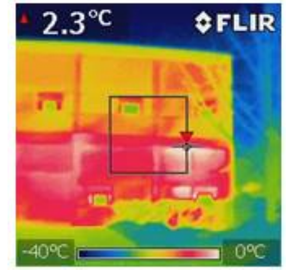
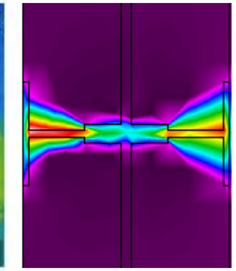
CAE 331/513
Building Science
Fall 2018





August 30, 2018
Heat transfer in buildings: Conduction (continued)

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Objectives for today's lecture

- Collect HW 1 (due today)

- Finish conduction

Last time

- Introduced steady-state heat conduction in buildings

$$q = -k\nabla T = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right) \qquad q = -k\frac{dT}{dx}$$

$$q = \frac{k}{L}\left(T_1 - T_2\right) = U\left(T_1 - T_2\right) = \frac{1}{R}\left(T_1 - T_2\right)$$

$$\frac{k}{L} = U \qquad R = \frac{1}{U}$$

R-SI

$$1\,\frac{\text{m}^2\text{K}}{\text{W}} = 5.678\,\frac{\text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F}}{\text{Btu}}$$

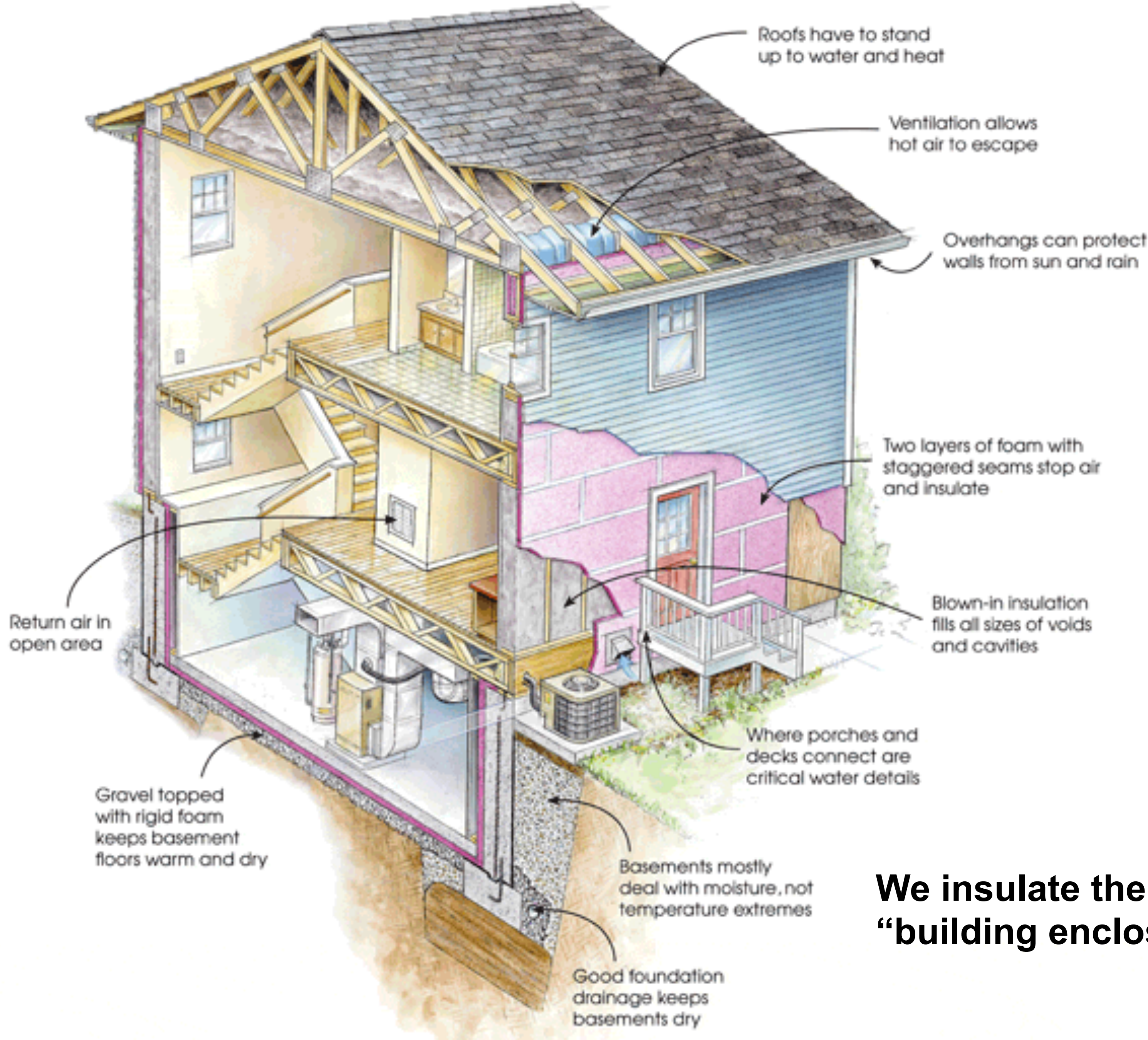
R-IP

$$R_{total} = R_1 + R_2 + R_3 + \ldots \qquad U_{total} = \frac{A_1}{A_{total}}U_1 + \frac{A_2}{A_{total}}U_2 + \ldots$$

Continuing conduction

- Other aspects of conduction to consider:
  - Practical considerations (e.g., where do we insulate?)
  - Steady-state conduction in different dimensions (e.g., pipes/cylindrical coordinates, corners, edges, slabs, basements, etc.)
  - Transient (i.e., dynamic) conduction

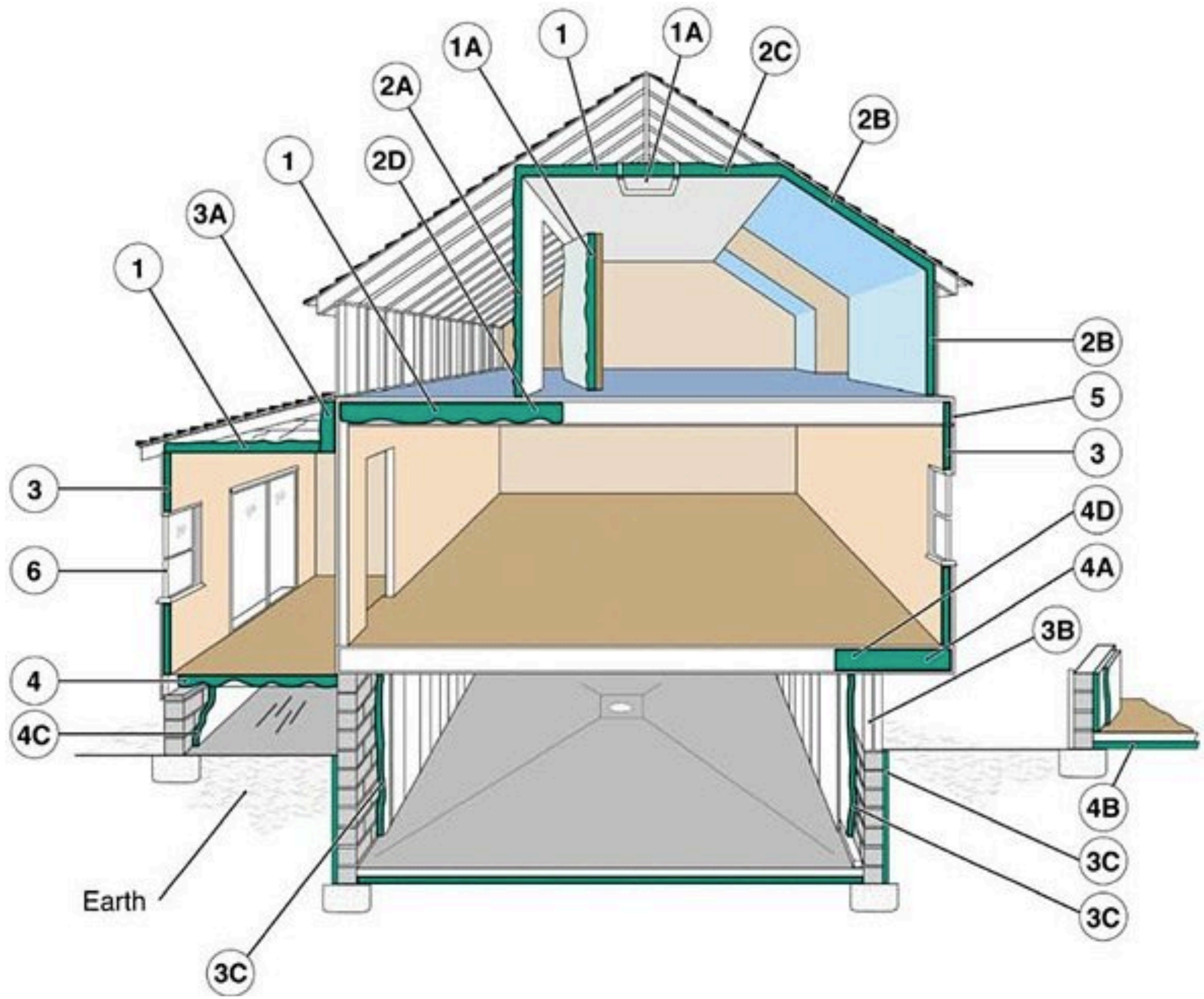
Where do we insulate buildings?



We insulate the "building enclosure"

https://www.greenbuildingadvisor.com/green-basics/the-building-envelope

Where do we insulate buildings?



https://www.energy.gov/energysaver/weatherize/insulation/where-insulate-home

Where do we insulate buildings?

1. In unfinished attic spaces, insulate between and over the floor joists to seal off living spaces below.
   1A attic access door

2. In finished attic rooms with or without dormer, insulate...
   2A between the studs of "knee" walls;
   2B between the studs and rafters of exterior walls and roof;
   2C ceilings with cold spaces above;
   2D extend insulation into joist space to reduce air flows.

3. All exterior walls, including...
   3A walls between living spaces and unheated garages, shed roofs, or storage areas;
   3B foundation walls above ground level;
   3C foundation walls in heated basements, full wall either interior or exterior.

4. Floors above cold spaces, such as vented crawl spaces and unheated garages. Also insulate...
   4A any portion of the floor in a room that is cantilevered beyond the exterior wall below;
   4B slab floors built directly on the ground;
   4C as an alternative to floor insulation, foundation walls of unvented crawl spaces;
   4D extend insulation into joist space to reduce air flows.

5. Band joists.

https://www.energy.gov/energysaver/weatherize/insulation/where-insulate-home

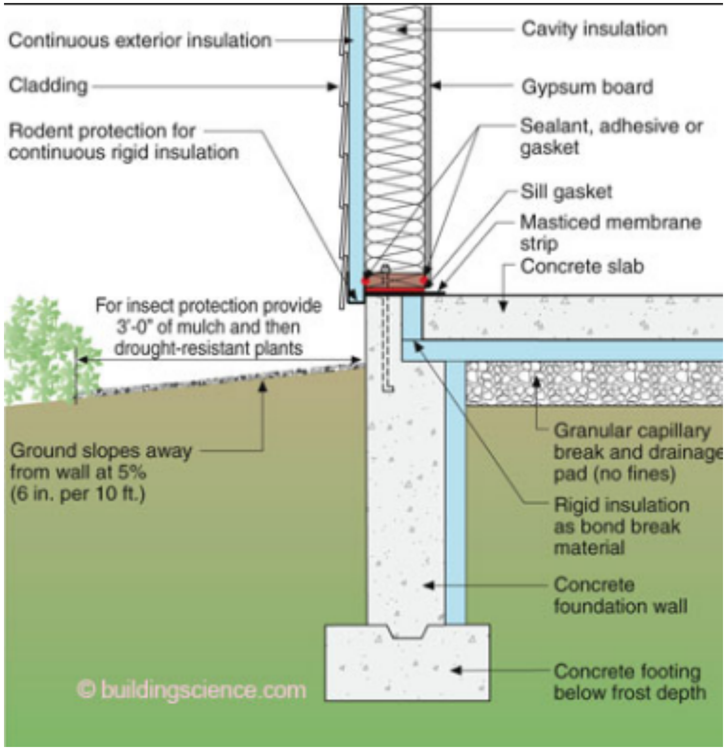
Where do we insulate buildings?

findinsulationcontractor.com

palletcover.co

Exterior walls

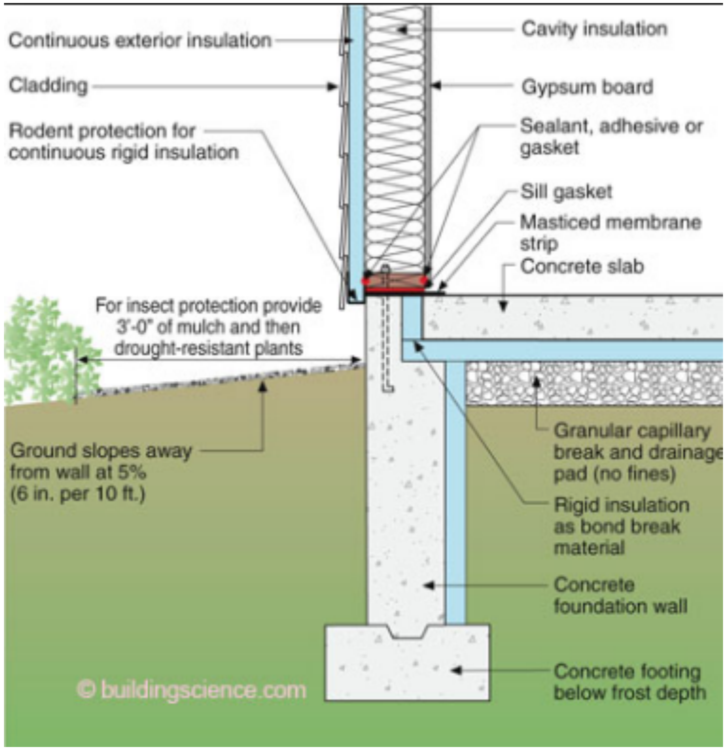
Where do we insulate buildings?



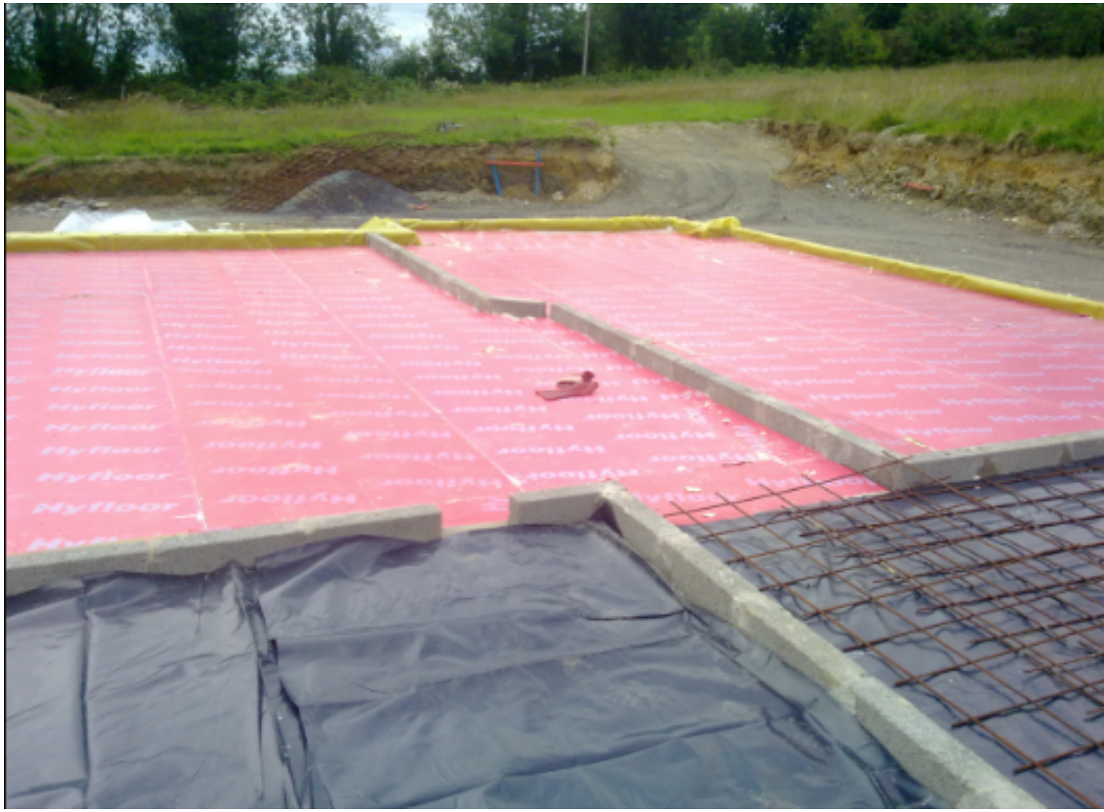
Slab and foundation insulation

hansenpolebuildings.com

Where do we insulate buildings?



Slab and foundation insulation



clipgoo.com

Where do we insulate buildings?

homeadvisor.com

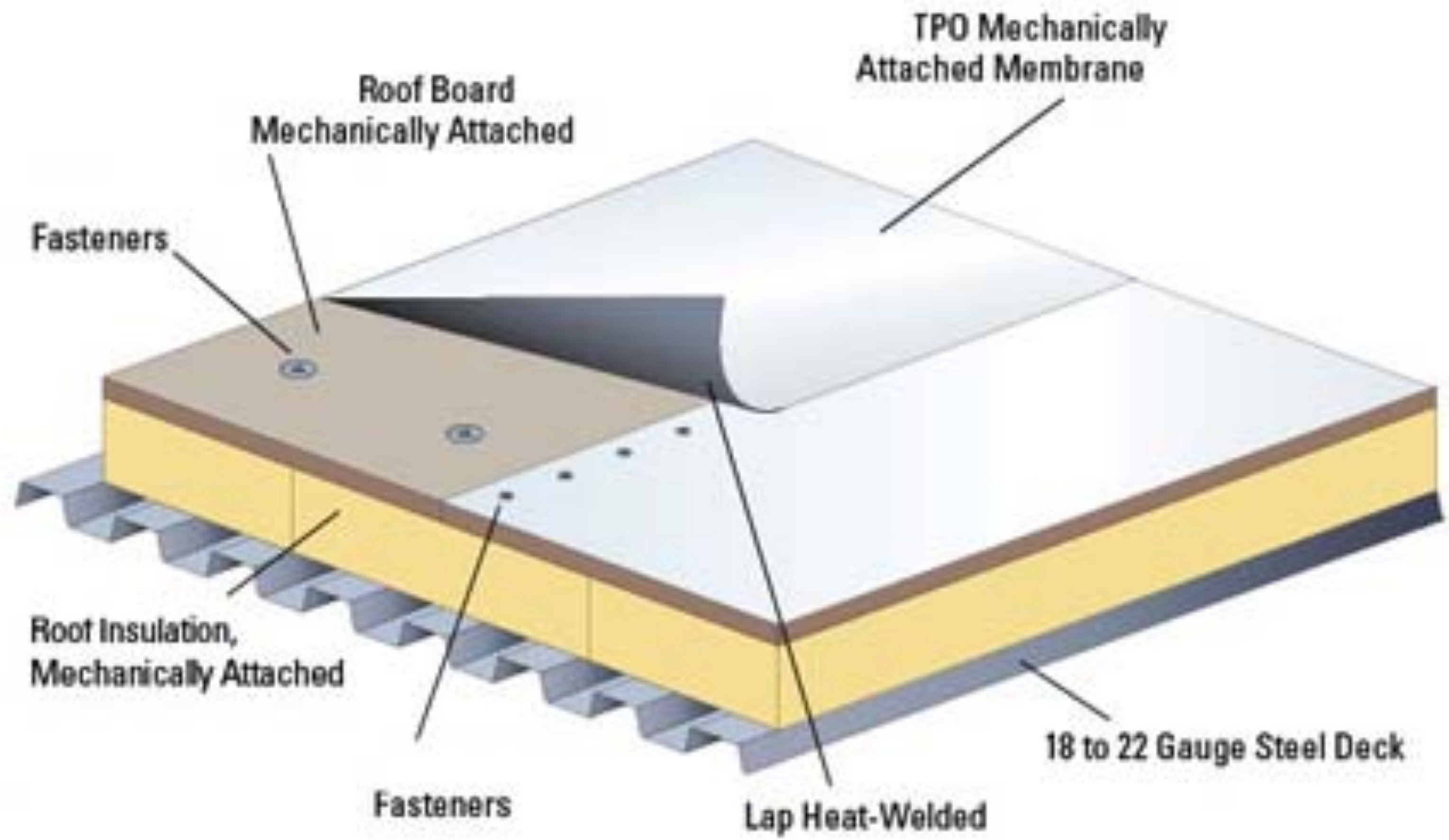
Roofs - Residential

Where do we insulate buildings?

unalansusam.com

Roofs - Residential

Where do we insulate buildings?



stormdamagerepairmn.wordpress.com

Roofs – Commercial - Over decking

Where do we insulate buildings?

chemtechroof.com

Roofs – Commercial - Over decking

Where do we insulate buildings?

finehomebuilding.com

Roofs – Commercial - Under decking

Where do we insulate buildings?

We also insulate ductwork and pipes:

https://www.aircreations.net/ductwork/duct-insulation

How much should we insulate?

- Recommendations vary
  - By authority
  - By climate zone
- Turn to building codes, standards, and guidelines
  - State and city building codes (e.g., State of IL; City of Chicago)
  - National codes (e.g., IECC)
  - Industry standards (e.g., ASHRAE)
- State of Illinois residential recommendations (IP):

Area	Recommended R-value
Attic	43
Wall (existing)	13
Wall (new)	21
Floor over unconditioned space	21
Basement wall	10
Crawl space wall	10

https://www2.illinois.gov/sites/KeepWarm/Documents/insulation.pdf

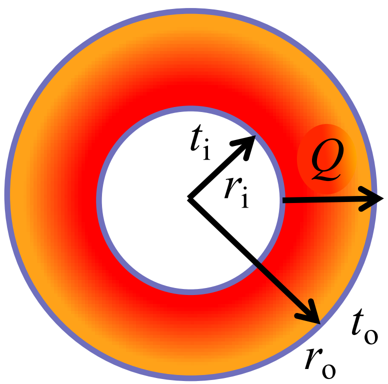
Example problem: How much insulation?

- To achieve a total of R-43 (IP) in a residential attic
- Assume: Blown-in cellulose, blown into 2x12 inch joists installed 16 inches on center

Conduction in cylindrical coordinates

- Fourier's law also applies to geometries other than plane walls
- Of particular interest in buildings is cylindrical geometry
  - For example: heat losses from piping in HVAC systems

- For a hollow cylinder with length L, inner radius r_i and outer radius r_o:
  - If you integrate $Q = -kA\dfrac{dT}{dx}$ in cylindrical coordinates, you get:



$$R'_{cyl} = \frac{\ln\left(r_o / r_i\right)}{2\pi kL}$$

$$Q = \frac{2\pi kL}{\ln\left(r_o / r_i\right)}\left(T_i - T_o\right)$$

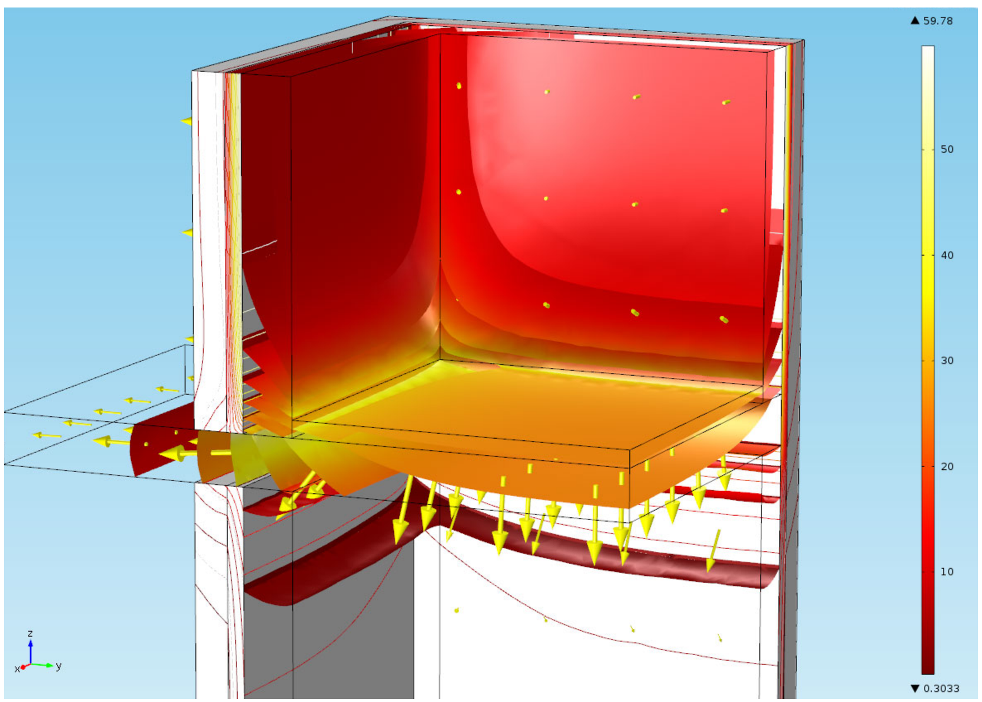
Conduction in cylindrical coordinates

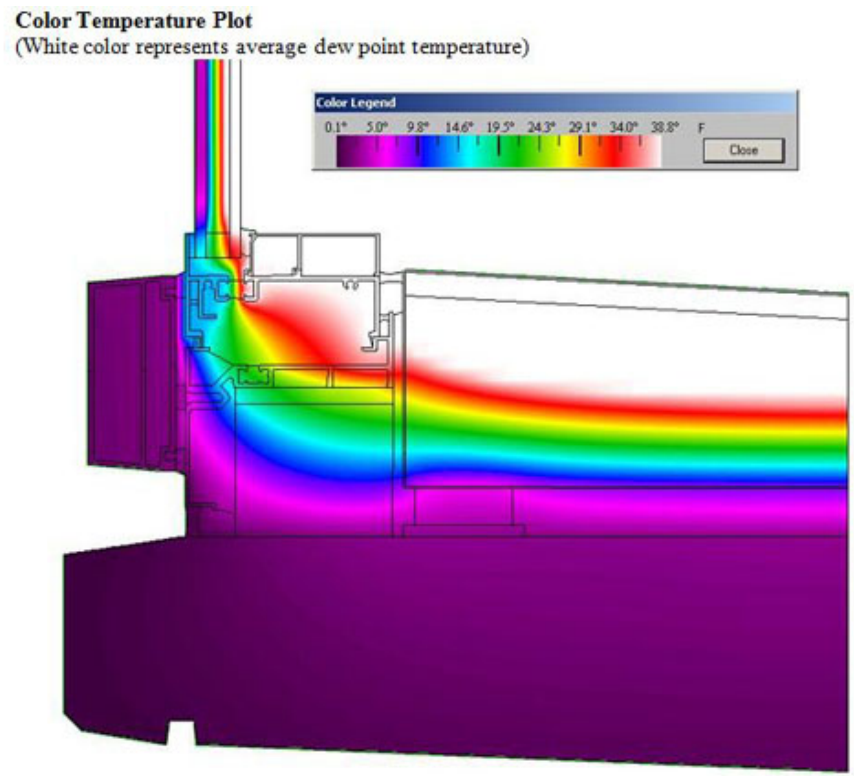
- **Example:** Steam at 260°C flows through a cast iron pipe (conductivity of 80 W/mK) with an outer diameter of 7.5 cm and an inner diameter of 7.0 cm. The pipe is wrapped with 3 cm thick of glass wool insulation (conductivity of 0.05 W/mK)

- What is the heat loss to the environment per meter length of pipe, assuming that the outer layer of insulation has a temperature of 20°C?

Conduction in other geometries: 2-D and 3-D effects

- There are often situations where heat conduction is not strictly 1-D
- The best approach is to conduct 2-D and/or 3-D analysis, although we don't cover in this class (we cover this in CAE 463/524 Building Enclosure Design)





THERM

BELOW- AND ON-GRADE CONDUCTION

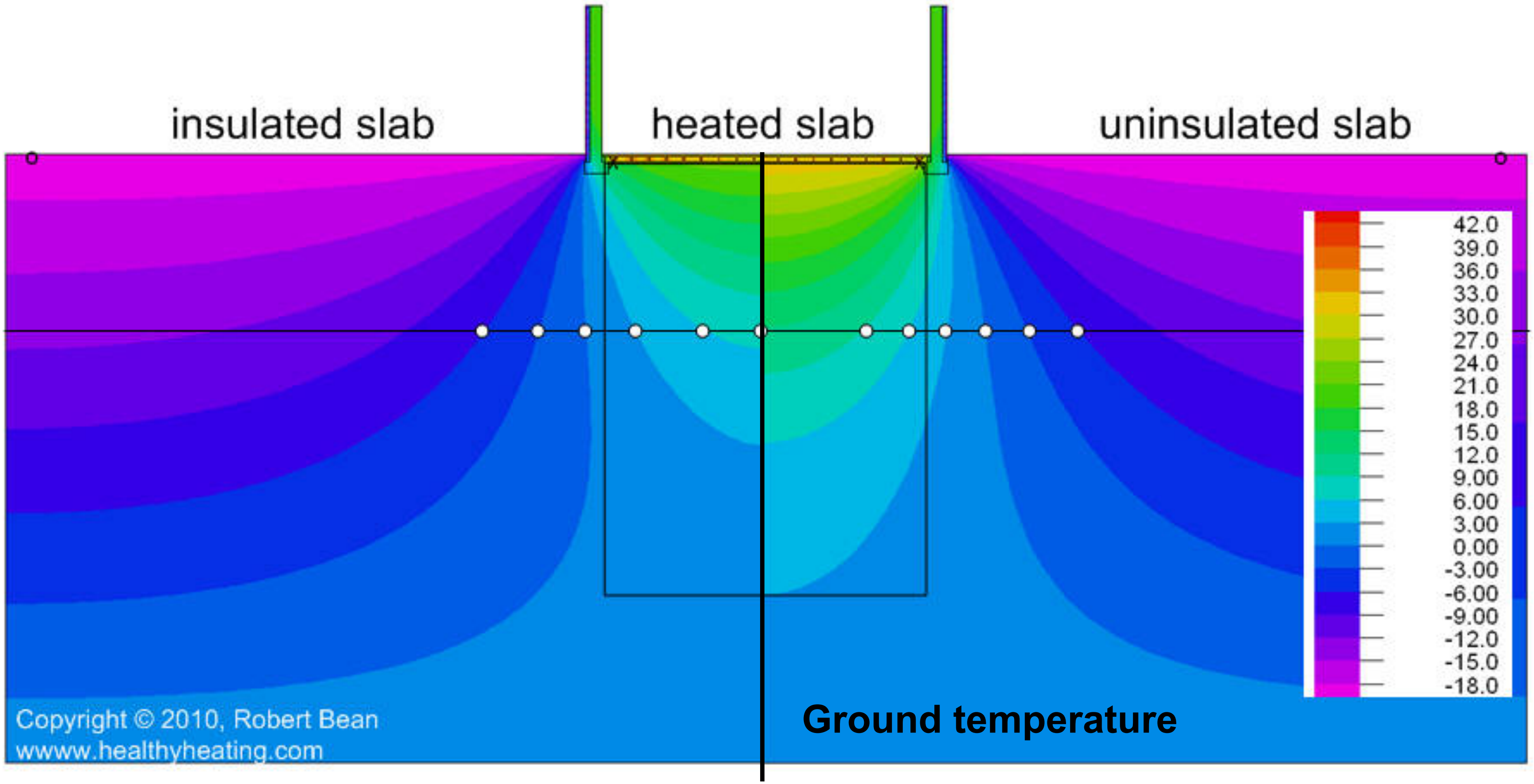
Below- and on-grade conduction

- Very often, building enclosure assemblies are located on or below ground level (i.e., “on-grade” or “below-grade”)
- Instead of exterior surfaces being in contact with the outside air, they are in contact with soil
- Where does heat flow?
  - Depends on surface and ground temperature distributions

Below-grade

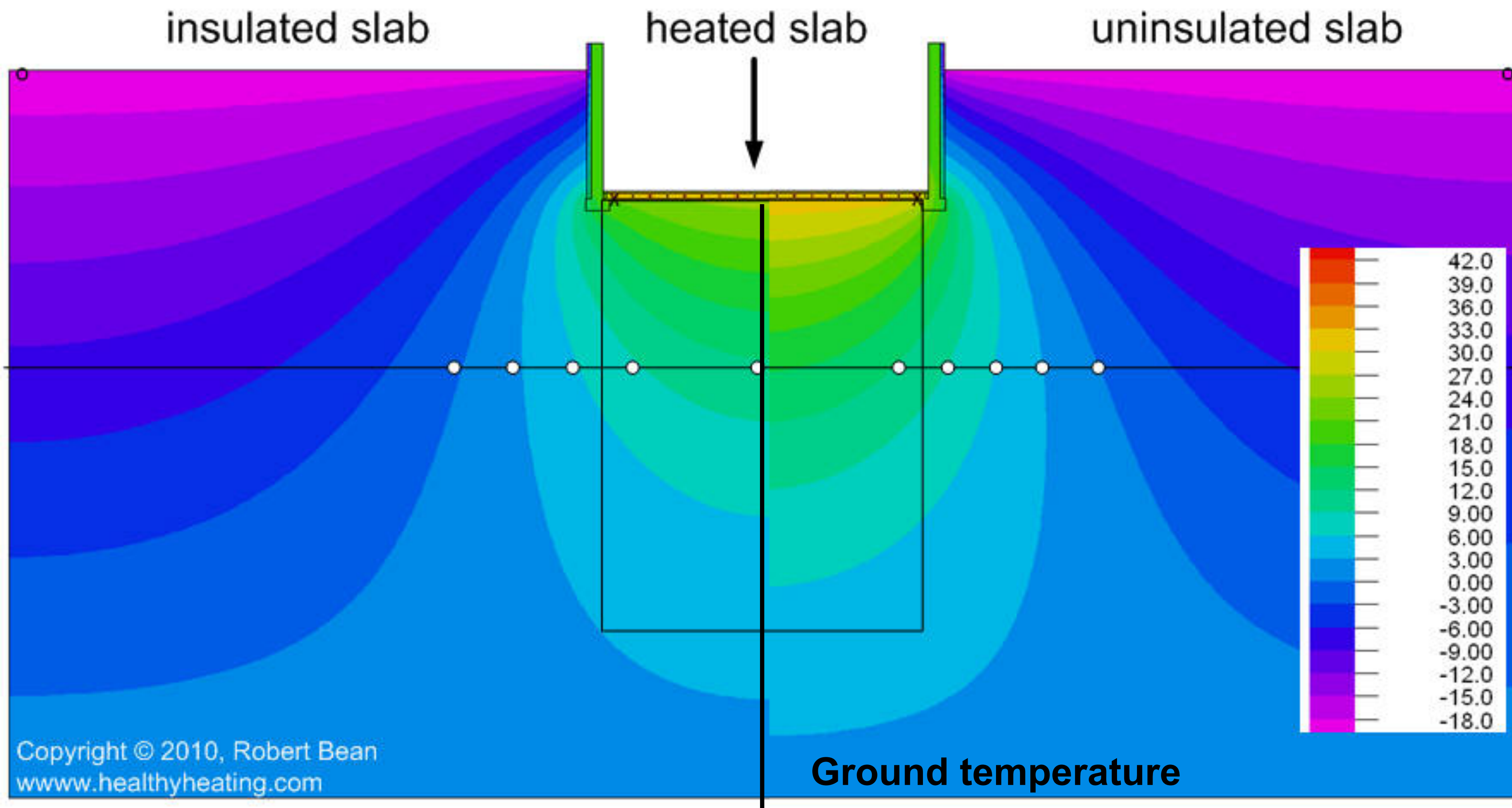
On-grade heat flow

- Often we have floors built directly on grade, in contact with the ground



Below-grade heat flow

- Often we have walls and floors built below-grade, or "submerged" within the soil



Average annual ground temperatures

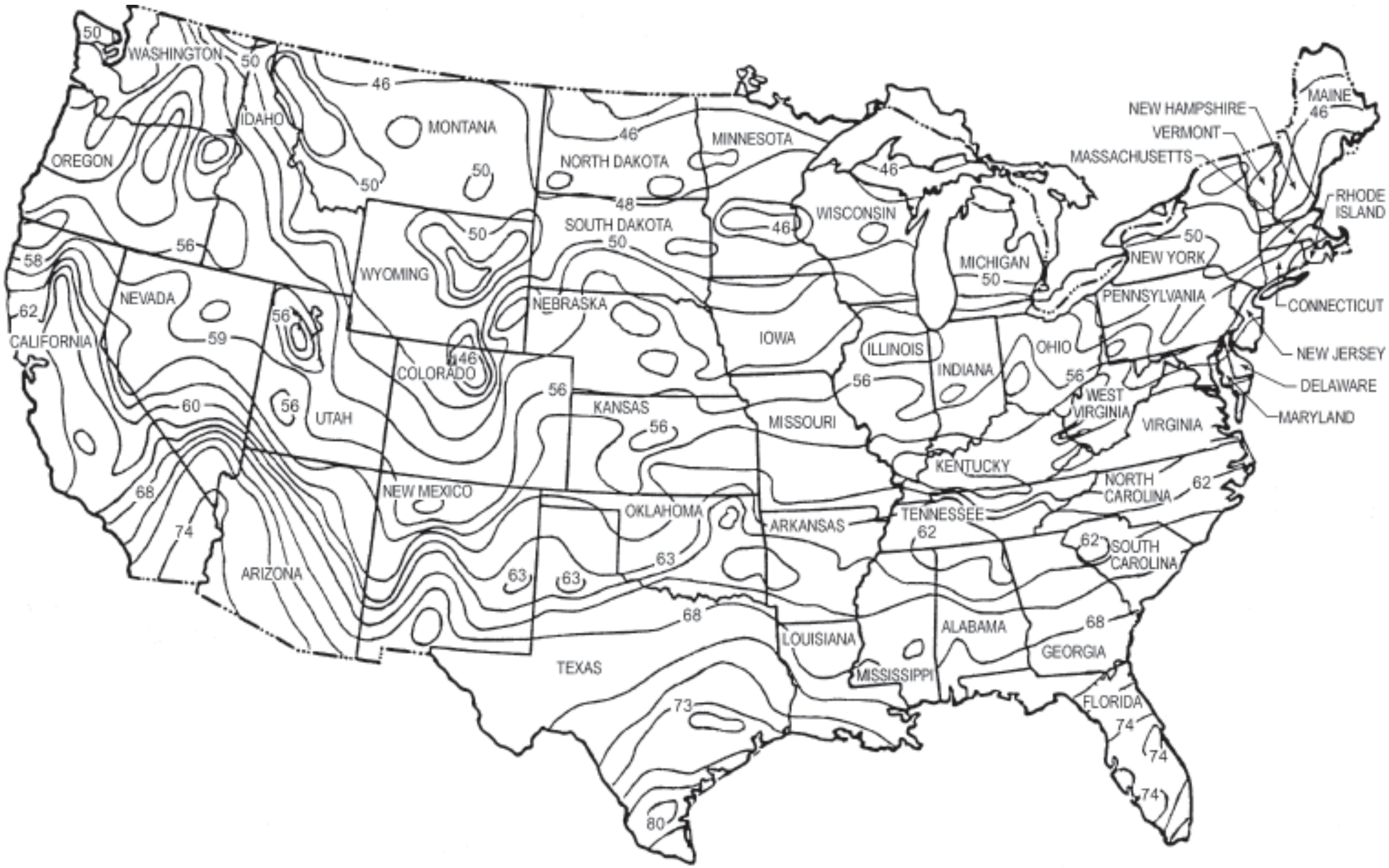
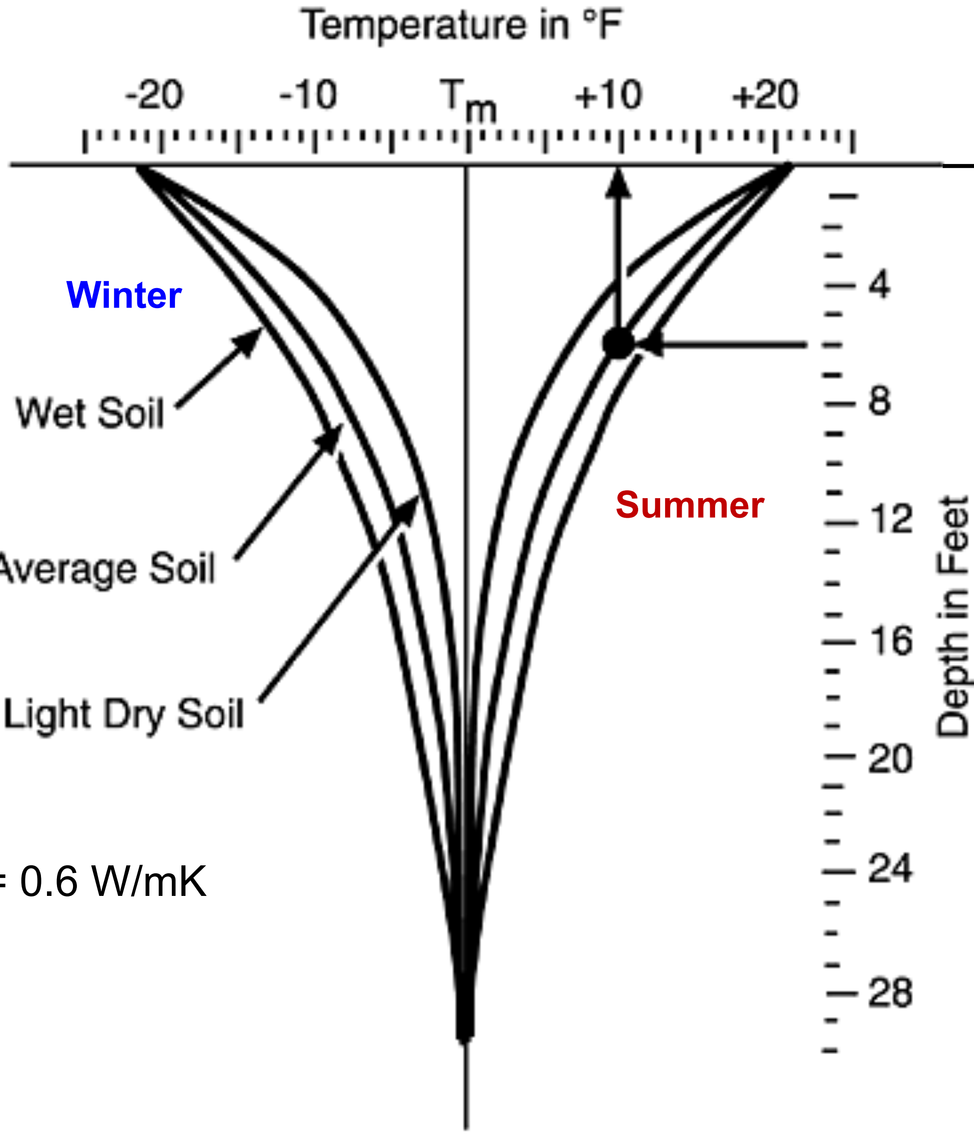
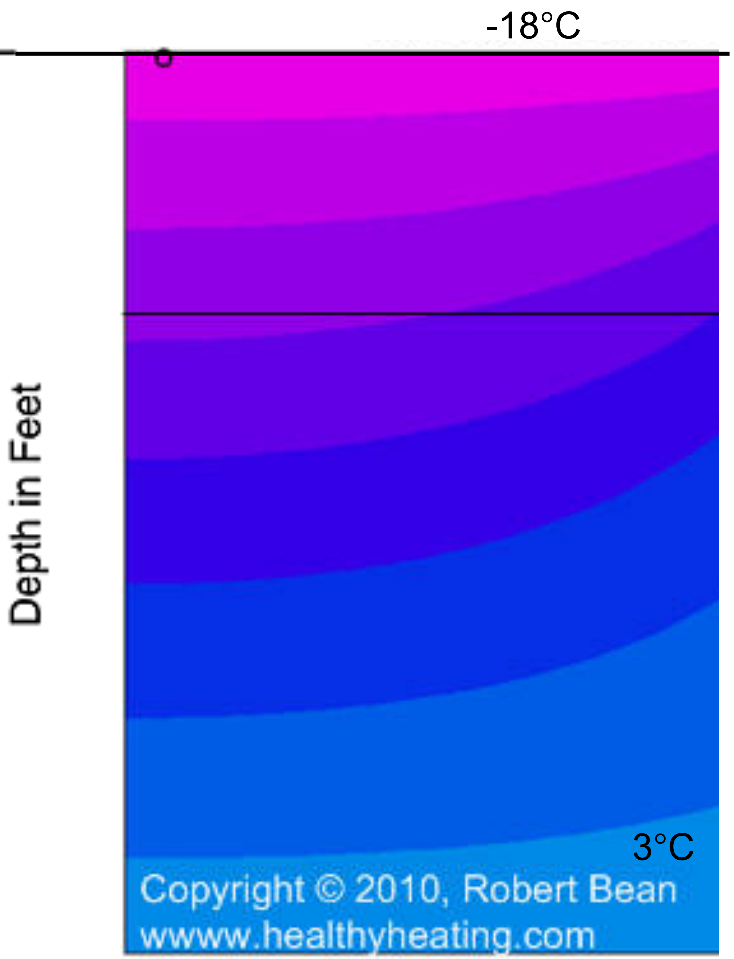


Fig. 17 Approximate Groundwater Temperatures (°F) in the Continental United States

Source: 2011 HVAC Apps Chapter 34, Figure 17

Ground temperatures vary with **depth** and **soil moisture**





k_{H2O} = 0.6 W/mK

Source: http://www.geo4va.vt.edu/A1/A1.htm

Simplified below-grade heat transfer

$$Q = AU_{avg}\,(T_i - T_{gr}) \quad \text{[W]}$$

$$q = U_{avg}\,(T_i - T_{gr}) \quad [\mathrm{W/m^2}]$$

where

A is the wall or floor area below grade [m^2] (analyze any wall portion above-grade in the normal way)

T_i is the below grade inside temperature [K]

T_{gr} is the ground surface temperature [K]

U_{avg} is the average U-value for the below grade surface [$W/(m^2K)$] (see following slides for ways to calculate)

Below grade depth parameters for estimating U-value

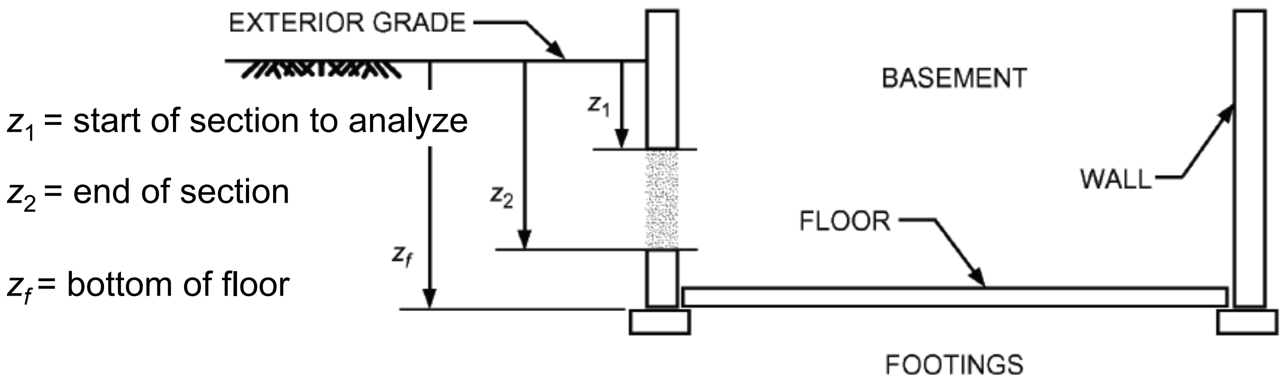


Fig. 14 Below-Grade Parameters

- For **average below-grade floor** value with a floor depth of height z_f from ground ("grade")

$$U_{avg,bf} = \frac{2k_{soil}}{\pi w_b} \times \left[\ln\left(\frac{w_b}{2} + \frac{z_f}{2} + \frac{k_{soil} R_{other}}{\pi} \right) - \ln\left(\frac{k_{soil} R_{other}}{\pi} \right) \right] \tag{40}$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
R_{other} = R value of floor assembly [m^2K/W]
w_b = shortest dimension of basement width [m]
z_f = floor depth below grade [m]

Pre-computed tables for $U_{avg,bf}$

- Assuming **un-insulated concrete** floor

Table 17 Average U-Factor for Basement Floors

z_f (depth of floor below grade), m	$U_{avg,bf}$, W/(m^2·K) — w_b (shortest width of basement), m: 6	7	8	9
0.3	0.370	0.335	0.307	0.283
0.6	0.310	0.283	0.261	0.242
0.9	0.271	0.249	0.230	0.215
1.2	0.242	0.224	0.208	0.195
1.5	0.220	0.204	0.190	0.179
1.8	0.202	0.188	0.176	0.166
2.1	0.187	0.175	0.164	0.155

Soil conductivity is 1.4 W/(m·K); floor is uninsulated. For other soil conductivities and insulation, use Equation (38).

$U_{avg,bw}$ for below-grade **walls**

$$U_{avg,bw} = \frac{2k_{soil}}{\pi(z_1 - z_2)} \times \left[\ln\left(z_2 + \frac{2k_{soil}R_{other}}{\pi}\right) - \ln\left(z_1 + \frac{2k_{soil}R_{other}}{\pi}\right)\right] \quad (39)$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
R_{other} = R value of wall, insulation and inside surface resistance [m^2K/W]
z_1, z_2 = depths of top and bottom of wall segment under consideration [m]

Table 16 Average U-Factor for Basement Walls with Uniform Insulation

Depth, m	$U_{avg,bw}$ from grade to depth, $W/(m^2 \cdot K)$ Uninsulated	R-0.88	R-1.76	R-2.64
0.3	2.468	0.769	0.458	0.326
0.6	1.898	0.689	0.427	0.310
0.9	1.571	0.628	0.401	0.296
1.2	1.353	0.579	0.379	0.283
1.5	1.195	0.539	0.360	0.272
1.8	1.075	0.505	0.343	0.262
2.1	0.980	0.476	0.328	0.252
2.4	0.902	0.450	0.315	0.244

Soil conductivity = 1.4 W/(m·K); insulation is over entire depth. For other soil conductivities and partial insulation, use Equation (37).

Assuming **concrete** walls with **uniform insulation**

Slab-on-grade floors

- Simplified heat transfer through slab-on-grade floors
  - Function of perimeter of slab (not area)

$$Q = pF_p\left(T_i - T_o\right)$$

where T_i and T_o are the inside and outside temperatures [K]

p is the perimeter of the exposed slab surface [m]

F_p is the heat loss coefficient per unit length of perimeter [W/mK]

Heat loss coefficient: F_p

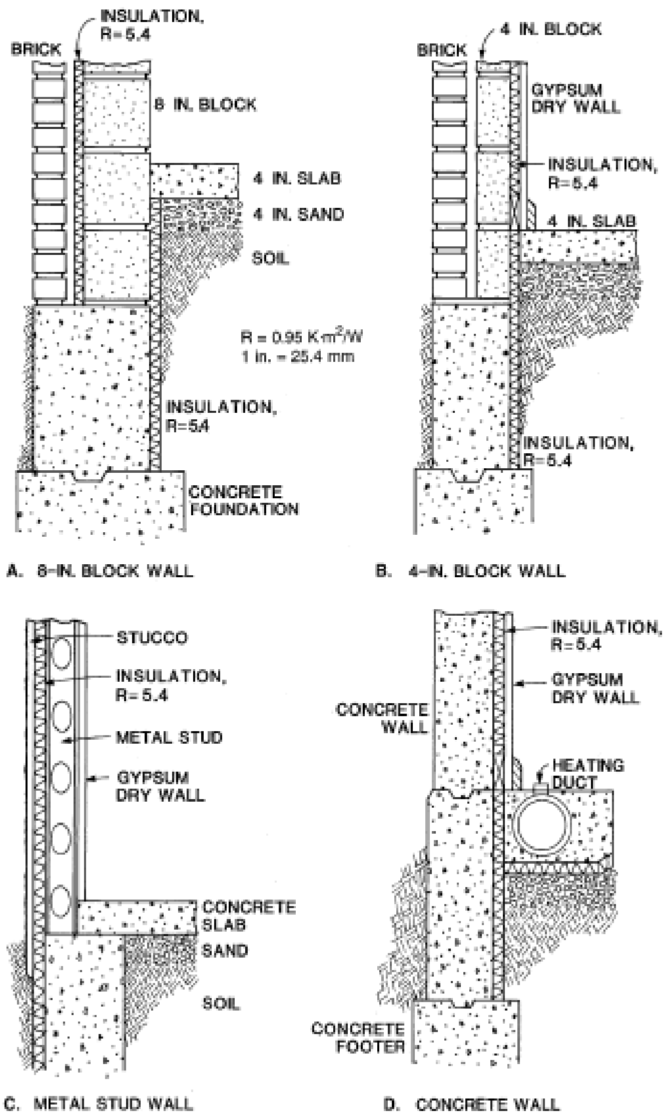


Fig. 8 Slab-on-Grade Foundation Insulation

Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

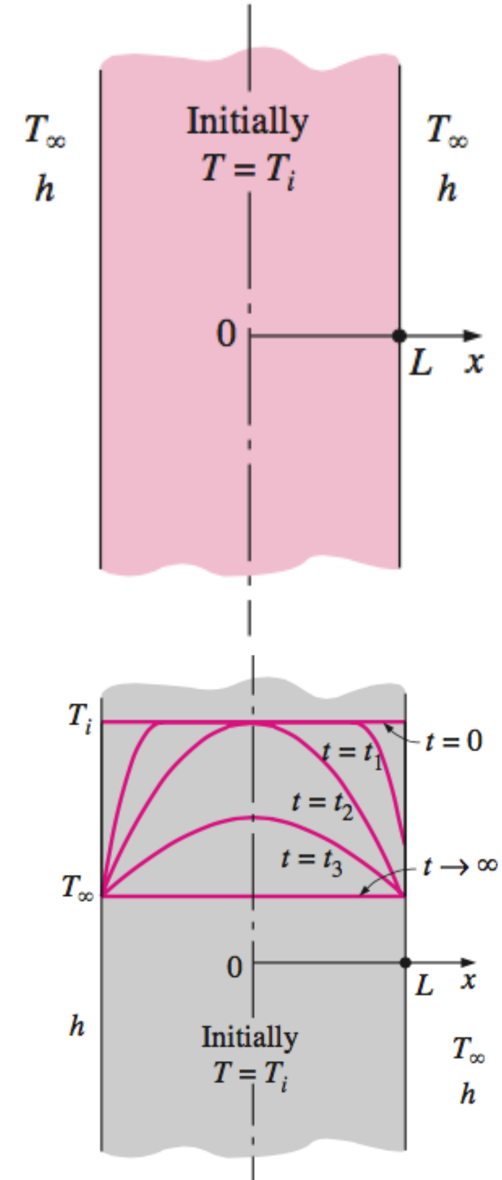
Construction	Insulation	F_p, W/(m·K)
200 mm block wall, brick facing	Uninsulated	1.17
	R-0.95 (m^2·K)/W from edge to footer	0.86
4 in. block wall, brick facing	Uninsulated	1.45
	R-0.95 (m^2·K)/W from edge to footer	0.85
Metal stud wall, stucco	Uninsulated	2.07
	R-0.95 (m^2·K)/W from edge to footer	0.92
Poured concrete wall with duct near perimeter*	Uninsulated	3.67
	R-0.95 (m^2·K)/W from edge to footer	1.24

*Weighted average temperature of the heating duct was assumed at 43°C during heating season (outdoor air temperature less than 18°C).

TRANSIENT HEAT CONDUCTION

Transient heat conduction

- Very often, conductive heat transfer in buildings does not occur at “steady-state”
- Instead, temperatures change in time at different places within and outside of a building object, so “non steady-state” conduction actually occurs
  - In other words: “dynamic” or “transient” conditions
- When temperature changes occur, the system changes in time toward a new equilibrium with the new conditions, but it takes time for that to happen

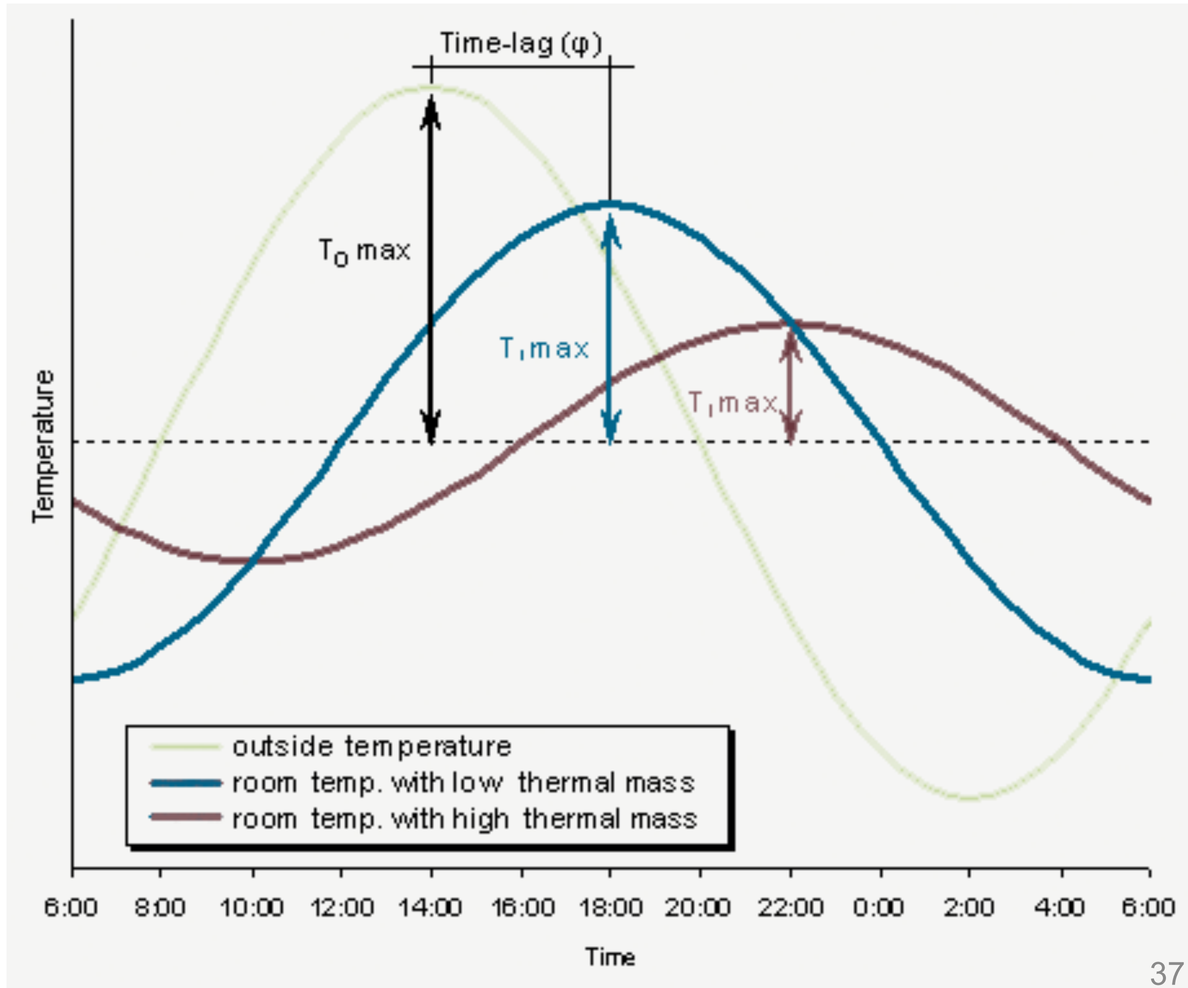


Transient heat conduction: Accounting for heat capacity

- All materials have at least capacity to store thermal energy for extended periods of time

- This is often referred to as "thermal mass"

- Thermal mass absorbs heat gains and release them at a later time



Heat capacity, HC

- The **heat capacity** (HC) of a material is a measure of the ability of a material to store energy under a temperature diff.
  - HC is the product of the **density** of the material and its **specific heat capacity**, with different thickness/area/volume formulations:

$HC = \rho L C_p$	$HCA = \rho L A C_p = \rho V C_p$
[J/m²K]	[J/K]

  - ρ = density [kg/m³]
  - C_p = specific heat capacity [J/kgK]
  - L = thickness of material [m]
  - A = projected surface area of material [m²]
  - V = volume of material [m³]

- Heat capacity is important to thermal mass, but needs to be compared with thermal conductivity to get the whole story

Thermal diffusivity, α

- Thermal diffusivity, α, is the measure of how fast heat can travel through an object
- α is proportional to conductivity but inversely proportional to density and specific heat capacity:

$$\alpha = \frac{k}{\rho C_p} \quad [\mathrm{m}^2/\mathrm{s}]$$

- The lower the α, the better the material is as a thermal mass (low conductivity relative to storage ability)
  - The time lag between peak internal and external temperature is related to the diffusivity of the walls
  - Steel has a high ρC_p but also a high k so it is not as good a thermal mass as concrete or brick

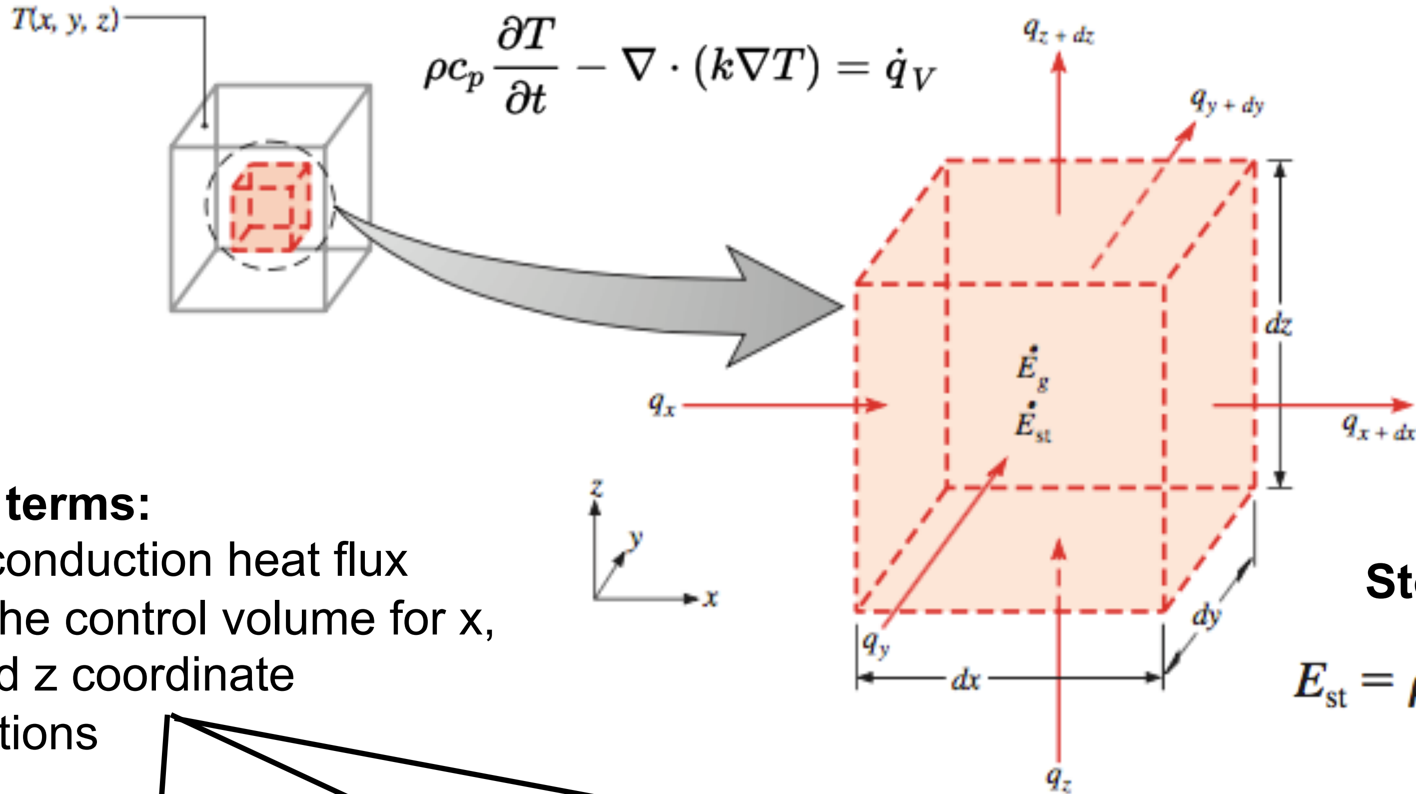
Thermal properties of building materials (ASHRAE)

- All three material properties can be found in the ASHRAE Handbook of Fundamentals chapter on thermal transmission data (Ch. 26)
  - Thermal conductivity, density, and specific heat

Description	Density, kg/m^3	Conductivity[b] (k), W/(m·K)	Conductance (C), W/(m^2·K)	Resistance[c] (R) 1/k, (m·K)/W	Resistance[c] (R) For Thickness Listed (1/C), (m^2·K)/W	Specific Heat, kJ/(kg·K)
Gypsum partition tile						
75 by 300 by 760 mm, solid	—	—	4.50	—	0.222	0.79
75 by 300 by 760 mm, 4 cells	—	—	4.20	—	0.238	—
100 by 300 by 760 mm, 3 cells	—	—	3.40	—	0.294	—
Concretes[o]						
Sand and gravel or stone aggregate concretes (concretes with more than 50% quartz or quartzite sand have conductivities in the higher end of the range)	2400	1.4-2.9	—	0.69-0.35	—	—
	2240	1.3-2.6	—	0.77-0.39	—	0.8-1.0
	2080	1.0-1.9	—	0.99-053	—	—
Limestone concretes	2240	1.60	—	0.62	—	—
	1920	1.14	—	0.88	—	—
	1600	0.79	—	1.26	—	—

The transient heat conduction equation

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = \dot{q}_V$$



Flux terms:
Net conduction heat flux into the control volume for x, y, and z coordinate directions

Storage term:

$$E_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + q = \rho c_p \frac{\partial T}{\partial t}$$

Heat energy source term:
Usually ignored

The transient heat conduction equation

- If thermal conductivity is constant throughout the material:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Thermal diffusivity

- In 1-dimension (e.g., heat flux through a solid wall), the Fourier transient heat conduction equation simplifies to:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

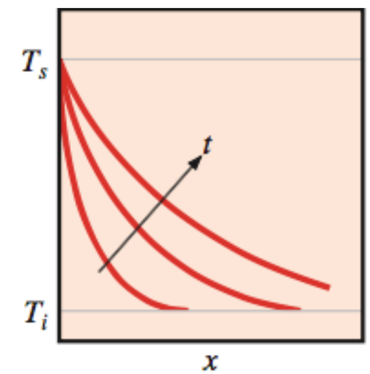
- Under steady-state conditions, $\rho C_p \partial T/\partial t = 0$:

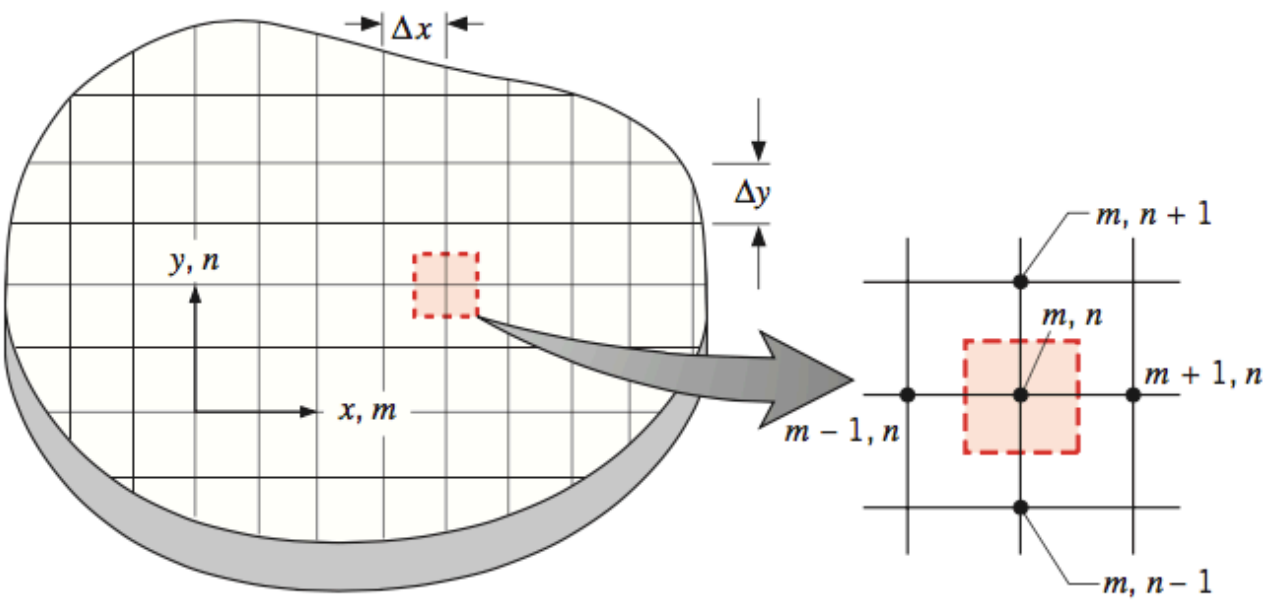
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + q = 0$$

Solutions to the transient heat conduction equation

- **Analytical solutions:**
  - Case specific
  - Simple geometries and boundary conditions
  - Mathematically more complicated
- **Numerical solutions:**
  - Finite-difference methods (explicit and implicit)

$$\frac{T(x,t)-T_s}{T_i-T_s}=\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$





$$\frac{1}{\alpha}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}$$

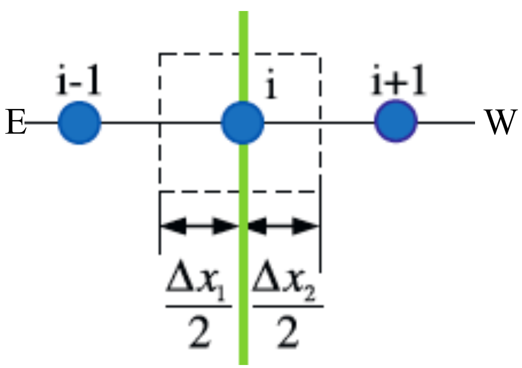
$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n}=\frac{T_{m,n}-T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n}=\frac{T_{m+1,n}-T_{m,n}}{\Delta x}$$

Transient conduction: Example numerical approach

- Conduction finite difference solution (**implicit**)

$$C_p \rho \Delta x \frac{T_i^{j+1}-T_i^{j}}{\Delta t} = \frac{1}{2}\left[\left(k_W \frac{\left(T_{i+1}^{j+1}-T_i^{j+1}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j+1}-T_i^{j+1}\right)}{\Delta x}\right) + \left(k_W \frac{\left(T_{i+1}^{j}-T_i^{j}\right)}{\Delta x} + k_E \frac{\left(T_{i-1}^{j}-T_i^{j}\right)}{\Delta x}\right)\right] \qquad (36)$$



Where:

T = node temperature

Subscripts:

i = node being modeled

i+1 = adjacent node to interior of construction

i-1 = adjacent node to exterior of construction

j+1 = new time step

j = previous time step

Δt = calculation time step

Δx = finite difference layer thickness (always less than construction layer thickness)

C_p = specific heat of material

k_W = thermal conductivity for interface between i node and i+1 node

k_E = thermal conductivity for interface between i node and i-1 node

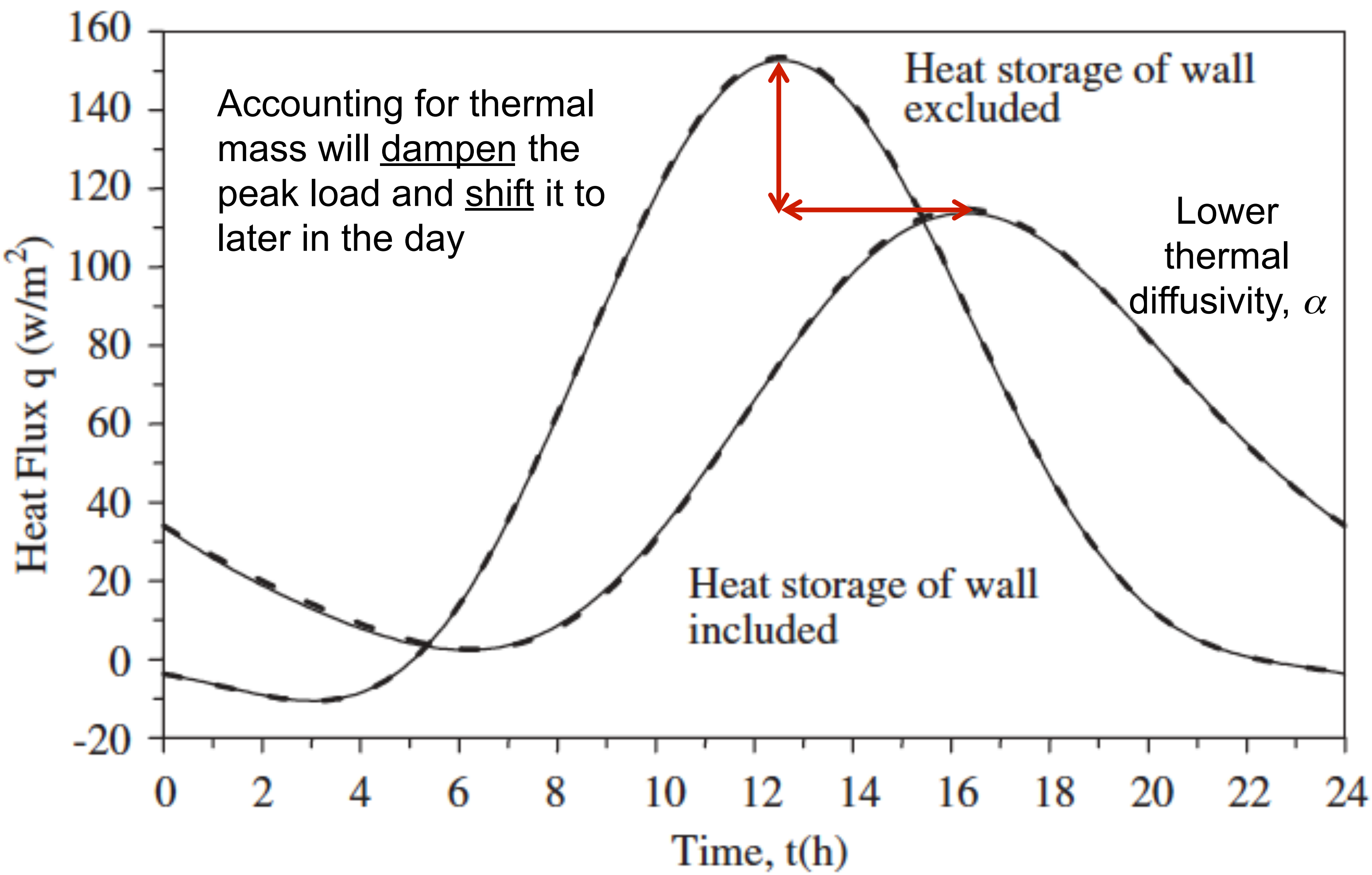
ρ = density of material

Selecting grid size:

$$\left(Fo = \alpha \Delta t / \Delta x^2\right) < 0.5$$

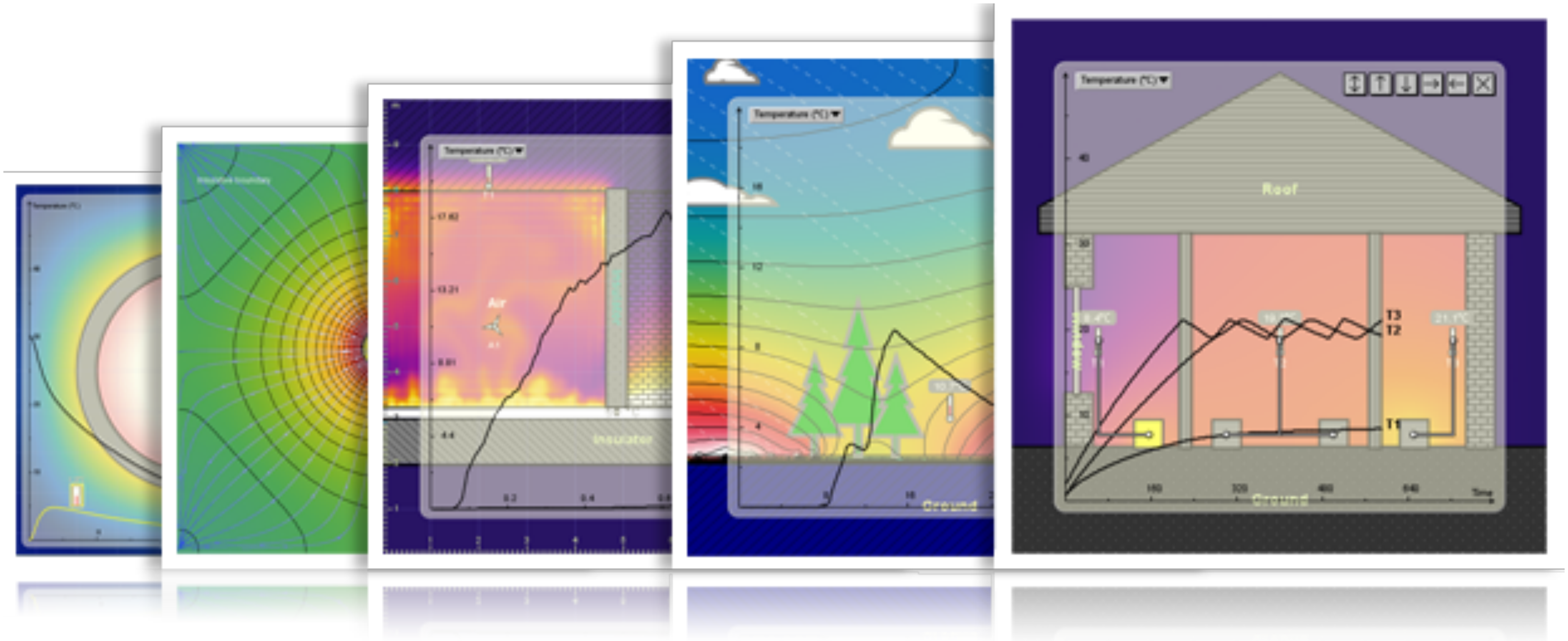
Implicit = temperatures are evaluated at time j+1 as a function of temperatures at time j

What happens when you account for thermal mass



Asan, H. 2006. Numerical computation of time lags an decrement factors for different building materials. *Building and Environment* 41:615-620.

Heat transfer visualizations



Energy2D

Interactive Heat Transfer Simulations for Everyone

Next time

- Continuing heat transfer (convection)