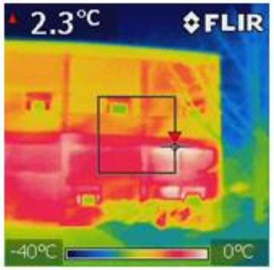
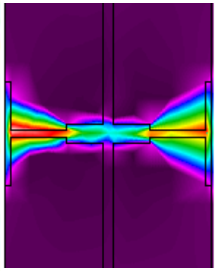
CAE 331/513
Building Science
Fall 2018





September 11, 2018
Combined modes of heat transfer and energy balances

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Last time

- Radiation
  - Long wave (surface to surface radiation)

$$Q_{1\to 2} = \frac{A_1 \sigma \left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{12}}}$$

$$Q_{1\to 2} = \varepsilon_{surf} A_{surf} \sigma F_{12} \left(T_1^4 - T_2^4\right)$$

  - Short wave (solar radiation)

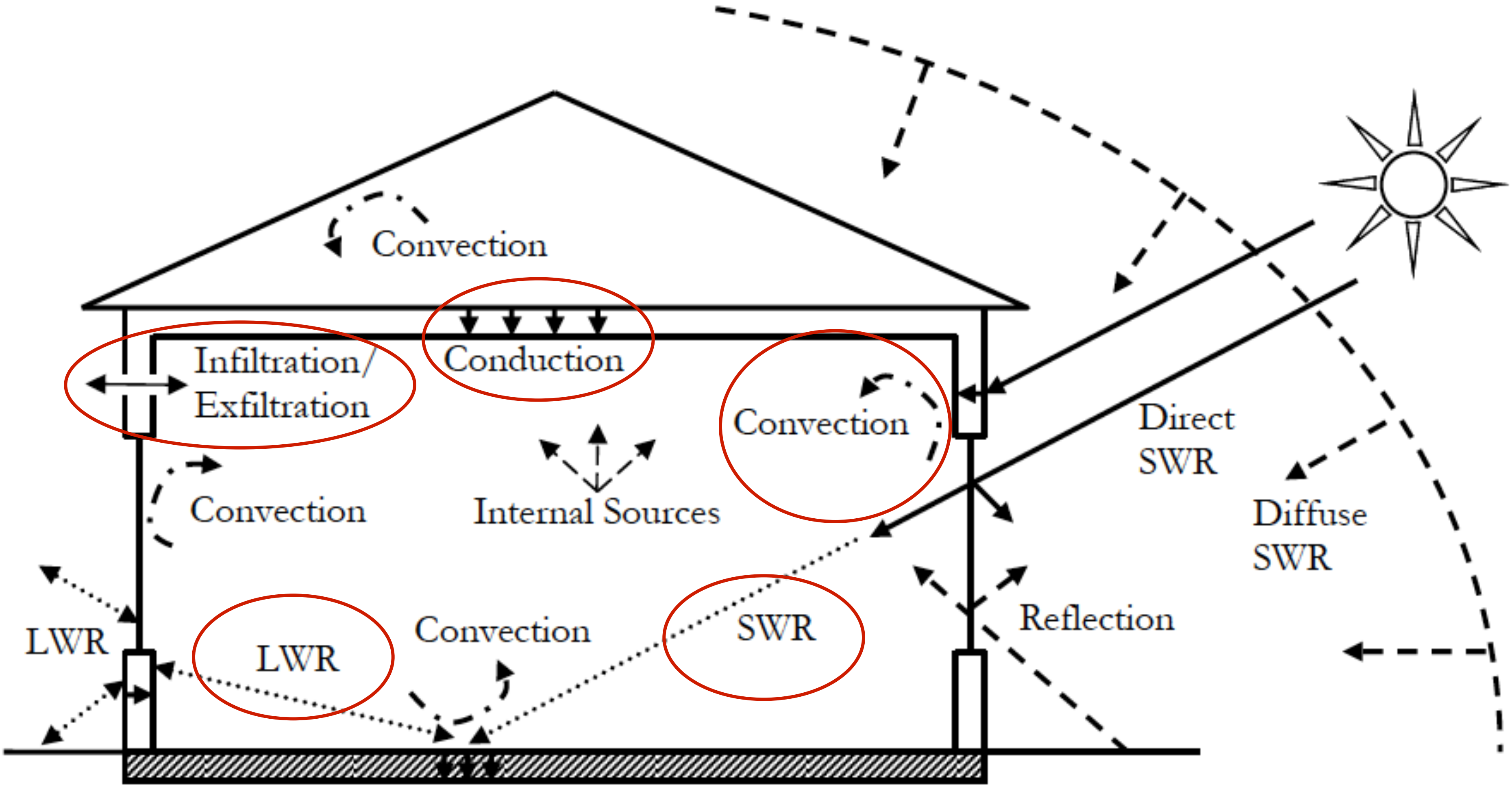
$$I_{solar} \quad \left[\frac{\text{W}}{\text{m}^2}\right]$$

Solar radiation: $q_{solar} = \alpha I_{solar}$
(opaque surface)

Transmitted solar radiation: $q_{solar} = \tau I_{solar}$
(transparent surface)

$$I_D = I_{DN} \cos\theta$$

Summary: Modes of heat transfer in a building



Summary to date: Modes of heat transfer in a building

Conduction

$$q = \frac{k}{L}\left(T_{surf,1} - T_{surf,2}\right)$$

$$\frac{k}{L} = U = \frac{1}{R}$$

$$R_{total} = \frac{1}{U_{total}}$$

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

For thermal bridges and combined elements:

$$U_{total} = \frac{A_1}{A_{total}} U_1 + \frac{A_2}{A_{total}} U_2 + \ldots$$

Convection

$$q_{conv} = h_{conv}\left(T_{fluid} - T_{surf}\right)$$

$$R_{conv} = \frac{1}{h_{conv}}$$

Advection

$$Q_{bulk} = \dot{m} C_p \Delta T$$

Radiation

Long-wave

$$q_{1\rightarrow 2} = \frac{\sigma\left(T_{surf,1}^4 - T_{surf,2}^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{12}}}$$

$$q_{rad,1\rightarrow 2} = h_{rad}\left(T_{surf,1} - T_{surf,2}\right)$$

$$h_{rad} = \frac{4\sigma T_{avg}^3}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad R_{rad} = \frac{1}{h_{rad}}$$

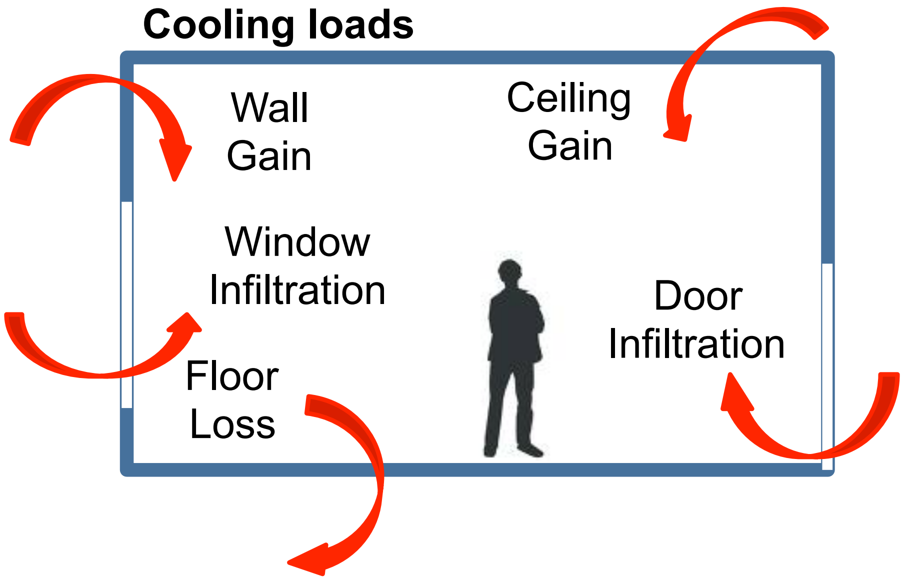
$$q_{1\rightarrow 2} = \varepsilon_{surf}\sigma F_{12}\left(T_{surf,1}^4 - T_{surf,2}^4\right)$$

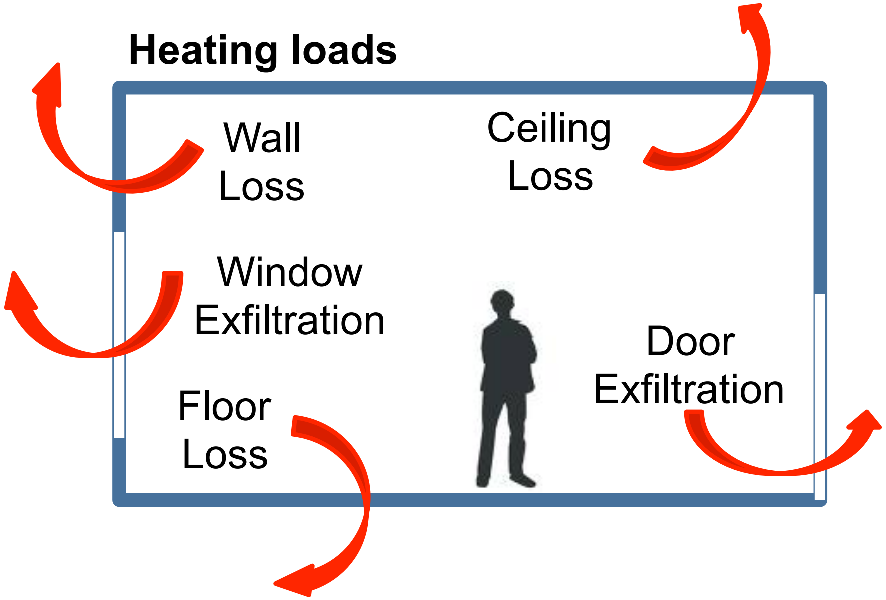
Solar radiation: (opaque surface) $q_{solar} = \alpha I_{solar}$

Transmitted solar radiation: (transparent surface) $q_{solar} = \tau I_{solar}$

Where are we going? Building energy balances

- Taken altogether, each of the heat transfer modes we've discussed can be combined with inputs for climate data, material properties, and geometry to make up a building's **energy balance**
  - We will also revisit this for heating and cooling load calculations





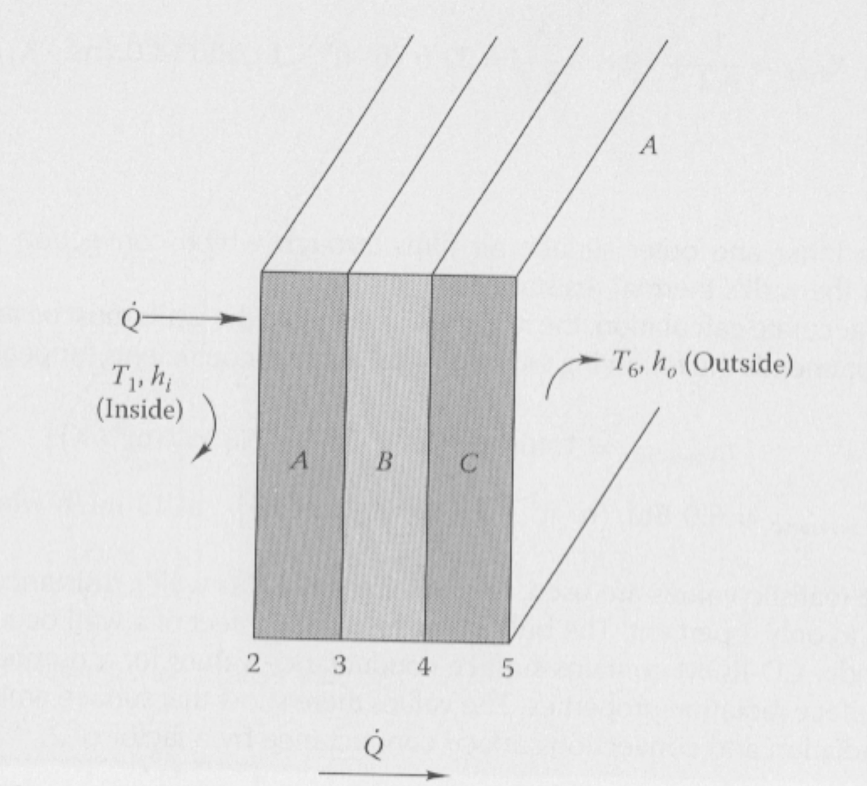
COMBINED MODE HEAT TRANSFER

Combined mode heat transfer: Series

- Nearly all heat transfer situations in buildings include more than one mode of heat transfer
- When more than one heat transfer mode is present, we can compute heat transfer (of all kinds) using resistances in series
  - **Sum resistances in series**

$$R_{tot} = R_A + R_B + R_C + R_{film,in} + R_{film,out}$$

$$q = \frac{1}{R_{tot}}\left(T_{air,in} - T_{air,out}\right)$$



Combined mode heat transfer: Series

- Remember our wood stud wall?
  - Fiberglass batts in 2x6 inch stud cavities, with brick exterior and gypsum board interior ($R_{assembly}$ = 18.7 IP)

  - Just need to add the "film resistances" to calculate heat transfer from indoor air to outdoor air

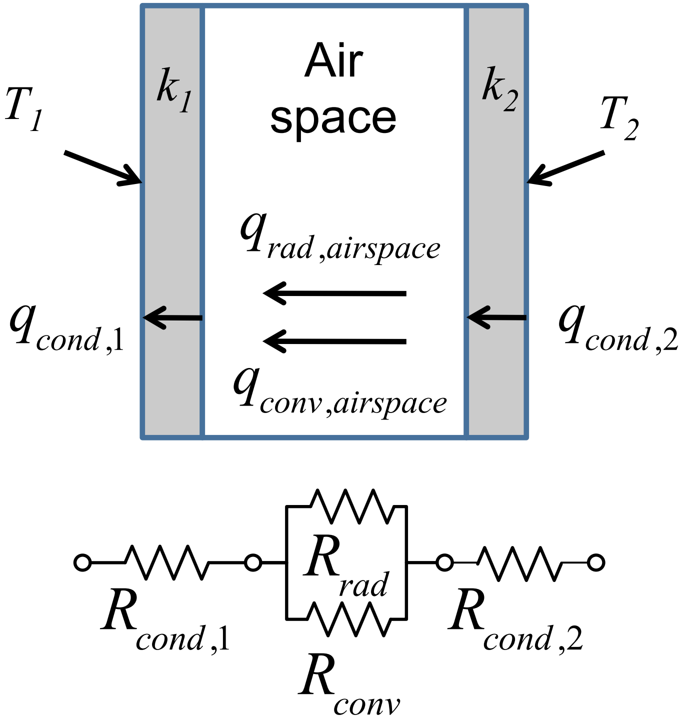
Typical convective "film resistances"

- We often use the values given below for most conditions

Surface Conditions	**Horizontal Heat Flow**	**Upwards Heat Flow**	**Downwards Heat Flow**
Indoors: R_{in}	0.12 m^{2}K/W (SI) 0.68 h·ft^2·°F/Btu (IP)	0.11 m^{2}K/W (SI) 0.62 h·ft^2·°F/Btu (IP)	0.16 m^{2}K/W (SI) 0.91 h·ft^2·°F/Btu (IP)
R_{out}: 6.7 m/s wind (Winter)	0.030 m^{2}K/W (SI) 0.17 h·ft^2·°F/Btu (IP)		
R_{out}: 3.4 m/s wind (Summer)	0.044 m^{2}K/W (SI) 0.25 h·ft^2·°F/Btu (IP)		

Combined mode heat transfer: Parallel

- When more than one mode of heat transfer exists at a location (e.g., convection and radiation), resistances get placed **in parallel**
  - Example: Heat transfer in a building cavity



$$\frac{1}{R_{airspace}} = \frac{1}{R_{rad,airspace}} + \frac{1}{R_{conv,airspace}}$$

$$R_{tot} = R_1 + R_{airspace} + R_2$$

$$q = \frac{1}{R_{tot}}\left(T_2 - T_1\right)$$

Combined modes of heat transfer: Parallel

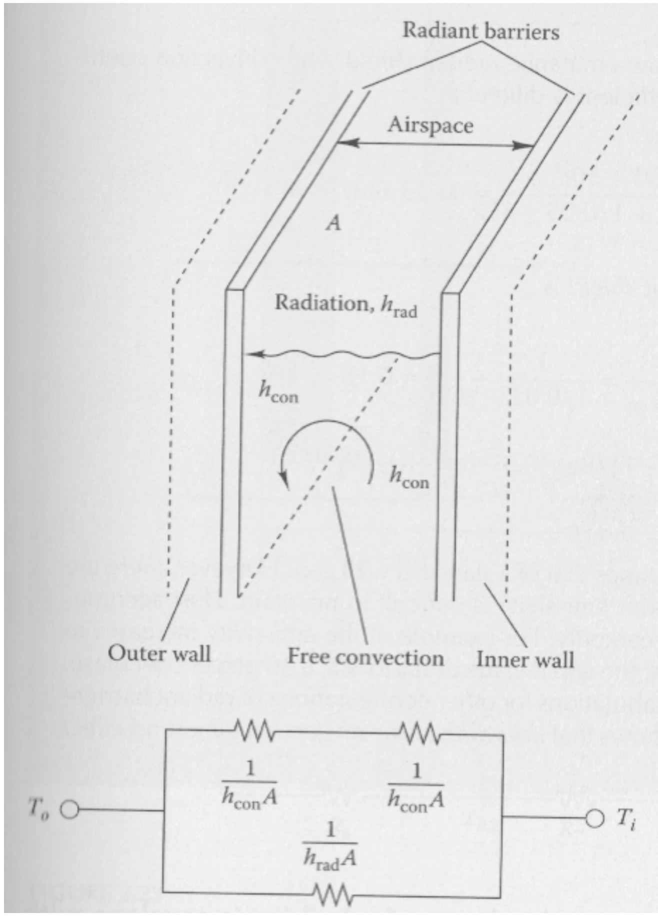
- Example problem: Radiant barrier in a residential wall

A building designer wishes to evaluate the R-value of a 1-inch wide *ventilated air gap* in a wall for its insulation effect

She finds the resistance to heat flow to be quite small, so she proposes lining the cavity's inner and outer surfaces with a highly reflecting aluminum foil film whose emissivity is 0.05

Find the R-value of this cavity with both emissivity conditions, including both radiation and convection effects in the cavity

Assume the surface temperatures facing the gap are 7.2°C and 12.8°C



Combined convection + conduction: Heat exchangers

- Heat exchangers are used widely in buildings
- Heat exchangers are devices in which two fluid streams, usually separated from each other by a solid wall, exchange thermal energy by both <u>convection</u> and <u>conduction</u>
  - One fluid is typically heated, one is typically cooled
    - Fluids may be gases, liquids, or vapors

$k_{aluminum}$ = 205 W/mK
k_{copper} = 385 W/mK

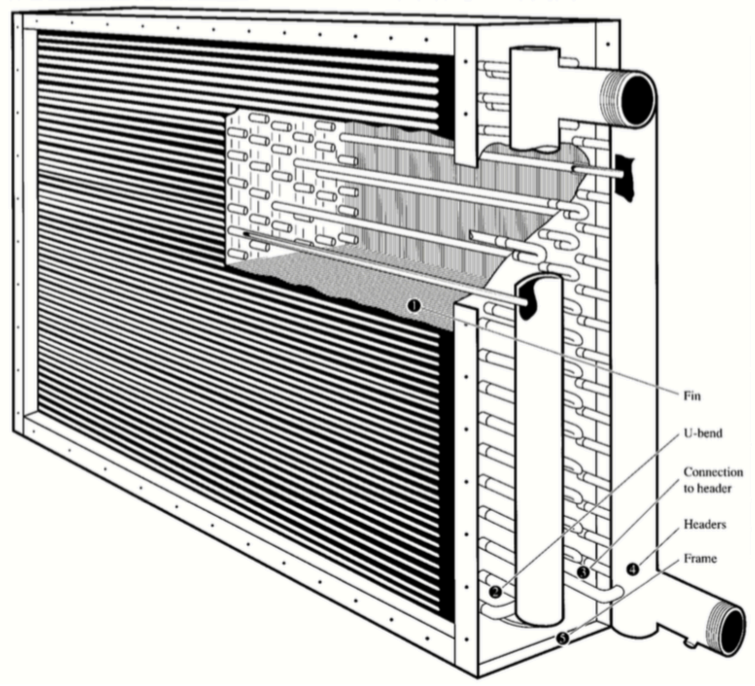
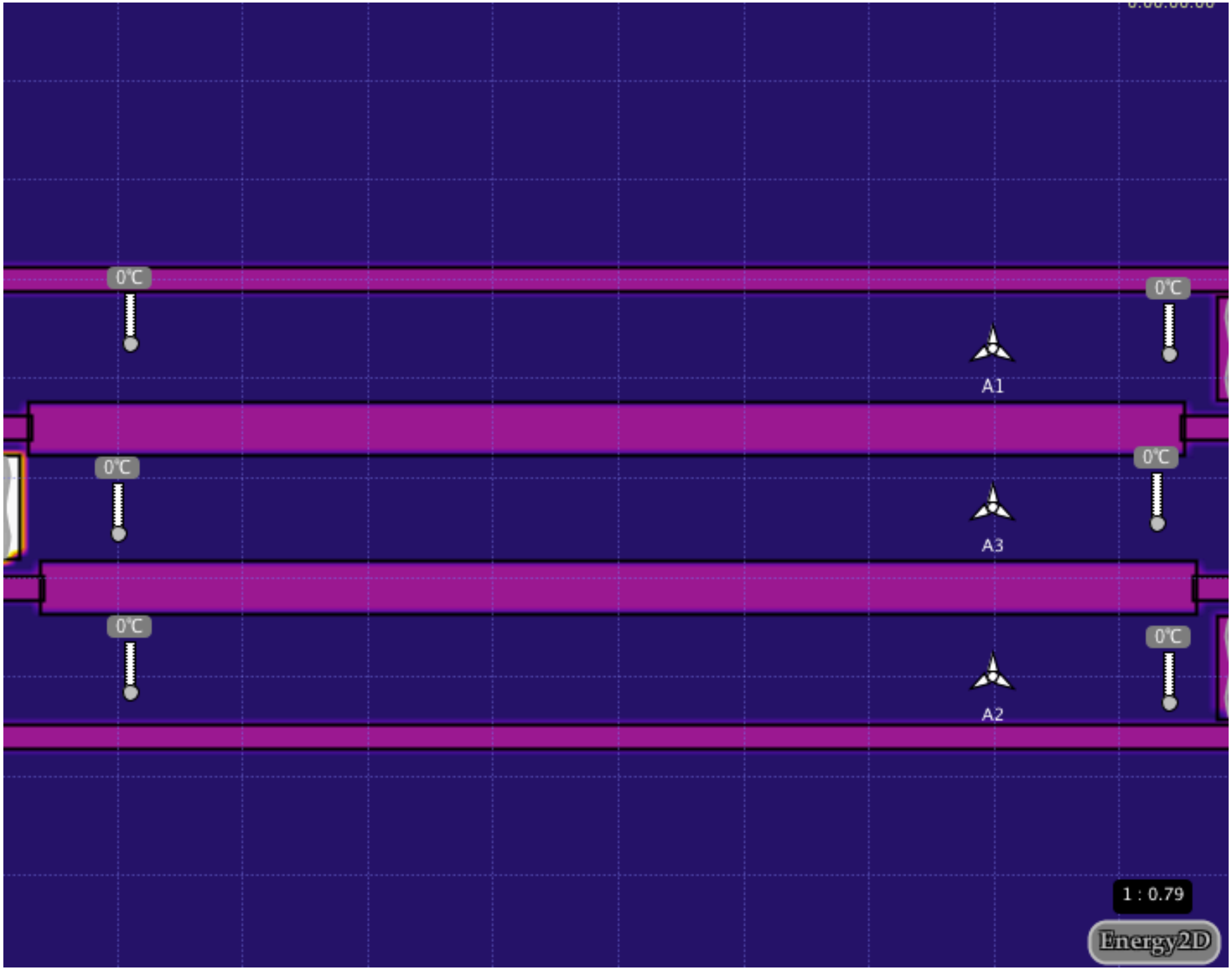


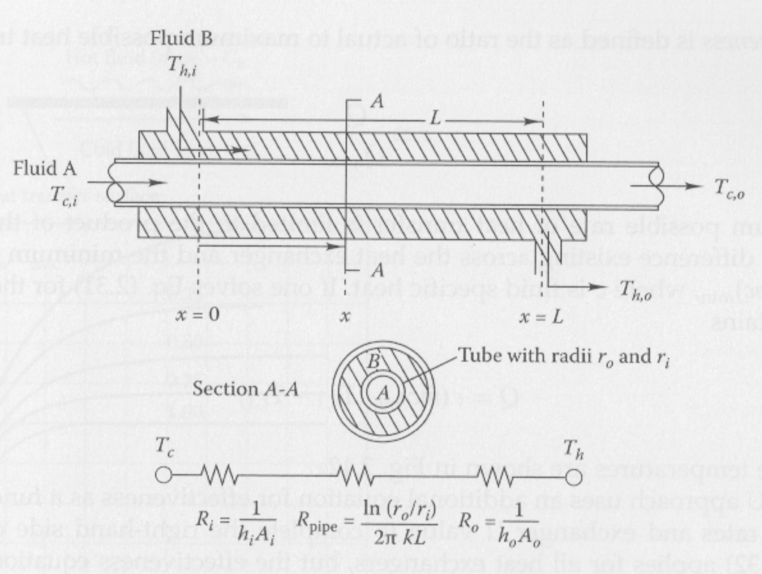
FIGURE 15.26a Structure of a water cooling coil. (*Source: York International Corporation. Reprinted with permission.*)

Combined convection + conduction: Heat exchangers



Combined convection + conduction: Heat exchangers

- The effectiveness of a heat exchanger depends on:
  - The flow rates of fluids in the heat exchanger
  - The overall UA-value of the heat exchanger
    - U is governed by convection and conduction resistance
    - A is governed by heat exchanger design (high surface A)



$$U_oA_o = \frac{1}{R'_{conv,i} + R'_{pipe} + R'_{conv,o}}$$

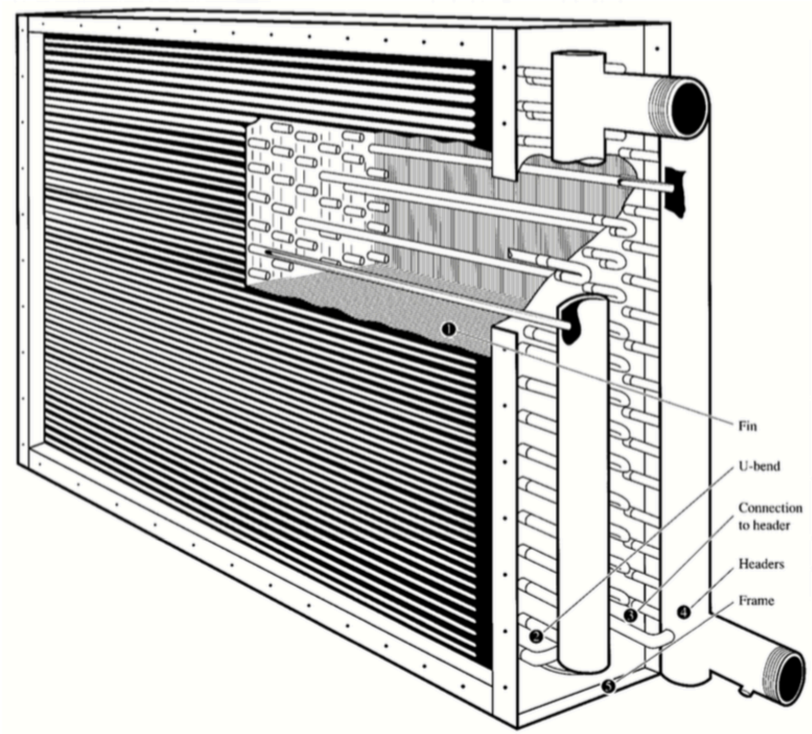
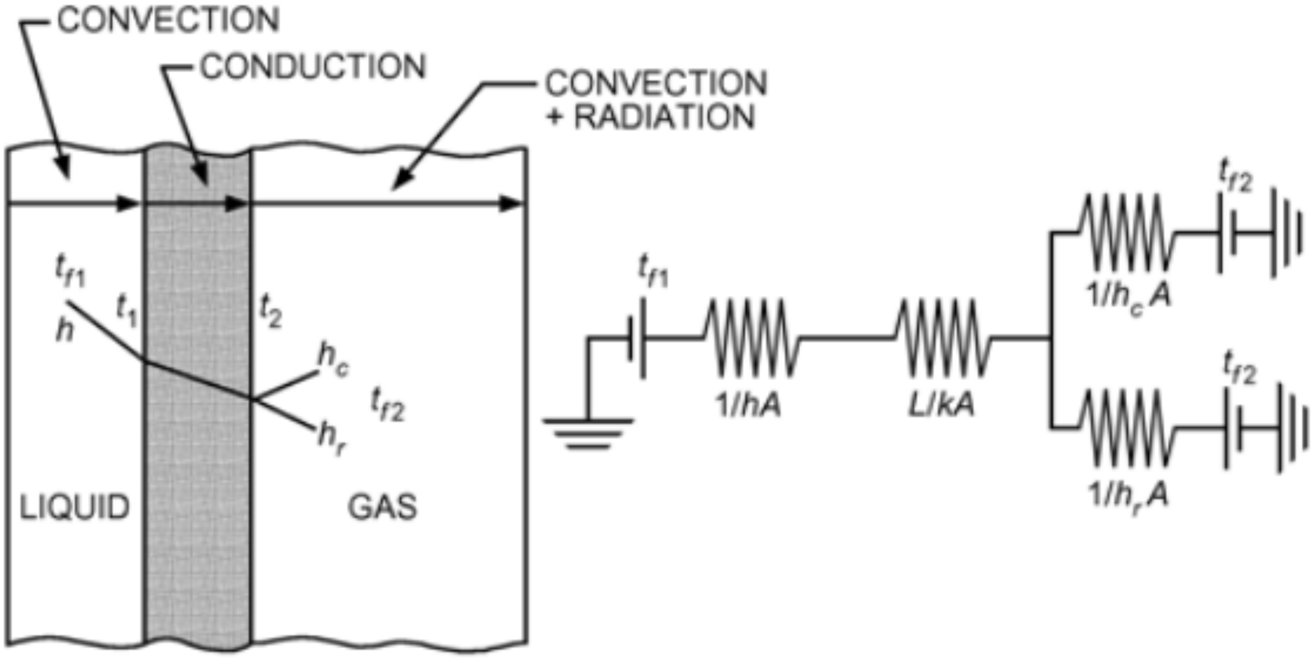


FIGURE 15.26a Structure of a water cooling coil. (*Source: York International Corporation. Reprinted with permission.*)

Combined convection + conduction: Heat exchangers

- Example from ASHRAE Handbook of Fundamentals (Ch. 4)



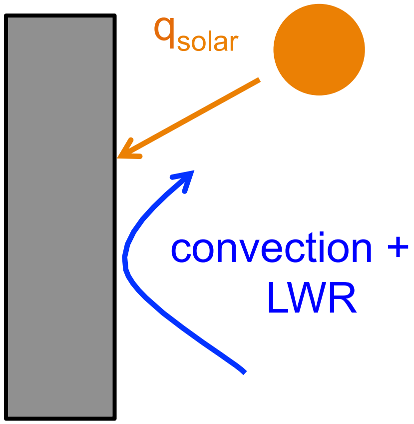
$$q = UA\left(T_{f,1} - T_{f,2}\right)$$

$$\frac{1}{UA} = R'_{tot} = R'_1 + R'_{1-2} + R'_2$$

$$\frac{1}{R'_2} = \frac{1}{R'_{2,conv}} + \frac{1}{R'_{2,rad}}$$

$$R'_1 = \frac{1}{h_{1,conv}A} \qquad R'_{1-2} = \frac{L}{kA}$$

$$R'_{2,conv} = \frac{1}{h_{2,conv}A} \qquad R'_{2,rad} = \frac{1}{h_{2,rad}A}$$



BUILDING ENERGY BALANCES

Building energy balances

- We know that multiple modes of heat transfer are typically acting at the same time at multiple points point within a building…

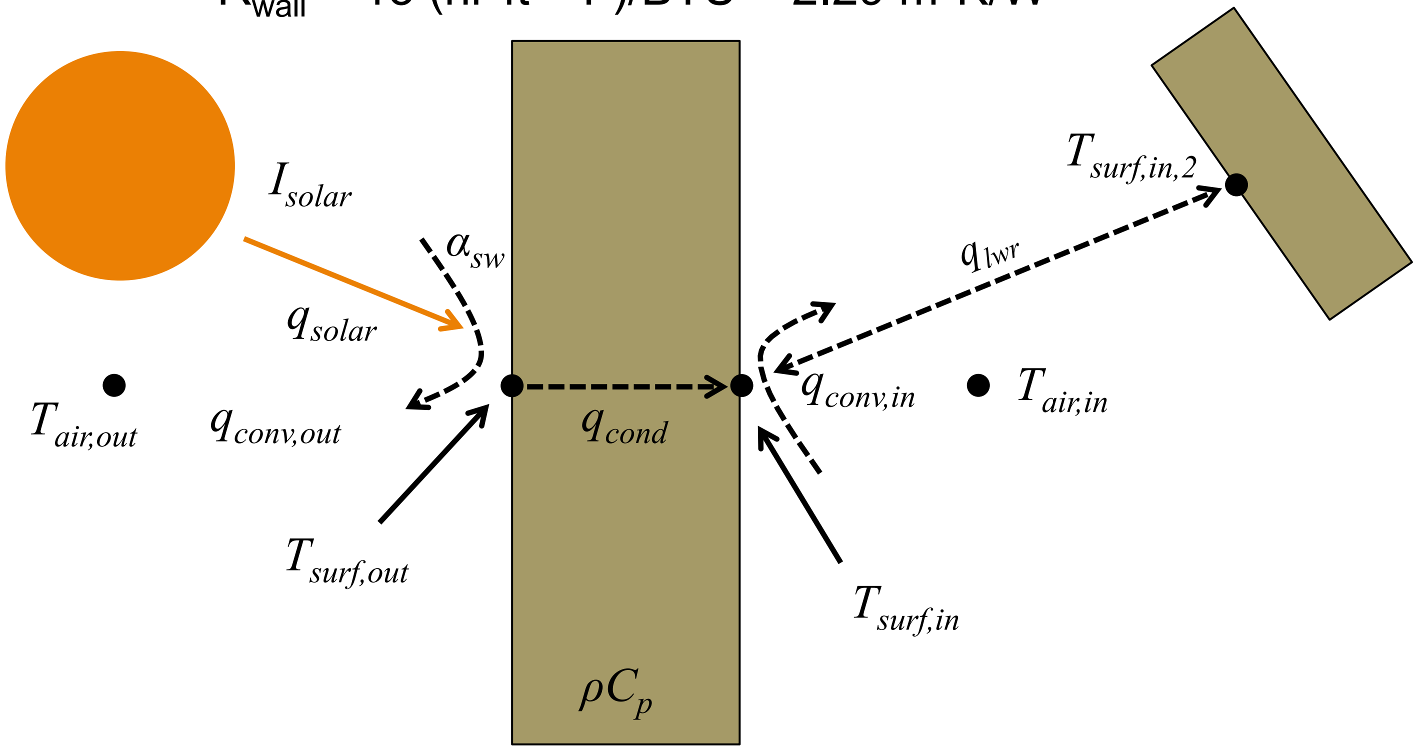
  … So we can also write expressions to quantify heat flow/ flux to/from these various points simultaneously by accounting for all relevant modes of heat transfer

  - Writing “building energy balances”
  - Solving systems of equations

Building energy balances: Simplified

Imagine an external wall of a building:

R_{wall} = 13 (hr-ft^2-°F)/BTU = 2.29 m^{2}K/W



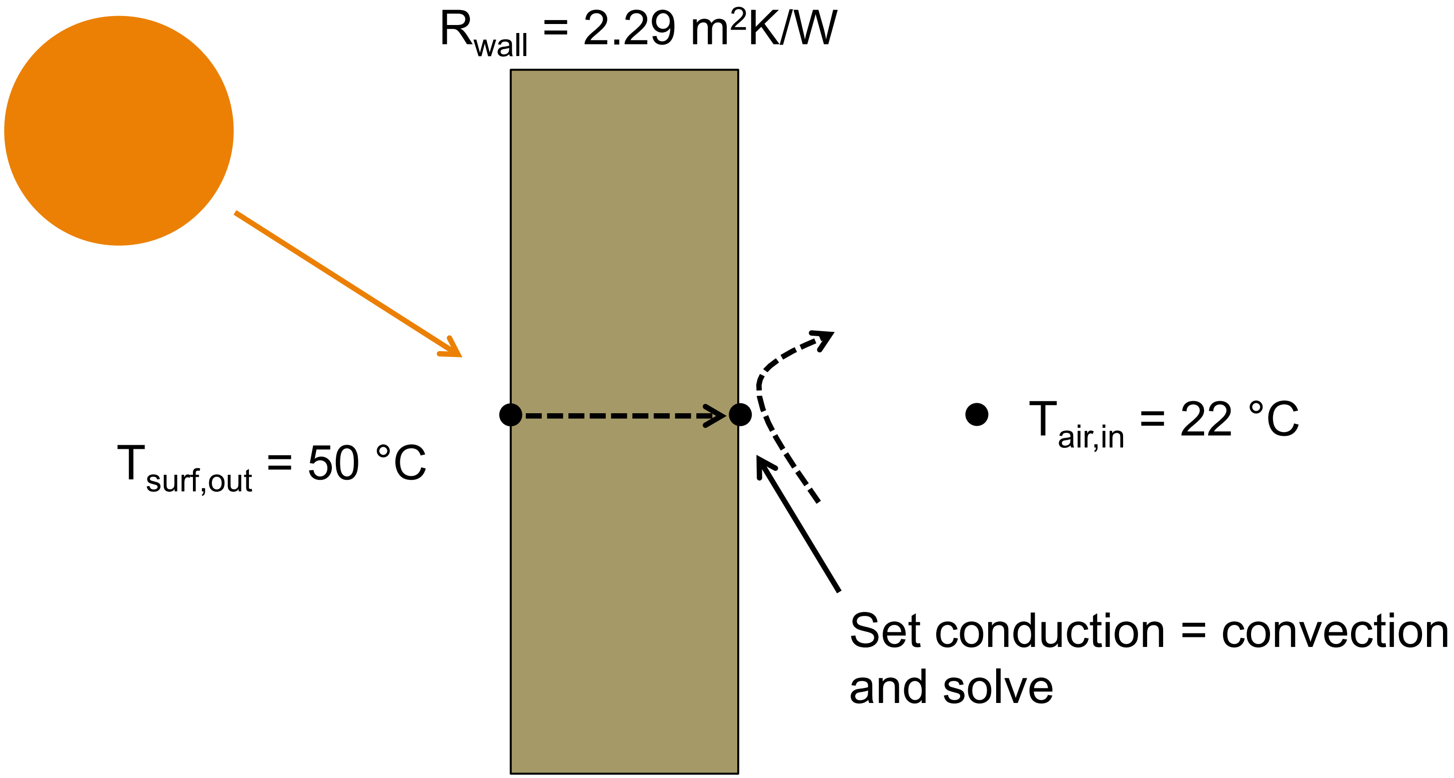
How is this helpful to us?

- Imagine the classroom wall behind me is being heated by the sun on the other side
- The exterior surface temperature is 122°F (50°C)
- The interior air temperature is 72°F (22°C)
- The R-value of the wall is R-13 (IP) (2.29 m^2K/W)
- What is the interior surface temperature of the wall?

- This interior surface temperature impacts the heat flux to indoor air, as well as the surrounding surface temperatures (via radiation), which all impact the building's energy balance

Building energy balance example

- Estimate the surface temperature of an interior wall whose exterior side is being warmed by the sun
  - Assume that LWR can be ignored and assume steady-state



R_{wall} = 2.29 m^2K/W

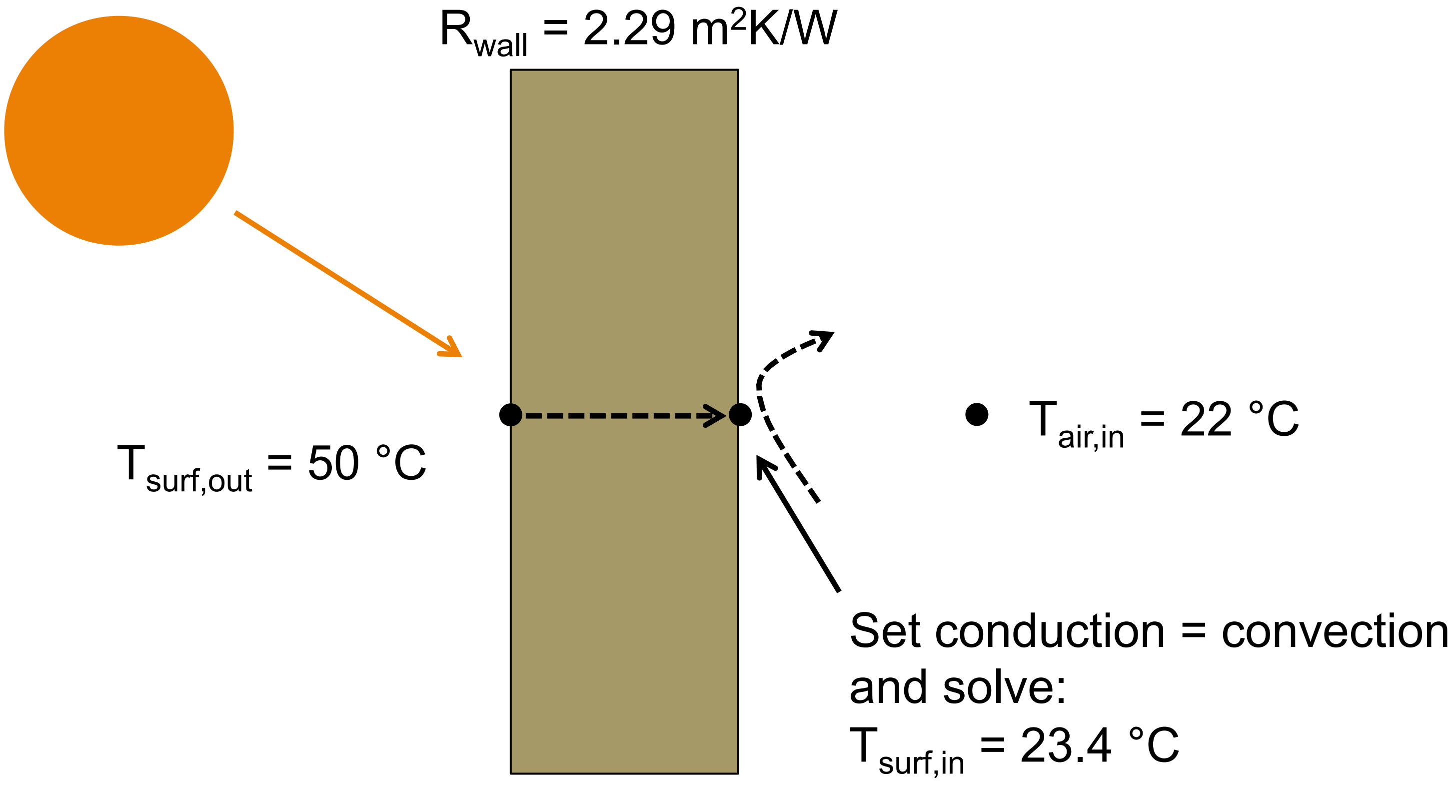
$T_{surf,out}$ = 50 °C

$T_{air,in}$ = 22 °C

Set conduction = convection and solve

Building energy balance example

- Estimate the surface temperature of an interior wall whose exterior side is being warmed by the sun
  - Assume that LWR can be ignored and assume steady-state



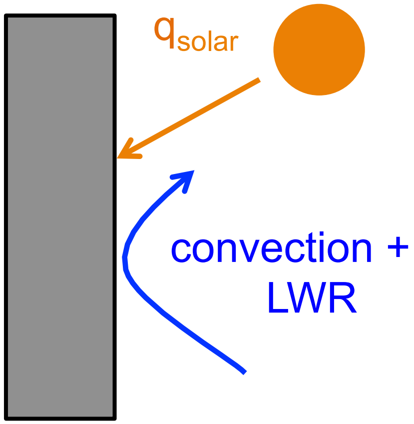
R_{wall} = 2.29 m^2K/W

$T_{surf,out}$ = 50 °C

$T_{air,in}$ = 22 °C

Set conduction = convection and solve:

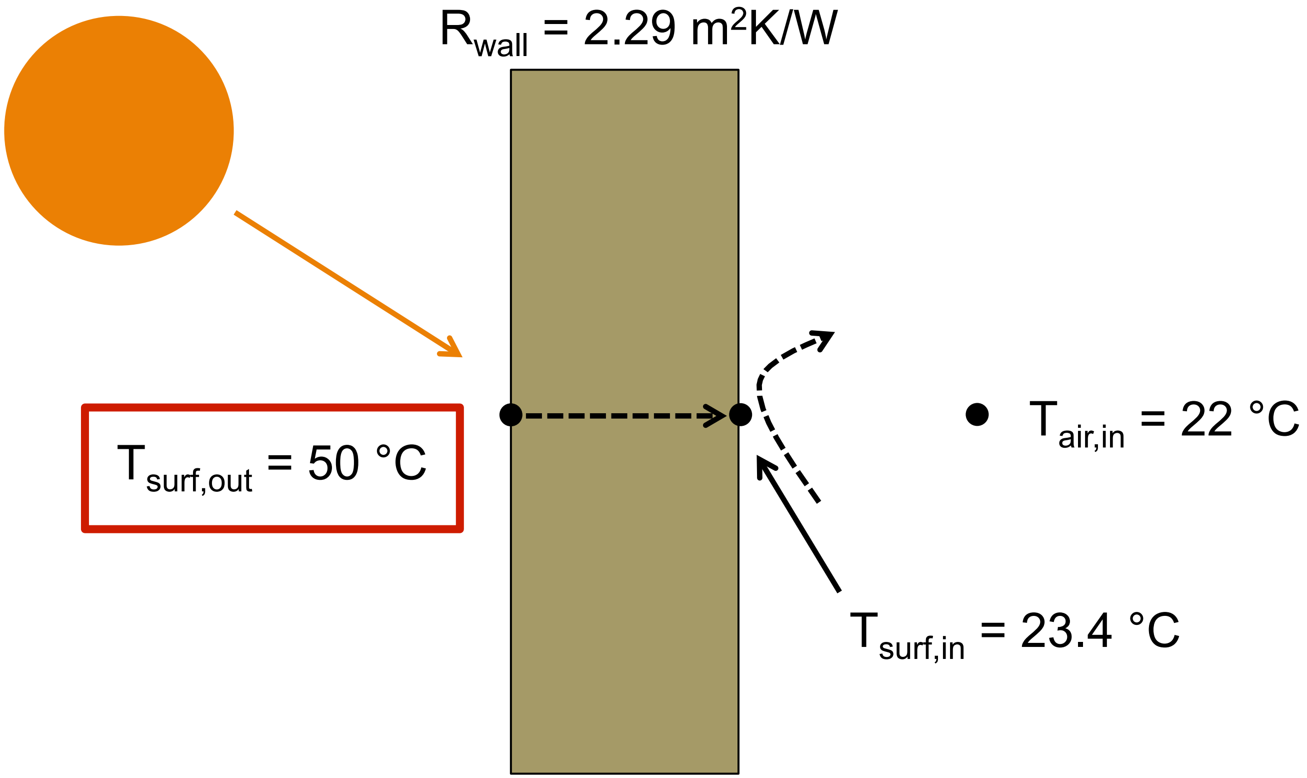
$T_{surf,in}$ = 23.4 °C



“SOL-AIR” TEMPERATURES

Sol-air temperatures

- In the last example, we were given that the exterior surface temperature was 122°F (50°C)
  - **How did we know that?**



Sol-air temperatures

- If we take an external surface with a combined convective and radiative heat transfer coefficient, $h_{conv+rad}$

$$q_{conv+rad} = h_{conv+rad}\left(T_{air} - T_{surf}\right)$$

- If that surface now absorbs solar radiation (αI_{solar}), the total heat flow at the exterior surface becomes:

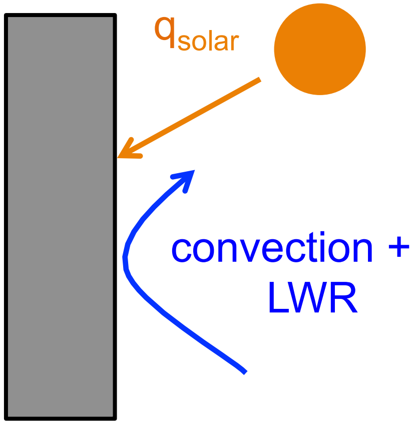
$$q_{conv+rad} = h_{conv+rad}\left(T_{air} - T_{surf}\right) + \alpha I_{solar}$$

- To simplify our calculations, we can define a "**sol-air**" temperature that accounts for all of these impacts:

$$T_{sol-air} = T_{air} + \frac{\alpha I_{solar}}{h_{conv+rad}}$$

- Now we can describe heat transfer at that surface as:

$$q_{total} = h_{conv+rad}\left(T_{sol-air} - T_{surf}\right)$$



Example **sol-air** temperatures

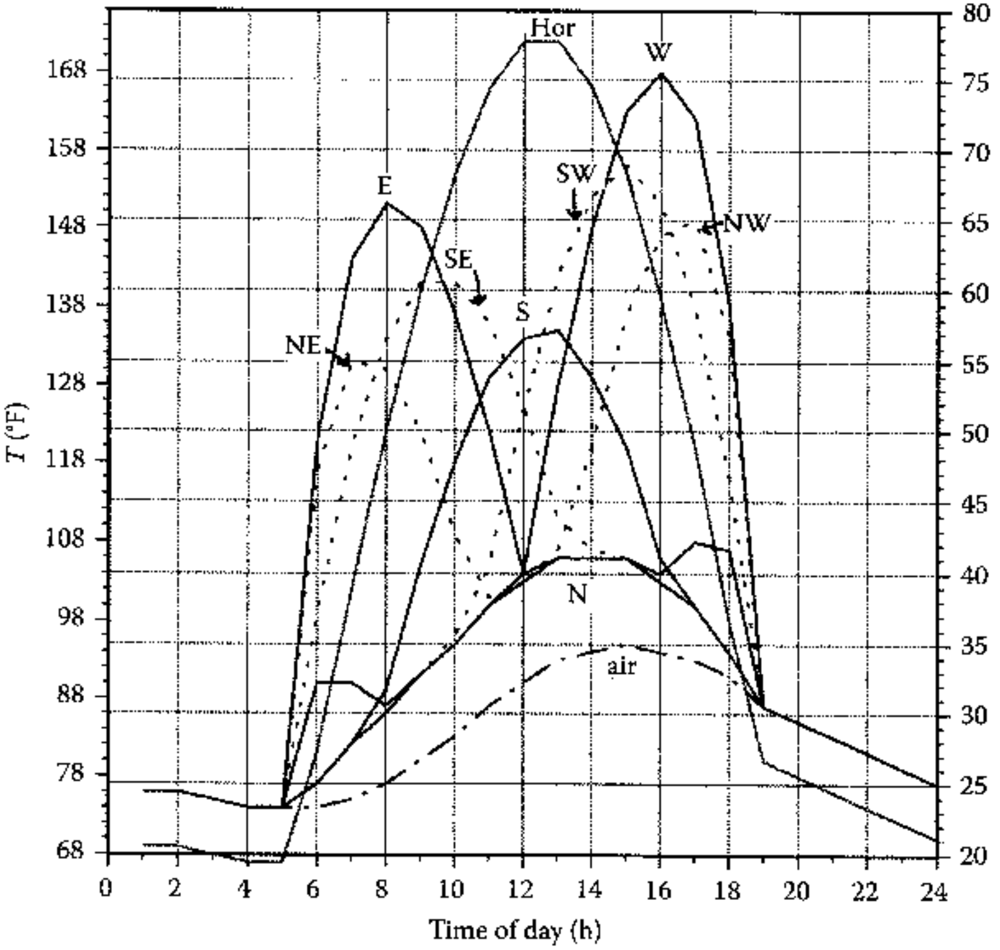
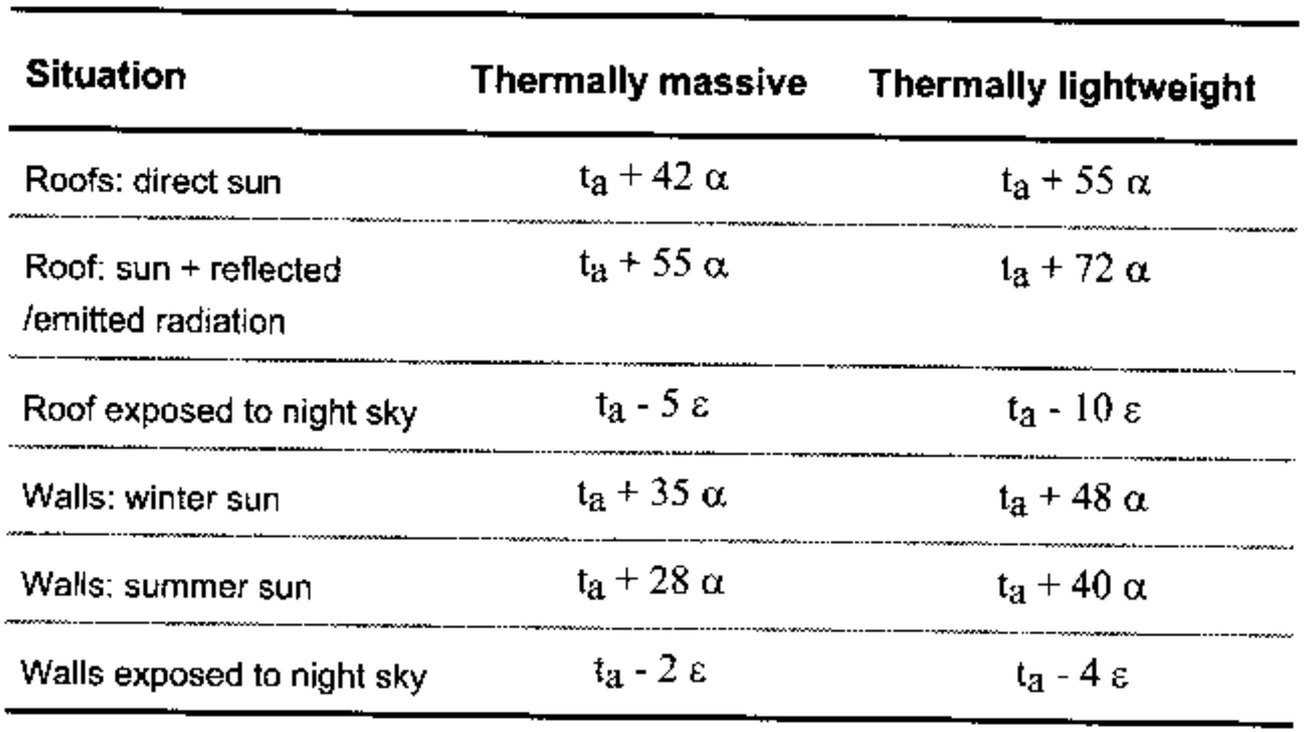


FIGURE 6.17
Sol-air temperature for horizontal and vertical surfaces as a function of time of day for summer design conditions, July 21 at 40° latitude, assuming $\alpha/h_o = 0.30$ (h · ft^2 · °F)/Btu [0.052 (m^2 · K)/W]. The curves overlap when there is no direct radiation on a surface. (Courtesy of ASHRAE, *Handbook of Fundamentals*, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Atlanta, GA, 1989, Table 26.1.)

Solar radiation and external surface temperatures

- We can also use air temperatures and material properties (absorptivity and emissivity) to estimate exterior surface temperatures that are exposed to radiation
  - These are not extremely accurate but provide a reasonable estimate



Situation	**Thermally massive**	**Thermally lightweight**
Roofs: direct sun	$t_a + 42\ \alpha$	$t_a + 55\ \alpha$
Roof: sun + reflected /emitted radiation	$t_a + 55\ \alpha$	$t_a + 72\ \alpha$
Roof exposed to night sky	$t_a - 5\ \varepsilon$	$t_a - 10\ \varepsilon$
Walls: winter sun	$t_a + 35\ \alpha$	$t_a + 48\ \alpha$
Walls: summer sun	$t_a + 28\ \alpha$	$t_a + 40\ \alpha$
Walls exposed to night sky	$t_a - 2\ \varepsilon$	$t_a - 4\ \varepsilon$

Source: Straube and Burnett

Next time

- HW #2 is due
- Fenestration (doors and windows) – applications of combined mode heat transfer