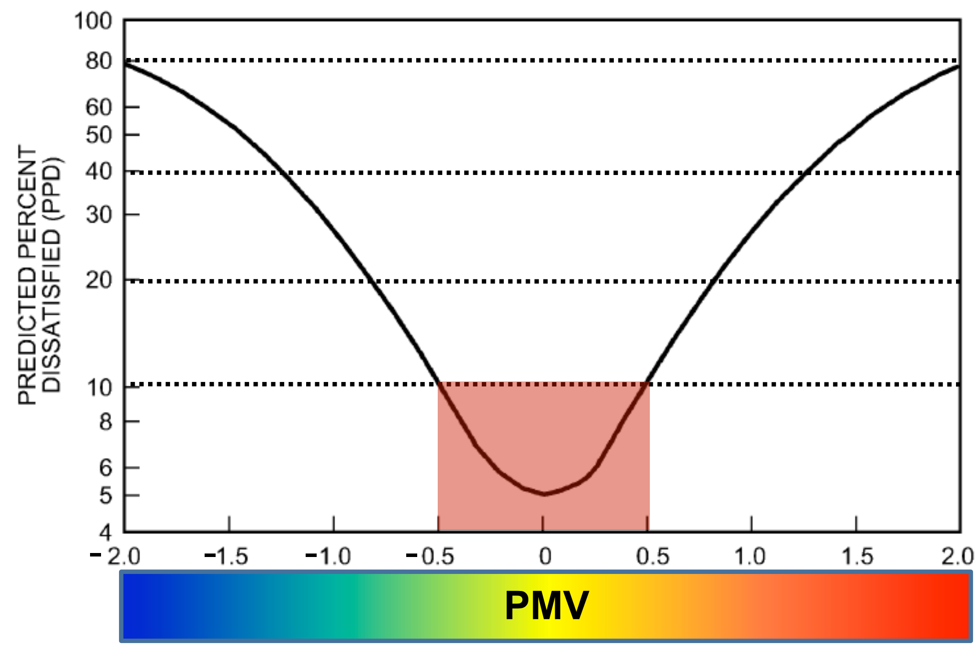
- Human thermal comfort

$$\dot{Q} = MA_{skin}$$

$$A_D = 0.202 m^{0.425} l^{0.725}$$

A_D = DuBois surface area, m^2
m = mass, kg
l = height, m

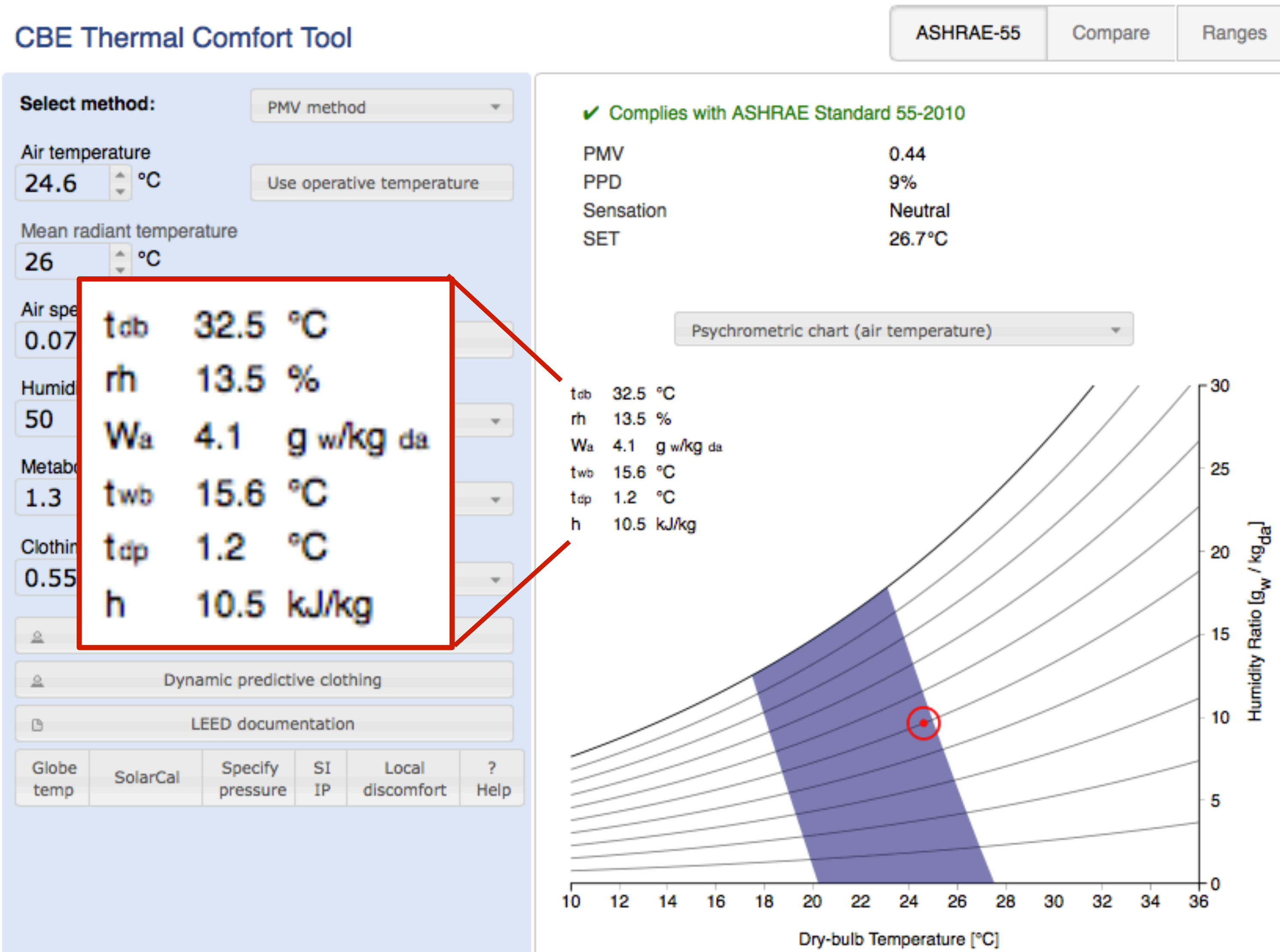




$$1 \text{ met} = 18.4 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} = 58 \frac{\text{W}}{\text{m}^2}$$

-3	-2	-1	0	+1	+2	+3
cold	cool	slightly cool	neutral	slightly warm	warm	hot

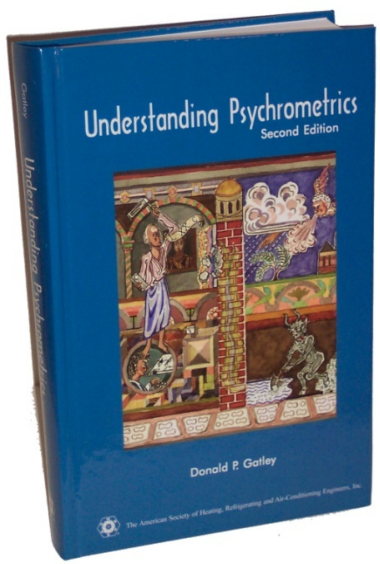
ASHRAE comfort zone: CBE Thermal Comfort Tool



http://smap.cbe.berkeley.edu/comforttool

PSYCHROMETRICS

Psychrometrics



Psychrometrics is the science and engineering of air/vapor mixtures

- For architectural engineers and building scientists, the vapor is usually **water vapor**
- We use psychrometrics to relate the thermodynamic and physical properties of moist air

Applying psychrometrics

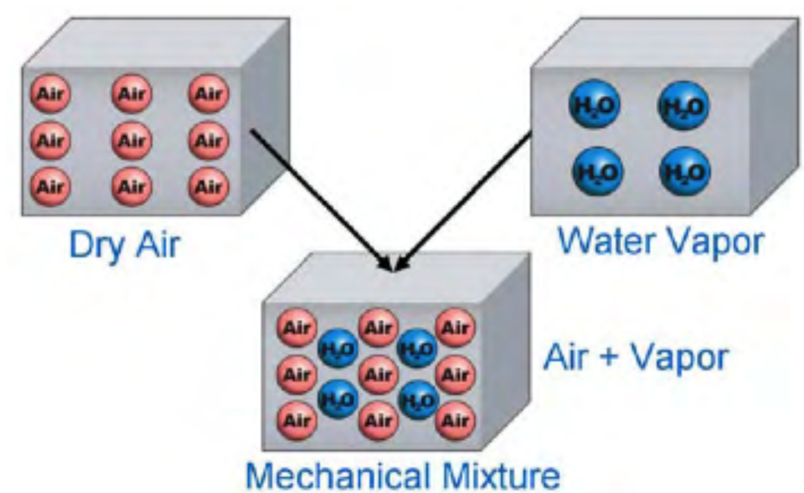
- We need to understand air temperature and moisture content to understand **human thermal comfort**
  - In hot, humid weather we design HVAC systems to remove moisture by dehumidification/cooling
  - In dry, cold weather, we add moisture by humidifiers

- We are also concerned about moisture for energy use, structural, aesthetic, and indoor air quality reasons

- Psychrometrics also involves learning how to use and combine a variety of moist air parameters

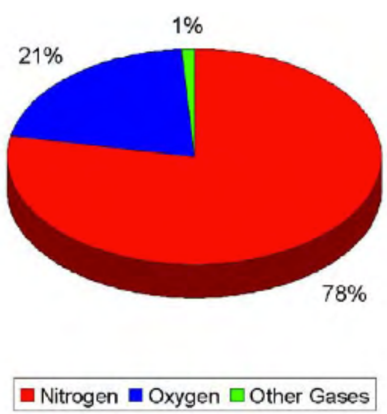
Some definitions for psychrometrics

- **Atmospheric air** contains:
  - Many gaseous components
  - Water vapor
  - Contaminants (particulate matter and gaseous pollutants)
- **Dry air** is atmospheric air with all of the water vapor removed
- **Moist air** is a two-component mixture of dry air and water vapor



Standard composition of dry air

Gas	Molecular weight (g/mol)	Volume %
Nitrogen (N_2)	32.000	78.084
Oxygen (O_2)	28.016	20.946
Argon (Ar)	39.444	0.9340
Carbon Dioxide (CO_2)	44.010	0.03697
Neon (Ne)	20.179	0.00182
Helium (He)	4.002	0.00052
Methane (CH_4)	16.042	0.00014
Krypton	83.800	0.00010



Where does water fit in?

Standard composition of moist air

Gas	Molecular weight (g/mol)	Volume %
Nitrogen (N_2)	32.000	78.084%
Oxygen (O_2)	28.016	20.946%
Water (H_2O)	**18.015**	**0 to 4%**
Argon (Ar)	39.444	0.9340%
Carbon Dioxide (CO_2)	44.010	0.03697%
Neon (Ne)	20.179	0.00182%
Helium (He)	4.002	0.00052%
Methane (CH_4)	16.042	0.00014%
Krypton	83.800	0.00010%

Key terms for describing moist air

- To describe and deal with moist air, we need to be able to describe the relative portions of <u>dry air</u> and <u>water vapor</u>

- There are several different equivalent measures…

- Which one you use depends on what data you have to start with and what quantity you are trying to find

- If you know two properties, you can usually get all the others

Key terms for describing moist air

Key terms to learn today:

1. Dry bulb temperature
2. Vapor pressure
3. Saturation
4. Relative humidity
5. Absolute humidity (or humidity ratio)
6. Dew point temperature
7. Wet bulb temperature
8. Enthalpy
9. Density
10. Specific volume

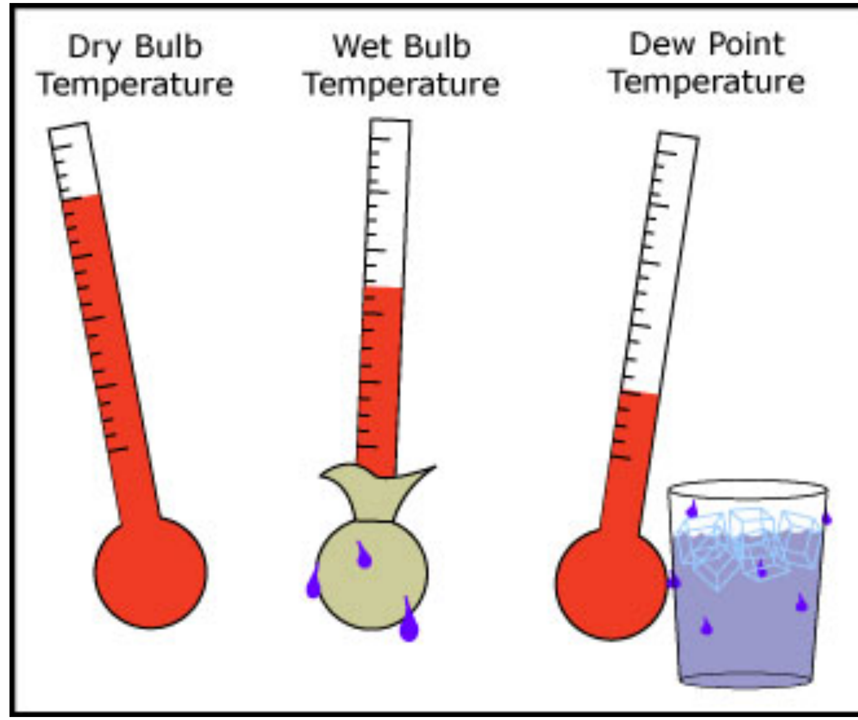
Three different temperatures: T, T_{dew}, and T_{wb}

The standard temperature, T, we are all familiar with is called the **dry-bulb** temperature, or T_d

- It is a measure of internal energy

We can also define:

- **Dew-point** temperature, T_{dew}
  - Temperature at which water vapor changes into liquid (condensation)
  - Air is maximally saturated with water vapor
- **Wet-bulb** temperature, T_{wb}
  - The temperature that a parcel of air would have if it were cooled to saturation (100% relative humidity) by the evaporation of water into it



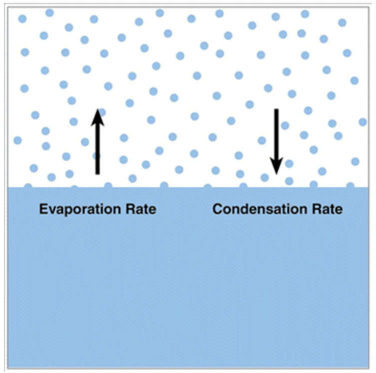
*Units of Celsius, Fahrenheit, or Kelvin

✓ The energy needed to evaporate liquid water (heat of vaporization) is taken from the air in the form of sensible heat and converted to latent heat, which lowers the temperature at constant enthalpy

Key concepts: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is the pressure exerted by a vapor in thermodynamic equilibrium with its condensed phases

p_w *Units of pressure, Pa or kPa
(aka "partial pressure")



- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
  - We call the limit ***saturation***

p_{ws} *Units of pressure, Pa or kPa
(aka "saturation vapor pressure")

Key concept: Relative humidity, ϕ

- **Relative humidity** (RH, or ϕ) is the ratio of the vapor pressure of water vapor in a sample of air to the **saturation** vapor pressure at the dry bulb temperature of the sample

- Relative humidity ≠ absolute humidity!

$$\phi = \frac{p_w}{p_{ws}}$$

Key concept: Saturation vapor pressure, p_{ws}

- The **saturation vapor pressure** is the partial pressure of water vapor at saturation (p_{ws}) *Units of pressure, Pa or kPa
  - Cannot absorb any more moisture at that temperature
- We can look up p_{ws} in tables (as a function of T)
  - Table 3 in Ch.1 of 2013 ASHRAE Fundamentals
- We can also use empirical equations

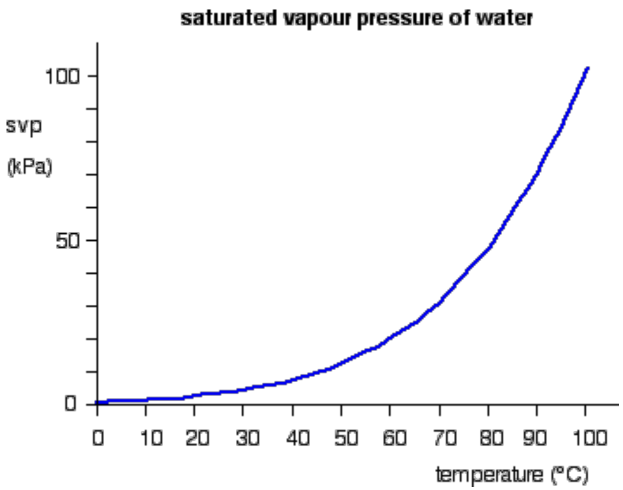
Table 3 Thermodynamic Properties of Water at Saturation (*Continued*)

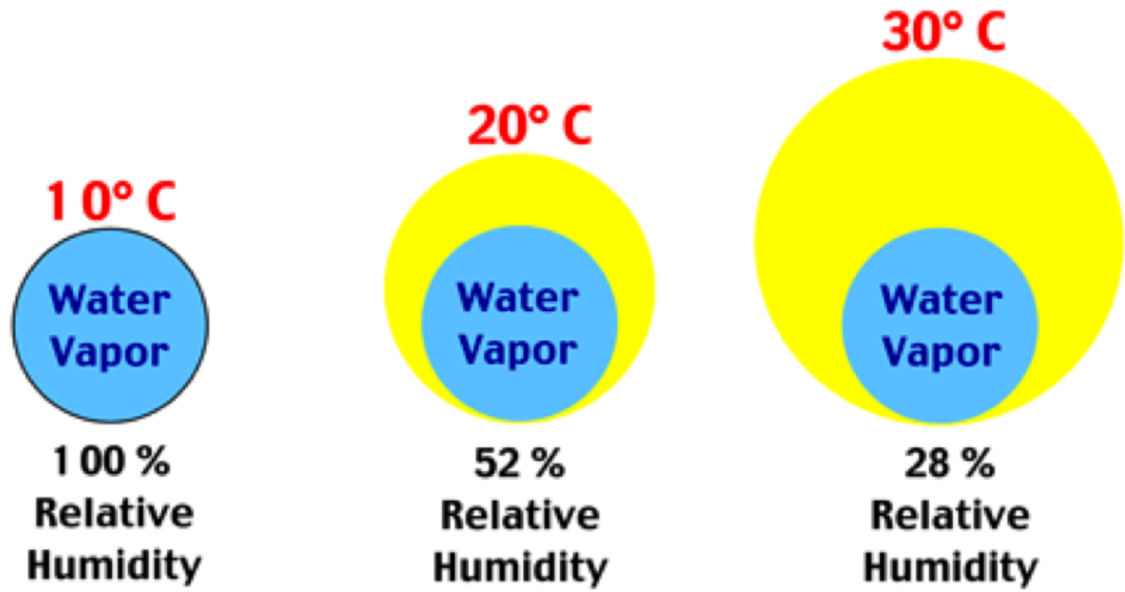
Temp., °C t	Absolute Pressure p_{ws}, kPa	Specific Volume, m^3/kg_w Sat. Liquid v_i/v_f	Evap. v_{ig}/v_{fg}	Sat. Vapor v_g	Specific Enthalpy, kJ/kg_w Sat. Liquid h_i/h_f	Evap. h_{ig}/h_{fg}	Sat. Vapor h_g	Specific Entropy, $kJ/(kg_w \cdot K)$ Sat. Liquid s_i/s_f	Evap. s_{ig}/s_{fg}	Sat. Vapor s_g	Temp., °C t
3	0.7581	0.001000	168.013	168.014	12.60	2493.80	2506.40	0.0459	9.0306	9.0765	3
4	0.8135	0.001000	157.120	157.121	16.81	2491.42	2508.24	0.0611	8.9895	9.0506	4
5	0.8726	0.001000	147.016	147.017	21.02	2489.05	2510.07	0.0763	8.9486	9.0249	5
6	0.9354	0.001000	137.637	137.638	25.22	2486.68	2511.91	0.0913	8.9081	8.9994	6
7	1.0021	0.001000	128.927	128.928	29.43	2484.31	2513.74	0.1064	8.8678	8.9742	7
8	1.0730	0.001000	120.833	120.834	33.63	2481.94	2515.57	0.1213	8.8278	8.9492	8
9	1.1483	0.001000	113.308	113.309	37.82	2479.58	2517.40	0.1362	8.7882	8.9244	9
10	1.2282	0.001000	106.308	106.309	42.02	2477.21	2519.23	0.1511	8.7488	8.8998	10
11	1.3129	0.001000	99.792	99.793	46.22	2474.84	2521.06	0.1659	8.7096	8.8755	11
12	1.4028	0.001001	93.723	93.724	50.41	2472.48	2522.89	0.1806	8.6708	8.8514	12
13	1.4981	0.001001	88.069	88.070	54.60	2470.11	2524.71	0.1953	8.6322	8.8275	13
14	1.5989	0.001001	82.797	82.798	58.79	2467.75	2526.54	0.2099	8.5939	8.8038	14
15	1.7057	0.001001	77.880	77.881	62.98	2465.38	2528.36	0.2245	8.5559	8.7804	15
16	1.8188	0.001001	73.290	73.291	67.17	2463.01	2530.19	0.2390	8.5181	8.7571	16
17	1.9383	0.001001	69.005	69.006	71.36	2460.65	2532.01	0.2534	8.4806	8.7341	17
18	2.0647	0.001001	65.002	65.003	75.55	2458.28	2533.83	0.2678	8.4434	8.7112	18
19	2.1982	0.001002	61.260	61.261	79.73	2455.92	2535.65	0.2822	8.4064	8.6886	19

Relative humidity and temperature

- **Relative humidity** (RH, or ϕ) is a <u>function of temperature</u>

$$\phi = \frac{p_w}{p_{ws}}$$





Key concept: Humidity ratio, W

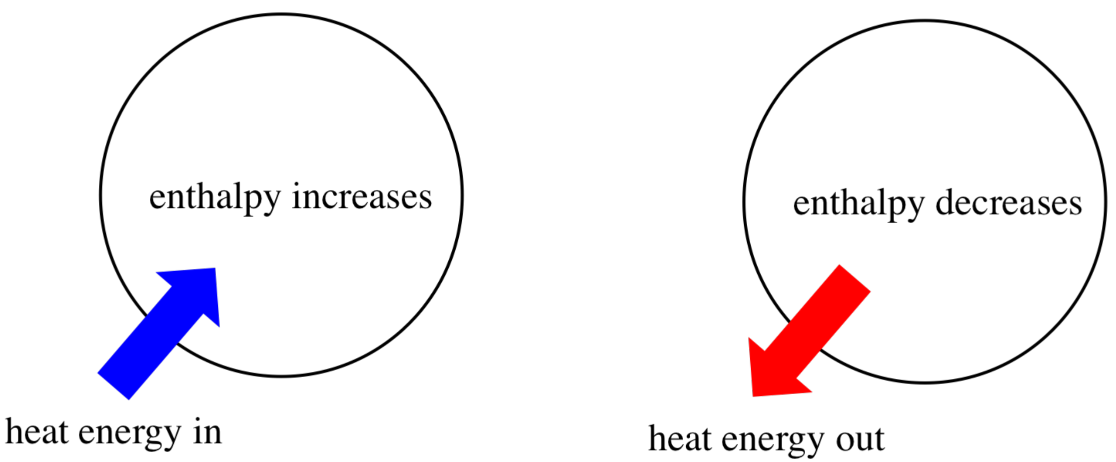
- The **humidity ratio** is a measure of the mass of water vapor present in a parcel of air (a measure of *absolute humidity*)

- The humidity ratio is simply the mass of water vapor that exists in a parcel of mass of *dry air*
  - Units of mass of water vapor per mass of dry air
    - kg/kg (kg_w/kg_{da})
    - g/kg (g_w/kg_{da})

$$W = \frac{\text{mass of water vapor}}{\text{mass of dry air}} \quad \left[\frac{kg_w}{kg_{da}}\right]$$

Key concept: Enthalpy

- **Enthalpy** is a measure of the amount of energy in a system
  - Units of Joules or BTU (or J/kg or BTU/lb)
- The enthalpy of moist air is the total enthalpy of the dry air plus the water vapor mixture per mass of moist air
- Includes:
  - Enthalpy of dry air, or sensible heat
  - Enthalpy of evaporated water, or latent heat



Key concept: Density and specific volume

Air density

- Density is a measure of the mass of moist air per unit volume of air
- Includes mass of dry air + water vapor

$$\rho = \frac{\text{mass of moist air}}{\text{volume of moist air}} \quad [\frac{\text{kg}}{\text{m}^3}]$$

Specific volume

- Specific volume is the volume of unit mass of dry air at a given temperature, expressed as m^3/kg (inverse of dry air density)

$$\upsilon = \frac{\text{volume of dry air}}{\text{mass of dry air}} \quad [\frac{\text{m}^3}{\text{kg}_{da}}]$$

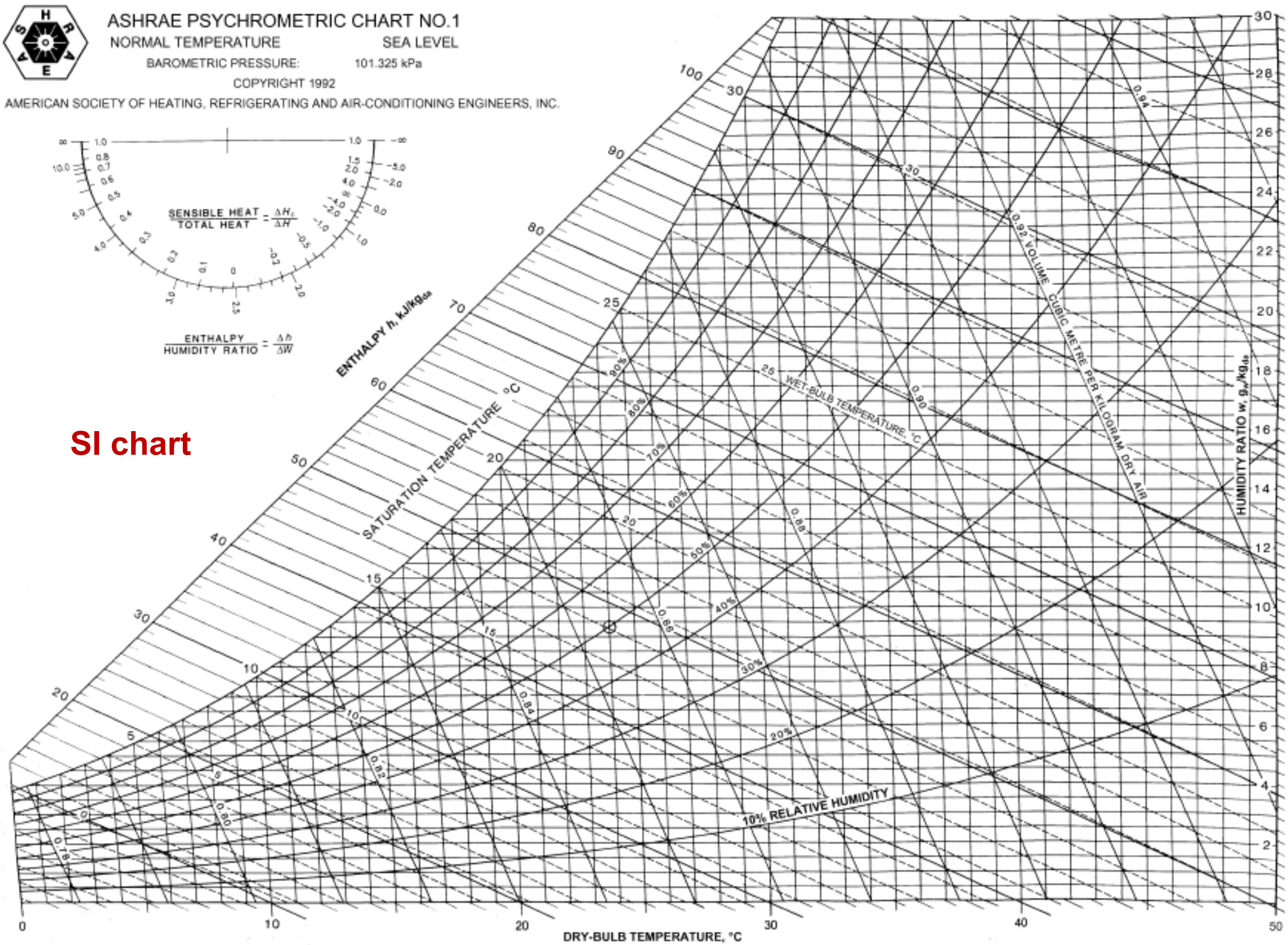
The Psychrometric Chart

- There are both simple and complex ways to estimate these properties
  - Equations and tables (more complex, save for next lecture)
  - Graphically using …

- **The Psychrometric Chart**
  - Plots dry bulb temperature (T) on the x-axis and humidity ratio (W) on the y-axis
    - Shows relationships between T and W and relative humidity, wet-bulb temperature, vapor pressure, specific volume, and enthalpy
  - Charts are unique at each value of atmospheric pressure (p)

- Both SI and IP versions are on BB in the ASHRAE materials folder

SI chart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SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
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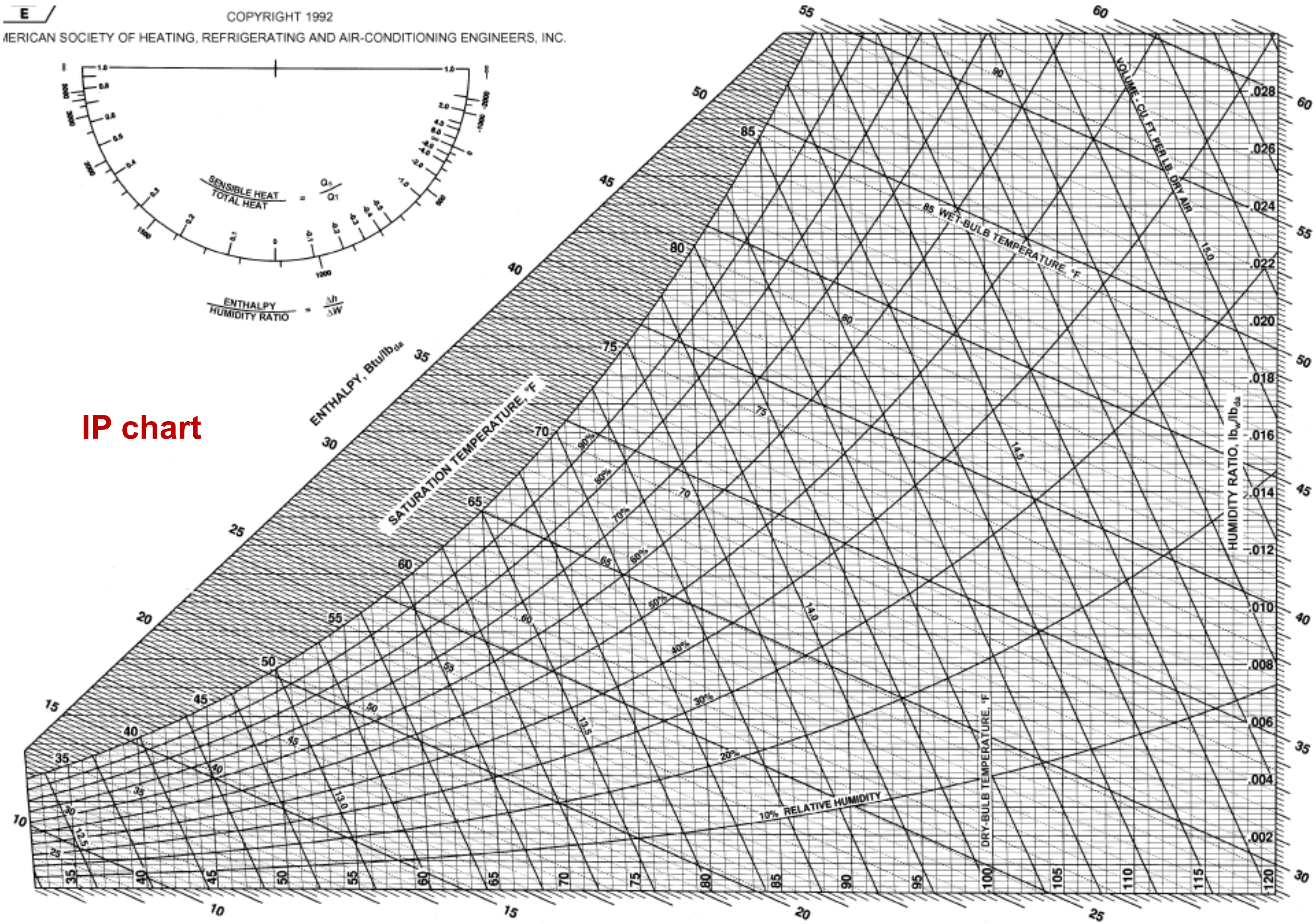
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM DRY AIR
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, gw/kg da
DRY-BULB TEMPERATURE, °C



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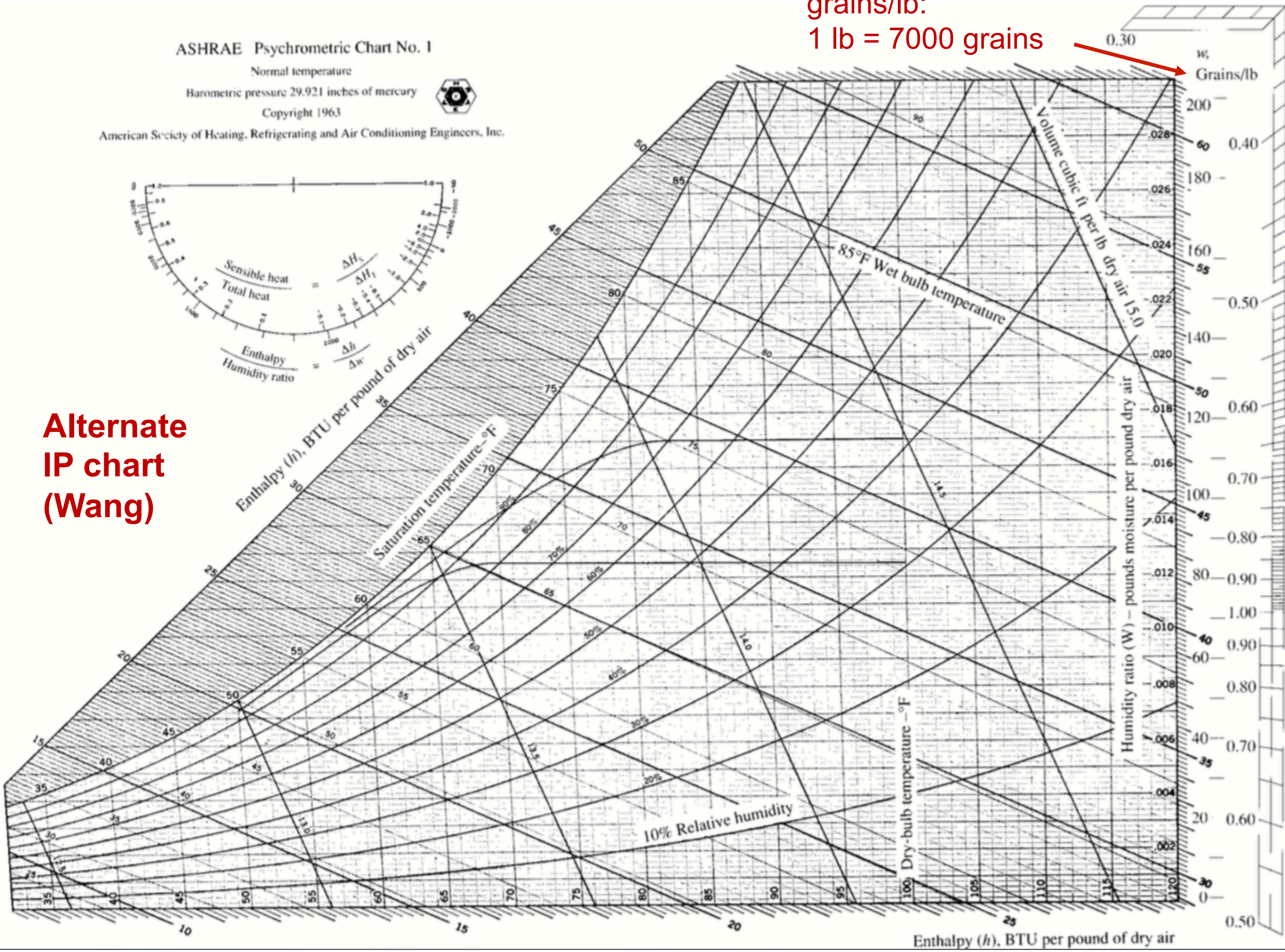
SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
IP chart
ENTHALPY, Btu/lbda
SATURATION TEMPERATURE, °F
WET-BULB TEMPERATURE, °F
VOLUME - CU. FT. PER LB. DRY AIR
HUMIDITY RATIO, lbw/lbda
DRY-BULB TEMPERATURE, °F
10% RELATIVE HUMIDITY

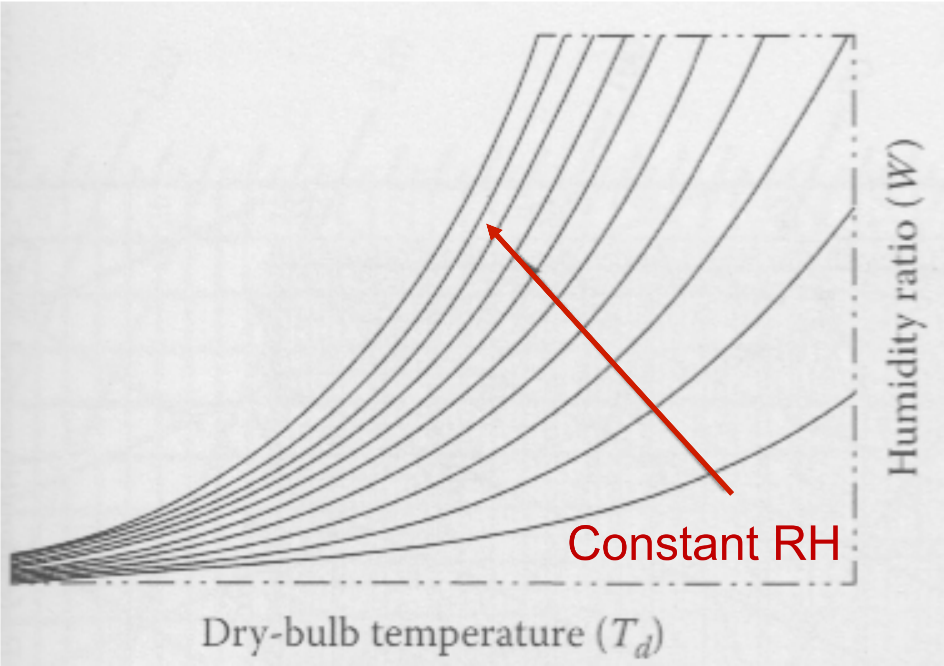
grains/lb:
1 lb = 7000 grains

Alternate IP chart (Wang)

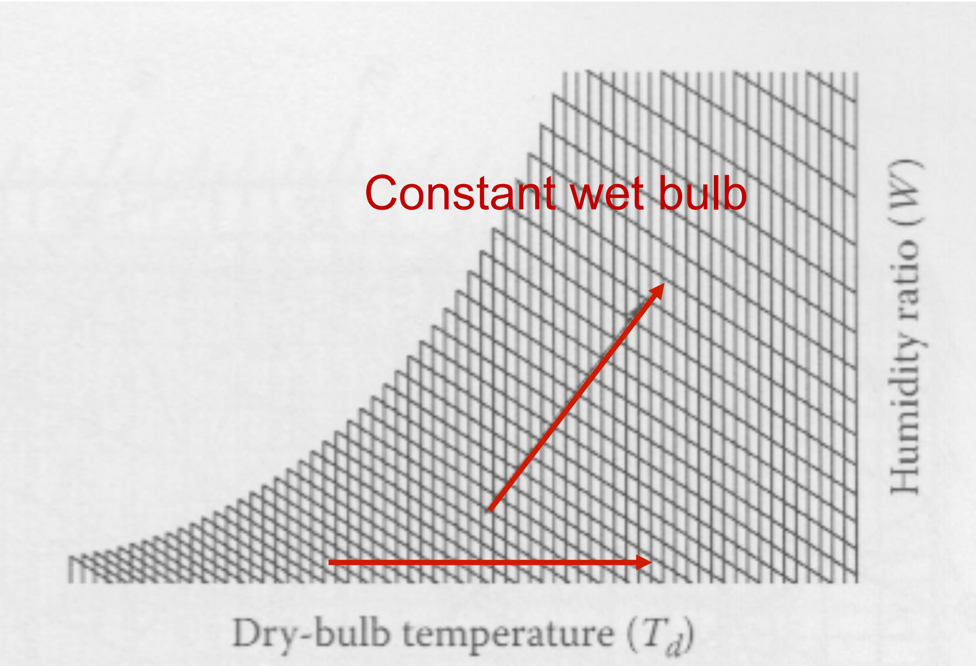


Deciphering the psychrometric chart

Lines of constant RH



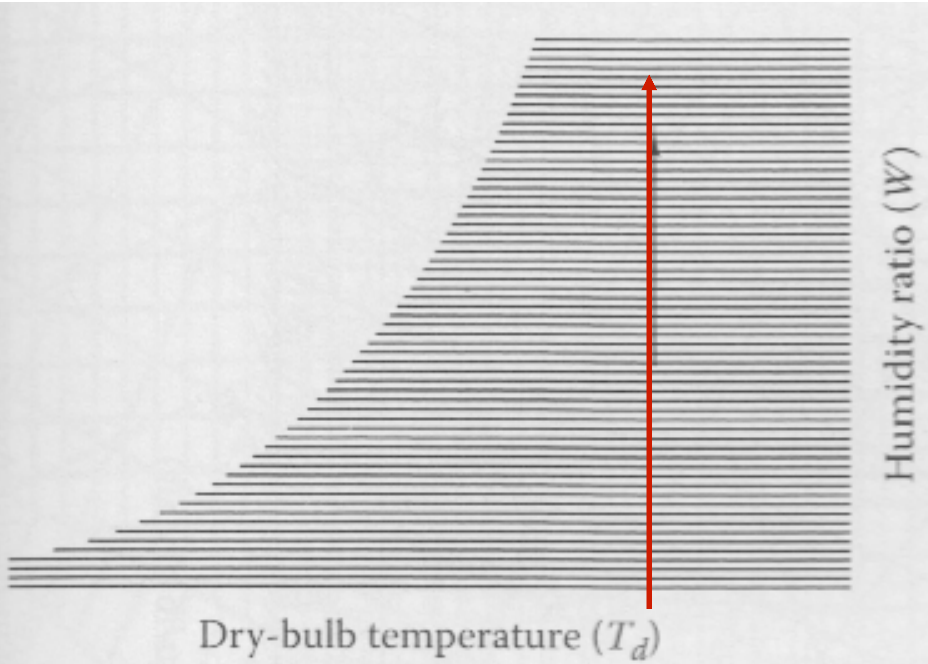
Lines of constant wet-bulb and dry-bulb



Constant dry bulb temperature

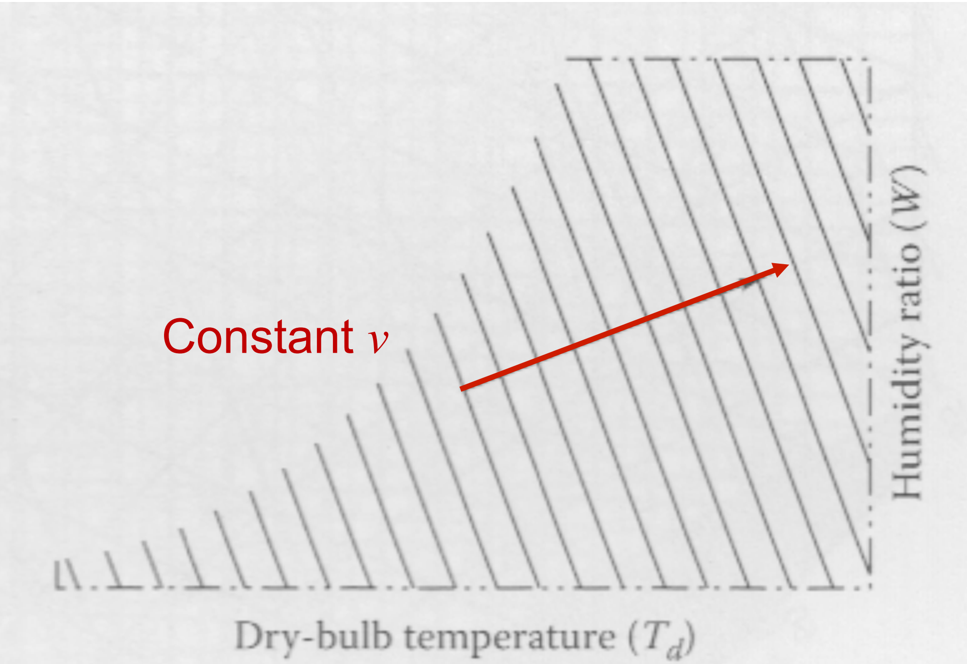
Deciphering the psychrometric chart

Lines of constant humidity ratio



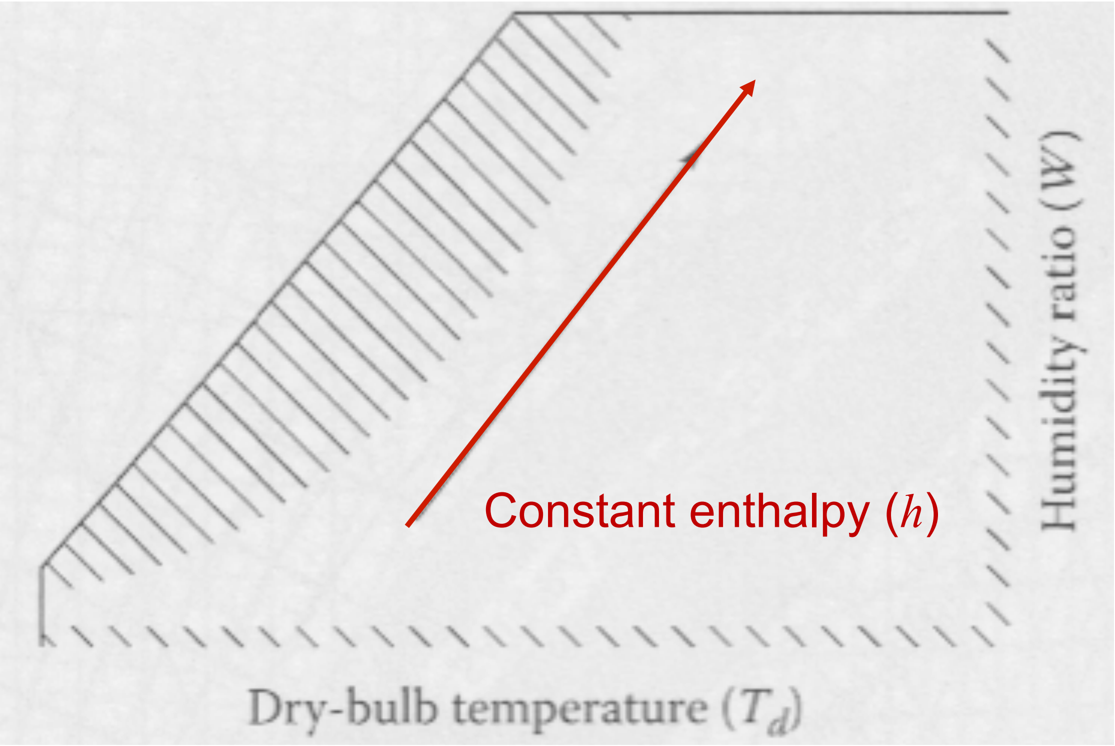
Constant W

Lines of constant specific volume



Deciphering the psychrometric chart

Lines of constant enthalpy



Some psychrometric examples

Moist air exists at 22°C dry-bulb temperature with 50% RH

Find the following:

(a) the humidity ratio, W

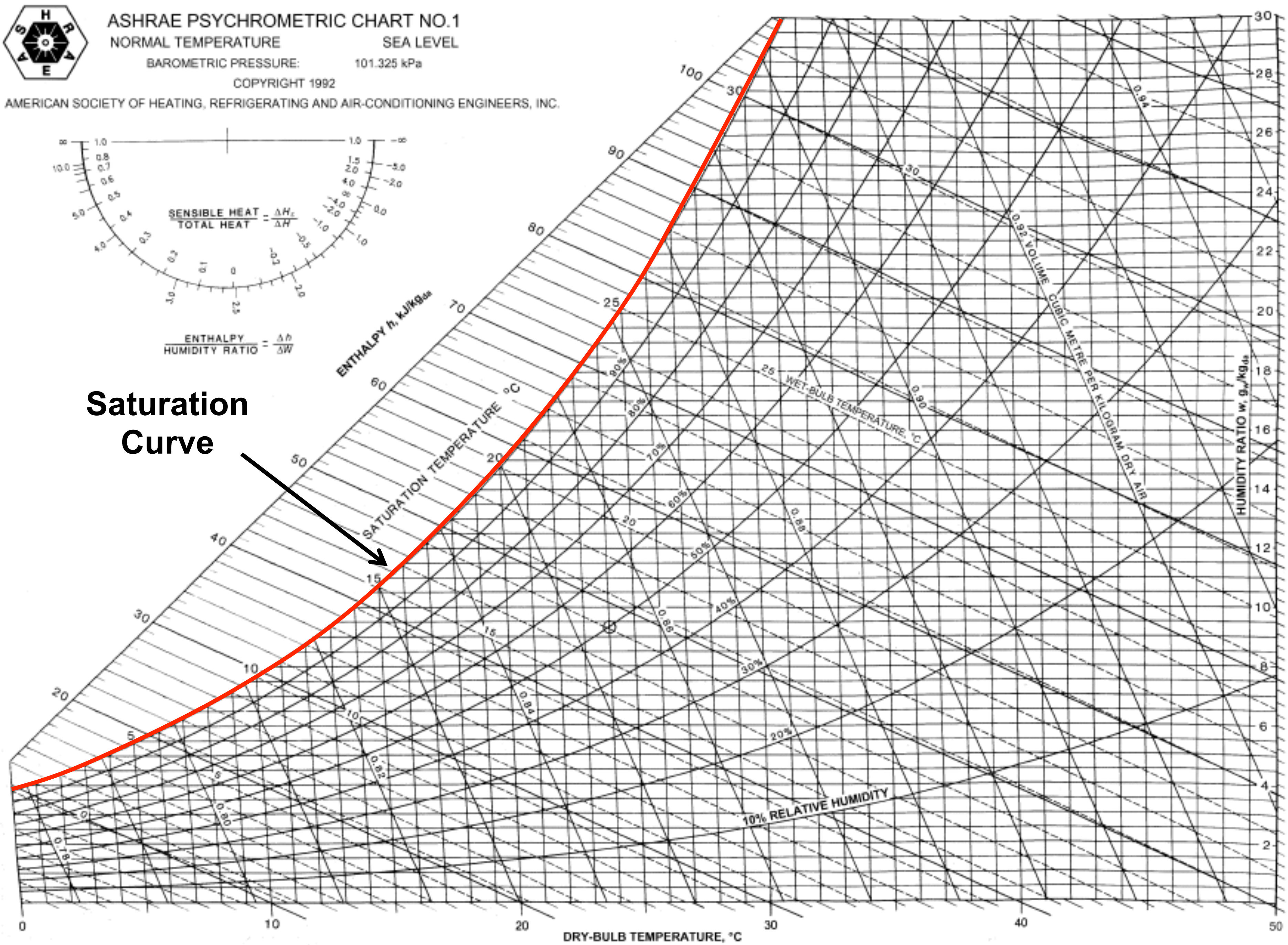
(b) dew point temperature, T_{dew}

(c) wet-bulb temperature, T_{wb}

(d) enthalpy, h

(e) specific volume, v

(f) dry air density, ρ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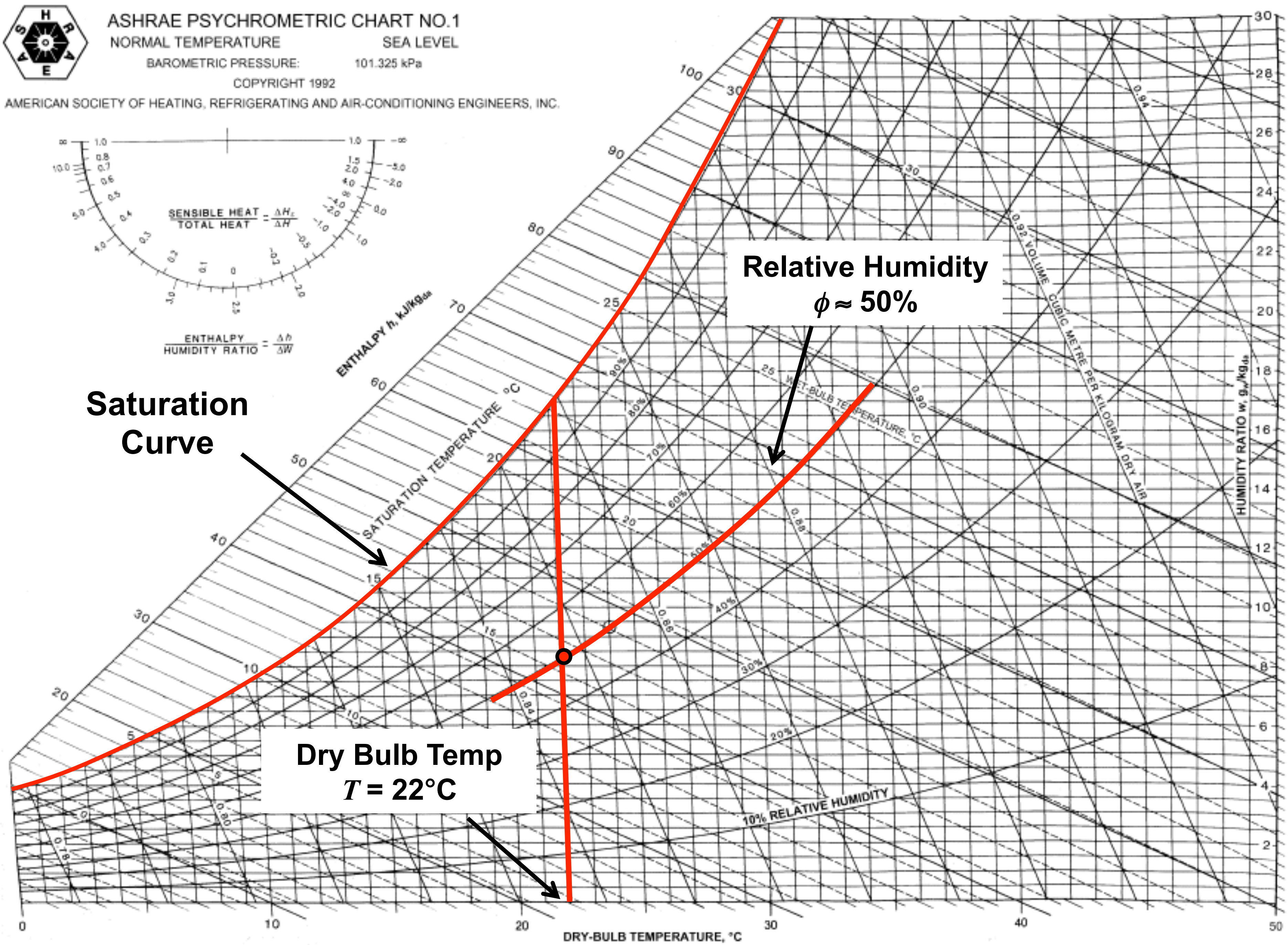
0.92 VOLUME CUBIC METRE PER KILOGRAM DRY AIR
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, gw/kgda
DRY-BULB TEMPERATURE, °C
Saturation Curve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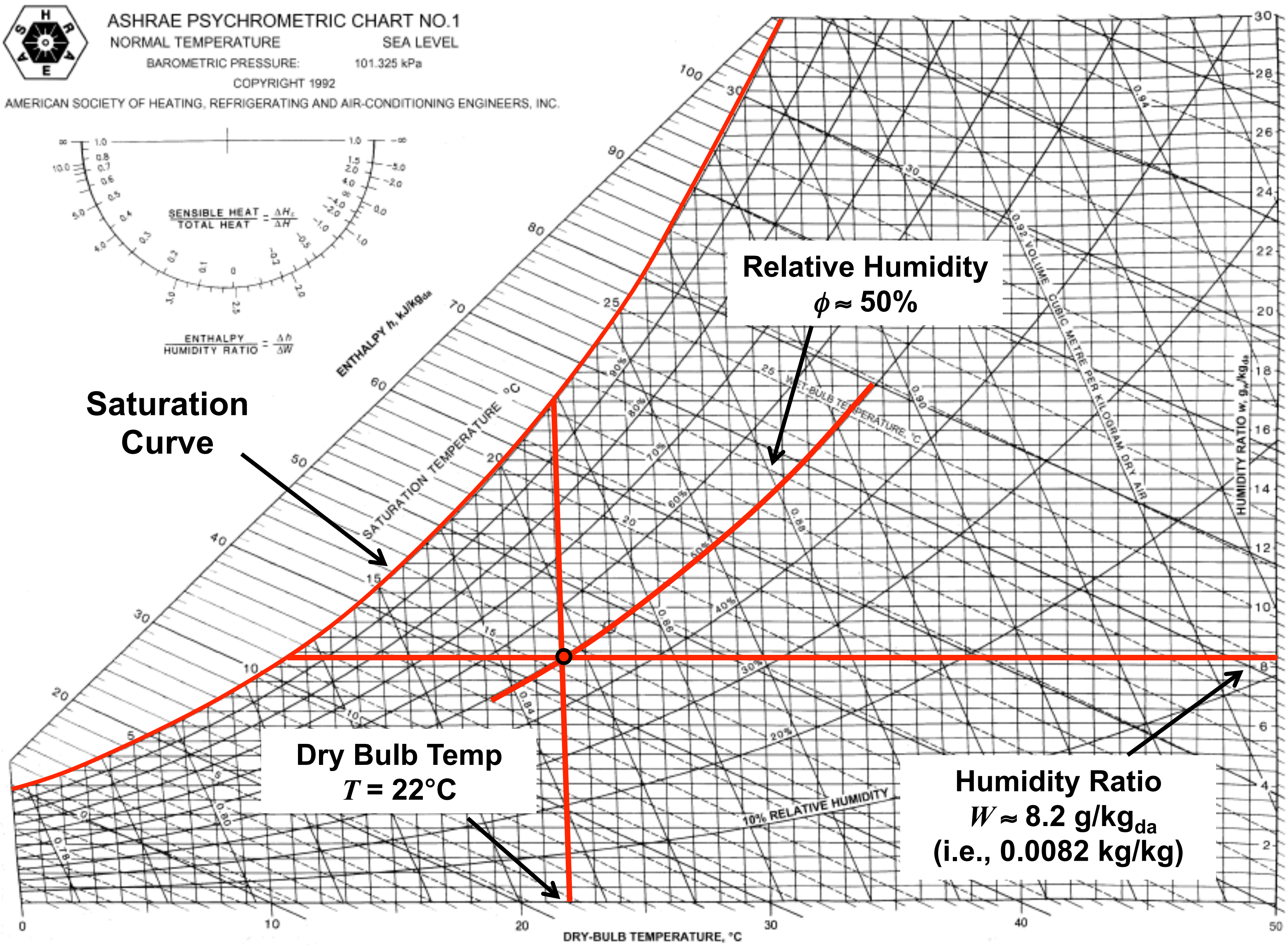
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Saturation Curve
Relative Humidity
φ ≈ 50%
Dry Bulb Temp
T = 22°C



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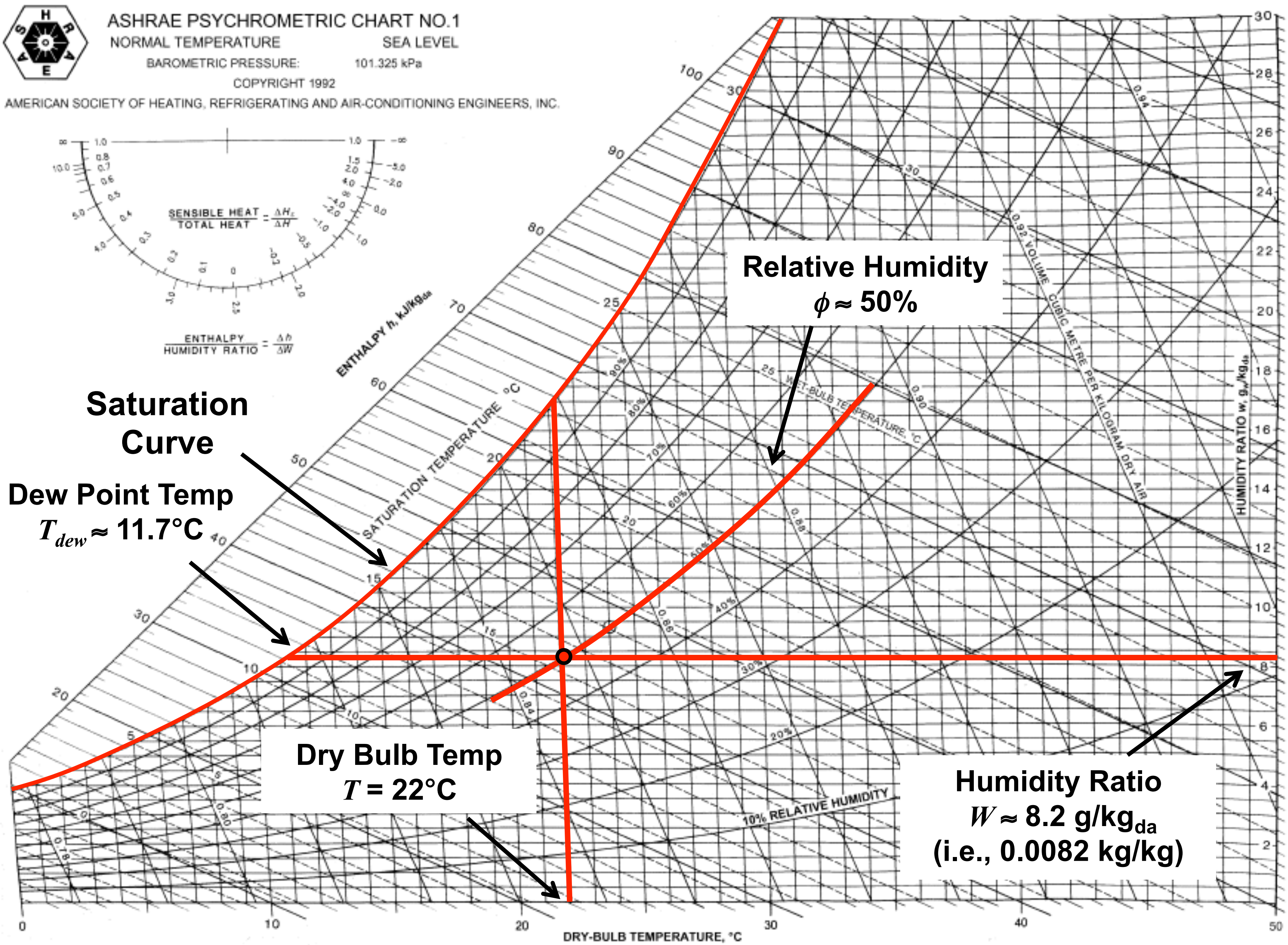
DRY-BULB TEMPERATURE, °C
Saturation Curve
Relative Humidity
φ ≈ 50%
Dry Bulb Temp
T = 22°C
Humidity Ratio
W ≈ 8.2 g/kg da
(i.e., 0.0082 kg/kg)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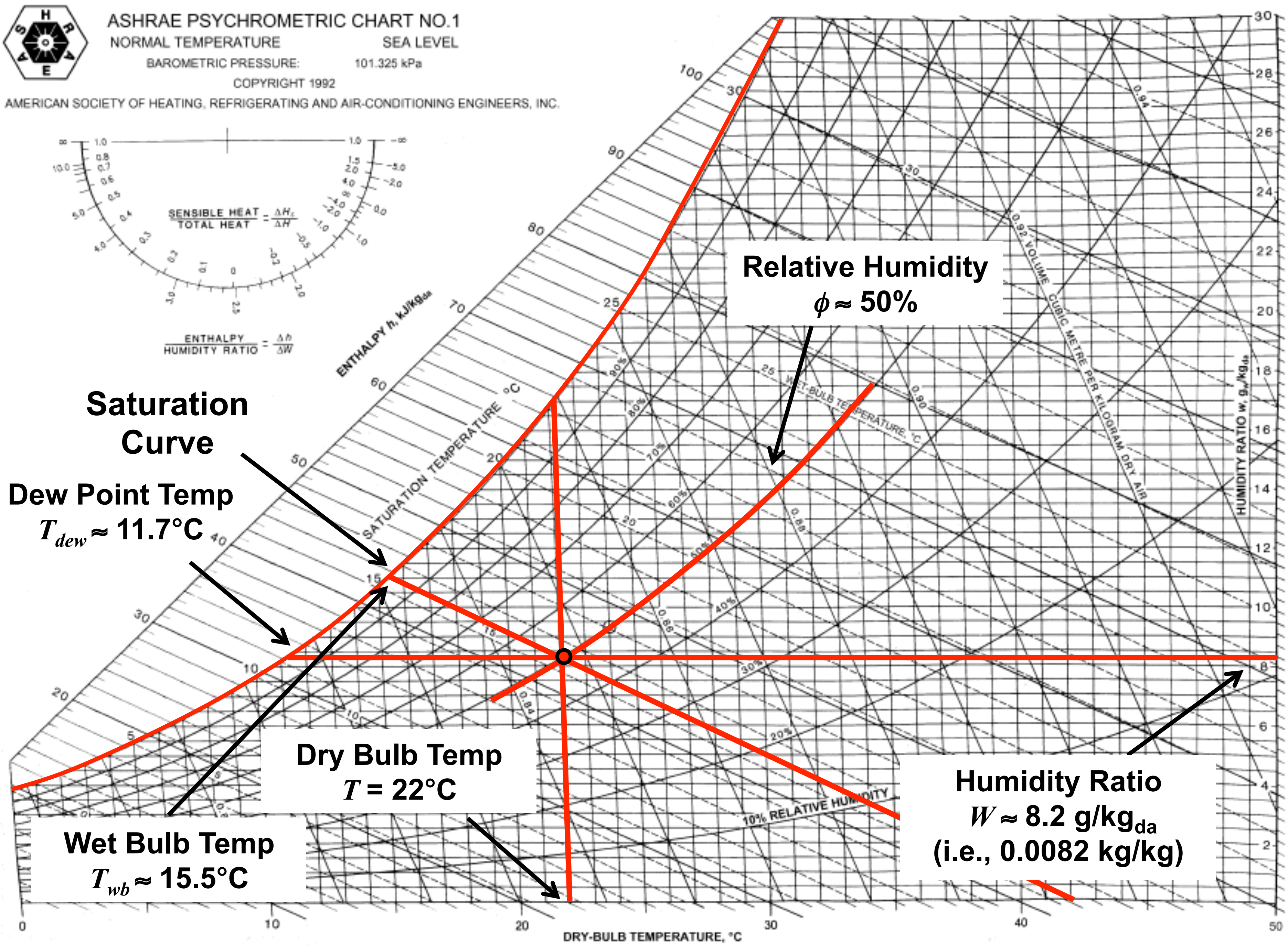
HUMIDITY RATIO w, g w/kg da
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Saturation Curve
Dew Point Temp
$T_{dew} \approx 11.7°C$
Relative Humidity
$\phi \approx 50\%$
Dry Bulb Temp
$T = 22°C$
Humidity Ratio
$W \approx 8.2\ g/kg_{da}$
(i.e., 0.0082 kg/kg)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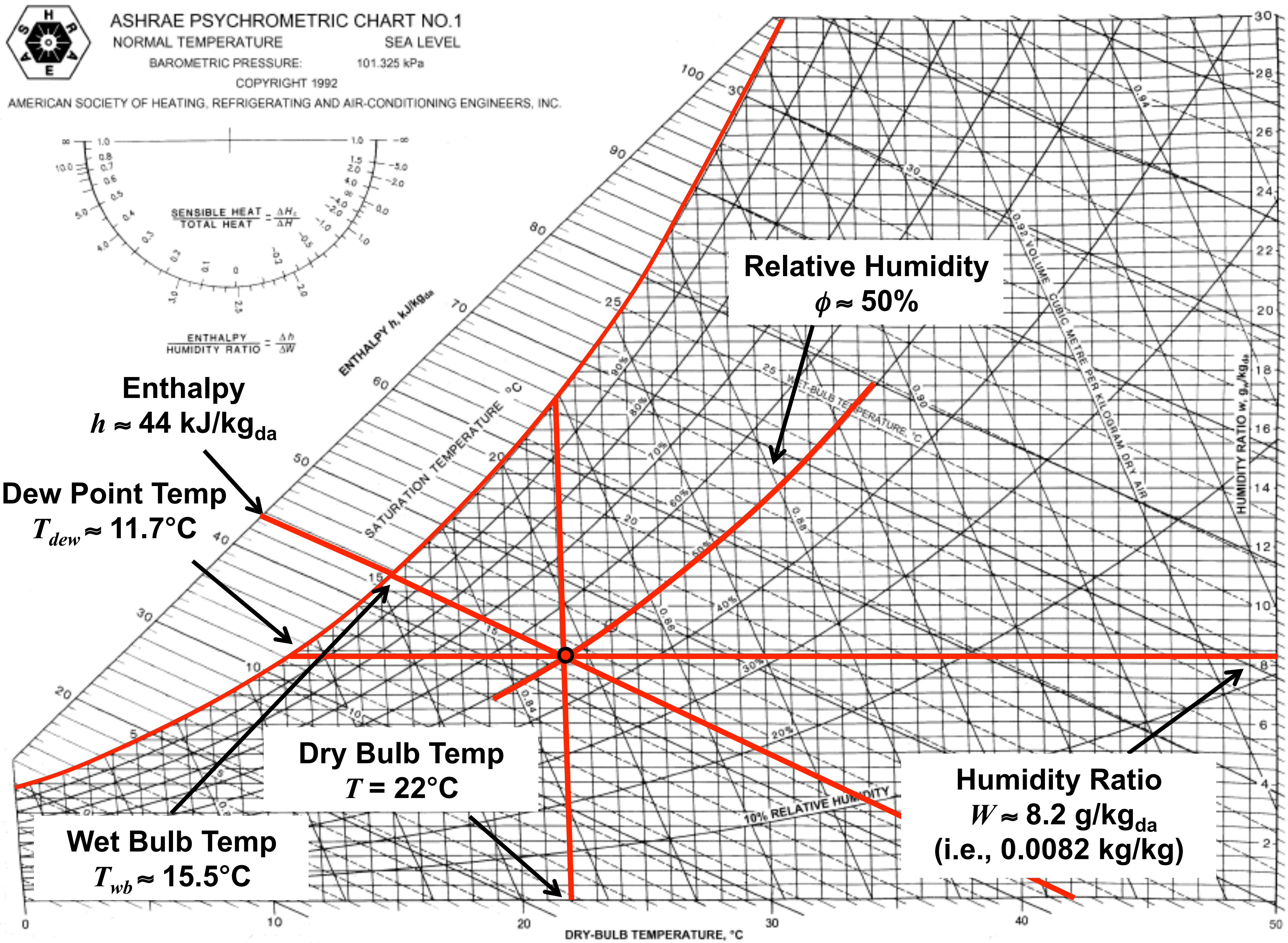
Saturation Curve
Dew Point Temp
$T_{dew} \approx 11.7$°C
Relative Humidity
$\phi \approx 50\%$
Dry Bulb Temp
$T = 22$°C
Wet Bulb Temp
$T_{wb} \approx 15.5$°C
Humidity Ratio
$W \approx 8.2$ g/kg$_{da}$
(i.e., 0.0082 kg/kg)


ASHRAE PSYCHROMETRIC CHART NO.1
NORMAL TEMPERATURE
SEA LEVEL
BAROMETRIC PRESSURE: 101.325 kPa
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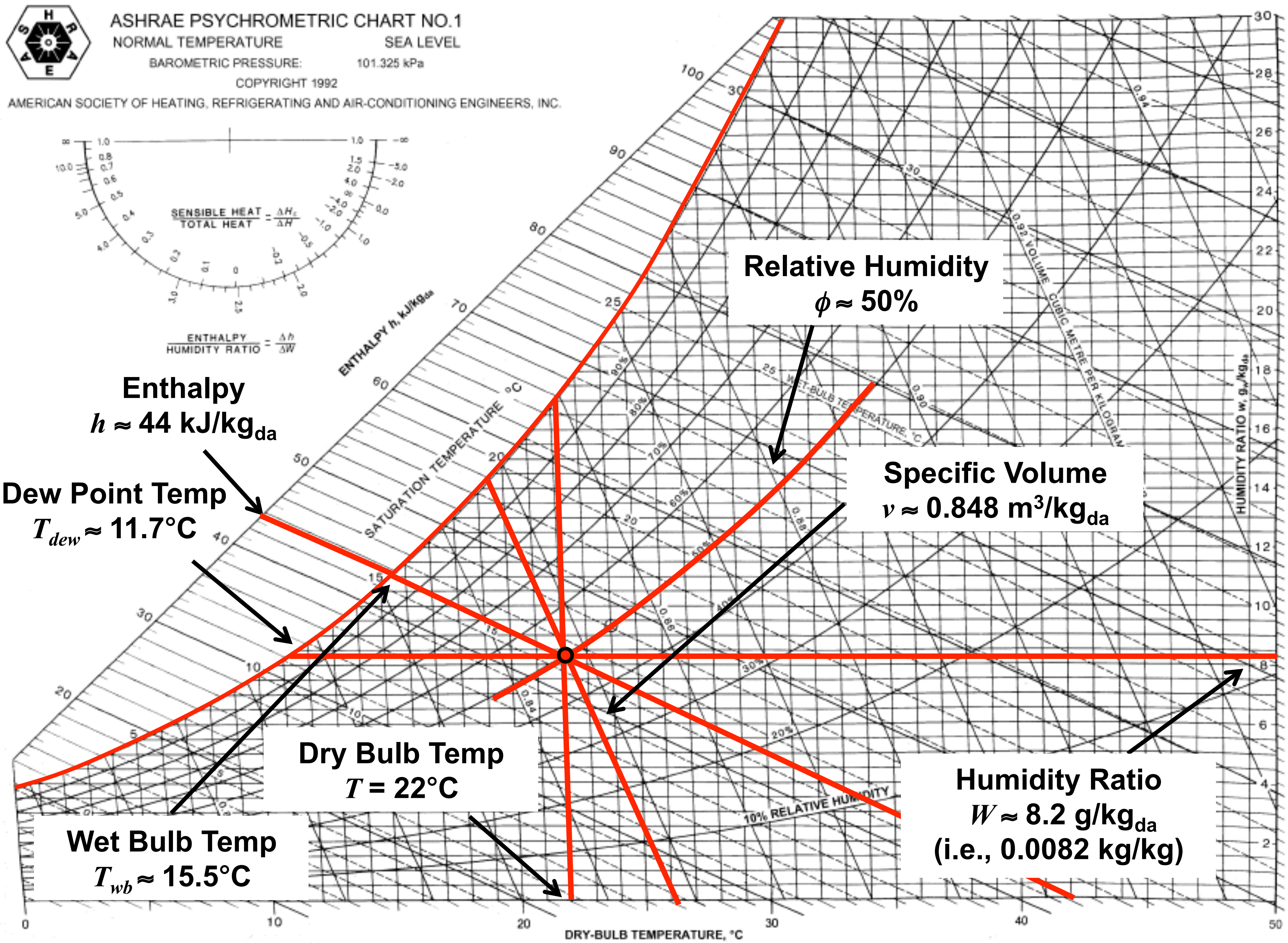
SENSIBLE HEAT / TOTAL HEAT = ΔHs / ΔH
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
0.92 VOLUME CUBIC METRE PER KILOGRAM DRY AIR
HUMIDITY RATIO w, gw/kgda
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Relative Humidity
ϕ ≈ 50%
Enthalpy
h ≈ 44 kJ/kgda
Dew Point Temp
Tdew ≈ 11.7°C
Dry Bulb Temp
T = 22°C
Wet Bulb Temp
Twb ≈ 15.5°C
Humidity Ratio
W ≈ 8.2 g/kgda
(i.e., 0.0082 kg/kg)



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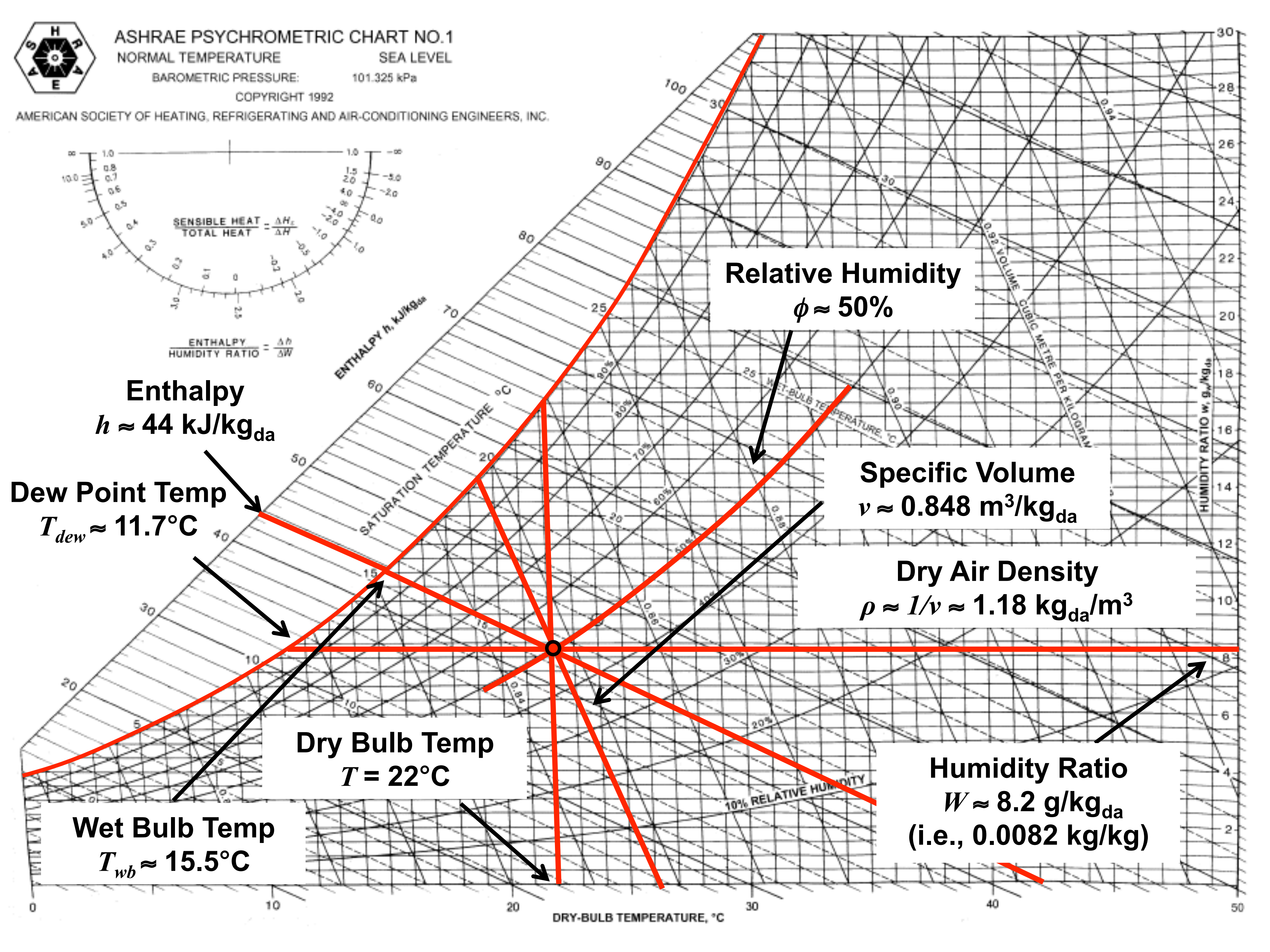
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM
HUMIDITY RATIO W, g/kg da
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Relative Humidity
$\phi \approx 50\%$
Enthalpy
$h \approx 44$ kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx 11.7$°C
Specific Volume
$v \approx 0.848$ m^3/kg$_{da}$
Dry Bulb Temp
$T = 22$°C
Wet Bulb Temp
$T_{wb} \approx 15.5$°C
Humidity Ratio
$W \approx 8.2$ g/kg$_{da}$
(i.e., 0.0082 kg/kg)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ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM
HUMIDITY RATIO w, gw/kgda
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Relative Humidity
$\phi \approx 50\%$
Enthalpy
$h \approx$ 44 kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx$ 11.7°C
Specific Volume
$v \approx$ 0.848 m^3/kg$_{da}$
Dry Air Density
$\rho \approx 1/v \approx$ 1.18 kg$_{da}$/m^3
Dry Bulb Temp
T = 22°C
Humidity Ratio
$W \approx$ 8.2 g/kg$_{da}$
(i.e., 0.0082 kg/kg)
Wet Bulb Temp
$T_{wb} \approx$ 15.5°C

Some psychrometric examples

Moist air exists at 30°C dry-bulb temperature with a 15°C dew point temperature

Find the following:

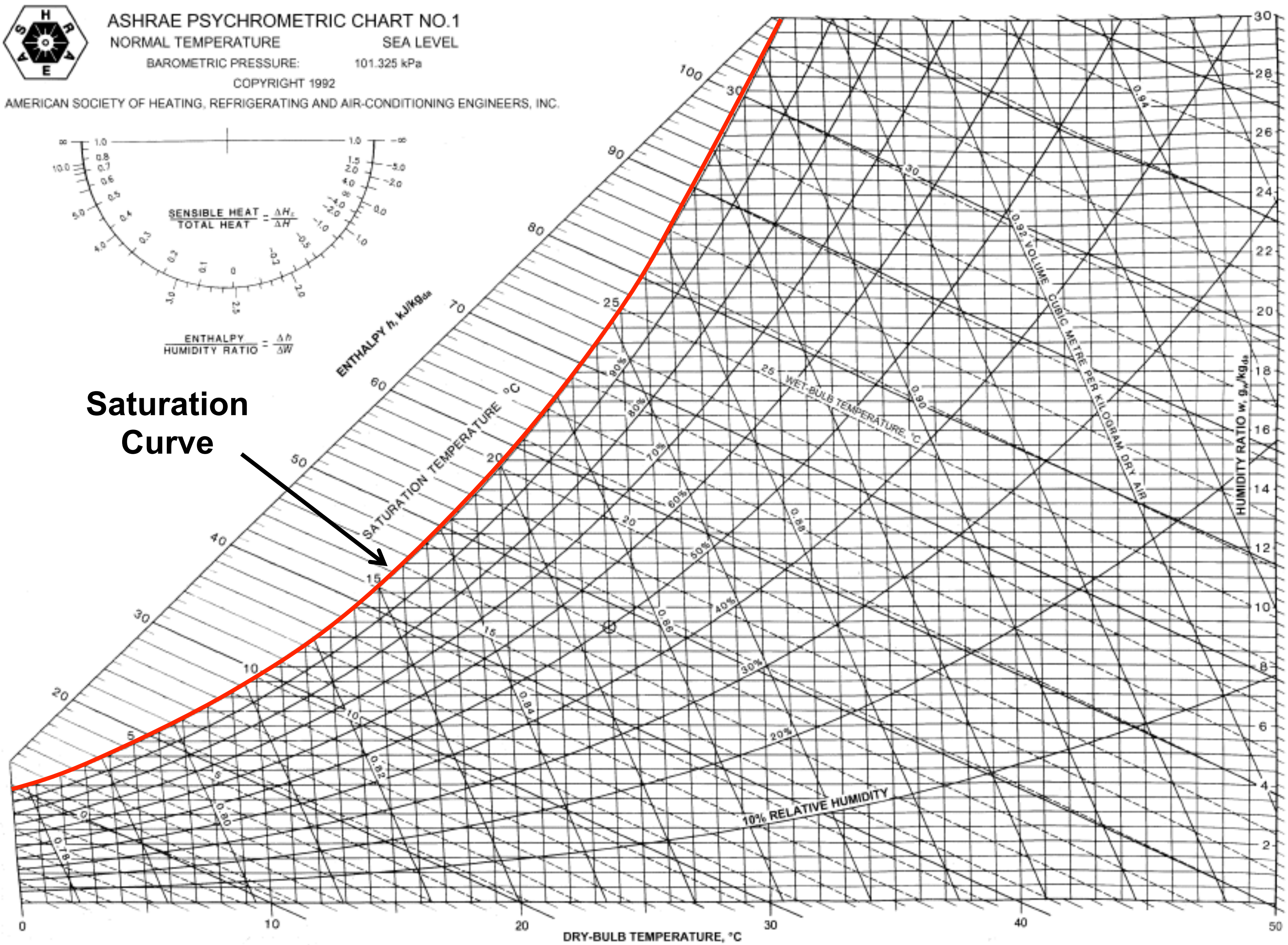
(a) the humidity ratio, W

(b) wet-bulb temperature, T_{wb}

(c) enthalpy, h

(d) specific volume, v

(e) relative humidity, ϕ



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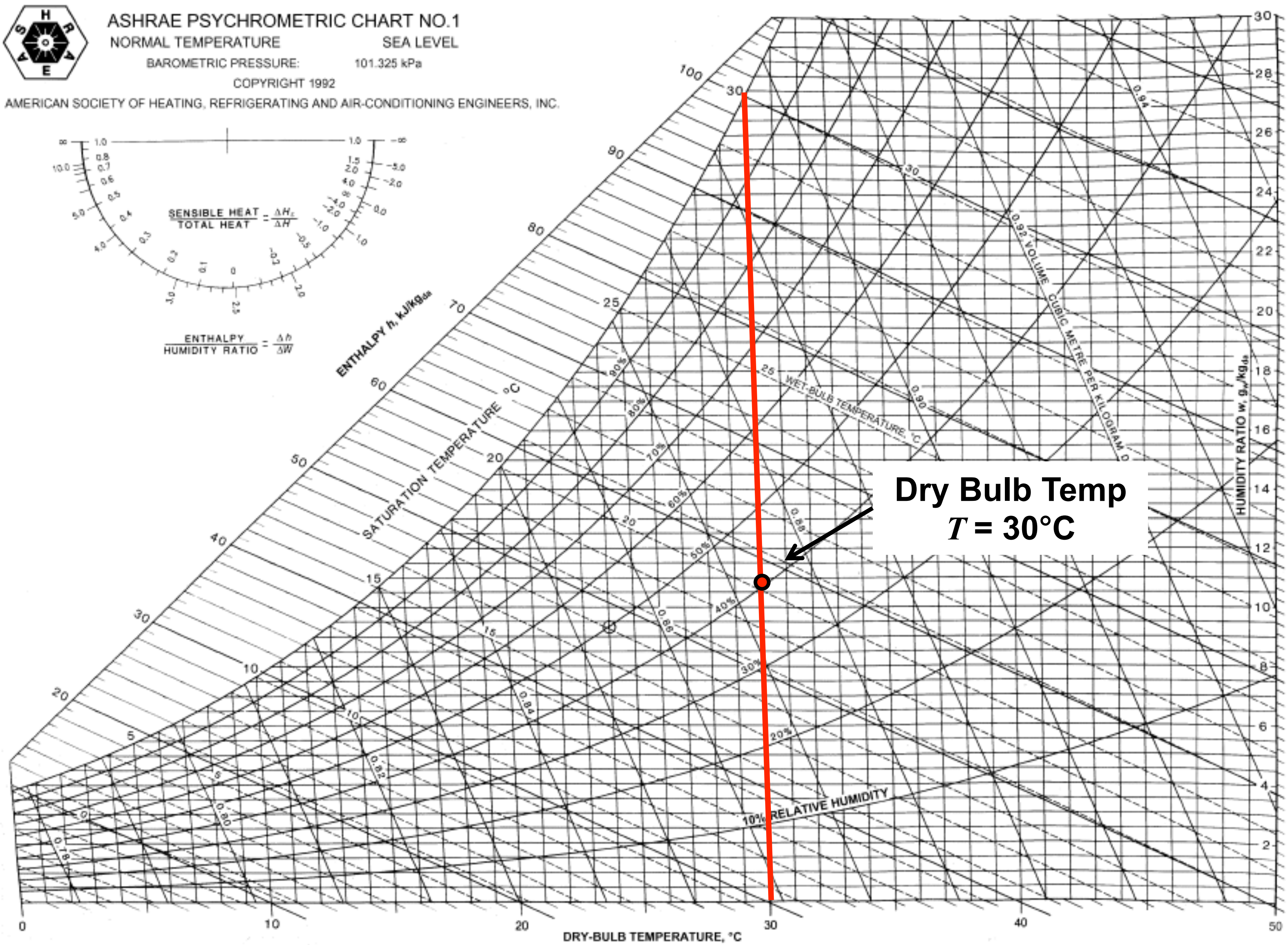
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY h, kJ/kg_da
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM DRY AIR
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, g_w/kg_da
DRY-BULB TEMPERATURE, °C
Saturation Curve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SATURATION TEMPERATURE °C
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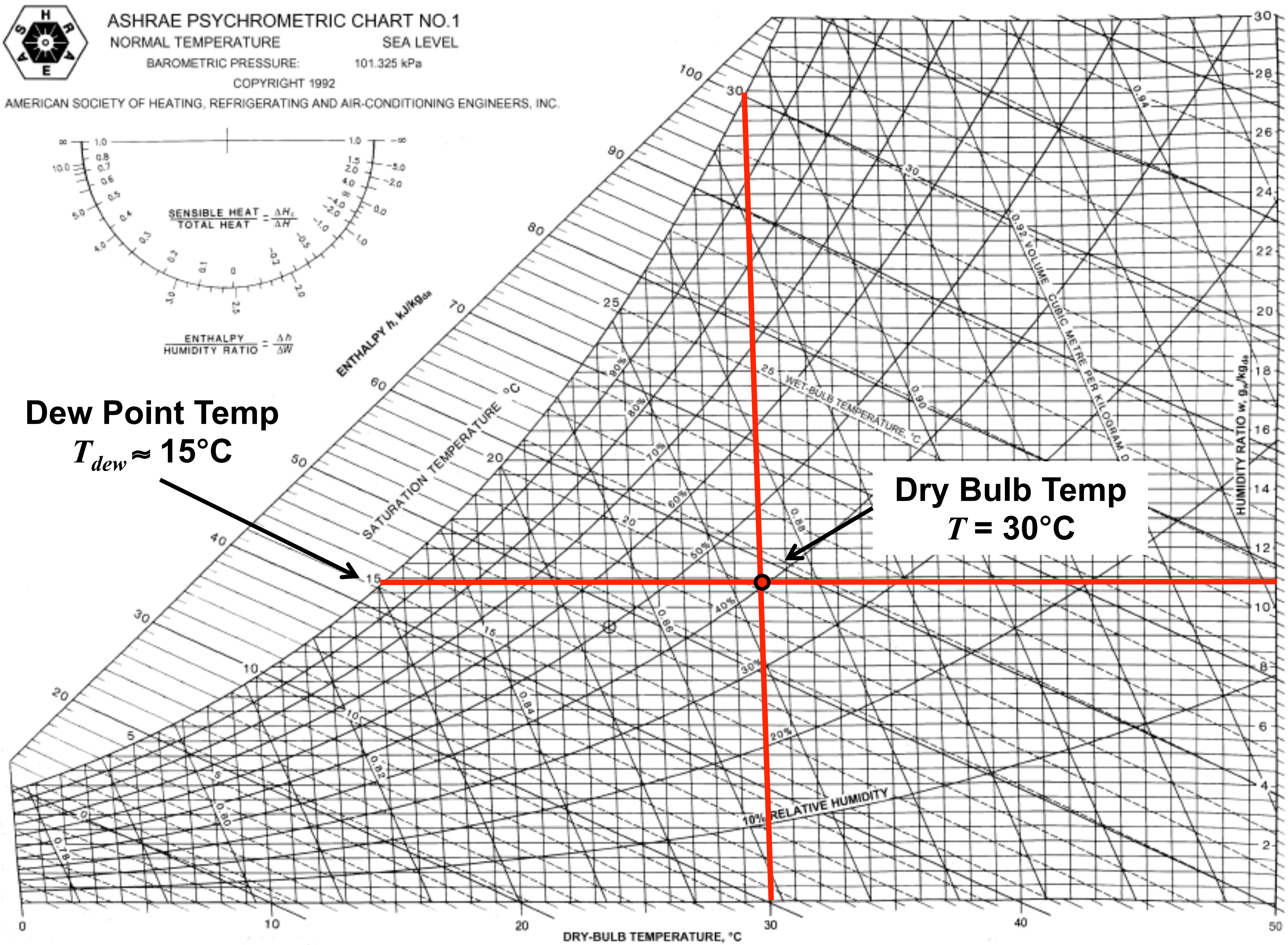
VOLUME CUBIC METRE PER KILOGRAM
HUMIDITY RATIO w, gw/kg da
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Dry Bulb Temp
T = 30°C



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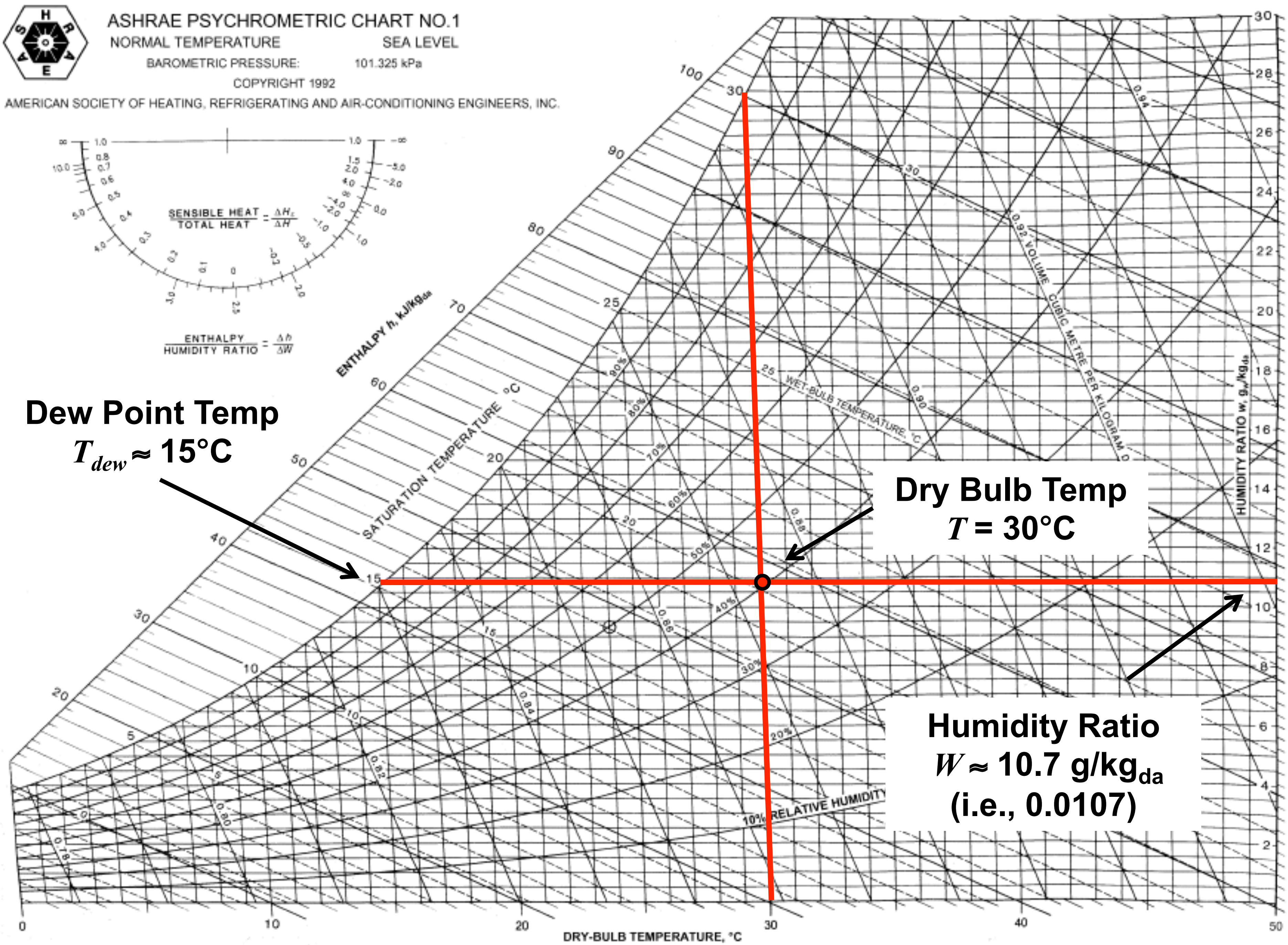
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM
HUMIDITY RATIO w, g w/kg da
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C
Dew Point Temp
T dew ≈ 15°C
Dry Bulb Temp
T = 30°C



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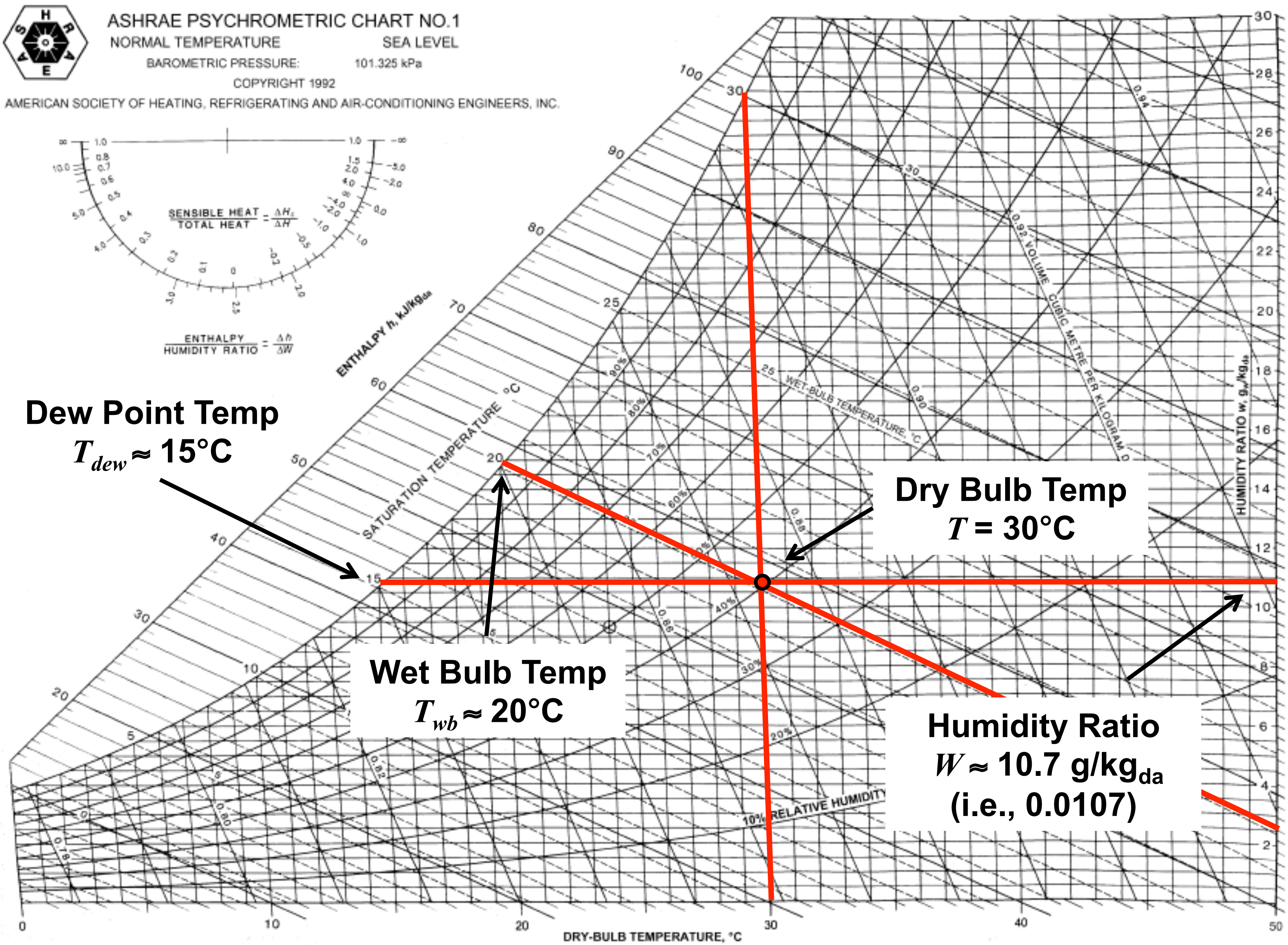
VOLUME CUBIC METRE PER KILOGRAM
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, gw/kgda
DRY-BULB TEMPERATURE, °C
Dew Point Temp
$T_{dew} \approx$ 15°C
Dry Bulb Temp
T = 30°C
Humidity Ratio
$W \approx$ 10.7 g/kg$_{da}$
(i.e., 0.0107)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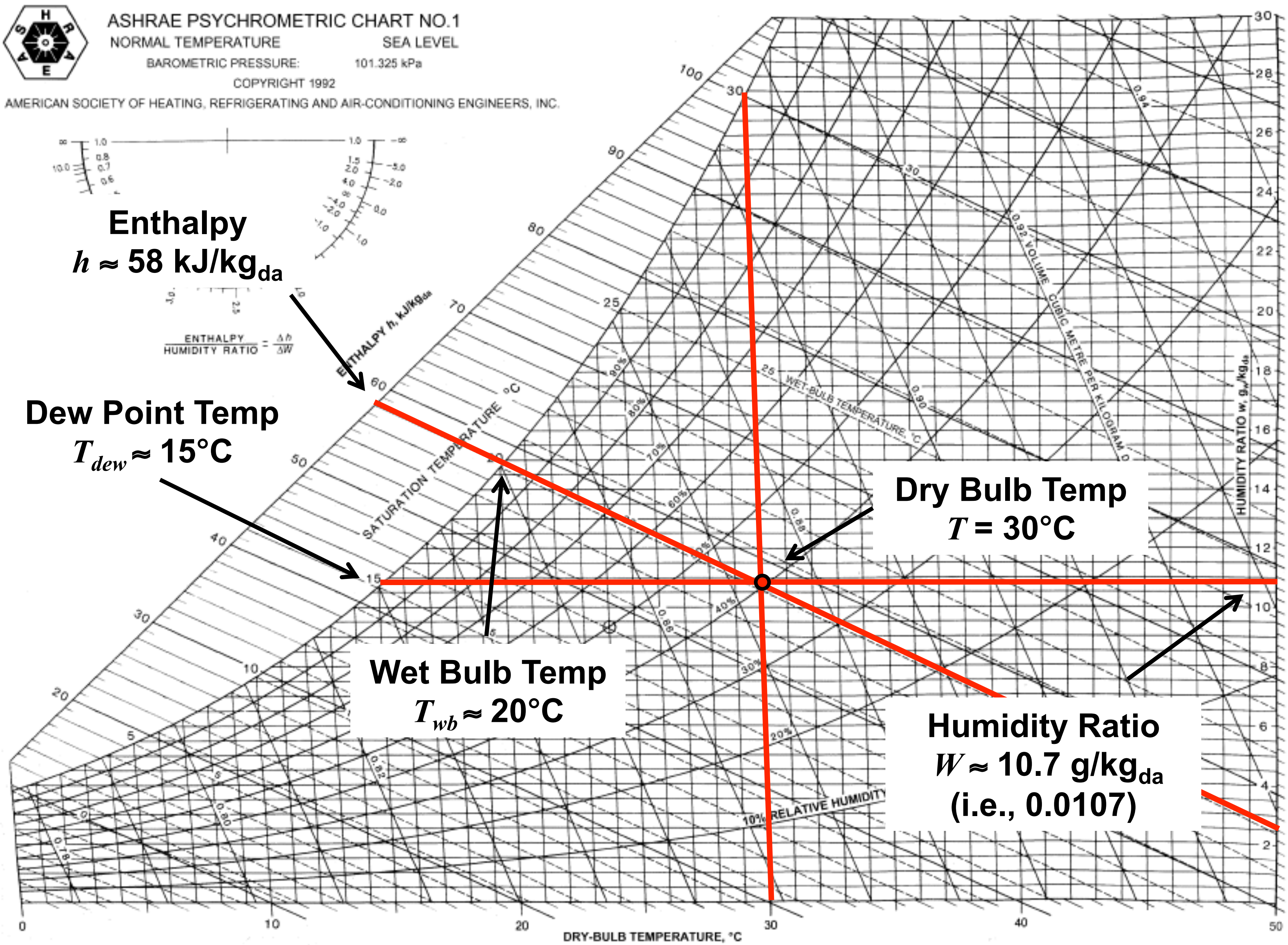
ENTHALPY h, kJ/kg da
SATURATION TEMPERATURE °C
WET BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM D
10% RELATIVE HUMIDITY
HUMIDITY RATIO W, gw/kgda
DRY-BULB TEMPERATURE, °C
Dew Point Temp
$T_{dew} \approx 15°C$
Wet Bulb Temp
$T_{wb} \approx 20°C$
Dry Bulb Temp
$T = 30°C$
Humidity Ratio
$W \approx 10.7\ g/kg_{da}$
(i.e., 0.0107)



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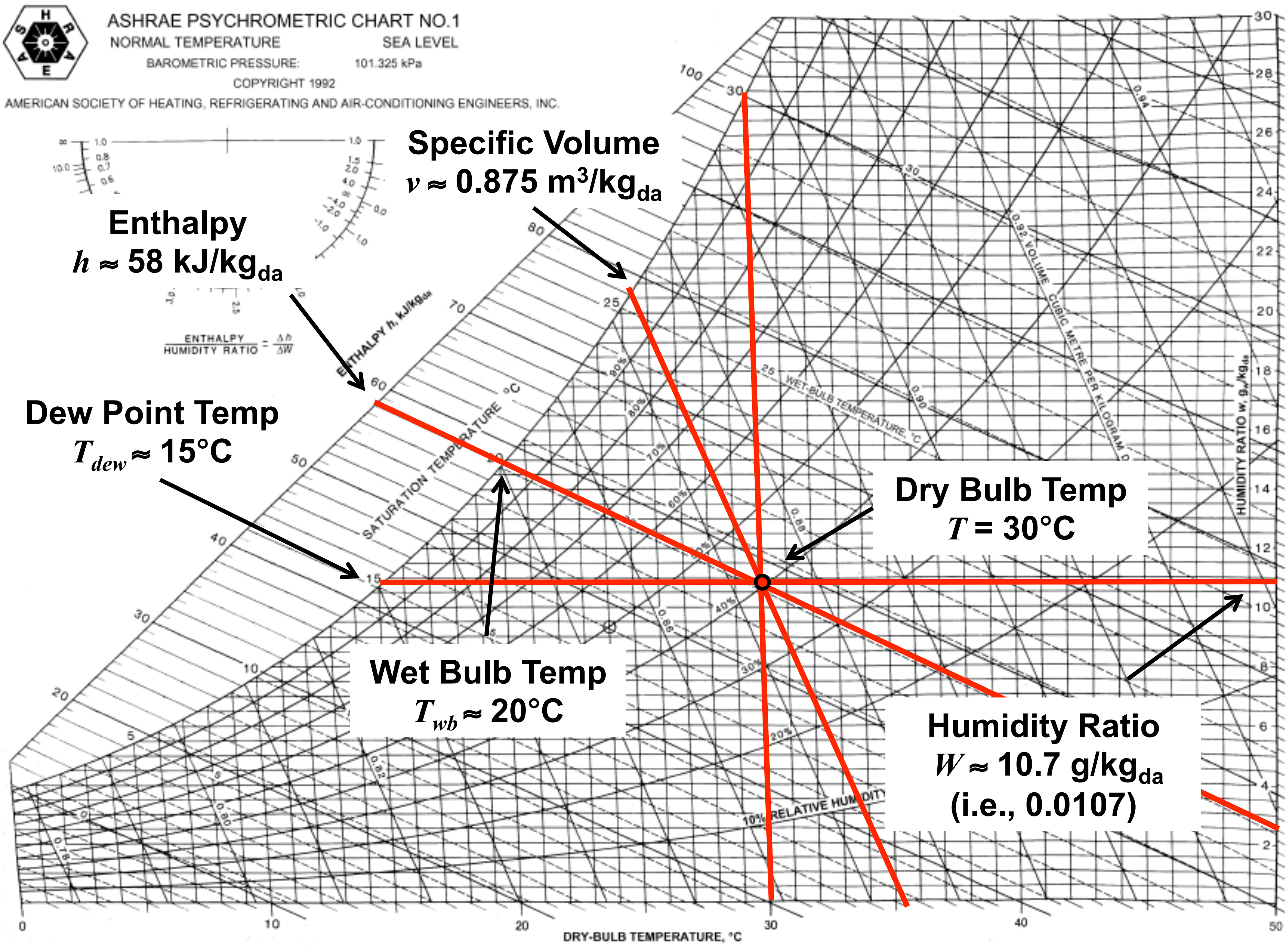
Enthalpy
$h \approx 58$ kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx 15$°C
Wet Bulb Temp
$T_{wb} \approx 20$°C
Dry Bulb Temp
$T = 30$°C
Humidity Ratio
$W \approx 10.7$ g/kg$_{da}$
(i.e., 0.0107)
ENTHALPY h, kJ/kg$_{da}$
SATURATION TEMPERATURE °C
WET BULB TEMPERATURE, °C
VOLUME CUBIC METRE PER KILOGRAM
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, g$_w$/kg$_{da}$
DRY-BULB TEMPERATURE, °C
ENTHALPY / HUMIDITY RATIO = Δh / ΔW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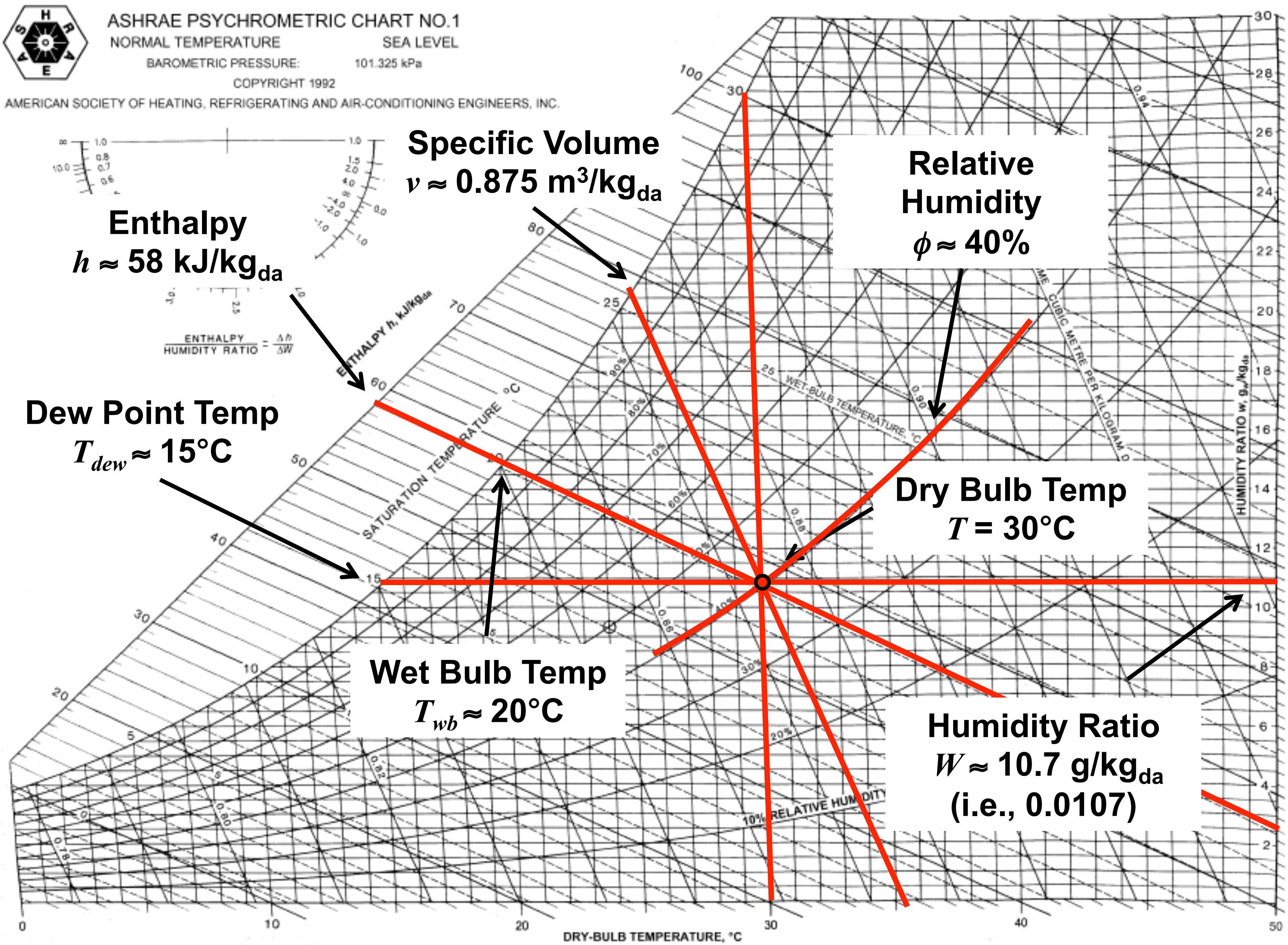
Specific Volume
$v \approx 0.875$ m^3/kg$_{da}$
Enthalpy
$h \approx 58$ kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx 15$°C
Dry Bulb Temp
$T = 30$°C
Wet Bulb Temp
$T_{wb} \approx 20$°C
Humidity Ratio
$W \approx 10.7$ g/kg$_{da}$
(i.e., 0.0107)
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY h, kJ/kg$_{da}$
SATURATION TEMPERATURE °C
WET-BULB TEMPERATURE, °C
0.92 VOLUME CUBIC METRE PER KILOGRAM
10% RELATIVE HUMIDITY
HUMIDITY RATIO w, g$_w$/kg$_{da}$
DRY-BULB TEMPERATURE, °C



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Specific Volume
$v \approx 0.875$ m³/kg$_{da}$
Relative Humidity
$\phi \approx 40\%$
Enthalpy
$h \approx 58$ kJ/kg$_{da}$
Dew Point Temp
$T_{dew} \approx 15°C$
Dry Bulb Temp
$T = 30°C$
Wet Bulb Temp
$T_{wb} \approx 20°C$
Humidity Ratio
$W \approx 10.7$ g/kg$_{da}$
(i.e., 0.0107)
ENTHALPY h, kJ/kg$_{da}$
SATURATION TEMPERATURE °C
WET BULB TEMPERATURE, °C
CUBIC METRE PER KILOGRAM
HUMIDITY RATIO W, g$_w$/kg$_{da}$
10% RELATIVE HUMIDITY
DRY-BULB TEMPERATURE, °C

Psychrometrics: IP units example

- Moist air exists at 100°F dry bulb, 65°F wet bulb and 14.696 psia

Find:

a) Humidity ratio
b) Enthalpy
c) Dew-point temperature
d) Relative humidity
e) Specific volume

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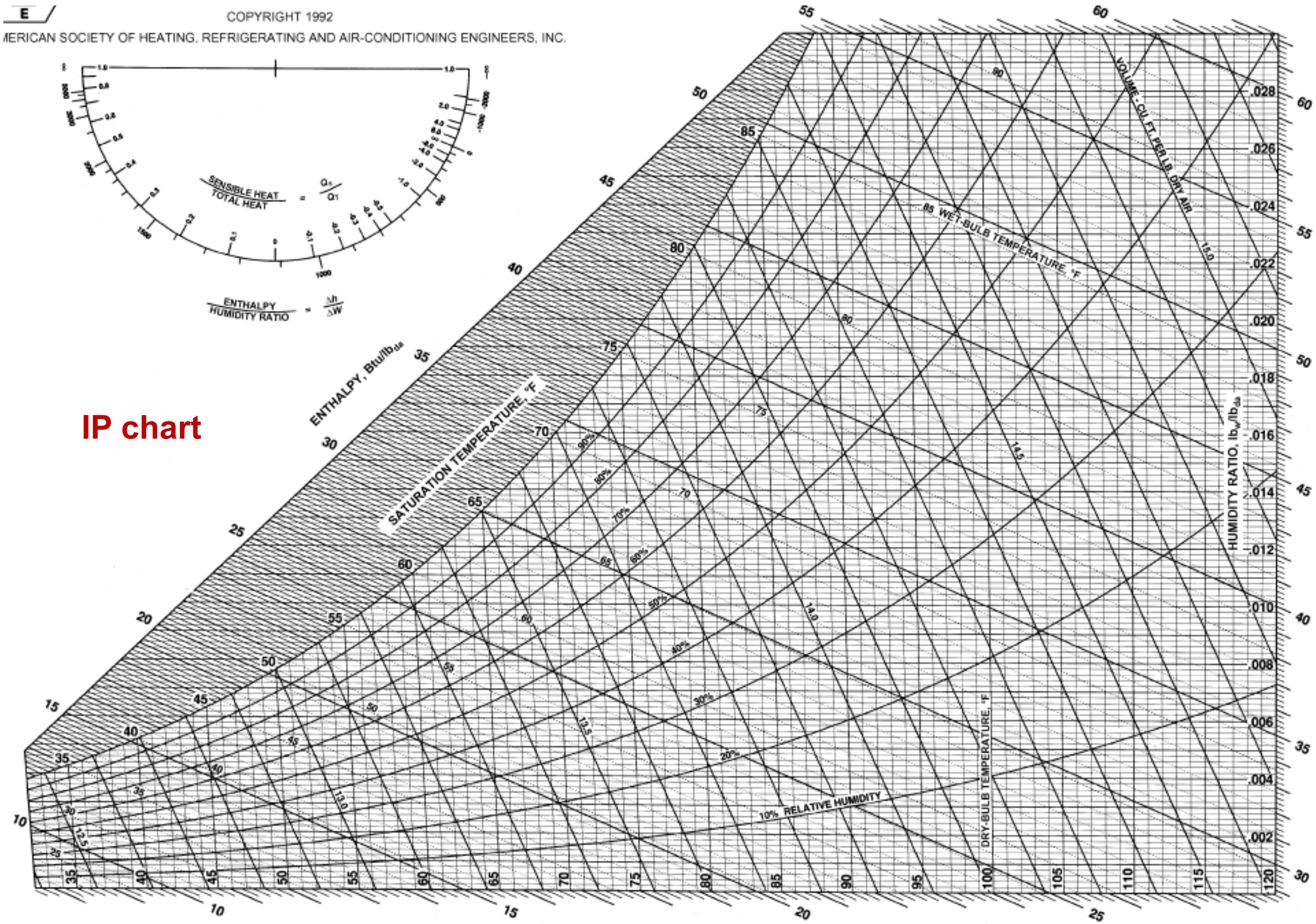

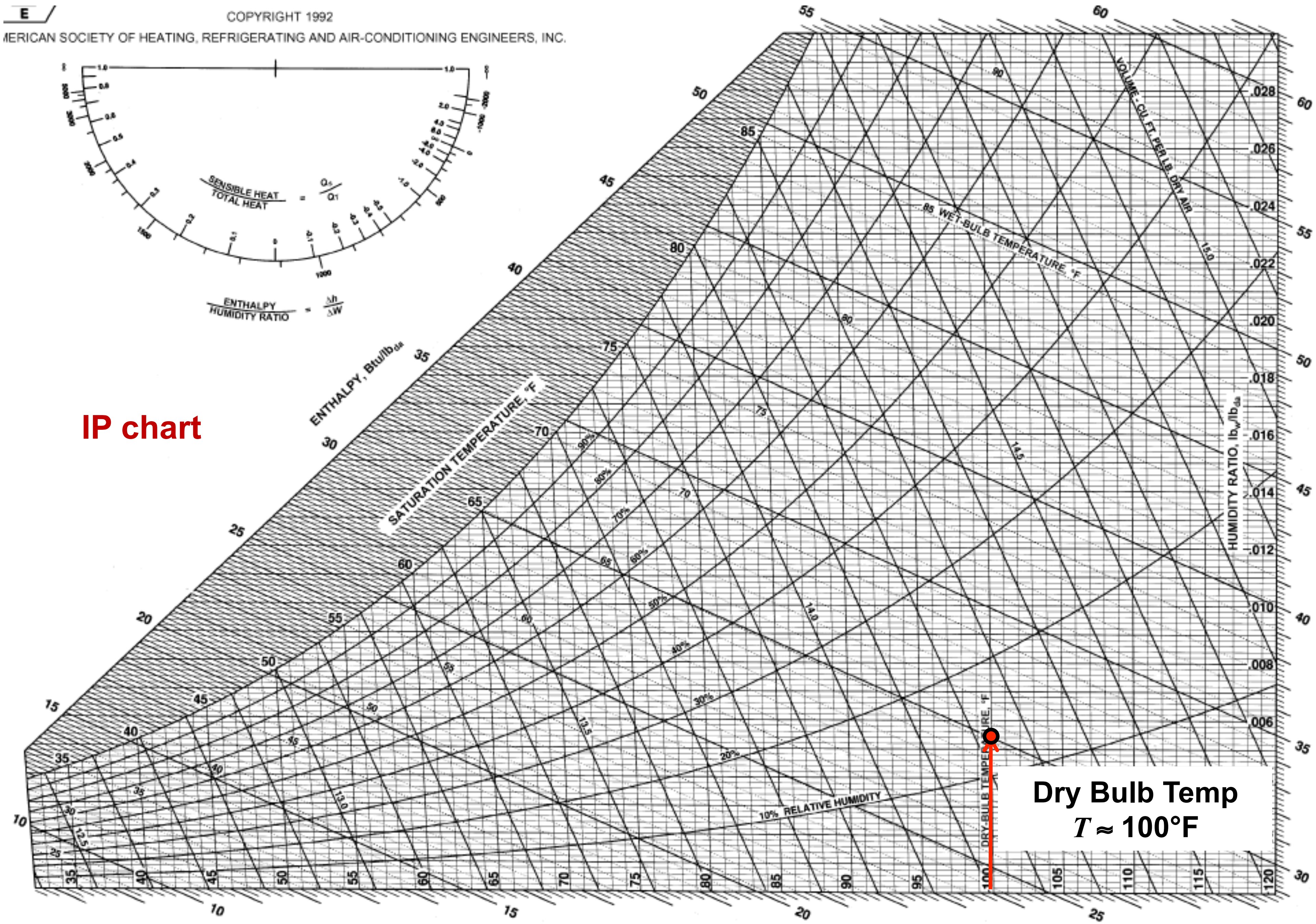
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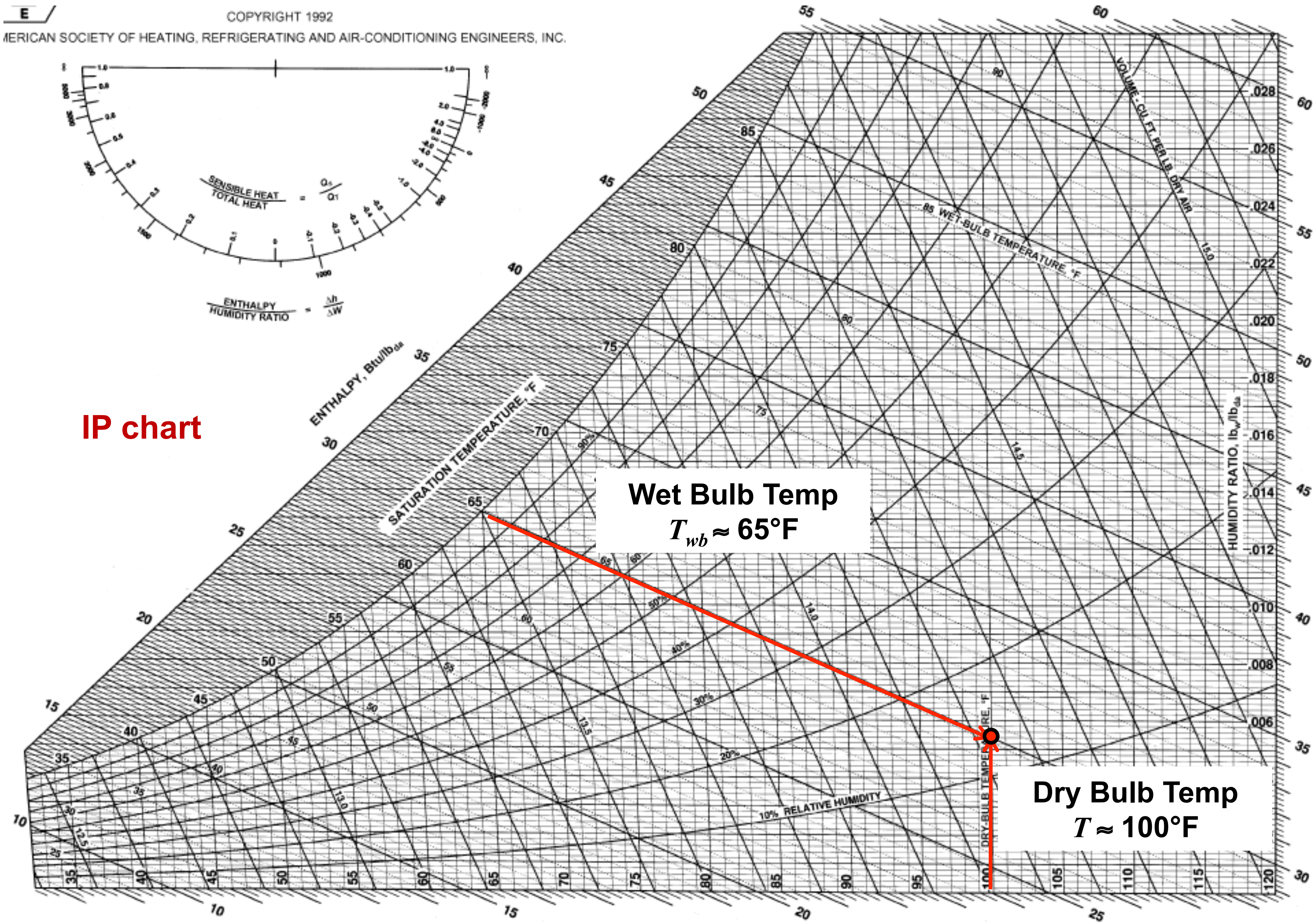
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
IP chart
ENTHALPY, Btu/lb_da
SATURATION TEMPERATURE, °F
WET-BULB TEMPERATURE, °F
VOLUME - CU.FT. PER LB. DRY AIR
HUMIDITY RATIO, lb_w/lb_da
10% RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F
Dry Bulb Temp
T ≈ 100°F



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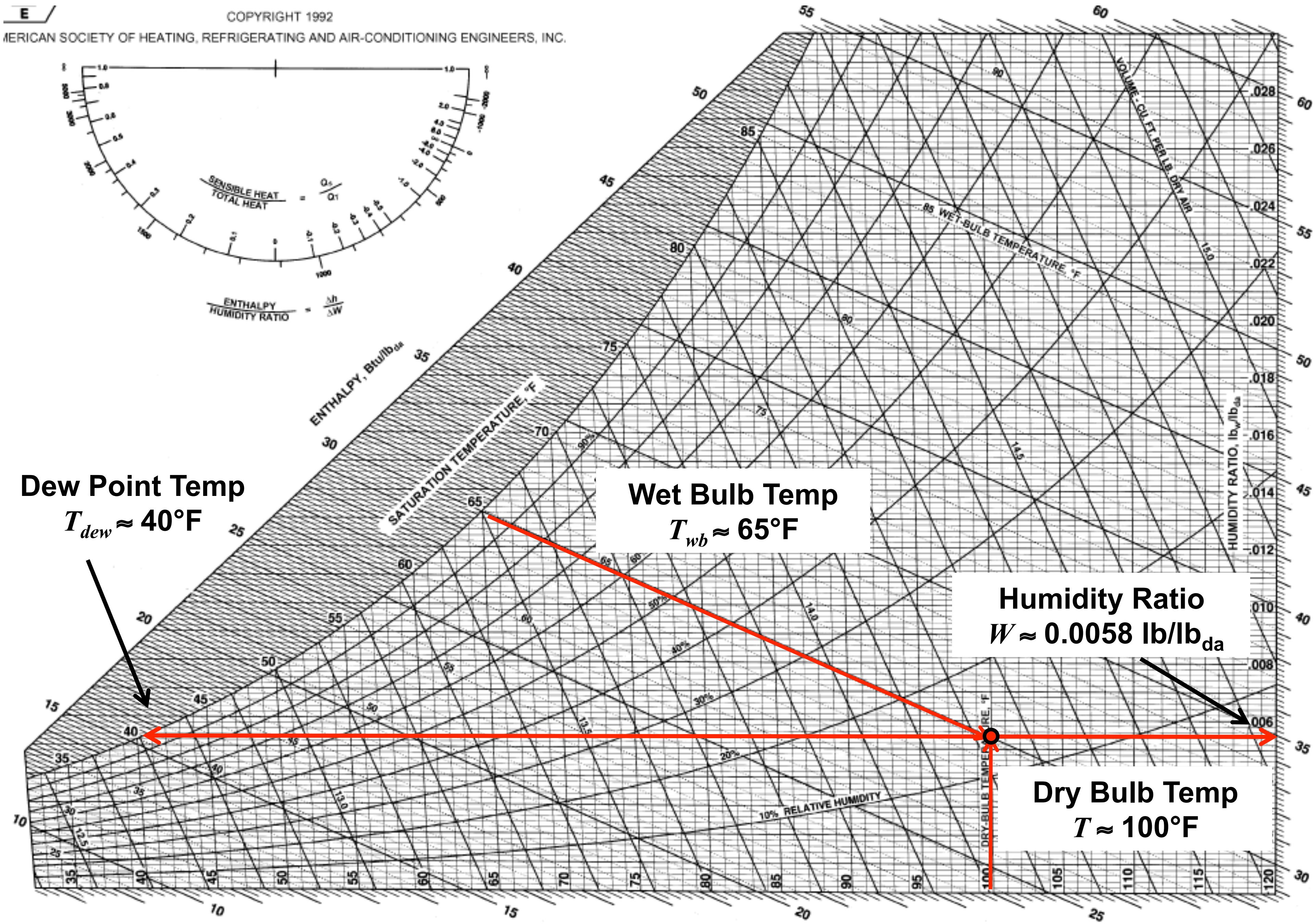
IP chart
ENTHALPY, Btu/lb_da
SATURATION TEMPERATURE, °F
WET-BULB TEMPERATURE, °F
VOLUME - CU.FT. PER LB. DRY AIR
HUMIDITY RATIO, lb_w/lb_da
RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F
Wet Bulb Temp
$T_{wb} \approx 65°F$
Dry Bulb Temp
$T \approx 100°F$



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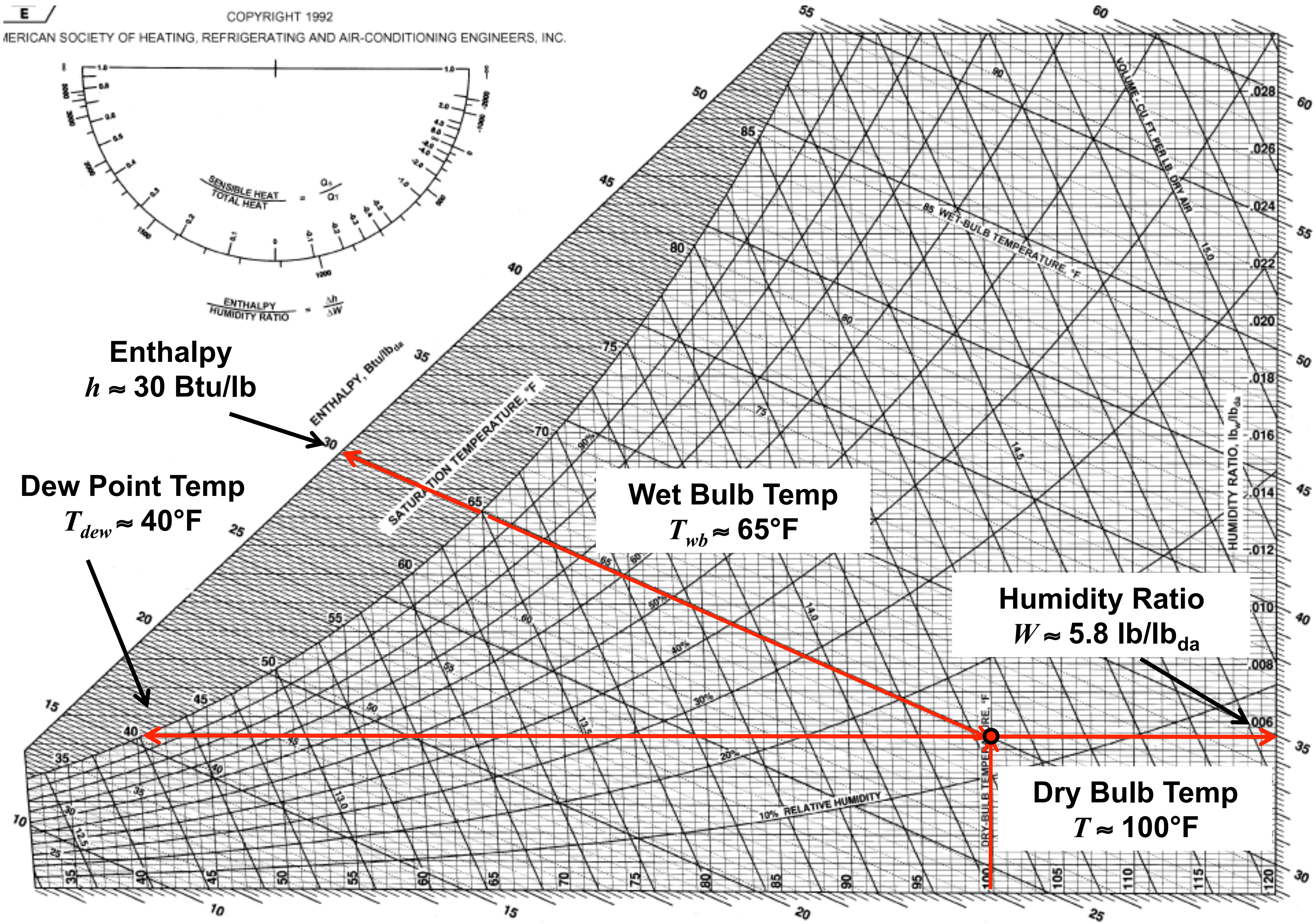
SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY, Btu/lb$_{da}$
SATURATION TEMPERATURE, °F
85 WET-BULB TEMPERATURE, °F
VOLUME, CU.FT. PER LB. DRY AIR
HUMIDITY RATIO, lb$_w$/lb$_{da}$
10% RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F
Dew Point Temp
$T_{dew} \approx$ 40°F
Wet Bulb Temp
$T_{wb} \approx$ 65°F
Humidity Ratio
$W \approx$ 0.0058 lb/lb$_{da}$
Dry Bulb Temp
$T \approx$ 100°F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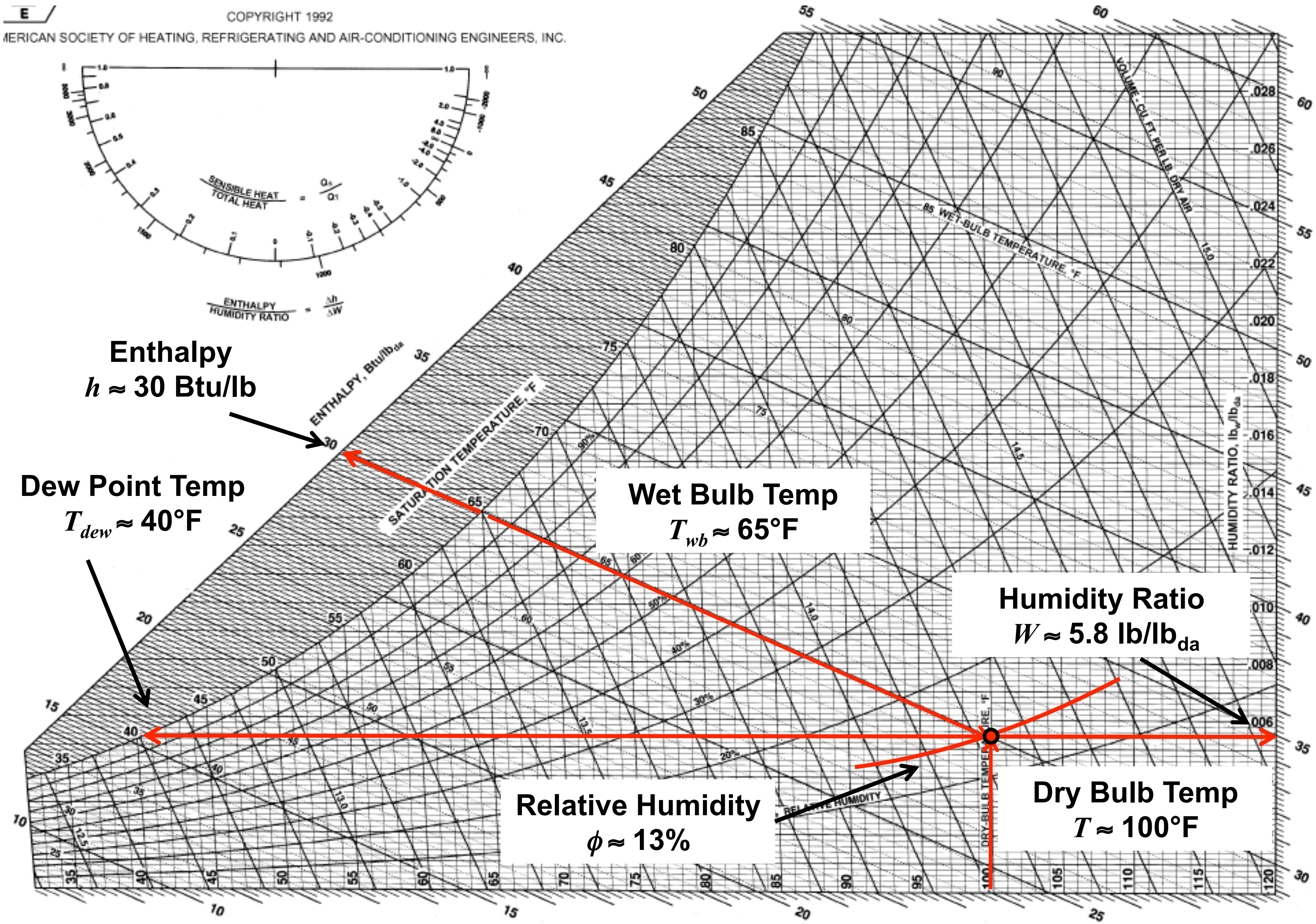
SENSIBLE HEAT / TOTAL HEAT = Qs / Qt
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
Enthalpy
h ≈ 30 Btu/lb
Dew Point Temp
Tdew ≈ 40°F
Wet Bulb Temp
Twb ≈ 65°F
Humidity Ratio
W ≈ 5.8 lb/lbda
Dry Bulb Temp
T ≈ 100°F
ENTHALPY, Btu/lbda
SATURATION TEMPERATURE, °F
85 WET-BULB TEMPERATURE, °F
VOLUME, CU.FT. PER LB. DRY AIR
HUMIDITY RATIO, lbw/lbda
10% RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F



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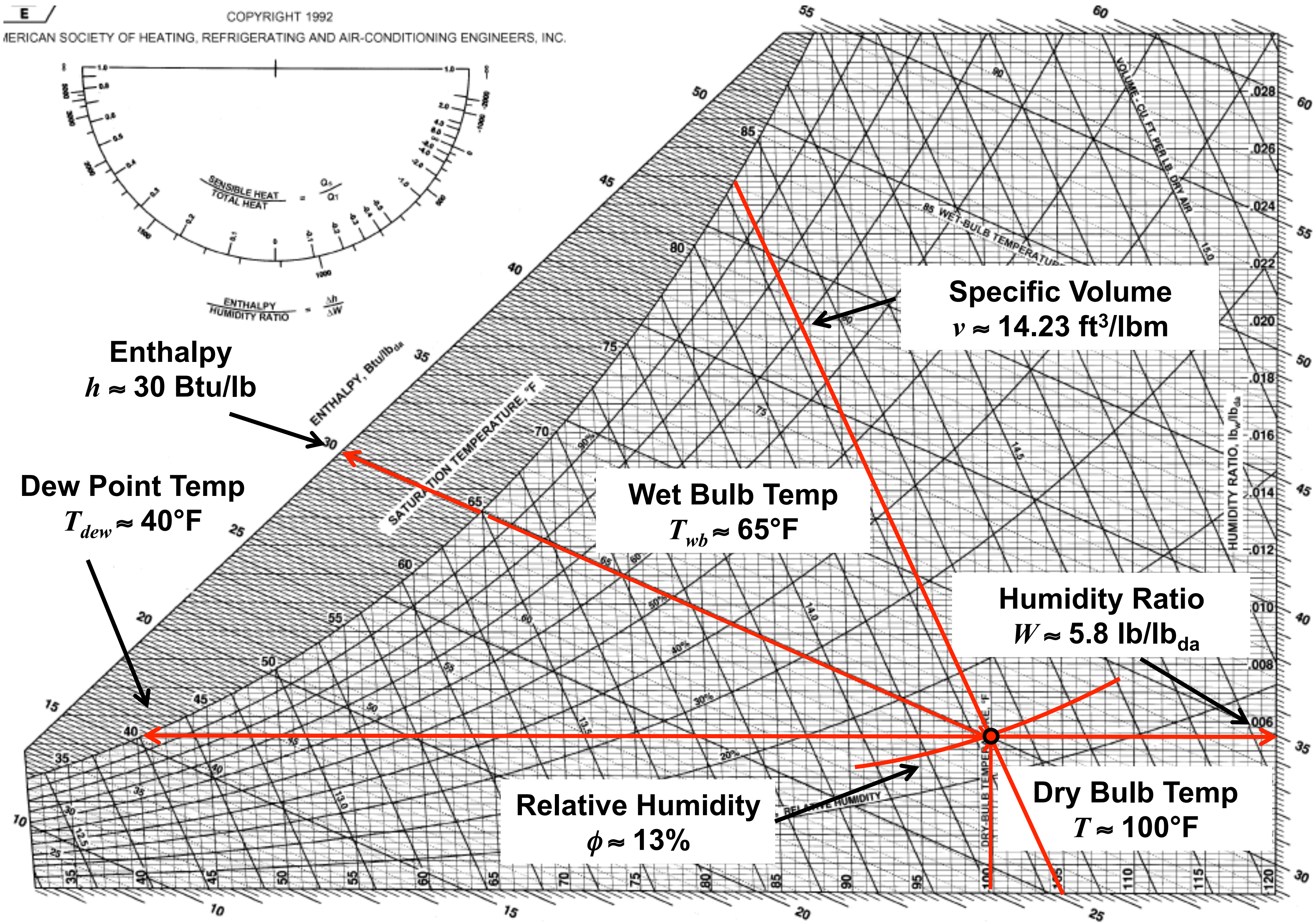
ENTHALPY / HUMIDITY RATIO = Δh / ΔW
ENTHALPY, Btu/lb$_{da}$
SATURATION TEMPERATURE, °F
85 WET-BULB TEMPERATURE, °F
VOLUME - CU. FT. PER LB. DRY AIR
HUMIDITY RATIO, lb$_w$/lb$_{da}$
RELATIVE HUMIDITY
DRY BULB TEMPERATURE, °F
Enthalpy
$h \approx$ 30 Btu/lb
Dew Point Temp
$T_{dew} \approx$ 40°F
Wet Bulb Temp
$T_{wb} \approx$ 65°F
Humidity Ratio
$W \approx$ 5.8 lb/lb$_{da}$
Relative Humidity
$\phi \approx$ 13%
Dry Bulb Temp
$T \approx$ 100°F



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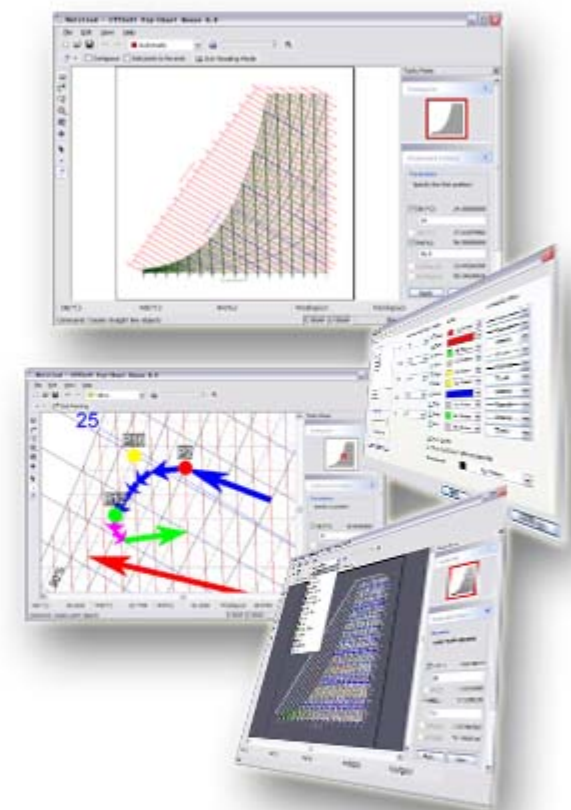
SENSIBLE HEAT / TOTAL HEAT
ENTHALPY / HUMIDITY RATIO
ENTHALPY, Btu/lb
SATURATION TEMPERATURE, °F
VOLUME, CU FT PER LB. DRY AIR
85 WET-BULB TEMPERATURE
HUMIDITY RATIO, lb/lb
DRY BULB TEMPERATURE, °F
RELATIVE HUMIDITY
Specific Volume
$v \approx$ 14.23 ft³/lbm
Enthalpy
$h \approx$ 30 Btu/lb
Dew Point Temp
$T_{dew} \approx$ 40°F
Wet Bulb Temp
$T_{wb} \approx$ 65°F
Humidity Ratio
$W \approx$ 5.8 lb/lb$_{da}$
Relative Humidity
$\phi \approx$ 13%
Dry Bulb Temp
$T \approx$ 100°F

Applying psychrometrics



- We can also use psychrometric charts or software
  - Psych and Psychpro
    - Very popular psych chart and analysis software
    - I think at least one of these is in the AM 218 lab
- There are a bunch of online calculators as well
  - http://www.psychrometric-calculator.com
  - http://www.sugartech.co.za/psychro/
  - http://www.wolframalpha.com/examples/Psychrometrics.html
- And smart phone apps too
- You can also make your own (i.e., in Excel)
  - You will have a HW problem where you have to do this

Psychrometrics also involves learning how to use and combine those quantities to determine things like sensible and latent heating and cooling loads (i.e., ***processes***) (covered in a future lecture)

Using these parameters

- Question:
  - What is the mass of water vapor in this classroom right now?

PSYCHROMETRIC EQUATIONS

Psychrometric equations

- When we need more precise answers, or when we need to automate engineering calculations, we must:
  - Use the underlying **equations** that govern moist air properties and processes and make up the psychrometric chart

- This begins by treating air as an **ideal gas**

Treating air as an ideal gas

- At typical temperatures and pressures within buildings, air and its constituents act approximately as ideal gases
- Each gas i in the mixture, as well as the entire mixture, will follow the ideal gas law:

Ideal Gas Law (Boyle's law + Charles's law)

$$pV = nRT$$

p = pressure (Pa)
V = volume (m^3)
n = number of moles (#)
R = gas constant ($Pa \cdot m^3/(mol\ K)$)*
T = absolute temperature (K)

*Units on R vary with units of pressure

Air as an ideal gas

- We can treat air as a composition of ideal gases
  - A bunch of ideal gases acting as an ideal gas

- For individual gases (e.g., N_2, O_2, Ar, H_2O, CO_2, pollutant i):

$$P_i V = n_i R T$$

P_i = partial pressure exerted by gas i
n_i = # of moles of gas i
R, V, T = gas constant, volume, temperature

$$P_i = \frac{n_i}{V} R T$$

Rearrange so that n_i/V is the molar concentration

$$P_i = y_i P_{tot}$$

P_{tot} = total pressure of air (atm, Pa, etc.)
y_i = mole fraction of gas i in air (moles i / moles air)

Air as an ideal gas

- Air as a composite mixture

$$P_i = y_i P_{tot}$$

$$P_{tot} = \sum P_i = \sum \frac{n_i}{V} RT = \frac{RT}{V} \sum n_i = \frac{RT}{V} n_{tot}$$

$$PV = nRT$$

Calculating the density of air at typical indoor conditions

$$PV = nRT \longrightarrow \frac{n}{V} = \frac{P}{RT}$$

$$\frac{n}{V} = \frac{P}{RT} = \frac{1 \text{ atm}}{\left(8.205 \times 10^{-5} \ \frac{\text{atm} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right) \times 293 \text{ K}}$$

20°C, 68°F

$$\frac{n}{V} = 41.6 \ \frac{\text{moles}}{\text{m}^3} = 0.0416 \ \frac{\text{moles}}{\text{L}}$$

$$\rho_{air} = \frac{n}{V} MW_{air} = MW_{air} \times 0.0416 \ \frac{\text{moles}}{\text{L}} \ \text{@20 degrees C}$$

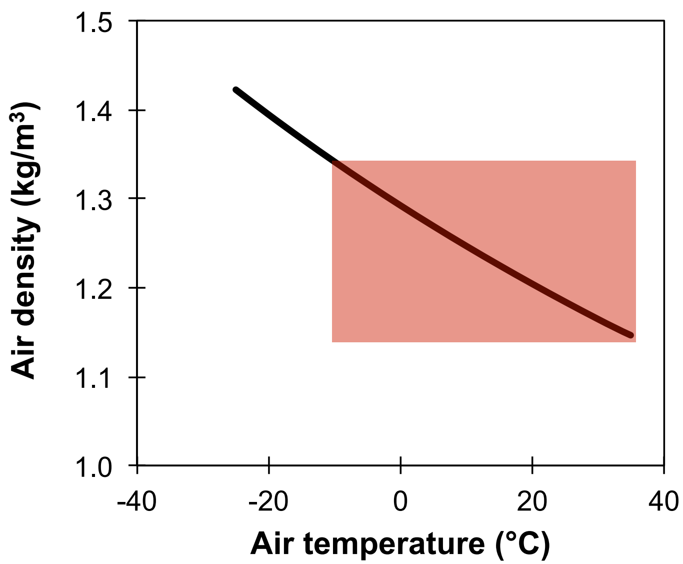
What is the molecular weight (MW) of air?

$$MW_{air} = \sum y_i MW_i = y_{N_2} MW_{N_2} + y_{O_2} MW_{O_2} + y_{H_2O} MW_{H_2O} + \ldots$$

$$MW_{air} = 0.781(28 \text{ g/mol}) + 0.209(32 \text{ g/mol}) + \ldots \approx 29 \text{ g/mol}$$

$$\rho_{air} = (29 \frac{\text{g}}{\text{mol}}) \times 0.0416 \frac{\text{mol}}{\text{L}} = 1.2 \frac{\text{g}}{\text{L}} = 1.2 \frac{\text{kg}}{\text{m}^3} \text{ @20 degrees C}$$

Remember this number: density of air is ~1.2 kg/m^3 at 20°C (~0.075 lb/ft^3 in IP units)



Density is a function of temperature:

$$\rho_{air} \approx 1.3 - 0.0046\left(T_{air}\right) \text{ where } T_{air} \text{ is in degrees C}$$

In building applications, where:
-15°C < T < 40°C
1.15 kg/m^3 < ρ_{air} < 1.3 kg/m^3

Universal gas constant

- The universal gas constant relates energy and temperature
  - It takes many forms depending on units

Universal gas constant

$$PV = nRT$$

Value of R	Units ($V\ P\ T^{-1}\ n^{-1}$)
8.314	J/(K·mol)
8.314	m^3·Pa/(K·mol)
0.08206	L·atm/(K·mol)
8.205×10^{-5}	m^3·atm/(K·mol)
10.731	ft^3·psi/(R·lb-mol)
1.986	Btu/(lb-mol·R)

Specific gas constants

- To work with air and water vapor we can also work with specific gas constants (which are functions of molecular weight)
- Dry air (no water vapor): MW_{da} = 28.965 g/mol

$$R_{da} = \frac{R}{MW_{da}} = \frac{8.314 \frac{\text{J}}{\text{K}\cdot\text{mol}}}{28.965 \frac{\text{g}}{\text{mol}}} \frac{1000\text{g}}{\text{kg}} = 287 \frac{\text{J}}{\text{kg}\cdot\text{K}}$$

$$pv = \frac{p}{\rho} = R_i T$$

Specific gas constant:

$$R_i = \frac{R}{MW_i}$$

- Water vapor alone: MW_w = 18.015 g/mol

$$R_w = \frac{R}{MW_w} = \frac{8.314 \frac{\text{J}}{\text{K}\cdot\text{mol}}}{18.015 \frac{\text{g}}{\text{mol}}} \frac{1000\text{g}}{\text{kg}} = 462 \frac{\text{J}}{\text{kg}_\text{w}\cdot\text{K}}$$

Air pressure variations

- The barometric (atmospheric) pressure and temperature of air vary with both altitude and local weather conditions
  - But there are standard values for pressure as a function of altitude that are normally used
- At sea level, the standard temperature is 15°C and the standard pressure is 101.325 kPa (1 atm)
  - Temperature is assumed to decrease linearly with altitude
    - Pressure is more complicated

$$T_{air} = 15 - 0.0065Z$$

$$p = 101.325\left(1 - \left(2.25577 \times 10^{-5}\right)Z\right)^{5.2559}$$

$$pv = \frac{p}{\rho} = RT$$

T = temperature (°C)

Z = altitude (m)

p = barometric pressure (kPa)

Air pressure variations

Table 1 Standard Atmospheric Data for Altitudes to 10 000 m

Altitude, m	Temperature, °C	Pressure, kPa	
–500	18.2	107.478	
0	15.0	101.325	*Chicago, IL*
500	11.8	95.461	
1000	8.5	89.875	
1500	5.2	84.556	*Denver, CO*
2000	2.0	79.495	*Big Sky, MT*
2500	–1.2	74.682	
3000	–4.5	70.108	*Breckenridge, CO*
4000	–11.0	61.640	
5000	–17.5	54.020	
6000	–24.0	47.181	
7000	–30.5	41.061	
8000	–37.0	35.600	
9000	–43.5	30.742	
10 000	–50	26.436	

Source: Adapted from NASA (1976).

Dalton's law of partial pressures for psychrometrics

- In an ideal gas, the total pressure can be considered to be the sum of the partial pressures of the constituent gases

$$p = p_{N_2} + p_{O_2} + p_{H_2O} + p_{CO_2} + p_{Ar} + \ldots$$

- We can consider moist air as dry air combined with water vapor and break the pressure into only two partial pressures:
  - Dry air (da)
  - Water vapor (w)

$$p = p_{da} + p_w$$

Dalton's law of partial pressures for psychrometrics

- We can analyze the dry air, the water vapor, and the mixture of each gas using the ideal gas law and assuming they are all at the same temperature

$$p_{da} v_{da} = R_{da} T \quad \& \quad p_w v_w = R_w T \quad \& \quad pv = RT$$

- For each individual gas, a mole fraction (Y_i) can be defined as the ratio of the partial pressure of gas i to the total pressure

$$\frac{n_i}{n} = \frac{p_i}{p} = Y_i$$

Specifying the state of moist air

In order to specify the state of moist air, we need total atmospheric pressure, p, the air temperature, T, and at least one other property

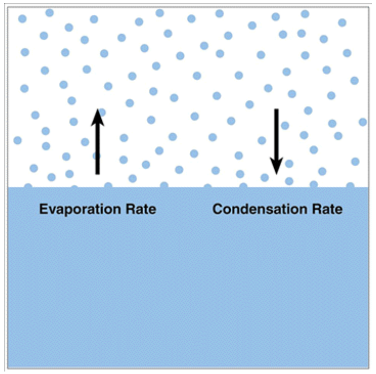
- W, ϕ, h, p_w, or T_{dew}

- We can use the psychrometric chart
- We can also use the **underlying equations** for greater accuracy and automation

Remember: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is the pressure exerted by a vapor in thermodynamic equilibrium with its condensed phases

p_w *Units of pressure, Pa or kPa
(aka "partial pressure")

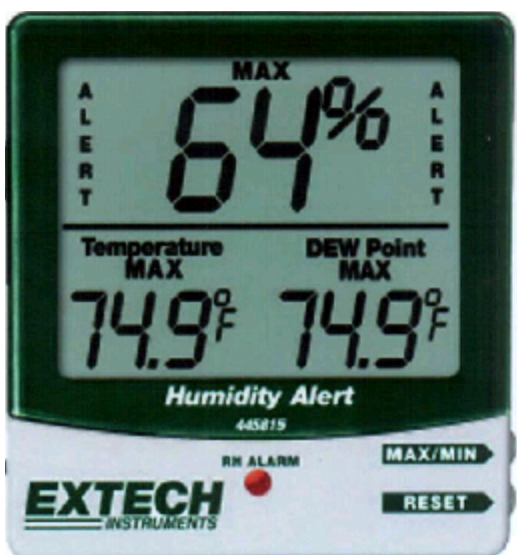


- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
  - We call the limit ***saturation***

p_{ws} *Units of pressure, Pa or kPa
(aka "saturation vapor pressure")

Relative humidity, ϕ (RH)

- The relative humidity ratio, ϕ, is the mole fraction of water vapor (x_w) relative to the water vapor that would be in the mixture if it were saturated at the given T and P (x_{ws})
  - We can also describe RH by partial pressures (ideal gas)
- Relative humidity is a common measure that relates well to how we perceive moisture in air



$$\phi = \left[\frac{x_w}{x_{ws}}\right]_{T,P} = \frac{p_w}{p_{ws}}$$

p_{ws} for 0°C< T <200°C (SI units)

For p_{ws}, the saturation pressure over **liquid water**:

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13}\ln T$$

where

$C_8 = -5.800\,220\,6\text{ E}+03$
$C_9 = 1.391\,499\,3\text{ E}+00$
$C_{10} = -4.864\,023\,9\text{ E}-02$
$C_{11} = 4.176\,476\,8\text{ E}-05$
$C_{12} = -1.445\,209\,3\text{ E}-08$
$C_{13} = 6.545\,967\,3\text{ E}+00$

Note:
These constants are only for SI units
IP units are different

Unit

p_{ws} = saturation pressure, Pa
T = absolute temperature, K = °C + 273.15

*We will use this equation for most conditions in building science (above 0°C)

p_{ws} for -100°C< T <0°C (SI units)

For p_{ws}, the saturation pressure over **ice**:

$$\ln p_{ws} = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T$$

where

C_1 = –5.674 535 9 E+03
C_2 = 6.392 524 7 E+00
C_3 = –9.677 843 0 E–03
C_4 = 6.221 570 1 E–07
C_5 = 2.074 782 5 E–09
C_6 = –9.484 024 0 E–13
C_7 = 4.163 501 9 E+00

Note:
These constants are only for SI units
IP units are different

Units:

p_{ws} = saturation pressure, Pa
T = absolute temperature, K = °C + 273.15

Humidity ratio, *W* (SI units)

- The humidity ratio, *W*, is ratio of the mass of water vapor to mass of dry air in a given volume
  - We use *W* when finding other mixture properties
  - Note 1: *W* is small (*W* < 0.03 for most real building conditions)
  - Note 2: *W* is sometimes expressed in grains/lb where 1 lb = 7000 grains (I don't use this but you will in CAE 464 HVAC Design)

Units:

$$W = \frac{m_w}{m_{da}} = \frac{MW_w p_w}{MW_{da} p_{da}} = 0.622 \frac{p_w}{p_{da}} = 0.622 \frac{p_w}{p_{total} - p_w} \quad [\frac{\mathrm{kg_w}}{\mathrm{kg_{da}}}]$$

where: $p_{total} = p_{da} + p_w = 101325$ Pa @ sea level

Saturation humidity ratio, W_s (SI units)

- At a given temperature T and pressure P there is a maximum W that can be obtained
- If we try to add any more moisture, it will just condense out
  - It is when the partial pressure of vapor has reached the saturation pressure
- This maximum humidity ratio is called the saturation humidity ratio, W_s
  - From our previous equation we can write:

UNITS

$$W_s = 0.622 \frac{p_{ws}}{p_{da}} = 0.622 \frac{p_{ws}}{p_{total} - p_{ws}} \qquad [\frac{kg_w}{kg_{da}}]$$

Degree of saturation, μ (SI units)

- The degree of saturation, μ (dimensionless), is the ratio of the humidity ratio W to that of a saturated mixture W_s at the same T and P
  - Note that μ and ϕ are not quite the same
  - Their values are very similar at lower temperatures but may differ a lot at higher temperatures

$$\mu = \left[\frac{W}{W_s}\right]_{T,P}$$

$$\mu = \frac{\phi}{1+(1-\phi)W_s/(0.6295)}$$

$$\phi = \frac{\mu}{1-(1-\mu)p_{ws}/p_{total}}$$

Specific volume, v, and density, ρ (SI units)

- The specific volume of moist air (or the volume per unit mass of air, m^3/kg) can be expressed as:

$$v = \frac{R_{da}T}{p_{total} - p_w} = \frac{R_{da}T(1+1.6078W)}{p_{total}}$$

where

v = specific volume, m^3/kg_{da}
t = dry-bulb temperature, °C
W = humidity ratio, kg_w/kg_{da}
p = total pressure, kPa

$$v \approx 0.287042(T+273.15)(1+1.6078W)\,/\,p_{total}$$

- If we have v we can also find moist air density, ρ (kg/m^3):

$$\rho = \frac{m_{da}+m_w}{V} = \frac{1}{v}\left(1+W\right)$$

Enthalpy, h (SI units)

- The enthalpy of a mixture of perfect gases equals the sum of the individual partial enthalpies of the components
- Therefore, the enthalpy (h) for moist air is: $$h = h_{da} + Wh_g$$

  h = enthalpy for moist air [kJ/kg]
  h_g = specific enthalpy for saturated water vapor (i.e., h_{ws}) [kJ/kg$_w$]
  h_{da} = specific enthalpy for dry air (i.e., h_{ws}) [kJ/kg$_{da}$]

- Some approximations: $h_{da} \approx 1.006T$ $\quad h_g \approx 2501 + 1.86T$

$$h \approx 1.006T + W(2501 + 1.86T)$$

*where T is in °C and h is in kJ/kg