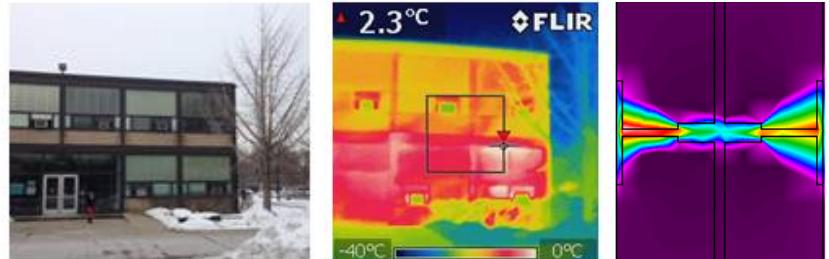


# CAE 331/513

# Building Science

## Fall 2018



**September 4, 2018**  
**Heat transfer in buildings: Convection**

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**Dr. Brent Stephens, Ph.D.**  
Civil, Architectural and Environmental Engineering  
Illinois Institute of Technology  
[brent@iit.edu](mailto:brent@iit.edu)

# Introducing our TA for this course

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- Afshin Faramarzi
  - PhD student in Civil (Architectural) Engineering
  - Office: Alumni Hall Room 217 (BERG Lab)
  - Email: [afaramar@hawk.iit.edu](mailto:afaramar@hawk.iit.edu)

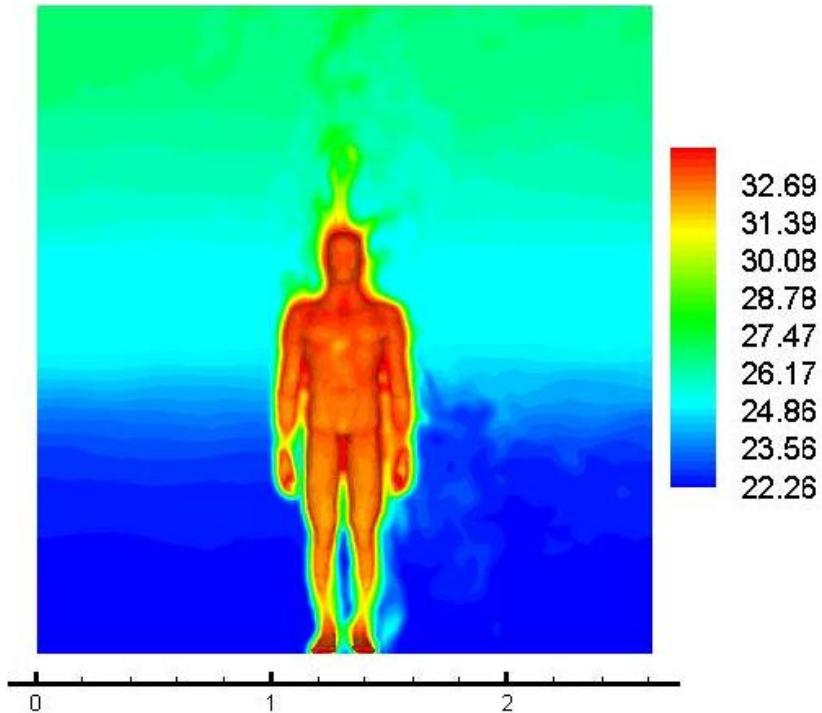
# Review from last time

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- Turned in HW 1
- Finished conduction
  - 2-D and 3-D simplifications
  - Below- and on-grade conduction
  - Transient conduction
    - Heat capacity
    - Thermal diffusivity

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

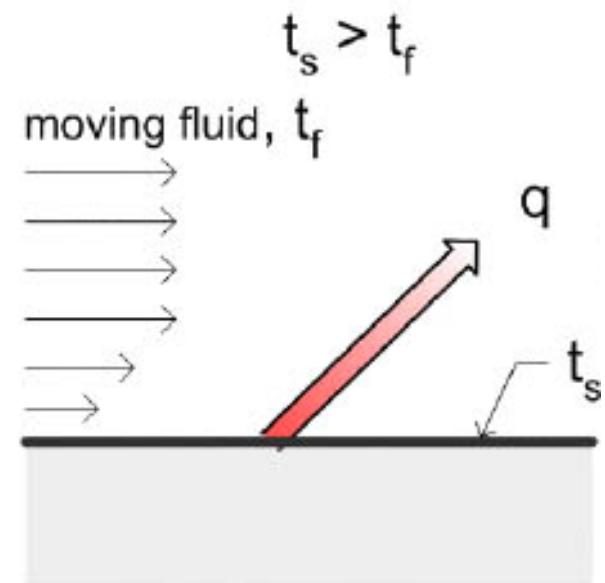
$$HCA = \rho L A C_p = \rho V C_p \quad \alpha = \frac{k}{\rho C_p} \quad [\text{m}^2/\text{s}]$$



# CONVECTION

# Convection

- **Convection** is **heat transfer** due to bulk movement of molecules within fluids (e.g., gases and liquids)
  - Also a mode of **mass transfer** in fluids



# Convective heat transfer in buildings

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2 main types of convective heat transfer in buildings:

1. Bulk convection, or “**advection**”

- The transport of heat between fluids by fluid flow (e.g., air or water)

2. What most refer to as “**convection**”:

- Convective heat transfer between a surface and a fluid

# Bulk convective heat transfer: Advection

- Bulk convective heat transfer, or **advection**, is more direct than convection between surfaces and fluids
- Fluids, such as air, have the capacity to store heat, so fluids flowing into or out of a control volume also carry heat with it

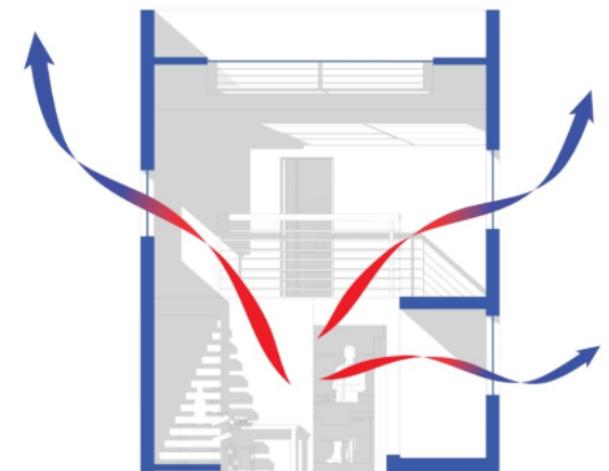
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$$Q_{bulk} = m C_p \Delta T$$

$m$  “dot” = mass flow rate of fluid (kg/s or lb/hr)

$C_p$  = specific heat capacity of fluid [J/(kgK) or BTU/(lb°F)]

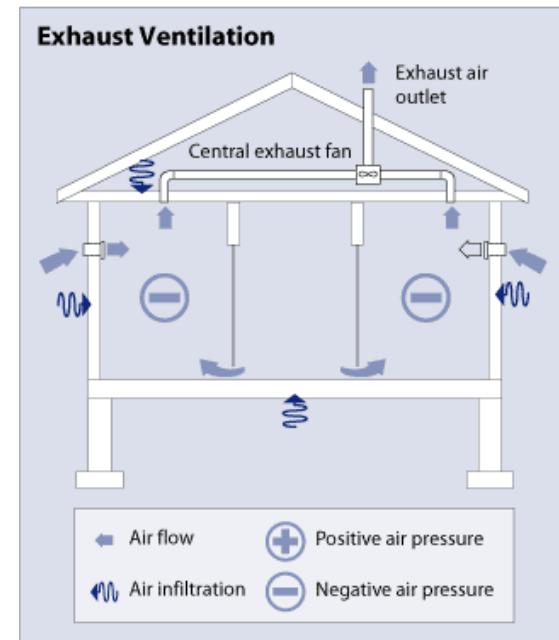
$$[W] = \left[ \frac{\text{kg}}{\text{s}} \cdot \frac{\text{J}}{\text{kg} \cdot \text{K}} \cdot \text{K} \right]$$



$$\left[ \frac{\text{BTU}}{\text{h}} \right] = \left[ \frac{\text{lb}}{\text{h}} \cdot \frac{\text{BTU}}{\text{lb} \cdot \text{°F}} \cdot \text{°F} \right]$$

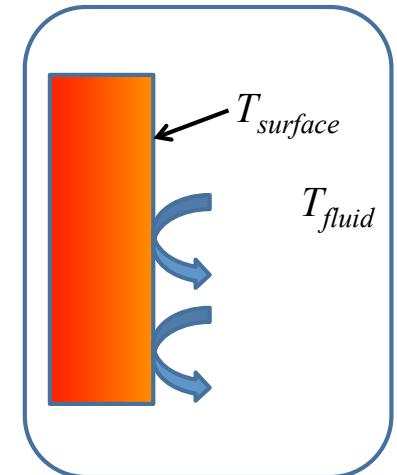
# Bulk convective heat transfer: Advection

- Every time you take a shower at home, you should use your bathroom exhaust fan to exhaust the hot/humid air generated by the shower
- If it is 68°F inside the house and 10°F outside, what is the rate of heat loss via bulk convection (i.e., advection) during these conditions?
  - Assume that the fan operates at an airflow rate of 100 CFM



# Convection (between a surface and a fluid)

- When a fluid comes in contact with a surface at a different temperature
- We use a heat transfer coefficient,  $h_{conv}$ , to relate the rate of heat transfer to the difference between the solid surface temperature,  $T_{surface}$ , and the temperature of the fluid far from the surface,  $T_{fluid}$



$$q_{conv} = h_{conv} (T_{fluid} - T_{surface}) = \frac{1}{R_{conv}} (T_{fluid} - T_{surface})$$

An application of  
Newton's law of cooling

where  $T_{fluid}$  = fluid temperature far enough not to be affected by  $T_{surface}$

$h_{conv}$  = convective heat transfer coefficient [ $\text{W}/(\text{m}^2 \cdot \text{K})$ ] or [ $\text{BTU}/(\text{hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F})$ ]

and  $R_{conv} = \frac{1}{h_{conv}}$  = convective thermal resistance [ $(\text{m}^2 \cdot \text{K})/\text{W}$ ] or [ $(\text{hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F})/\text{BTU}$ ]

# Q versus q for convection

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- Same story as conduction...

$$q_{conv} = h_{conv} (T_{fluid} - T_{surface}) \quad \left[ \frac{\text{W}}{\text{m}^2} \right]$$

- 

- To get  $Q$ , just multiply by surface area,  $A$

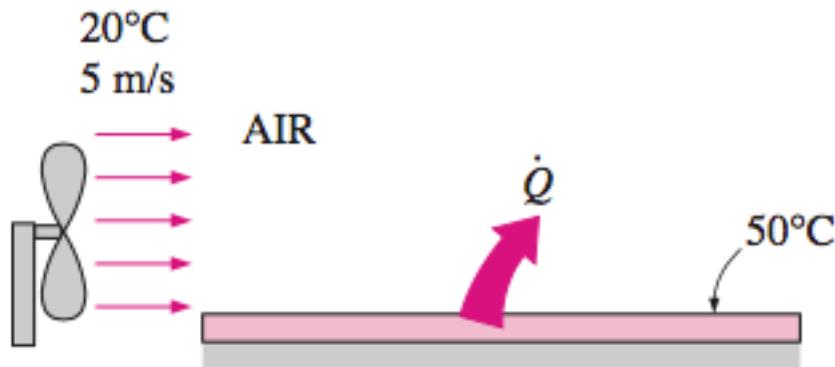
$$Q_{conv} = h_{conv} A (T_{fluid} - T_{surface}) \quad [\text{W}]$$

# Two types of (surface/fluid) convective heat transfer

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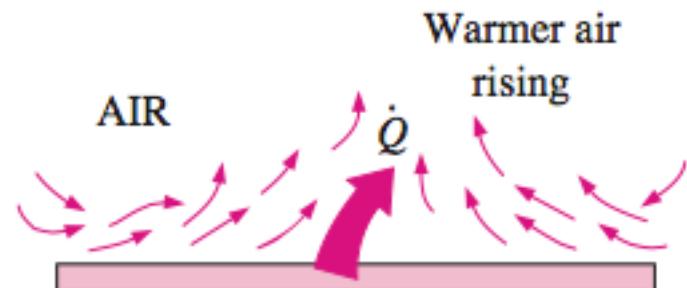
- Two types of (surface/fluid) convection exist:
  - **Natural (or free) convection:** Results from **density differences** in the fluid caused by contact with the surface to or from which the heat transfer occurs
    - **Buoyancy** is the main driver
      - Temperature dependent density differences
    - Example: The circulation of air in a room caused by the presence of a solar-warmed window or wall (without a mechanical system) is a manifestation of natural/free convection
  - **Forced convection:** Results from a force external to the problem (other than gravity or other body forces) moves a fluid past a warmer or cooler surface
    - Usually much higher velocities and more random and chaotic flow
    - Driven by mechanical forces (e.g. **fans and wind**)
    - Example: Heat transfer between cooling coils and an air stream

# Two types of convective heat transfer



(a) Forced convection

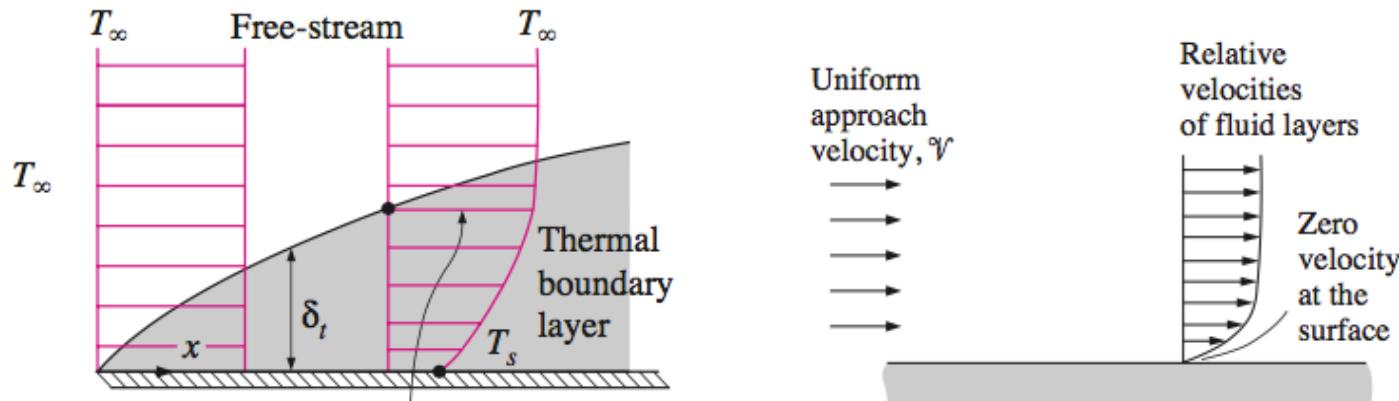
$$\rho = \frac{n}{V} MW = \frac{P}{RT} MW \quad T \downarrow \rho \uparrow \quad T \uparrow \rho \downarrow$$



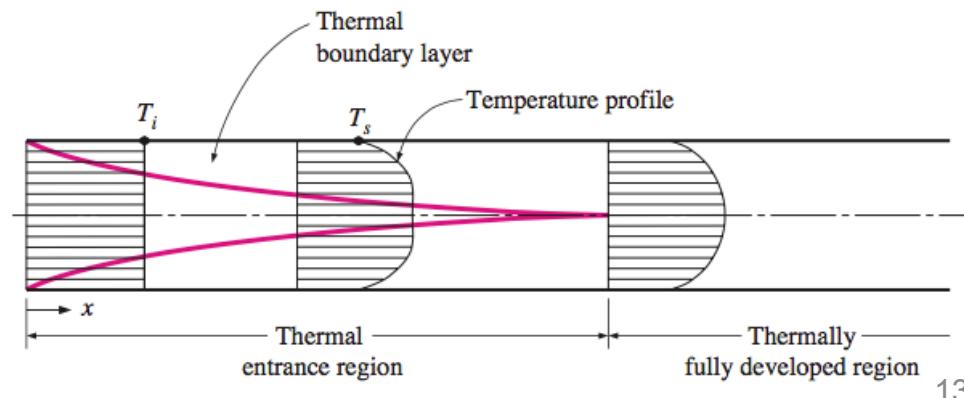
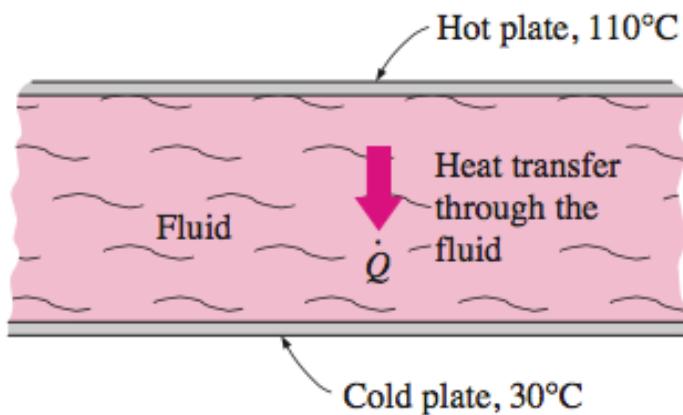
(b) Free convection

# Two forms of convection in buildings

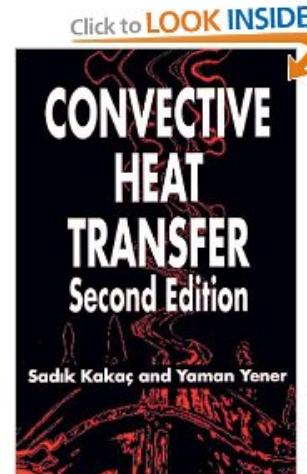
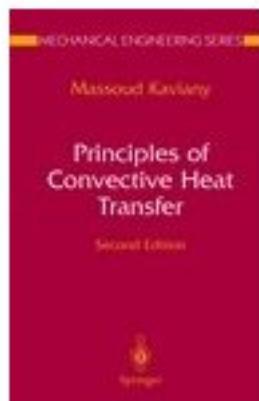
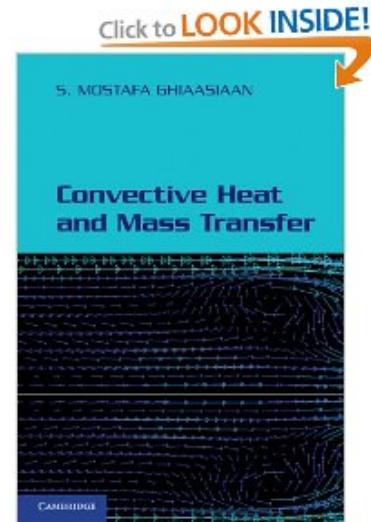
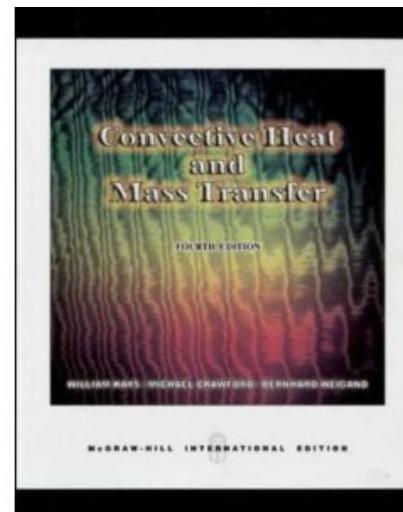
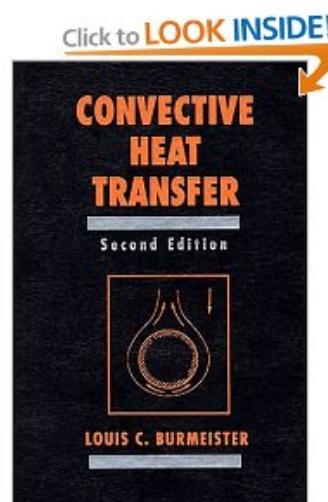
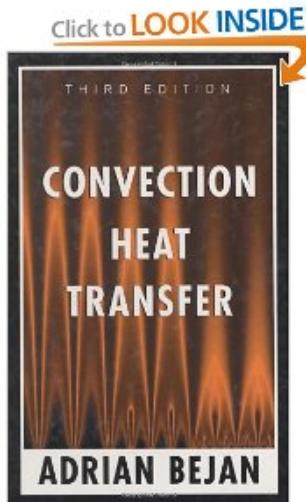
- **External flows**
  - Fluid flow over objects (building surfaces, pipes, etc.)



- **Internal flows**
  - Fluid flow inside channels (e.g., pipes, ducts, etc.)



# Convection is really a field of its own



# Convective heat transfer coefficient, $h_{conv}$

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- The convective heat transfer coefficient,  $h_{conv}$ , will take many forms depending upon whether the convection is forced or natural
  - $h_c$  is also known as the surface conductance
  - $R_c = 1/h_c$  is the surface or “film” resistance
- $h_c$  is typically determined *empirically* (i.e., it is *measured*)
  - It can also be estimated based on a dimensionless group of fluid properties
  - We can express convection coefficients as a function of:

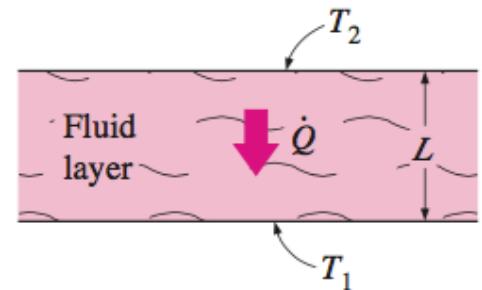
$$Nu = f(Re, Pr)$$

Nu = Nusselt #  
Re = Reynolds #  
Pr = Prandtl #

# Convective heat transfer coefficient, $h_{conv}$

- Nusselt # (Nu)
  - Ratio of convection to conduction heat transfer
  - Ratio of heat transfer when fluid is in motion to when it is motionless

$$Nu = \frac{hL_c}{k}$$



$$\Delta T = T_2 - T_1$$

Nu = Nusselt number (dimensionless)

$k$  = thermal conductivity of the fluid (W/mK)

$L_c$  = characteristic length (m)

$h$  = convective heat transfer coefficient (W/m<sup>2</sup>K)

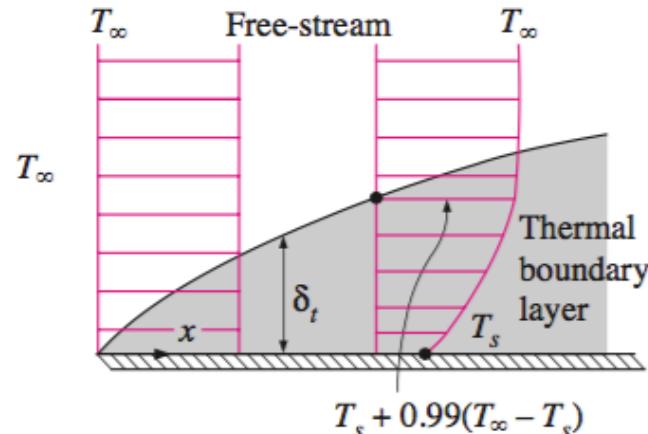
$$\dot{q}_{\text{conv}} = h\Delta T \quad \dot{q}_{\text{cond}} = k \frac{\Delta T}{L}$$

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = Nu$$

The larger the Nusselt number, the more effective the convective heat transfer

# Convective heat transfer coefficient, $h_{conv}$

- Thermal boundary layer
  - Defines a flow region over which the temperature variation between the free-stream fluid flow and the surface temperature is significant
- Prandtl # (Pr)
  - We can describe the relative thickness of the velocity and thermal boundary layers by another dimensionless parameter: Pr



$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

$\nu$  = fluid kinematic viscosity =  $\mu/\rho$  ( $m^2/s$ )

$\mu$  = fluid dynamic viscosity ( $kg/m \cdot s$ )

$C_p$  = specific heat capacity of the fluid ( $J/kgK$ )

$k$  = thermal conductivity of fluid ( $W/mK$ )

Typical ranges of Prandtl numbers for common fluids

Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.19–1.0
Water	1.19–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

Pr ~ 1 for gases → both momentum and heat dissipate at about the same rate

# Convective heat transfer coefficient, $h_{conv}$

- Reynolds # (Re)
  - Transition from laminar to turbulent flow depends on the surface geometry, surface roughness, upstream velocity, surface temperature, and the type of fluid
  - This is best described by Re:

$$Re_x = \frac{\rho V x}{\mu} = \frac{V x}{v}$$

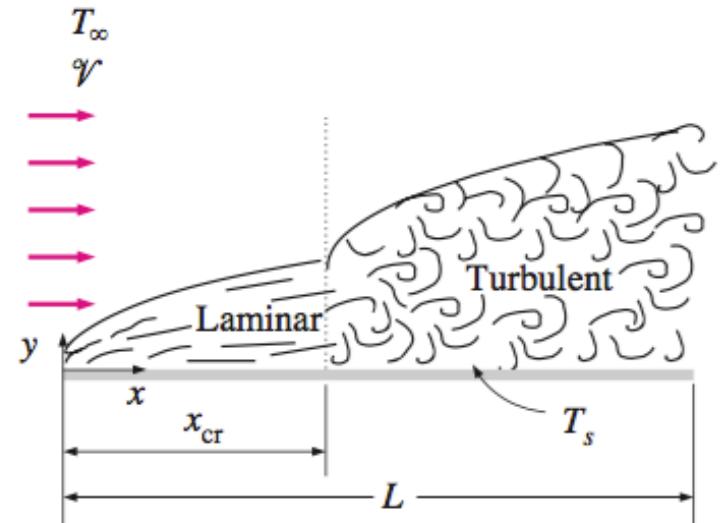
$V$  = upstream fluid velocity (m/s)

$x$  = distance along a plate from the upstream velocity (m)

$\mu$  = fluid dynamic viscosity (kg/m-s)

$\rho$  = fluid density (kg/m<sup>3</sup>)

$v$  = fluid kinematic viscosity =  $\mu/\rho$  (m<sup>2</sup>/s)



- Re will vary over  $x$
- Transition from laminar to turbulent is typically around Re =  $5 \times 10^5$  (may vary)

# Convective heat transfer coefficient, $h_{conv}$

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How do we use these values to estimate convective heat transfer coefficients?

$$\text{Nu} = \frac{hL_c}{k} \quad \text{Nu} = f(\text{Re}, \text{Pr}) \quad \text{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{v} \quad \text{Pr} = \frac{\mu C_p}{k}$$

It depends on the scenario:

External flows:

Forced convective flow over a flat plate

*Laminar:*       $\text{Nu}_x = \frac{h_x x}{k} = 0.332 \text{ Re}_x^{0.5} \text{ Pr}^{1/3} \quad \text{Pr} > 0.60$

The corresponding relation for turbulent flow is

*Turbulent:*       $\text{Nu}_x = \frac{h_x x}{k} = 0.0296 \text{ Re}_x^{0.8} \text{ Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$   
 $5 \times 10^5 \leq \text{Re}_x \leq 10^7$

This gives us a “local” Nu #

# Convective heat transfer coefficient, $h_{conv}$

---

How do we use these values to estimate convective heat transfer coefficients?

$$\text{Nu} = \frac{hL_c}{k} \quad \text{Nu} = f(\text{Re}, \text{Pr}) \quad \text{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\nu} \quad \text{Pr} = \frac{\mu C_p}{k}$$

It depends on the scenario:

**External flows:** Forced convective flow over a flat plate

**The average Nu # over the whole plate, which is more helpful, is:**

*Laminar:*  $\text{Nu} = \frac{hL}{k} = 0.664 \text{Re}_L^{0.5} \text{Pr}^{1/3} \quad \text{Re}_L < 5 \times 10^5$

*Turbulent:*  $\text{Nu} = \frac{hL}{k} = 0.037 \text{Re}_L^{0.8} \text{Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$   
 $5 \times 10^5 \leq \text{Re}_L \leq 10^7$

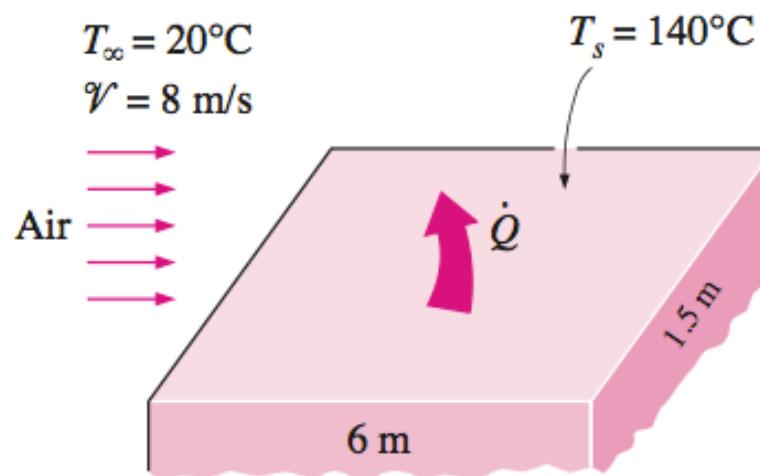
*Transition:*  $\text{Nu} = \frac{hL}{k} = (0.037 \text{Re}_L^{0.8} - 871) \text{Pr}^{1/3} \quad 0.6 \leq \text{Pr} \leq 60$   
 $5 \times 10^5 \leq \text{Re}_L \leq 10^7$

There are many different conditions that each have to be analyzed separately!

# Example problem

## Cooling of a hot block by forced air

- A fan moves air at  $20^{\circ}\text{C}$  and a velocity of 8 m/s over a 1.5-m x 6-m flat plate whose temperature is  $140^{\circ}\text{C}$
- Determine the rate of heat transfer if the air flows parallel to the 6-m long side



Assume: 1) steady state operation; 2) critical  $\text{Re} = 5 \times 10^5$ ; 3) radiation effects are negligible; 4) you are at sea level

# Convective heat transfer coefficient, $h_{conv}$

- There are many more scenarios applicable to buildings
  - From Chapter 4 of the ASHRAE Handbook of Fundamentals:

**Table 8 Forced-Convection Correlations**

I. General Correlation	$Nu = f(Re, Pr)$	
<b>II. Internal Flows for Pipes and Ducts:</b> Characteristic length = $D$ , pipe diameter, or $D_h$ , hydraulic diameter.		
$Re = \frac{\rho V_{avg} D_h}{\mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{Q D_h}{A_c v} = \frac{4 \dot{m}}{\mu P_{wet}} = \frac{4 Q}{v P_{wet}}$	where $\dot{m}$ = mass flow rate, $Q$ = volume flow rate, $P_{wet}$ = wetted perimeter, $A_c$ = cross-sectional area, and $v$ = kinematic viscosity ( $\mu/\rho$ ).	
<i>Laminar</i> : $Re < 2300$	$\frac{Nu}{Re^{1/3}} = \frac{f}{2}$ $Nu = 1.86 \left( \frac{Re}{L/D} \right)^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}$	Colburn's analogy (turbulent) <span style="float: right;">(T8.1)</span>
Developing	$Nu = 3.66 + \frac{0.065(D/L)Re Pr}{1 + 0.04[(D/L)Re Pr]^{2/3}}$	$\frac{L}{D} < \frac{Re}{8} \left( \frac{\mu}{\mu_s} \right)^{0.42}$ <span style="float: right;">(T8.2)<sup>a</sup></span>
Fully developed, round	$Nu = 3.66$	Uniform surface temperature <span style="float: right;">(T8.4a)</span>
	$Nu = 4.36$	Uniform heat flux <span style="float: right;">(T8.4b)</span>
<i>Turbulent</i> :	$Nu = 0.023 Re^{4/5} Pr^{0.4}$	Heating fluid <span style="float: right;">(T8.5a)<sup>b</sup></span>
Fully developed	$Nu = 0.023 Re^{4/5} Pr^{0.3}$	Cooling fluid <span style="float: right;">(T8.5b)<sup>b</sup></span>
Evaluate properties at bulk temperature $t_b$ except $\mu_s$ and $t_s$ at surface temperature	$Nu = \frac{(f_s/2)(Re - 1000)Pr}{1 + 12.7(f_s/2)^{1/2}(Pr^{2/3} - 1)} \left[ 1 + \left( \frac{D}{L} \right)^{2/3} \right]$ <p>For fully developed flows, set <math>D/L = 0</math>.</p> $Nu = 0.027 Re^{4/5} Pr^{1/3} \left( \frac{\mu}{\mu_s} \right)^{0.14}$	$f_s = \frac{1}{(1.58 \ln Re - 3.28)^2}$ <span style="float: right;">(T8.6)<sup>c</sup></span>
		Multiply Nu by $(T/T_s)^{0.45}$ for gases and by $(Pr/Pr_s)^{0.11}$ for liquids
		For viscous fluids <span style="float: right;">(T8.7)<sup>a</sup></span>
	For noncircular tubes, use hydraulic mean diameter $D_h$ in the equations for Nu for an approximate value of $h$ .	

# Convective heat transfer coefficient, $h_{conv}$

- There are many more scenarios applicable to buildings
  - From Chapter 4 of the ASHRAE Handbook of Fundamentals:

**III. External Flows for Flat Plate:** Characteristic length =  $L$  = length of plate.  $Re = VL/v$ .

All properties at arithmetic mean of surface and fluid temperatures.

Laminar boundary layer: $Re < 5 \times 10^5$	$Nu = 0.332 Re^{1/2} Pr^{1/3}$	Local value of $h$	(T8.8)
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	$Nu = 0.664 Re^{1/2} Pr^{1/3}$	Average value of $h$	(T8.9)
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Turbulent boundary layer: $Re > 5 \times 10^5$	$Nu = 0.0296 Re^{4/5} Pr^{1/3}$	Local value of $h$	(T8.10)
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Turbulent boundary layer beginning at leading edge: All $Re$	$Nu = 0.037 Re^{4/5} Pr^{1/3}$	Average value of $h$	(T8.11)
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Laminar-turbulent boundary layer: $Re > 5 \times 10^5$	$Nu = (0.037 Re^{4/5} - 871) Pr^{1/3}$	Average value $Re_c = 5 \times 10^5$	(T8.12)
-----------------------------------------------------------	----------------------------------------	--------------------------------------	---------

**IV. External Flows for Cross Flow over Cylinder:** Characteristic length =  $D$  = diameter.  $Re = VD/v$ .

All properties at arithmetic mean of surface and fluid temperatures.

$Nu = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282\,000}\right)^{5/8}\right]^{4/5}$	Average value of $h$	(T8.14) <sup>d</sup>
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**V. Simplified Approximate Equations:**  $h$  is in  $W/(m^2 \cdot K)$ ,  $V$  is in  $m/s$ ,  $D$  is in  $m$ , and  $t$  is in  $^\circ C$ .

Flows in pipes $Re > 10\,000$	Atmospheric air (0 to $200^\circ C$ ): $h = (3.76 - 0.00497t)V^{0.8}/D^{0.2}$	(T8.15a) <sup>e</sup>
	Water (3 to $200^\circ C$ ): $h = (1206 + 23.9t)V^{0.8}/D^{0.2}$	(T8.15b) <sup>e</sup>
	Water (4.4 to $104^\circ C$ ): $h = (1431 + 20.9t)V^{0.8}/D^{0.2}$ (McAdams 1954)	(T8.15c) <sup>g</sup>

Flow over cylinders  
Atmospheric air:  $0^\circ C < t < 200^\circ C$ , where  $t$  = arithmetic mean of air and surface temperature.

$h = 2.755V^{0.471}/D^{0.529}$	$35 < Re < 5000$	(T8.16a)
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$h = (4.22 - 0.00257t)V^{0.633}/D^{0.367}$	$5000 < Re < 50\,000$	(T8.16b)
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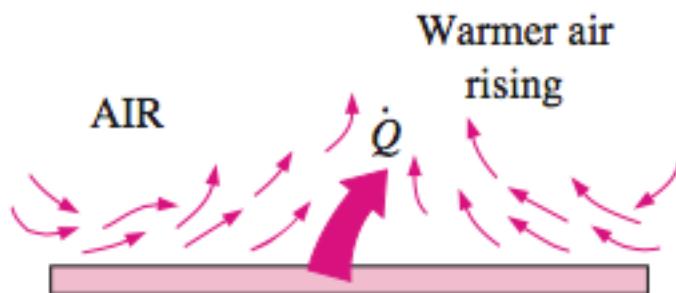
Water:  $5^\circ C < t < 90^\circ C$ , where  $t$  = arithmetic mean of water and surface temperature.

$h = (461.8 + 2.01t)V^{0.471}/D^{0.529}$	$35 < Re < 5000$	(T8.17a)
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$h = (1012 + 9.19t)V^{0.633}/D^{0.367}$	$5000 < Re < 50\,000$	(T8.17b) <sup>f</sup>
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# Convective heat transfer coefficient, $h_{conv}$

- There are similar looking (albeit different) relationships for natural convection



(b) Free convection

$$Nu = \frac{hL_c}{k} = f(Ra_{Lc}, Pr)$$

$Ra_{Lc}$  = Rayleigh number =  $g\beta \Delta t L_c^3 / \nu \alpha$

$$\Delta t = |t_s - t_\infty|$$

$g$  = gravitational acceleration

$\beta$  = coefficient of thermal expansion

$\nu$  = fluid kinematic viscosity =  $\mu/\rho$

$\alpha$  = fluid thermal diffusivity =  $k/\rho c_p$

Pr = Prandtl number =  $\nu/\alpha$

$$Ra = Gr Pr$$

Gr = Grashof # (relationship between buoyancy and viscosity in a fluid)

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \text{ for vertical flat plates}$$

$$Gr_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} \text{ for pipes}$$

$$(\beta \approx 0.0034/K \text{ for air})$$

# Convective heat transfer coefficient, $h_{conv}$

- Equations for natural convection
  - From Chapter 4 of the ASHRAE Handbook of Fundamentals:

## I. General relationships

Characteristic length depends on geometry

$$Nu = f(Ra, Pr) \text{ or } f(Ra) \quad (T9.1)$$

$$Ra = Gr \Pr \quad Gr = \frac{g\beta p^2 |\Delta t| L^3}{\mu^2} \quad \Pr = \frac{c_p \mu}{k} \quad \Delta t = |t_s - t_\infty|$$

## II. Vertical plate

$t_s = \text{constant}$

$$Nu = 0.68 + \frac{0.67 Ra^{1/4}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{4/9}} \quad 10^{-1} < Ra < 10^9 \quad (T9.2)^a$$

Characteristic dimension:  $L = \text{height}$

Properties at  $(t_s + t_\infty)/2$  except  $\beta$  at  $t_\infty$

$$Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{\left[1 + (0.492/\Pr)^{9/16}\right]^{8/27}} \right\}^2 \quad 10^9 < Ra < 10^{12} \quad (T9.3)^a$$

$q''_s = \text{constant}$

Characteristic dimension:  $L = \text{height}$

Properties at  $t_{s,L/2} - t_\infty$  except  $\beta$  at  $t_\infty$

$$Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{\left[1 + (0.437/\Pr)^{9/16}\right]^{8/27}} \right\}^2 \quad 10^{-1} < Ra < 10^{12} \quad (T9.4)^a$$

Equations (T9.2) and (T9.3) can be used for vertical cylinders if  $D/L > 35/\text{Gr}^{1/4}$  where  $D$  is diameter and  $L$  is axial length of cylinder

## III. Horizontal plate

Characteristic dimension =  $L = A/P$ , where  $A$  is plate area and  $P$  is perimeter

Properties of fluid at  $(t_s + t_\infty)/2$

Downward-facing cooled plate and upward-facing heated plate

$$Nu = 0.96 Ra^{1/6} \quad 1 < Ra < 200 \quad (T9.5)^b$$

$$Nu = 0.59 Ra^{1/4} \quad 200 < Ra < 10^4 \quad (T9.6)^b$$

$$Nu = 0.54 Ra^{1/4} \quad 2.2 \times 10^4 < Ra < 8 \times 10^6 \quad (T9.7)^b$$

$$Nu = 0.15 Ra^{1/3} \quad 8 \times 10^6 < Ra < 1.5 \times 10^9 \quad (T9.8)^b$$

Downward-facing heated plate and upward-facing cooled plate

$$Nu = 0.27 Ra^{1/4} \quad 10^5 < Ra < 10^{10} \quad (T9.9)^b$$

# Convective heat transfer coefficient, $h_{conv}$

- Equations for natural convection
  - From Chapter 4 of the ASHRAE Handbook of Fundamentals:

## IV. Horizontal cylinder

Characteristic length =  $d$  = diameter

Properties of fluid at  $(t_s + t_\infty)/2$  except  $\beta$  at  $t_\infty$

$$Nu = \left\{ 0.6 + \frac{0.387 Ra^{1/6}}{\left[ 1 + (0.559/\text{Pr})^{9/16} \right]^{8/27}} \right\}^2 \quad 10^9 < Ra < 10^{13} \quad (\text{T9.10})^c$$

## V. Sphere

Characteristic length =  $D$  = diameter

Properties at  $(t_s + t_\infty)/2$  except  $\beta$  at  $t_\infty$

$$Nu = 2 + \frac{0.589 Ra^{1/4}}{\left[ 1 + (0.469/\text{Pr})^{9/16} \right]^{4/9}} \quad Ra < 10^{11} \quad (\text{T9.11})^d$$

## VI. Horizontal wire

Characteristic dimension =  $D$  = diameter

Properties at  $(t_s + t_\infty)/2$

$$\frac{2}{Nu} = \ln \left( 1 + \frac{3.3}{c Ra^n} \right) \quad 10^{-8} < Ra < 10^6 \quad (\text{T9.12})^e$$

## VII. Vertical wire

Characteristic dimension =  $D$  = diameter;  $L$  = length of wire

Properties at  $(t_s + t_\infty)/2$

$$Nu = c (Ra D/L)^{0.25} + 0.763 c^{(1/6)} (Ra D/L)^{(1/24)} \quad c (Ra D/L)^{0.25} > 2 \times 10^{-3} \quad (\text{T9.13})^e$$

In both Equations (T9.12) and (T9.13),  $c = \frac{0.671}{[1 + (0.492/\text{Pr})^{(9/16)}]^{(4/9)}}$  and

$$n = 0.25 + \frac{1}{10 + 5(Ra)^{0.175}}$$

## VIII. Simplified equations with air at mean temperature of 21°C: $h$ is in $\text{W}/(\text{m}^2 \cdot \text{K})$ , $L$ and $D$ are in m, and $\Delta t$ is in °C.

Vertical surface

$$h = 1.33 \left( \frac{\Delta t}{L} \right)^{1/4} \quad 10^5 < Ra < 10^9 \quad (\text{T9.14})$$

$$h = 1.26 (\Delta t)^{1/3} \quad Ra > 10^9 \quad (\text{T9.15})$$

Horizontal cylinder

$$h = 1.04 \left( \frac{\Delta t}{D} \right)^{1/4} \quad 10^5 < Ra < 10^9 \quad (\text{T9.16})$$

$$h = 1.23 (\Delta t)^{1/3} \quad Ra > 10^9 \quad (\text{T9.17})$$

# Simplifications of convective heat transfer coefficients

- For practical purposes in building science, we usually simplify convective heat transfer coefficients to common values for relatively common cases
  - Sometimes these are fundamentally estimated
  - Sometimes these are empirical (measured) in different scenarios

TABLE 2.9

Magnitude of Convection Coefficients

Arrangement	W/(m <sup>2</sup> · K)	Btu/(h · ft <sup>2</sup> · F)
Air, free convection	6–30	1–5
Superheated steam or air, forced convection	30–300	5–50
Oil, forced convection	60–1800	10–300
Water, forced convection	300–6000	50–1000
Water, boiling	3000–60,000	500–10,000
Steam, condensing	6000–120,000	1000–20,000

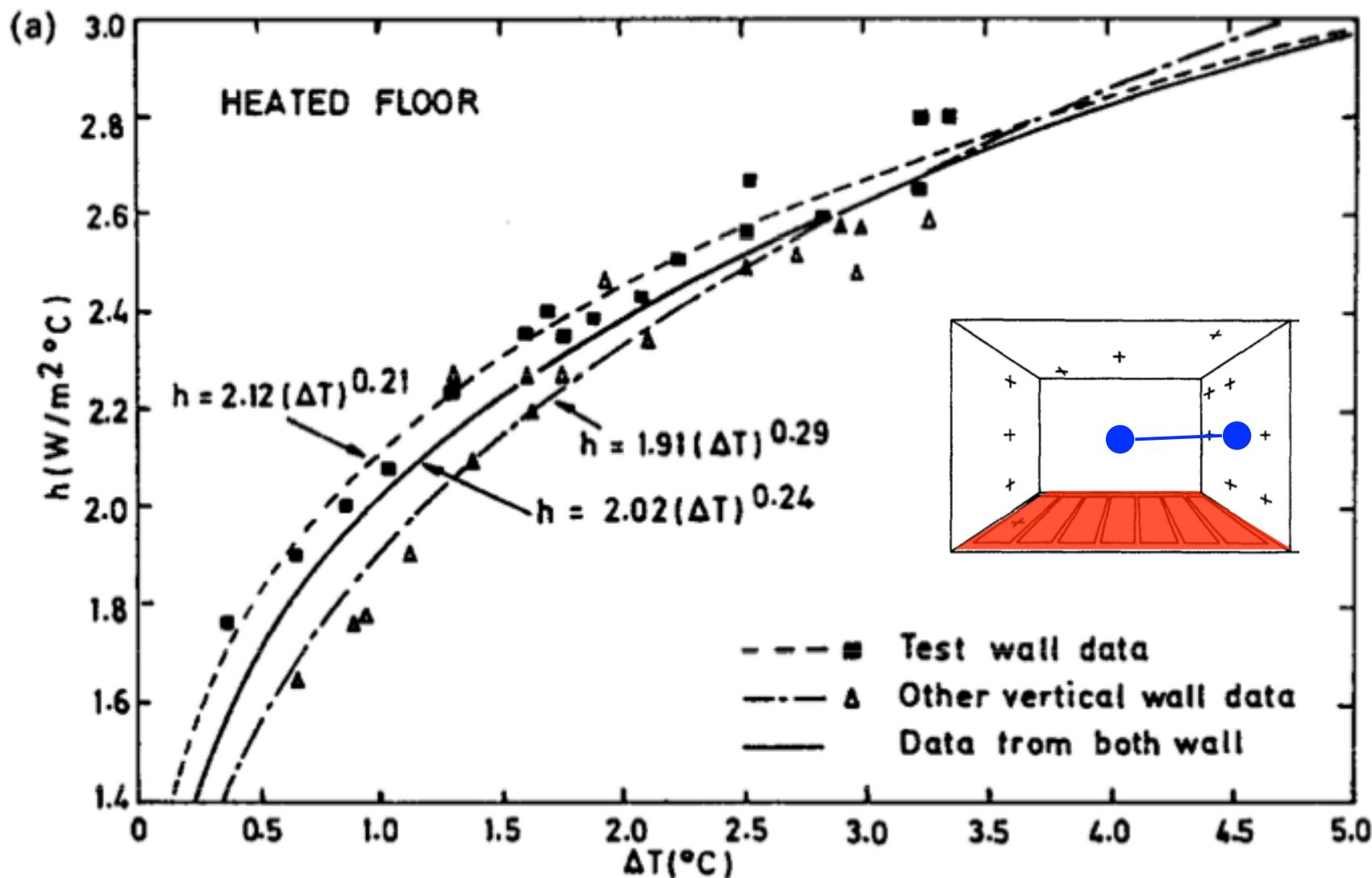
The conversion between SI and USCS units is  $5.678 \text{ W}/(\text{m}^2 \cdot \text{K}) = 1 \text{ Btu}/(\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F})$ .

# Simplifications of convective heat transfer coefficients

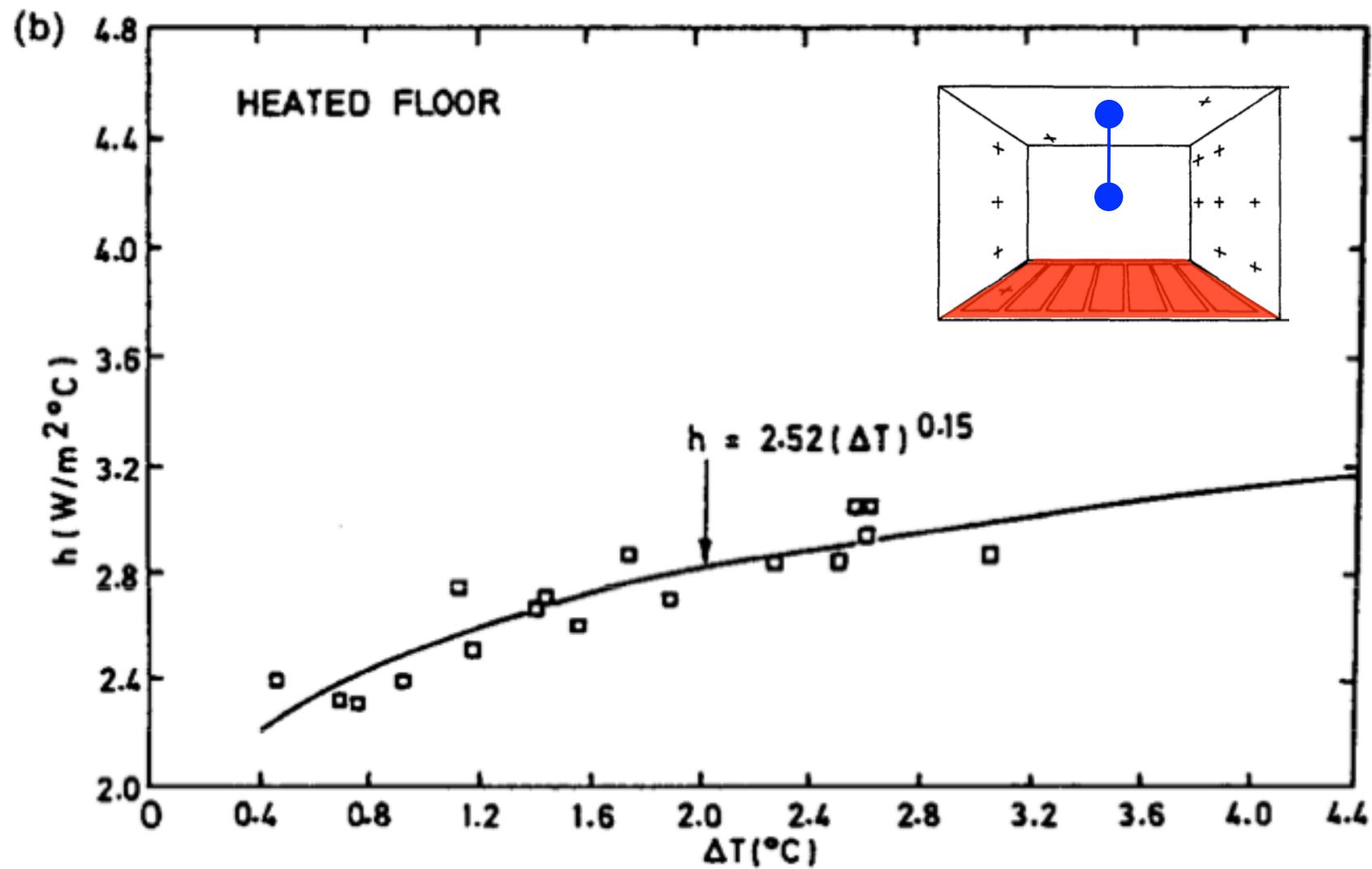
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- Convective heat transfer coefficients can depend upon details of the surface-fluid interface
  - Rough surfaces have higher rates of convection
  - Orientation is important for natural convection
  - Convective heat transfer coefficients for natural convection can depend upon the actual fluid temperature and not just the temperature difference

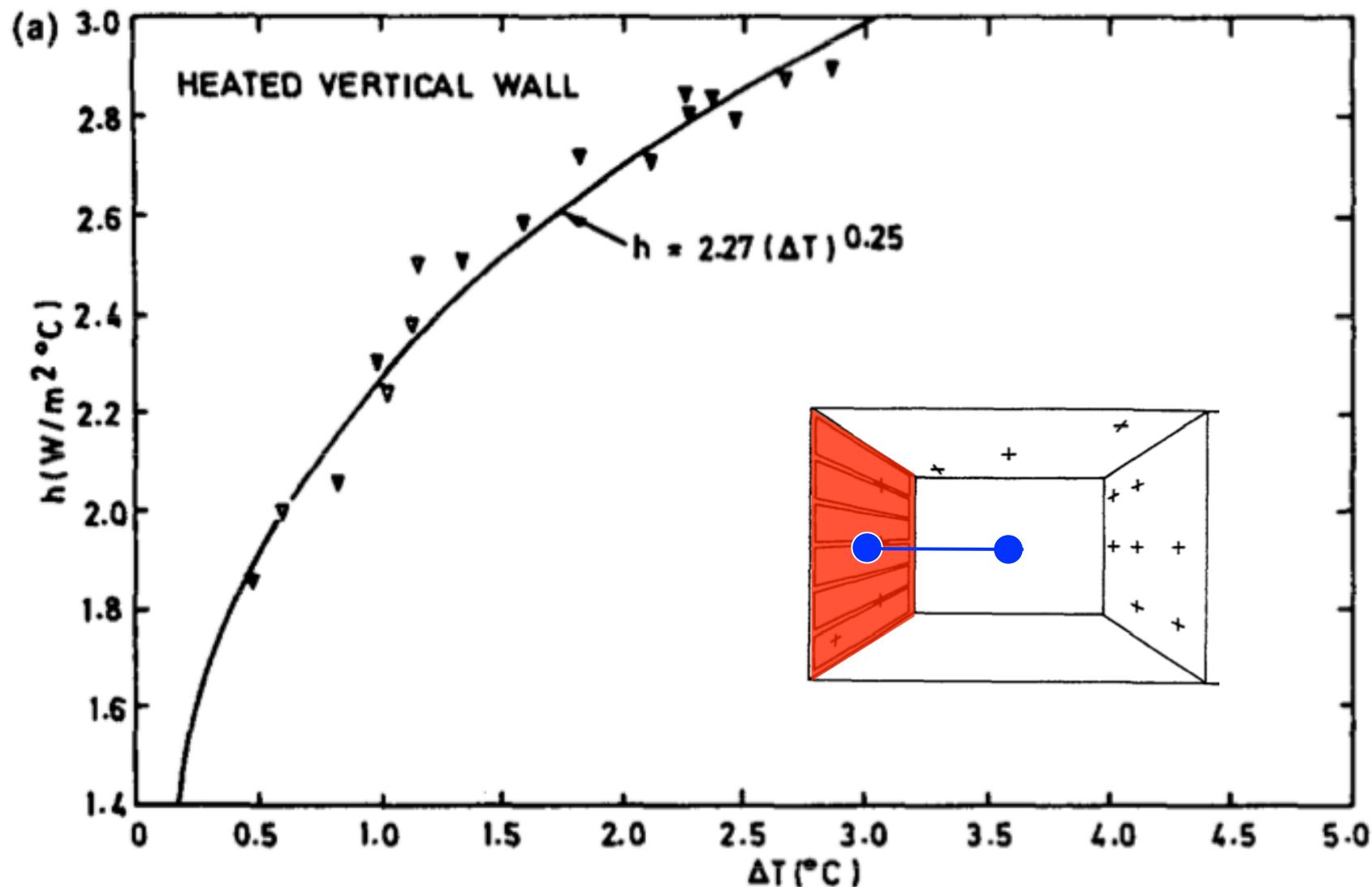
# Empirical: $h_{conv}$ vs. $\Delta T$ for vertical walls and a heated floor



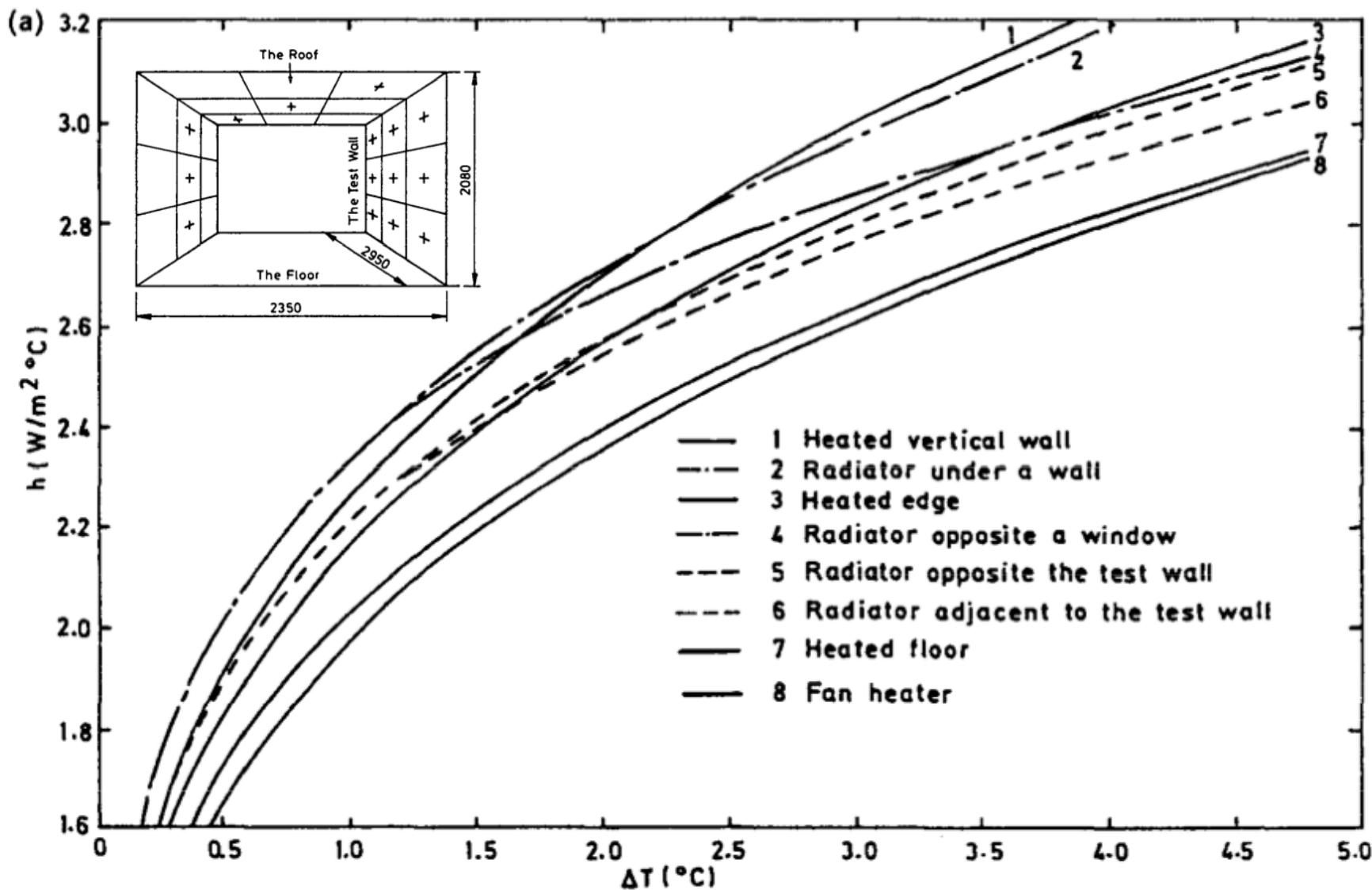
## Empirical: $h_{conv}$ vs. $\Delta T$ for a ceiling and a heated floor



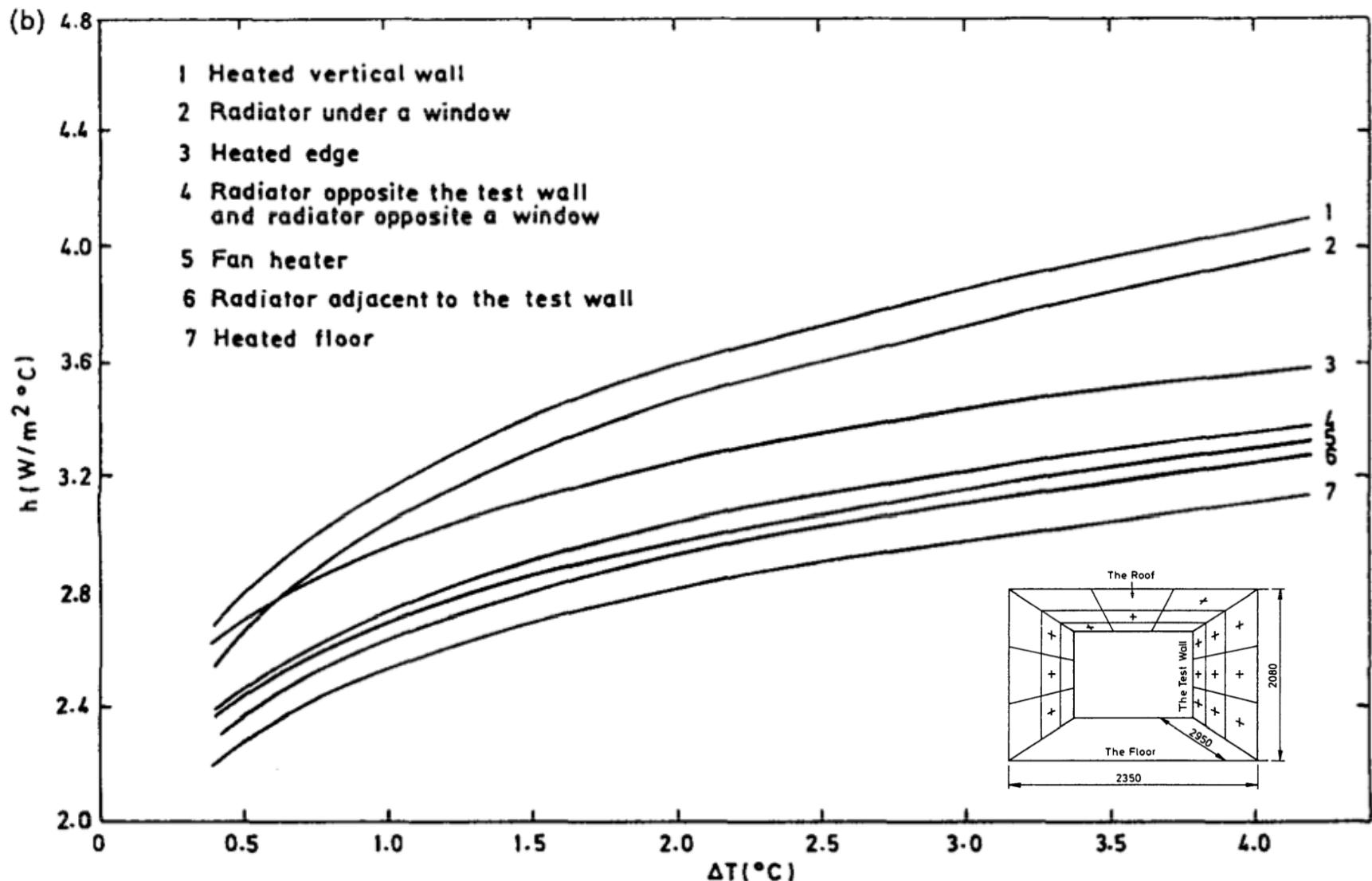
## Empirical: $h_{conv}$ vs. $\Delta T$ for heated walls



# Empirical: $h_{conv}$ vs. $\Delta T$ for interior walls

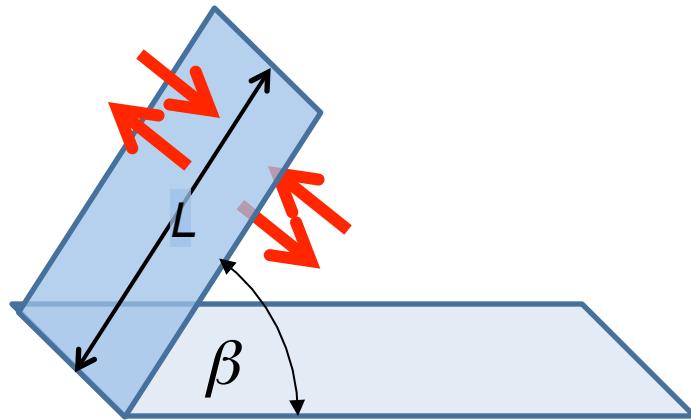


# Empirical: $h_{conv}$ vs. $\Delta T$ for interior ceilings



# Free convection in air from a tilted surface: Simplified

SI units (IP equations are different!)



$h_{conv}$  in [W/(m<sup>2</sup> K)]

For natural convection to or from either side of a vertical surface or a sloped surface with  $\beta > 30^\circ$

For laminar: 
$$h_{conv} = 1.42 \left( \frac{\Delta T}{L} \sin \beta \right)^{\frac{1}{4}}$$
 [Kreider 2.18SI]

For turbulent: 
$$h_{conv} = 1.31 (\Delta T \sin \beta)^{\frac{1}{3}}$$
 [Kreider 2.19SI]

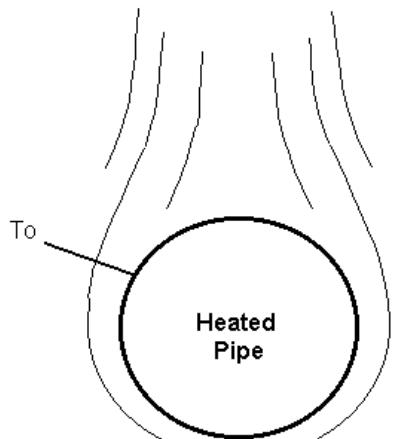
Note that these equations are **dimensional**, so they are different for IP and SI

# Free convection from horizontal pipes in air

- For cylindrical pipes of outer diameter,  $D$ , in [m]

For laminar: 
$$h_{conv} = 1.32 \left( \frac{\Delta T}{D} \right)^{\frac{1}{4}}$$
 [Kreider 2.20SI]

For turbulent: 
$$h_{conv} = 1.24 \left( \Delta T \right)^{\frac{1}{3}}$$
 [Kreider 2.21SI]

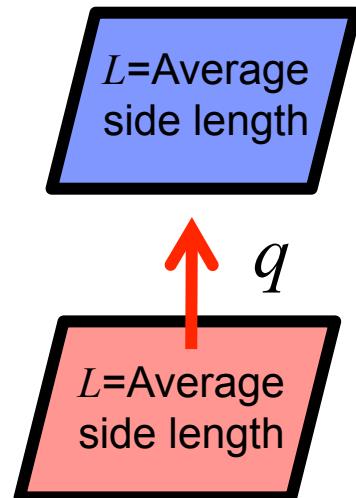


SI units  
(IP equations are different!)

Free Convection Heat Transfer

# Free convection for surfaces: Simplified

- Warm horizontal surfaces facing up
  - e.g. up from a **warm floor** to a **cold ceiling**

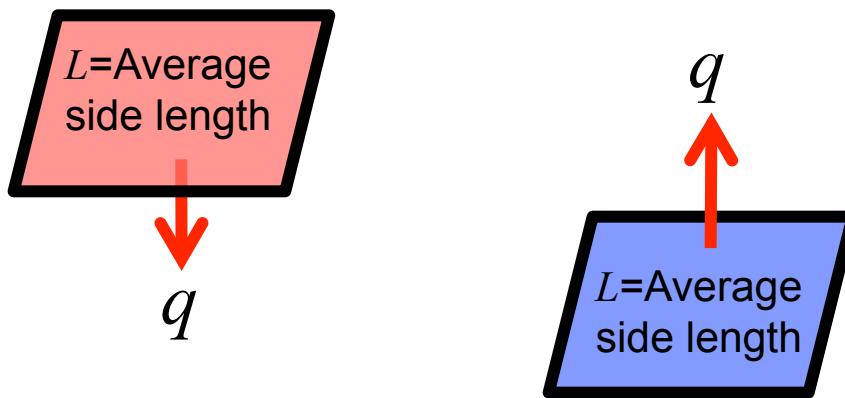


laminar:  $h_{conv} \approx 1.32 \left( \frac{\Delta T}{L} \right)^{1/4}$  [Kreider 2.22SI]

turbulent:  $h_{conv} \approx 1.52 \left( \Delta T \right)^{1/3}$  [Kreider 2.23SI]

# Free convection for surfaces: Simplified

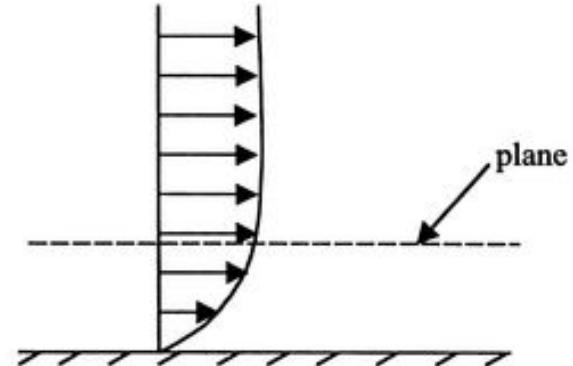
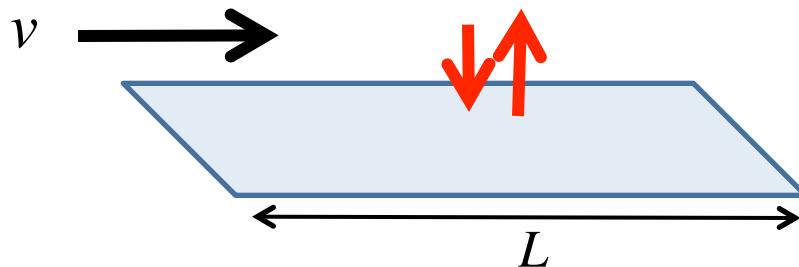
- Warm horizontal surface facing down
  - Convection is reduced because of stratification
    - e.g. a **warm ceiling facing down** (works against buoyancy)
    - Also applies for **cooled flat surfaces facing up** (like a cold floor)



$$h_{conv} \approx 0.59 \left( \frac{\Delta T}{L} \right)^{1/4} \quad \text{both laminar and turbulent}$$

# Forced convection over planes: Simplified

- Does not depend on orientation



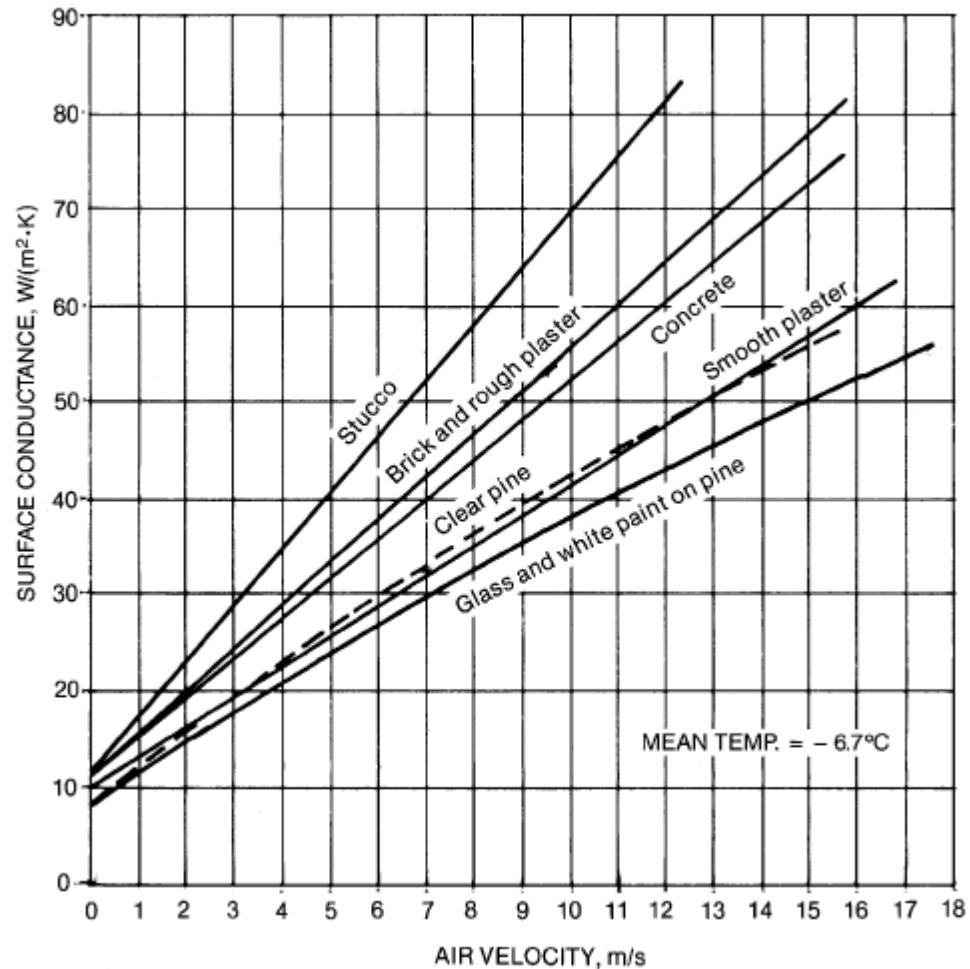
laminar:  $h_{conv} \approx 2.0 \left( \frac{v}{L} \right)^{1/2}$  [Kreider 2.24SI]

turbulent:  $h_{conv} \approx 6.2 \left( \frac{v^4}{L} \right)^{1/5}$  [Kreider 2.25SI]

\*Velocity is in m/s

# $h_{conv}$ for exterior forced convection

- For forced convection,  $h_{conv}$  depends upon surface roughness and air velocity but not orientation



## Most used $h_{conv}$ for exterior forced convection

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There are two relationships for  $h_{conv}$  (forced convection) which are commonly used, depending on wind speed:

- For  $1 < v_{wind} < 5$  m/s

$$h_c = 5.6 + 3.9v_{wind} \quad [\text{W}/(\text{m}^2 \cdot \text{K})] \quad [\text{Straube 5.15}]$$

- For  $5 < v_{wind} < 30$  m/s

$$h_c = 7.2v_{wind}^{0.78} \quad [\text{W}/(\text{m}^2 \cdot \text{K})] \quad [\text{Straube 5.16}]$$

\*Good for use with external surfaces like walls and windows

# Convective “R-value”

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- Convective heat transfer can also be translated to an ‘effective conductive layer’ in contact with air
  - Allows us to assign an R-value to it
  - aka “film resistance”

$$R_{conv} = \frac{1}{h_{conv}}$$

# Typical convective “film resistances”

- We often use the values given below for most conditions

Surface Conditions	Horizontal Heat Flow	Upwards Heat Flow	Downwards Heat Flow
Indoors: $R_{in}$	0.12 m <sup>2</sup> K/W (SI) 0.68 h·ft <sup>2</sup> ·°F/Btu (IP)	0.11 m <sup>2</sup> K/W (SI) 0.62 h·ft <sup>2</sup> ·°F/Btu (IP)	0.16 m <sup>2</sup> K/W (SI) 0.91 h·ft <sup>2</sup> ·°F/Btu (IP)
$R_{out}$ : 6.7 m/s wind (Winter)		0.030 m <sup>2</sup> K/W (SI) 0.17 h·ft <sup>2</sup> ·°F/Btu (IP)	
$R_{out}$ : 3.4 m/s wind (Summer)		0.044 m <sup>2</sup> K/W (SI) 0.25 h·ft <sup>2</sup> ·°F/Btu (IP)	

We can still sum resistances in series,  
even if it involves different modes of heat transfer

## Internal convection within building HVAC systems

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- Flows of fluids confined by boundaries (such as the sides of a duct) are called internal flows
- Mechanisms of convection are different
  - And so are the equations for  $h_c$



# Forced convection for fully developed turbulent flow

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- Airflow through ducts:

$$h_{conv} \approx 8.8 \left( \frac{v^4}{D_h} \right)^{1/5} \quad [\text{Kreider 2.26SI}]$$

$D_h$  = the hydraulic diameter: 4 times the ratio of the flow conduit's cross-sectional area divided by the perimeter of the conduit

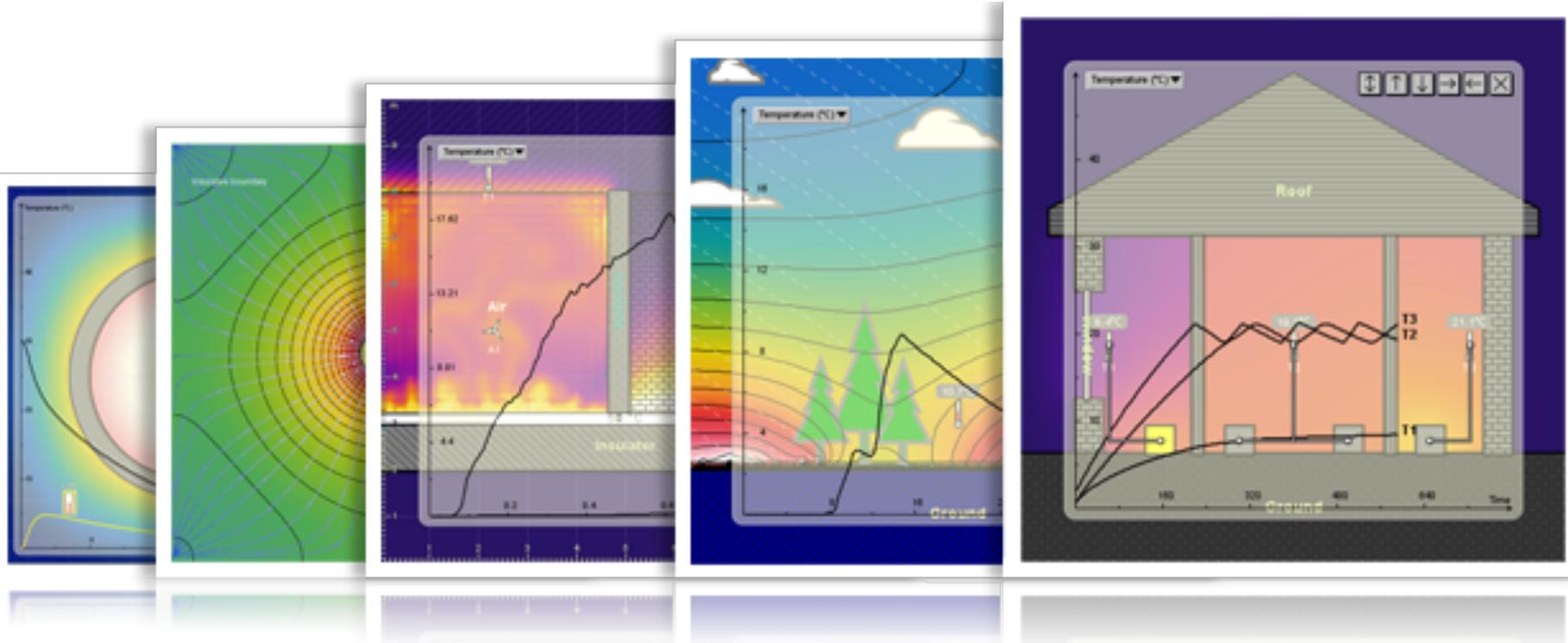
$$D_h = \frac{4 \left( \frac{\pi D^2}{4} \right)}{\pi D} \quad [\text{Kreider 2.27SI}]$$

- Water flow through pipes:

$$h_{conv} \approx 3580(1 + 0.015T) \left( \frac{v^4}{D_h} \right)^{1/5} \quad [\text{Kreider 2.28SI}]$$

**SI units (IP equations are different!)**

# Convection visualizations



# Energy2D

Interactive Heat Transfer Simulations for Everyone