CAE 208 / MMAE 320: Thermodynamics

Fall 2023

October 19, 2023

Mass & energy analysis of control volumes (2)

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ANNOUNCEMENTS

Announcements

- Assignment 5 is due tonight
- Assignment 6 is posted (due next Thursday)

Announcements

- Next week will start talking about the bonus activities:
  - ❑ Using Python for thermodynamics calculations
  - ❑ Using Energy Equation Solver (EES) for thermodynamics calculations
  - ❑ Hands on activities

RECAP

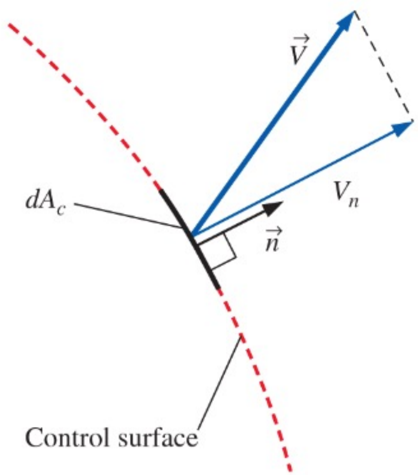
Recap

- In using control volumes, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume

Recap

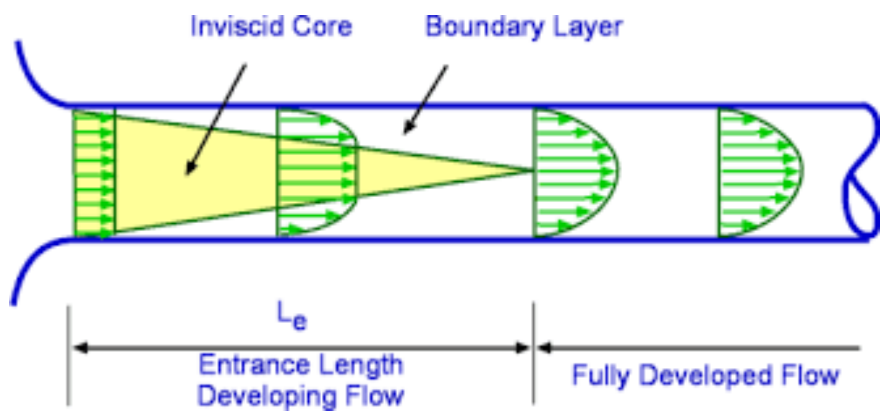
- Mass and volume flow rates

$$\dot{m} = \int \delta\dot{m} = \int \rho V_n dA_c$$



$$\dot{m} = \rho V_{avg} A_c$$

$$\dot{m} = \rho\dot{\forall} = \frac{\dot{\forall}}{v}$$



Recap

- The conservation of mass for a control volume:

$$\Delta m_{CV} = m_{in} - m_{out}$$

$$\Delta m_{CV} = m_{final} - m_{initial}$$

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

These equations are often referred to as the ***mass balance*** *and are applicable to any control volume undergoing any kind of process.*

Recap

- We can write the net flow rate:

$$\frac{d}{dt}\int_{CS} \rho d\forall + \int_{CS} \rho\left(\vec{V}.\vec{n}\right) dA = 0$$

$$\frac{d}{dt}\int_{CS} \rho d\forall = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

Recap

- Steady–flow for a single stream:

$$\dot{m}_1 = \dot{m}_2 \qquad \rightarrow \qquad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

- Steady, incompressible flow (single stream):

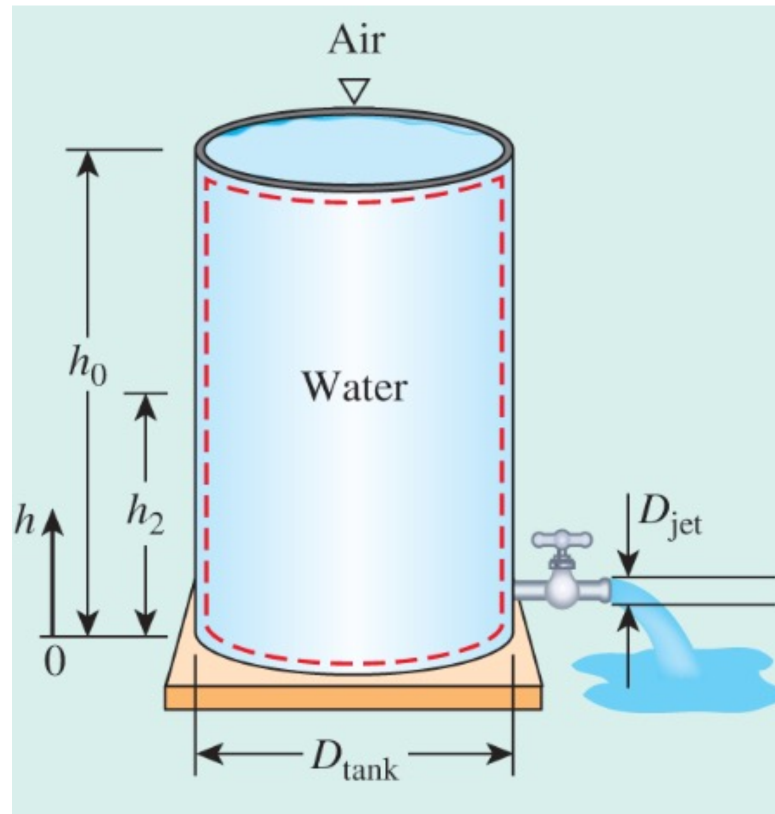
$$\dot{\forall}_1 = \dot{\forall}_2 \quad \rightarrow \quad V_1 A_1 = V_2 A_2$$

CLASS ACTIVITY

Class Activity

- A 4-ft high, 3-ft diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams.

The average velocity of the jet is approximated as $V = \sqrt{2gh}$ where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop 2 ft from the bottom.



Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m_{CV} = \rho\forall = \rho A_{tank} h$$

Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m}_{in} = 0$$

Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m}_{out} = (\rho V A)_{out} = \rho\sqrt{2gh}\,A_{jet}$$

Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \frac{d}{dt}(\rho A_{tank} h)$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \rho(\frac{\pi D_{tank}^2}{4})\frac{dh}{dt}$$

$$dt = \frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

Class Activity

- Solution:

$$dt = -\frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

$$\begin{cases} t = 0 \;\rightarrow \qquad h = h_0 \\ t = t \;\rightarrow \qquad h = h_2 \end{cases}$$

$$\int_0^t dt = -\frac{D_{tank}^2}{D_{jet}^2\sqrt{2g}}\int_{h_1}^{h_2}\frac{dh}{\sqrt{h}}$$

$$t = \frac{\left(\sqrt{h_0} - \sqrt{h_2}\right)}{\sqrt{\frac{g}{2}}}\left(\frac{D_{tank}}{D_{jet}}\right)^2$$

Class Activity

- Solution:

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{2\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times 12\ in}{0.5\ in}\right)^2 = 757\ s = 12.6\ min$$

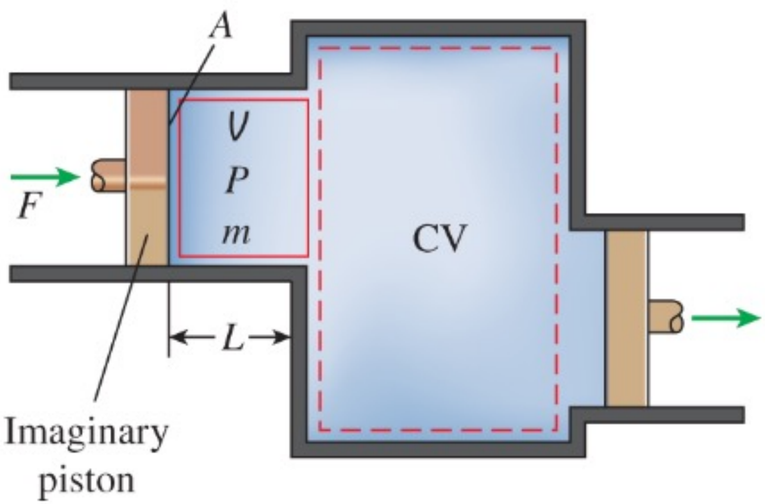
What does the entire flow exit the system?

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{0\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times 12\ in}{0.5\ in}\right)^2 = 43.1\ min$$

FLOW WORK AND ENERGY OF FLOWING FLUID

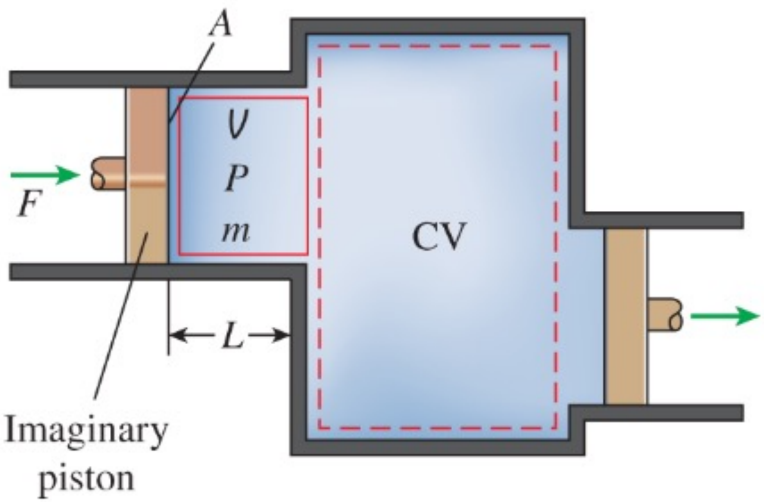
Flow Work and Energy of Flowing Fluid

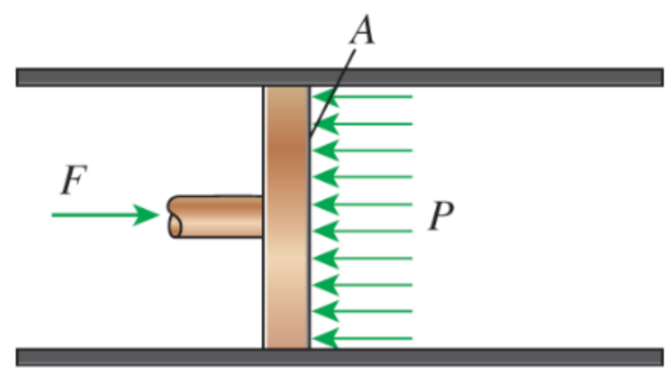
- Flow work (or flow energy): The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume



Flow Work and Energy of Flowing Fluid

- If the fluid pressure is P and the cross-sectional area of the fluid element is A, the force applied on the fluid by the imaginary piston is:





$$F = PA$$

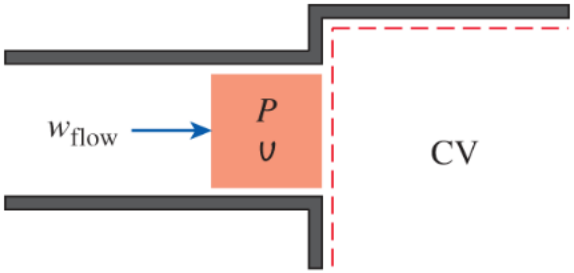
Flow Work and Energy of Flowing Fluid

- To push the entire fluid element into the control volume, this force must act through a distance L

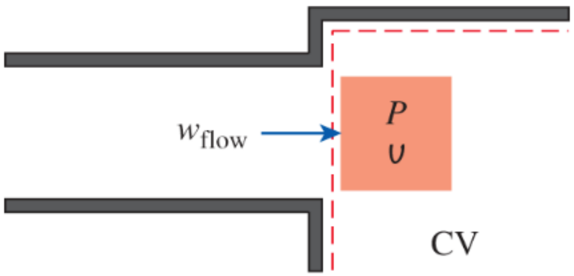
$$F = PA$$

$$W_{flow} = FL = PAL = P\forall$$

$$w_{flow} = Pv$$



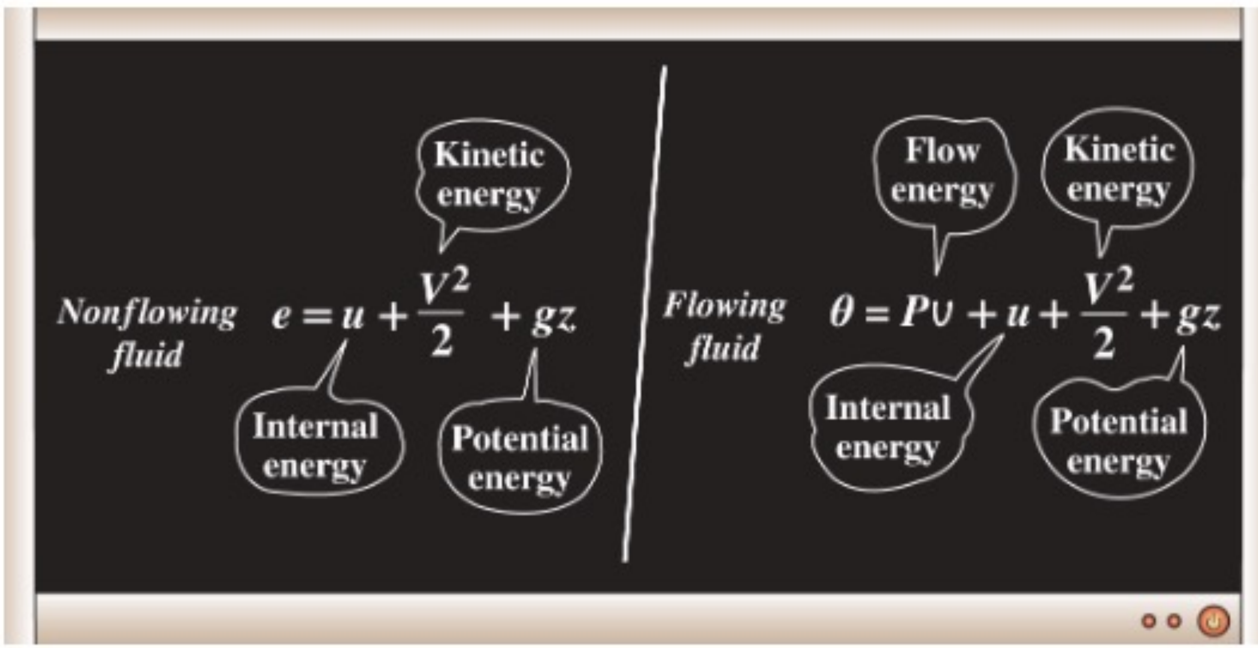
(a) Before entering



(b) After entering

Flow Work and Energy of Flowing Fluid

- Total energy of a flowing fluid is:



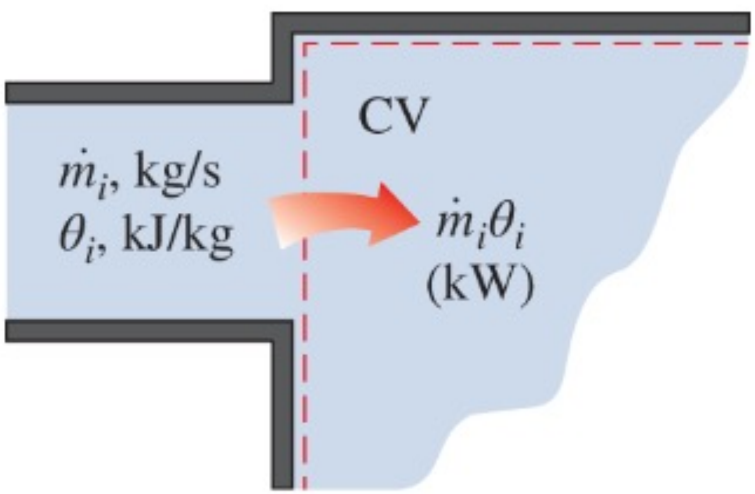
$$\theta = Pv + e = Pv + (u + ke + pe) = (u + Pv) + ke + pe$$

Flow Work and Energy of Flowing Fluid

- Energy transport by mass:

$$E_{mass} = m\theta = m(h + \frac{V^2}{2} + gz)$$

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}(h + \frac{V^2}{2} + gz)$$



Flow Work and Energy of Flowing Fluid

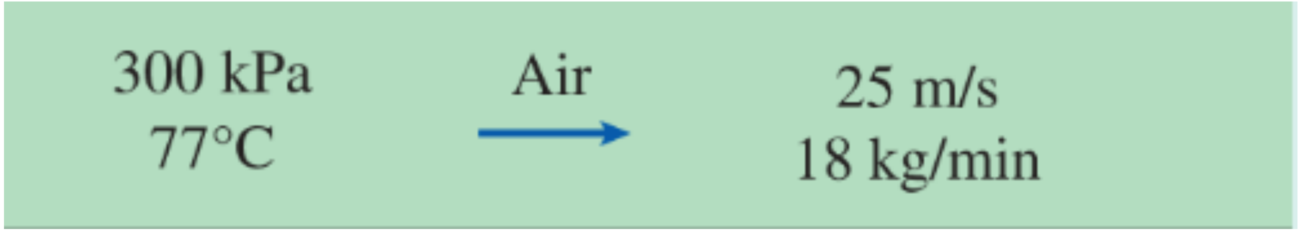
- What does happen when mass is not constant:

$$E_{in,mass} = \int_{m_i} \theta_i \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

CLASS ACTIVITY

Class Activity

- Air flows steadily in a pipe at 300 kPa, 77 °C, and 25 m/s at a rate of 18 kg/min. Determine:
  a) The diameter of the pipe
  b) The rate of flow energy
  c) The rate of energy transport by mass
  d) The error involved in part c if the kinetic energy is neglected.



Class Activity

- Solution (assumptions):
  - ❑ The flow is steady
  - ❑ The potential energy is negligible
  - ❑ R = 0.287 kJ/kg-K
  - ❑ c_p=1.008 kJ/kg-K (at 350 K from Table A-2b)

Class Activity

- Solution (a):

$$v = \frac{RT}{P} = \frac{(0.287\,\frac{kJ}{kg-K})(77+273)}{300\,kPa} = 0.3349\,\frac{m^3}{kg}$$

$$A = \frac{\dot{m}v}{\forall} = \frac{\left(\frac{18}{60}\frac{kg}{s}\right)(0.3349\,\frac{m^3}{kg})}{25\,\frac{m}{s}} = 0.004018\,m^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.004018\,m^2)}{\pi}} = 0.0715\,m$$

Class Activity

- Solution (b):

$$\dot{W}_{flow} = \dot{m}Pv = \left(\frac{18}{60}\frac{kg}{s}\right)(300\ kPa)\left(0.3349\frac{m^3}{kg}\right) = 30.14\ kW$$

Class Activity

- Solution (c):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}(c_p T + \frac{V^2}{2})$$

$$\dot{E}_{mass} = \left(\frac{18}{60}\frac{kg}{s}\right)[\left(1.008\frac{kJ}{kg-K}\right)(77 + 273\ K) + \left(\frac{1}{2}\right)\left(25\frac{m}{s}\right)^2 (\frac{1\frac{kJ}{kg}}{1000\frac{m^2}{s^2}})]$$

$$\dot{E}_{mass} = 105.94\ kW$$

Class Activity

- Solution (d):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}h = \dot{m}c_pT$$

$$\dot{E}_{mass} = \dot{m}c_pT = \left(\frac{18}{60}\frac{kg}{s}\right)\left(1.005\,\frac{kJ}{kg-K}\right)(77 + 273K) = 105.84\ kW$$

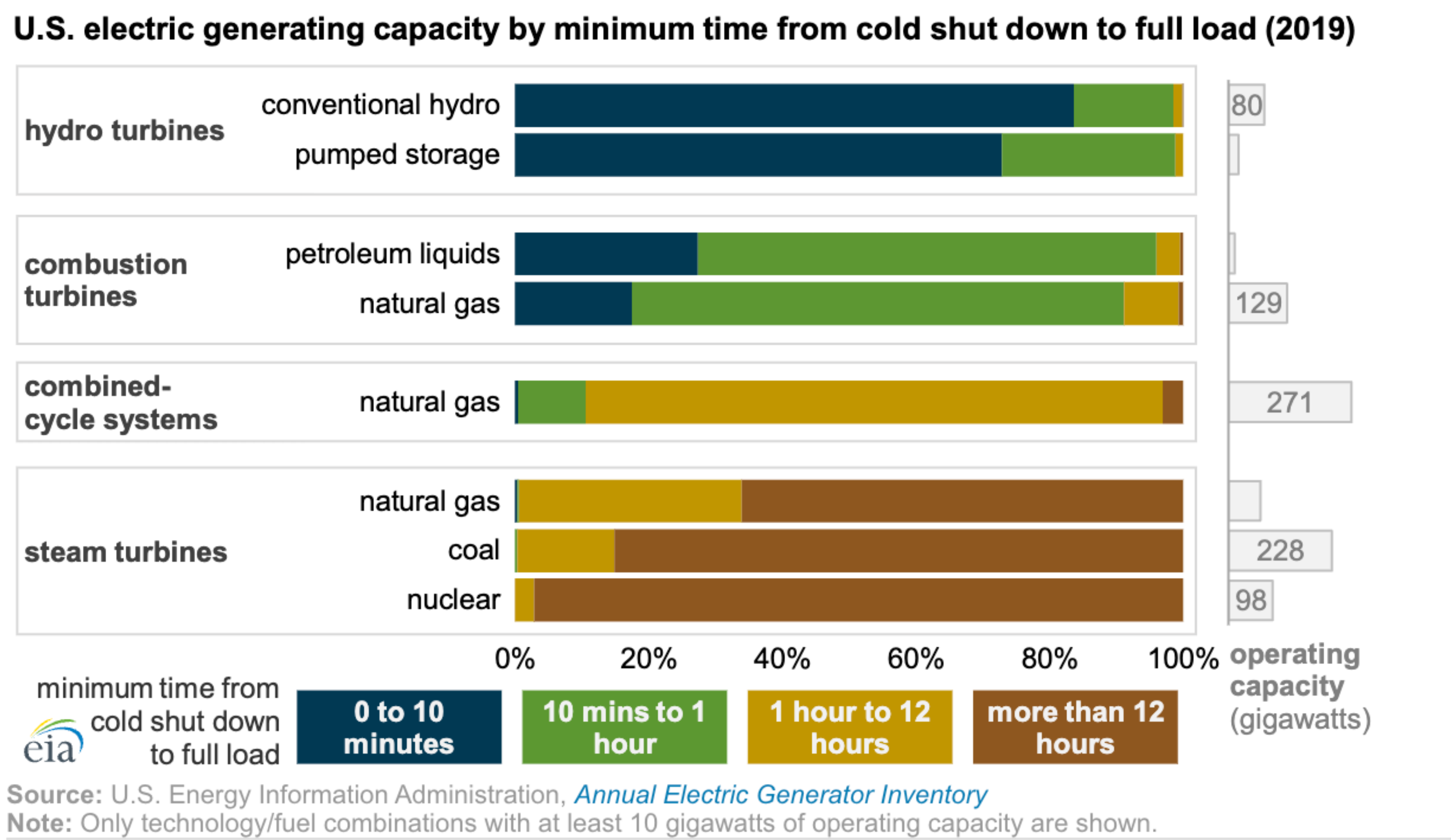
ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

Energy Analysis of Steady-Flow Systems

- A large number of engineering devices such as turbines, compressors, and nozzles operate for long periods of time under the same conditions once the transient start-up period is completed and steady operation is established, and they are classified as *steady-flow devices*.

Energy Analysis of Steady-Flow Systems

- For example, power plants:



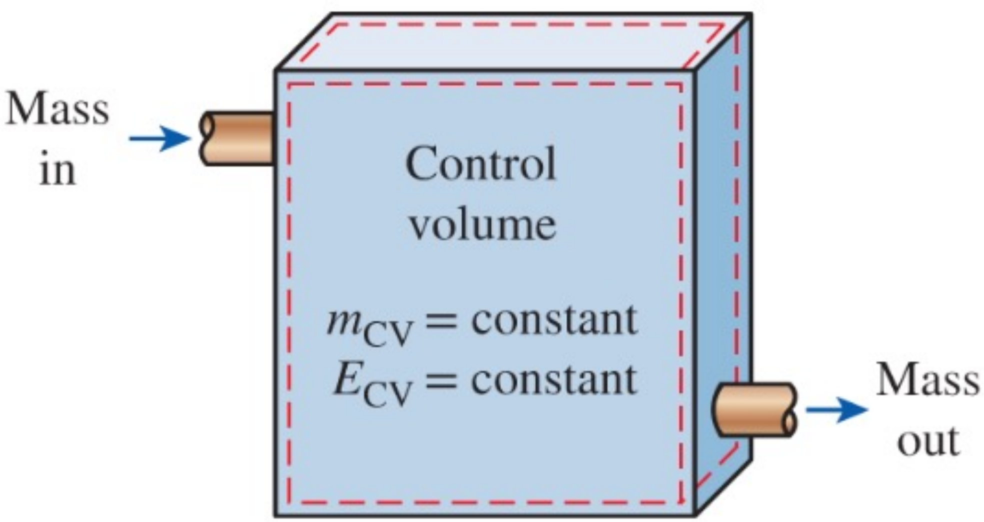
Energy Analysis of Steady-Flow Systems

- Process involving such devices can be represented reasonably well by a somewhat idealized process, called steady-flow process which was defined as a process during which a fluid flows through a control volume steadily

What do you think about a spatial and temporal change in a tank with a steady-flow?

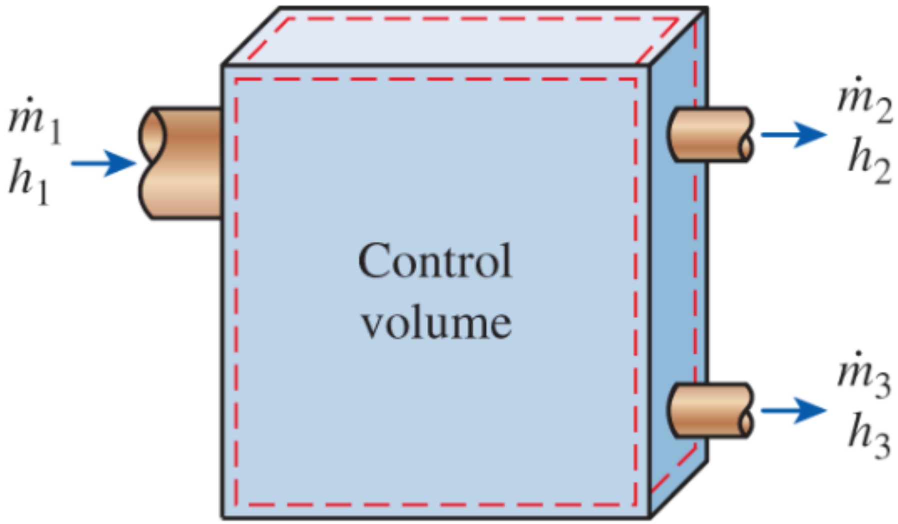
Energy Analysis of Steady-Flow Systems

- Steady-flow process:
  - ❑ No intensive or extensive properties within the control volume change with time
  - ❑ Boundary work is zero
  - ❑ $\Delta E_{CV} = 0$



Energy Analysis of Steady-Flow Systems

- Steady-flow process:
  - ❑ Power remain constant



Energy Analysis of Steady-Flow Systems

- Steady-flow process:

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\dot{m}_1 = \dot{m}_2 \;\rightarrow\; \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Energy Analysis of Steady-Flow Systems

- Steady-flow process:

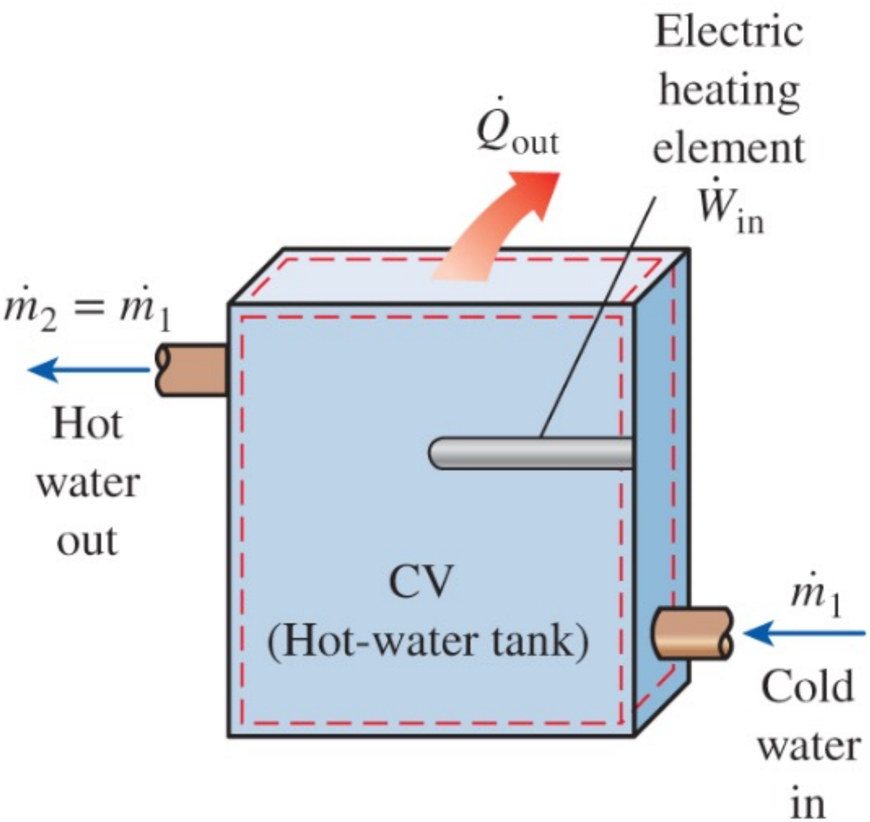
$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} = 0 \qquad \rightarrow \qquad \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + W_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + W_{out} + \sum_{out} \dot{m}\theta$$

$$\dot{Q}_{in} + W_{in} + \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz) = \dot{Q}_{out} + W_{out} + \sum_{out} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Consider an electric hot water heater under steady condition



Energy Analysis of Steady-Flow Systems

- Consider an electric hot water heater under steady condition

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

$$\dot{Q} - \dot{W} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1$$

Energy Analysis of Steady-Flow Systems

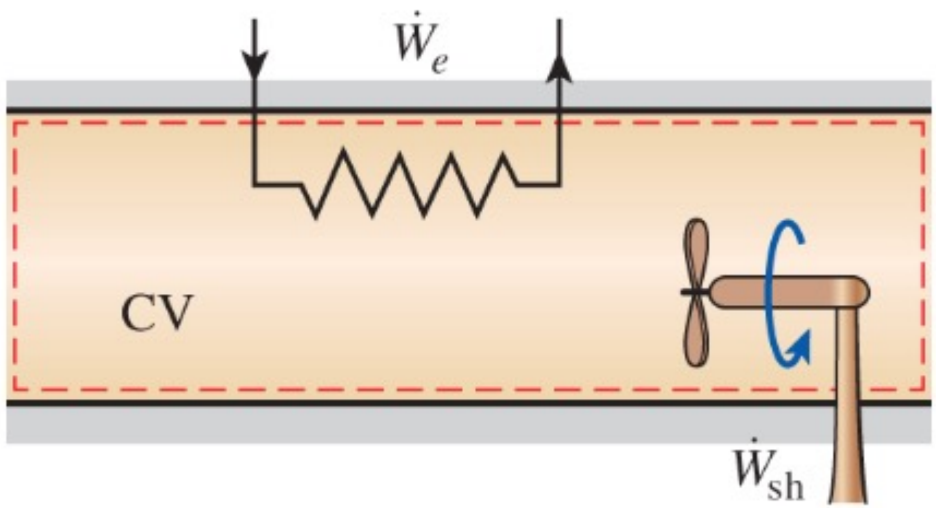
- Let's look at $\dot{Q}$

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Let's look at $\dot{W}$:

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$



Energy Analysis of Steady-Flow Systems

- Let's look at $\Delta h = h_2 - h_1 = c_{p,avg}(T_2 - T_1)$

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Let's look at Δke

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

V_1 m/s	V_2 m/s	Δke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

$$\frac{\text{J}}{\text{kg}} \equiv \frac{\text{N}\cdot\text{m}}{\text{kg}} \equiv \left(\text{kg}\frac{\text{m}}{\text{s}^2}\right)\frac{\text{m}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2}$$

$$\left(\text{Also, } \frac{\text{Btu}}{\text{lbm}} \equiv 25{,}037\ \frac{\text{ft}^2}{\text{s}^2}\right)$$

Energy Analysis of Steady-Flow Systems

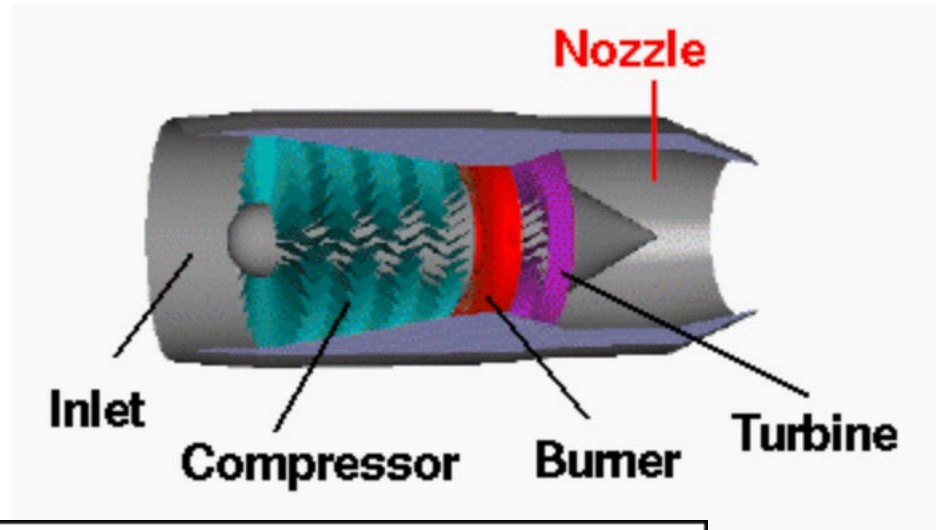
- Let's look at Δpe (when do you think it becomes important?)

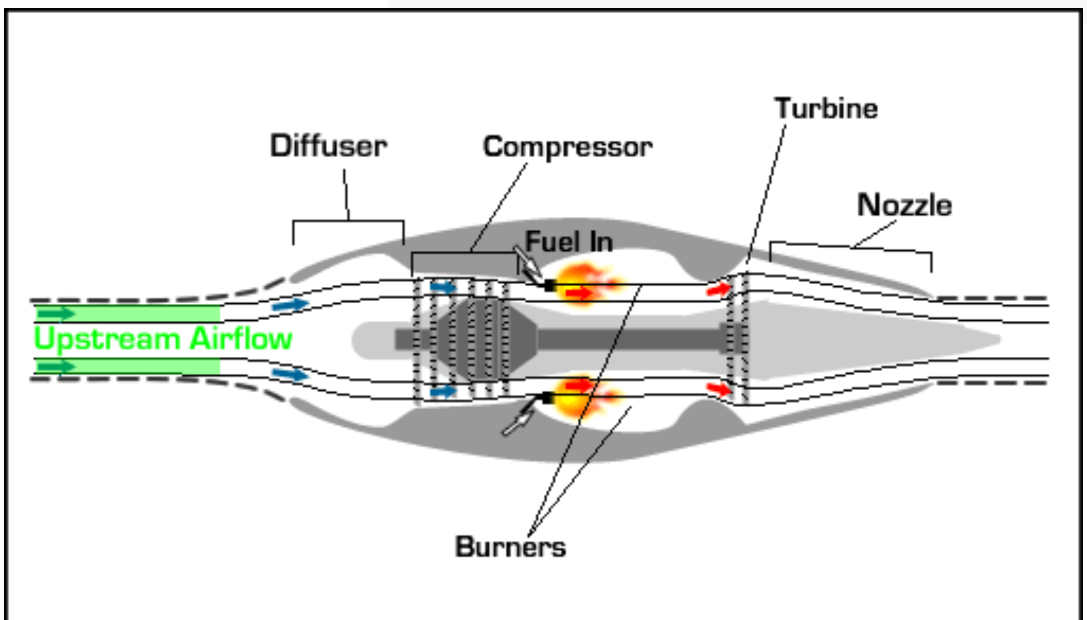
$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

SOME STEADY-FLOW ENGINEERING DEVICES

Some Steady-Flow Energy Devices

- **Nozzles and Diffusers**





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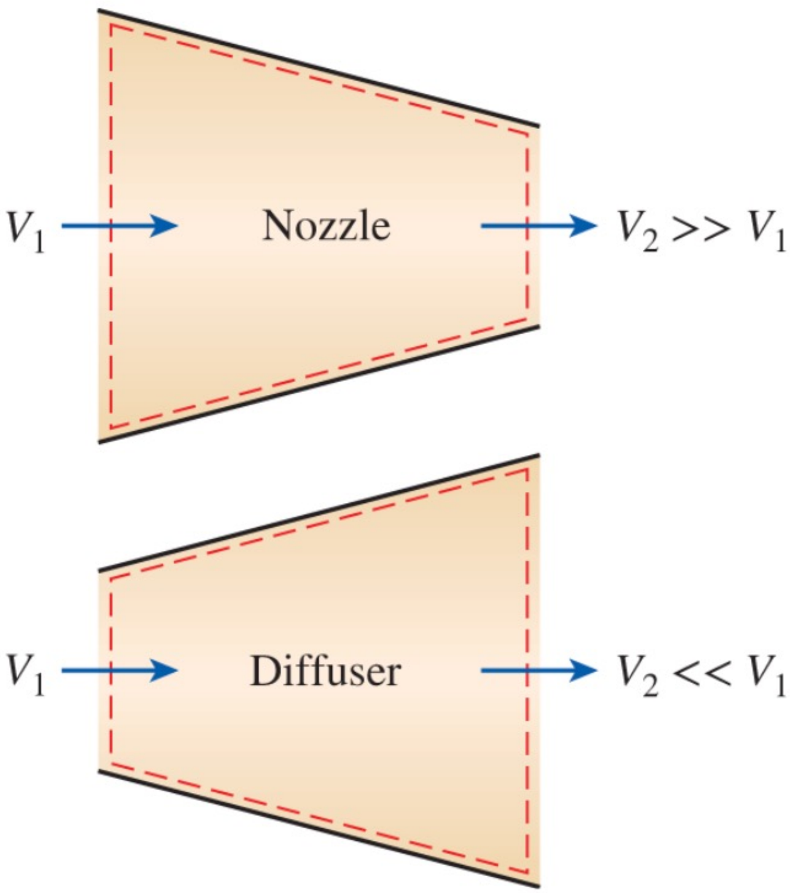
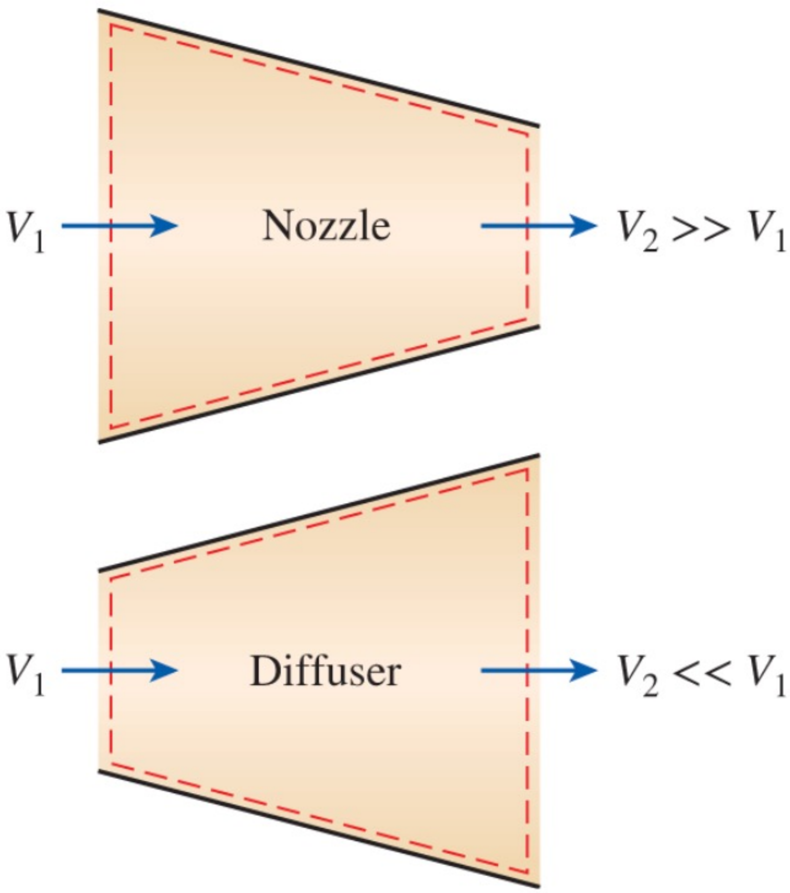
Some Steady-Flow Energy Devices

- **Nozzles and Diffusers**



Some Steady-Flow Energy Devices

- **Nozzles and Diffusers**



$\dot{Q} \cong 0$

$\dot{W} = 0 \; (most\ times)$

$\Delta pe \cong 0$

$\Delta ke \neq 0$

CLASS ACTIVITY

Class Activity

- Air at 10 °C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m^2. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine:
  a) The mass flow rate of the air
  b) The temperature of the air leaving the diffuser

Class Activity

- Solution (assumptions):
  1. This is a steady-flow process ($\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$)
  2. Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values
  3. The potential energy balance change is zero
  4. $\Delta pe = 0$
  5. Heat transfer is negligible
  6. Kinetic energy at the diffuser exit is negligible
  7. There are no work interactions

Class Activity

- Solution (a):

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287\ kPa - \frac{m^3}{kg - K}\right)(283\ K)}{80\ kPa} = 1.015\,\frac{m^3}{kg}$$

Class Activity

- Solution (b):

$$\dot{E}_{in} = \dot{E}_{out} = \frac{dE_{system}}{dt} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}(h_2 + V_2^2)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2} \qquad (V_2^2 \ll V_1^2)$$

Class Activity

- Solution (b):

$$Using\ Table\ A-21\ \rightarrow\ \ h_1 = h_{@\ 283\ K} = 283.14\frac{kJ}{kg}$$

TABLE A–21

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559
210	209.97	0.3987	149.69	1512.0	1.34444
220	219.97	0.4690	156.82	1346.0	1.39105
230	230.02	0.5477	164.00	1205.0	1.43557
240	240.02	0.6355	171.13	1084.0	1.47824
250	250.05	0.7329	178.28	979.0	1.51917
260	260.09	0.8405	185.45	887.8	1.55848
270	270.11	0.9590	192.60	808.0	1.59634
280	280.13	1.0889	199.75	738.0	1.63279
285	285.14	1.1584	203.33	706.1	1.65055

Class Activity

- Solution (b):

$$Using\ Table\ A-21\ \rightarrow\ \ h_1 = h_{@\,283\,K} = 283.14\,\frac{kJ}{kg}$$

$$h_2 = 283.14\,\frac{kJ}{kg} - \frac{0-\left(200\,\frac{m}{s}\right)^2}{2}\left(\frac{1\,\frac{kJ}{kg}}{1000\,\frac{m^2}{s^2}}\right) = 303.14\,\frac{kJ}{kg}$$

$$From\ Table\ A-21\ \rightarrow T_2 = 303\ K$$

Class Activity

- Solution (b):

$$h_2 = 283.14\frac{kJ}{kg} - \frac{0-\left(200\frac{m}{s}\right)^2}{2}\left(\frac{1\frac{kJ}{kg}}{1000\frac{m^2}{s^2}}\right) = 303.14\frac{kJ}{kg}$$

TABLE A–21

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	υ_r	$s°$ kJ/kg · K
298	298.18	1.3543	212.64	631.9	1.69528
300	300.19	1.3860	214.07	621.2	1.70203
305	305.22	1.4686	217.67	596.0	1.71865

$$From\ Table\ A-21\ \rightarrow T_2 = 303\ K$$

Some Steady-Flow Energy Devices

- **Turbines, Compressors, fans, and Pumps**
  - Turbine produce power output whereas compressors, pumps, and fans require power input
  - Heat Transfer is usually negligible ($\dot{Q} \cong 0$)
  - Potential energy is negligible ($pe \cong 0$)
  - Kinetic energy is negligible ($ke \cong 0$) - except for fans and turbines but the change in enthalpy is significant compared to the velocity change

CLASS ACTIVITY

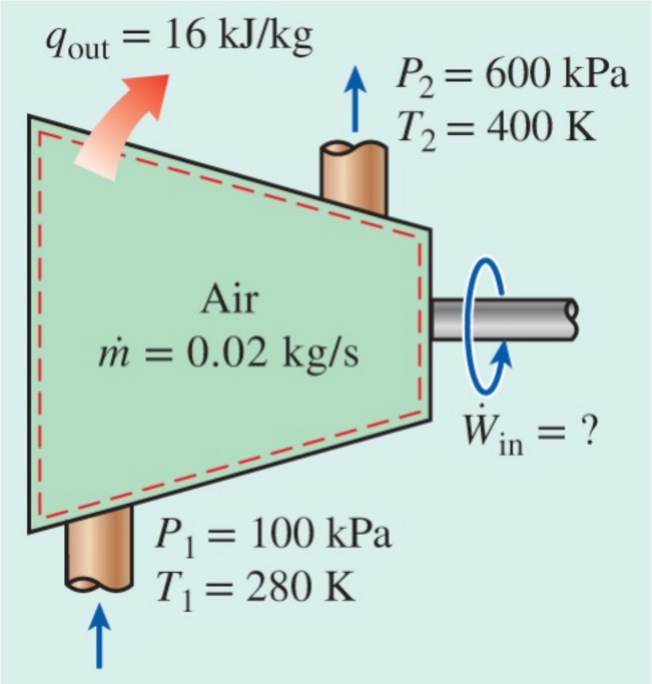
Class Activity

- Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

Class Activity

- Solution (assumptions):
  1. Steady-flow ($\Delta m_{CV} = 0$ and $\Delta \mathrm{E}_{\mathrm{CV}} = 0$)
  2. Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point value
  3. The kinetic and potential energy changes are zero ($\Delta \mathrm{ke} = \Delta pe = 0$)

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$



Class Activity

- Solution:

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} = 0 \;\; \rightarrow \;\; \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$\dot{W}_{in} = \dot{Q}_{out} + \dot{m}(h_2 - h_1)$$

Class Activity

- Solution (From Table A-21):

$$h_1 = h_{@\,280\,K} = 280.13\,\frac{kJ}{kg}$$

$$h_2 = h_{@\,400\,K} = 400.98\,\frac{kJ}{kg}$$

TABLE A–21

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
270	270.11	0.9590	192.60	808.0	1.59634
280	280.13	1.0889	199.75	738.0	1.63279
285	285.14	1.1584	203.33	706.1	1.65055
290	290.16	1.2311	206.91	676.1	1.66802
390	390.88	3.481	278.93	321.5	1.96633
400	400.98	3.806	286.16	301.6	1.99194
410	411.12	4.153	293.43	283.3	2.01699

Class Activity

- Solution:

$$\dot{W}_{in} = \left(0.02\,\frac{kg}{s}\right)\left(16\,\frac{kJ}{kg}\right) + \left(0.02\,\frac{kg}{s}\right)\left(400.98 - 280.13\,\frac{kJ}{kg}\right) = 2.74\ kW$$

Some Steady-Flow Energy Devices

- **Throttling valves**
  - Throttling valves are any kind of flow-restricting devices that cause a significant pressure drop in the fluid (e.g., valves, capillary tubes, porous plugs)
  - Unlike turbines, they produce a pressure drop without involving any work
  - The pressure drop in the fluid is often accompanied b a large drop in temperature, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications

(*a*) An adjustable valve

(*b*) A porous plug

(*c*) A capillary tube

Some Steady-Flow Energy Devices

- **Throttling valves**

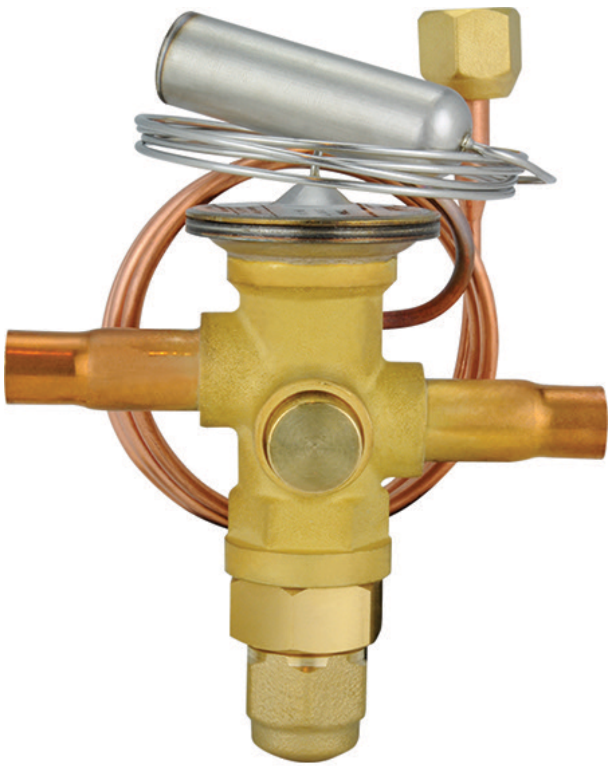
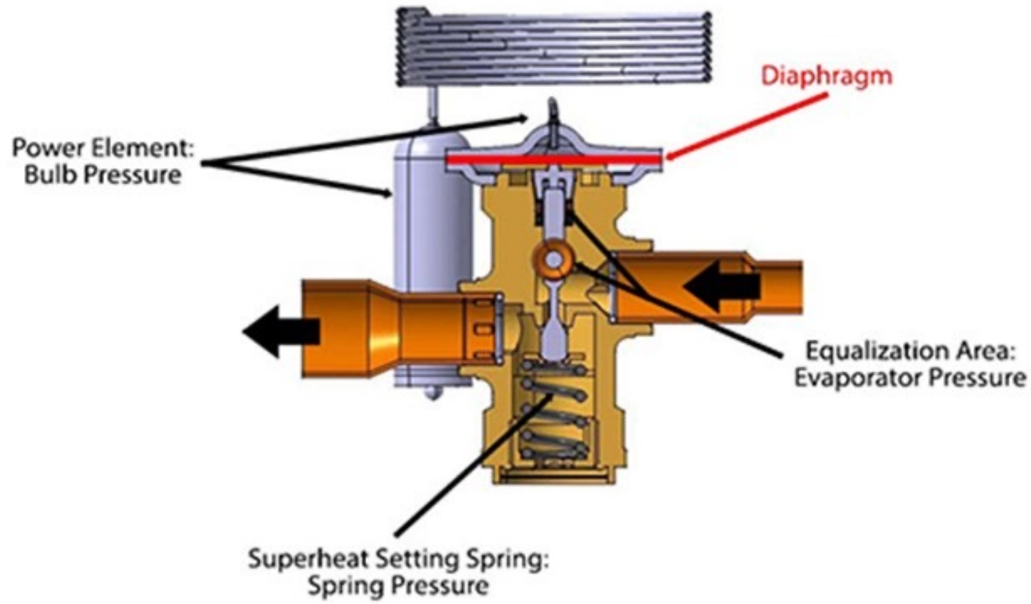


Figure 1



TXV Parts

Credit: Danfoss

Some Steady-Flow Energy Devices

- **Throttling valves**
  - They are usually small, the process can be adiabatic ($q \cong 0$)
  - No work is done ($w \cong 0$)
  - $\Delta pe \cong 0$
  - $\Delta ke \cong 0$

$$h_2 \cong h_1$$ *(Isenthalpic device)*

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

$$Internal\ energy + Flow\ energy = Constant$$

Some Steady-Flow Energy Devices

- **Throttling valves**
  - In case of an ideal gas, $h = h(T)$, and thus the temperature has to remain constant during a throttling process: