

CAE 208 / MMAE 320: Thermodynamics

Fall 2023

November 16, 2023
Entropy (2)

Built
Environment
Research
@ IIT



*Advancing energy, environmental, and
sustainability research within the built environment*

www.built-envi.com

Dr. Mohammad Heidarinejad, Ph.D., P.E.
Civil, Architectural and Environmental Engineering
Illinois Institute of Technology

muh182@iit.edu

ANNOUNCEMENTS

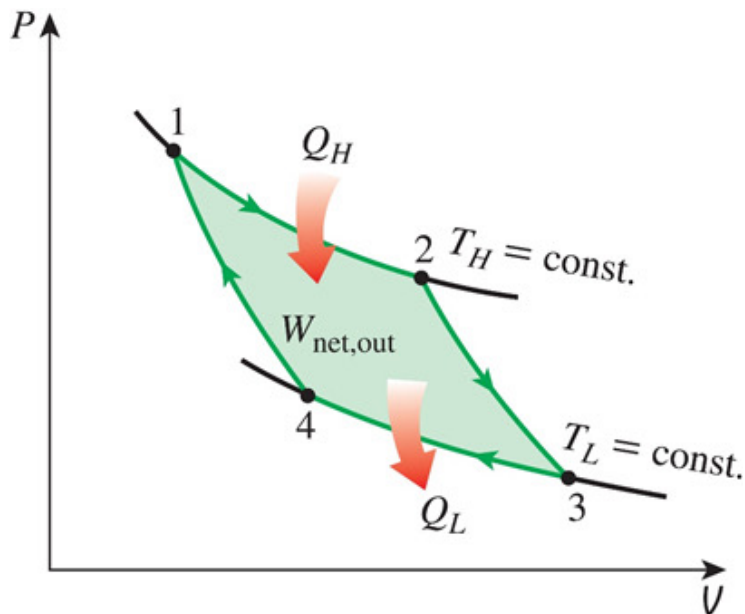
Announcements

- The extended deadline for Assignment 8 is tonight
- Assignment 9 is posted (2-3 days more extension is fine to submit it on Friday after the Thanksgiving)
- The bonus activities document is posted. Please pay attention to the deadlines and also the updates about this task

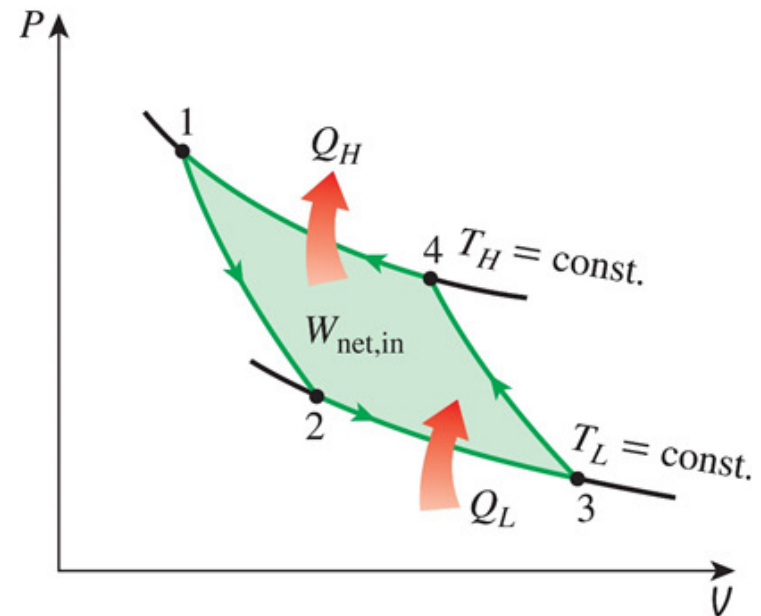
RECAP

Recap

- The Reversed Carnot Cycle
 - The Carnot heat-engine cycle is a totally reversible cycle
 - Therefore, all the processes that comprise it can be reversed, in which case it becomes the Carnot refrigeration cycle



P-V diagram of the Carnot cycle



P-V diagram of the reversed Carnot cycle

Recap

- Any heat engine:

$$\eta_{th} = \frac{W}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

- Any Carnot heat engine:

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H}$$

Recap

- We can say:

$$\eta_{th} = \begin{cases} < \eta_{th,rev} & \textit{irreversible heat engine} \\ = \eta_{th,rev} & \textit{reversible heat engine} \\ > \eta_{th,rev} & \textit{impossible heat engine} \end{cases}$$

QUIZZ

ENTROPY

Entropy

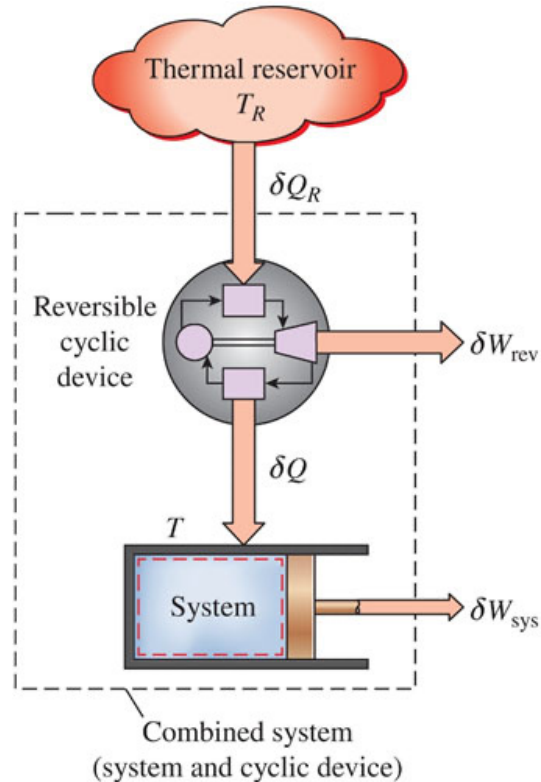
- Objectives of Chapter 8:
 - ❑ Apply the second law of thermodynamics to processes
 - ❑ Define a new property called entropy to quantify the second-law effects
 - ❑ Establish the increase of entropy principle
 - ❑ Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases
 - ❑ Examine a special class of idealized processes, called isentropic processes, and develop the property relations for these processes
 - ❑ Derive the reversible steady-flow work relations
 - ❑ Develop the isentropic efficiencies for various steady-flow devices
 - ❑ Introduce and apply the entropy balance to various systems

Entropy

- While the first law of thermodynamics deals with the property “energy” and “the conservation of it”, the second law leads to the definition of a new property called “entropy”
- Entropy is somewhat an abstract property, and it is difficult to give a physical description of it without considering the microscopic state of the system
- Entropy is best understood and appreciated by studying *its uses in commonly encountered engineering processes*, and this is what we intend to do

Entropy

- The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones



$$\oint \frac{\delta Q}{T} \leq 0$$

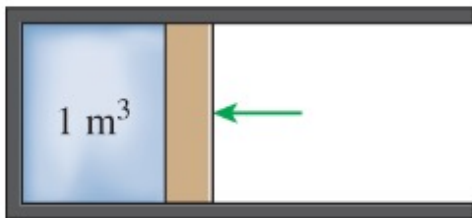
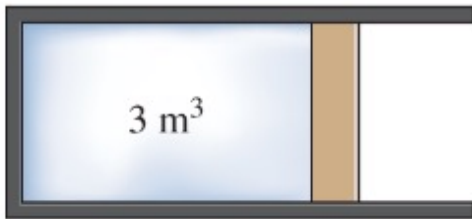
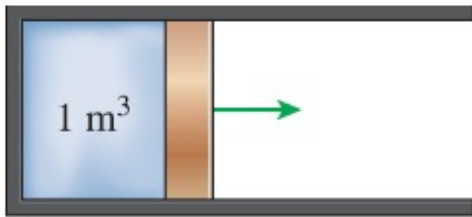
Entropy

- The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones

$$\left(\oint \frac{\delta Q}{T} \right)_{int,rev} = 0$$

Entropy

- Let's try to find out more about entropy with looking at work in a cycle:



$$\oint dV = ?$$

$$\oint dV = \Delta V_{cycle} = 0$$

How about δW ?

Entropy

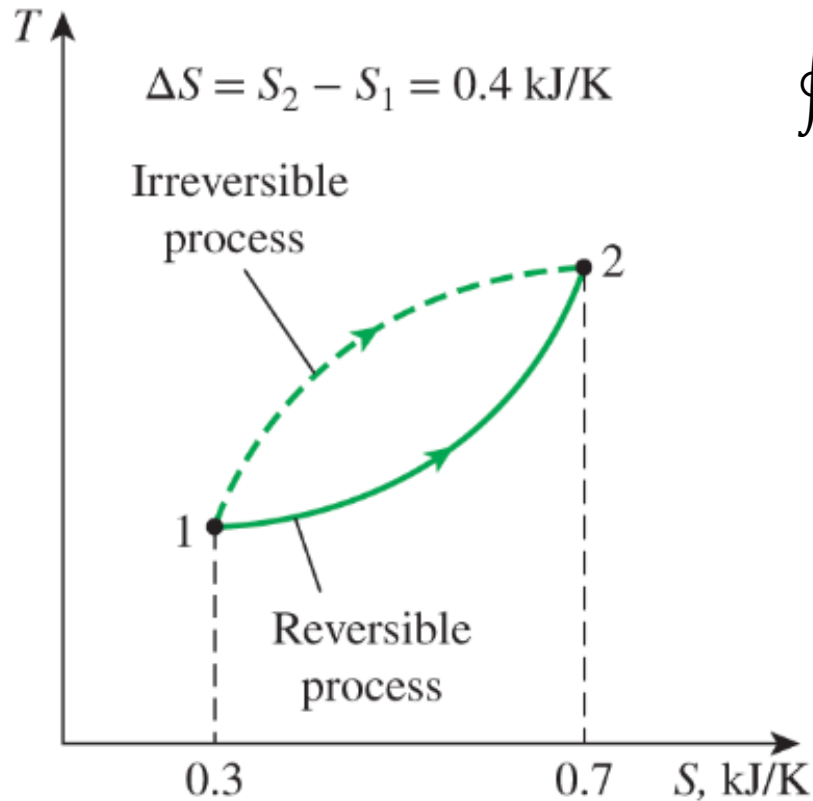
- Let's try to find out more about entropy with looking into a cycle:

$$dS = \oint \frac{\delta Q}{T} \quad \left(\frac{kJ}{K}\right)$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,rev}$$

Entropy

- Pay attention to reversible and irreversible integration:



$$\oint \frac{\delta Q}{T} = \text{if internally reversible} = S$$

Entropy

- A special case: *Internally reversible isothermal* heat transfer processes:

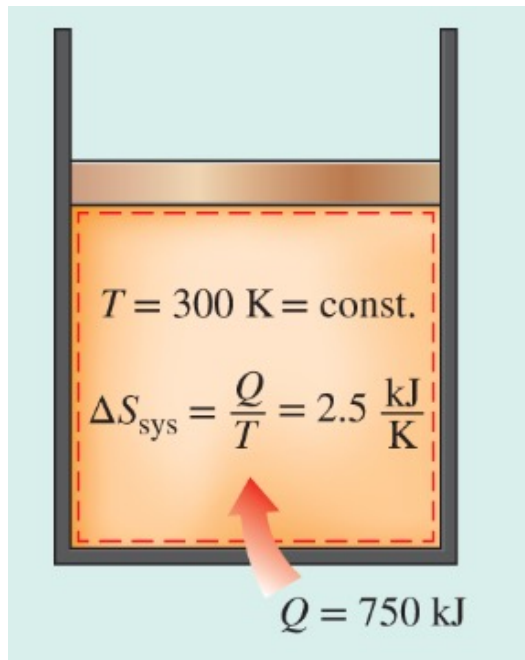
$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{int,rev} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{int,rev} = \frac{1}{T_0} \int_1^2 \delta Q_{int,rev}$$

$$\Delta S_{isothermal} = \frac{Q}{T_0} \quad \left(\frac{kJ}{K} \right)$$

(A reservoir can absorb or supply heat indefinitely at a constant temperature)

Entropy

- For example, a piston-cylinder device contains a liquid-vapor mixture of water at 300 K. During a constant pressure process, 750 kJ of heat is transferred to the water. As a result of the liquid in the cylinder vaporizes. Determine the entropy change of water during this process.



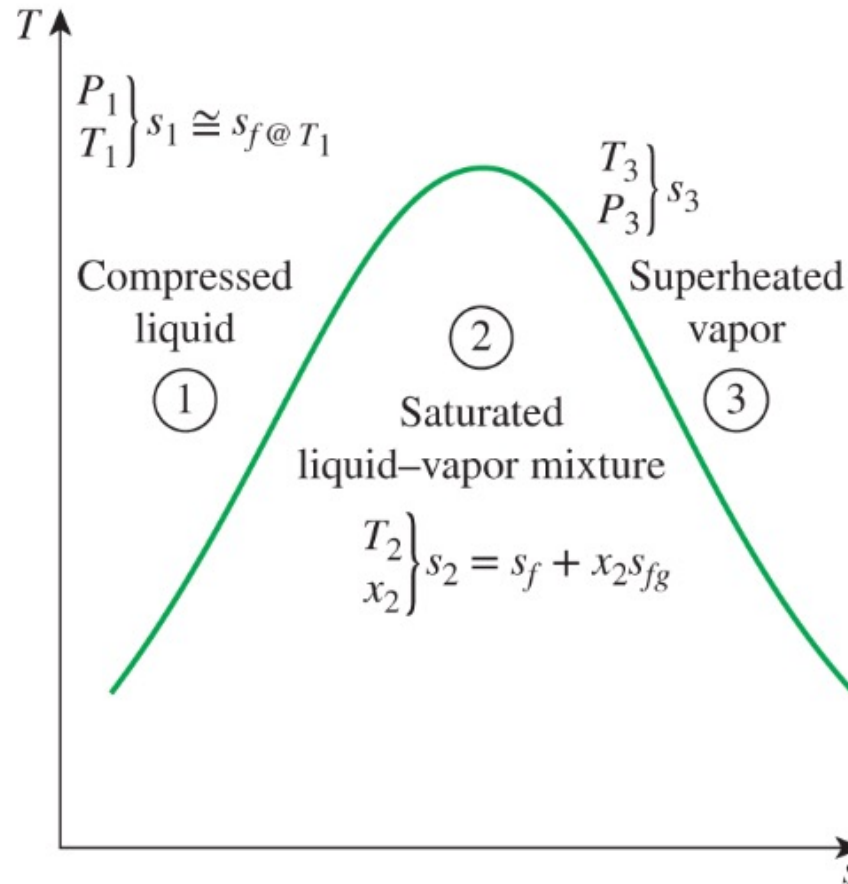
$$\Delta S_{\text{isothermal}} = \frac{Q}{T_0} \quad \left(\frac{\text{kJ}}{\text{K}} \right)$$

$$\Delta S = \frac{750 \text{ kJ}}{300 \text{ K}} = 2.5 \frac{\text{kJ}}{\text{K}}$$

ENTROPY CHANGE OF PURE SUBSTANCES

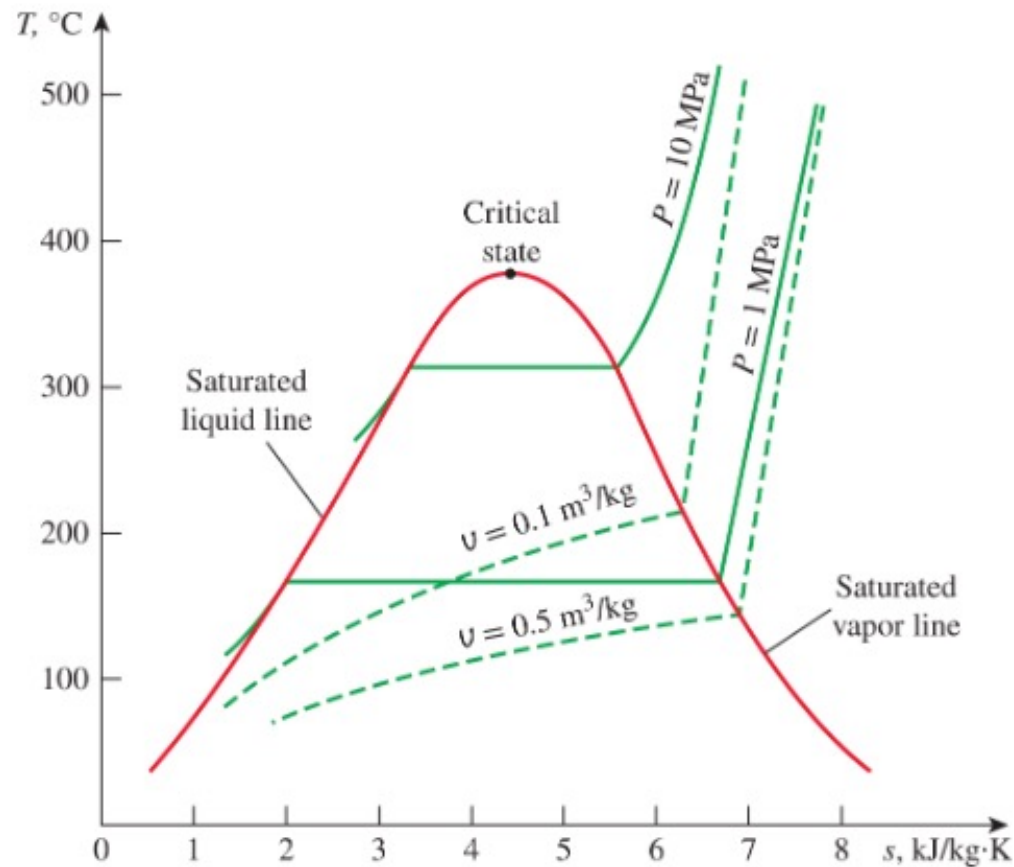
Entropy Change of Pure Substances

- Entropy is a property:



Entropy Change of Pure Substances

- Entropy is a property:



Entropy Change of Pure Substances

- For a closed system ($m = \text{constant}$), during a process we have:

$$\Delta S = m\Delta s = m(s_2 - s_1)$$

THE INCREASE OF ENTROPY PRINCIPLE

The Increase of Entropy Principle

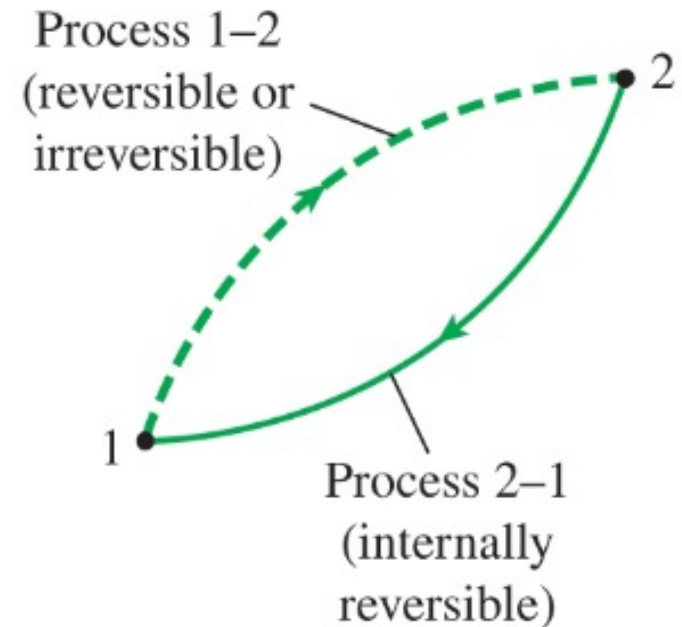
- For processes we can write:

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + \left(\int_2^1 \frac{\delta Q}{T} \right)_{int,rev} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} \leq S_2 - S_1$$

$$\oint \frac{\delta Q}{T} \leq dS$$



The Increase of Entropy Principle

- For entropy, we can say

$$\oint_1^2 \frac{\delta Q}{T} \leq \Delta S$$

$$\Delta S_{\text{sys}} = S_2 - S_1 = \oint_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

The Increase of Entropy Principle

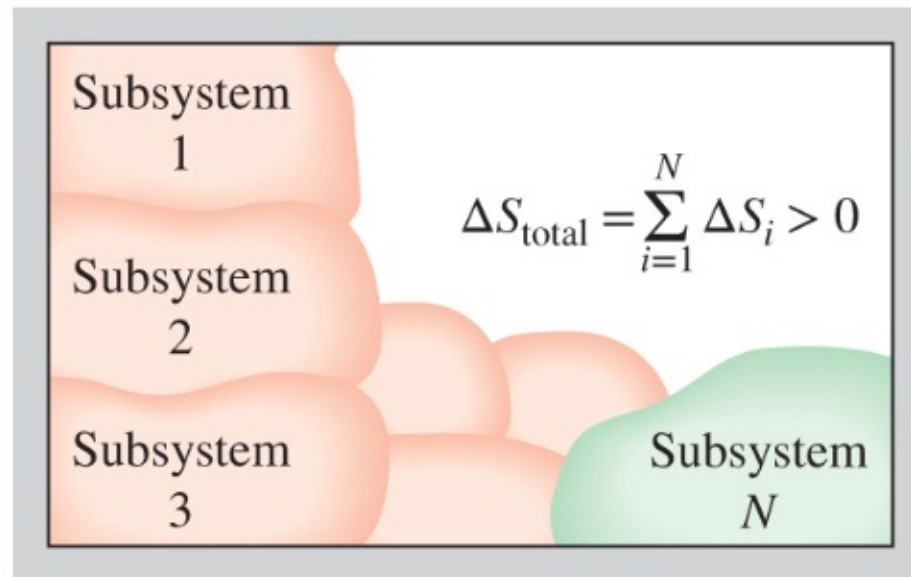
- Increase of entropy principle:

$$\Delta S_{isolated} \geq 0$$

The Increase of Entropy Principle

- Entropy is an extensive property (not entropy per unit mass), so the total entropy of a system is equal to the sum of the entropies of the parts of the system (i.e., an isolated system may consist of any number of subsystem)

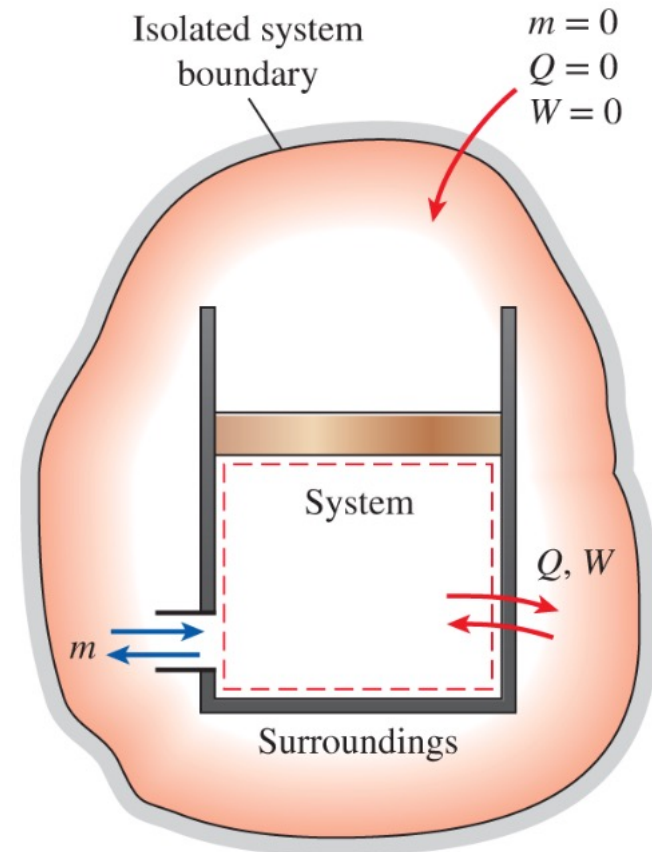
(Isolated)



The Increase of Entropy Principle

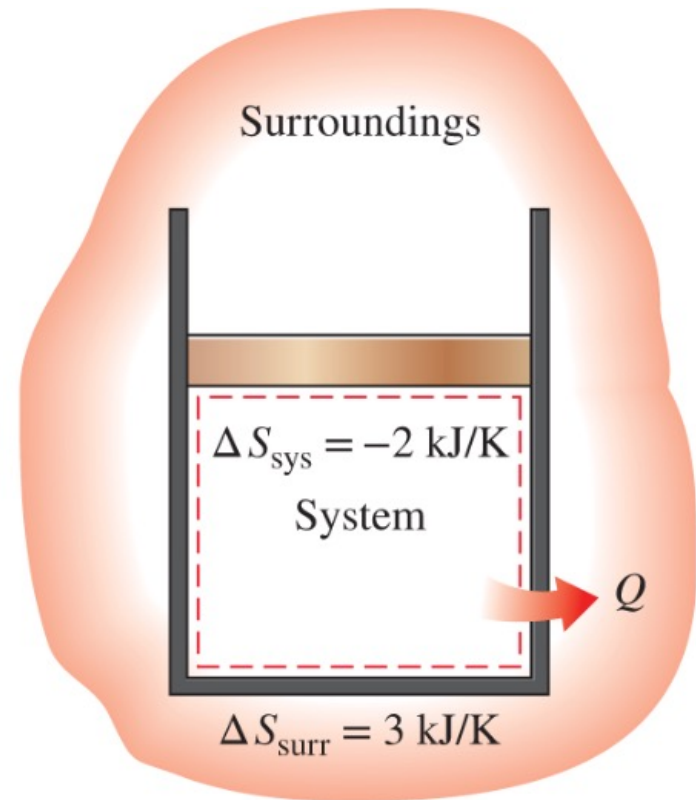
- For an isolated system:

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \geq 0$$



The Increase of Entropy Principle

- For an isolated system:



The Increase of Entropy Principle

- The increase of entropy principle can be summarized as:

$$S_{gen} = \begin{cases} > 0. & \text{irreversible process} \\ = 0 & \text{reversible process} \\ < 0. & \text{impossible process} \end{cases}$$

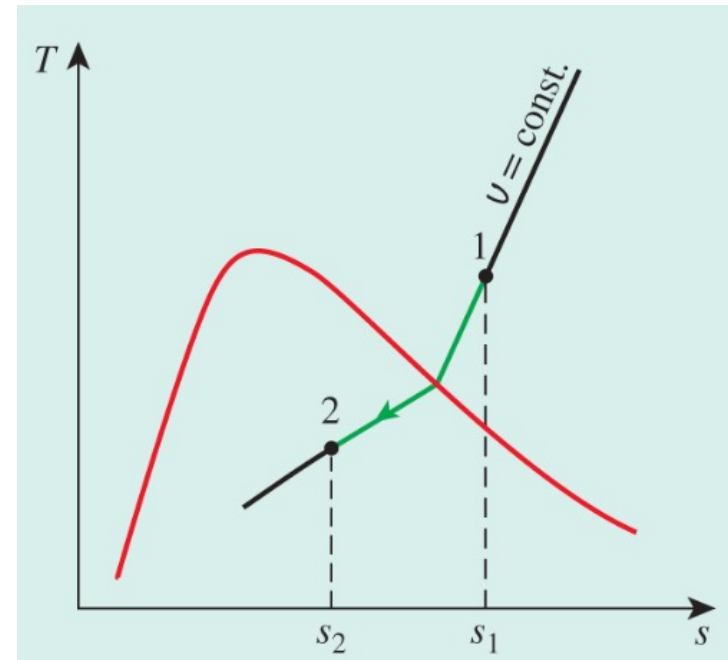
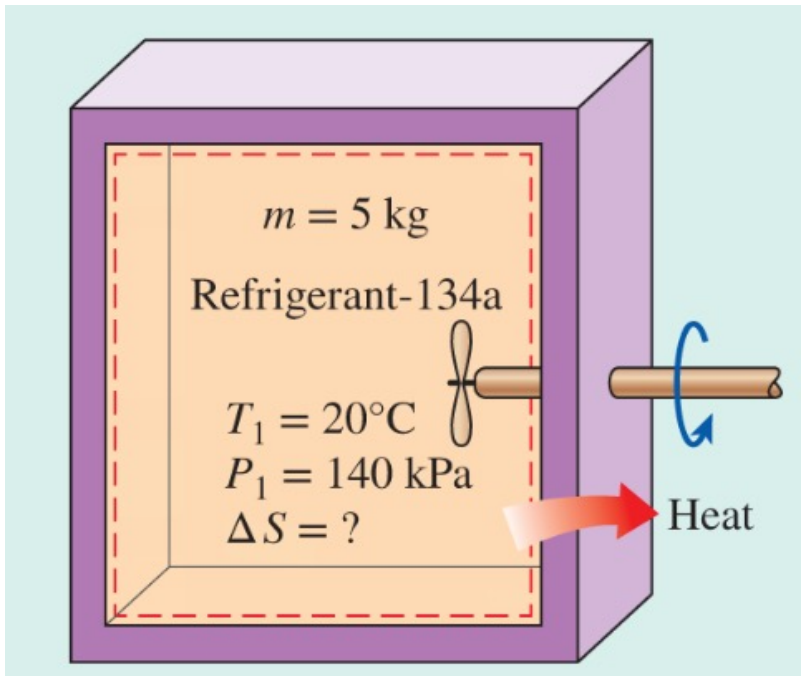
CLASS ACTIVITY

Class Activity

- A rigid tank contains 5-kg of refrigerant 134-a at 20 °C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.

Class Activity

- Solutions (assumptions):
 - Closed system ($m = \text{constant}$)



Class Activity

- Solutions (Calculations):

$$\left\{ \begin{array}{l} P_1 = 140 \text{ kPa} \\ T_1 = 20 \text{ }^\circ\text{C} \end{array} \right. \rightarrow \begin{array}{l} s_1 = 1.0625 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \\ v_1 = 0.16544 \frac{\text{m}^3}{\text{kg}} \end{array}$$

$$\left\{ \begin{array}{l} P_2 = 100 \text{ kPa} \\ v_2 = v_1 \end{array} \right. \rightarrow \begin{array}{l} v_f = 0.0007258 \frac{\text{m}^3}{\text{kg}} \\ v_g = 0.19255 \frac{\text{m}^3}{\text{kg}} \end{array} \quad (v_f < v_2 < v_g)$$

Class Activity

- Solutions (Calculations):

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007258}{0.19255 - 0.0007258} = 0.859$$

$$s_2 = s_f + x_2 s_{fg} = (0.07182) + (0.859)(0.88008) = 0.8278 \frac{kJ}{kg - K}$$

$$\Delta S = m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0625 \frac{kJ}{kg - K}) = -1.173 \text{ kJ/K}$$

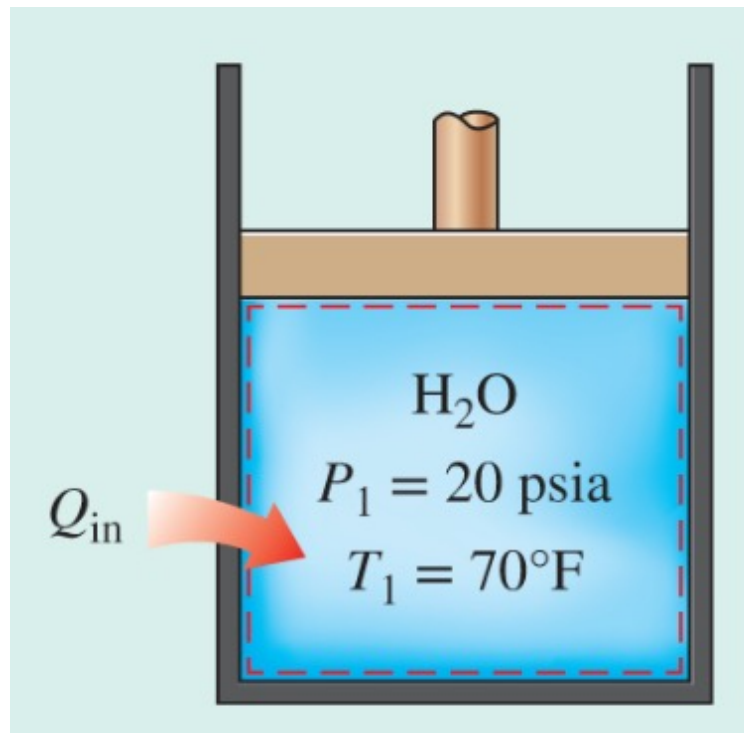
CLASS ACTIVITY

Class Activity

- A piston-cylinder device initially contains 3 lbm of liquid water at 20 psia and 70 °F. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.

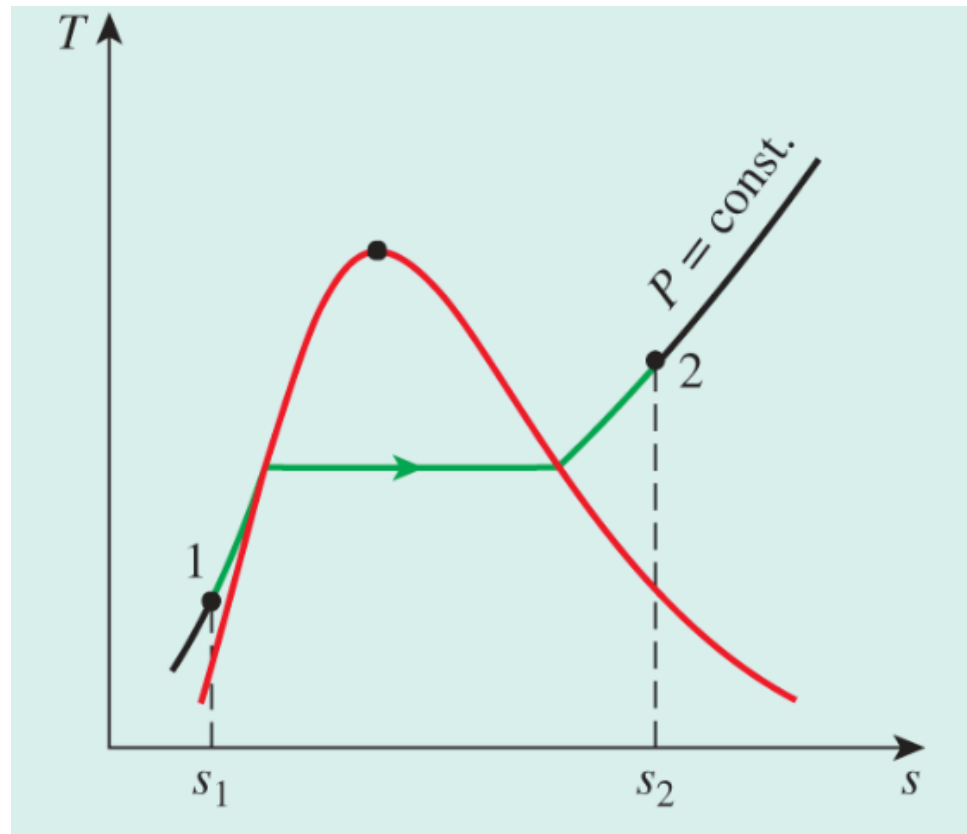
Class Activity

- Solutions (assumptions):
 - ❑ The tank is stationary and thus the kinetic and potential energy changes are zero ($\Delta KE = \Delta PE = 0$)
 - ❑ The process is quasi-equilibrium
 - ❑ The pressure remains constant during this process ($P_1 = P_2$)



Class Activity

- Solutions (processes):



at $70^\circ\text{F} \rightarrow P_{sat} = 0.3632 \text{ psia}$

Class Activity

- Solutions (Calculation):

$$\begin{array}{l} P_1 = 20 \text{ psia} \\ T_1 = 70^\circ F \end{array} \rightarrow \begin{array}{l} s_1 \cong s_f @ 70^\circ F = 0.07459 \frac{\text{Btu}}{\text{lbm} - R} \\ h_1 \cong h_f @ 70^\circ F = 38.08 \frac{\text{Btu}}{\text{lbm}} \end{array}$$

$$\begin{array}{l} P_1 = 20 \text{ psia} \\ \text{Another property (????)} \end{array} \rightarrow$$

Class Activity

- Solutions (Calculation):

$$E_{in} - E_{out} = \Delta E_{system}$$

$$Q_{in} - W_b = \Delta U$$

$$Q_{in} = \Delta H = m(h_2 - h_1)$$

$$3450 \text{ Btu} = (3 \text{ lbm})(h_2 - 38.08 \frac{\text{Btu}}{\text{lbm}})$$

$$h_2 = 1188.1 \frac{\text{Btu}}{\text{lbm}}$$

Class Activity

- Solutions (Calculation):

$$\begin{array}{l} P_1 = 20 \text{ psia} \\ h_2 = 1188.1 \frac{\text{Btu}}{\text{lbm}} \end{array} \rightarrow \begin{array}{l} s_2 = 1.7761 \frac{\text{Btu}}{\text{lbm} - R} \\ \text{(From Table A - 6A - interpolation)} \end{array}$$

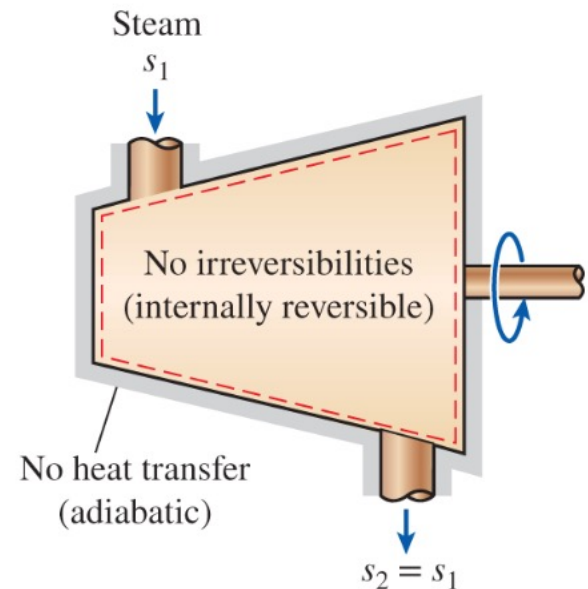
$$\Delta S = m(s_2 - s_1) = (3 \text{ lbm}) \left(1.7761 - 0.07459 \frac{\text{Btu}}{\text{lbm} - R} \right) = 5.105 \frac{\text{Btu}}{R}$$

ISENTROPIC PROCESSES

Isentropic Processes

- The entropy of a fixed mass can be changed by:
 - ❑ Heat Transfer
 - ❑ Irreversibilities
- Entropy of a fixed mass does not change during a process that is internally reversible and adiabatic. During this process entropy remains constant and we call it *isentropic* process

$$\Delta s = 0 \text{ or } s_2 = s_1 \quad \left(\frac{kJ}{kg - K} \right)$$



Isentropic Processes

- A substance will have the same entropy value at the end of the process as it does the beginning if the process is carried out in an isentropic manner
- Many engineering systems or devices such as pumps, turbines, nozzles, and diffusers are essentially adiabatic in their operation, and they perform best when the irreversibilities are minimized (idealized conditions)

Isentropic Processes

- A reversible adiabatic process is necessarily isentropic ($s_1 = s_2$), but an isentropic process is not necessarily a reversible adiabatic process (the entropy increase of a substance during a process as a result of irreversibilities may be offset by a decrease in entropy as a result of heat losses, for example)
- The term isentropic process is customarily used in thermodynamics to imply an internally reversible, adiabatic process

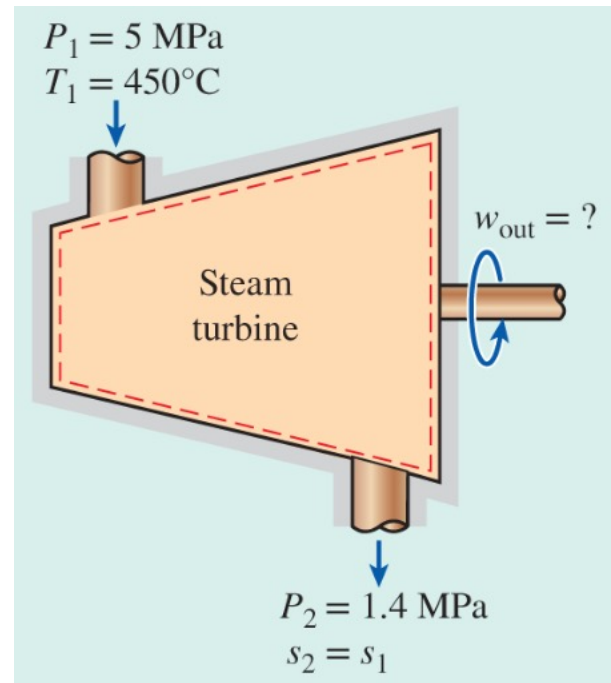
CLASS ACTIVITY

Class Activity

- Steam enters an adiabatic turbine at 5 MPa and 450 °C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.

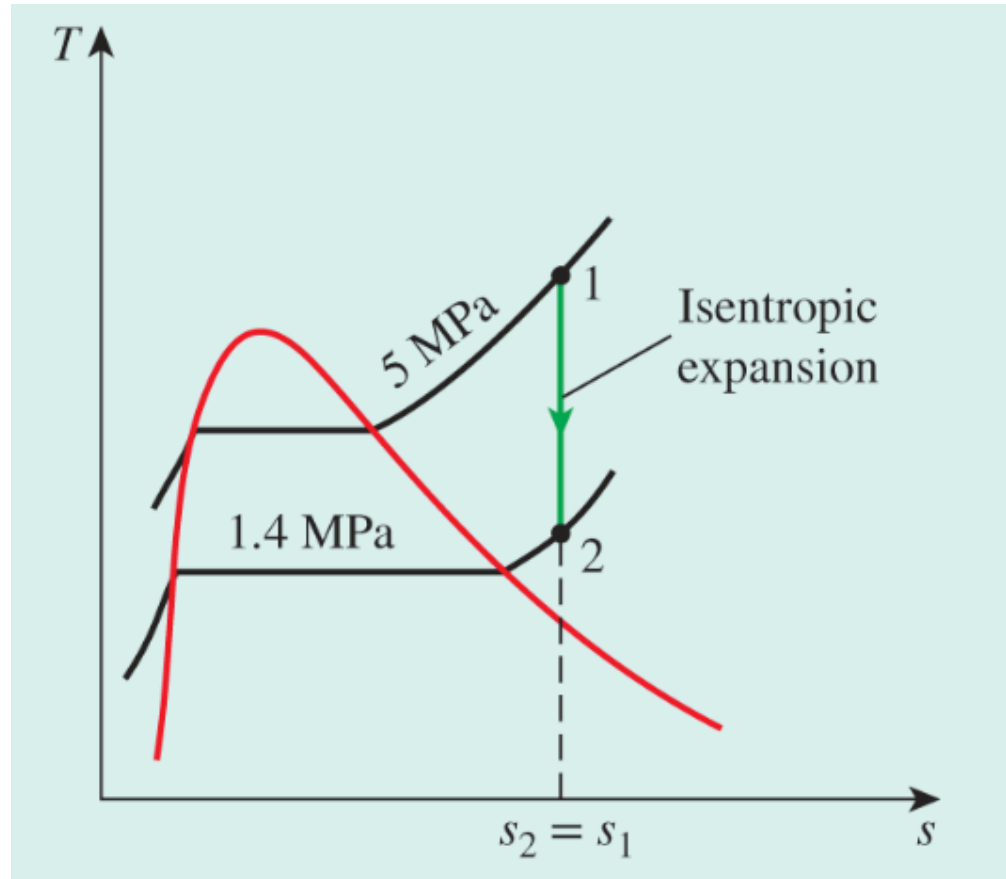
Class Activity

- Solutions (assumptions):
 - ❑ This is a steady flow process (no change with respect to time), meaning $\Delta m_{CV} = 0, \Delta E_{CV} = 0, \Delta S_{CV} = 0$)
 - ❑ The kinetic and potential energy changes are negligible ($\Delta KE = \Delta PE = 0$)
 - ❑ The process is adiabatic and thus there is no heat transfer
 - ❑ The process is reversible



Class Activity

- Solutions (processes):



Class Activity

- Solutions (calculations):

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}h_1 = \dot{W}_{out} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, ke \cong 0, pe \cong 0)$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

Class Activity

- Solutions (calculations):

$$\left\{ \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450 \text{ }^\circ\text{C} \end{array} \right. \rightarrow \begin{array}{l} h_1 = 3317.2 \frac{\text{kJ}}{\text{kg}} \\ s_1 = 6.8210 \frac{\text{kJ}}{\text{kg} - \text{K}} \end{array}$$

$$\left\{ \begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array} \right. \rightarrow h_2 = 2967.4 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_{out} = h_1 - h_2 = 3317.2 - 2967.4 = 349.8 \frac{\text{kJ}}{\text{kg}}$$

PROPERTY DIAGRAMS INVOLVING ENTROPY

Property Diagrams Involving Entropy

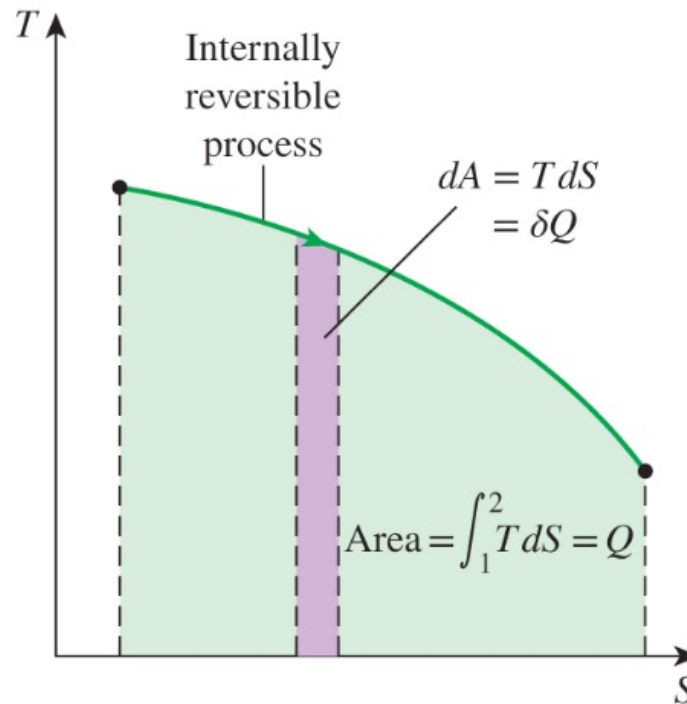
- Property diagrams serve as great visual aids in the thermodynamic analysis of processes
- Based on the 2nd law, we can plot new diagrams that involve entropy:
 - Temperature-entropy
 - Enthalpy-entropy

Property Diagrams Involving Entropy

- We can rearrange our entropy equation:

$$\delta Q_{int,rev} = T dS$$

$$Q_{int,rev} = \int_1^2 T dS \quad (kJ)$$

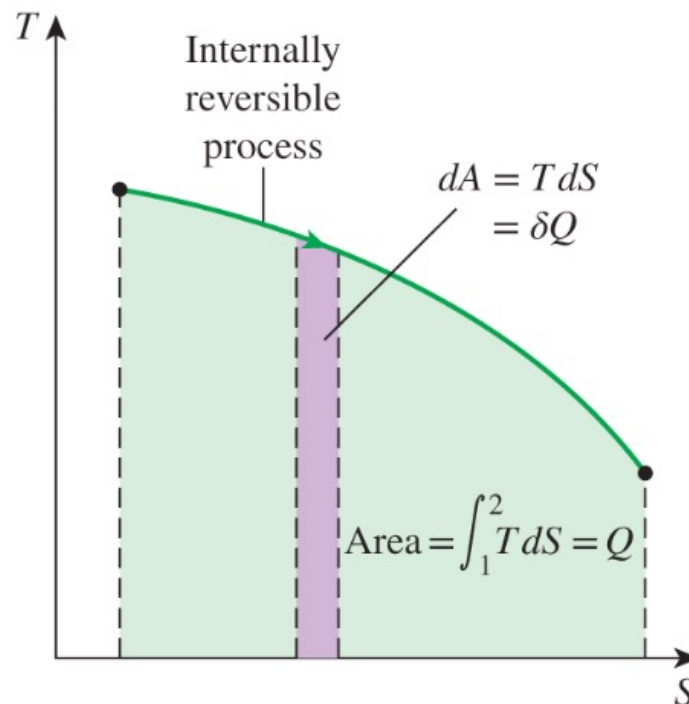


(The area under the process curve on a T-S diagram represents heat transfer during an internally reversible process)

Property Diagrams Involving Entropy

- We can use the per-unit mass equation:

$$\delta q_{int,rev} = T ds \qquad q_{int,rev} = \int_1^2 T ds \quad (kJ)$$



(The area under the process curve on a T-S diagram represents heat transfer during an internally reversible process)

Property Diagrams Involving Entropy

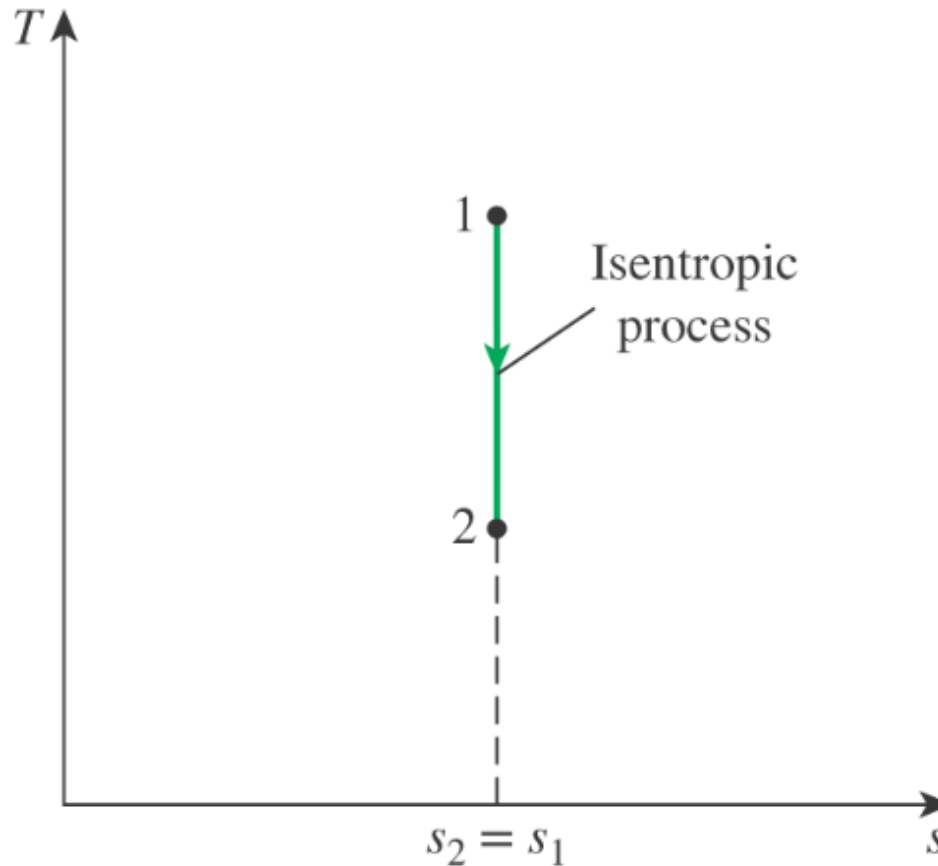
- One special case (internally reversible isothermal process):

$$Q_{int,rev} = T_0 \Delta S$$

$$q_{int,rev} = T_0 \Delta s$$

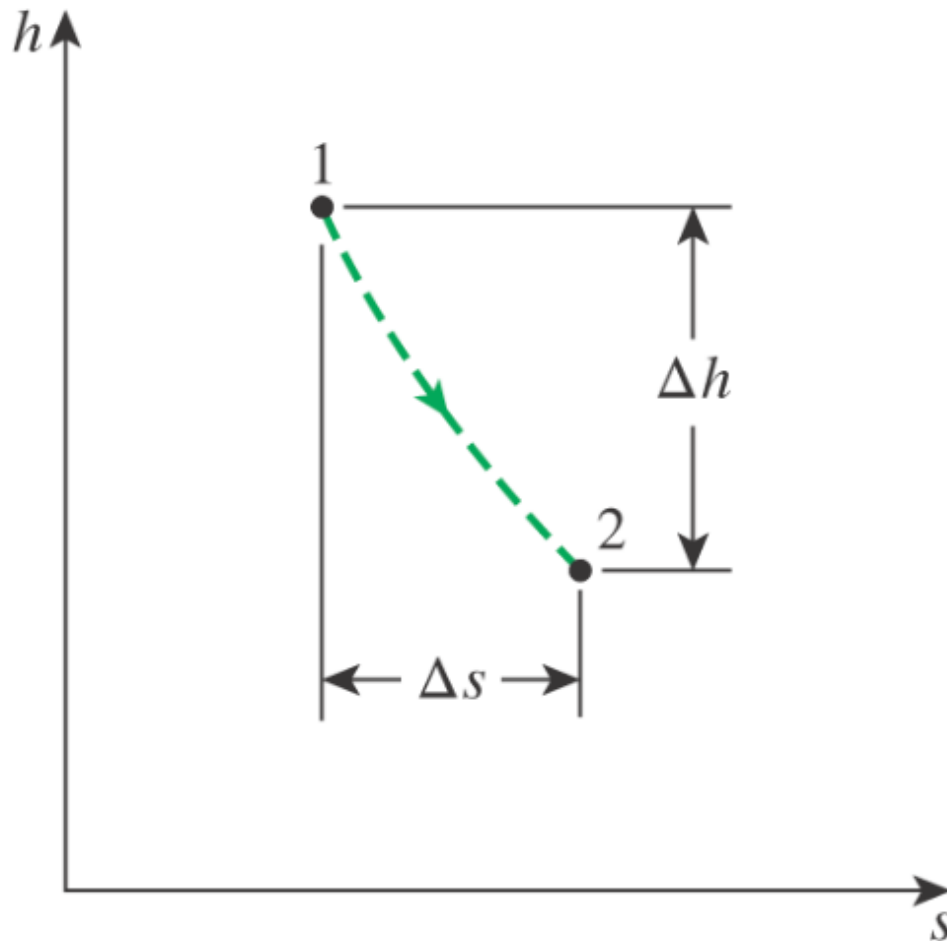
Property Diagrams Involving Entropy

- T-s diagram for an isentropic process:



Property Diagrams Involving Entropy

- h-s diagram (could be helpful for steady flow of devices such as nozzles, compressors, turbines):



Property Diagrams Involving Entropy

- T-s diagram of water is given in the appendix:

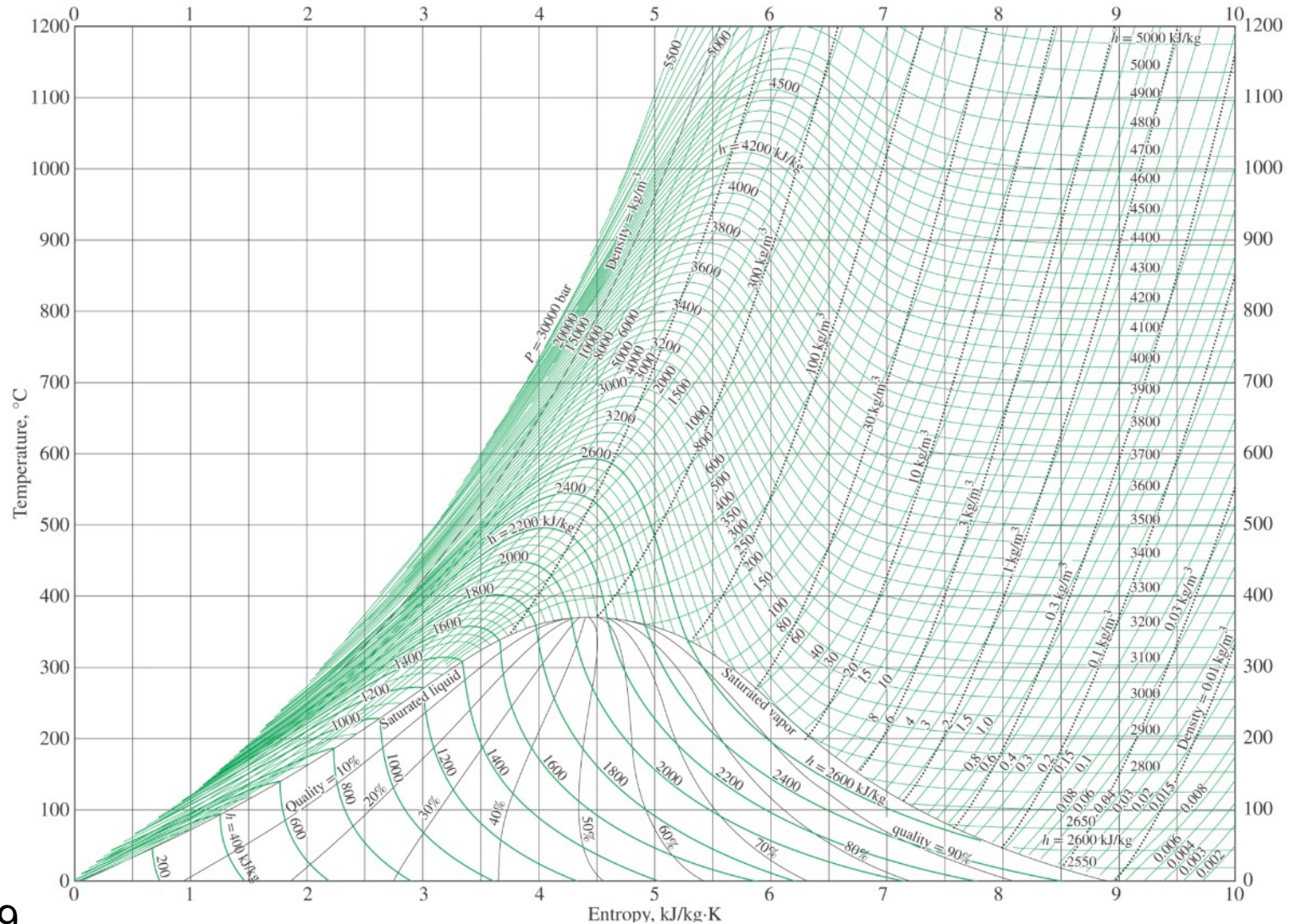


Figure A-9

Property Diagrams Involving Entropy

- h-s diagram of steam is given in the appendix:

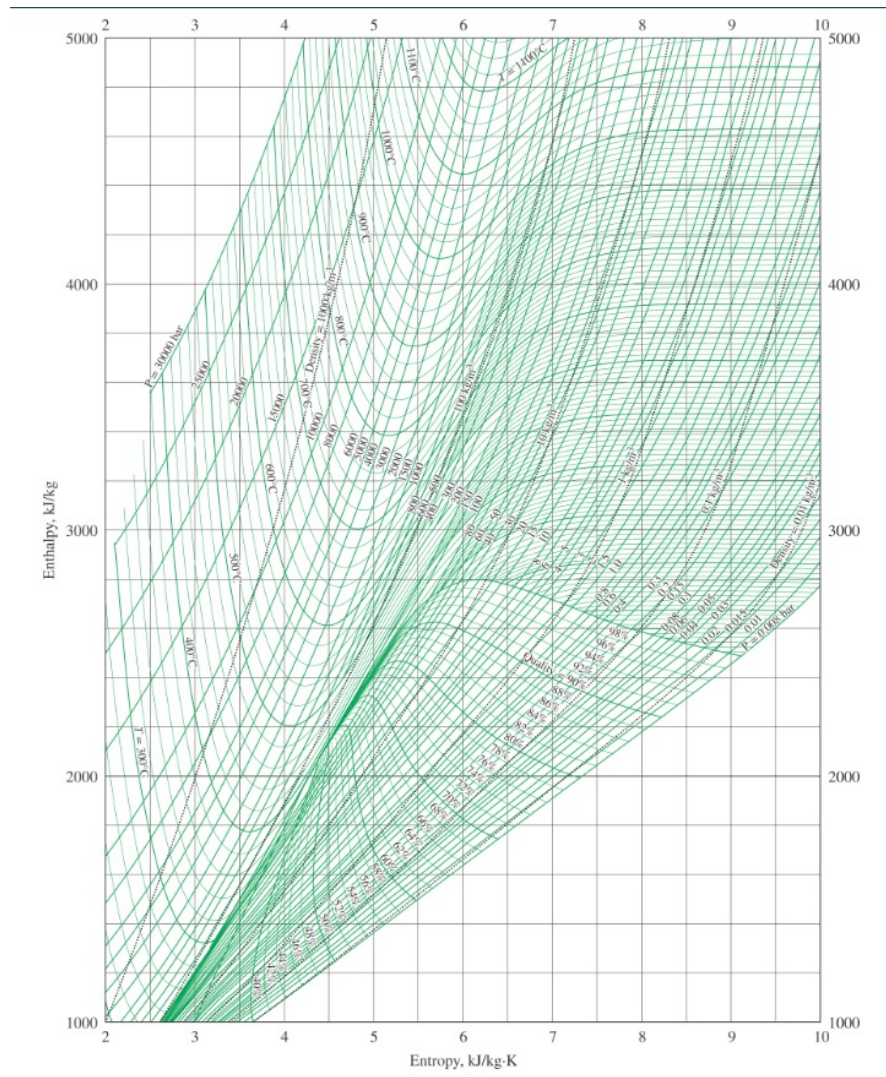


Figure A-10

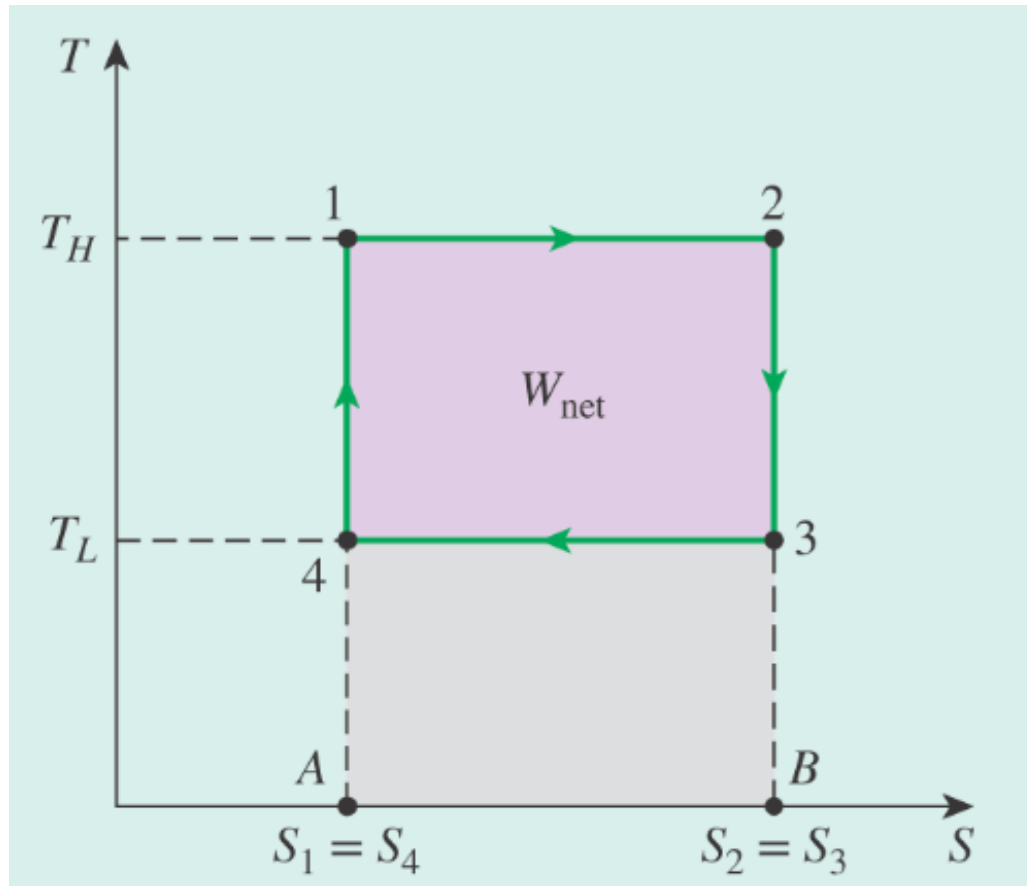
CLASS ACTIVITY

Class Activity

- Show the Carnot cycle on a T-S diagram and indicate the areas that represent the heat supplied and rejected and the network in the diagram.

Class Activity

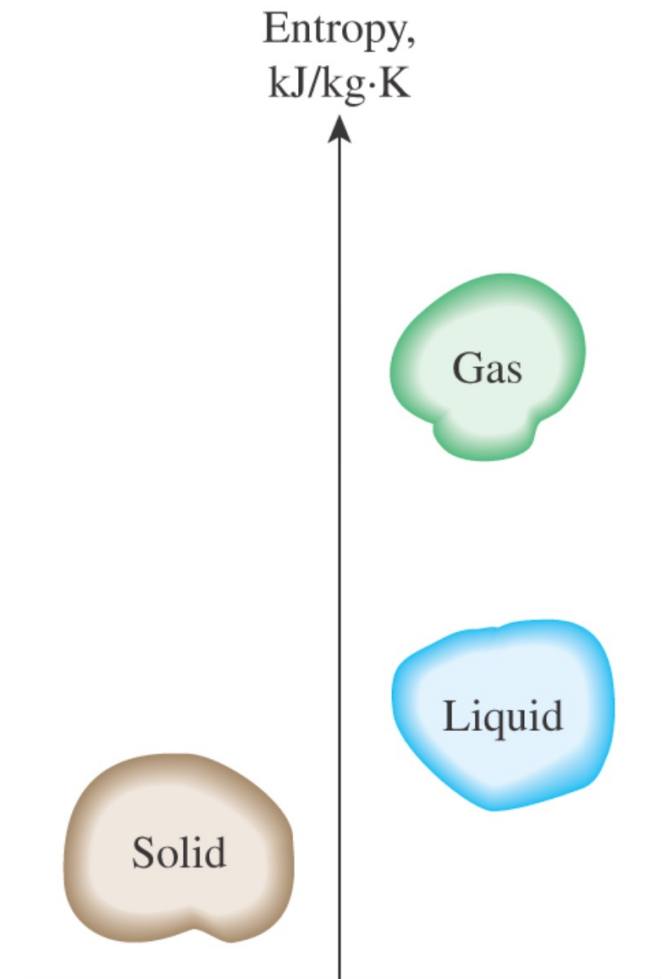
- Solution:



WHAT IS ENTROPY

What Is Entropy

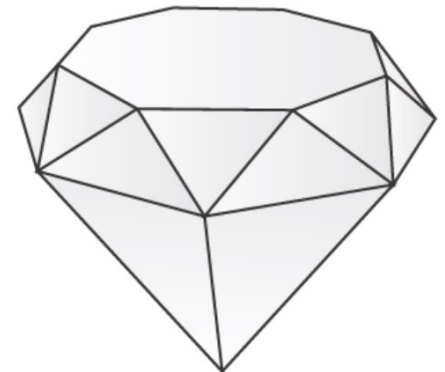
- Entropy can be viewed as a measure of molecular disorder or molecular randomness (lowest in solid phase and highest in the gas phase)



What Is Entropy

- Oscillations in solid phase fade as temperature is decreased, and the **molecules supposedly become motionless at absolute zero**. This represents a state of ultimate molecular order (and ***minimum energy***).
- The entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant (**3rd law of thermodynamics**)

Pure crystal
 $T = 0 \text{ K}$
Entropy = 0

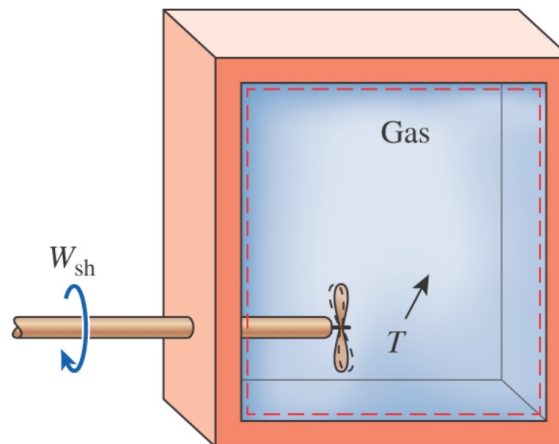
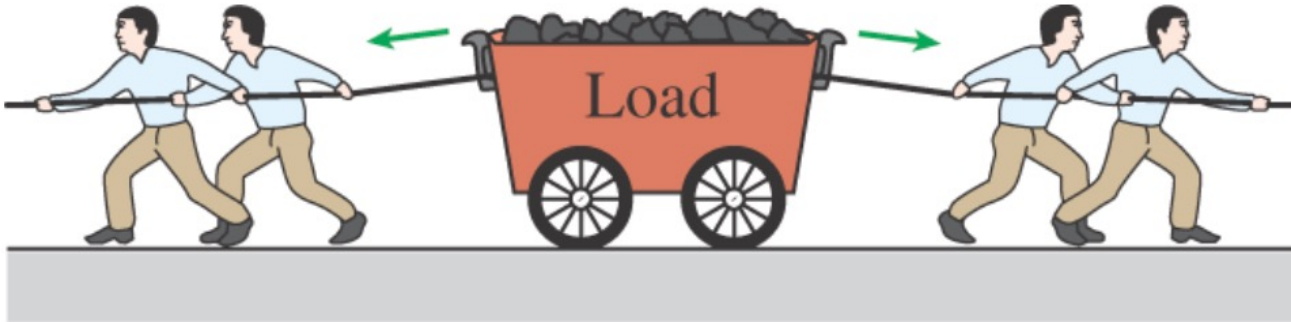


What Is Entropy

- The third law of thermodynamics provide an absolute reference point for the determination of entropy
- The entropy determined relative to this point is called absolute entropy and it is extremely useful in the thermodynamics analysis of entropy
- The entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature

What Is Entropy

- A few examples:



THE TDS RELATIONS

The T ds Relations

- The first T ds (or Gibbs) equation (closed stationary system):

$$\delta Q_{int,rev} - \delta W_{int,rev,out} = dU$$

$$TdS = dU + PdV \quad (kJ)$$

$$\delta Q_{int,rev} = Tds$$

$$Tds = du + Pdv \quad \left(\frac{kJ}{kg}\right)$$

$$\delta W_{int,rev,out} = PdV$$

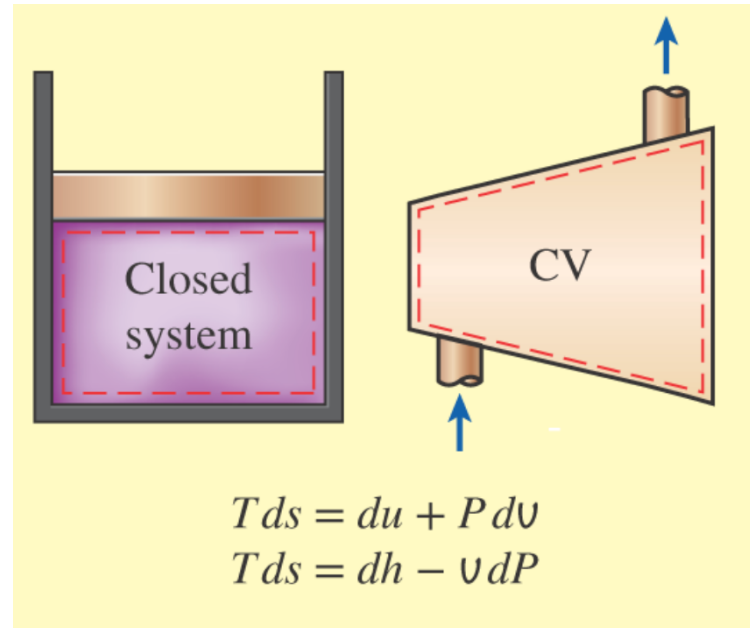
$$\left. \begin{array}{l} Tds = du + Pdv \\ h = u + Pv \rightarrow dh = du + Pdv + vdP \end{array} \right\} \rightarrow Tds = dh - vdP$$

The T ds Relations

- Use the first T ds (or Gibbs) equation to solve for entropy changes

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = \frac{du}{T} - \frac{vdP}{T}$$



ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Entropy Change of Liquids and Solids

- Liquids and solids can be approximated as incompressible substances ($dv \cong 0$ & $c_p = c_v = c_p = c$):

$$ds = \frac{du}{T} - \frac{vdP}{T}$$

$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{avg} \ln\left(\frac{T_2}{T_1}\right)$$

$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{avg} \ln\left(\frac{T_2}{T_1}\right) = 0 \quad \rightarrow \quad T_2 = T_1 \quad \text{(For isentropic)}$$

CLASS ACTIVITY

Class Activity

- Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or ~ -82 °C) and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperature and pressure are given in Table below. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa.
 - Using tabulated properties
 - Approximating liquid methane as an incompressible substance?

Properties of liquid methane

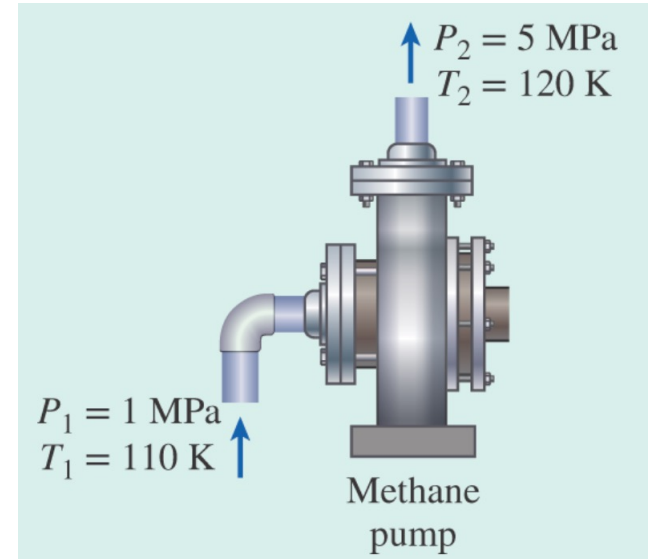
Temp., T , K	Pressure, P , MPa	Density, ρ , kg/m ³	Enthalpy, h , kJ/kg	Entropy, s , kJ/kg · K	Specific heat, c_p , kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

Class Activity

- Solutions (a):

$$\left. \begin{array}{l} P_1 = 1 \text{ MPa} \\ T_1 = 110 \text{ K} \end{array} \right\} \rightarrow \begin{array}{l} s_1 = 4.875 \frac{\text{kJ}}{\text{kg-K}} \\ c_{p1} = 3.471 \frac{\text{kJ}}{\text{kg-K}} \end{array}$$

$$\left. \begin{array}{l} P_2 = 5 \text{ MPa} \\ T_2 = 120 \text{ K} \end{array} \right\} \rightarrow \begin{array}{l} s_1 = 5.145 \frac{\text{kJ}}{\text{kg-K}} \\ c_{p1} = 3.486 \frac{\text{kJ}}{\text{kg-K}} \end{array}$$



$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = 0.270 \frac{\text{kJ}}{\text{kg-K}}$$

Class Activity

- Solutions (b):

$$c_{avg} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \frac{kJ}{kg - K}$$

$$\Delta S = c_{avg} \ln\left(\frac{T_2}{T_1}\right) = \left(3.4785 \frac{kJ}{kg - K}\right) \ln\left(\frac{120K}{110K}\right) = 0.303 \frac{kJ}{kg - K}$$

Class Activity

- Solutions (error):

$$Error = \frac{\Delta S_{actual} - \Delta S_{ideal}}{\Delta S_{actual}} = \frac{|0.27 - 0.303|}{0.270} = 0.122 \text{ (or 12.2\%)}$$