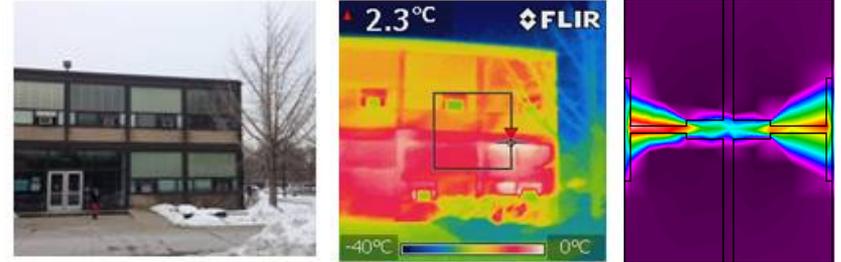


CAE 331/513

Building Science

Fall 2016



Week 7: October 4, 2016

Finish psychrometric equations

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Scheduling update

- Instead of coming to class on Thursday (Oct 6th), I would like you to attend my lecture at 1:45 pm in MTCC Auditorium



Talk title: “Outdoor pollutant penetration through building envelopes”

- You are already registered:
<http://events.r20.constantcontact.com/register/event?oeidk=a07ed4dw98b70b651d5&llr=ykrp4sdab>

Psychrometric equations summary (SI units)

$$pV = nRT \quad W = 0.622 \frac{p_w}{p - p_w} \left[\frac{\text{kg}_w}{\text{kg}_{da}} \right] \quad \rho = \frac{m_{da} + m_w}{V} = \frac{1}{v} (1 + W)$$

$$p = p_{da} + p_w$$

$$\phi = \frac{p_w}{p_{ws}} \quad pv = \frac{p}{\rho} = RT \quad R_i = \frac{R}{MW_i}$$

Dew point temperature:

Between dew points of 0 and 93°C,

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}(p_w)^{0.1984}$$

Below 0°C,

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

$$t_d = 6.09 + 12.608\alpha + 0.4959\alpha^2$$

where

$$C_8 = -5.800\ 220\ 6\ \text{E}+03$$

$$C_9 = 1.391\ 499\ 3\ \text{E}+00$$

$$C_{10} = -4.864\ 023\ 9\ \text{E}-02$$

$$C_{11} = 4.176\ 476\ 8\ \text{E}-05$$

$$C_{12} = -1.445\ 209\ 3\ \text{E}-08$$

$$C_{13} = 6.545\ 967\ 3\ \text{E}+00$$

p_{ws} = saturation pressure, Pa

T = absolute temperature, K = °C + 273.15

where

t_d = dew-point temperature, °C

α = $\ln p_w$

p_w = water vapor partial pressure, kPa

C_{14} = 6.54

C_{15} = 14.526

C_{16} = 0.7389

C_{17} = 0.09486

C_{18} = 0.4569

Psychrometric equations summary (SI units)

Wet bulb temperature (iterative solver):

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = \text{actual } W$$

*Where T_{wb} and T are in Celsius

Specific volume:

$$v = \frac{R_{da} T}{p - p_w} = \frac{R_{da} T (1 + 1.6078W)}{p}$$

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p \quad \text{where}$$

v = specific volume, $\text{m}^3/\text{kg}_{da}$
 t = dry-bulb temperature, $^{\circ}\text{C}$
 W = humidity ratio, $\text{kg}_w/\text{kg}_{da}$
 p = total pressure, kPa

Specific enthalpy:

$$h \approx 1.006T + W(2501 + 1.86T)$$

*where T is in $^{\circ}\text{C}$

Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 2 gives us W_s , v_{da} , v_s , h_{da} , and h_s directly at different temperatures:

Table 2 Thermodynamic Properties of Moist Air at Standard Atmospheric Pressure

Temp., °C <i>t</i>	Humidity Ratio W_s , kg _w /kg _{da}	Specific Volume, m ³ /kg _{da}			Specific Enthalpy, kJ/kg _{da}		
		v_{da}	v_{as}	v_s	h_{da}	h_{as}	h_s
15	0.010694	0.8159	0.0140	0.8299	15.087	27.028	42.115
16	0.011415	0.8188	0.0150	0.8338	16.093	28.873	44.966
17	0.012181	0.8216	0.0160	0.8377	17.099	30.830	47.929
18	0.012991	0.8245	0.0172	0.8416	18.105	32.906	51.011
19	0.013851	0.8273	0.0184	0.8457	19.111	35.107	54.219
20	0.014761	0.8301	0.0196	0.8498	20.117	37.441	57.558
21	0.015724	0.8330	0.0210	0.8540	21.124	39.914	61.037
22	0.016744	0.8358	0.0224	0.8583	22.130	42.533	64.663

Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 3 gives us p_{ws} at different temperatures:

Table 3 Thermodynamic Properties of Water at Saturation

Temp., °C <i>t</i>	Absolute Pressure p_{ws} , kPa	Specific Volume, m ³ /kg _w			Specific Enthalpy, kJ/kg _w		
		Sat. Liquid v_i/v_f	Evap. v_{ig}/v_{fg}	Sat. Vapor v_g	Sat. Liquid h_i/h_f	Evap. h_{ig}/h_{fg}	Sat. Vapor h_g
3	0.7581	0.001000	168.013	168.014	12.60	2493.80	2506.40
4	0.8135	0.001000	157.120	157.121	16.81	2491.42	2508.24
5	0.8726	0.001000	147.016	147.017	21.02	2489.05	2510.07
6	0.9354	0.001000	137.637	137.638	25.22	2486.68	2511.91
7	1.0021	0.001000	128.927	128.928	29.43	2484.31	2513.74
8	1.0730	0.001000	120.833	120.834	33.63	2481.94	2515.57
9	1.1483	0.001000	113.308	113.309	37.82	2479.58	2517.40
10	1.2282	0.001000	106.308	106.309	42.02	2477.21	2519.23

Revisit example from a previous class

Moist air exists at 22°C dry-bulb temperature with 50% RH at sea level

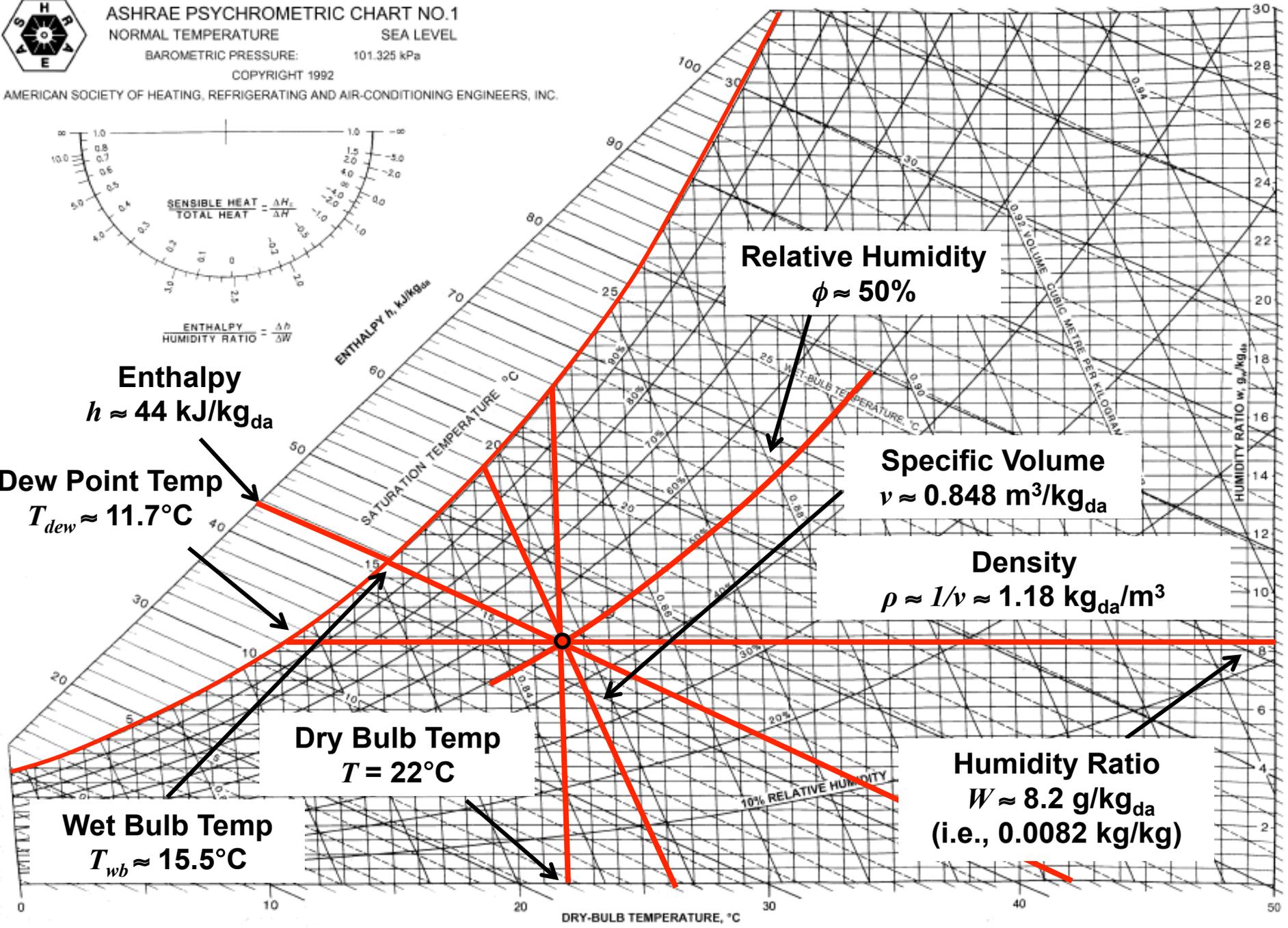
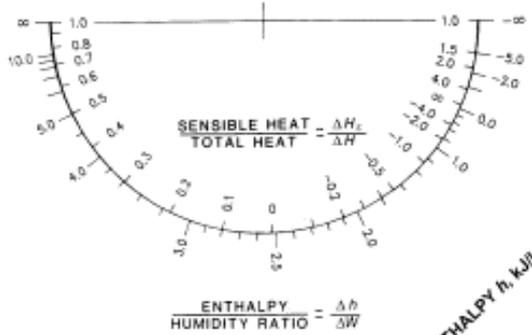
Find the following using psychrometric equations (SI units):

- (a) the humidity ratio, W
- (b) dew point temperature, T_{dew}
- (c) wet-bulb temperature, T_{wb}
- (d) enthalpy, h
- (e) specific volume, v
- (f) density, ρ
- (g) degree of saturation, μ



ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Relative Humidity
 $\phi \approx 50\%$

Specific Volume
 $v \approx 0.848 \text{ m}^3/\text{kg}_{da}$

Density
 $\rho \approx 1/v \approx 1.18 \text{ kg}_{da}/\text{m}^3$

Humidity Ratio
 $W \approx 8.2 \text{ g}/\text{kg}_{da}$
 (i.e., $0.0082 \text{ kg}/\text{kg}$)

Dry Bulb Temp
 $T = 22^\circ\text{C}$

Wet Bulb Temp
 $T_{wb} \approx 15.5^\circ\text{C}$

Dew Point Temp
 $T_{dew} \approx 11.7^\circ\text{C}$

Enthalpy
 $h \approx 44 \text{ kJ}/\text{kg}_{da}$

Revisit another example from a previous class

Moist air exists at 30°C dry-bulb temperature with a 15°C dew point temperature

Find the following:

- (a) the humidity ratio, W
- (b) degree of saturation, μ
- (c) relative humidity, ϕ
- (d) enthalpy, h
- (e) specific volume, v
- (f) density, ρ
- (g) wet bulb temperature, T_{wb}

Humidity ratio

$$W = 0.622 \frac{p_w}{p - p_w} \Big|_{@T=30^\circ C} \quad \text{Assume } p = 101.325 \text{ kPa (sea level)}$$

- For a known $T_{dew} = 15^\circ C$, we know that the actual humidity ratio in the air, W , is by definition the same as the saturation humidity ratio, W_s , at an air temperature of $15^\circ C$

$$W_{@T=30^\circ C} = W_{s@T=15^\circ C} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T=15^\circ C}$$

Temp., °C <i>t</i>	Absolute Pressure <i>p_{ws}</i> kPa
14	1.5989
15	1.7057

$$p_{ws@15C} = 1.7057 \text{ kPa}$$

$$W_{@T=30^\circ C} = W_{s@T=15^\circ C} = 0.622 \frac{1.7057}{101.325 - 1.7057} = 0.01065 \frac{\text{kg}_w}{\text{kg}_{da}}$$

Degree of saturation

- Need the saturation humidity ratio @ $T = 30^\circ\text{C}$: $\mu = \left[\frac{W}{W_s} \right]_{@T=30^\circ\text{C}}$

$$W_{s@T=30^\circ\text{C}} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T=30^\circ\text{C}}$$

Temp., °C <i>t</i>	Absolute Pressure <i>p_{ws}</i> , kPa
30	4.2467
31	4.4966

$p_{ws@15^\circ\text{C}} = 4.2467 \text{ kPa}$

$$W_{s@T=30^\circ\text{C}} = 0.622 \frac{4.2467}{101.325 - 4.2467} = 0.02720 \frac{\text{kg}_w}{\text{kg}_{\text{da}}}$$

$$\mu = \frac{W}{W_s} = \frac{0.01065}{0.02720} = 0.39$$

Relative humidity

$$\phi = \frac{p_w}{p_{ws}}$$

- From previous:

$$p_{w@T=30^\circ C} = p_{ws@T=15^\circ C} = 1.7057 \text{ kPa}$$

$$p_{ws@T=30^\circ C} = 4.2467 \text{ kPa}$$

$$\phi = \frac{1.7057}{4.2467} = 0.40 = 40\%$$

Enthalpy

$$h \approx 1.006T + W(2501 + 1.86T)$$

*where T is in °C

$$h \approx 1.006(30) + (0.01065)(2501 + 1.86(30)) = 57.4 \frac{\text{kJ}}{\text{kg}}$$

Specific volume and density

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

$$v \approx 0.287042(30 + 273.15)(1 + 1.6078(0.01065)) / (101.325)$$

$$v \approx 0.873 \frac{\text{m}^3}{\text{kg}_{\text{da}}}$$

$$\rho = \frac{1}{v}(1 + W) = \frac{1}{0.873}(1 + 0.01065) = 1.157 \frac{\text{kg}}{\text{m}^3}$$

Wet-bulb temperature

- Wet-bulb temperature is the T_{wb} that fits this equation:

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = 0.01065$$

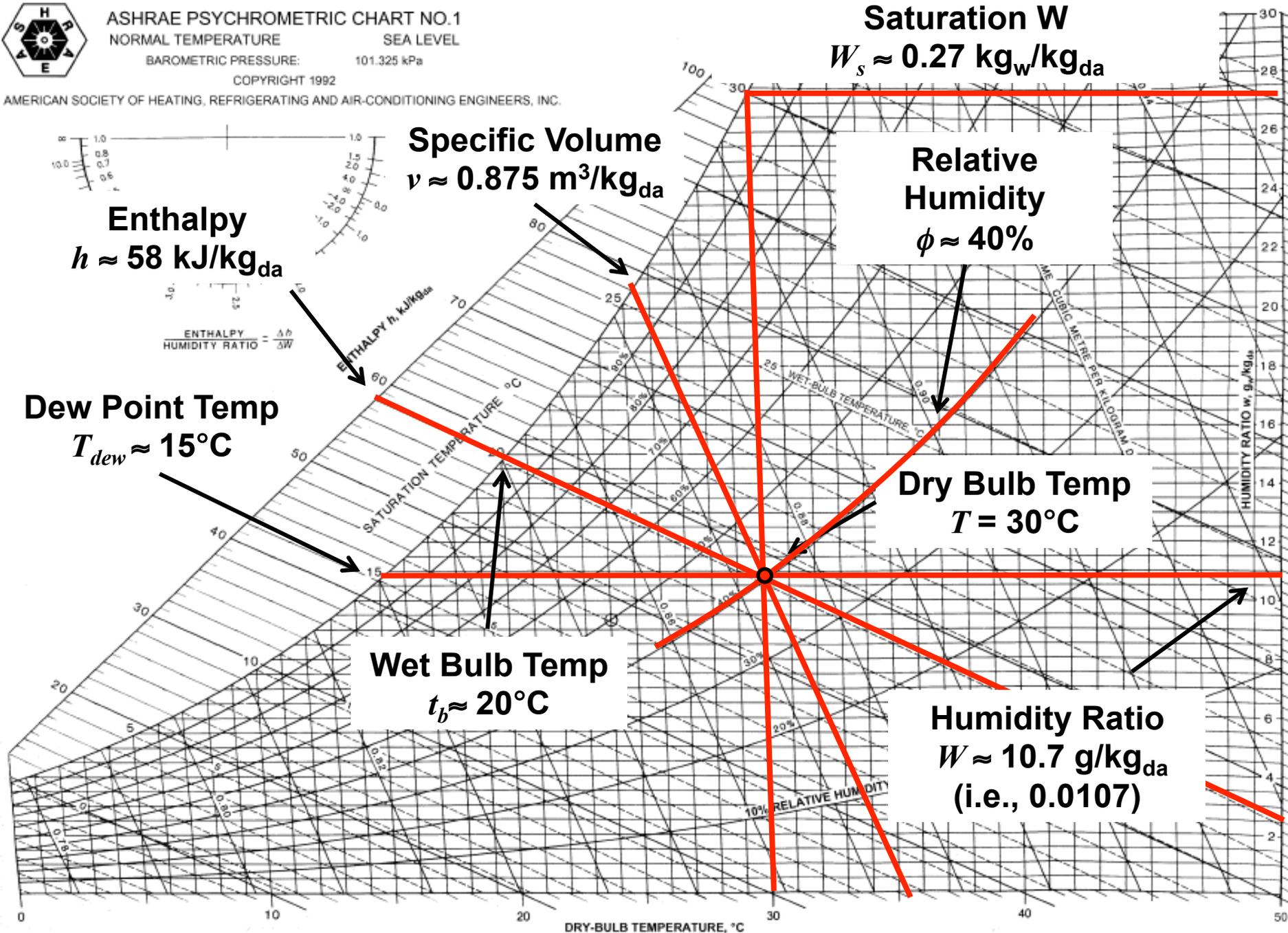
where: $T = 30^\circ\text{C}$
 $T_{wb} = ?^\circ\text{C}$

$$W_{s@T_{wb}=?} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T_{wb}=?}$$

Procedure:

- Guess T_{wb} , calculate p_{ws} for that T , calculate W_s for that T
 - Repeat until W calculated based on those values (and original T) in equation above is equal to actual W (0.01065 in our case)

$$T_{wb} = 20.1^\circ\text{C}$$



Dew Point Temp
 $T_{dew} \approx 15^\circ\text{C}$

Enthalpy
 $h \approx 58 \text{ kJ/kg}_{da}$

Specific Volume
 $v \approx 0.875 \text{ m}^3/\text{kg}_{da}$

Relative Humidity
 $\phi \approx 40\%$

Dry Bulb Temp
 $T = 30^\circ\text{C}$

Wet Bulb Temp
 $t_b \approx 20^\circ\text{C}$

Humidity Ratio
 $W \approx 10.7 \text{ g/kg}_{da}$
 (i.e., 0.0107)

Saturation W
 $W_s \approx 0.27 \text{ kg}_w/\text{kg}_{da}$

ENTHALPY HUMIDITY RATIO = $\frac{\Delta h}{\Delta W}$

DRY-BULB TEMPERATURE, °C

HUMIDITY RATIO $w, \text{ g}_w/\text{kg}_{da}$

30
28
26
24
22
20
18
16
14
12
10
8
6
4
2

IP units example

Moist air exists at 68°F dry-bulb temperature with 50% RH at sea level

Find the following using psychrometric equations (IP units):

- (a) the humidity ratio, W
- (b) the saturation humidity ratio, W_s
- (c) degree of saturation, μ
- (d) specific volume, v
- (e) density, ρ
- (f) enthalpy, h

Why is this stuff helpful?

PSYCHROMETRIC PROCESSES

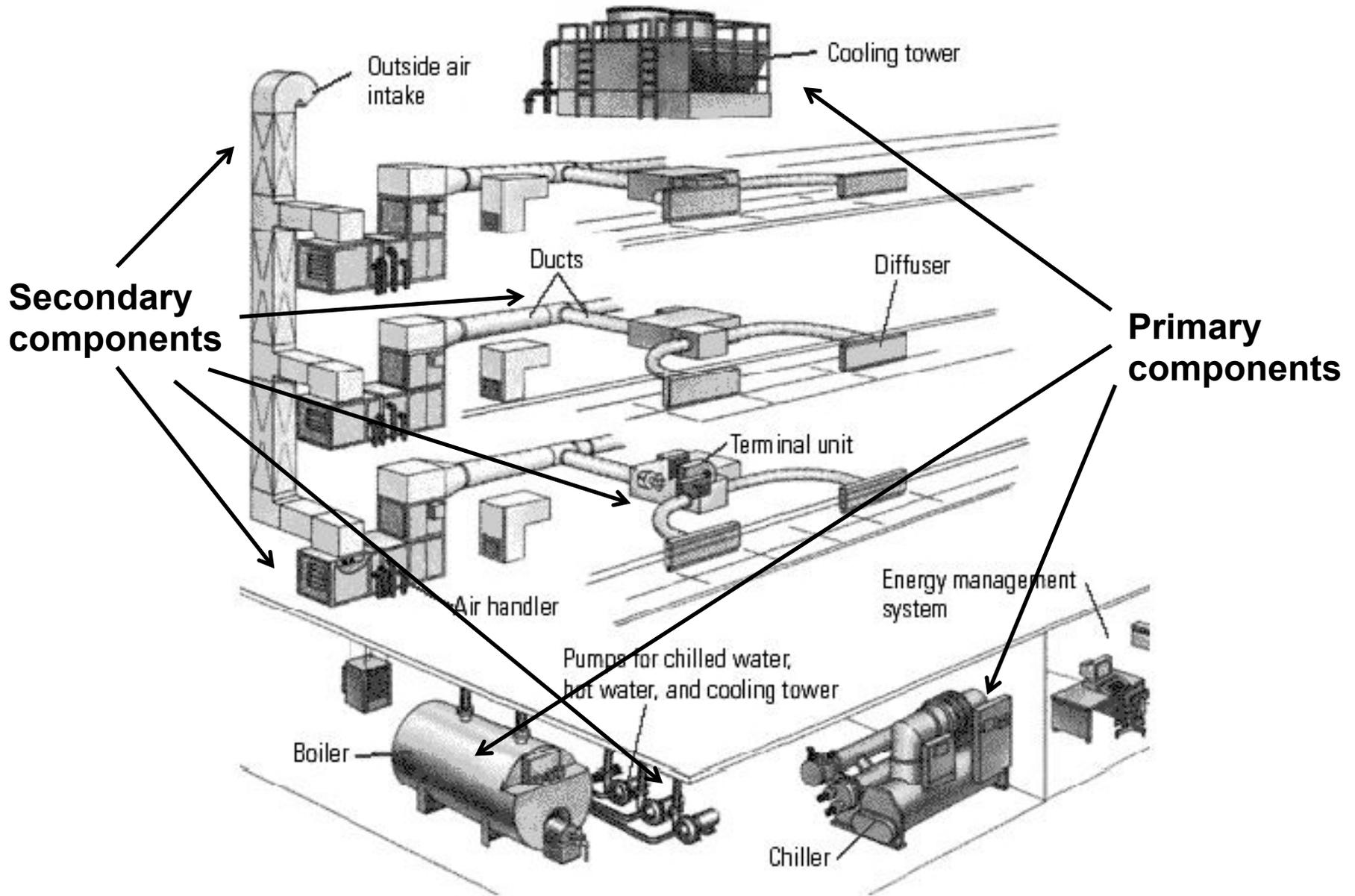
Use of the psychrometric chart for *processes*

We can use the psychrometric chart (and equations) not only to describe states of moist air, but for a number of processes that are important for building science and HVAC applications

Examples:

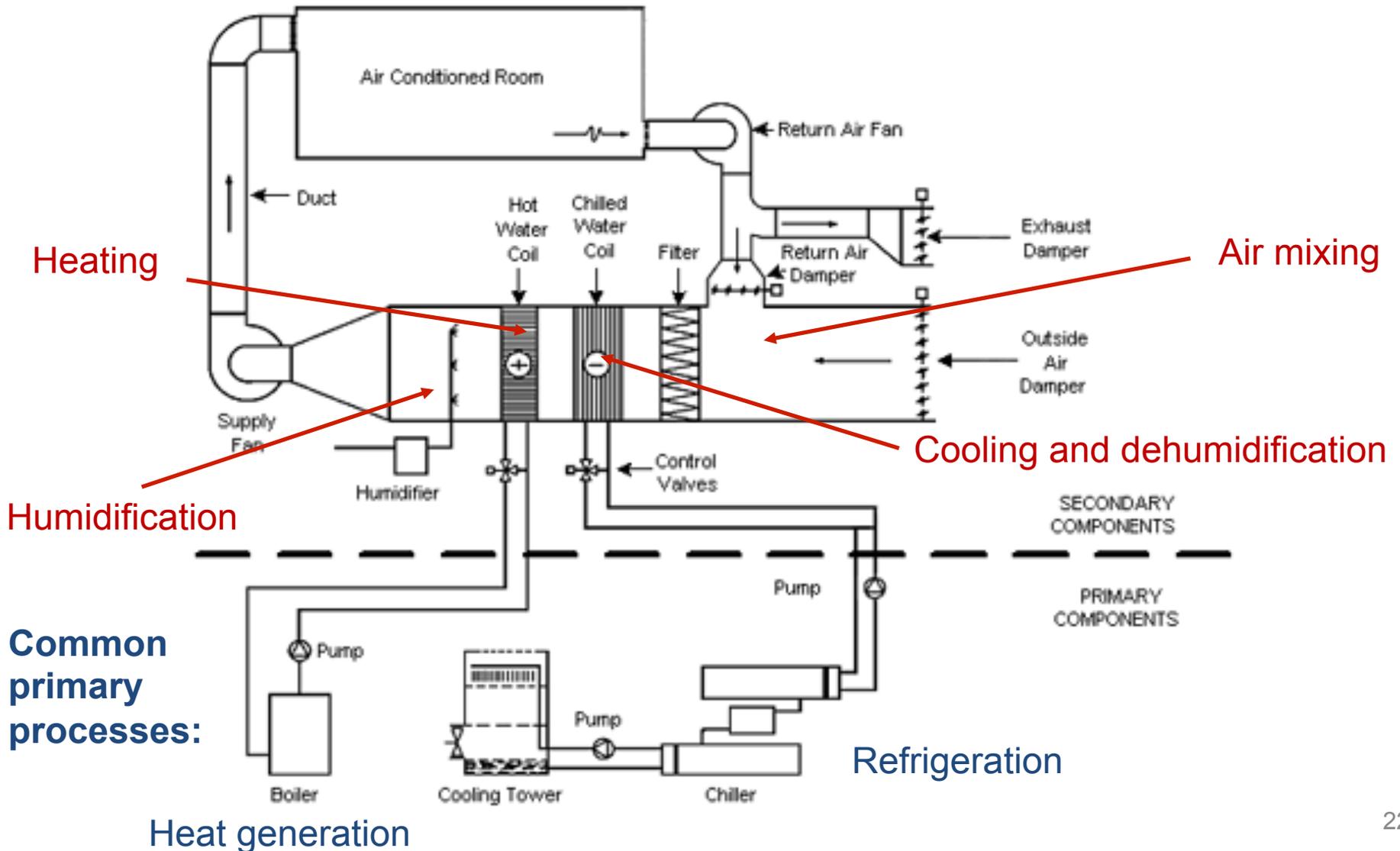
- Sensible cooling or heating
- Warming and humidification of cold, dry air
- Cooling and dehumidification of warm, humid air
 - Sensible + latent cooling
- Evaporative cooling
- Mixing of airstreams

Typical components of an HVAC system



Typical HVAC processes

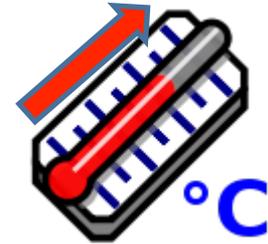
Some common psychrometric processes:



Definitions: Sensible and latent heat

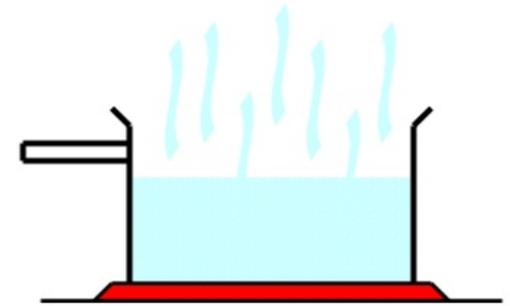
- **Sensible** heat transfer

- Increase or decrease in temperature of a substance *without* undergoing a phase change



- **Latent** heat transfer

- Heat transfer required to change the phase of a substance (e.g., heat required to change liquid to vapor)



$$Q_{total} = Q_{sensible} + Q_{latent}$$

Units of [W], [BTU/hr], or [ton]

Sensible and latent heat transfer equation

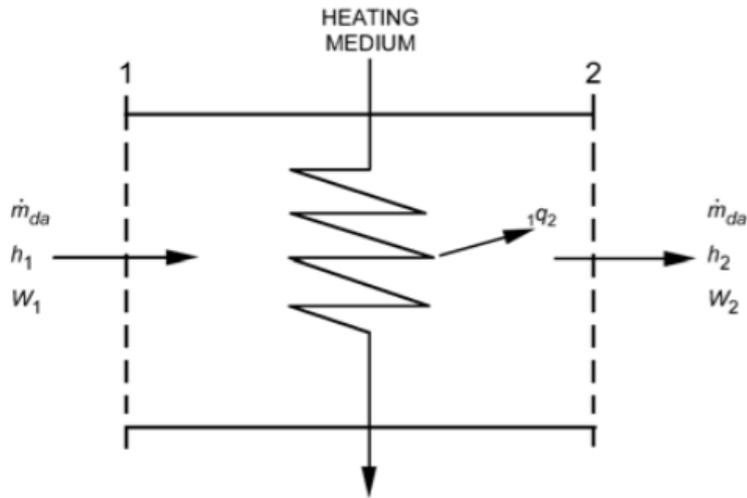


Fig. 2 Schematic of Device for Heating Moist Air

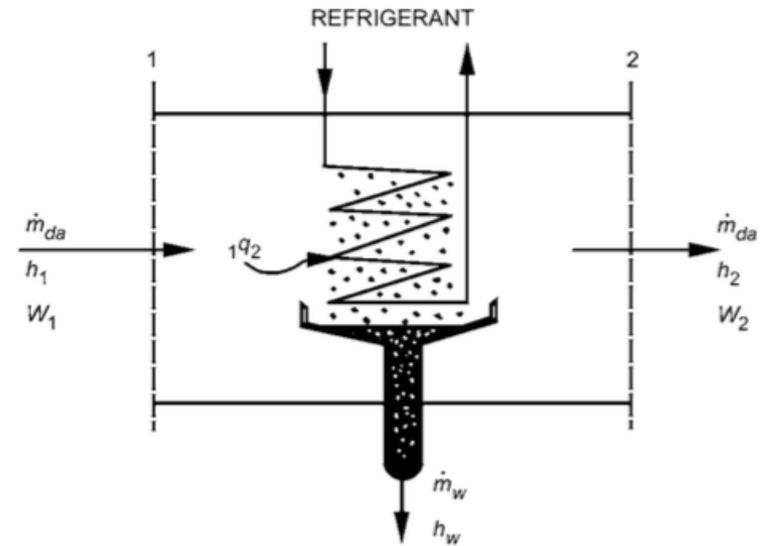


Fig. 3 Schematic of Device for Cooling Moist Air

Generic equations for both heating and cooling processes:

$$Q_{1 \rightarrow 2} = \dot{m}_{da} (h_2 - h_1) \quad Q_{total} = \dot{m}_{da} (h_{exit} - h_{inlet})$$

$Q_{1 \rightarrow 2}$ = total rate of heat transfer from state 1 to state 2 (W or BTU/hr or ton)

\dot{m}_{da} = mass flow rate of dry air (kg_{da}/s or lb_{da}/hr)

$h_{exit,2}$ = enthalpy at the exit (J/kg_{da} or BTU/lb_{da})

$h_{inlet,1}$ = enthalpy at the inlet (J/kg_{da} or BTU/lb_{da})

Sensible heat transfer equation

$$Q_{sensible} = \dot{m}_{da} C_p (T_{exit} - T_{inlet}) = \rho_{da} \dot{V}_{da} C_p (T_{exit} - T_{inlet})$$

$Q_{sensible}$ = rate of sensible heat transfer (W or BTU/hr or ton)

C_p = specific heat of air (J/kgK or BTU/lb°F)

ρ_{da} = dry air density (kg/m³ or lb/ft³)

T_{inlet} = inlet temperature (K or °F)

T_{exit} = exit temperature (K or °F)

For heating: $Q_{sensible} > 0$

For cooling: $Q_{sensible} < 0$



Latent heat transfer equation

$$Q_{latent} = \dot{m}_{da} h_{fg} (W_{exit} - W_{inlet}) = \rho_{da} \dot{V}_{da} h_{fg} (W_{exit} - W_{inlet})$$

Q_{latent} = rate of latent heat transfer (W or BTU/hr or ton)

m_w = mass flow rate of water vapor (kg_w/s or lb_w/hr)

h_{fg} = enthalpy, or latent heat, of vaporization (J/kg or BTU/lb)

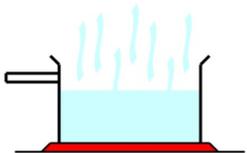
* h_{fg} = 2260 kJ/kg or 970 BTU/lb for water

W_{inlet} = inlet humidity ratio (kg_w/kg_{da} or lb_w/lb_{da})

W_{exit} = exit humidity ratio (kg_w/kg_{da} or lb_w/lb_{da})

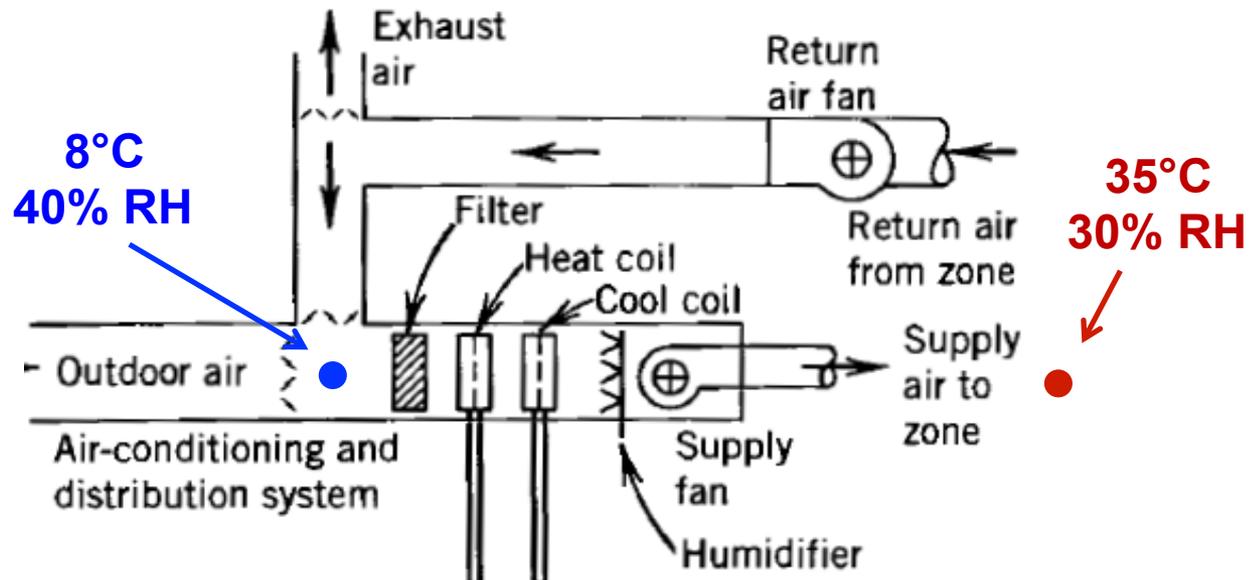
For humidification: $Q_{latent} > 0$

For dehumidification: $Q_{latent} < 0$



Heating and humidification of cold, dry air

- **Example:** Heating and humidification of air
 - Process: Adding moisture and heat (sensible + latent heating)



Q1: What is the enthalpy change required?

Q2: What is the total rate of heat transfer if the airflow rate is $10 \text{ m}^3/\text{s}$?

Q2: What is the split between sensible and latent transfer?



ASHRAE PSYCHROMETRIC CHART NO.1

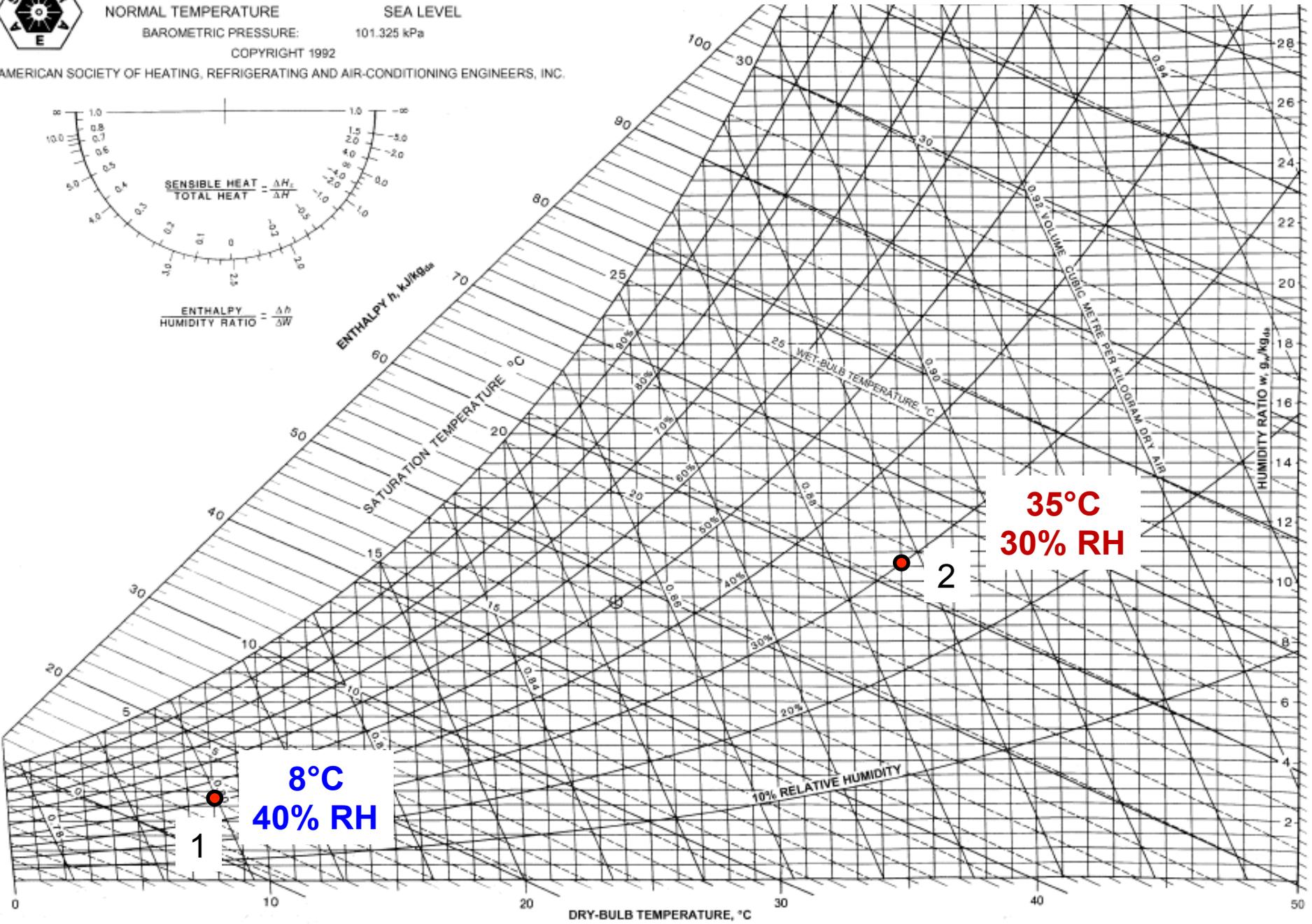
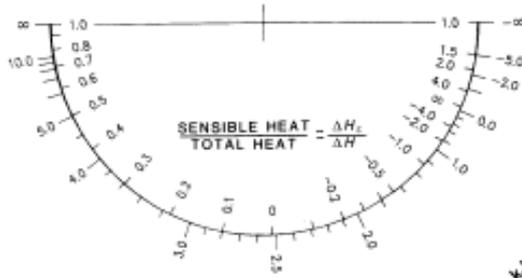
NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

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Heating and humidification of cold, dry air



1

8°C
40% RH

2

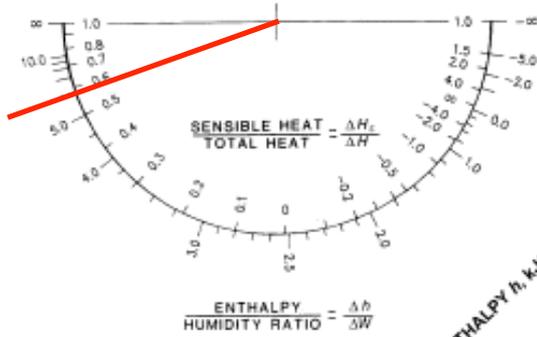
35°C
30% RH



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 BAROMETRIC PRESSURE: 101.325 kPa
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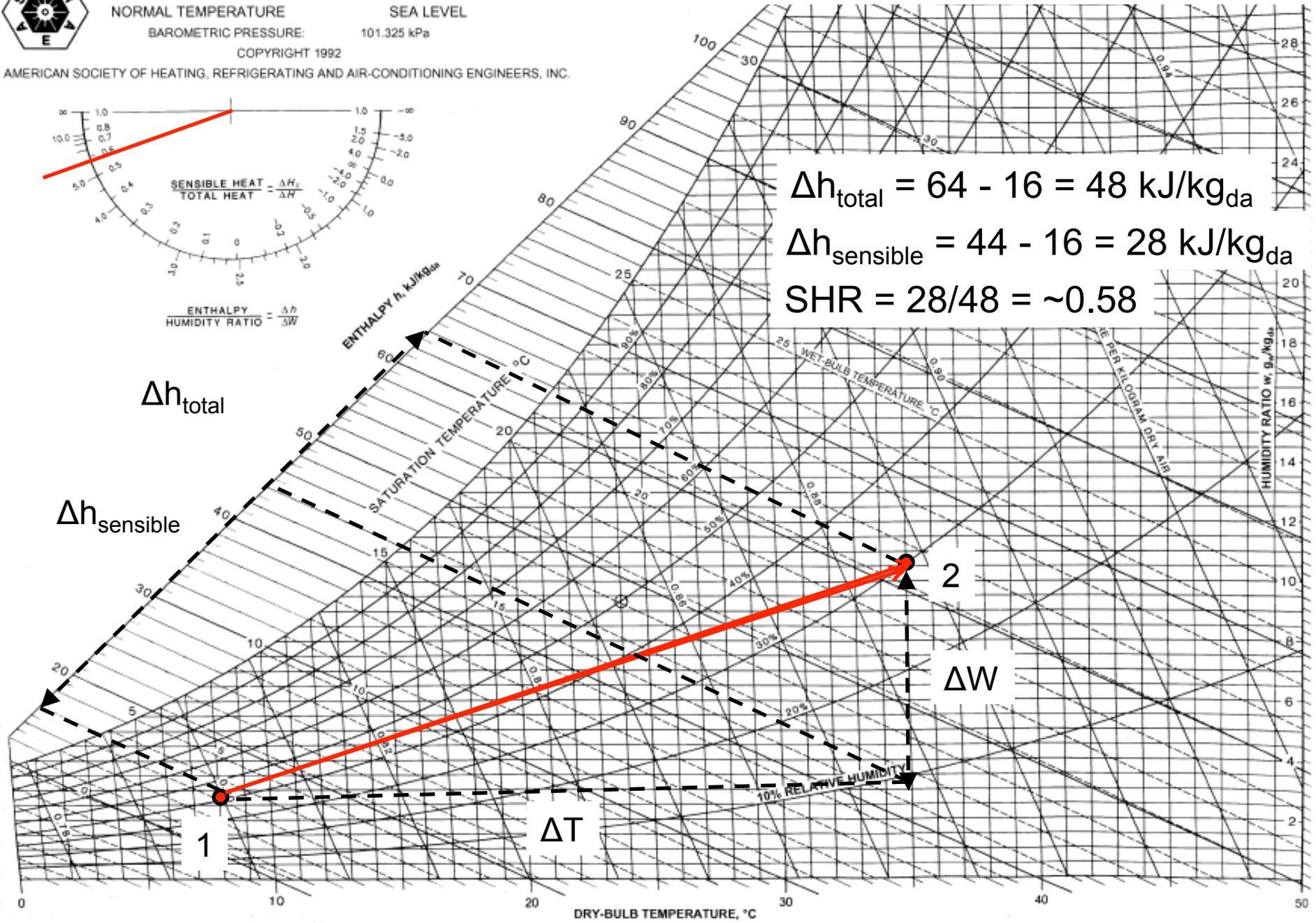
Heating and humidification of cold, dry air



$$\Delta h_{\text{total}} = 64 - 16 = 48 \text{ kJ/kg}_{\text{da}}$$

$$\Delta h_{\text{sensible}} = 44 - 16 = 28 \text{ kJ/kg}_{\text{da}}$$

$$\text{SHR} = 28/48 = \sim 0.58$$



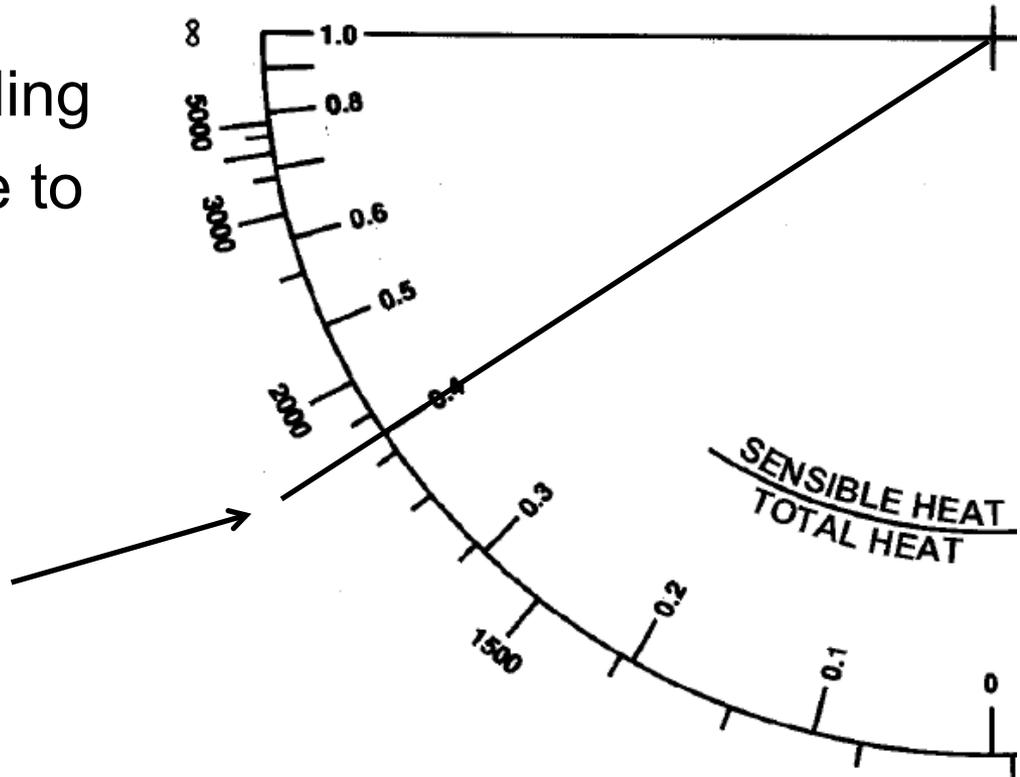
Sensible heat ratio (SHR)

- The sensible heat ratio is defined as:

$$SHR = \frac{\dot{q}_{sens}}{\dot{q}_{total}} = \frac{\dot{q}_{sens}}{\dot{q}_{sens} + \dot{q}_{latent}} = \frac{\Delta h_{sens}}{\Delta h_{total}}$$

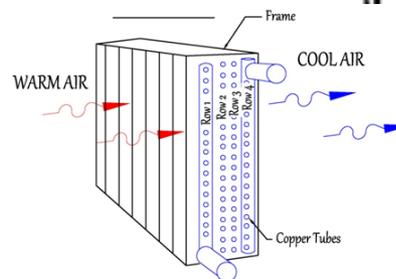
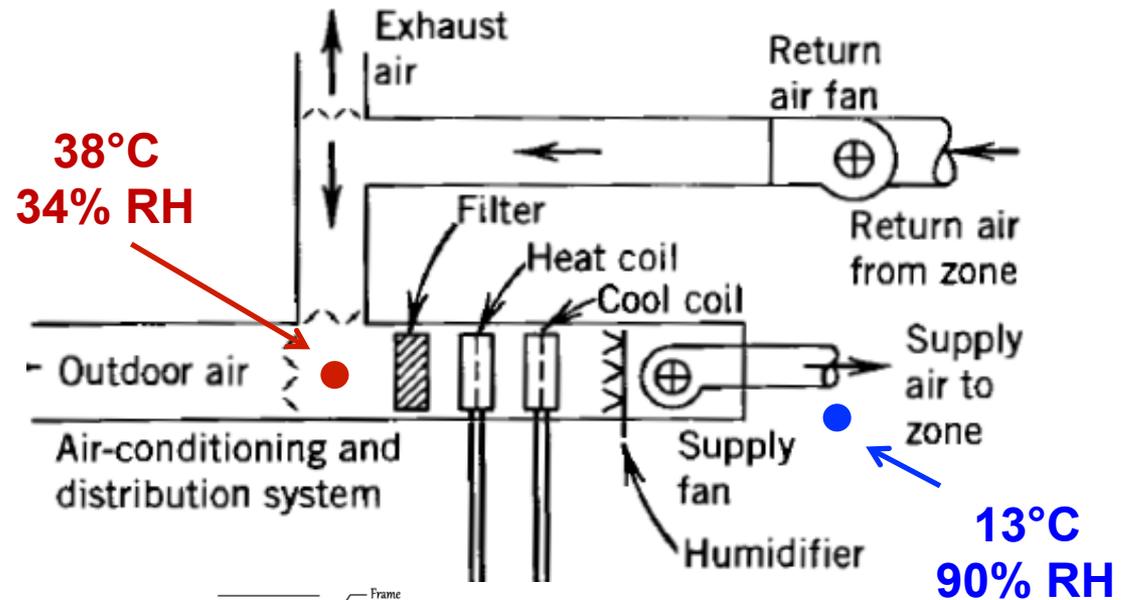
- Allows for understanding sensible load relative to latent load

Here is a process
with an SHR ≈ 0.4



Cooling and dehumidification of **warm**, humid air

- **Example:** Air flowing over a cooling coil
- Removing both moisture and heat
 - Sensible + latent **cooling**

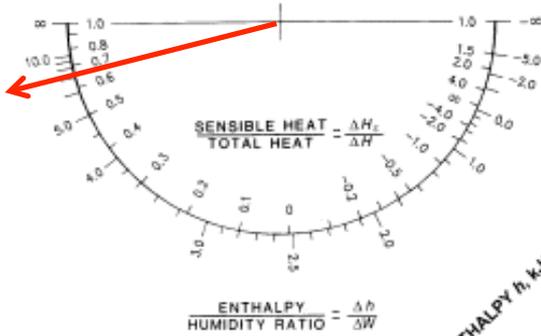




ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Cooling and dehumidification of warm, humid air

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SHR ~ 0.61

Δh

38°C
34% RH

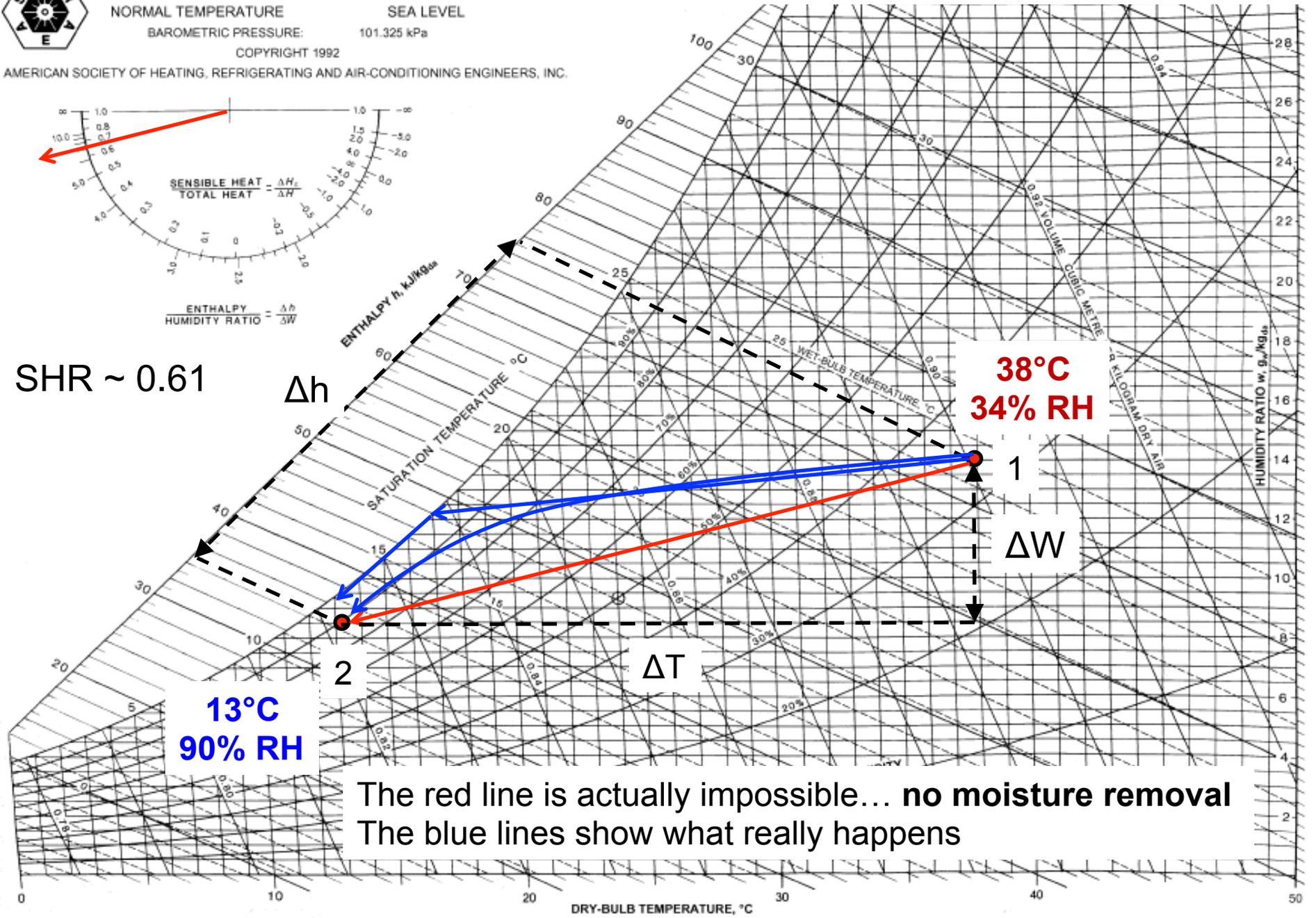
1
 ΔW

13°C
90% RH

2

ΔT

The red line is actually impossible... no moisture removal
 The blue lines show what really happens



Example: *Sensible* cooling

- Moist air is cooled from 40°C and 30% RH to 30°C without condensation
 - What is the RH at W at the process end point?



ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

SEA LEVEL

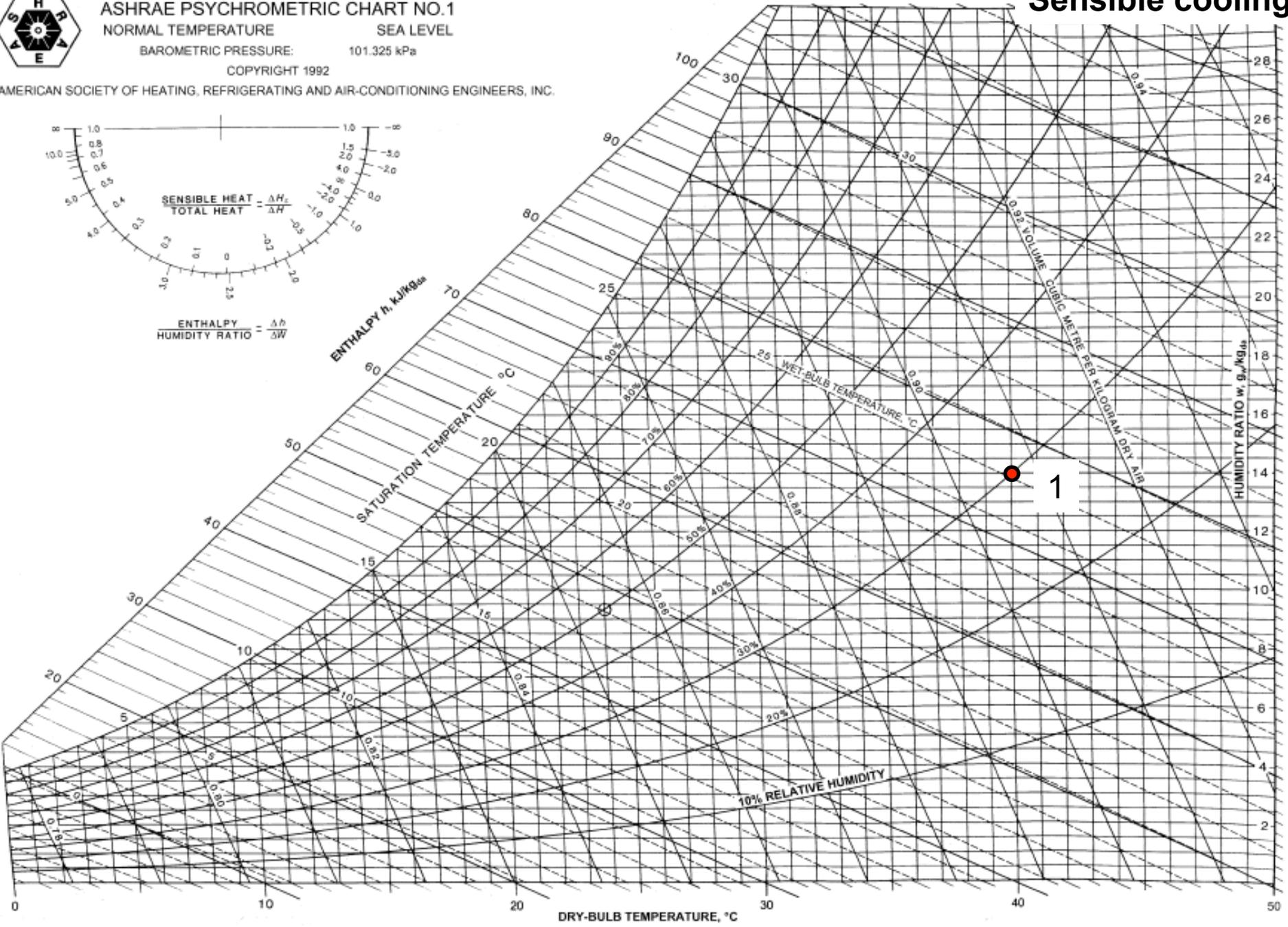
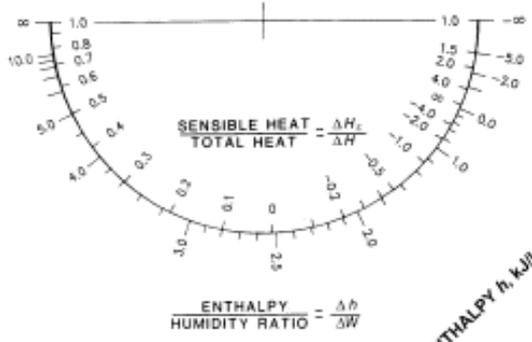
BAROMETRIC PRESSURE:

101.325 kPa

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Sensible cooling

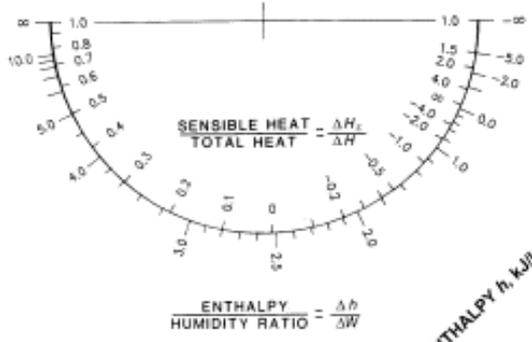


1

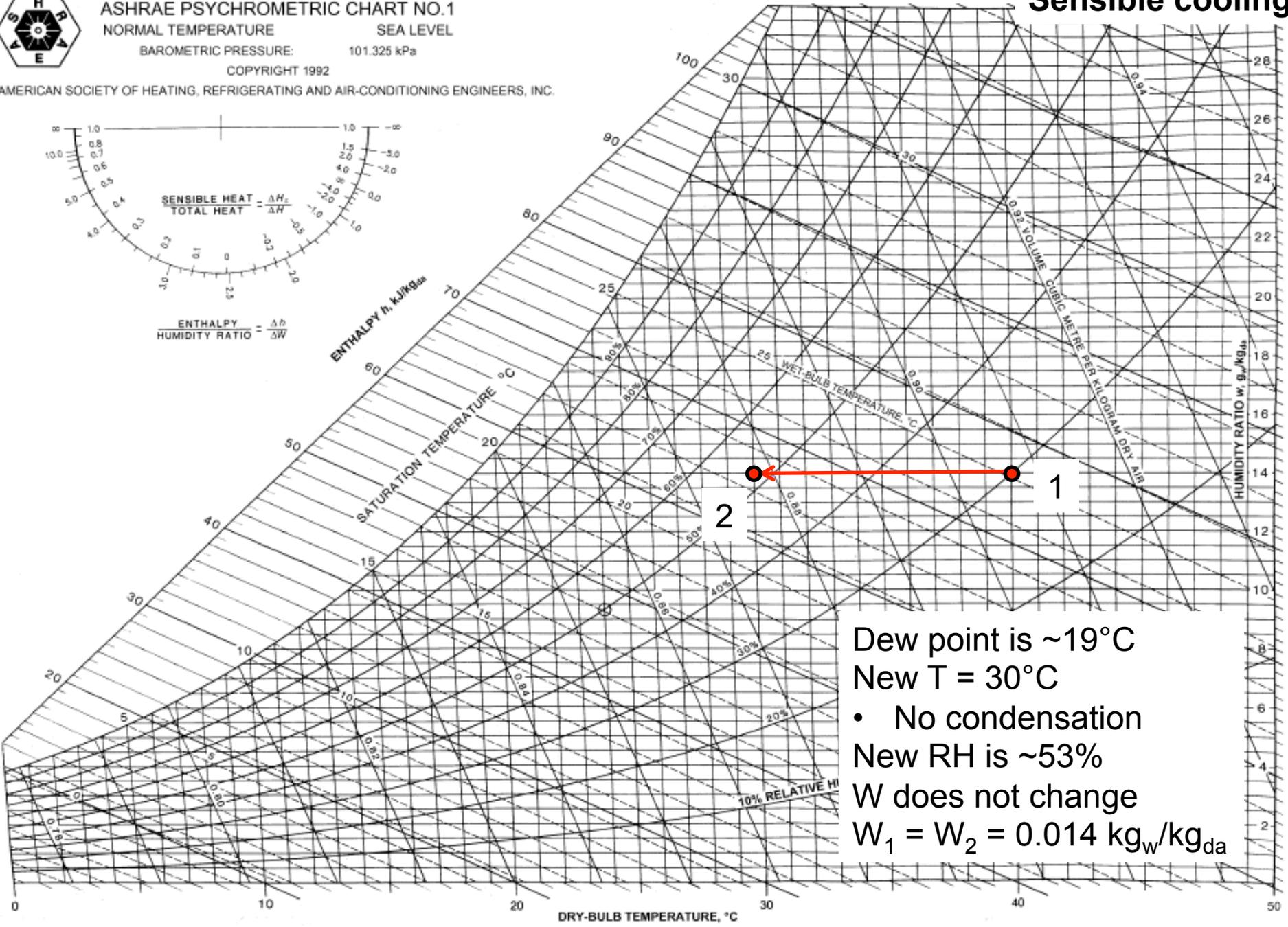


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 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Sensible cooling



Dew point is $\sim 19^\circ\text{C}$
 New $T = 30^\circ\text{C}$

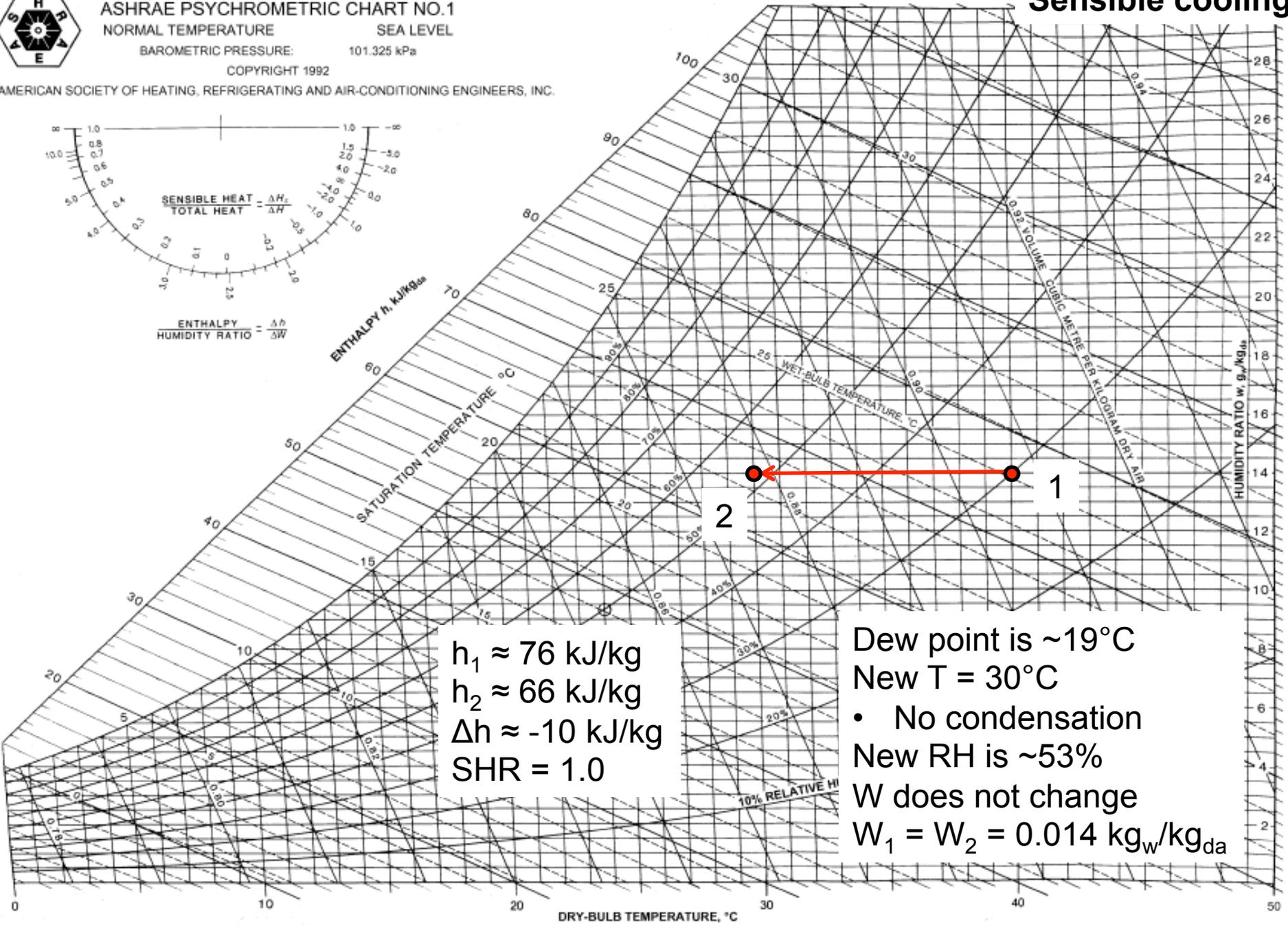
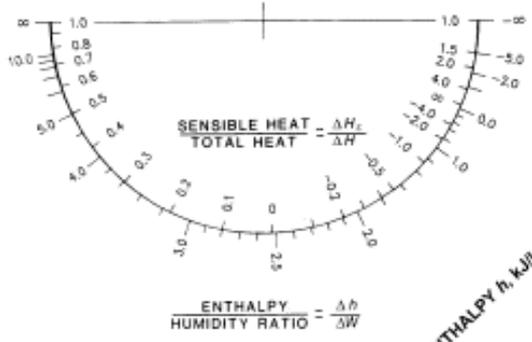
- No condensation
- New RH is $\sim 53\%$
- W does not change
- $W_1 = W_2 = 0.014 \text{ kg}_w/\text{kg}_{da}$



ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE
 SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
 COPYRIGHT 1992

Sensible cooling

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$h_1 \approx 76 \text{ kJ/kg}$
 $h_2 \approx 66 \text{ kJ/kg}$
 $\Delta h \approx -10 \text{ kJ/kg}$
 $\text{SHR} = 1.0$

Dew point is $\sim 19^\circ\text{C}$
 New $T = 30^\circ\text{C}$
 • No condensation
 New RH is $\sim 53\%$
 W does not change
 $W_1 = W_2 = 0.014 \text{ kg}_w/\text{kg}_{da}$

Example: *Sensible + latent cooling*

- Moist air is cooled from 40°C and 30% RH to 15°C
 - Q1: Does the water vapor condense?
 - Q2: What is RH at W at the process end point?



ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

SEA LEVEL

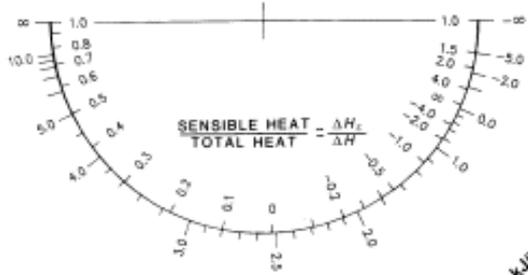
BAROMETRIC PRESSURE:

101.325 kPa

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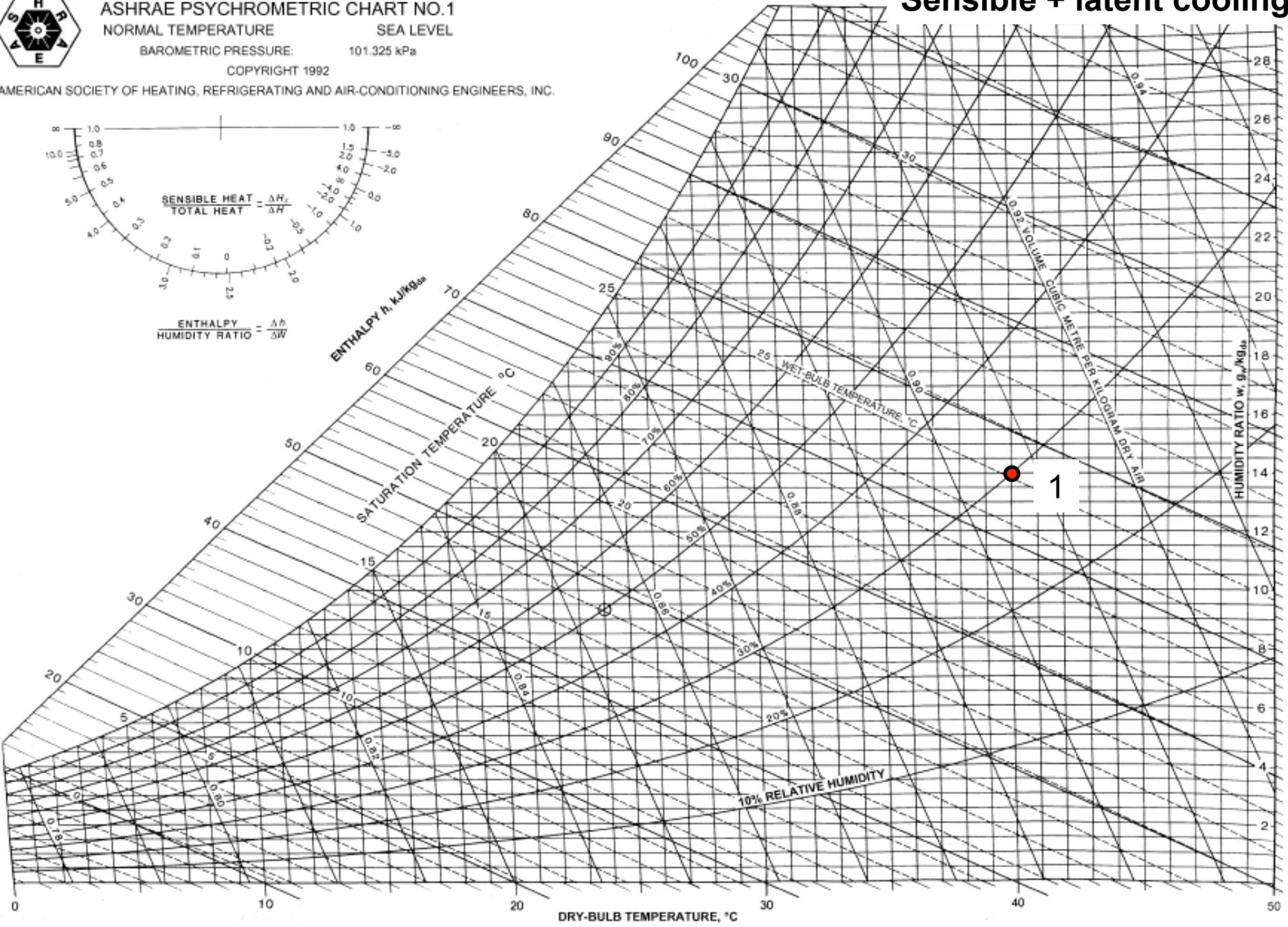
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Sensible + latent cooling



$$\frac{\text{SENSIBLE HEAT}}{\text{TOTAL HEAT}} = \frac{\Delta H_s}{\Delta H}$$

$$\frac{\text{ENTHALPY}}{\text{HUMIDITY RATIO}} = \frac{\Delta h}{\Delta W}$$



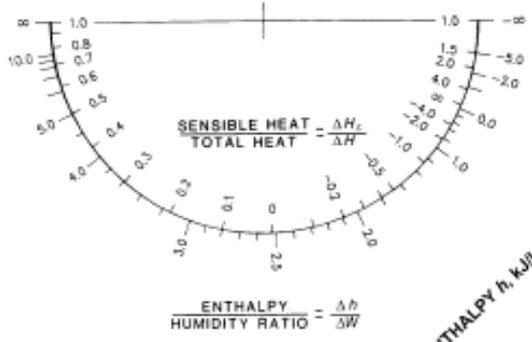
1



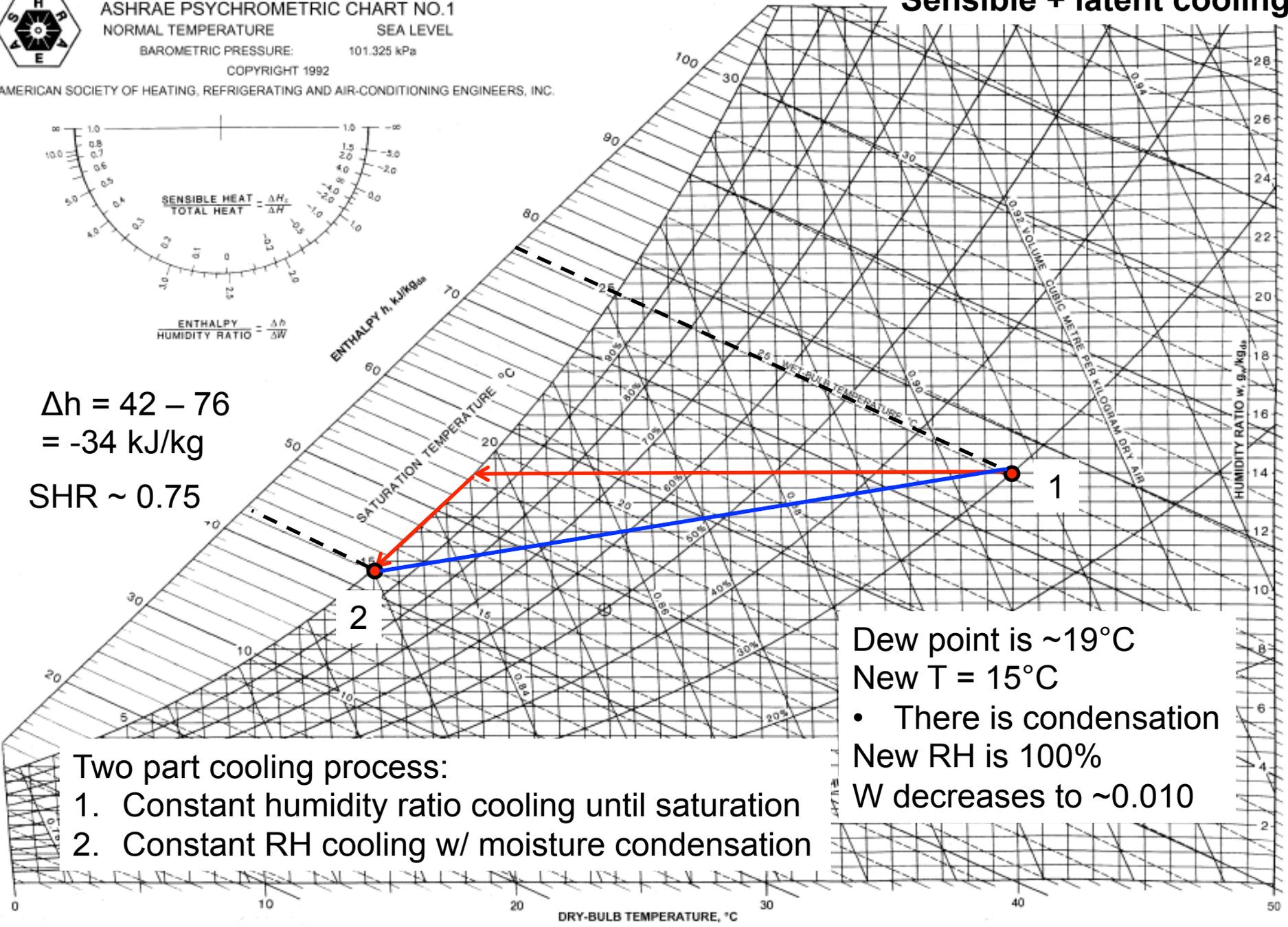
ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Sensible + latent cooling

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.



$\Delta h = 42 - 76$
 $= -34 \text{ kJ/kg}$
 SHR ~ 0.75



Dew point is $\sim 19^\circ\text{C}$
 New $T = 15^\circ\text{C}$
 • There is condensation
 New RH is 100%
 W decreases to ~ 0.010

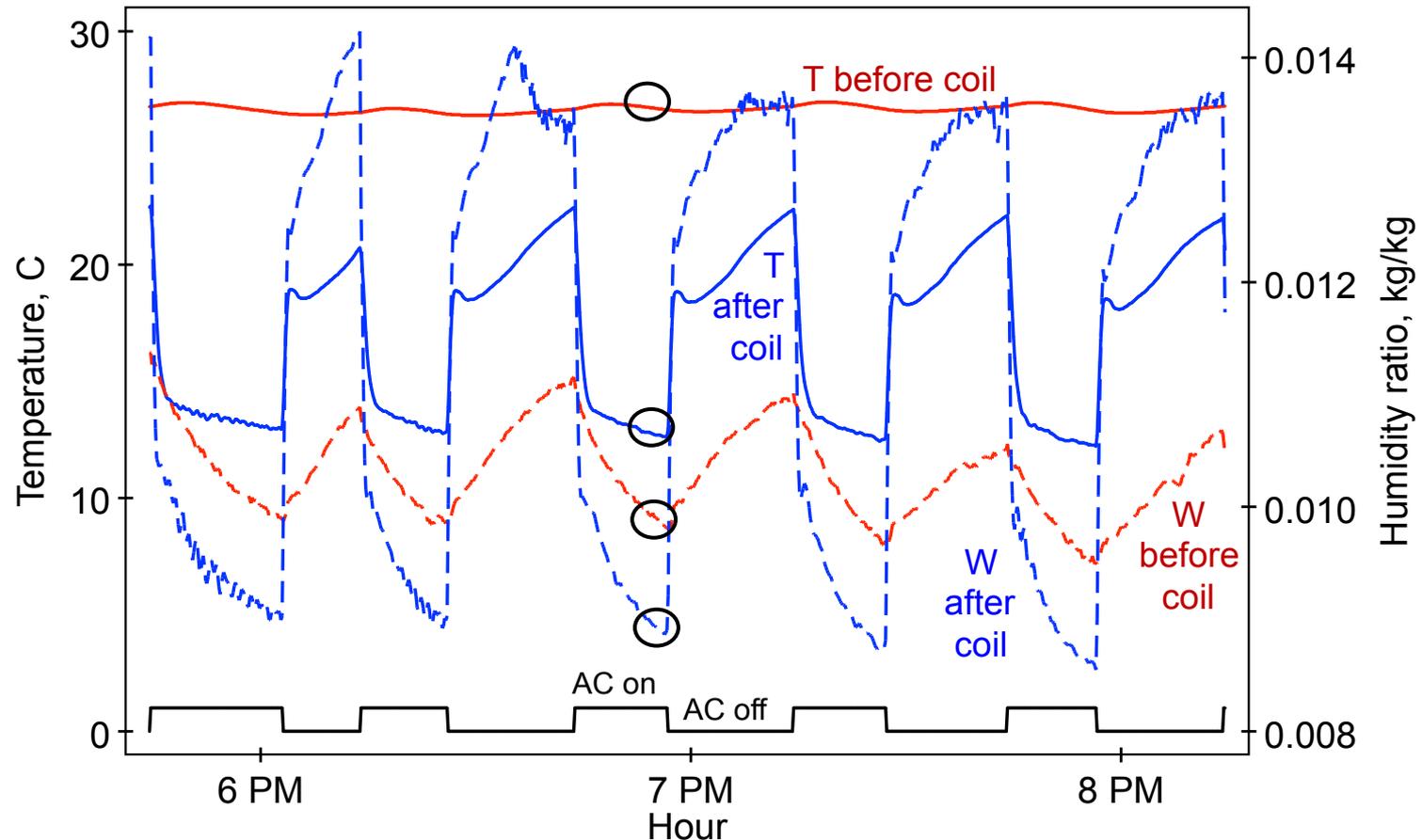
- Two part cooling process:
1. Constant humidity ratio cooling until saturation
 2. Constant RH cooling w/ moisture condensation

DRY-BULB TEMPERATURE, °C

Real data: ASHRAE RP-1299

Energy implications of filters

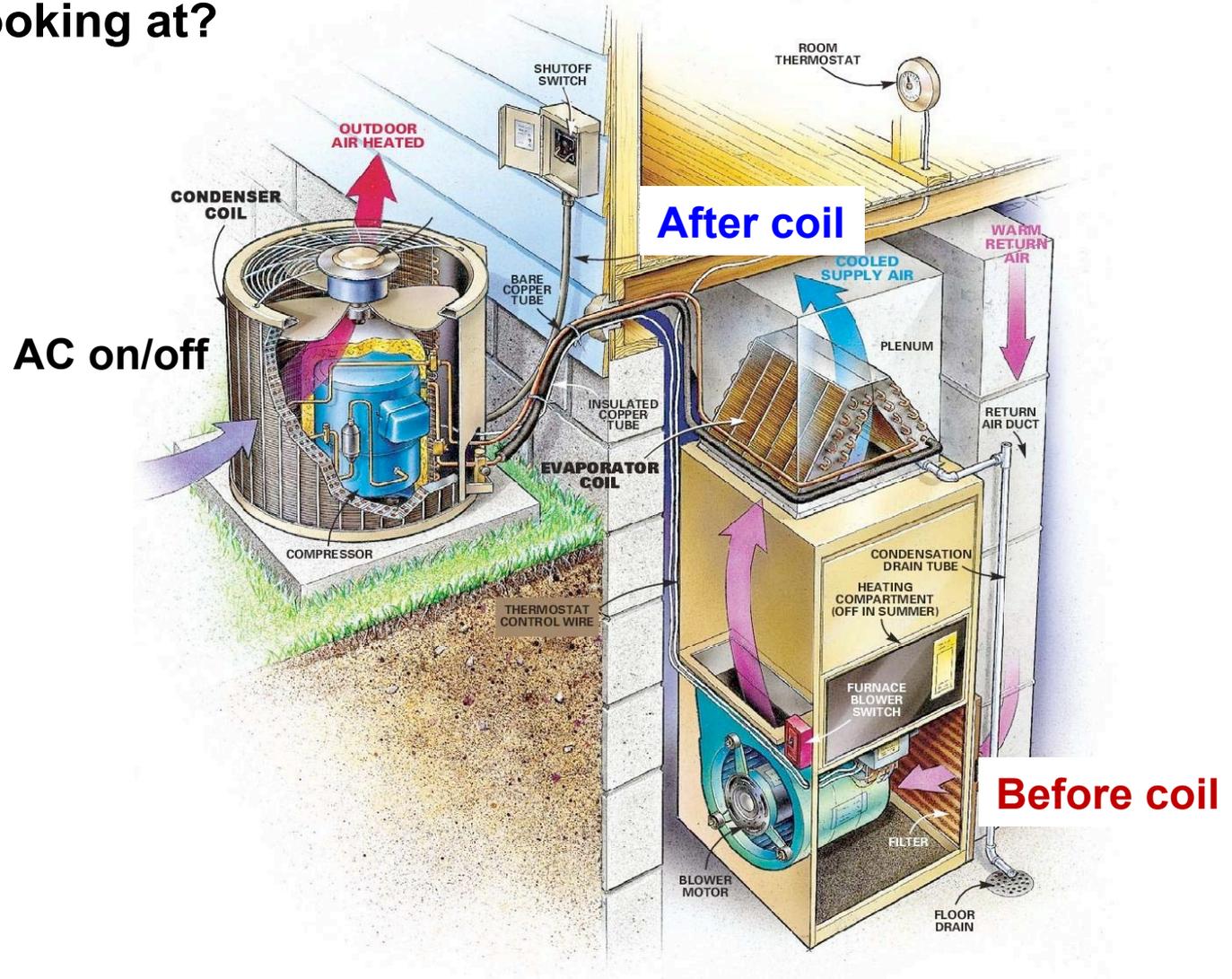
Temperature and humidity ratio differences across AC coils in homes



Real data: ASHRAE RP-1299

Energy implications of filters

What are we looking at?

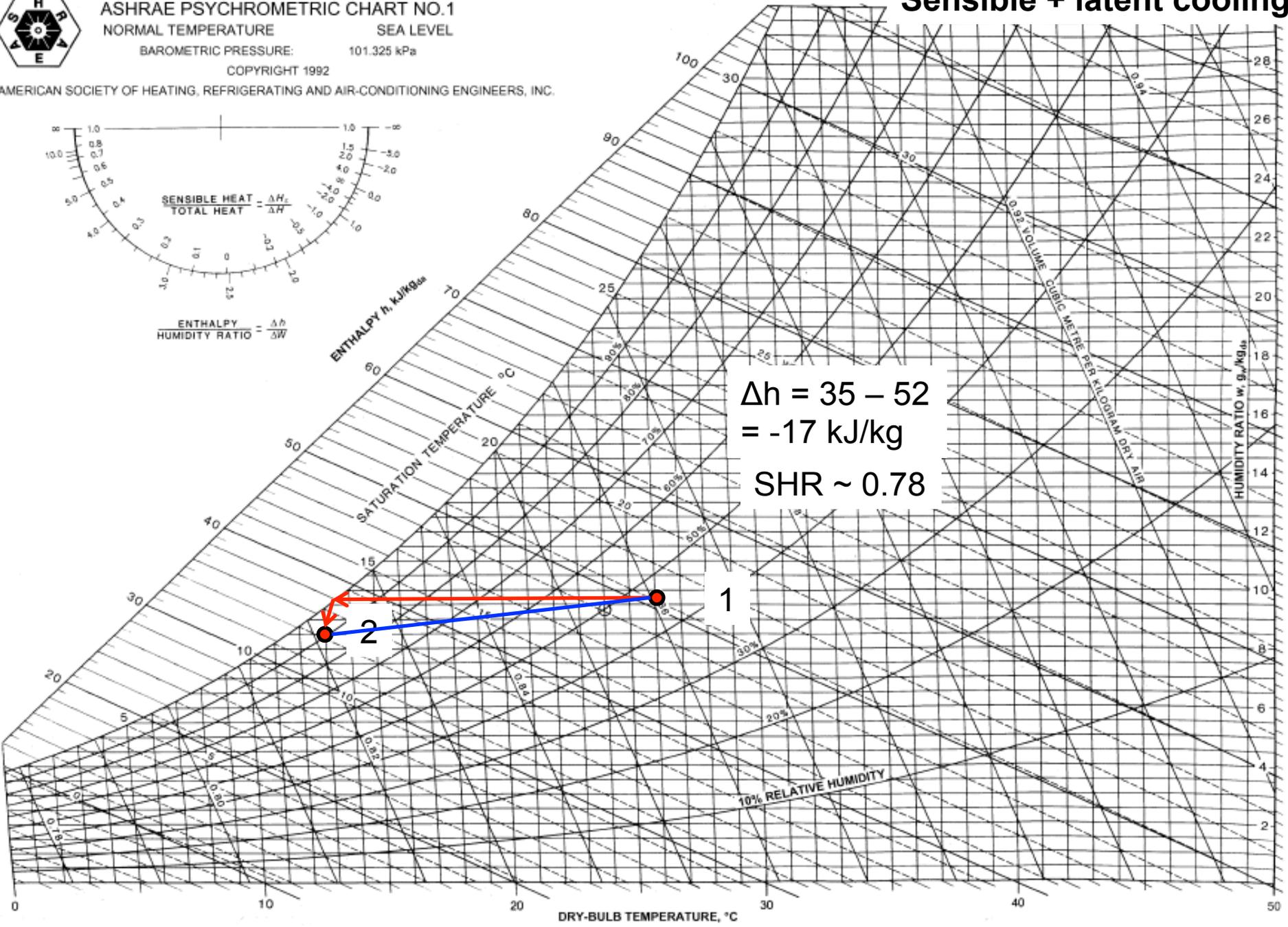
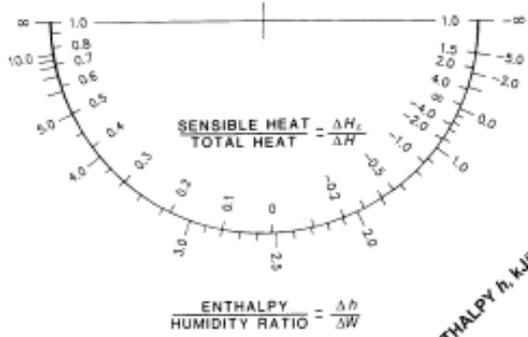




ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Sensible + latent cooling

AMERICAN SOCIETY OF HEATING, REFRIGERATING AND AIR-CONDITIONING ENGINEERS, INC.



$\Delta h = 35 - 52$
 $= -17 \text{ kJ/kg}$
 SHR ~ 0.78

1

2

10% RELATIVE HUMIDITY