CAE 208 / MMAE 320: Thermodynamics

Fall 2023

October 12, 2023

Energy analysis of closed systems (3)

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Dr. Mohammad Heidarinejad, Ph.D., P.E.
Civil, Architectural and Environmental Engineering
Illinois Institute of Technology
muh182@iit.edu

ANNOUNCEMENTS

Announcement

LUNCH WITH ARCHITECTURAL ENGINEERING PROFESSORS

Are you interested in building systems? Join us to discuss HVAC, lighting fixtures, and building design. Open to any major!

- Location: AM 120
- Date: October 12
- Time: 12:45 - 1:40
- Food is Provided!!

For more information, feel free to email ashrae_iit@iit.edu or send a Instagram dm to @ashrae_iit.

Announcement

Announcement

SmithGroup
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We're excited to open the 2024 SmithGroup Justice, Equity, Diversity & Inclusion Scholarship program for applications. The scholarship supports students from underrepresented backgrounds who are studying architecture, engineering, interior design, landscape architecture or urban planning.

Each scholarship includes both tuition support and a paid summer internship at one of SmithGroup's locations. Applications are due November 30.

Learn more about eligibility, requirements and the application process: https://lnkd.in/dJuNTEKt

https://www.smithgroup.com/jedi-scholarship

Announcements

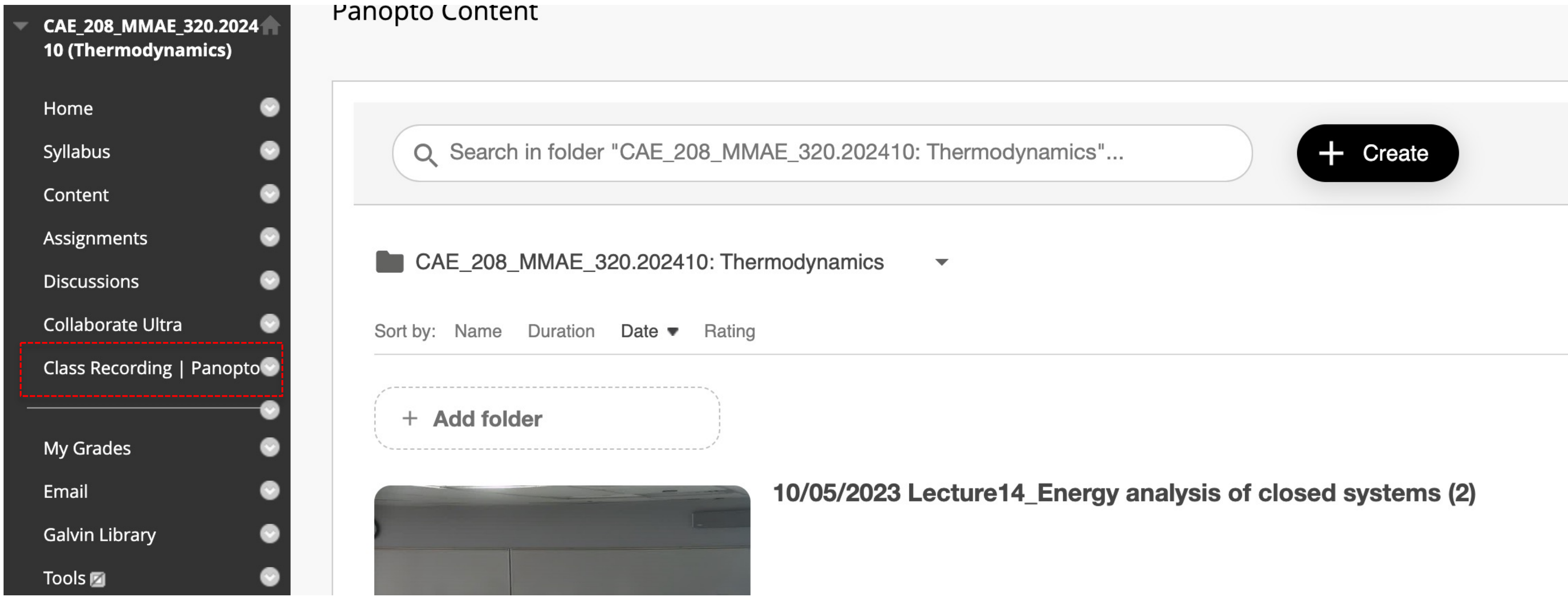
- Assignment 5 is posted (due next Thursday)

Announcements

- Midterm exam 1:
  - Solutions will be uploaded
  - The exams will be graded by the next lecture

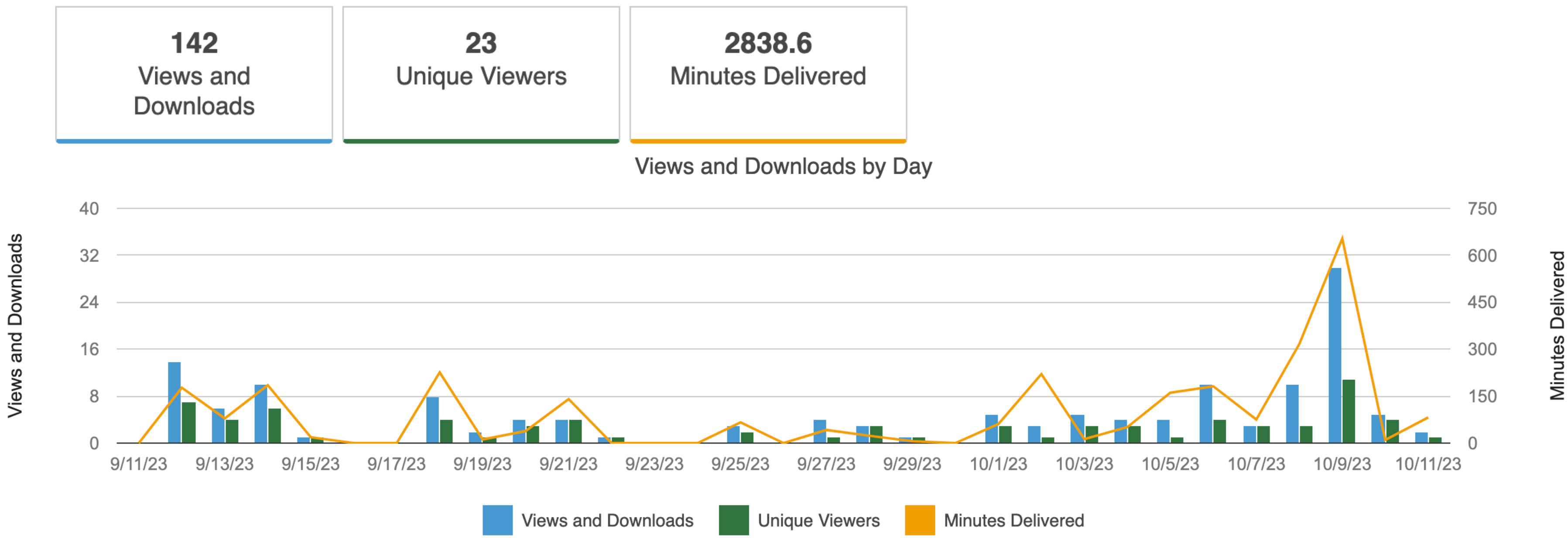
Announcements

- Please review the lecture recordings on Blackboard:



Announcements

- I usually review the number of views:



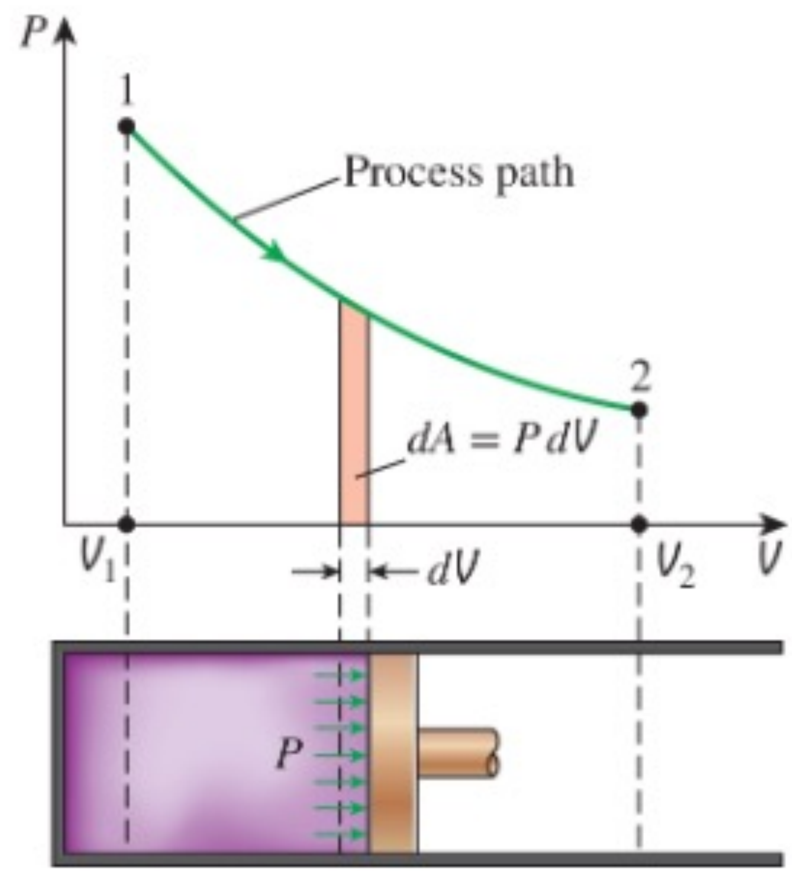
RECAP

Recap

- For a quasi-equilibrium expansion process, we can write:

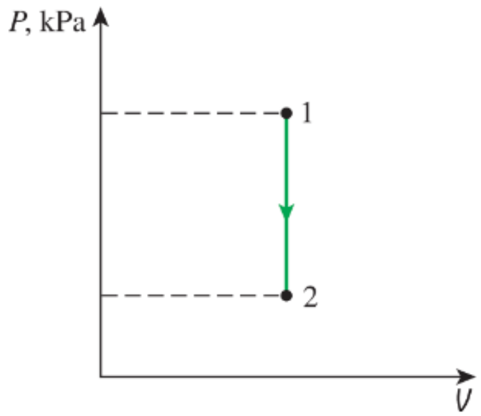
$$Area = A = \int_1^2 dA = \int_1^2 P dV$$

$$W_b = \int_1^2 P_i dV$$

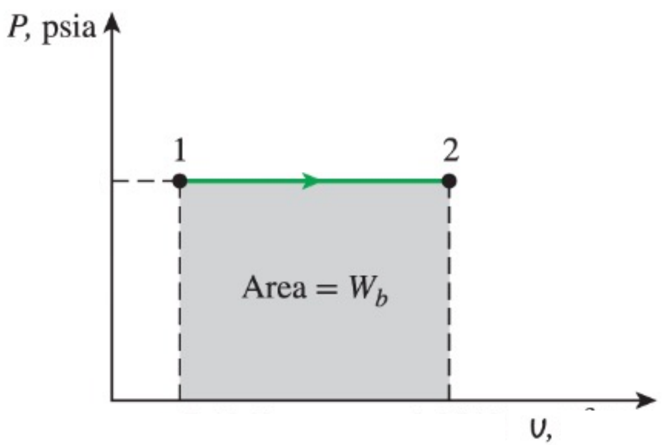


Recap

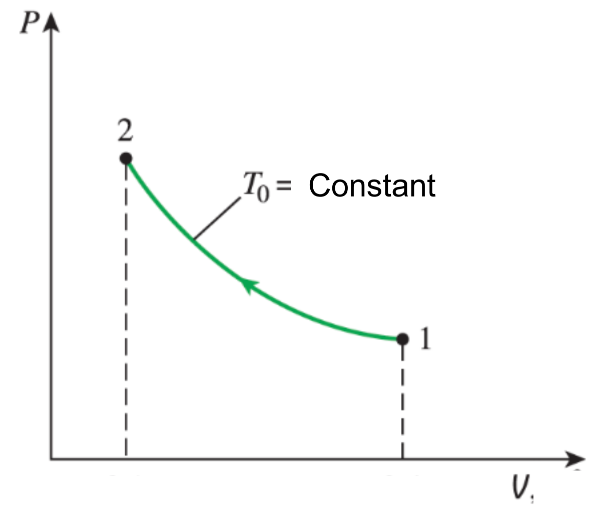
- Constant Volume



- Constant Pressure



- Constant Temperature



Recap

- Moving boundary work under different processes

Process	**Moving boundary work**
Constant volume	0
Constant pressure	$P_0(V_2 - V_1)$
Isothermal	$P_1V_1 \times Ln(\frac{V_2}{V_1})$ $P_1V_1 \times Ln(\frac{P_1}{P_2})$ $mRT_o \times Ln(\frac{V_2}{V_1})$
Polytropic	$\frac{P_2V_2 - P_1V_1}{1-n}$ $\frac{mR(T_2 - T_1)}{1-n}$

Recap

- We can sum the heat, work, and mass, and the heat transfer:

Net energy transfer by heat, work ↓

Change in internal, kinetic, potential, …, energies ↓

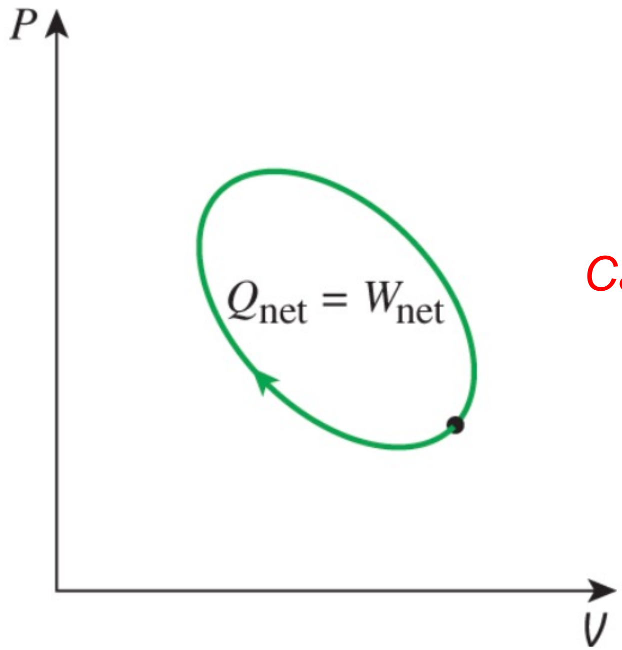
$$E_{in} - E_{out} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = \Delta E_{system}$$

Recap

- For a closed system undergoing a cycle, the initial and final states are identical:

$$\Delta E = E_{in} - E_{out} = 0 \quad \rightarrow E_{in} = E_{out}$$

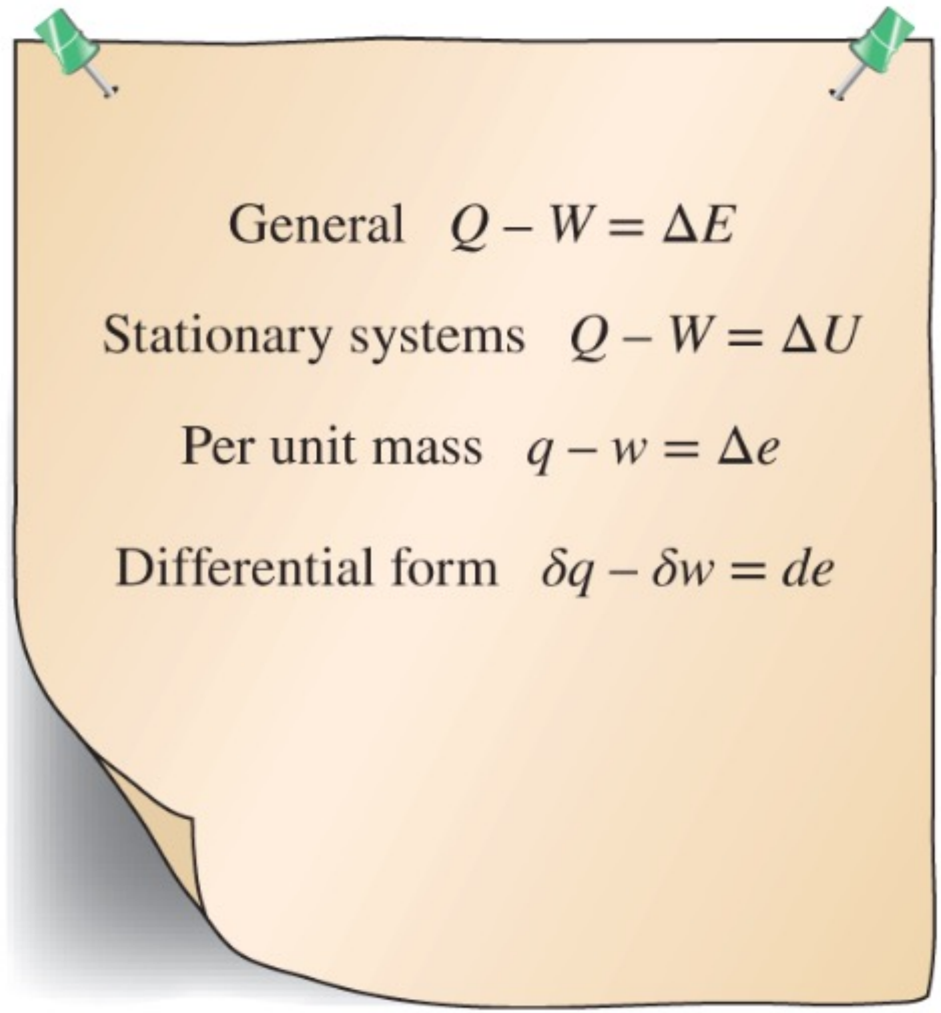
$$W_{net,out} = Q_{net,in} \rightarrow \dot{W}_{net,out} = \dot{Q}_{net,in}$$



Can we comment about the sign of the work?

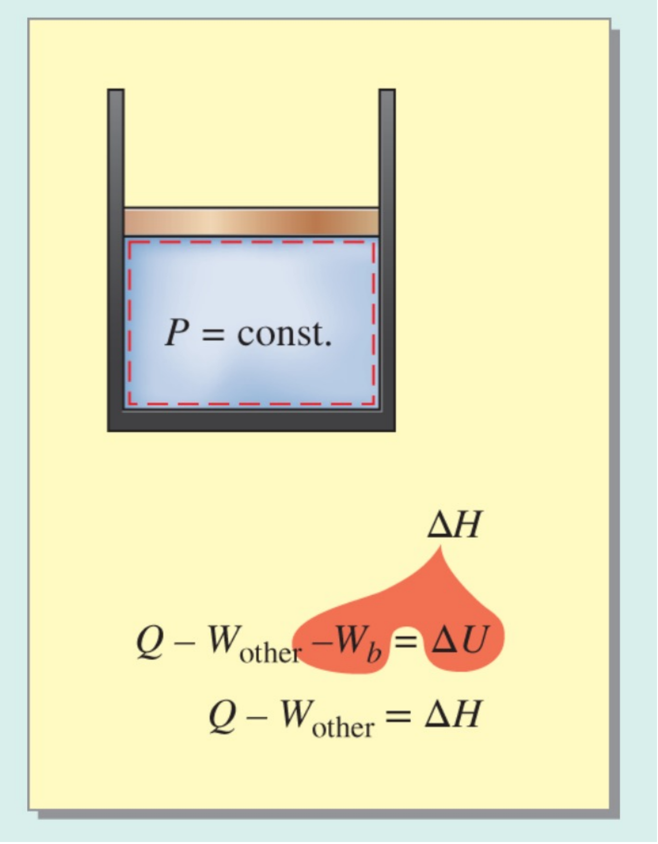
Recap

- We can write:



Recap

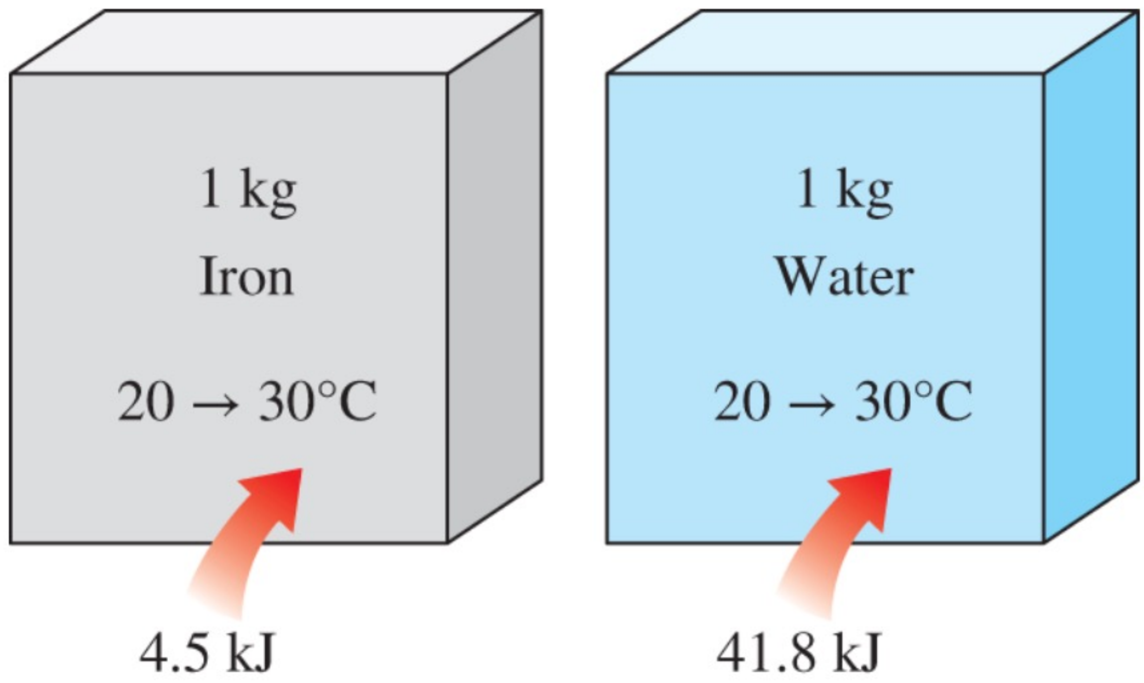
- For a constant pressure process with constant mass, we have:



SPECIFIC HEATS

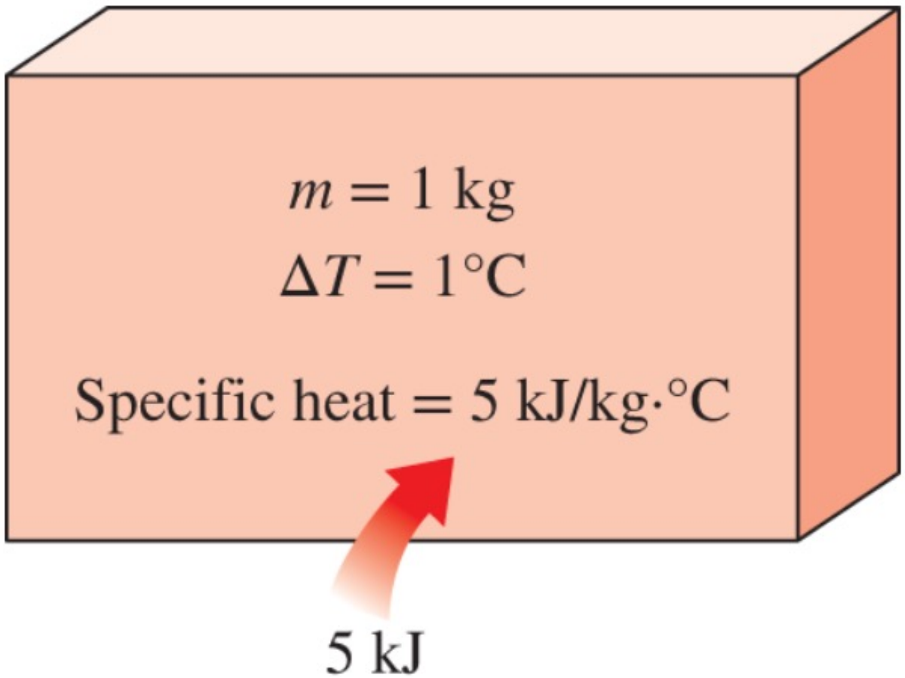
Specific Heats

- How much heat do we need to add to increase temperature of 1 kg iron vs water for 10 °C?



Specific Heats

- Specific heat is defined as the energy required to raise temperature of a unit mass of substance by one degree



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  - Specific heat at constant volume (c_v)
  - Specific heat at constant pressure (c_p)

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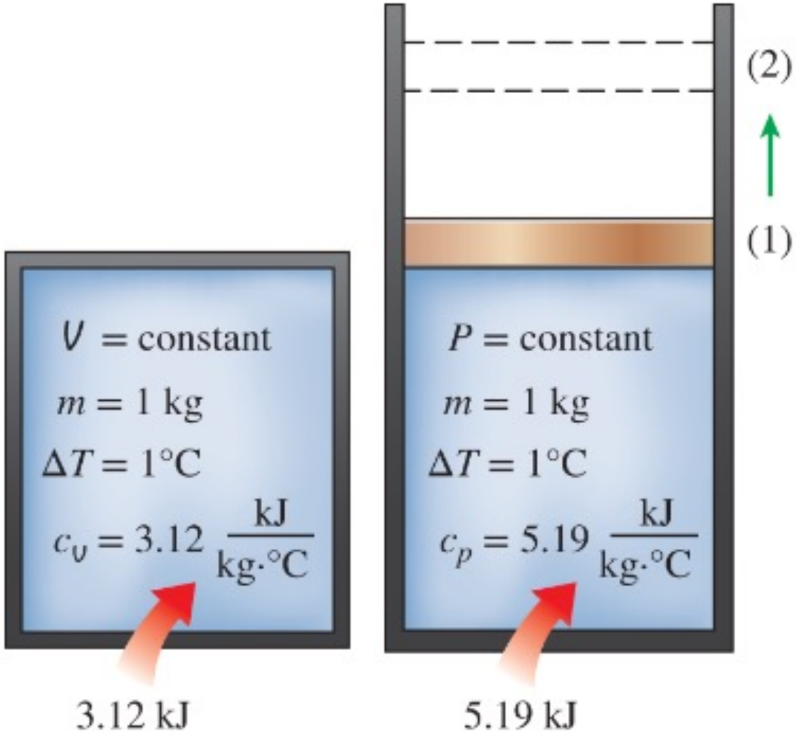


Specific Heats

- Let's start from the fixed mass in a stationary closed system that undergoes a constant volume process:

$$dE_{in} - dE_{out} = dU + dKE + dPE = \delta Q - \delta W$$

$$\delta Q = dU + \delta W = dU + PdV$$

$$c_v = \frac{1}{m}\left(\frac{\delta Q}{dT}\right)_v = \frac{1}{m}\left(\frac{\partial U}{\partial T}\right)_v = \left(\frac{\partial u}{\partial T}\right)_v$$

$$c_v dT = du$$

Specific Heats

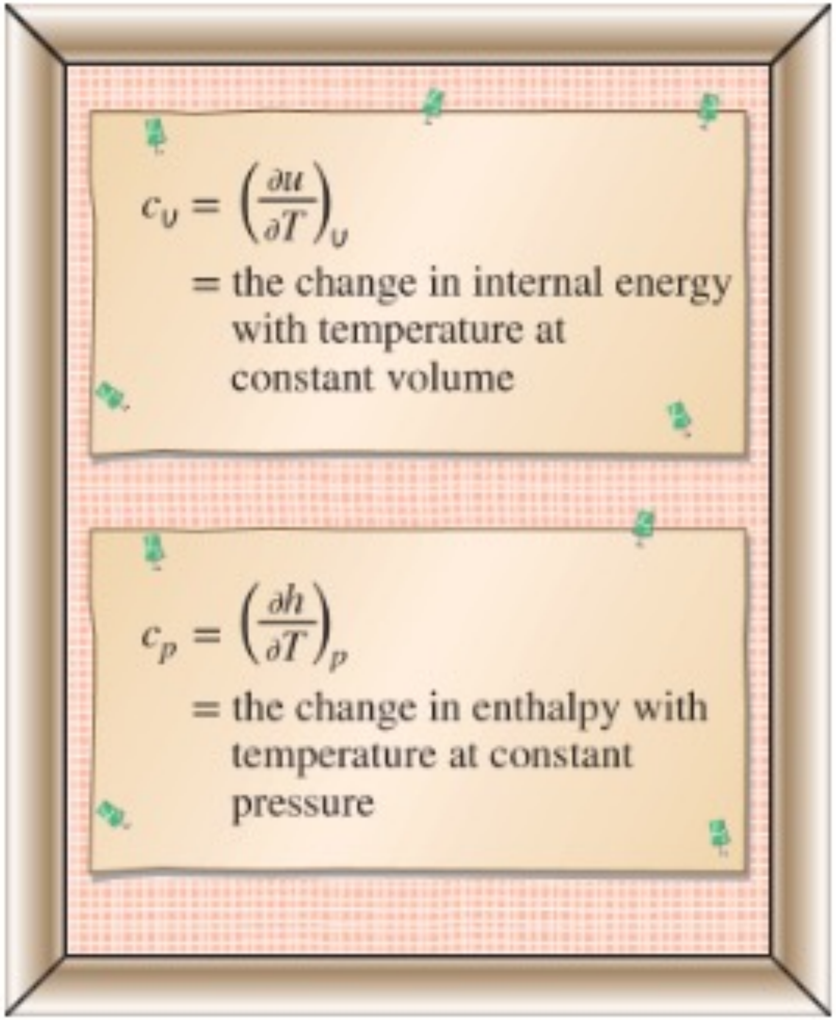
- Similarly, we can write the following for a constant pressure process:

$$\delta Q = dU + \delta W = dU + PdV$$

$$c_p = \frac{1}{m}\left(\frac{\delta Q}{dT}\right)_p = \frac{1}{m}\left(\frac{\partial H}{\partial T}\right)_p = \left(\frac{\partial h}{\partial T}\right)_p$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

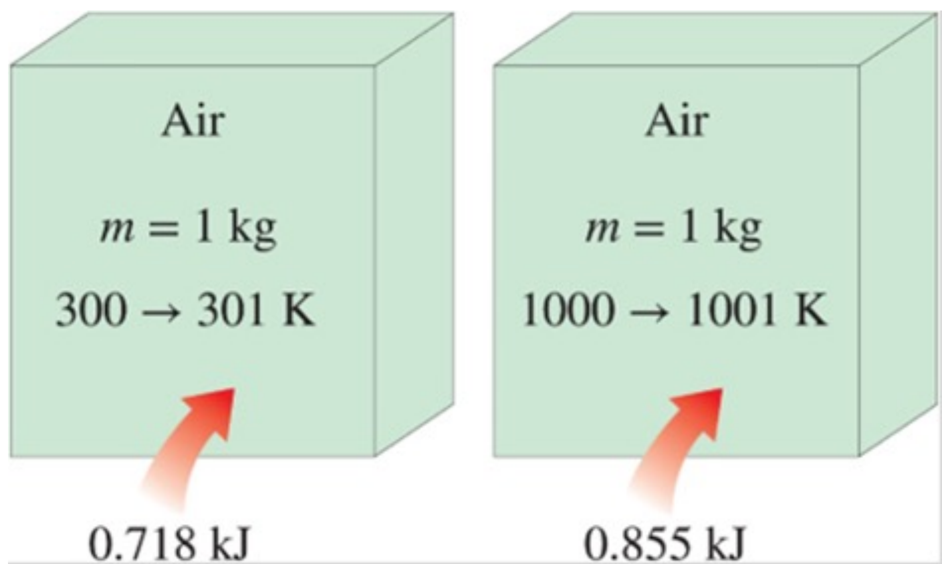
Specific Heats



Specific Heats

- c_p and c_v are defined based on properties. They must be properties too

- The energy required to raise the temperature of a substance by one degree is different at different temperatures and pressures



Specific Heats

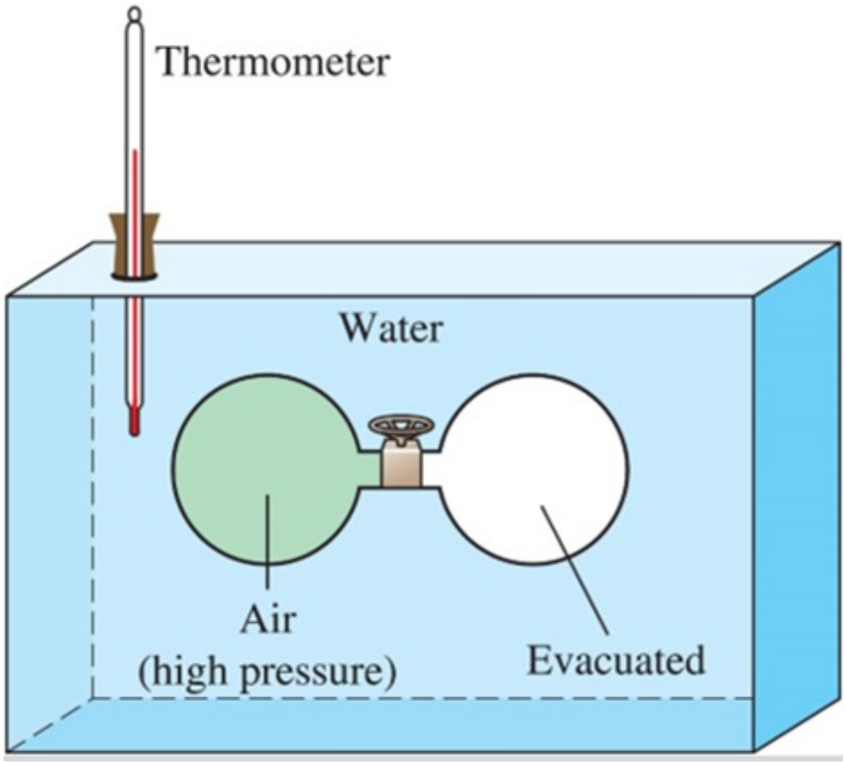
- A common unit for specific heats is $\frac{kJ}{kg-°C}$ or $\frac{kJ}{kg-K}$ (why?)

- We can write them in the molar basis too $\frac{kJ}{kmol-°C}$ or $\frac{kJ}{kmol-K}$ for $\overline{c_p}$ and $\overline{c_v}$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES

Internal/Energy/Enthalpy/Heats of Ideal Gases

- It has demonstrated mathematically and experimentally that internal energy is a function of temperature:



$$u = u(T)$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

- Using the definition of enthalpy and the equation of state of an ideal gas, we have:

$$u = u(T)$$

$$\left.\begin{array}{r} h = u + Pv \\ Pv = RT \end{array}\right\} \rightarrow \quad h = u + RT \quad \rightarrow h = h(T)$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

- We have:

$$u = u(T)$$
$$h = h(T)$$
$$c_v = c_v(T)$$
$$c_p = c_p(T)$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

- The change in internal energy or enthalpy for an ideal gas during a process from state 1 to state 2 is determined by integrating these equations

$$\Delta u = u_2 - u_1 = \int_1^2 c_v\,(T)dt$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p\,(T)dt$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

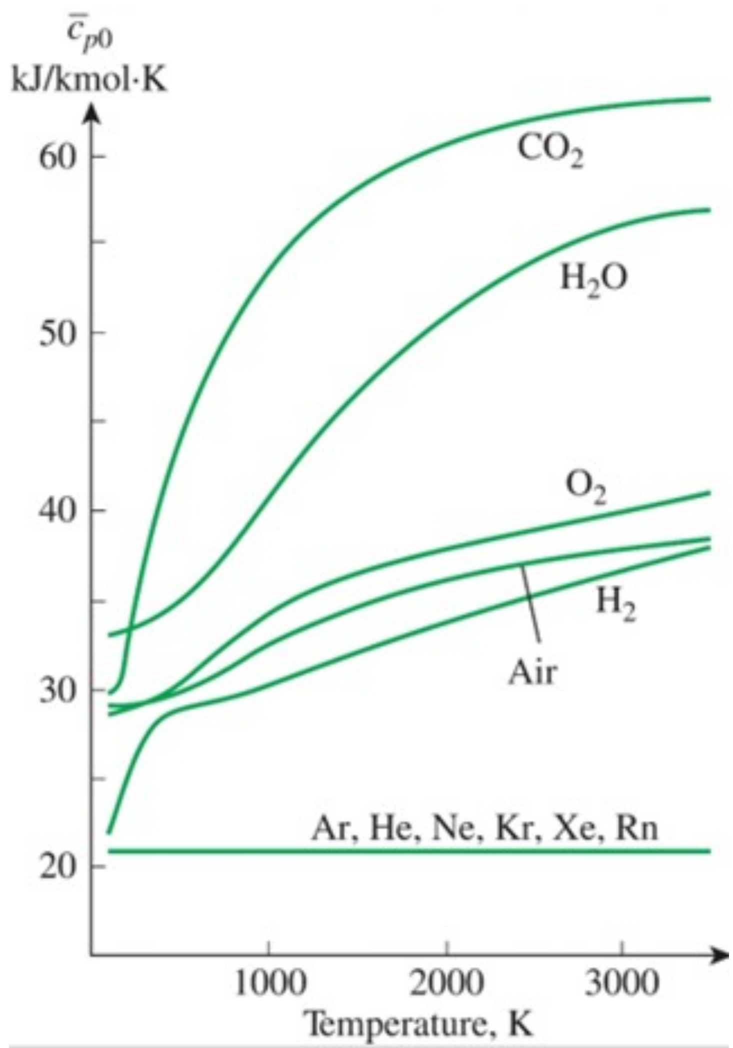
- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only

- The specific heats of real gases at low pressures are called ideal-gas specific heats, or zero-pressure specific heats, and are often denoted c_{p0} and c_{v0}

Internal/Energy/Enthalpy/Heats of Ideal Gases

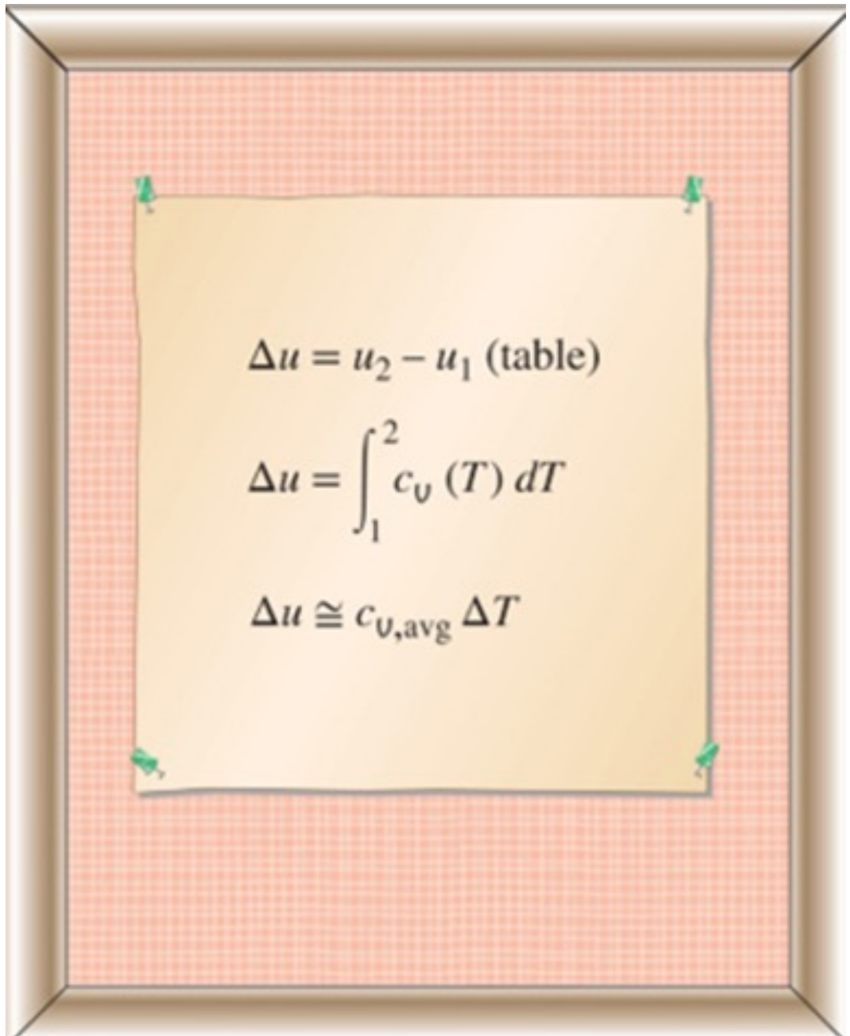
- Ideal gas constant pressure specific heats for some gases:

$\overline{c_{p0}}$



Internal/Energy/Enthalpy/Heats of Ideal Gases

- Three ways to calculate Δh and Δu:



Internal/Energy/Enthalpy/Heats of Ideal Gases

- Approach 1: By using the tabulated u and h data. This is the easiest and most accurate way when tables are readily available

Internal/Energy/Enthalpy/Heats of Ideal Gases

- u and h data for a number of gases have been tabulated
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state

Air

T, K	u, kJ/kg	h, kJ/kg
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

Internal/Energy/Enthalpy/Heats of Ideal Gases

- We can use the Table A-21

TABLE A–21

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K	T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
210	209.97	0.3987	149.69	1512.0	1.34444	590	596.52	15.31	427.15	110.6	2.39140
220	219.97	0.4690	156.82	1346.0	1.39105	600	607.02	16.28	434.78	105.8	2.40902
230	230.02	0.5477	164.00	1205.0	1.43557	610	617.53	17.30	442.42	101.2	2.42644
240	240.02	0.6355	171.13	1084.0	1.47824	620	628.07	18.36	450.09	96.92	2.44356
250	250.05	0.7329	178.28	979.0	1.51917	630	638.63	19.84	457.78	92.84	2.46048
260	260.09	0.8405	185.45	887.8	1.55848	640	649.22	20.64	465.50	88.99	2.47716
270	270.11	0.9590	192.60	808.0	1.59634	650	659.84	21.86	473.25	85.34	2.49364
280	280.13	1.0889	199.75	738.0	1.63279	660	670.47	23.13	481.01	81.89	2.50985
285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589

Internal/Energy/Enthalpy/Heats of Ideal Gases

- Approach 2: By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are very accurate

Internal/Energy/Enthalpy/Heats of Ideal Gases

- Approach 2: Table A-2c

APPENDIX 1

PROPERTY TABLES AND CHARTS (SI UNITS)

(*c*) As a function of temperature

$$\bar{c}_p = a + bT + cT^2 + dT^3$$
$$(T \text{ in K, } c_p \text{ in kJ/kmol} \cdot \text{K})$$

Substance	Formula	a	b	c	d	Temper range,
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	273
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	273
Air	–	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	273

Internal/Energy/Enthalpy/Heats of Ideal Gases

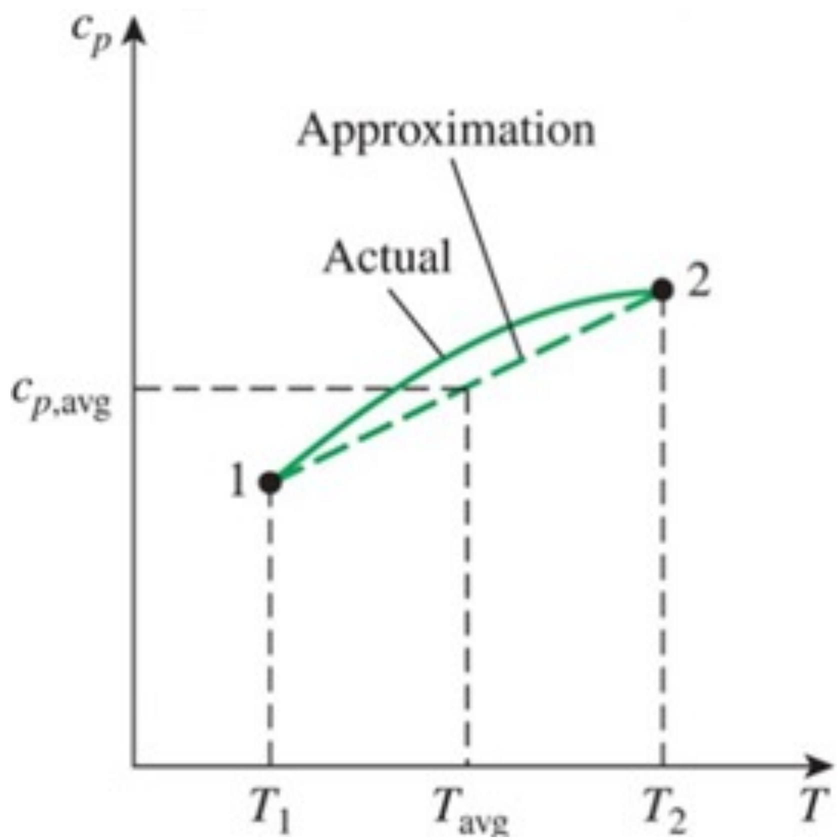
- Approach 3: By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are reasonably accurate if the temperature interval is not very large

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

- For small temperature intervals, the specific heats may be assumed to vary linearly with temperature



Internal/Energy/Enthalpy/Heats of Ideal Gases

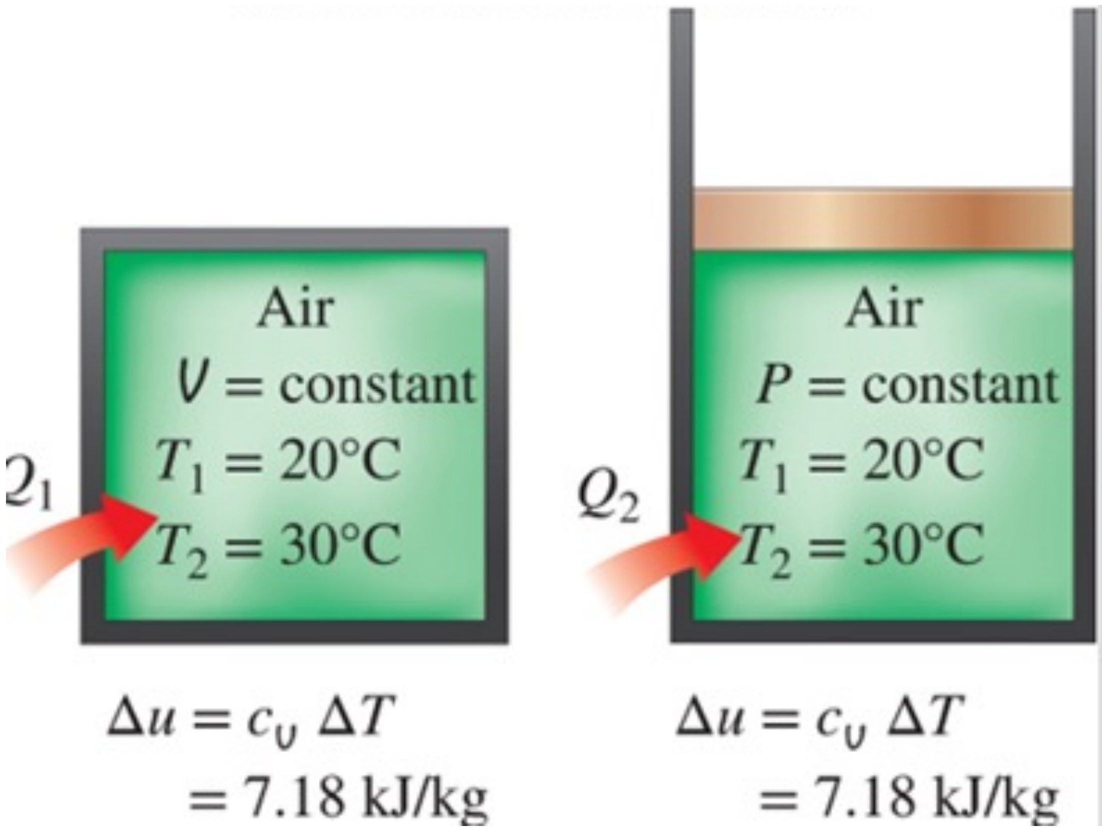
- Interpolation:

(*b*) At various temperatures

Temperature, K	c_p kJ/kg · K	c_v kJ/kg · K	k	c_p kJ/kg · K	c_v kJ/kg · K	k	c_p kJ/kg · K
	Air			*Carbon dioxide, CO_2*			*Carb*
250	1.003	0.716	1.401	0.791	0.602	1.314	1.039
300	1.005	0.718	1.400	0.846	0.657	1.288	1.040
350	1.008	0.721	1.398	0.895	0.706	1.268	1.043
400	1.013	0.726	1.395	0.939	0.750	1.252	1.047
450	1.020	0.733	1.391	0.978	0.790	1.239	1.054
500	1.029	0.742	1.387	1.014	0.825	1.229	1.063

Internal/Energy/Enthalpy/Heats of Ideal Gases

- The relation $\Delta u = c_v \Delta T$ is valid for any kind of process, constant-volume or not



Internal/Energy/Enthalpy/Heats of Ideal Gases

- Can we find a relation between c_p and c_v?

$$h = u + RT$$

$$dh = du + RdT$$

$$c_p - c_v = R$$

$$dh = c_p dT$$

$$du = c_v dT$$

$$c_p dT = c_v dT + RdT$$

Internal/Energy/Enthalpy/Heats of Ideal Gases

- Specific heat ratio (k):
  - ❑ For monatomic (e.g., Helium) gases the value is 1.667
  - ❑ For many diatomic gases, including air, the value is 1.4

$$k = \frac{c_p}{c_v}$$

CLASS ACTIVITY

Class Activity

- Air at 300 K and 200 kPa is heated at constant pressure to 600 K. Determine the change in internal energy of air per unit mass from
  a) Data from the air table (Table A-21)
  b) The functional form of the specific heat (Table A-2c)
  c) The average specific heat value (Table A-2b)

Class Activity

- Solution (a):

TABLE A–21

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K	T K	h kJ/kg	P_r	u kJ/kg	v_r	$s°$ kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
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285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589
290	290.16	1.2311	206.91	676.1	1.66802	680	691.82	25.85	496.62	75.50	2.54175
295	295.17	1.3068	210.49	647.9	1.68515	690	702.52	27.29	504.45	72.56	2.55731
298	298.18	1.3543	212.64	631.9	1.69528	700	713.27	28.80	512.33	69.76	2.57277
300	300.19	1.3860	214.07	621.2	1.70203	710	724.04	30.38	520.23	67.07	2.58810

$$\Delta u = u_2 - u_1 = 434.78 - 214.07 = 220.71 \frac{kJ}{kg}$$

Class Activity

- Solution (b): Table 2A-c

(*c*) As a function of temperature

			$\bar{c}_p = a + bT + cT^2 + dT^3$ (*T* in K, c_p in kJ/kmol · K)			
Substance	Formula	*a*	*b*	*c*	*d*	Te
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	
Air	–	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	
Hydrogen	H_2	29.11	-0.1916×10^{-2}	0.4003×10^{-5}	-0.8704×10^{-9}	

$$\bar{c_p} = a + bT + cT^2 + dT^3$$

$$\bar{c_v} = \bar{c_p} - R_u = a + bT + cT^2 + dT^3$$

Class Activity

- Solution (b): Table 2A-c

$$\bar{c_p} = a + bT + cT^2 + dT^3$$

$$\bar{c_v} = \bar{c_p} - R_u = a + bT + cT^2 + dT^3$$

$$\Delta\bar{u} = \int_{T_1}^{T_2} \bar{c_v}(T)\, dT = \bar{c_p} - R_u = \int_{T_1}^{T_2} [(a - R_u) + bT + cT^2 + dT^3] dT$$

$$\Delta\bar{u} = 6447 \, \frac{\text{kJ}}{\text{kmol}}$$

Class Activity

- Solution (b): Table A-1

$$\Delta\bar{u} = 6447\,\frac{\text{kJ}}{\text{kmol}}$$

TABLE A–1

Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, *M* kg/kmol	Gas constant, *R* kJ/kg · K*
Air	–	28.97	0.2870
Ammonia	NH_3	17.03	0.4882
Argon	Ar	39.948	0.2081
Benzene	C_6H_6	78.115	0.1064

$$\Delta\text{u} = \frac{\Delta\bar{u}}{M} = \frac{6447\,\frac{\text{kJ}}{\text{kmol}}}{28.97\,\frac{kg}{kmol}} = 222.5\,\frac{kJ}{kg}$$

Class Activity

- Solution (c): Table A-2b

$$Average\ Temp = \frac{T_1 + T_2}{2}$$

$$c_{v,avg} = c_{v,450\ K} = 0.7333\,\frac{kJ}{kg-K}$$

$$\Delta u = \left(0.7333\,\frac{kJ}{kg-K}\right)(600-300) = 220\,\frac{kJ}{kg}$$

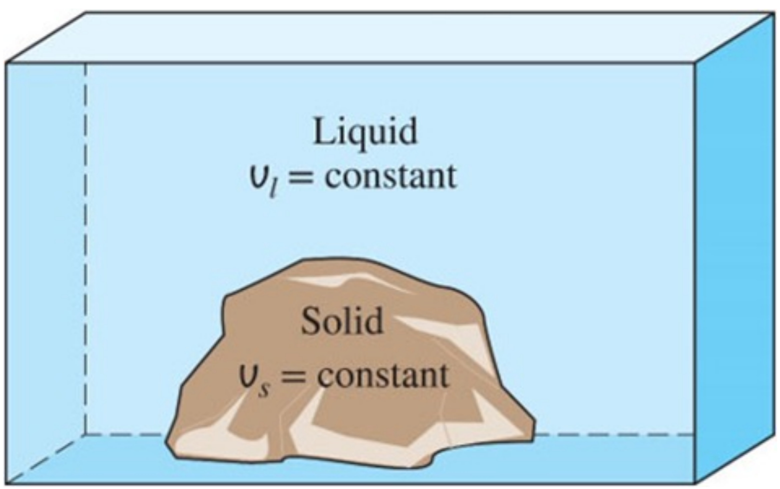
(*b*) At various temperatures

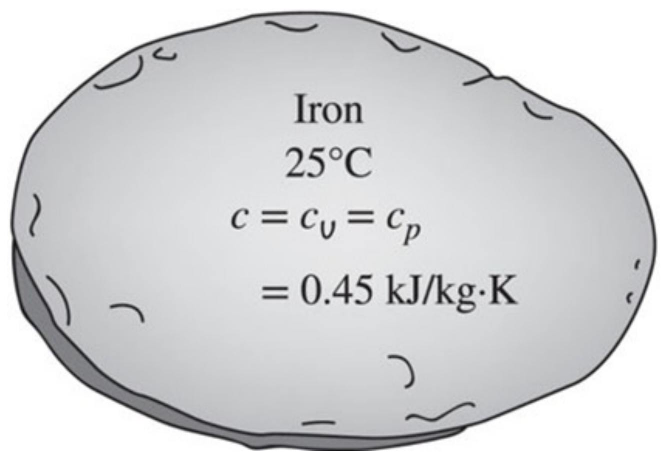
Temperature, K	c_p kJ/kg · K	c_v kJ/kg · K	k
	Air		
250	1.003	0.716	1.401
300	1.005	0.718	1.400
350	1.008	0.721	1.398
400	1.013	0.726	1.395
450	1.020	0.733	1.391
500	1.029	0.742	1.387
550	1.040	0.753	1.381
600	1.051	0.764	1.376
650	1.063	0.776	1.370
700	1.075	0.788	1.364
750	1.087	0.800	1.359
800	1.099	0.812	1.354
900	1.121	0.834	1.344
1000	1.142	0.855	1.336

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Internal/Energy/Enthalpy/Heats of Solids/Liquids

- Incompressible substance is a substance whose specific volume (or density) is constant.
- Solids and liquids are incompressible substances.





$$c_p = c_v = c$$

Internal/Energy/Enthalpy/Heats of Solids/Liquids

- Internal energy changes:

$$du = c_v dT = c(T)dt$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)dt$$

$$\Delta u \cong c_{avg}(T_2 - T_1)$$

Internal/Energy/Enthalpy/Heats of Solids/Liquids

- Enthalpy changes:

$$h = u + PV$$

$$dh = du + dP \times V + P \times dV$$

$$\Delta h \cong \Delta u + V \times \Delta P \sim c_{avg} \times \Delta T + v \times \Delta P$$

Internal/Energy/Enthalpy/Heats of Solids/Liquids

- We can say:
  - For solids:

$$\Delta h = \Delta u + V \times \Delta P \cong c_{avg} \times \Delta T + v \times \Delta P \cong c_{avg} \times \Delta T$$

  - For liquids (constant pressure process such as heaters):

$$h = \Delta u \cong c_{avg} \times \Delta T$$

  - For liquids (constant temperature process such as pumps):

$$h = v \times \Delta P$$

Internal/Energy/Enthalpy/Heats of Solids/Liquids

- The enthalpy of a compressed liquid:

$$h_{@P,T} \cong h_{f@T} + v_{f@T}(P - P_{sat\ @T})$$

$$h_{@P,T} \cong h_{f@T}$$

EXTRA EXAMPLE

Extra Example

- Determine the internal energy change Δu of hydrogen, in kJ/kg, as it is heated from 200 to 800 K, using
  a) The empirical specific heat equation as a function of temperature (Table A–2c)
  b) The cv value at the average temperature (Table A–2b)
  c) The cv value at room temperature (Table A–2a)

Extra Example

- Solution (a):

$$\bar{c}_v(T) = \bar{c}_p(T) - R = (a - R) + bT + cT^2 + dT^3$$

- From Table A-2c:
  - a = 29.11
  - b = -0.1916×10-[2]
  - c = 0.4003×10^{-5}
  - d = -0.8704×10^{-9}

$$\Delta\bar{u} = \int_1^2 \bar{c}_v(T) = [(a - R) + bT + cT^2 + dT^3]dT = 12{,}487\ \frac{kJ}{kmol}$$

$$\Delta u = \frac{12{,}487\,\frac{kJ}{kmol}}{2.016\ kmol} = 6{,}194\ kJ$$

Extra Example

- Solution (b): Using a constant c_p value from Table A-2b at the average temperature of 500 K

$$c_{v,avg} = c_{v\,@\,500K} = 10.389\,\frac{kJ}{kg-K}$$

$$\Delta u = c_{v\,@\,500K}(\mathrm{T}_2 - \mathrm{T}_1) = 10.389\,\frac{kJ}{kg-K}(800-200K) = 6{,}233\,\frac{kJ}{kg}$$

Extra Example

- Solution (c): Using a constant c_p value from Table A-2a at room temperature

$$c_{v,avg} = c_{v\,@\,300K} = 10.183\,\frac{kJ}{kg-K}$$

$$\Delta u = c_{v\,@\,500K}(\mathrm{T}_2 - \mathrm{T}_1) = 10.183\,\frac{kJ}{kg-K}(800-200K) = 6{,}110\,\frac{kJ}{kg}$$