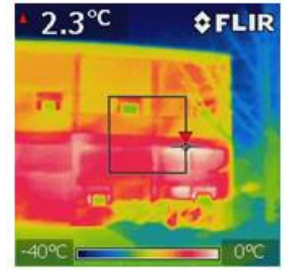
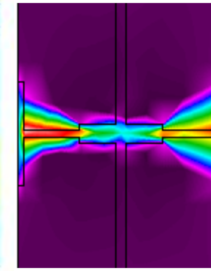
CAE 331/513
Building Science
Fall 2017





August 31, 2017
Heat transfer in buildings: Conduction (continued)

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Objectives for today's lecture

- Collect HW 1 (due today)

- Finish conduction

Last time

- Introduced steady-state heat conduction in buildings

$$q = -k\nabla T = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right) \qquad q = -k\frac{dT}{dx}$$

$$q = \frac{k}{L}\left(T_1 - T_2\right) = U\left(T_1 - T_2\right) = \frac{1}{R}\left(T_1 - T_2\right)$$

$$\frac{k}{L} = U \qquad R = \frac{1}{U}$$

R-SI

$$1\,\frac{\text{m}^2\text{K}}{\text{W}} = 5.678\,\frac{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}{\text{Btu}}$$

R-IP

$$R_{total} = R_1 + R_2 + R_3 + \ldots \qquad U_{total} = \frac{A_1}{A_{total}}U_1 + \frac{A_2}{A_{total}}U_2 + \ldots$$

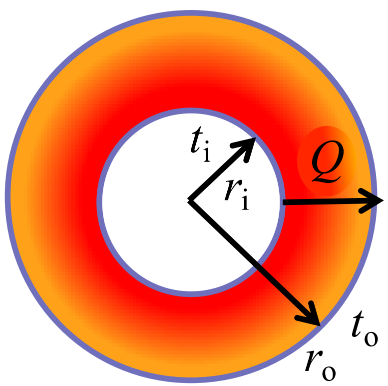
Continuing conduction

- Other aspects of conduction to consider:
  - Steady-state conduction in cylindrical coordinates (e.g., pipes)
  - Steady-state conduction in multiple dimensions (e.g., at corners and edges)
  - Steady-state conduction in below- and on-grade floors and walls (e.g., basements and slabs)
  - Transient (i.e., dynamic) conduction

Conduction in cylindrical coordinates

- Fourier's law also applies to geometries other than plane walls
- Of particular interest in buildings is cylindrical geometry
  - For example: heat losses from piping in HVAC systems

- For a hollow cylinder with length L, inner radius r_i and outer radius r_o:
  - If you integrate $Q = -kA\frac{dT}{dx}$ in cylindrical coordinates, you get:



$$R'_{cyl} = \frac{\ln\left(r_o/r_i\right)}{2\pi kL}$$

$$Q = \frac{2\pi kL}{\ln\left(r_o/r_i\right)}\left(T_i - T_o\right)$$

Conduction in cylindrical coordinates

- **Example:** Steam at 260°C flows through a cast iron pipe (conductivity of 80 W/mK) with an outer diameter of 7.5 cm and an inner diameter of 7.0 cm. The pipe is wrapped with 3 cm thick of glass wool insulation (conductivity of 0.05 W/mK)
- What is the heat loss to the environment per meter length of pipe, assuming that the outer layer of insulation has a temperature of 20°C?

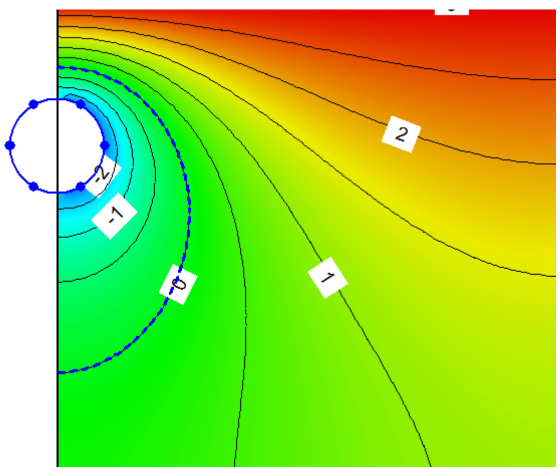
Conduction in other geometries: 2-D effects

- There are often situations where heat conduction is not strictly 1-D
- One way to account for this is the use of a "shape factor" below:
  - Shape factors account for 2-D effects (easier than 2-D analysis)
- Another approach is a full 2-D analysis, although we don't cover in this class

$$Q = kS\Delta T$$

Where S = shape factor [m]

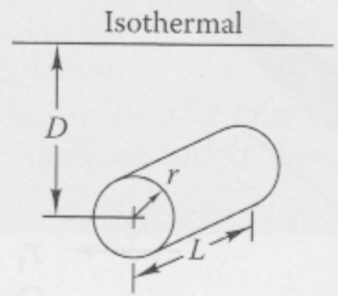
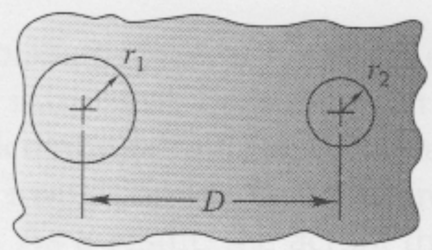
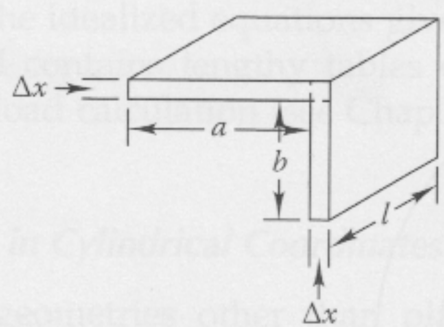
<u>Example:</u>
underground pipe carrying a heated or cooled fluid from a central plant to a building



Example shape factors

TABLE 2.1

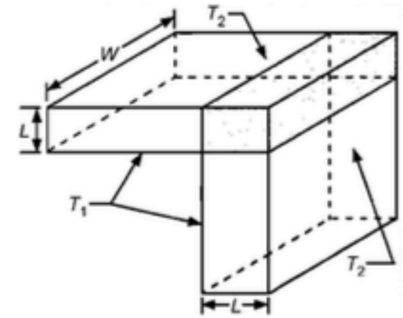
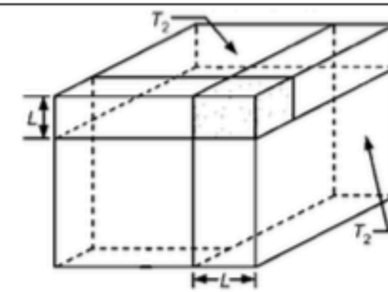
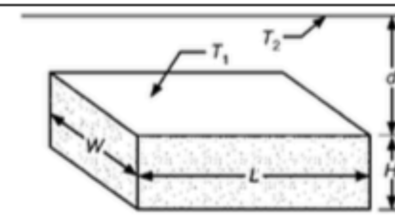
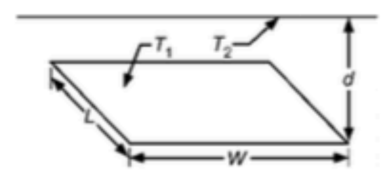
Conduction Shape Factors

Physical System	Schematic	Shape Factor	Restrictions
Isothermal cylinder of radius r buried in semi-infinite medium having isothermal surface		$\dfrac{2\pi L}{\cosh^{-1}(D/r)}$	$L \gg r$
		$\dfrac{2\pi L}{\ln(2D/r)}$	$L \gg r$ $D > 3r$
		$\dfrac{2\pi L}{\ln\left(\dfrac{L}{r}\right)\left\{1 - \dfrac{\ln[L/2D)]}{\ln(L/r)}\right\}}$	$D \gg r$ $L \gg D$
Conduction between two isothermal cylinders buried in infinite medium		$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 - r_1^2 - r_2^2}{2r_1r_2}\right)}$	$L \gg r_1, r_2$ $L \gg D$
Conduction through two plane sections and the edge section of two walls of thermal conductivity k—inner and outer surface temperatures uniform		$\dfrac{al}{\Delta x} + \dfrac{bl}{\Delta x} + 0.54l$	

Source: Courtesy of Holman, J.P., *Heat Transfer*, 8th edn, McGraw-Hill, New York, 1997. With permission.

Example shape factors

Table 3 Multidimensional Conduction Shape Factors

Configuration	Shape Factor S, m	Restriction	
Edge of two adjoining walls	$0.54W$	$W > L/5$	
Corner of three adjoining walls (inner surface at T_1 and outer surface at T_2)	$0.15L$	$L <<$ length and width of wall	
Isothermal rectangular block embedded in semi-infinite body with one face of block parallel to surface of body	$\dfrac{2.756L}{\left[\ln\left(1+\dfrac{d}{W}\right)\right]^{0.59}}\left(\dfrac{H}{d}\right)^{0.078}$	$L > W$ $L >> d, W, H$	
Thin isothermal rectangular plate buried in semi-infinite medium	$\dfrac{\pi W}{\ln(4W/L)}$	$d = 0,\ W > L$	
	$\dfrac{2\pi W}{\ln(4W/L)}$	$d >> W$ $W > L$	
	$\dfrac{2\pi W}{\ln(2\pi d/L)}$	$d > 2W$ $W >> L$	

Source: ASHRAE HOF 2013

Example shape factor calculations

- **Example:** The walls and roof of a house are made of 8 inch thick concrete with k = 5.2 Btu·in/h·ft^2·°F. The inner surface temperature is 68°F, and the outer surface temperature is 46°F. The roof is 33 × 33 ft, and the walls are 16 ft high.
- Find the rate of heat loss from the house through its walls and roof, including edge and corner effects.

BELOW- AND ON-GRADE CONDUCTION

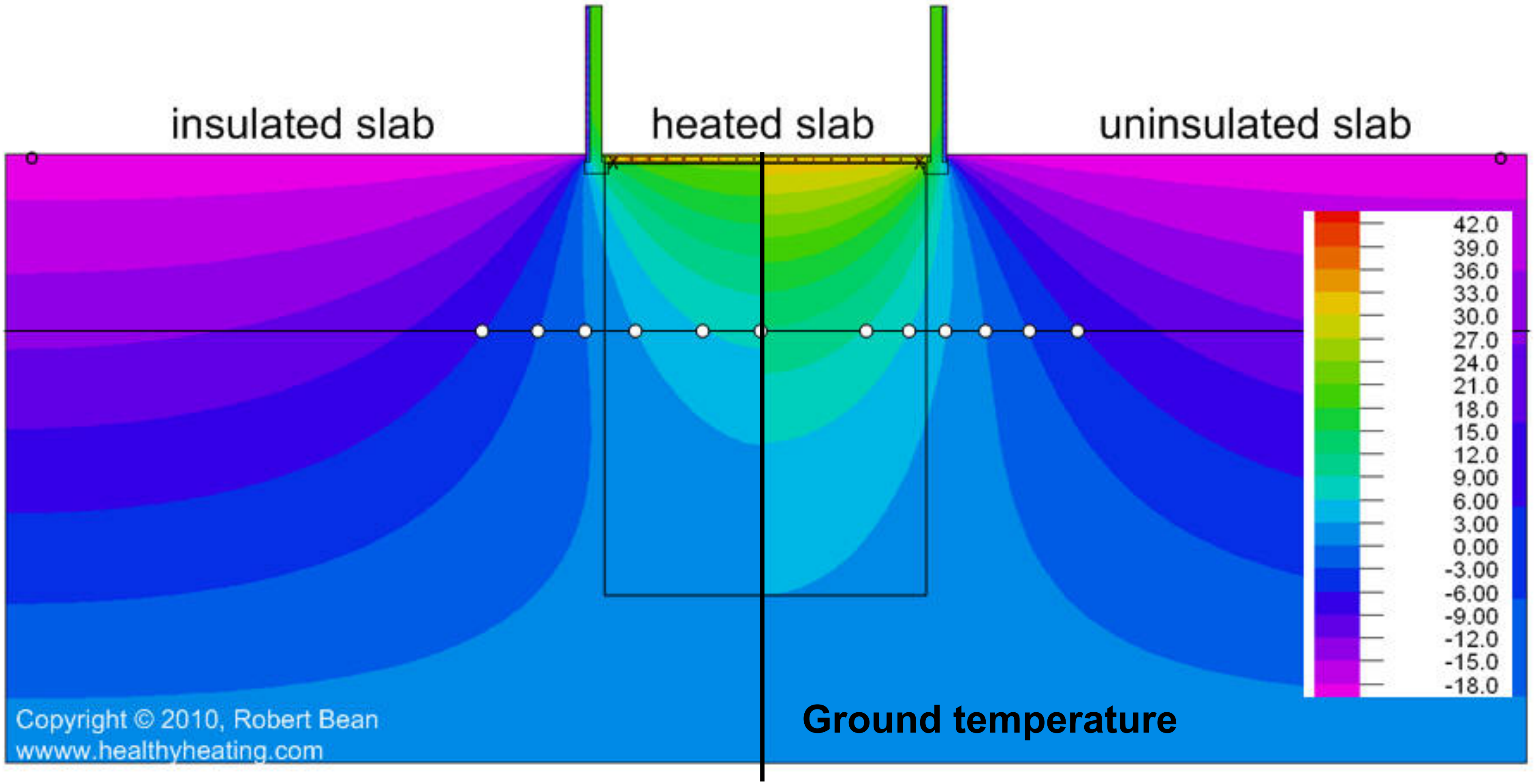
Below- and on-grade conduction

- Very often, building enclosure assemblies are located on or below ground level (i.e., “on-grade” or “below-grade”)
- Instead of exterior surfaces being in contact with the outside air, they are in contact with soil
- Where does heat flow?
  - Depends on surface and ground temperature distributions

Below-grade

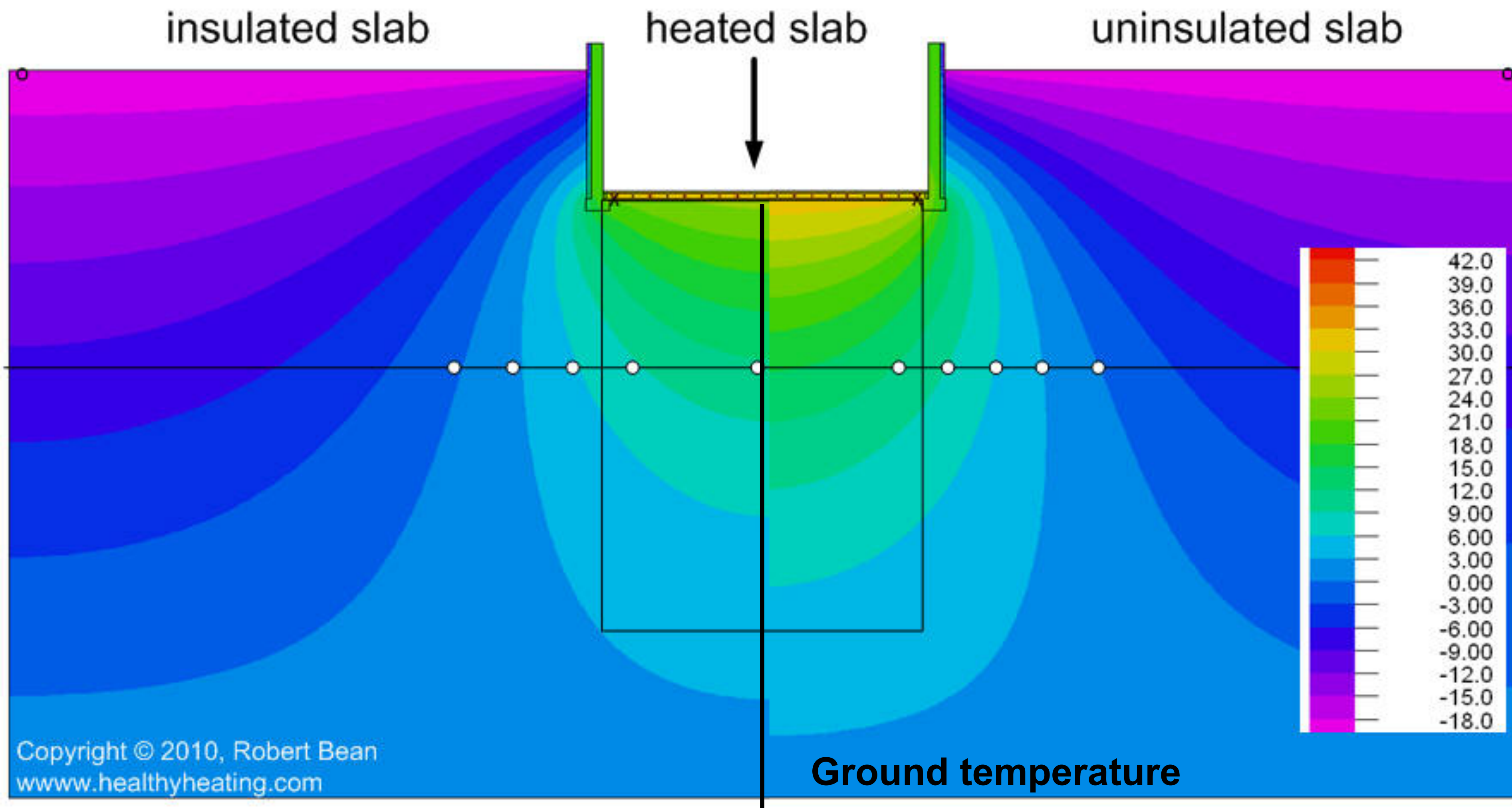
On-grade heat flow

- Often we have floors built directly on grade, in contact with the ground



Below-grade heat flow

- Often we have walls and floors built below-grade, or "submerged" within the soil



Average annual ground temperatures

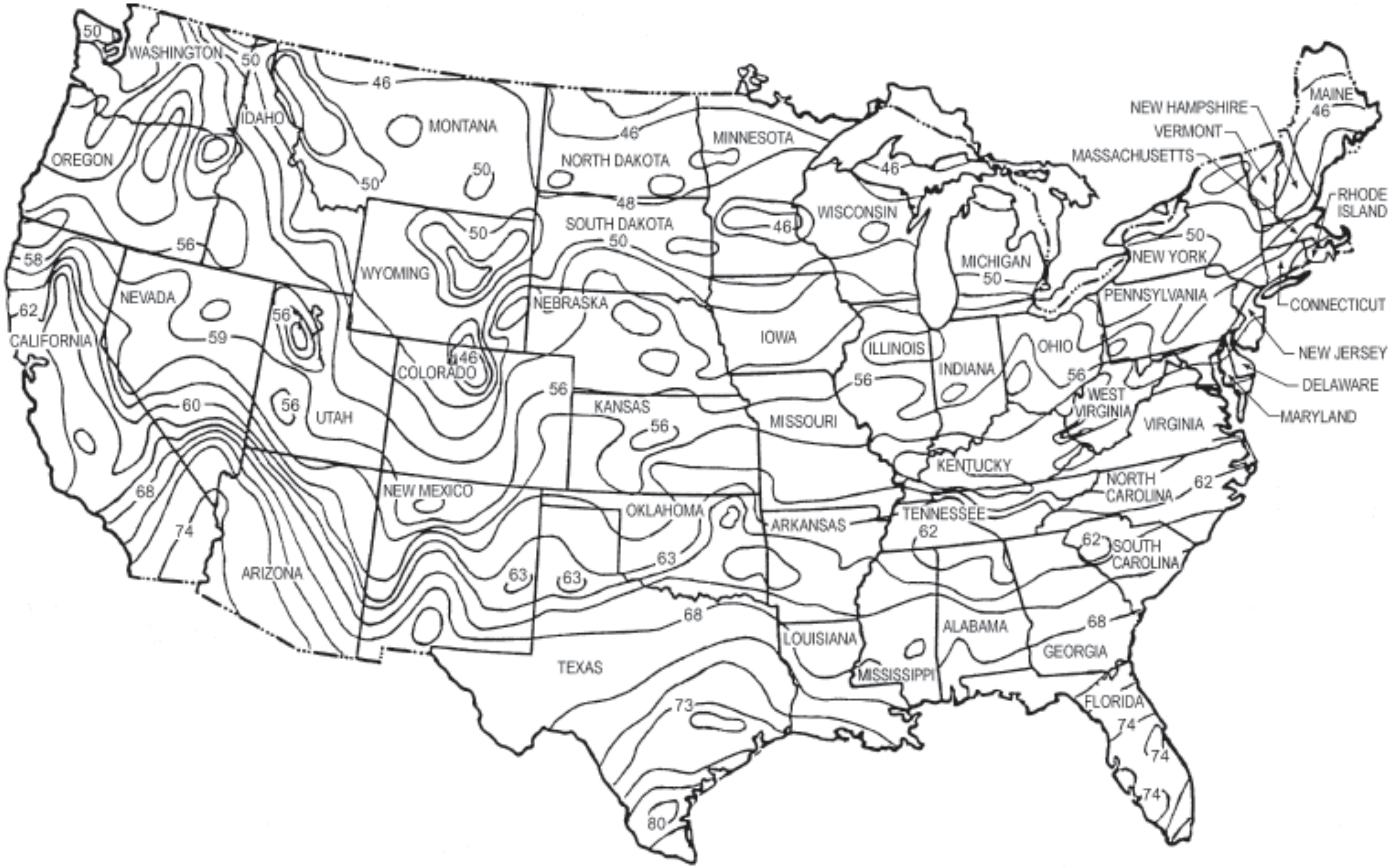
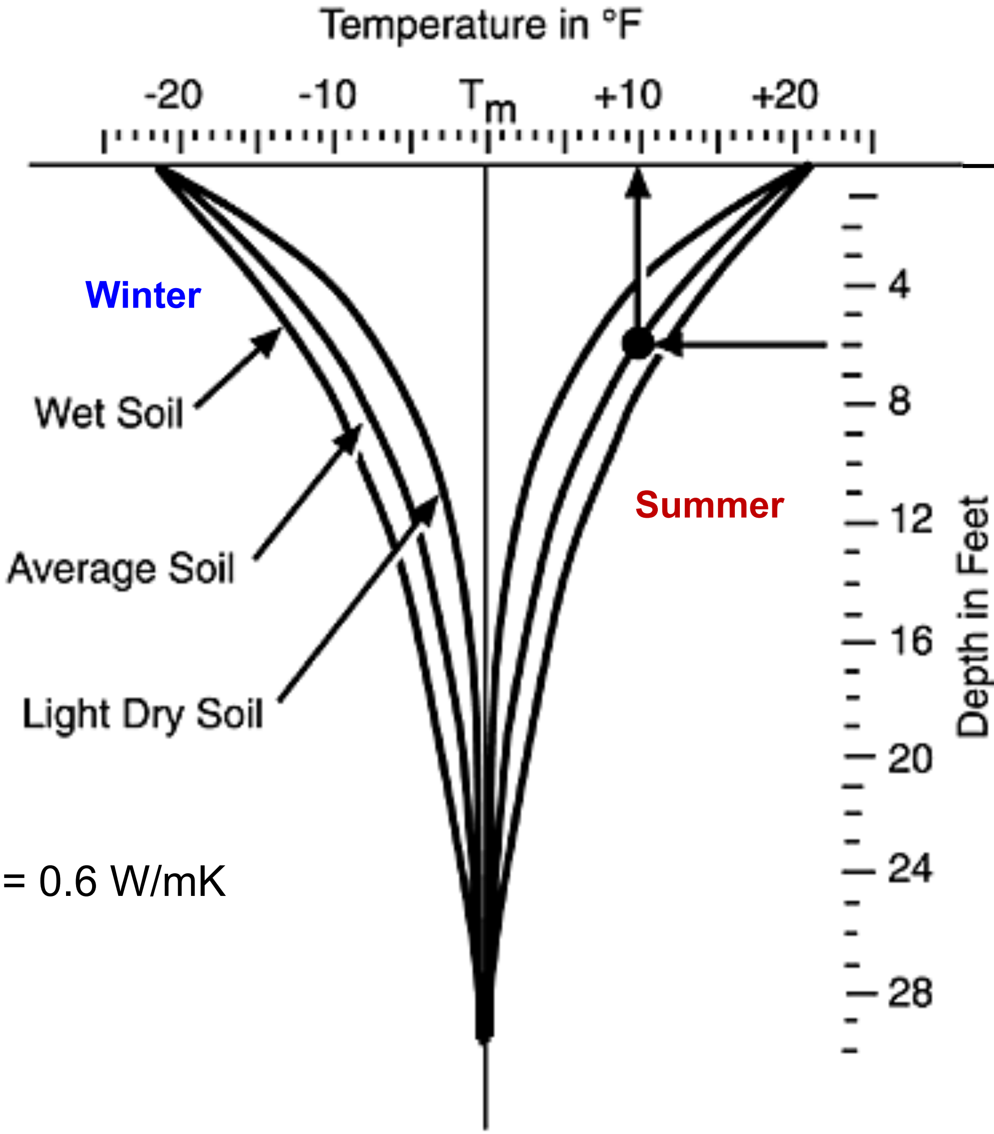
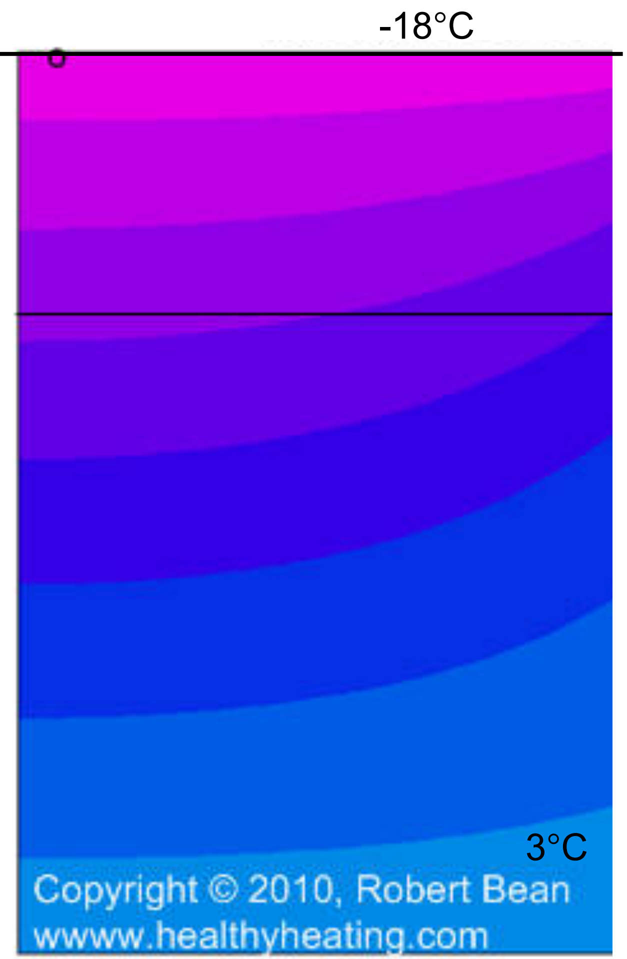


Fig. 17 Approximate Groundwater Temperatures (°F) in the Continental United States

Source: 2011 HVAC Apps Chapter 34, Figure 17

Ground temperatures vary with depth and soil moisture





k_{H2O} = 0.6 W/mK

Source: http://www.geo4va.vt.edu/A1/A1.htm

Simplified below-grade heat transfer

$$Q = AU_{avg}\,(T_i - T_{gr}) \quad \text{[W]}$$

$$q = U_{avg}\,(T_i - T_{gr}) \quad \text{[W/m}^2\text{]}$$

where

A is the wall or floor area below grade [m^2] (analyze any wall portion above-grade in the normal way)

T_i is the below grade inside temperature [K]

T_{gr} is the ground surface temperature [K]

U_{avg} is the average U-value for the below grade surface [$W/(m^2K)$] (see following slides for ways to calculate)

Below grade depth parameters for estimating U-value

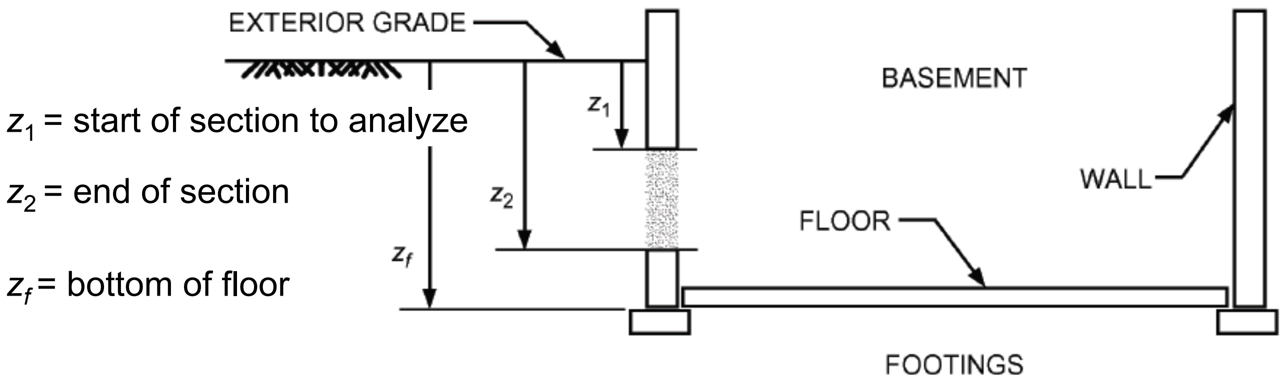


Fig. 14 Below-Grade Parameters

- For **average below-grade floor** value with a floor depth of height z_f from ground ("grade")

$$U_{avg,bf} = \frac{2k_{soil}}{\pi w_b} \times \left[\ln\left(\frac{w_b}{2} + \frac{z_f}{2} + \frac{k_{soil}R_{other}}{\pi}\right) - \ln\left(\frac{k_{soil}R_{other}}{\pi}\right)\right] \quad (40)$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
R_{other} = R value of floor assembly [m^2K/W]
w_b = shortest dimension of basement width [m]
z_f = floor depth below grade [m]

Pre-computed tables for $U_{avg,bf}$

- Assuming **un-insulated concrete** floor

Table 17 Average U-Factor for Basement Floors

z_f (depth of floor below grade), m	$U_{avg,bf}$, W/(m²·K)			
	w_b (shortest width of basement), m			
	6	7	8	9
0.3	0.370	0.335	0.307	0.283
0.6	0.310	0.283	0.261	0.242
0.9	0.271	0.249	0.230	0.215
1.2	0.242	0.224	0.208	0.195
1.5	0.220	0.204	0.190	0.179
1.8	0.202	0.188	0.176	0.166
2.1	0.187	0.175	0.164	0.155

Soil conductivity is 1.4 W/(m·K); floor is uninsulated. For other soil conductivities and insulation, use Equation (38).

$U_{avg,bw}$ for below-grade **walls**

$$U_{avg,bw} = \frac{2k_{soil}}{\pi(z_1 - z_2)} \times \left[\ln\left(z_2 + \frac{2k_{soil}R_{other}}{\pi}\right) - \ln\left(z_1 + \frac{2k_{soil}R_{other}}{\pi}\right)\right] \quad (39)$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
R_{other} = R value of wall, insulation and inside surface resistance [m^2K/W]
z_1, z_2 = depths of top and bottom of wall segment under consideration [m]

Table 16 Average U-Factor for Basement Walls with Uniform Insulation

Depth, m	$U_{avg,bw}$ from grade to depth, W/($m^2 \cdot K$)			
	Uninsulated	R-0.88	R-1.76	R-2.64
0.3	2.468	0.769	0.458	0.326
0.6	1.898	0.689	0.427	0.310
0.9	1.571	0.628	0.401	0.296
1.2	1.353	0.579	0.379	0.283
1.5	1.195	0.539	0.360	0.272
1.8	1.075	0.505	0.343	0.262
2.1	0.980	0.476	0.328	0.252
2.4	0.902	0.450	0.315	0.244

Soil conductivity = 1.4 W/(m·K); insulation is over entire depth. For other soil conductivities and partial insulation, use Equation (37).

Assuming **concrete** walls with **uniform insulation**

Slab-on-grade floors

- Simplified heat transfer through slab-on-grade floors
  - Function of perimeter of slab (not area)

$$Q = pF_p\left(T_i - T_o\right)$$

where T_i and T_o are the inside and outside temperatures [K]
p is the perimeter of the exposed slab surface [m]
F_p is the heat loss coefficient per unit length of perimeter [W/mK]

Heat loss coefficient: F_p

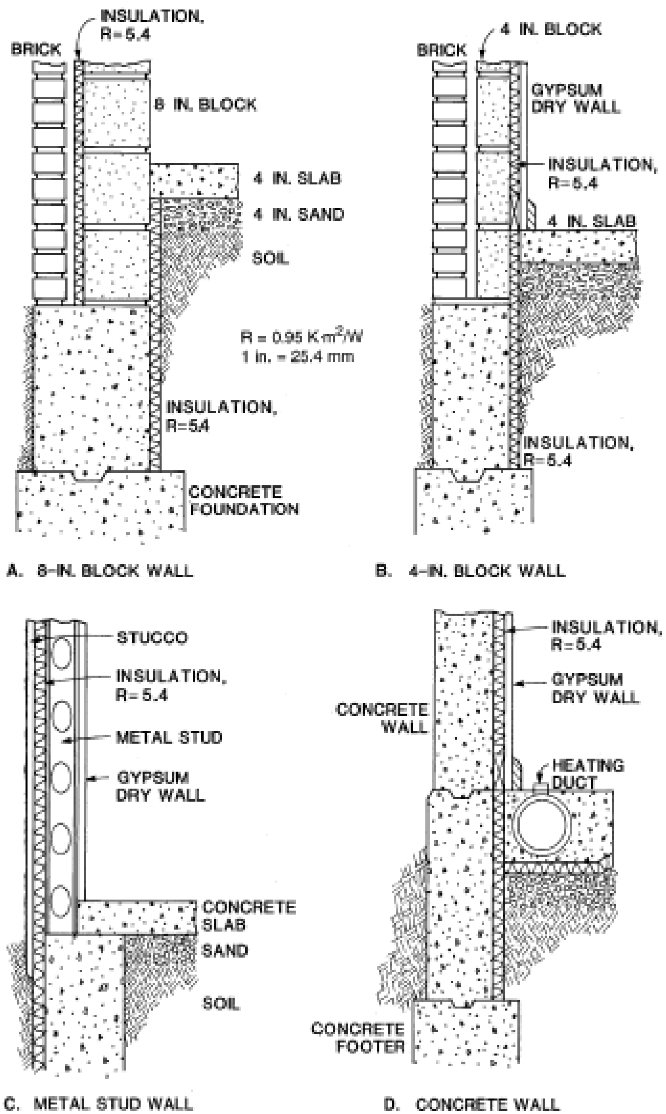


Fig. 8 Slab-on-Grade Foundation Insulation

Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

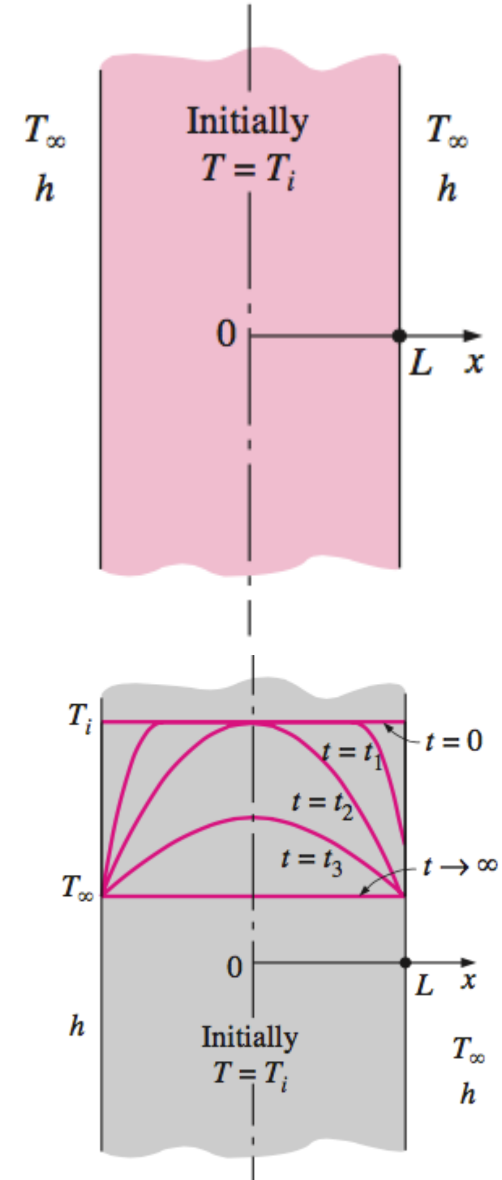
Construction	Insulation	F_p, W/(m·K)
200 mm block wall, brick facing	Uninsulated	1.17
	R-0.95 (m^2·K)/W from edge to footer	0.86
4 in. block wall, brick facing	Uninsulated	1.45
	R-0.95 (m^2·K)/W from edge to footer	0.85
Metal stud wall, stucco	Uninsulated	2.07
	R-0.95 (m^2·K)/W from edge to footer	0.92
Poured concrete wall with duct near perimeter*	Uninsulated	3.67
	R-0.95 (m^2·K)/W from edge to footer	1.24

*Weighted average temperature of the heating duct was assumed at 43°C during heating season (outdoor air temperature less than 18°C).

TRANSIENT HEAT CONDUCTION

Transient heat conduction

- Very often, conductive heat transfer in buildings does not occur at "steady-state"
- Instead, temperatures change in time at different places within and outside of a building object, so "non steady-state" conduction actually occurs
  - In other words: "dynamic" or "transient" conditions
- When temperature changes occur, the system changes in time toward a new equilibrium with the new conditions, but it takes time for that to happen

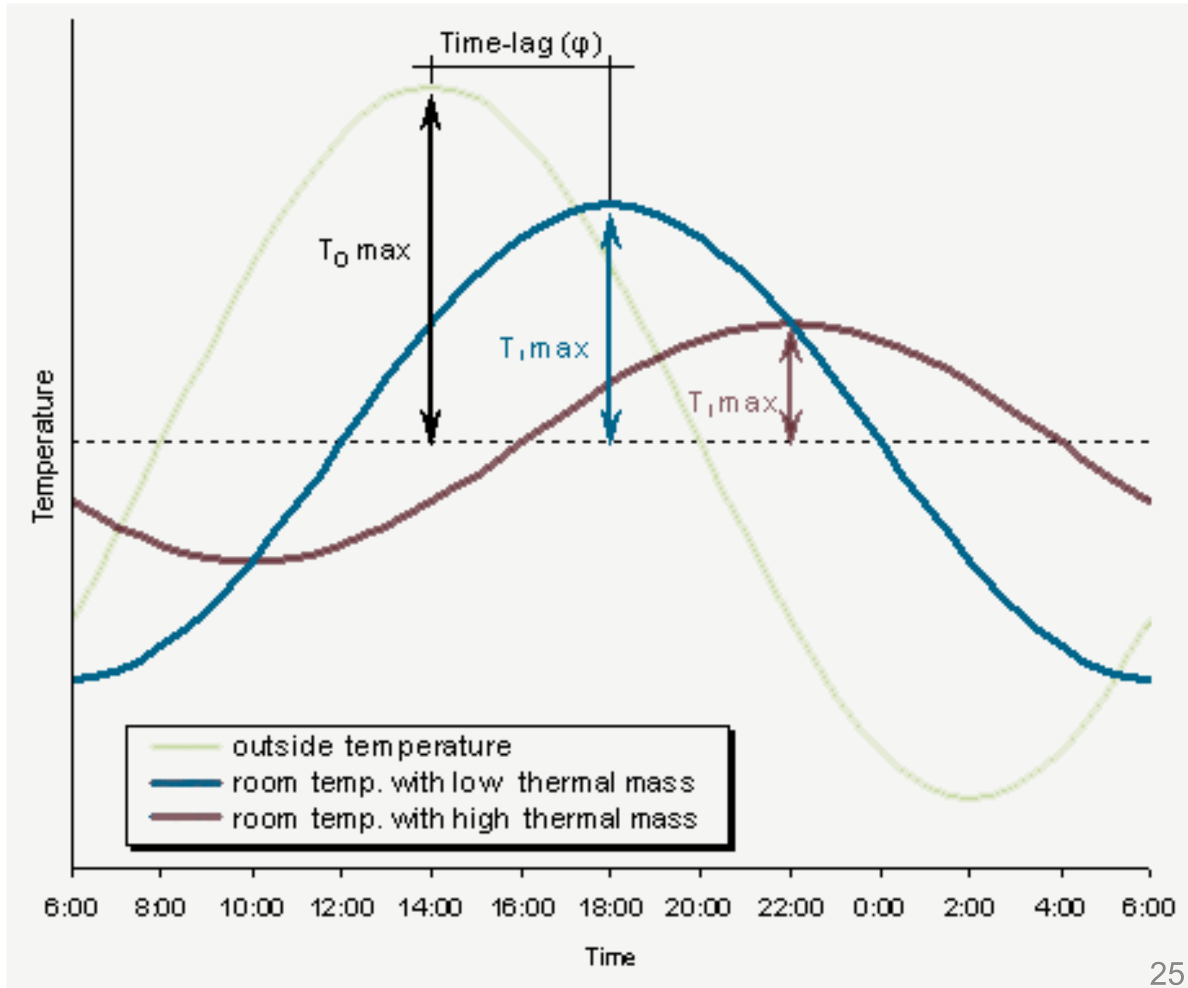


Transient heat conduction: Accounting for heat capacity

- All materials have at least capacity to store thermal energy for extended periods of time

- This is often referred to as "thermal mass"

- Thermal mass absorbs heat gains and release them at a later time



Heat capacity, *HC*

- The **heat capacity** (*HC*) of a material is a measure of the ability of a material to store energy under a temperature diff.
  - *HC* is the product of the **density** of the material and its **specific heat capacity**, with different thickness/area/volume formulations:

$HC = \rho L C_p$	$HCA = \rho LAC_p = \rho VC_p$
[J/m^{2}K]	[J/K]

  - ρ = density [kg/m^3]
  - C_p = specific heat capacity [J/kgK]
  - L = thickness of material [m]
  - A = projected surface area of material [m^2]
  - V = volume of material [m^3]

- Heat capacity is important to thermal mass, but needs to be compared with thermal conductivity to get the whole story

Thermal diffusivity, α

- Thermal diffusivity, α, is the measure of how fast heat can travel through an object
- α is proportional to conductivity but inversely proportional to density and specific heat capacity:

$$\alpha = \frac{k}{\rho C_p} \quad [\mathrm{m^2/s}]$$

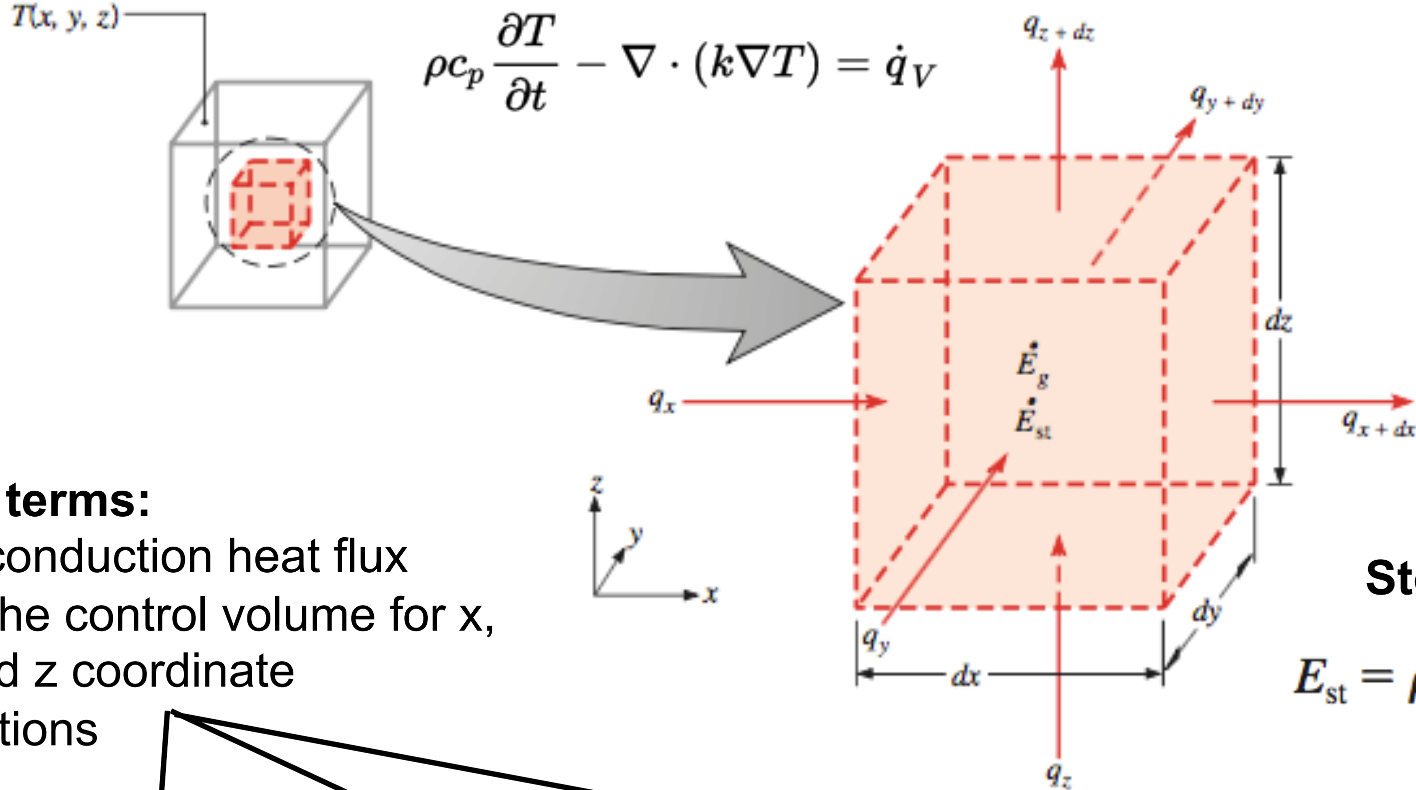
- The lower the α, the better the material is as a thermal mass (low conductivity relative to storage ability)
  - The time lag between peak internal and external temperature is related to the diffusivity of the walls
  - Steel has a high ρC_p but also a high k so it is not as good a thermal mass as concrete or brick

Thermal properties of building materials (ASHRAE)

- All three material properties can be found in the ASHRAE Handbook of Fundamentals chapter on thermal transmission data (Ch. 26 in 2013 version)
  - Thermal conductivity, density, and specific heat

Description	Density, kg/m³	Conductivity[b] (k), W/(m·K)	Conductance (C), W/(m²·K)	Resistance[c] (R) 1/k, (m·K)/W	Resistance[c] (R) For Thickness Listed (1/C), (m²·K)/W	Specific Heat, kJ/(kg·K)
Gypsum partition tile						
75 by 300 by 760 mm, solid	—	—	4.50	—	0.222	0.79
75 by 300 by 760 mm, 4 cells	—	—	4.20	—	0.238	—
100 by 300 by 760 mm, 3 cells	—	—	3.40	—	0.294	—
Concretes[o]						
Sand and gravel or stone aggregate concretes (concretes	2400	1.4-2.9	—	0.69-0.35	—	—
with more than 50% quartz or quartzite sand have	2240	1.3-2.6	—	0.77-0.39	—	0.8-1.0
conductivities in the higher end of the range)	2080	1.0-1.9	—	0.99-053	—	—
Limestone concretes	2240	1.60	—	0.62	—	—
	1920	1.14	—	0.88	—	—
	1600	0.79	—	1.26	—	—

The transient heat conduction equation



$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = \dot{q}_V$$

Flux terms:
Net conduction heat flux into the control volume for x, y, and z coordinate directions

Storage term:

$$E_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q = \rho c_p \frac{\partial T}{\partial t}$$

Heat energy source term:
Usually ignored

The transient heat conduction equation

- If thermal conductivity is constant throughout the material:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Thermal diffusivity

- Under steady-state conditions, $\rho C_p \partial T/\partial t = 0$:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q = 0$$

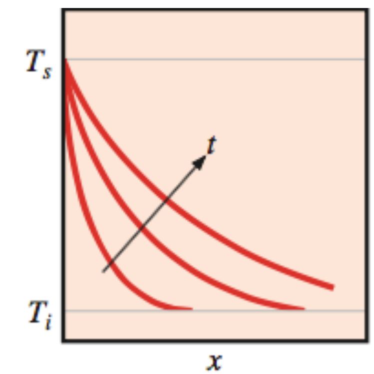
- In 1-dimension (e.g., heat flux through a solid wall), the Fourier transient heat conduction equation simplifies to:

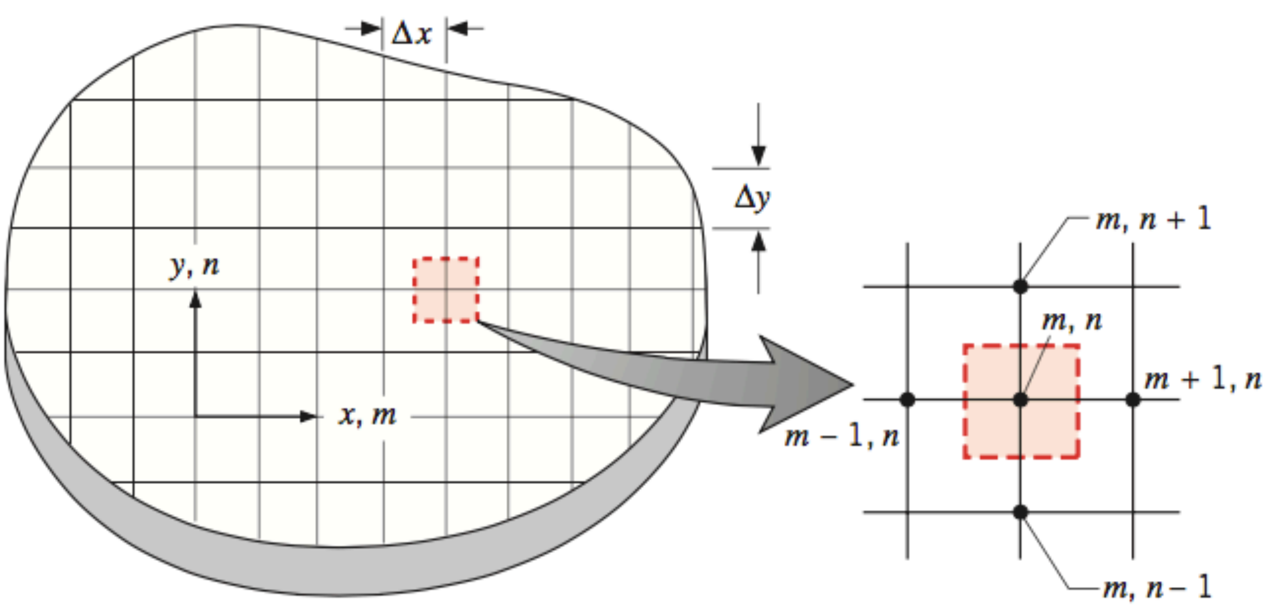
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Solutions to the transient heat conduction equation

- **Analytical solutions:**
  - Case specific
  - Simple geometries and boundary conditions
  - Mathematically more complicated
- **Numerical solutions:**
  - Finite-difference methods (explicit and implicit)

$$\frac{T(x,t)-T_s}{T_i-T_s}=\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$





$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}$$

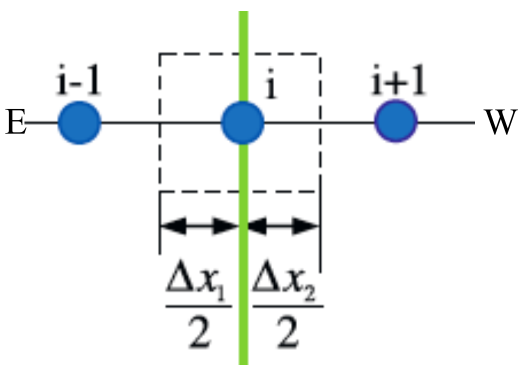
$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n}=\frac{T_{m,n}-T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n}=\frac{T_{m+1,n}-T_{m,n}}{\Delta x}$$

Transient conduction: Example numerical approach

- Conduction finite difference solution (**implicit**)

$$C_p \rho \Delta x \frac{T_i^{j+1} - T_i^{j}}{\Delta t} = \frac{1}{2}\left[\left(k_W \frac{\left(T_{i+1}^{j+1} - T_i^{j+1}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j+1} - T_i^{j+1}\right)}{\Delta x}\right) + \left(k_W \frac{\left(T_{i+1}^{j} - T_i^{j}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j} - T_i^{j}\right)}{\Delta x}\right)\right] \quad (36)$$



Where:

T = node temperature

Subscripts:

i = node being modeled

i+1 = adjacent node to interior of construction

i-1 = adjacent node to exterior of construction

j+1 = new time step

j = previous time step

Δt = calculation time step

Δx = finite difference layer thickness (always less than construction layer thickness)

C_p = specific heat of material

k_W = thermal conductivity for interface between i node and i+1 node

k_E = thermal conductivity for interface between i node and i-1 node

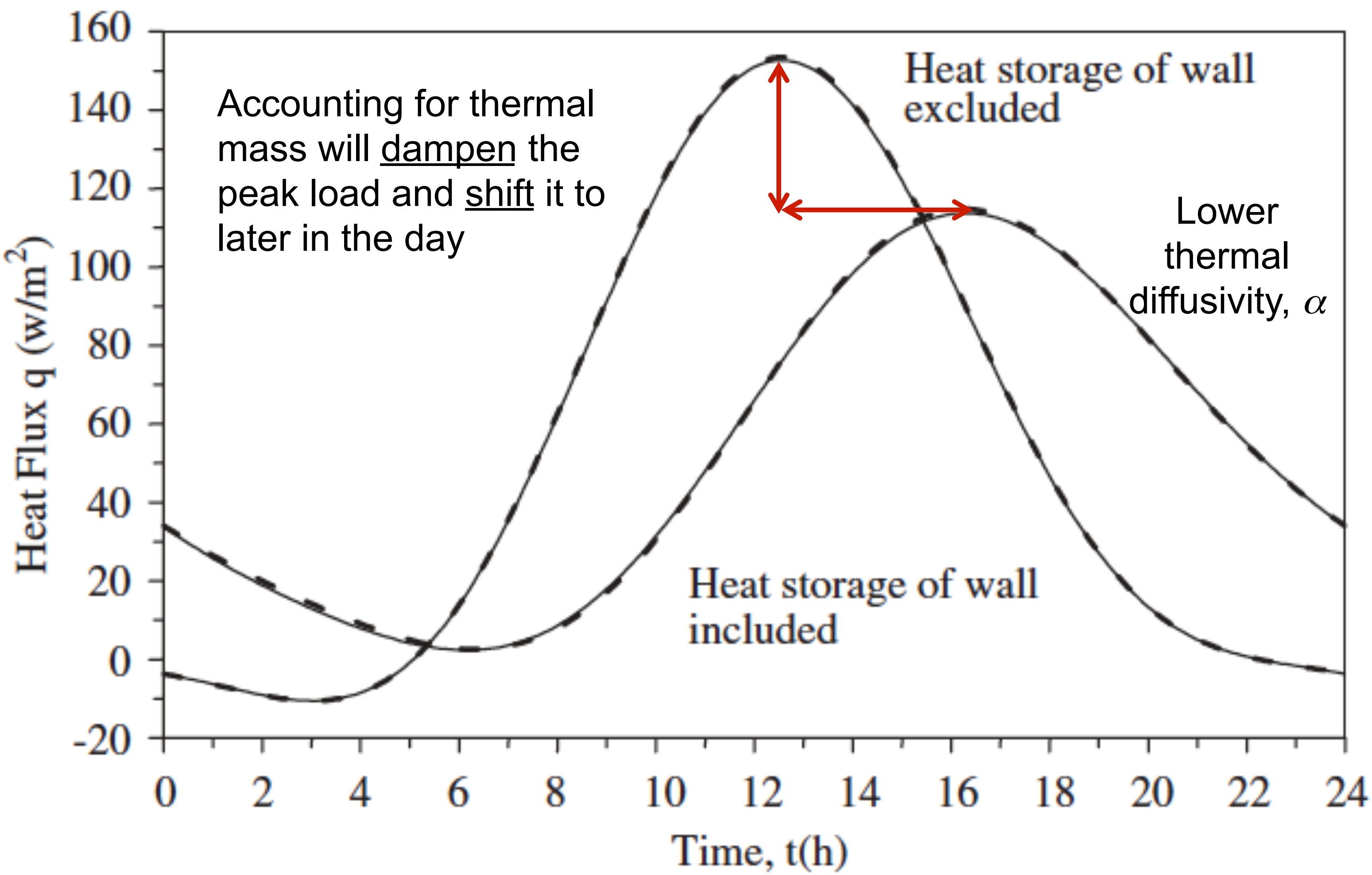
ρ = density of material

Selecting grid size:

$$\left(Fo = \alpha \Delta t / \Delta x^2\right) < 0.5$$

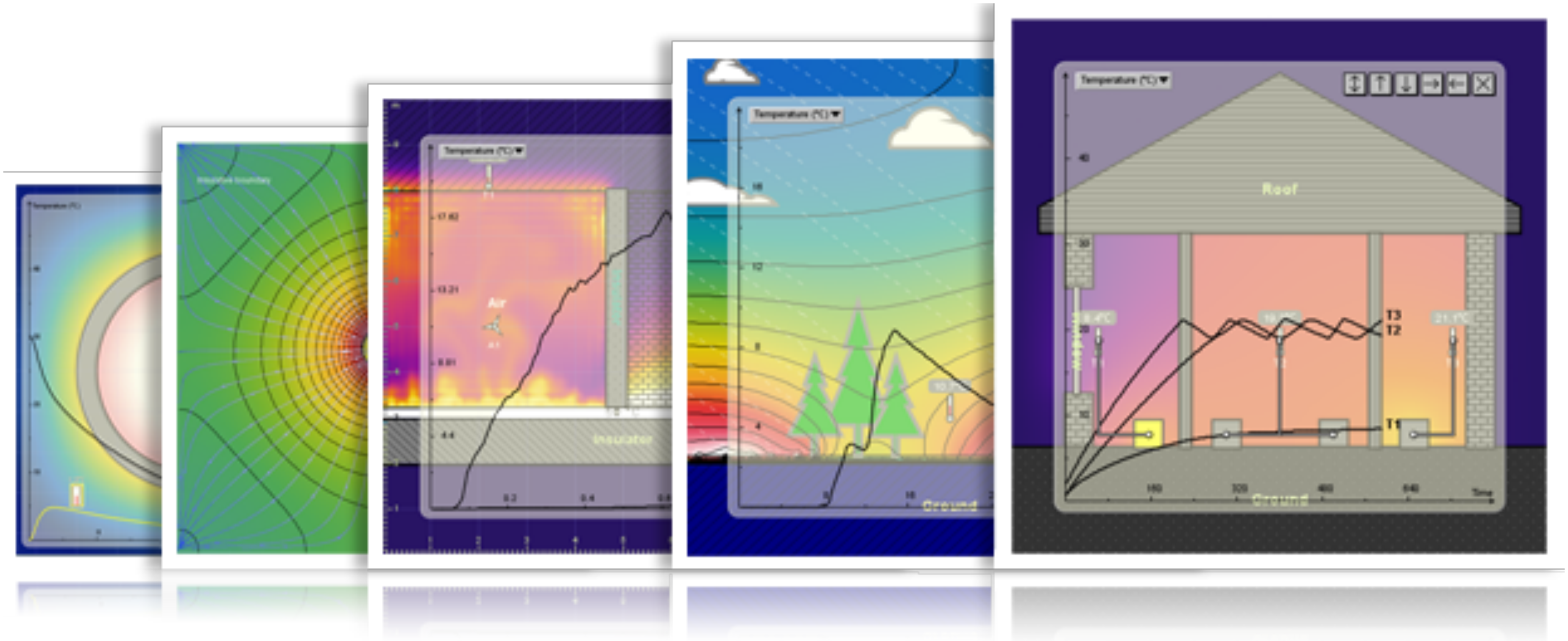
Implicit = temperatures are evaluated at time j+1 as a function of temperatures at time j

What happens when you account for thermal mass



Asan, H. 2006. Numerical computation of time lags an decrement factors for different building materials. *Building and Environment* 41:615-620.

Heat transfer visualizations



Energy2D

Interactive Heat Transfer Simulations for Everyone

Next time

- Continuing heat transfer (convection)