CAE 208 / MMAE 320: Thermodynamics Fall 2023

November 28, 2023 Entropy (4) and vapor compression cycles (1)

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ANNOUNCEMENTS

Announcements

- Assignment 10 is posted (Due 12/01/23 for those who need to submit it).
- The bonus activities document is posted. Please pay attention to the deadlines and also the updates about this task (an idea submission by the 11/29/2023 is required)

RECAP

Use the first T-ds (or Gibbs) equation to solve for entropy changes



• Approach 1: Constant Specific Heats (Approximate Analysis):

$$s_2 - s_1 = c_{\nu,a\nu g} \ln\left(\frac{T_2}{T_1}\right) + R \times \ln\left(\frac{\nu_2}{\nu_1}\right)$$



$$s_2 - s_1 = c_{p,avg} \times \ln\left(\frac{T_2}{T_1}\right) - R \times \ln\left(\frac{P_2}{P_1}\right)$$

Recap

• Approach 2: Variable Specific Heats (Exact Analysis):

$$s^0 = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_0^T c_p(T) \frac{dT}{T} = s_2^0 - s_1^0$$

$$s_2 - s_1 = s_2^0 - s_1^0 - R \times \ln(\frac{P_2}{P_1})$$

$$\overline{s_2} - \overline{s_1} = \overline{s_2^0} - \overline{s_1^0} - R_u \times \ln(\frac{P_2}{P_1})$$

• For Isentropic Processes of Ideal Gases, we have:

 $Tv^{k-1} = Constant$ $TP^{\frac{1-k}{k}} = Constant$ $Pv^{k} = Constant$

$$\begin{pmatrix} T_2 \\ T_1 \end{pmatrix}_{s=\text{const.}} = \begin{pmatrix} P_2 \\ P_1 \end{pmatrix}^{(k-1)/k} = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}^{k-1}$$

*ideal gas
*ideal gas
*isentropic process
*constant specific heats

CLASS ACTIVITY

Air enters an isentropic turbine at 150 psia and 900 °F through a 0.5 ft² inlet section with a velocity of 100 ft/s. It leaves at 15 psia with a velocity of 500 ft/s. Calculate the air temperature at the turbine exit and the power produced, in hp, by this turbine.

- Solution (assumptions):
 - □ Steady flow
 - □ The process is isentropic (both reversible and adiabatic)
 - □ Ideal gas with a constant specific heat



• Solution (Tables): \Box Table A-2Eb: @900 °F $\rightarrow c_p = 0.259 \frac{Btu}{lbm-R}$ and k = 1.358

Temp., °F	c_p Btu/lbm $\cdot \mathbb{R}$	$c_{\rm v}$ Btu/lbm $\cdot{\rm R}$	k	c_p Btu/lbm $\cdot {\rm R}$	c_{v} Btu/lbm \cdot R	k
		Air		Carbo	on dioxide, CO ₂	
40	0.240	0.171	1.401	0.195	0.150	1.300
100	0.240	0.172	1.400	0.205	0.160	1.283
200	0.241	0.173	1.397	0.217	0.172	1.262
300	0.243	0.174	1.394	0.229	0.184	1.246
400	0.245	0.176	1.389	0.239	0.193	1.233
500	0.248	0.179	1.383	0.247	0.202	1.223
600	0.250	0.182	1.377	0.255	0.210	1.215
700	0.254	0.185	1.371	0.262	0.217	1.208
800	0.257	0.188	1.365	0.269	0.224	1.202
900	0.259	0.191	1.358	0.275	0.230	1.197
1000	0.263	0.195	1.353	0.280	0.235	1.192
1500	0.276	0.208	1.330	0.298	0.253	1.178
2000	0.286	0.217	1.312	0.312	0.267	1.169

• Solution (Tables): \Box Table A-1E: $R = 0.3704 \frac{psia-ft^3}{lbm-R}$

TABLE A-1E								
Molar mass, gas constant, and critical-point properties								
	Formula	Molar mass M	Gas c	onstant, R*	Critical-point properties			
Substance		lbm/lbmol	Btu/lbm · R	psia · ft ³ /lbm · R	Temperature, R	Pressure, psia	Volume, ft ³ /lbmol	
Air	-	28.97	0.06855	0.3704	238.5	547	1.41	

• Solution (Problem solving):

 $\dot{m} = \dot{m}_1 = \dot{m}_2$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{d\dot{E}_{system}}{dt} = 0$$

$$\dot{m}(h_1 + V_1^2) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m} \left(h_1 - h_1 + \frac{V_1^2 - V_2}{2} \right)$$



• Solution (Calculations):

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \to T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (900 + 460 R) \left(\frac{15 \text{ psia}}{150 \text{ psia}}\right)^{\frac{1.358-1}{1.358}} = 741 R$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.3704\frac{psia - ft^3}{lbm - R}\right)(900 + 460\,R)}{150\,psia} = 3.358\frac{ft^3}{lbm}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(0.5 f t^2) \left(500 \frac{f t}{s}\right)}{3.358 \frac{f t^3}{l b m}} = 74.45 \frac{l b m}{s}$$

• Solution (Calculations):

$$\dot{W}_{out} = \dot{m} \left(h_1 - h_1 + \frac{V_1^2 - V_2}{2} \right)$$

$$\dot{W}_{out} = \left(74.45 \frac{lbm}{s}\right) \left[\left(0.250 \frac{Btu}{lbm - R}\right) (1360 - 724R) + \left(\frac{\left(500 \frac{ft}{s}\right)^2}{2} - \frac{\left(100 \frac{ft}{s}\right)^2}{2}\right) \left(\frac{1 \frac{Btu}{lbm}}{25.037} ft^2 - \frac{100 \frac{ft}{s}}{s^2}\right) \right] \left(\frac{1 \frac{Btu}{lbm}}{s^2} - \frac{100 \frac{ft}{s}}{s^2}\right) \left(\frac{1 \frac{Btu}{lbm}}{s^2} - \frac{100 \frac{ft}$$

$$\dot{W}_{out} = 12,194 \frac{Btu}{s} \left(\frac{1 \ hp}{0.7068 \frac{Btu}{s}} \right) = 17,250 \ hp$$

REVERSIBLE STEADY-FLOW WORK

Reversible Steady-Flow Work

• Recall we had this for the closed system:

$$W_b = \int_1^2 P \ dV$$

• For steady flow:

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

$$\longrightarrow -\delta w_{rev} = vdP + dke + dpe$$

$$\begin{cases} \delta q_{rev} = Tds \\ Tds = dh - vdP \end{cases} \rightarrow \delta q_{rev} = dh - vdP$$

$$w_{rev} = -\int_{1}^{2} v dP - \Delta ke - \Delta pe$$

Reversible Steady-Flow Work

• For steady flow:

$$w_{rev} = -\int_{1}^{2} v dP - \Delta ke - \Delta pe$$



(a) Steady-flow system



ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

 Steady-flow devices deliver the most and consume the least work when the process is reversible:



Isentropic Efficiencies

• Isentropic efficiency of a turbine can be written as:

$$\eta_T = \frac{Actual \ turbine \ work}{Isentropic \ turbine \ work} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$





Isentropic efficiency of compressors and pumps

$$\eta_{C} = \frac{Isentropic\ compressor\ work}{Actual\ compressor\ work} = \frac{w_{s}}{w_{a}}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$



Isentropic Efficiencies

Isentropic efficiency of nozzles

$$\eta_N = \frac{Actual \ KE \ nozzle \ exit}{Isentropic \ KE \ at \ nozzle \ exit} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



CLASS ACTIVITY

- Steam enters an adiabatic turbine steadily at 3 MPa and 400 C and leaves at 50 kPa and 100 C. If the power output of the turbine is 2 MW, determine:
 - a) The isentropic efficiency of the turbine
 - b) The mass flow rate of the steam flowing through the turbine

- Solution (assumptions):
 - □ Steady operating conditions exist
 - □ The kinetic and potential energies are negligible





• Solution (Tables):

State 1:
$$\begin{cases} P_1 = 3 MPa \\ T_1 = 400 \ ^{\circ}C \end{cases} \rightarrow \begin{cases} h_1 = 3231.7 \ kJ/kg \\ s_1 = 6.9235 \ kJ/(kg - K) \end{cases}$$

State 2a:
$$\begin{cases} P_1 = 50 \ kPa \\ T_{2a} = 100 \ ^{\circ}C \end{cases} \rightarrow h_{2a} = 2682.4 \ kJ/kg$$

State 2s:
$$\begin{cases} P_1 = 50 \ kPa \\ s_1 = s_2 \end{cases} \rightarrow \begin{cases} s_f = 1.0912 \ kJ/(kg - K) \\ s_g = 7.5931 \ kJ/(kg - K) \end{cases}$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

$$h_{2s} = h_f + x_{2s} \times h_{fg} = 340.54 + 0.897 \times (2304.7) = 2407.9 \, kJ/kg$$

• Solution (a): The isentropic efficiency is:

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = 0.667 \ (or \ 66.7\%)$$

• Solution (b): The mass flow rate is:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}h_1 = \dot{W}_{a,out} + \dot{m}h_{2a}$$
$$\dot{W}_{a,out} = \dot{m}(h_1 - h_{2a})$$

$$2 MW\left(\frac{1000 kJ}{1 MW}\right) = \dot{m}(3231.7 - 2682.4)\frac{kJ}{kg}$$

 $\dot{m} = 3.64 \, kg/s$

Chapter 8 Summary

• We did not cover 8-12 (Entropy Balance)

THE CARNOT CYCLE AND ITS VALUE IN ENGINEERING

The Carnot Cycle and Its Value in Engineering

- Carnot cycle has four main processes:
 - 1. Isothermal heat addition
 - 2. Isentropic expansion
 - 3. Isothermal heat rejection
 - 4. Isentropic compression



The Carnot Cycle and Its Value in Engineering

 Property diagrams such as T-s and P-V diagrams can serve as valuable aids in understanding and analysis of thermodynamics process:



The Carnot Cycle and Its Value in Engineering

• The thermal efficiency of a Carnot cycle operating between limits of T_H and T_L is solely function of these two temperatures is equal to $\eta_{thermal,Carnot} = 1 - \frac{T_L}{T_H}$



REFRIGERATORS AND HEAT PUMPS (SECTION 9-14 AND 9-15)

Refrigerators and Heat Pumps

• We looked at this in Chapter 7



Refrigerators and Heat Pumps

• The Carnot cycle includes:



Refrigerators and Heat Pumps

• The T-s diagram for the Carnot cycle is:



IDEAL VAPOR COMPRESSION REFRIGERATION CYCLE (SECTION 9-16)

- In practice, there are several issues that limit the use of Carnot vapor compression cycle:
 - □ 1-2: Isentropic compression in a compressor
 - □ 2-3: Constant pressure heat rejection in a condenser
 - □ 3-4: Throttling in an expansion valve
 - □ 4-1: Constant pressure heat absorption in an evaporator

 In practice, there are several issues that limit the use of Carnot vapor compression cycle:



• An ordinary refrigerator, has all the four main components:



• P-h diagram is very helpful in analyzing the performance:



$$COP_{HP} = \frac{q_H}{w_{net,in}} = \frac{h_2 - h_3}{h_2 - h_1}$$

$$COP_R = \frac{q_L}{w_{net,in}} = \frac{h_1 - h_4}{h_2 - h_1}$$

 $(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i$

CLASS ACTIVITY

- A refrigerator uses refrigerant 134-a as the working fluid and operates on an ideal vapor-compression cycle between 0.14 and 0.8 MPa. If the mass flow rate of the refrigerant is 0.05 kg/s, determine
 - a) The rate of heat removal from the refrigerated space and the power input to the compressor
 - b) The rate of heat rejection to the environment
 - c) The COP of the refrigerator

- Solution (assumption):
 - □ Steady operating condition exist
 - □ Kinetic and potential energy are negligible
- Understanding the states:



• Solution: Reading properties from the tables:

$$\begin{cases} P_1 = 0.14 MPa \rightarrow h_1 = h_{g @ 0.14 MPa} = 239.19 \frac{kJ}{kg} \\ s_1 = s_{g @ 0.14 MPa} = 0.94467 \frac{kJ}{kg - K} \end{cases}$$

TABLE A-12

Saturated	refrigerant-134a—Pressure table	
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		<i>Specij</i> n	fic volume, n ³ /kg	Internal energy, kJ/kg		Enthalpy, kJ/kg				Entropy, kJ/kg · K		
Press., P kPa	Sat. temp., T _{sat} °C	Sat. liquid, U _f	Sat. vapor, U _g	Sat. liquid, <i>u_f</i>	Evap., <i>u</i> _{fg}	Sat. vapor, u _g	Sat. liquid, h _f	Evap., h _{fg}	Sat. vapor, h _g	Sat. liquid, <i>s_f</i>	Evap., ^S fg	Sat. vapor, <i>s</i> g
60	-36.95	0.0007097	0.31108	3.795	205.34	209.13	3.837	223.96	227.80	0.01633	0.94812	0.96445
70	-33.87	0.0007143	0.26921	7.672	203.23	210.90	7.722	222.02	229.74	0.03264	0.92783	0.96047
80	-31.13	0.0007184	0.23749	11.14	201.33	212.48	11.20	220.27	231.47	0.04707	0.91009	0.95716
90	-28.65	0.0007222	0.21261	14.30	199.60	213.90	14.36	218.67	233.04	0.06003	0.89431	0.95434
100	-26.37	0.0007258	0.19255	17.19	198.01	215.21	17.27	217.19	234.46	0.07182	0.88008	0.95191
120	-22.32	0.0007323	0.16216	22.38	195.15	217.53	22.47	214.52	236.99	0.09269	0.85520	0.94789
140	-18.77	0.0007381	0.14020	26.96	192.60	219.56	27.06	212.13	239.19	0.11080	0.83387	0.94467

• Solution: Reading properties from the tables:

$$\begin{cases} P_3 = 0.8 MPa \\ s_2 = s_1 = 0.94467 \frac{kJ}{kg - K} & \rightarrow . \ h_2 = 275.40 \frac{kJ}{kg} \end{cases}$$

TABLE A-13								
Superheated refrigerant-134a								
T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg ∙ K				
	P	= 0.80 MPa	$T_{\rm sat} = 31.$.31°C)				
Sat.	0.025645	246.82	267.34	0.9185				
40	0.027035	254.84	276.46	0.9481				
50	0.028547	263.87	286.71	0.9803				
60	0.029973	272.85	296.82	1.0111				
70	0.031340	281.83	306.90	1.0409				

• Solution: Reading properties from the tables:

$$P_3 = 0.8 MPa \rightarrow h_3 = h_{f @ 0.8 MPa} = 95.48 \frac{kJ}{kg}$$

TABLE A	TABLE A-12									
Saturated refrigerant-134a—Pressure table										
		Specific volume, m ³ /kg			Internal energy, kJ/kg			Enthalpy, kJ/kg		
Press., <i>P</i> kPa	Sat. temp., T _{sat} °C	Sat. liquid, V _f	Sat. vapor, U _g	Sat. liquid, <i>u_f</i>	Evap., u _{fg}	Sat. vapor, u _g	Sat. liquid, <i>h_f</i>	Evap., h _{fg}	Sat. vapor, h _g	
650	24.20	0.0008265	0.031680	84.72	158.51	243.23	85.26	178.56	263.82	
700	26.69	0.0008331	0.029392	88.24	156.27	244.51	88.82	176.26	265.08	
750	29.06	0.0008395	0.027398	91.59	154.11	245.70	92.22	174.03	266.25	
800	31.31	0.0008457	0.025645	94.80	152.02	246.82	95.48	171.86	267.34	
850	33.45	0.0008519	0.024091	97.88	150.00	247.88	98.61	169.75	268.36	
900	35.51	0.0008580	0.022703	100.84	148.03	248.88	101.62	167.69	269.31	

$$h_4 \cong h_3 \ (throttling) \rightarrow h_4 = 95.48 \frac{kJ}{kg}$$

 Solution (a): The rate of heat removal from the refrigerated space and the power input to the compressor is

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = \left(0.05 \frac{kg}{s}\right) \left((239.19 - 95.48) \frac{kJ}{kg}\right) = 7.19 \, kW$$

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = \left(0.05 \frac{kg}{s}\right) \left((275.40 - 239.19) \frac{kJ}{kg}\right) = 1.18 \, kW$$

 Solution (b): The rate of heat rejection from the refrigerant to the environment is:

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = \left(0.05 \frac{kg}{s}\right) \left((275.40 - 95.48) \frac{kJ}{kg}\right) = 9.00 \ kW$$

 $\dot{Q}_H = \dot{Q}_L + \dot{W}_{in} = 7.19 + 1.81 = 9.00 \ kW$

• Solution (c): The coefficient of performance of the refrigerator is:

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.19 \ kW}{1.81 \ kW} = 3.97$$

ACTUAL VAPOR-COMPRESSION REFRIGERATION CYCLE (SECTION 9-17)

Actual Vapor-Compression Refrigeration Cycle

 An actual vapor-compression refrigeration cycle varies from the ideal one because of two common sources of irreversibilities:





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CLASS ACTIVITY

- (The actual vapor-compression refrigeration cycle almost similar inputs to the previous class activity): Refrigerant 134-a enters the compressor of a refrigerator as superheated vapor at 0.14 MPa and -10 °C at a rate of 0.05 kg/s and leaves at 0.8 MPa and 50 °C. The refrigerant is cooled in the condenser to 26 °C and 0.72 MPa and id throttled to 0.15 MPa. Disregarding any heat transfer and pressure drops in the connecting lines between the components determine
 - a) The rate of heat removal from the refrigerated space and the power pressure drops in the connecting lines between the components
 - b) The isentropic efficiency of the compressor
 - c) The coefficient of performance of the refrigerator

- Solution (assumption):
 - □ Steady operating condition exist
 - □ Kinetic and potential energy are negligible

• Solution (T-s diagram)



• Solution (Tables and Calculations):

$$\begin{cases} P_1 = 0.14 \ MPa \\ T_1 = -10 \ ^\circ C \end{cases} \rightarrow h_1 = 246.37 \ \frac{kJ}{kg} \end{cases}$$

TABLE A-12

Saturated refrigerant-134a—Pressure table

		<i>Specific volume,</i> m ³ /kg			Internal ener kJ/kg	gy,	Enthalpy, kJ/kg		
Press., <i>P</i> kPa	Sat. temp., T _{sat} °C	Sat. liquid, V _f	Sat. vapor, U _g	Sat. liquid, <i>u_f</i>	Evap., u _{fg}	Sat. vapor, u _g	Sat. liquid, h _j	Evap., h _{fg}	Sat. vapor, h _g
60	-36.95	0.0007097	0.31108	3.795	205.34	209.13	3.837	223.96	227.80
70	-33.87	0.0007143	0.26921	7.672	203.23	210.90	7.722	222.02	229.74
80	-31.13	0.0007184	0.23749	11.14	201.33	212.48	11.20	220.27	231.47
90	-28.65	0.0007222	0.21261	14.30	199.60	213.90	14.36	218.67	233.04
100	-26.37	0.0007258	0.19255	17.19	198.01	215.21	17.27	217.19	234.46
120	-22.32	0.0007323	0.16216	22.38	195.15	217.53	22.47	214.52	236.99
140	-18.77	0.0007381	0.14020	26.96	192.60	219.56	27.06	212.13	239.19

TABLE A-13

Superheated refrigerant-134a

	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/kg ∙ K
	Р	r = 0.14 MPa	$(T_{\text{sat}} = -1)$	8.77°C)
Sat.	0.14020	219.56	239.19	0.9447
-20	0.14605	225.02	246 27	0.0724
-10	0.14603	233.25	240.37	1.0032

• Solution (Tables and Calculations):

$$\begin{cases} P_1 = 0.14 MPa \\ T_1 = -10 \ ^\circ C \end{cases} \rightarrow h_1 = 246.37 \frac{kJ}{kg} \end{cases}$$

$$\begin{cases} P_2 = 0.8 MPa \\ T_2 = -50 \ ^{\circ}C \end{cases} \rightarrow h_2 = 286.71 \frac{kJ}{kg} \end{cases}$$

$$\begin{cases} P_3 = 0.72 MPa \\ T_3 = 26 \,^{\circ}C \end{cases} \rightarrow h_3 \cong h_{f @ 26 \,^{\circ}C} = 87.83 \frac{kJ}{kg} \end{cases}$$

$$\begin{cases} h_4 \cong h_3 = 87.83 \frac{kJ}{kg} \end{cases}$$

 Solution (a): The rate of heat removal from the refrigerated space and the power input to the compressor are:

$$\dot{Q}_L = \dot{m}(h_1 - h_4) = \left(0.05 \frac{kg}{s}\right) \left((246.37 - 87.83) \frac{kJ}{kg}\right) = 7.93 \ kW$$

$$\dot{W}_{in} = \dot{m}(h_2 - h_1) = \left(0.05 \frac{kg}{s}\right) \left((286.71 - 246.37) \frac{kJ}{kg}\right) = 2.02 \ kW$$

 Solution (b): The isentropic efficiency of the compressor is determined from:

$$\eta_C \cong \frac{h_{2s} - h_1}{h_2 - h_1}$$

• Where the enthalpy at state 2s ($P_{2s} = 0.8 MPa$ and $s_{2s} = s_1 = 0.9724 \frac{kJ}{kg-K}$) is 284.20 $\frac{kJ}{kg}$. Thus:

$$\eta_C \cong \frac{284.20 - 246.37}{286.71 - 246.37} = 0.938 \ or \ 93.8\%$$

• Solution (c): The coefficient of performance of the refrigerator is:

$$COP_R = \frac{\dot{Q}_L}{\dot{W}_{in}} = \frac{7.93 \ kW}{2.02 \ kW} = 3.93$$

CLASS ACTIVITY





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• Solve the previous example using P-h diagram (ASHRAE)



Fig. 8 Pressure-Enthalpy Diagram for Refrigerant 134a

Solve the previous example using P-h diagram (ASHRAE)



Fig. 8 Pressure-Enthalpy Diagram for Refrigerant 134a