CAE 208 / MMAE 320: Thermodynamics
Fall 2023

October 3, 2023
Energy analysis of closed systems (1)

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ANNOUNCEMENTS

Announcements

- The solution to Assignment 4 is posted
  - ❑ Please submit one file only
  - ❑ Always look at the solutions posted

- Assignment 5 will be posted next Tuesday (Time to focus on the exam)

- Midterm is scheduled for October 10

- Today's lecture will be in the exam (Content in our upcoming Thursday lecture will not be included in the exam)

- A problem-solving lecture was posted on Blackboard

Announcements

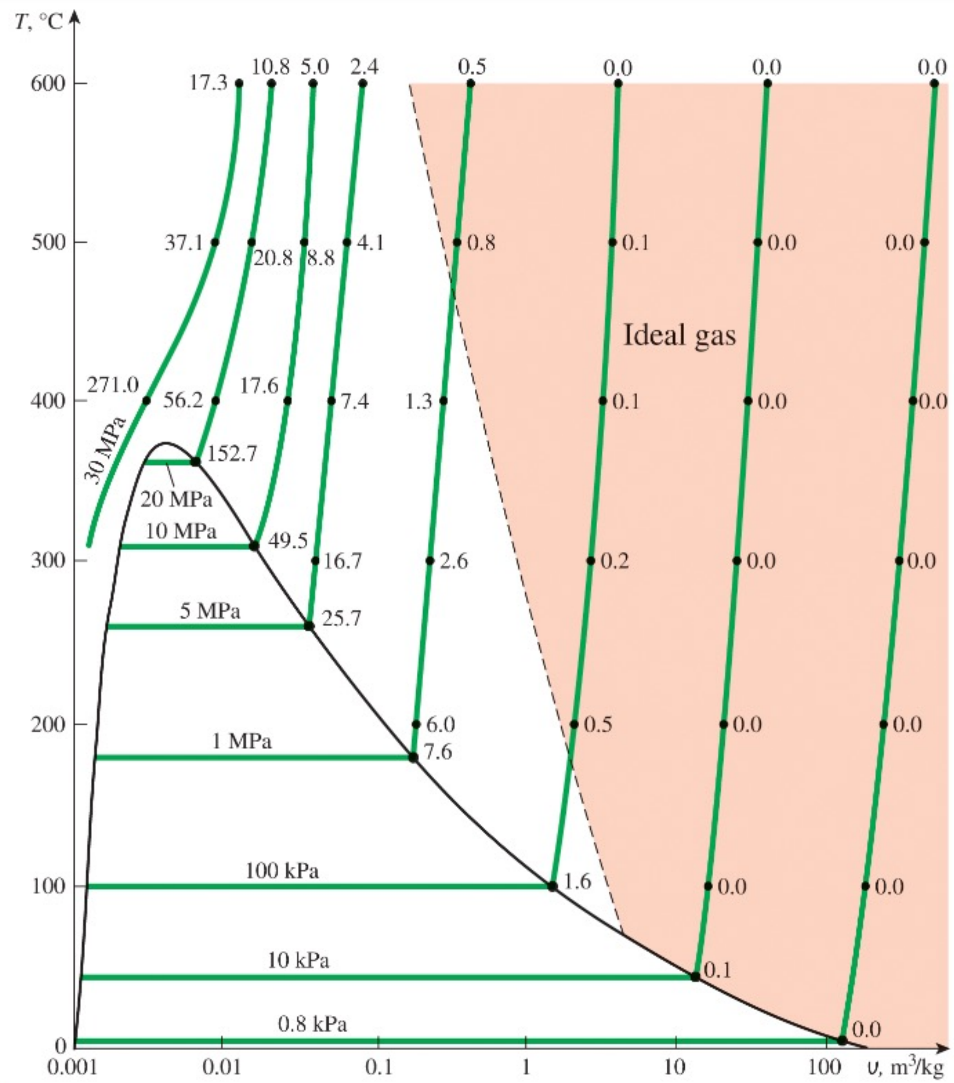
- For the first midterm exam:
  - ❑ The first part of the exam is closed book (You are allowed to have one page for your cheat sheet to include equations)
  - ❑ The second part of the exam is open book (only a thermodynamics book or thermodynamics property tables)

CLASS ACTIVITY

COMPRESSIBILITY FACTOR – A MEASURE OF OF DEVIATION FROM IDEAL-GAS BEHAVIOR

Compressibility Factor

- The figure shows $\left|\frac{v_{table}-v_{ideal}}{v_{table}}\right|\times 100$



Compressibility Factor

- Z factor for all gases is approximately the same at the same reduced temperature and pressure due to the principle of corresponding states

$$v = \frac{RT}{P}$$

$$T_R = \frac{T}{T_{Cr}}$$

Compressibility Factor

- Z factor for all gases is approximately the same at the same reduced temperature and pressure due to the principle of corresponding states

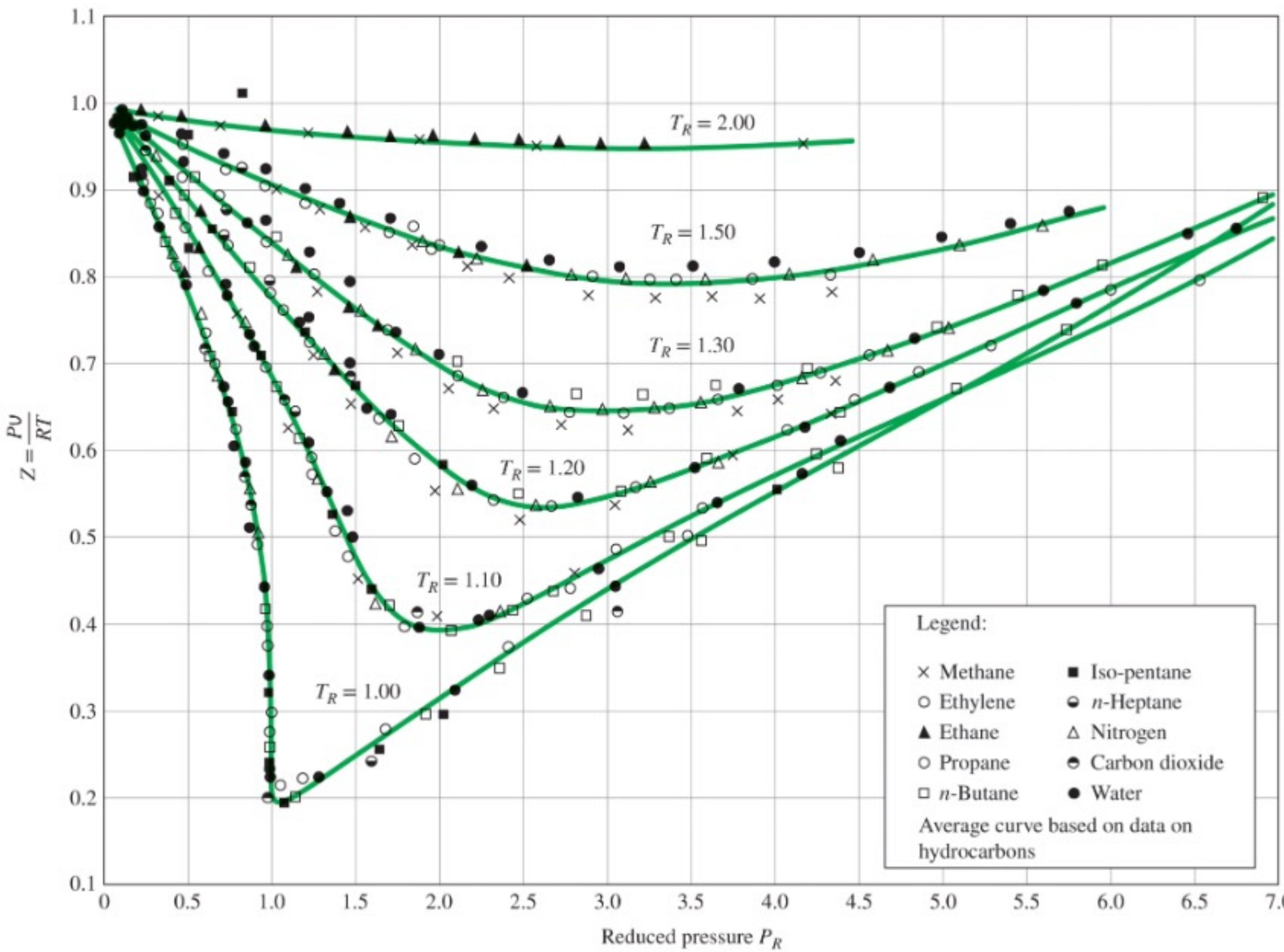
$$Pv = ZRT$$

$$v = \frac{ZRT}{P}$$

$$Z = \frac{v_{actual}}{v_{idea}}$$

Compressibility Factor

- We can define a “generalized compressibility chart”
- Let’s look at a few observations:



CLASS ACTIVITY

Class Activity

- Determine specific volume of refrigeratnt-134a at 1 MPa and 50 °C using (a) the ideal-gas equation of state and (b) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 m^3/kg and determine the error involved in each case.

Class Activity

- Solution (a):

TABLE A–1

Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg · K*	Critical-point properties: Temperature, K	Pressure, MPa	Volume, m^3/kmol
Propane	C_3H_8	44.097	0.1885	370	4.26	0.1998
Propylene	C_3H_6	42.081	0.1976	365	4.62	0.1810
Sulfur dioxide	SO_2	64.063	0.1298	430.7	7.88	0.1217
Tetrafluoroethane (R-134a)	CF_3CH_2F	102.03	0.08149	374.2	4.059	0.1993
Trichlorofluoromethane (R-11)	CCl_3F	137.37	0.06052	471.2	4.38	0.2478
Water	H_2O	18.015	0.4615	647.1	22.06	0.0560
Xenon	Xe	131.30	0.06332	289.8	5.88	0.1186

$$v = \frac{RT}{P} = \frac{\left(0.0815\,\frac{kJ}{kg.K}\right)(50 + 273.15\ K)}{1000\ kPa} = 0.026325\,\frac{m^3}{kg}$$

$$Error = \frac{0.026325 - 0.021796}{0.021796} = 0.208$$

Class Activity

- Solution (b):

$$P_R = \frac{P}{P_{cr}} = \frac{1\ MPa}{4.059\ MPa} = 0.246$$

$$Z = 0.84$$

$$T_R = \frac{T}{T_{cr}} = \frac{323\ K}{374.2\ K} = 0.863$$

$$v_{actual} = Zv_{ideal} = (0.84)\left(0.026325\,\frac{m^3}{kg}\right) = 0.022113\,\frac{m^3}{kg}$$

$$Error = \frac{0.022113 - 0.021796}{0.021796} \sim 0.02$$

CLASS ACTIVITY

Class Activity

- Determine the specific volume of refrigerant-134a vapor at 0.9 MPa and 70°C based on
  a) The ideal-gas equation
  b) The generalized compressibility chart
  c) Data from tables. Also, determine the error involved in the first two cases.

Class Activity

- Solution (a):

TABLE A–1

Molar mass, gas constant, and critical-point properties

Substance	Formula	Molar mass, M kg/kmol	Gas constant, R kJ/kg · K*	Critical-point properties: Temperature, K	Critical-point properties: Pressure, MPa	Critical-point properties: Volume, m^3/kmol
Propane	C_3H_8	44.097	0.1885	370	4.26	0.1998
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Xenon	Xe	131.30	0.06332	289.8	5.88	0.1186

$$R = 0.08149 \frac{kJ}{kg - K}$$

$$T_{cr} = 374.2\ K$$

$$P_{cr} = 4.049\ MPa$$

Class Activity

- Solution (a):

$$Pv = RT$$

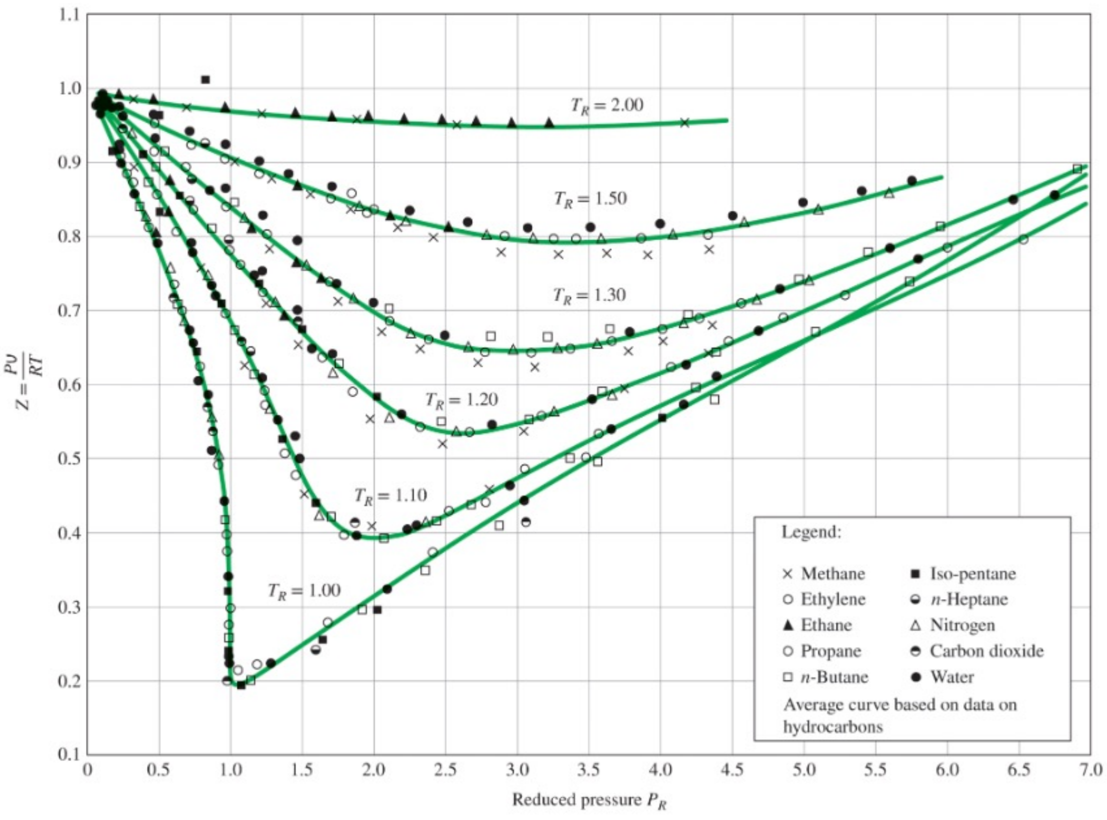
$$v = \frac{RT}{P} = \frac{(0.08149\,\frac{kJ}{kg-K})(273.15 + 70\ K)}{0.9\times10^{3}\ kPa} = 0.03105\,\frac{m^{3}}{kg}$$

Class Activity

- Solution (b):

$$P_R = \frac{P}{P_{cr}} = \frac{0.9}{4.049} = 0.222$$

$$T_R = \frac{T}{T_{cr}} = \frac{343}{374.2} = 0.917$$



$$Z = 0.894$$

Class Activity

- Solution (b):

$$v = Zv_{ideal} = (0.894)\left(0.03105\,\frac{m^3}{kg}\right) = 0.02776\,\frac{m^3}{kg}$$

Class Activity

- Solution (c):

TABLE A–13

Superheated refrigerant-134a

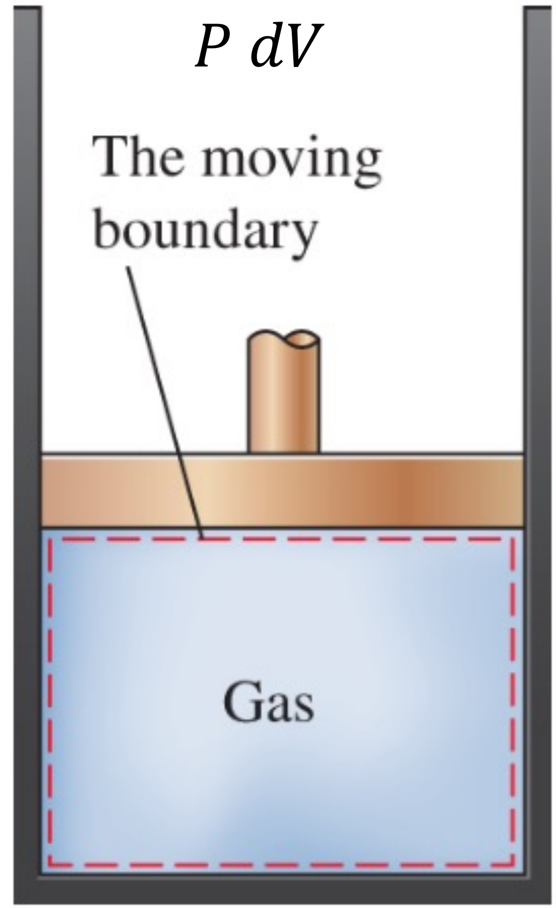
T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	$P = 0.80$ MPa ($T_{sat} = 31.31$°C)				$P = 0.90$ MPa ($T_{sat} = 35.51$°C)			
Sat.	0.025645	246.82	267.34	0.9185	0.022686	248.82	269.25	0.9169
40	0.027035	254.84	276.46	0.9481	0.023375	253.15	274.19	0.9328
50	0.028547	263.87	286.71	0.9803	0.024809	262.46	284.79	0.9661
60	0.029973	272.85	296.82	1.0111	0.026146	271.62	295.15	0.9977
70	0.031340	281.83	306.90	1.0409	0.027413	280.74	305.41	1.0280
80	0.032659	290.86	316.99	1.0699	0.028630	289.88	315.65	1.0574
90	0.033941	299.97	327.12	1.0982	0.029806	299.08	325.90	1.0861

QUIZ

MOVING BOUNDARY WORK

Moving Boundary Work

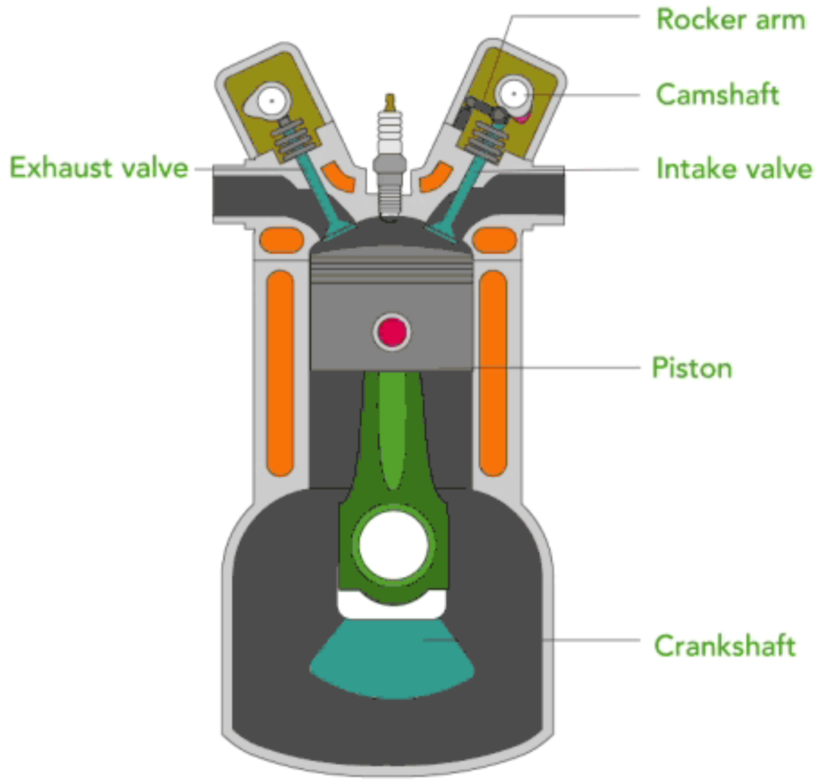
- The expansion or compression work is often called *moving boundary work* or simply *boundary work*



Think about internal combustion engines

Moving Boundary Work

- The expansion or compression work is often called *moving boundary work* or simply *boundary work*

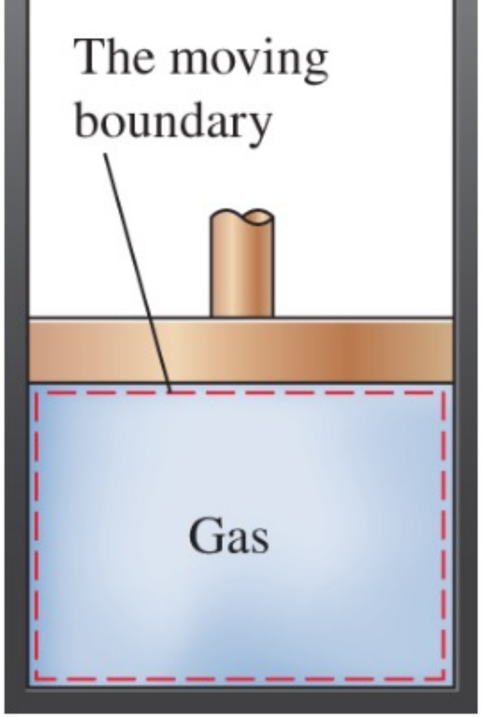


DOE

Moving Boundary Work

- The moving work associated with real engines or compressors cannot be determined exactly from a thermodynamic analysis alone. Why?

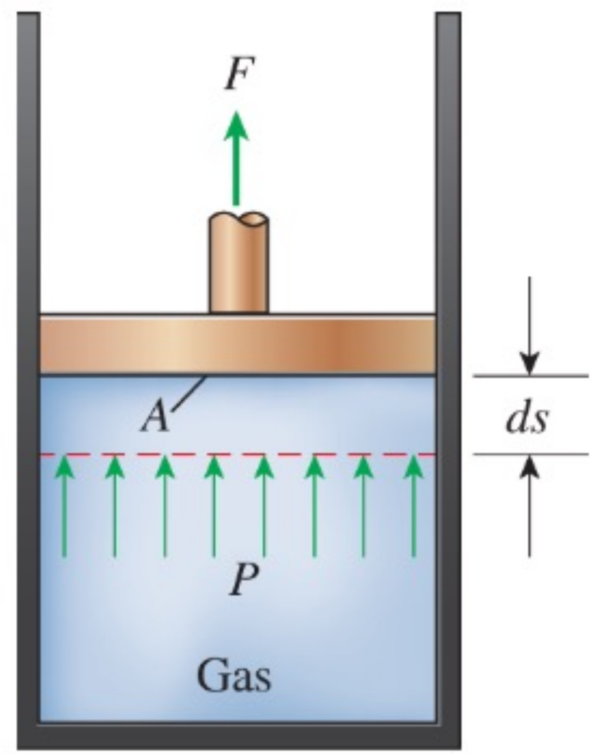
$P\,dV$



Moving Boundary Work

- Consider the gas enclosed in the cylinder-piston device
- The process is quasi equilibrium (ds)

$$\delta W_b = Fds = PAds = P\,dV$$



Moving Boundary Work

- Let’s think about dV in terms of expansion and compression

Moving Boundary Work

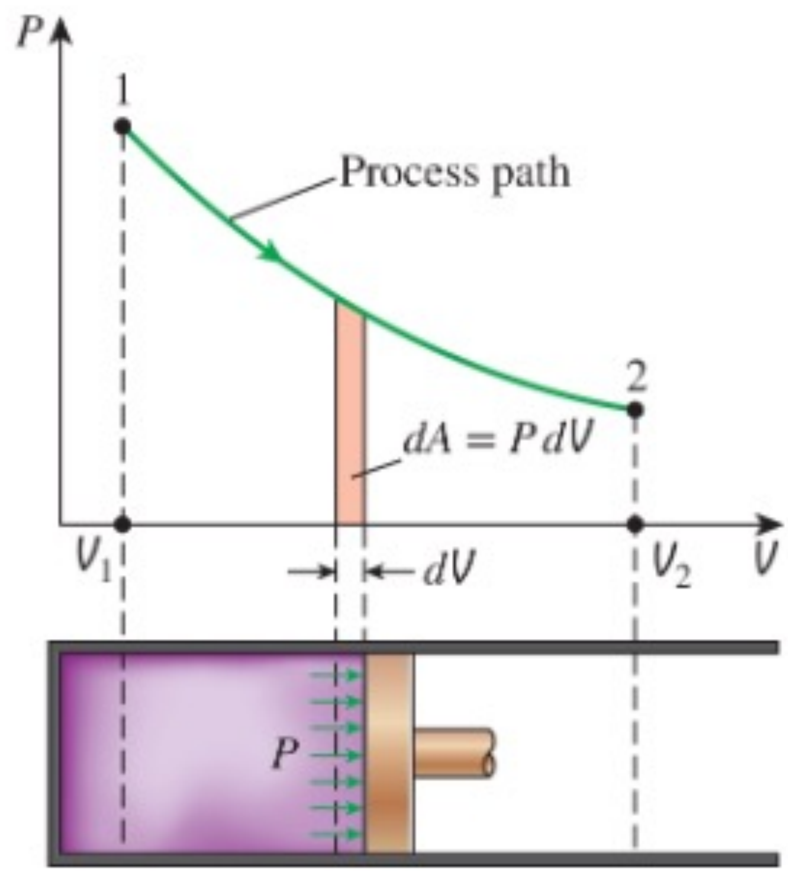
- The total boundary work done during the entire process as the piston moves is obtained by adding all the differential works from the initial state to the final state

$$W_b = \int_1^2 P dV$$

Moving Boundary Work

- For a quasi-equilibrium expansion process, we can write:

$$Area = A = \int_1^2 dA = \int_1^2 PdV$$



Moving Boundary Work

- Let's look at some basic integration

$$Area = A = \int_1^2 dA = W_b = \int_1^2 PdV$$

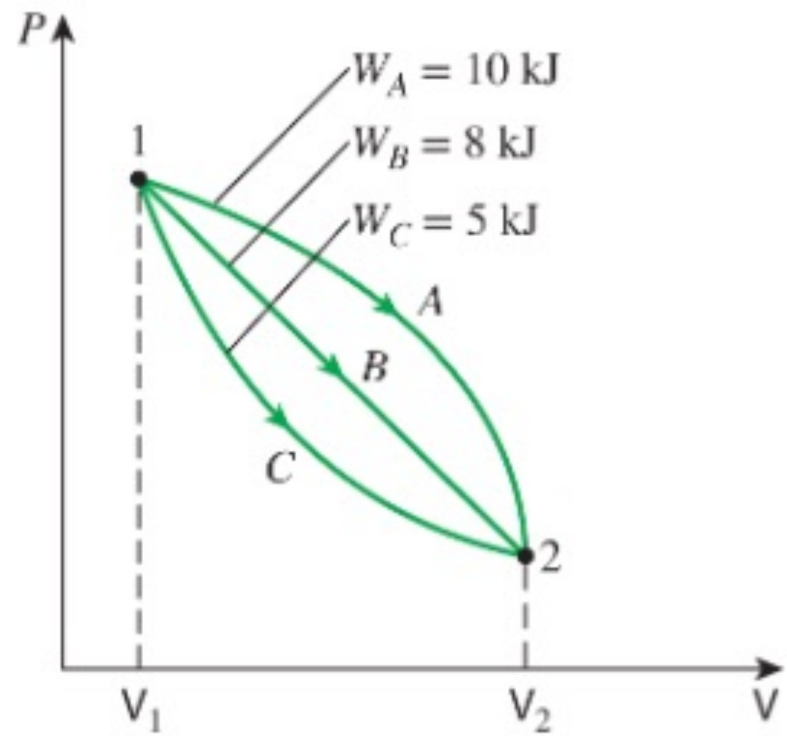
Moving Boundary Work

- Basically, we can say:

The area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system (on the P-V diagram, in presents the boundary work done per unit mass)

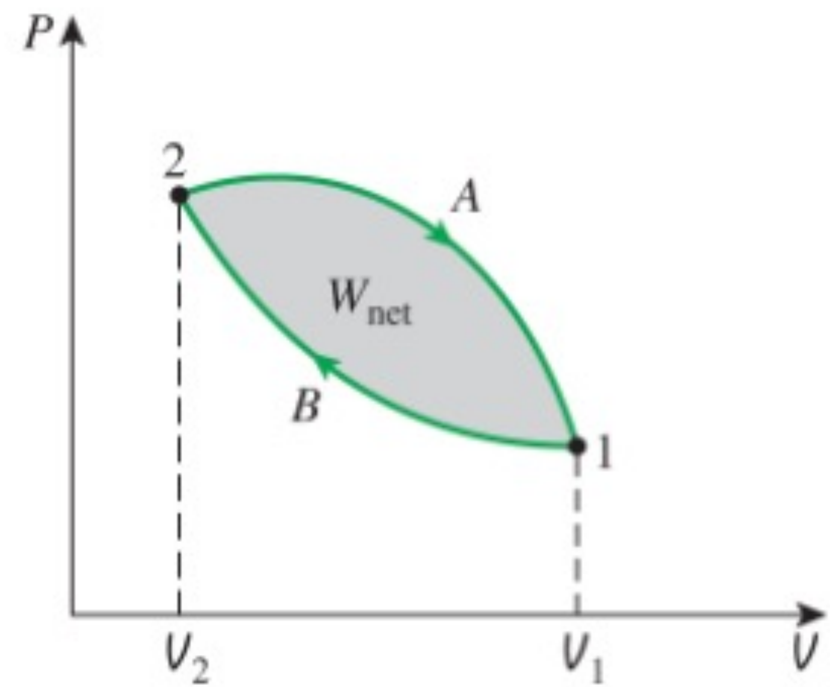
Moving Boundary Work

- The boundary work done during a process depends on the path followed as well as the end states:



Moving Boundary Work

- The net work done during a cycle is the difference between the work done by the system and the work done on the system. Why?



Moving Boundary Work

- We can use the equation for nonquasi-equilibrium process since properties are defined for equilibrium states only

$$W_b = \int_1^2 P_i dV$$

Moving Boundary Work

- We can use the equation for nonquasi-equilibrium process since properties are defined for equilibrium states only

$$W_b = W_{friction} + W_{atm} + W_{crank} = \int_1^2 \left(F_{friction} + P_{atm}A + F_{crank}\right)dx$$

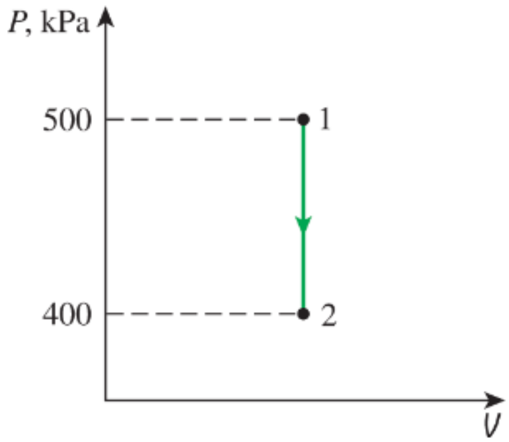
CLASS ACTIVITY

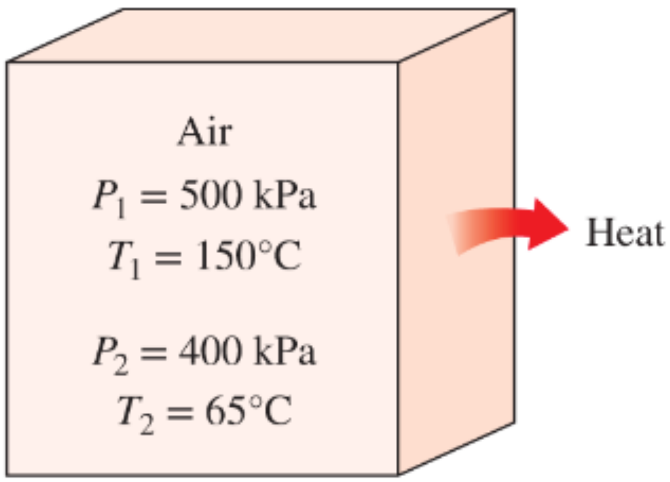
Class Activity

- A rigid tank contains air at 500 kPa and 150 °C. As a result of heat to the surroundings, the temperature and pressure inside the tank drop to 65 °C and 400 kPa, respectively. Determine the boundary work done during this process

Class Activity

- Solution





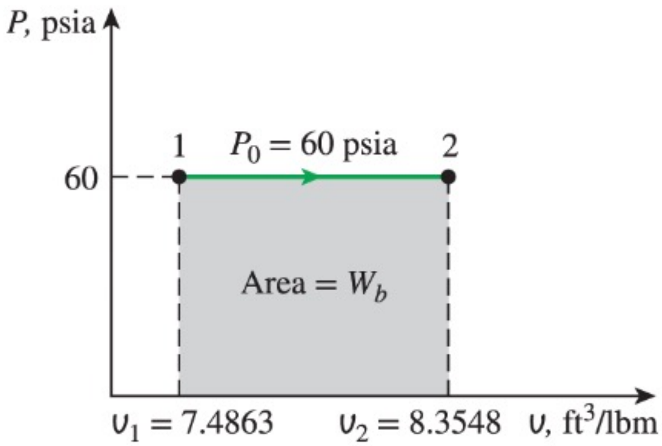
CLASS ACTIVITY

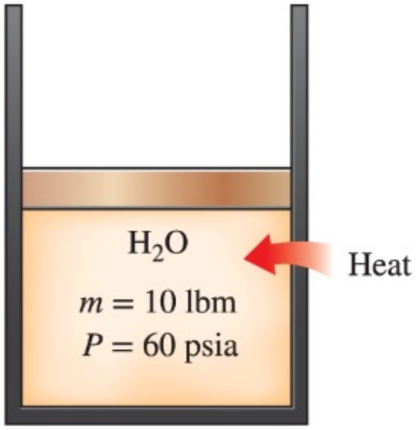
Class Activity

- A frictionless piston-cylinder device contains 10 lbm of steam at 60 psia and 320 °F. Heat is now transferred to the steam until the temperature reaches 400 °F. If the piston is not attached to a shaft and its mass is constant, determine work done by the steam during this process.

Class Activity

- Solution:





$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

$$W_b = mP_0(v_2 - v_1)$$

Class Activity

- Solution:

$$\begin{cases} P_1 = 60\ psia \\ T_1 = 320\ °F \end{cases} \qquad v_1 = 7.4863\,\frac{ft^3}{lbm}$$

Use Table A-6

$$\begin{cases} P_2 = 60\ psia \\ T_2 = 400\ °F \end{cases} \qquad v_2 = 8.3548\,\frac{ft^3}{lbm}$$

$$W_b = mP_0(v_2 - v_1)$$
$$= (10\ lbm)(8.3548 - 7.4863)\left(\frac{ft^3}{lbm}\right)\left(\frac{1Btu}{5.404\ psia - ft^3}\right) = 96.4\ Btu$$

Class Activity

Conversion Factors

Dimension	Metric	English
Acceleration	$1\ m/s^2 = 100\ cm/s^2$	$1\ m/s^2 = 3.2808\ ft/s^2$ $1\ ft/s^2 = 0.3048\ m/s^2$
Area	$1\ m^2 = 104\ cm^2 = 10^6\ mm^2 = 10^{-6}\ km^2$	$1\ m^2 = 1550\ in.^2 = 10.764\ ft^2$ $1\ ft^2 = 144\ in.^2 = 0.0929034\ m^2$
Density	$1\ g/cm^3 = 1\ kg/L = 1000\ kg/m^3$	$1\ g/cm^3 = 62.428\ lbm/ft^3 = 0.036127\ lbm/in.^3$ $1\ lbm/in.^3 = 1728\ lbm/ft^3$ $1\ kg/m^3 = 0.062428\ lbm/ft^3$
Energy, Heat, Work, Internal Energy, Enthalpy	$1\ kJ = 1000\ J = 1000\ N{\cdot}m = 1\ kPa{\cdot}m^3$ $1\ kJ/kg = 1000\ m^2/s^2$ $1\ kWh = 3600\ kJ$ $1\ Wh = 3600\ J$ $1\ cal = 4.1868\ J$ $1\ Cal = 4.1868\ kJ$	$1\ kJ = 0.94782\ Btu$ $1\ Btu = 1.055056\ kJ$ $= 5.40395\ psia{\cdot}ft^3$ $= 778.169\ lbf{\cdot}ft$ $1\ Btu/lbm = 25.037\ ft^2/s^2$ $= 2.326\ kJ/kg$ $1\ kJ/kg = 0.430\ Btu/lbm$ $1\ kWh = 3412.14\ Btu$ $1\ therm = 10^5\ Btu = 1.055 \times 10^5\ kJ$ (natural gas)

https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781118848944.app8

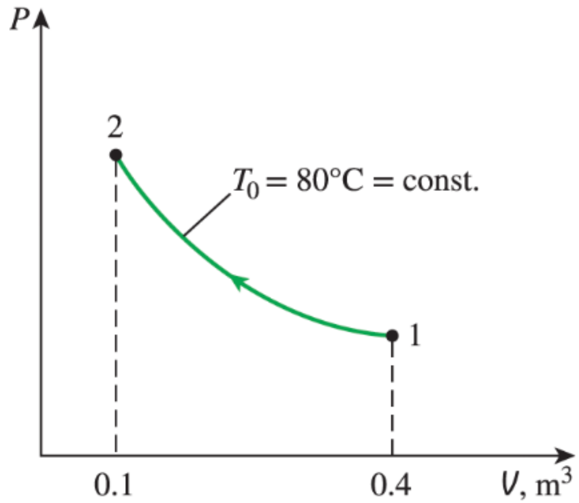
CLASS ACTIVITY

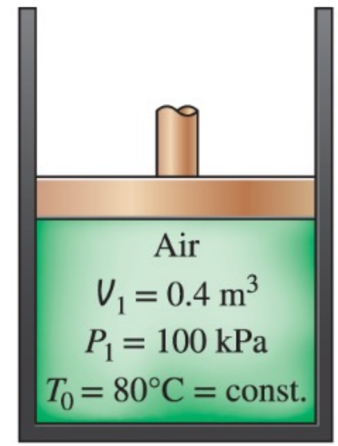
Class Activity

- A piston-cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80 °C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process

Class Activity

- Solution:



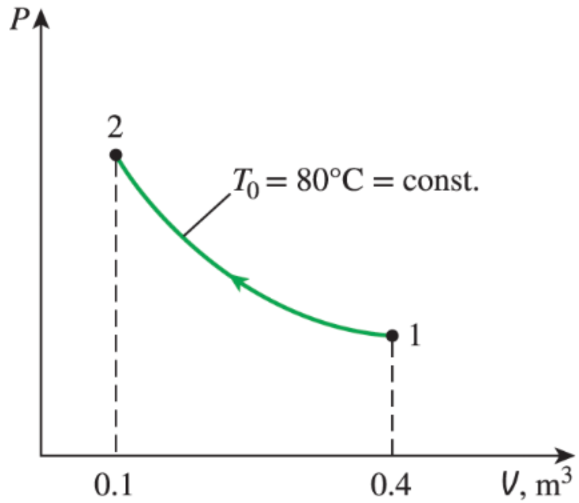


$$PV = mRT_0 \rightarrow P = \frac{C}{V}$$

$$W_b = \int_1^2 P dV = \int_1^2 (\frac{C}{V}) dV = C \int_1^2 (\frac{dV}{V})$$

Class Activity

- Solution:

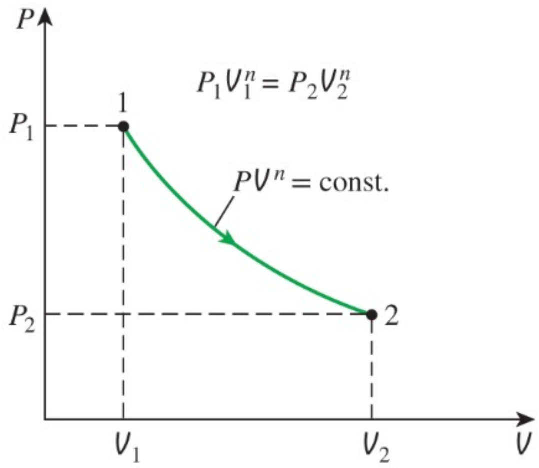


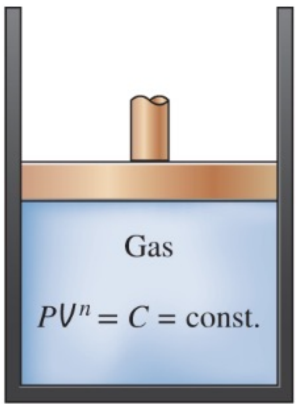
$$W_b = C \times Ln(V)\Big|_{V_1}^{V_2} = C[Ln(V_2) - Ln(V_1)] = P_1V_1 \times Ln(\frac{V_2}{V_1})$$

$$W_b = (100\ kPa)(0.4\ m^3)\left(\mathrm{Ln}\left(\frac{0.1}{0.4}\right)\right)\left(\frac{1kJ}{1\ kPa.m^2}\right) = -55.5\ kJ$$

Moving Boundary Work

- During expansion or compression processes of gases and volume are often related in $PV^n = C$ which is known as a Polytropic process





$$P = CV^{-n}$$

Moving Boundary Work

- Polytropic process

$$P = CV^{-n}$$

$$W_b = \int_1^2 P\,dV = \int_1^2 (CV^{-n})dV = \frac{C\left((V^{-n+1} - V^{-n+1}\right)}{(-n+1)} = \frac{P_2V_2 - P_1V_1}{1-n}$$

Moving Boundary Work

- Polytropic process

$$C = P_1 V_1^n = P_2 V_2^n$$

$$W_b = \frac{mR(T_2 - T_1)}{1 - n} \quad n \neq 1 \ (kJ)$$

Moving Boundary Work

- For the case of n =1

$$W_b = \int_1^2 PdV = \int_1^2 CV^{-1}dV = PV \times Ln(\frac{V_2}{V_1})$$

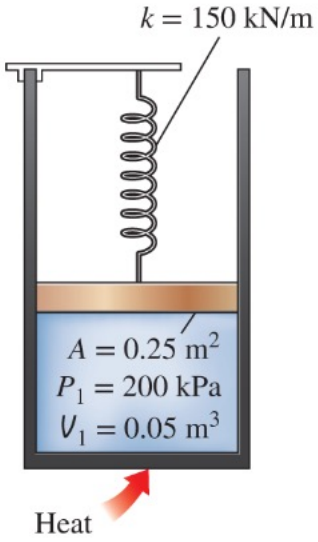
CLASS ACTIVITY

Class Activity

- A piston-cylinder device contains 0.05 m^3 of a gas initially at 200 kPa. At this state, a linear spring that has a spring constant of 150 kN/m is touching the piston but exerting no force on it. Now heat is transferred to the gas, causing the piston to rise and to compress the spring until the volume inside the cylinder doubles. If the cross-sectional area of the piston is 0.25 m^2, determine
  a) The final pressure inside the cylinder
  b) The total work done by the gas
  c) The fraction of this work done against the spring to compress it

Class Activity

- Solution

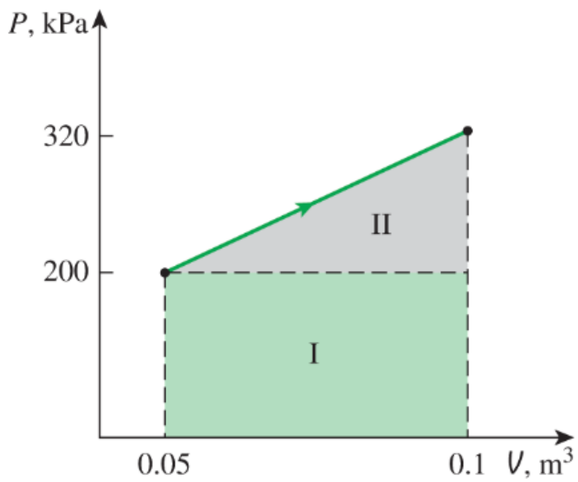


$$V_2 = 2V_1 = 2(0.05\ m^3) = 0.1\ m^3$$

$$x = \frac{\Delta V}{A} = \frac{(0.1 - 0.05)m^3}{0.25\ m^2} = 0.2\ m$$

Class Activity

- Solution



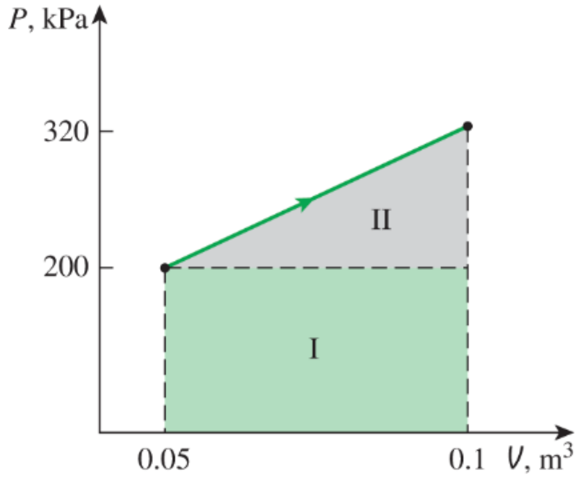
$$F = kx = \left(150\,\frac{kN}{m}\right)(0.2\ m) = 30\ kN$$

$$P = \frac{F}{A} = \frac{30\ kN}{0.25\ m^2} = 120\ kN$$

$$200 + 120 = 320\ kN$$

Class Activity

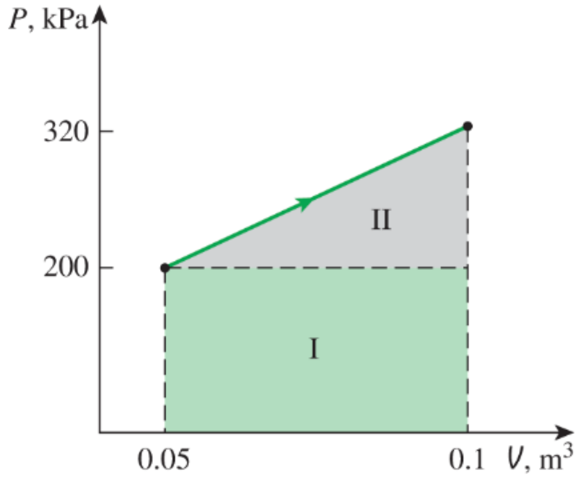
- Solution (b)



$$W = area = \frac{(200+320)kPa}{2}[0.1^2 - 0.05^2]\left(\frac{1kJ}{1\ kPa.m^2}\right) = 13\ kJ$$

Class Activity

- Solution (c)



$$W_{spring} = \frac{1}{2}[320 - 200]kPa(0.05\ m^3) = 3\ kJ$$

$$W_{spring} = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2}(150)(0.2^2 - 0^2) = 3\ kJ$$

Moving Boundary Work

- Moving boundary work under different processes

Process	**Moving boundary work**
Constant volume	0
Constant pressure	$P_0(V_2 - V_1)$
Isothermal	$P_1 V_1 \times Ln(\frac{V_2}{V_1})$ $P_1 V_1 \times Ln(\frac{P_1}{P_2})$ $mRT_o \times Ln(\frac{V_2}{V_1})$
Polytropic	$\frac{P_2 V_2 - P_1 V_1}{1-n}$ $\frac{mR(T_2 - T_1)}{1-n}$

ENERGY BALANCE FOR CLOSED SYSTEMS

Energy Balance for Closed Systems

- The net change (increase or decrease) in the total energy of the system during a process is equal to the difference between the total energy entering and the total energy leaving the system during the process

$$(Total\ energy\ entering\ the\ system) - (Total\ energy\ leaving\ the\ system) = (Change\ in\ the\ total\ energy\ of\ the\ sysem)$$

$$E_{in} - E_{out} = \Delta E_{system}$$

This is known as the energy balance

Energy Balance for Closed Systems

- Energy change of a system ΔE_{system}

$$Energy\ change = Energy\ at\ final\ state\ - Energy\ at\ initial\ state$$

$$\Delta E_{system} = E_{final} - E_{initial} = E_2 - E_1$$

Energy is a property, and the value of a property does not change unless the state of the system changes

Energy Balance for Closed Systems

- We can sum the heat, work, and mass, and the heat transfer:

$$E_{in} - E_{out} = (Q_{in} - Q_{out}) + (W_{in} - W_{out}) = \Delta E_{system}$$

Net energy transfer by heat, work

Change in internal, kinetic, potential, …, energies

Energy Balance for Closed Systems

- We can sum the heat, work, and mass, and the heat transfer in the rate form:

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system}$$

Rate of net energy transfer by heat, work

Rate of change in internal, kinetic, potential, …, energies

Energy Balance for Closed Systems

- The energy balance can be expressed on a per unit mass basis as

$$e_{in} - e_{out} = \Delta e_{system}$$

Energy Balance for Closed Systems

- For constant rates, we can write:

$$Q = \dot{Q}\Delta t$$

$$W = \dot{W}\Delta t$$

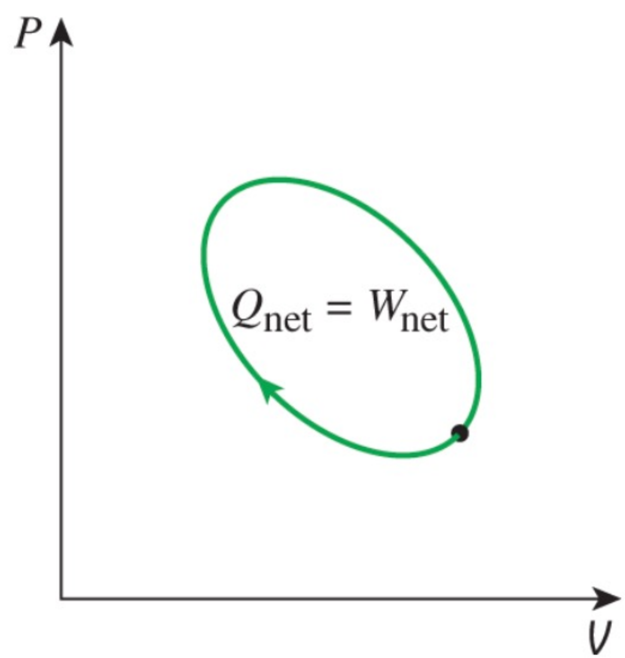
$$E = \left(\frac{dE}{dt}\right)\Delta t$$

Energy Balance for Closed Systems

- For a closed system undergoing a cycle, the initial and final states are identical:

$$\Delta E = E_{in} - E_{out} = 0 \quad \rightarrow E_{in} = E_{out}$$

$$W_{net,out} = Q_{net,in} \quad \rightarrow \dot{W}_{net,out} = \dot{Q}_{net,in}$$



Energy Balance for Closed Systems

- We can write: