CAE 208 / MMAE 320: Thermodynamics
Fall 2023

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Entropy (3)

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ANNOUNCEMENTS

Announcements

- Exam 2 solutions are posted

- Assignment 9 is posted (2-3 days more extension is fine to submit it on Friday or the weekend after the Thanksgiving)

- I will post assignment 10 due 12/01/23 (those who need to submit it)

- The bonus activities document is posted. Please pay attention to the deadlines and also the updates about this task (an idea submission by the 11/29/2023 is required)

RECAP

Recap

- The equality in the Clausius inequality holds for totally or jut internally reversible cycles and the inequality for the irreversible ones:
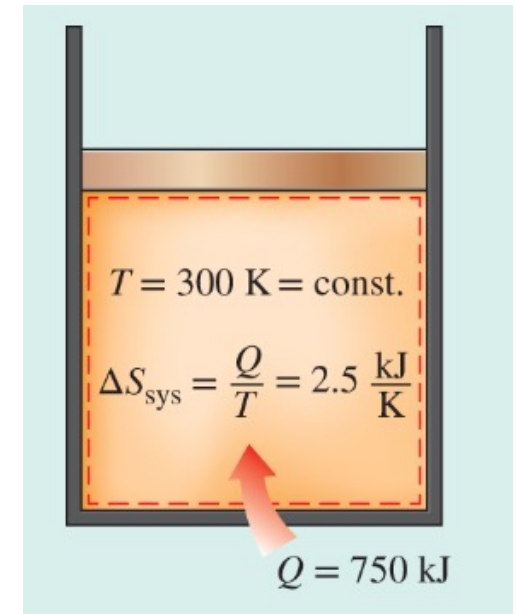
$$\left(\oint \frac{\delta Q}{T} \right)_{int,rev} = 0$$

Recap

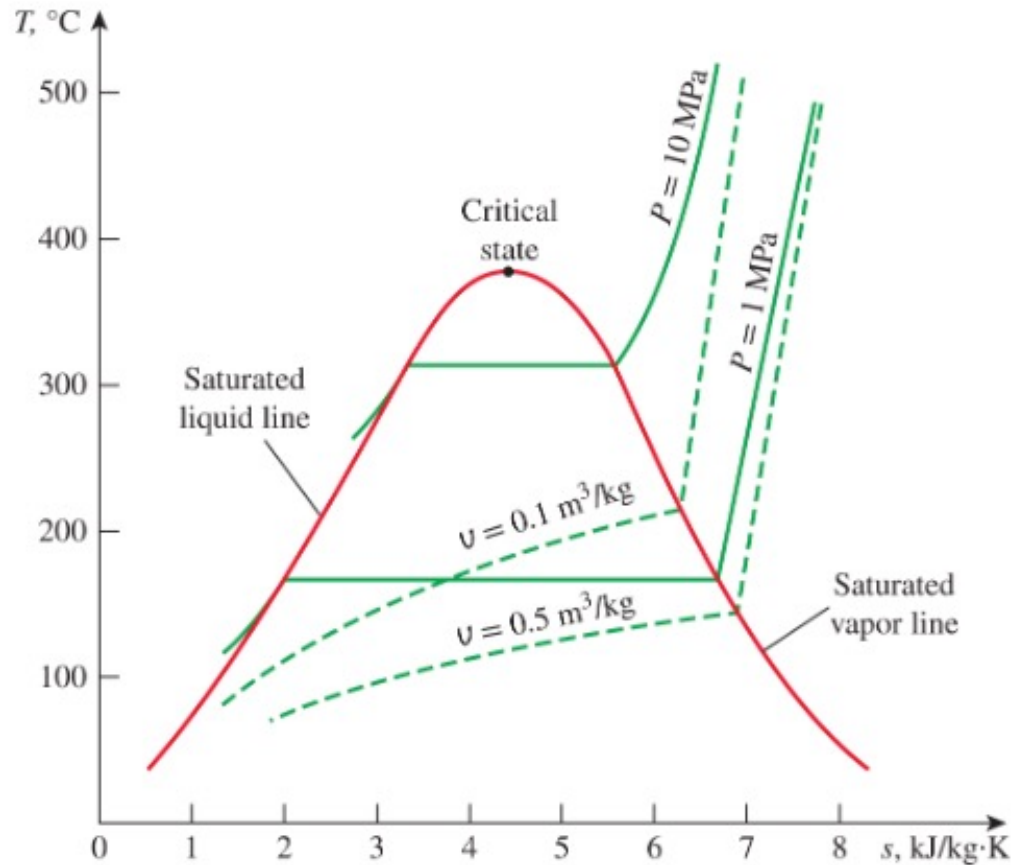- A special case: *Internally reversible isothermal* heat transfer processes:

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,rev} = \int_1^2 \left(\frac{\delta Q}{T_0}\right)_{int,rev} = \frac{1}{T_0}\int_1^2 \delta Q_{int,rev}$$

$$\Delta S_{isothermal} = \frac{Q}{T_0} \qquad \left(\frac{kJ}{K}\right)$$

$T = 300 \text{ K} = \text{const.}$

$\Delta S_{sys} = \dfrac{Q}{T} = 2.5 \ \dfrac{kJ}{K}$

$Q = 750 \text{ kJ}$

Recap

- Entropy is a property:

Recap

- For entropy, we can say "the increase of entropy principle":

$$\oint_1^2 \frac{\delta Q}{T} \leq \Delta S$$

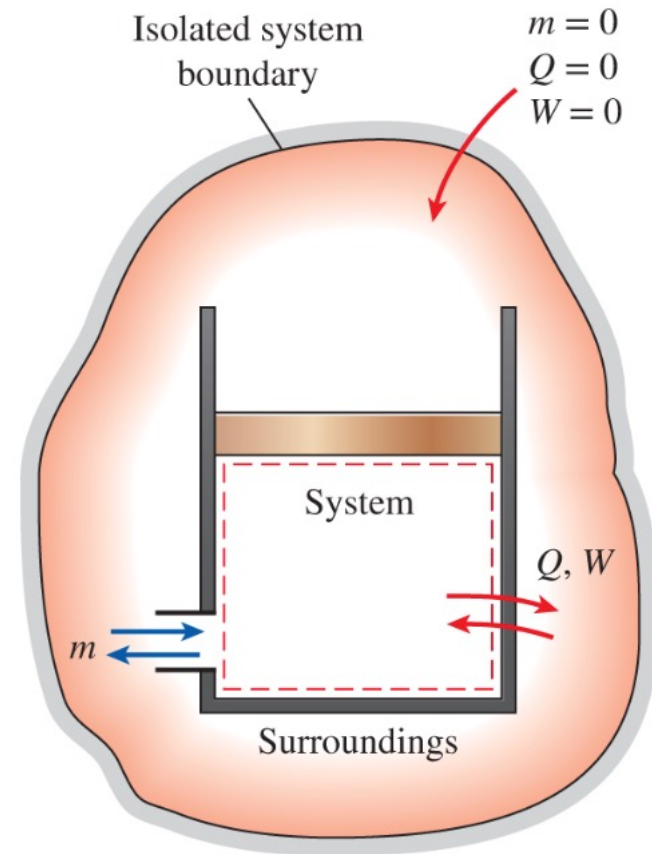$$\Delta S_{sys} = S_2 - S_1 = \oint_1^2 \frac{\delta Q}{T} + S_{gen}$$

$$\Delta S_{isolated} \geq 0$$

Recap

- For an isolated system:

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \geq 0$$

$$S_{gen} = \begin{cases} > 0. & irreversible\ process \\ = 0 & reversible\ process \\ < 0. & impossible\ process \end{cases}$$

Recap

- The entropy of a fixed mass can be changed by:
  - ❑ Heat Transfer
  - ❑ Irreversibilities

- Entropy of a fixed mass does not change during a process that is internally reversible and adiabatic. During this process entropy remains constant and we call it *isentropic* process

$$\Delta s = 0 \;\; or \;\; s_2 = s_1 \;\;\; (\frac{kJ}{kg - K})$$

Steam
s_1

No irreversibilities
(internally reversible)
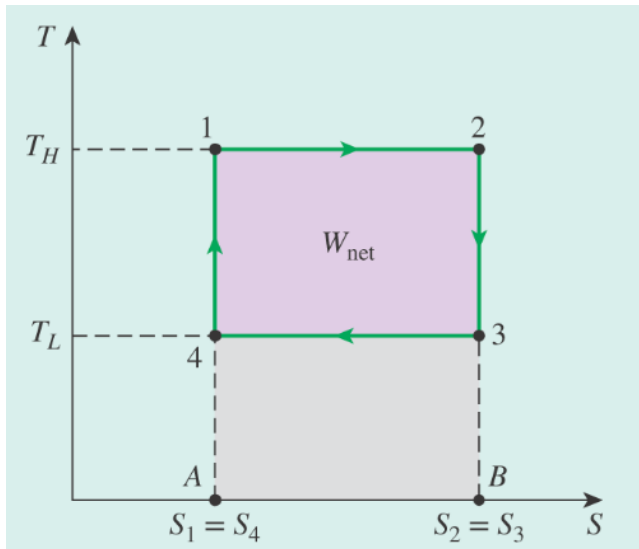
No heat transfer
(adiabatic)

$s_2 = s_1$
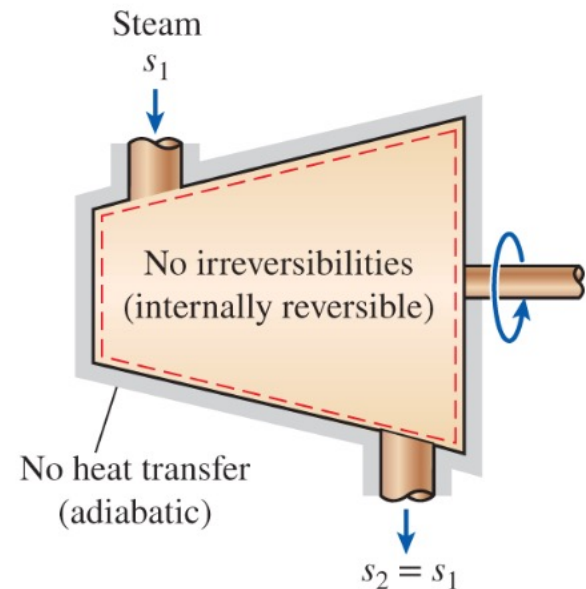
Recap

- We can rearrange our entropy equation:

$$\delta Q_{int,rev} = T \, dS$$
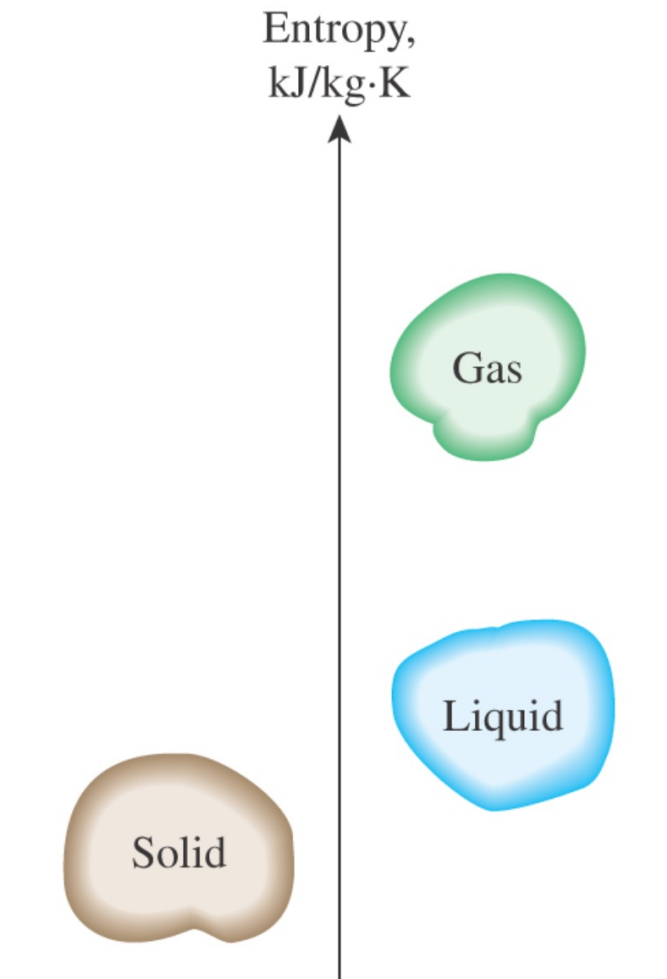
$$Q_{int,rev} = \int_1^2 T \, dS \quad (kJ)$$

(The area under the process curve on a T-S diagram represents heat transfer during an internally reversible process)

WHAT IS ENTROPY

What Is Entropy

- Entropy can be viewed as a measure of molecular disorder or molecular randomness (lowest in solid phase and highest in the gas phase)

Entropy, kJ/kg·K

Gas

Liquid

Solid

What Is Entropy

- Oscillations in solid phase fade as temperature is decreased, and the molecules supposedly become motionless at absolute zero. This represents a state of ultimate molecular order (and *minimum energy*).

- The entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant (3rd law of thermodynamics)

Pure crystal
$T = 0$ K
Entropy $= 0$

What Is Entropy

- The third law of thermodynamics provide an absolute reference point for the determination of entropy

- The entropy determined relative to this point is called absolute entropy and it is extremely useful in the thermodynamics analysis of entropy

- The entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature

What Is Entropy

- A few examples:
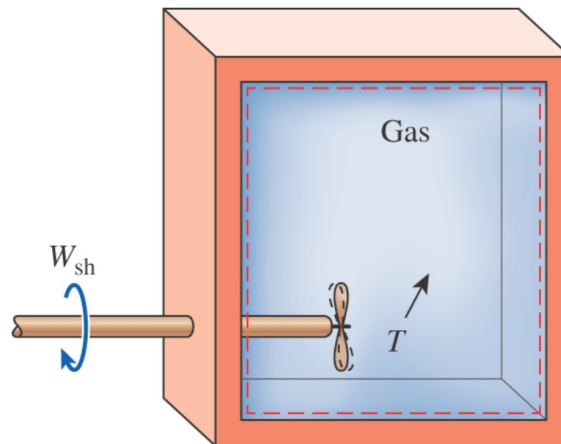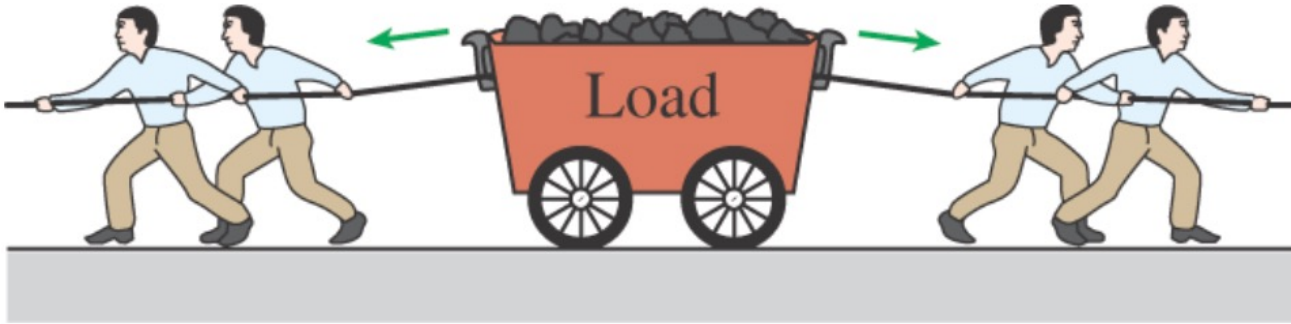
THE T DS RELATIONS

The T ds Relations

- The first T ds (or Gibbs) equation (closed stationary system):

$$\delta Q_{int,rev} - \delta W_{int,rev,out} = dU$$

$$TdS = dU + PdV \qquad (kJ)$$

$$\delta Q_{int,rev} = Tds$$

$$Tds = du + Pdv \qquad (\frac{kJ}{kg})$$

$$\delta W_{int,rev,out} = PdV$$

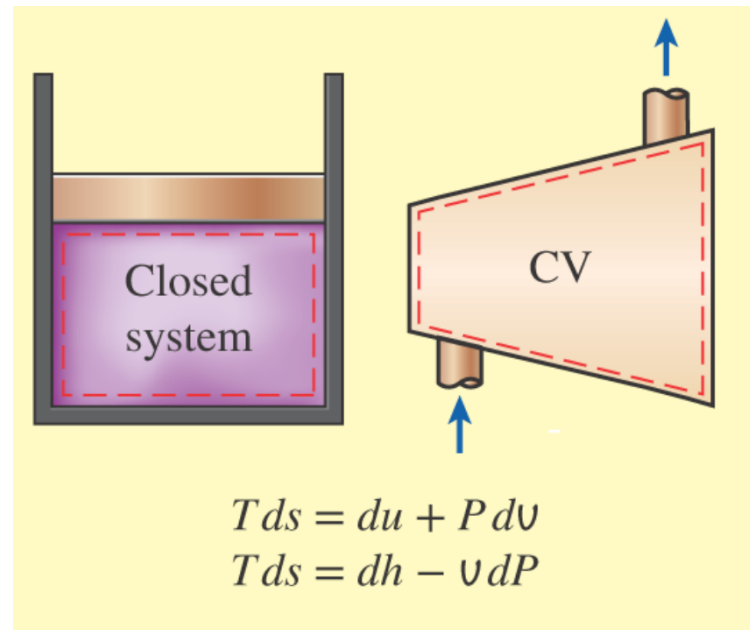$$\left.\begin{array}{l} Tds = du + Pdv \\ h = u + Pv \quad \rightarrow dh = du + Pdv + vdP \end{array}\right\} \rightarrow Tds = dh - vdP$$

The T ds Relations

- Use the first T ds (or Gibbs) equation to solve for entropy changes

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = \frac{dh}{T} - \frac{vdP}{T}$$

$$T\,ds = du + P\,dv$$
$$T\,ds = dh - v\,dP$$

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Entropy Change of Liquids and Solids

- Liquids and solids can be approximated as incompressible substances ($dv \cong 0$ & $c_p = c_v = c_p = c$):

$$ds = \frac{dh}{T} - \frac{vdP}{T}$$

$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg} \ln(\frac{T_2}{T_1})$$

$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg} \ln\left(\frac{T_2}{T_1}\right) = 0 \quad \rightarrow \quad T_2 = T_1 \qquad \text{(For isentropic)}$$

THE ENTROPY CHANGE OF IDEAL GASES

The Entropy Change of Ideal Gases

- For gases, we can write:

$$ds = \frac{du}{T} + \frac{Pdv}{T} = c_v \frac{dT}{T} + R \frac{dv}{v}$$

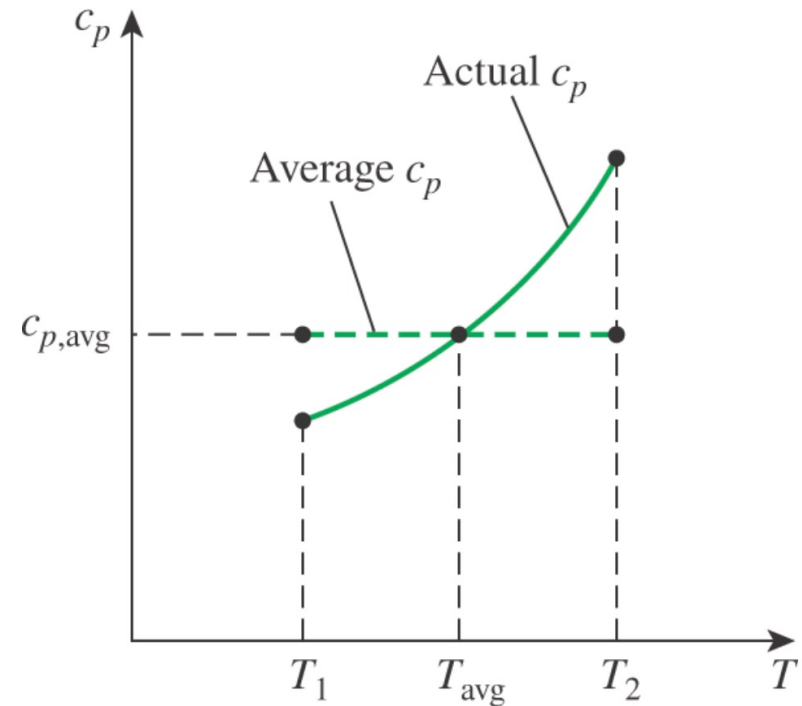$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln(\frac{v_2}{v_1})$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} + R \ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- **Approach 1:** Constant Specific Heats (Approximate Analysis):

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln(\frac{v_2}{v_1})$$

$$s_2 - s_1 = c_{v,avg}\ln\left(\frac{T_2}{T_1}\right) + R\times ln\left(\frac{v_2}{v_1}\right)$$

$$\begin{cases} dh = c_p dT \\ v = \dfrac{RT}{P} \end{cases} \rightarrow s_2 - s_1 = c_{p,avg}\times\ln\left(\frac{T_2}{T_1}\right) - R\times\ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- **Approach 2:** Variable Specific Heats (Exact Analysis):

$$s^0 = \int_0^T c_p(T)\frac{dT}{T}$$

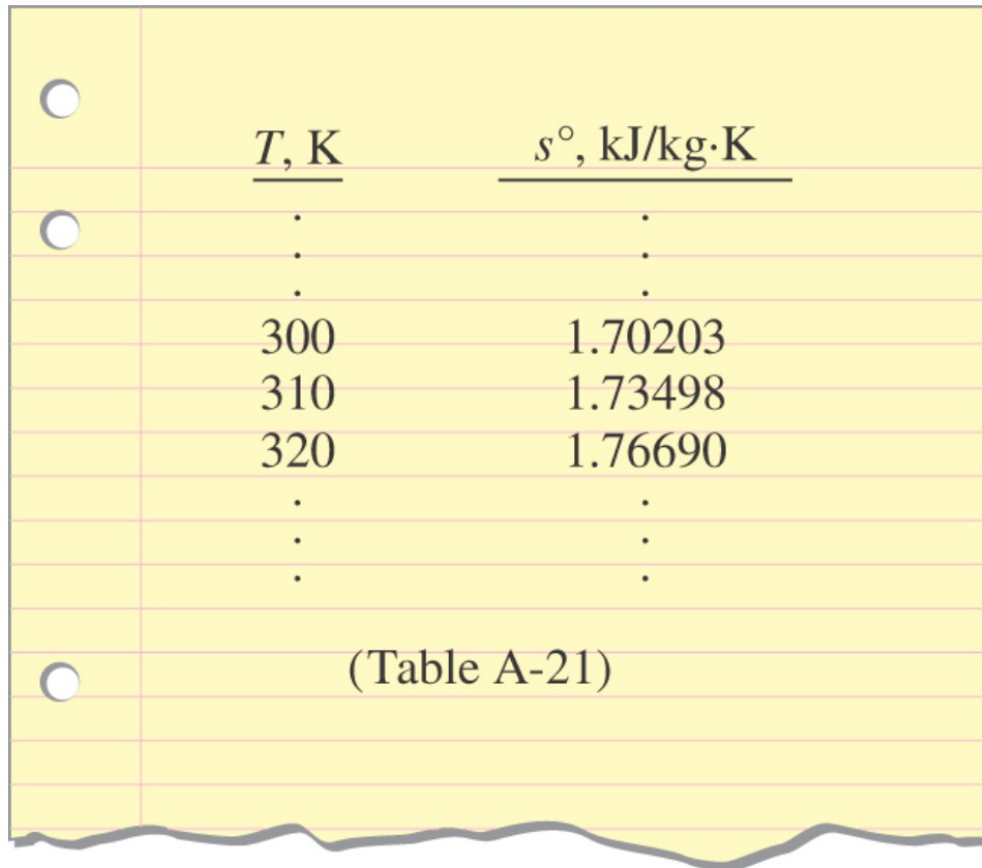$$\int_0^T c_p(T)\frac{dT}{T} = s_2^0 - s_1^0$$

$$s_2 - s_1 = s_2^0 - s_1^0 - R\times\ln(\frac{P_2}{P_1})$$

$$\overline{s_2} - \overline{s_1} = \overline{s_2^0} - \overline{s_1^0} - R_u\times\ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- Approach 2: Variable Specific Heats (Exact Analysis):

| T, K	$s°$, kJ/kg·K
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

(Table A-21)

CLASS ACTIVITY

Class Activity

- Air is compressed from an initial state of 100 kPa and 17 °C to a final state of 600 kPa and 57 °C. Determine the entropy change of air during this compression process by using:
    - a) Property values from the air table
    - b) Average specific heats

Class Activity

- Solution (assumptions):
  - ❑ Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values
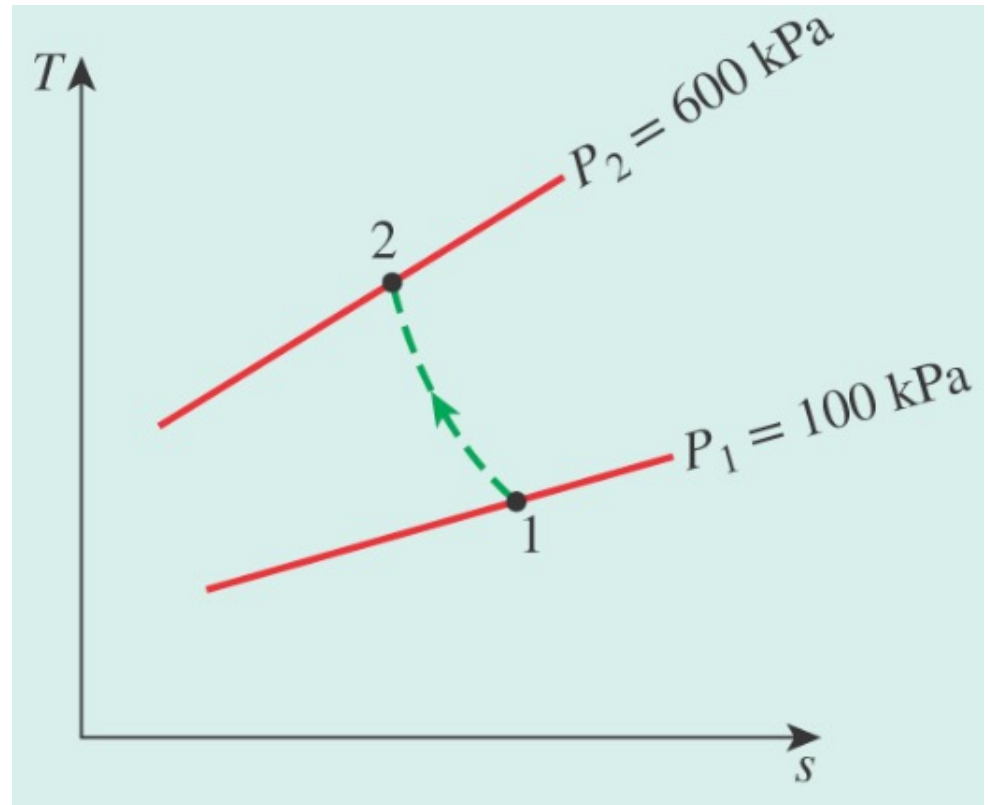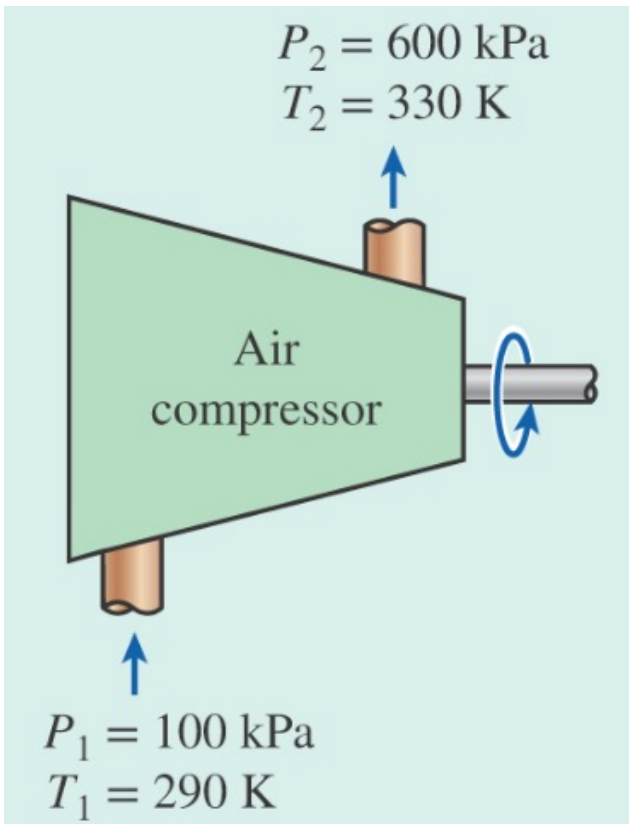
Class Activity

- Solution (calculations):
  - ❑ Part (a): Table A-21

$$s_2 - s_1 = s_2^0 - s_1^0 - R \times \ln\left(\frac{P_2}{P_1}\right) = \left[(1.79783 - 1.66802)\frac{kJ}{kg - K}\right] \times \ln(\frac{600\ kPa}{100\ kPa})$$

$$s_2 - s_1 = -0.3844 \frac{kJ}{kg - K}$$

  - ❑ Part (b): Using a c_p value at the average temperature of 37 °C (Table A-2b)

$$s_2 - s_1 = c_{p,avg} \ln\left(\frac{T_2}{T_1}\right) - R \times \ln\left(\frac{P_2}{P_1}\right)$$

$$= \left(1.006\frac{kJ}{kg - K}\right)\ln\left(\frac{330\ K}{290\ K}\right) - \left(0.287\frac{kJ}{kg - K}\right)\ln\left(\frac{600\ kPa}{100\ kPa}\right) = -0.3842\frac{kJ}{kg - K}$$

ISENTROPIC PROCESSES OF IDEAL GASES

Isentropic Processes of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$s_2 - s_1 = 0 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \times ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{c_v}\ln(\frac{v_2}{v_1}) \qquad \begin{cases} c_p - c_v = R \\ k = \dfrac{c_p}{c_v} \end{cases} \rightarrow \frac{R}{c_v} = k - 1$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{\frac{R}{c_v}}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Isentropic Processes of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$\left(\frac{T_2}{T_1}\right)_{s=constant} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=constant} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\left(\frac{P_2}{P_1}\right)_{s=constant} = \left(\frac{v_1}{v_2}\right)^{k}$$

Isentropic Processes of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$Tv^{k-1} = Constant$$

$$TP^{\frac{1-k}{k}} = Constant$$

$$Pv^k = Constant$$

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Valid for *ideal gas
*isentropic process
*constant specific heats

The Entropy Change of Ideal Gases

- Approach 2: Variable Specific Heats (Exact Analysis) for Isentropic Processes of Ideal Gases

$$0 = s_2^0 - s_1^0 - R \times \ln(\frac{P_2}{P_1})$$

$$s_2^0 = s_1^0 + R \times \ln(\frac{P_2}{P_1})$$

$$s_2^0 = s_1^0 + R \times \ln\left(\frac{P_2}{P_1}\right) \quad \rightarrow \quad \frac{P_2}{P_1} = \exp(\frac{s_2^0 - s_1^0}{R})$$
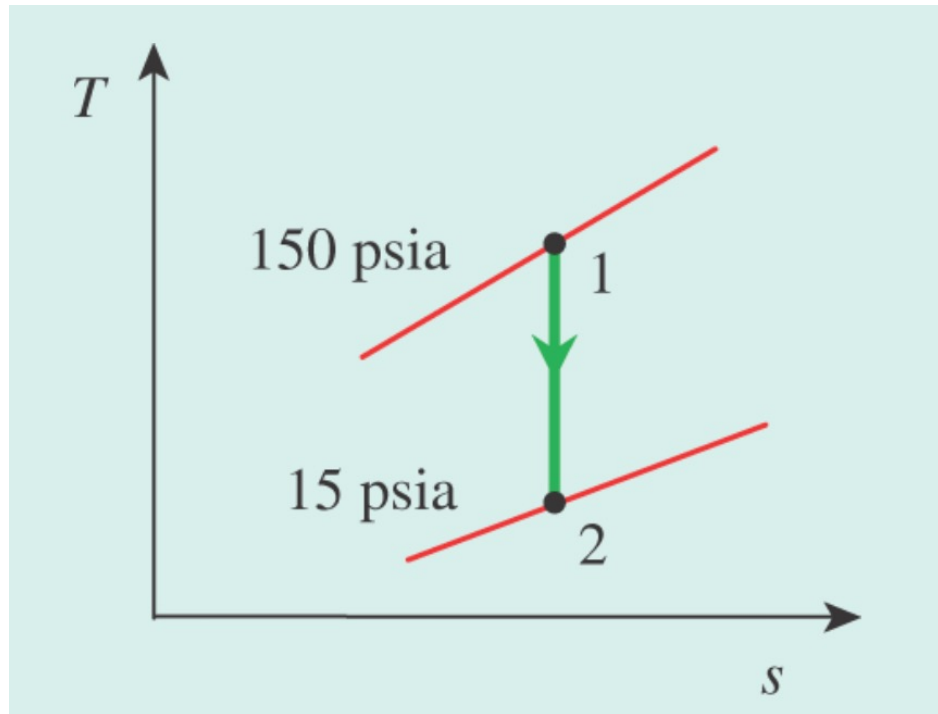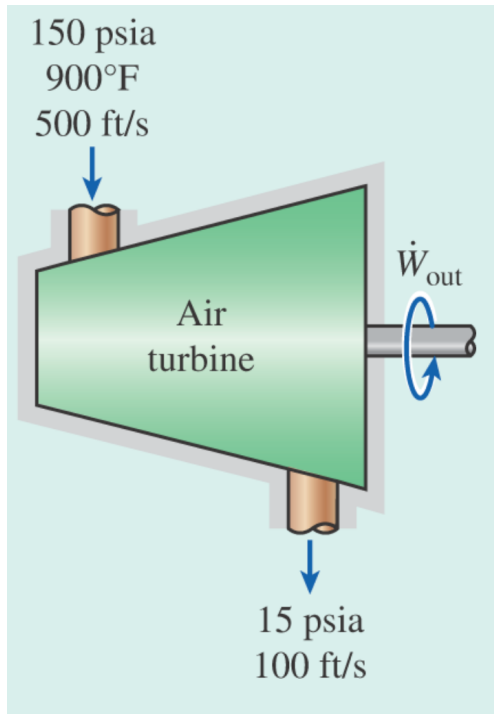
CLASS ACTIVITY

Class Activity

- Air enters an isentropic turbine at 150 psia and 900 °F through a 0.5 ft^2 inlet section with a velocity of 500 ft/s. It leaves at 15 psia with a velocity of 100 ft/s. Calculate the air temperature at the turbine exit and the power produced, in hp, by this turbine.

Class Activity

- Solution (assumptions):
  - ❑ Steady flow
  - ❑ The process is isentropic (both reversible and adiabatic)
  - ❑ Ideal gas with a constant specific heat

150 psia
900°F
500 ft/s

$\dot{W}_{out}$

Air turbine

15 psia
100 ft/s

T

150 psia

1

15 psia

2

s

Class Activity

- Solution (Tables):
  - ❑ Table A-2Eb: @$600\,°F \rightarrow c_p = 0.250\,\dfrac{Btu}{lbm-R}\; and\; k = 1.3777$

Temp., °F	c_p Btu/lbm · R	c_v Btu/lbm · R	k	c_p Btu/lbm · R	c_v Btu/lbm · R	k
		Air			Carbon dioxide, CO_2	
40	0.240	0.171	1.401	0.195	0.150	1.300
100	0.240	0.172	1.400	0.205	0.160	1.283
200	0.241	0.173	1.397	0.217	0.172	1.262
300	0.243	0.174	1.394	0.229	0.184	1.246
400	0.245	0.176	1.389	0.239	0.193	1.233
500	0.248	0.179	1.383	0.247	0.202	1.223
600	0.250	0.182	1.377	0.255	0.210	1.215
700	0.254	0.185	1.371	0.262	0.217	1.208
800	0.257	0.188	1.365	0.269	0.224	1.202
900	0.259	0.191	1.358	0.275	0.230	1.197
1000	0.263	0.195	1.353	0.280	0.235	1.192
1500	0.276	0.208	1.330	0.298	0.253	1.178
2000	0.286	0.217	1.312	0.312	0.267	1.169

Ideal-gas specific heats of various common gases (b) At various temperatures

Class Activity

- Solution (Tables):
  - Table A-1E: $R = 0.3704 \frac{psia - ft^3}{lbm - R}$

TABLE A–1E

Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, M lbm/lbmol	Gas constant, R^*		Critical-point properties		
			Btu/lbm · R	psia · ft³/lbm · R	Temperature, R	Pressure, psia	Volume, ft³/lbmol
Air	—	28.97	0.06855	0.3704	238.5	547	1.41

Class Activity

- Solution (Problem solving):

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{d\dot{E}_{system}}{dt} = 0$$

150 psia
900°F
500 ft/s

$\dot{W}_{out}$

Air
turbine

15 psia
100 ft/s

$$\dot{m}(h_1 + V_1^2) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_1 + \frac{V_1^2 - V_2}{2}\right)$$
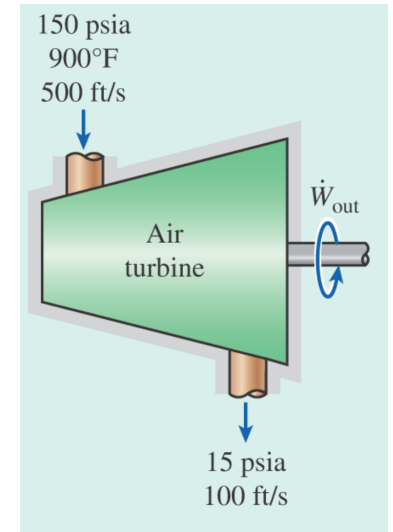
Class Activity

- Solution (Calculations):

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \rightarrow T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (900 + 460 \, R)\left(\frac{15 \, psia}{150 \, psia}\right)^{\frac{0.3777}{1.377}} = 724 \, R$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.3704 \frac{psia - ft^3}{lbm - R}\right)(900 + 460 \, R)}{150 \, psia} = 3.358 \frac{ft^3}{lbm}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(0.5 \, ft^2)\left(500 \frac{ft}{s}\right)}{3.358 \frac{ft^3}{lbm}} = 74.45 \frac{lbm}{s}$$

Class Activity

- Solution (Calculations):

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_1 + \frac{V_1^2 - V_2}{2}\right)$$

$$\dot{W}_{out} = \left(74.45\,\frac{lbm}{s}\right)\left[\left(0.250\,\frac{Btu}{lbm-R}\right)(1360 - 724R) + \left(\frac{\left(500\,\frac{ft}{s}\right)^2}{2} - \frac{\left(100\,\frac{ft}{s}\right)^2}{2}\right)\left(\frac{1\frac{Btu}{lbm}}{25.037}\frac{ft^2}{s^2}\right)\right.$$

$$\dot{W}_{out} = 12{,}194\,\frac{Btu}{s}\left(\frac{1\,hp}{0.7068\,\frac{Btu}{s}}\right) = 17{,}250\,hp$$

REVERSIBLE STEADY-FLOW WORK

Reversible Steady-Flow Work

- Recall we had

$$W_b = \int_1^2 P \, dV$$

- For steady flow:

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$
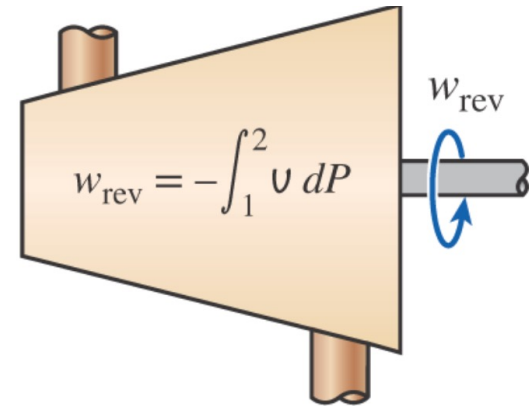
$$-\delta w_{rev} = vdP + dke + dpe$$

$$\left. \begin{array}{l} \delta q_{rev} = Td \\ Tds = dh - vdP \end{array} \right\} \ \rightarrow \delta q_{rev} = dh - vdP$$
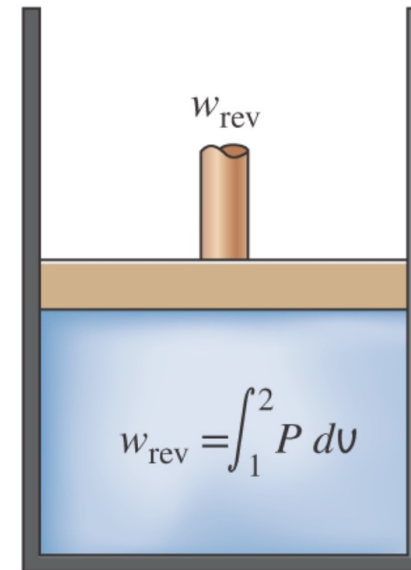
$$w_{rev} = -\int_1^2 vdP - \Delta ke - \Delta pe$$

Reversible Steady-Flow Work

- For steady flow:

$$w_{rev} = -\int_1^2 v\, dP - \Delta ke - \Delta pe$$

$$w_{rev} = -\int_1^2 \upsilon\, dP$$

w_{rev}

(a) Steady-flow system

w_{rev}

$$w_{rev} = \int_1^2 P\, d\upsilon$$

(b) Closed system
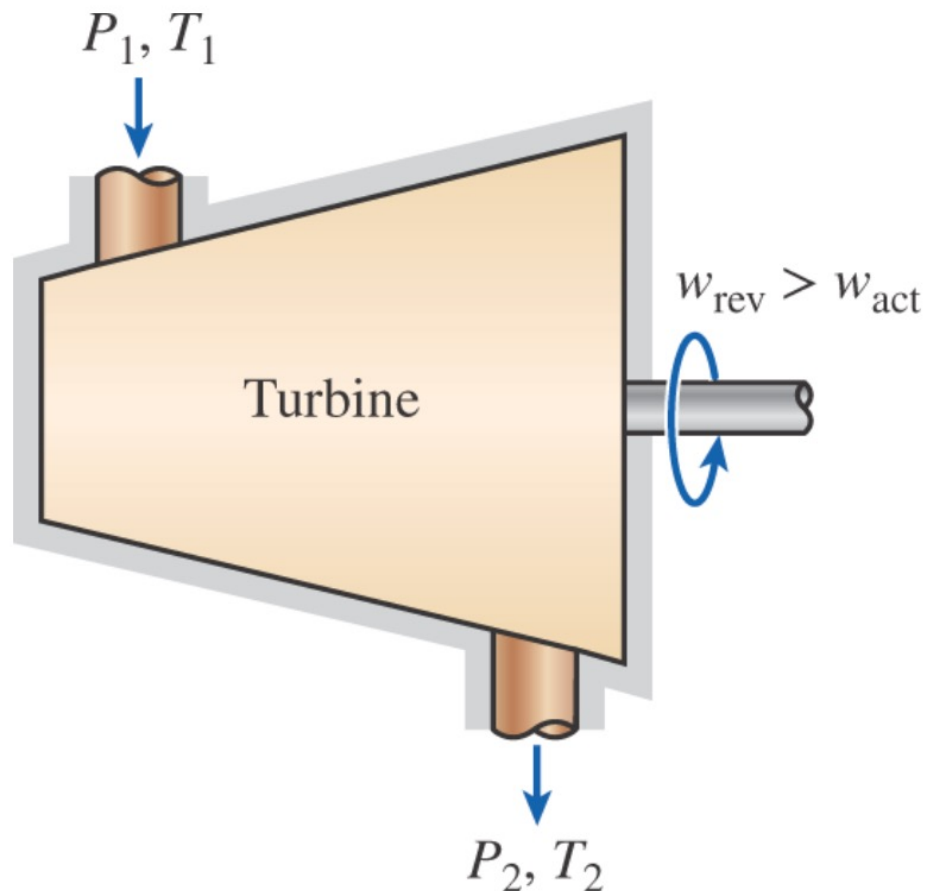
Reversible Steady-Flow Work

- We write the *Bernoulli equation*:

$$w_{rev} = 0 = v(P_2 - P_1) + \left(\frac{V_2^2 - V_1^2}{2}\right) + g(z_2 - z_1)$$

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

Isentropic Efficiencies

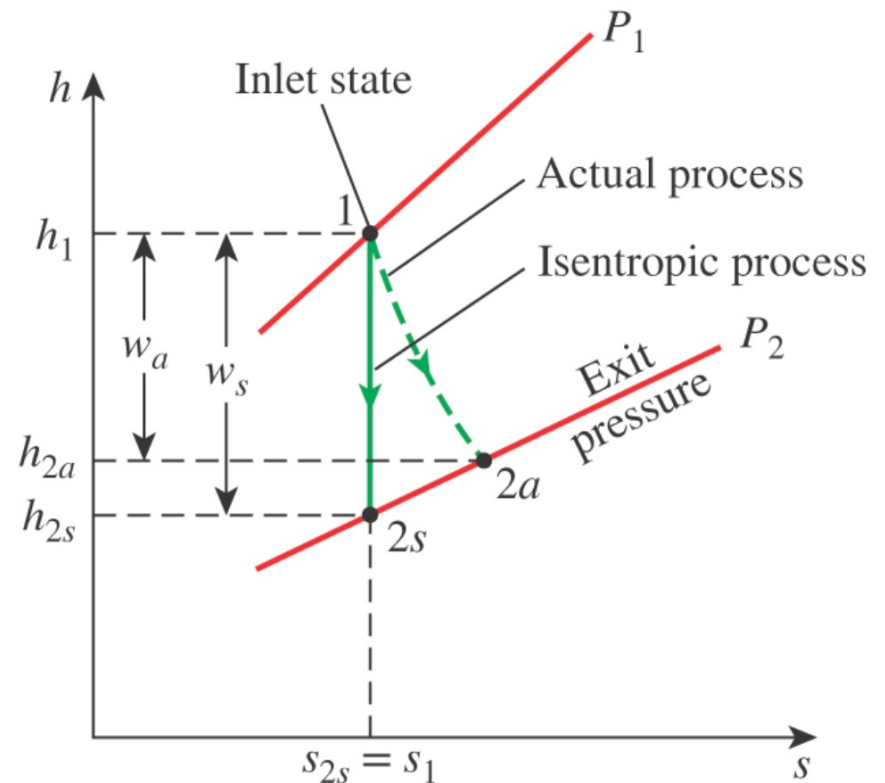- Steady-flow devices deliver the most and consume the least work when the process is reversible:
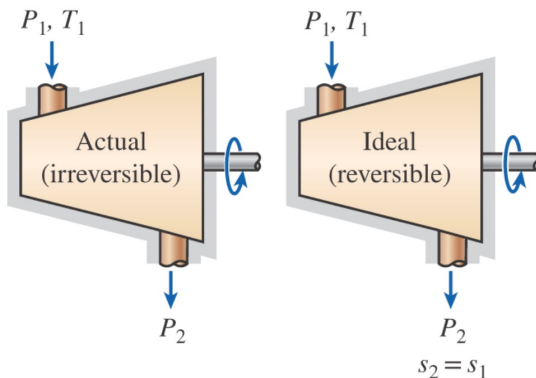
Isentropic Efficiencies

- Isentropic efficiency of a turbine can be written as:

$$\eta_T = \frac{Actual\ turbine\ work}{Isentropic\ turbine\ work} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

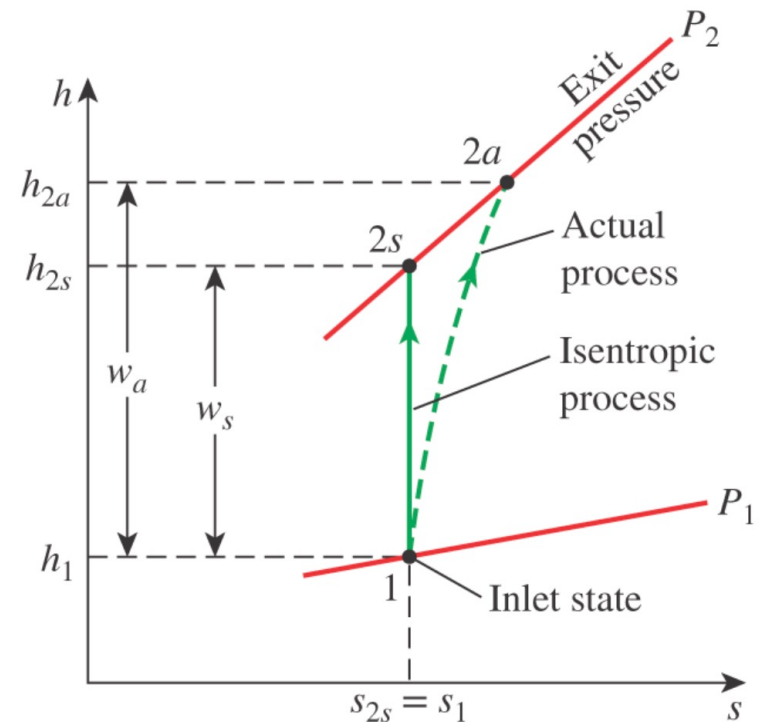# Isentropic Efficiencies

- Isentropic efficiency of compressors and pumps

$$\eta_C = \frac{Isentropic\ compressor\ work}{Actual\ compressor\ work} = \frac{w_s}{w_a}$$

$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

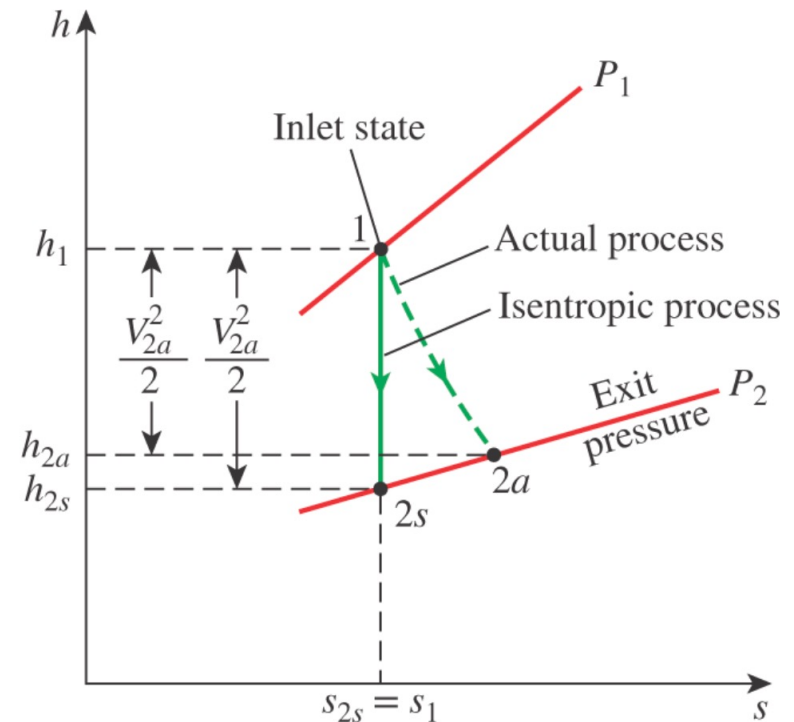# Isentropic Efficiencies

- Isentropic efficiency of nozzles

$$\eta_N = \frac{Actual\ KE\ nozzle\ exit}{Isentropic\ KE\ at\ nozzle\ exit} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

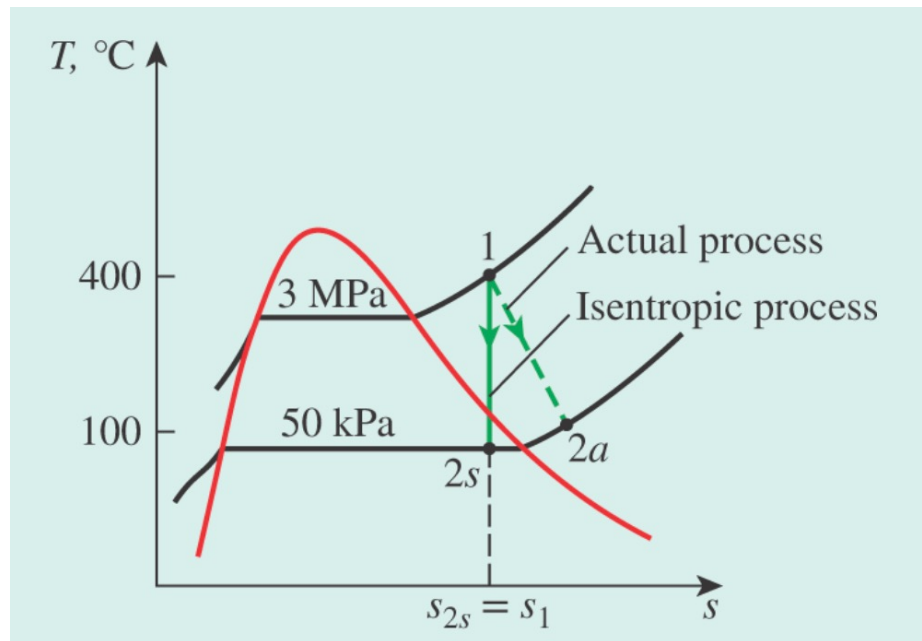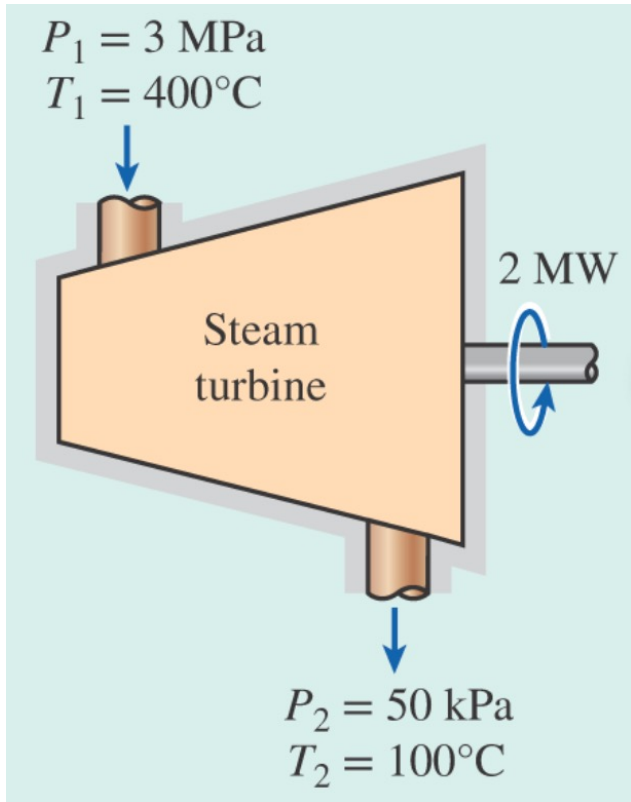$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

CLASS ACTIVITY

Class Activity

- Steam enters an adiabatic turbine steadily at 3 MPa and 400 C and leaves at 50 kPa and 100 C. If the power output of the turbine is 2 MW, determine:
    a) The isentropic efficiency of the turbine
    b) The mass flow rate of the steam flowing through the turbine

Class Activity

- Solution (assumptions):
  - ❑ Steady operating conditions exist
  - ❑ The kinetic and potential energies are negligible

$P_1 = 3$ MPa
$T_1 = 400°C$

2 MW

Steam turbine

$P_2 = 50$ kPa
$T_2 = 100°C$

T, °C

400

3 MPa

1   Actual process

Isentropic process

100   50 kPa

2s   2a

$s_{2s} = s_1$ s

Class Activity

- Solution (Tables):

State 1:
$$\begin{cases} P_1 = 3\ MPa \\ T_1 = 400\ °C \end{cases} \rightarrow \begin{cases} h_1 = 3231.7\ kJ/kg \\ s_1 = 6.9235\ kJ/(kg-K) \end{cases}$$

State 2a:
$$\begin{cases} P_1 = 50\ kPa \\ T_{2a} = 100\ °C \end{cases} \rightarrow h_{2a} = 2682.4\ kJ/kg$$

State 2s:
$$\begin{cases} P_1 = 50\ kPa \\ s_1 = s_2 \end{cases} \rightarrow \begin{cases} s_f = 1.0912\ kJ/(kg-K) \\ s_g = 7.5931\ kJ/(kg-K) \end{cases}$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

$$h_{2s} = h_f + x_{2s} \times h_{fg} = 340.54 + 0.897 \times (2304.7) = 2407.9\ kJ/kg$$

Class Activity

- Solution (a): The isentropic efficiency is:

$$\eta_T = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = 0.667 \; (or \; 66.7\%)$$

Class Activity

- Solution (b): The mass flow rate is:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{a,out} + \dot{m}h_{2a}$$

$$\dot{W}_{a,out} = \dot{m}(h_1 - h_{2a})$$

$$2\,MW\left(\frac{1000\,kJ}{1\,MW}\right) = \dot{m}(3231.7 - 2682.4)\frac{kJ}{kg}$$

$$\dot{m} = 3.64\,kg/s$$

CLASS ACTIVITY

Class Activity

- Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 110 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (assume constant specific heats of air):
  - a) The maximum possible exit velocity
  - b) The exit temperature
  - c) The actual exit velocity of the air

Class Activity

- Solution (assumptions):
  - ❑ Steady operating conditions exist
  - ❑ Air is an ideal gas
  - ❑ The kinetic and potential energies are negligible

$P_1 = 200$ kPa
$T_1 = 950$ K
$V_1 \ll V_2$

Air nozzle
$\eta_N = 0.92$
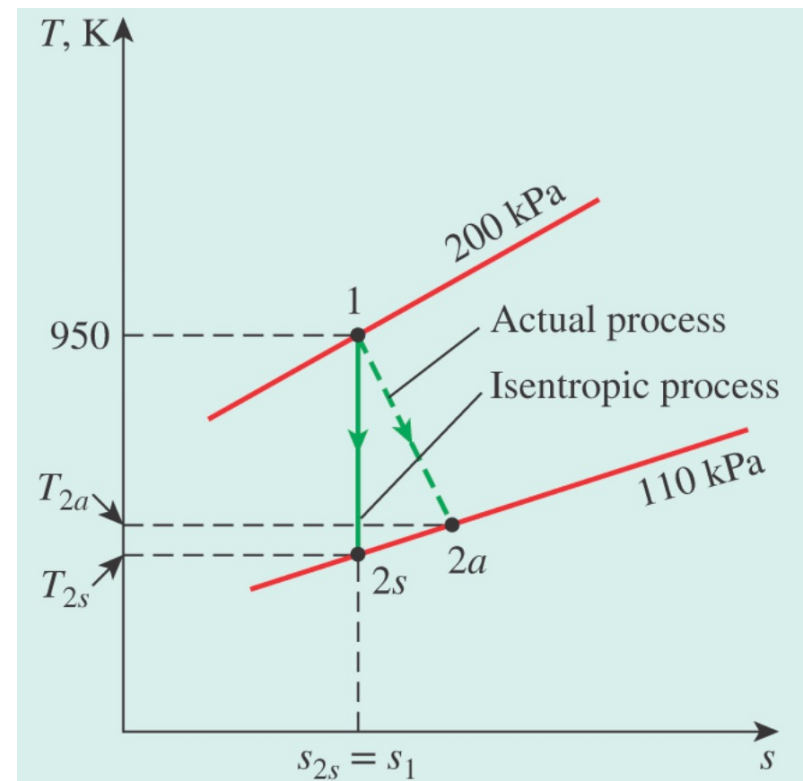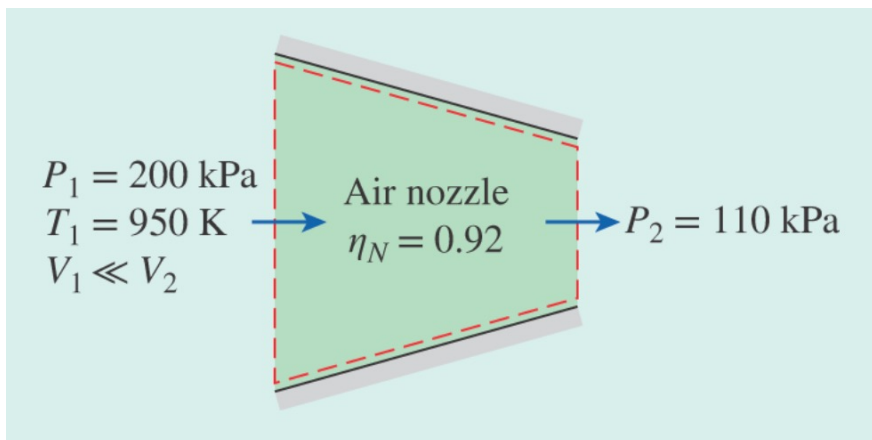
$P_2 = 110$ kPa

Class Activity

- Solution (a):

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_{2s} = T_1 \left(\frac{P_{2s}}{P_1}\right)^{\frac{k-1}{k}} = (950\ K)\left(\frac{110\ kPa}{200\ kPa}\right)^{\frac{(0.349)}{1.349}} = 814\ K$$

Class Activity

- Solution (a):

$$e_{in} = e_{out}$$

$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

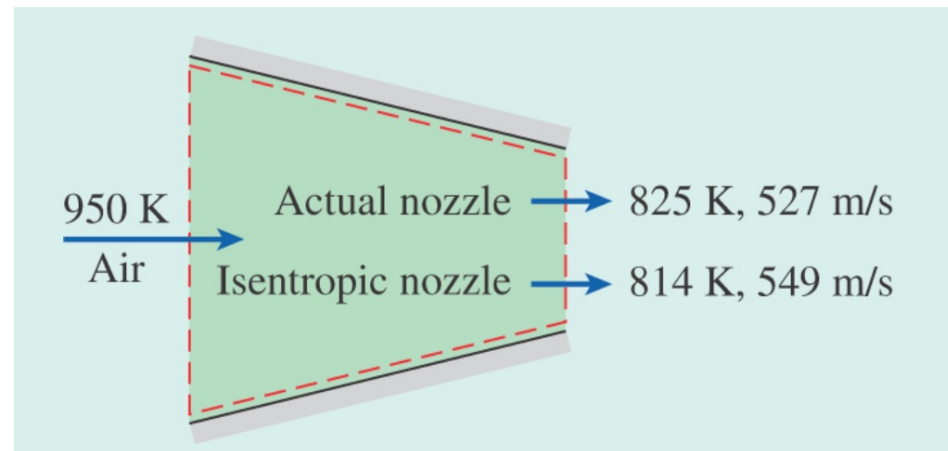$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p,avg}(T_1 - T_2)} = \sqrt{2(1.11\frac{kJ}{kg-K})(950-814)K(\frac{1000\frac{m^2}{s^2}}{1\ kJ/kg})}$$

$$V_{2s} = 549\frac{m}{s}$$

Class Activity

- Solution (b):

$$\eta_N = \frac{Actual\ KE\ nozzle\ exit}{Isentropic\ KE\ at\ nozzle\ exit} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$0.92 = \frac{950 - T_{2a}}{950 - 814} \quad \rightarrow \quad T_{2a} = 825\ K$$

950 K	Actual nozzle	825 K, 527 m/s
Air	Isentropic nozzle	814 K, 549 m/s

Class Activity

- Solution (c):

$$\eta_N = \frac{Actual\ KE\ nozzle\ exit}{Isentropic\ KE\ at\ nozzle\ exit} = \frac{V_{2a}^2}{V_{2s}^2}$$

$$V_{2a} = \sqrt{\eta_N \times V_{2s}^2} = \sqrt{(0.92)\left(549\,\frac{m}{s}\right)^2} = 527\,\frac{m}{s}$$

Chapter 8 Summary

- We did not cover 8-12 (Entropy Balance)