CAE 208 Thermal-Fluids Engineering I
MMAE 320: Thermodynamics
Fall 2022

November 22, 2022
Entropy (iii)

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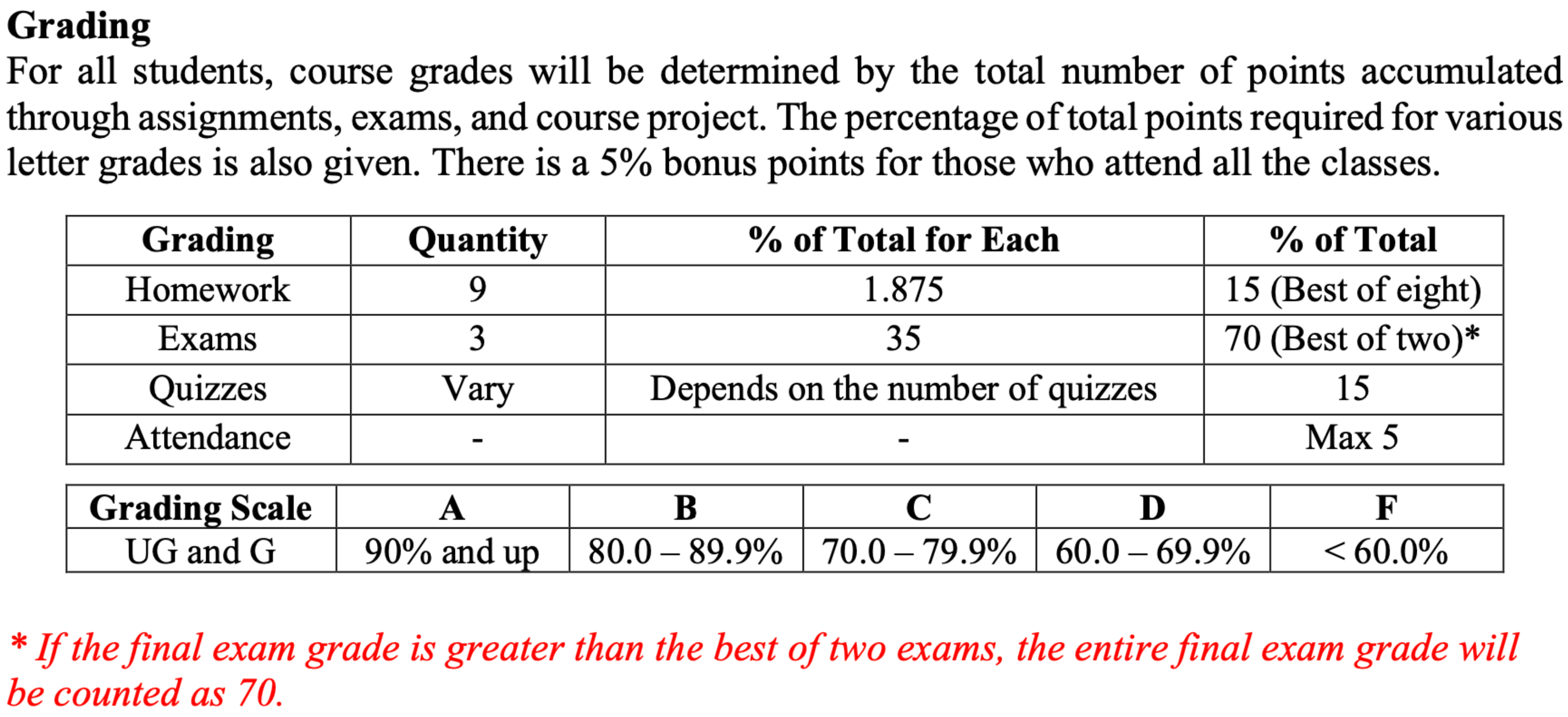
ANNOUNCEMENTS

Announcements

- Updated syllabus:



Grading

For all students, course grades will be determined by the total number of points accumulated through assignments, exams, and course project. The percentage of total points required for various letter grades is also given. There is a 5% bonus points for those who attend all the classes.

Grading	**Quantity**	**% of Total for Each**	**% of Total**
Homework	9	1.875	15 (Best of eight)
Exams	3	35	70 (Best of two)*
Quizzes	Vary	Depends on the number of quizzes	15
Attendance	-	-	Max 5

Grading Scale	**A**	**B**	**C**	**D**	**F**
UG and G	90% and up	80.0 – 89.9%	70.0 – 79.9%	60.0 – 69.9%	< 60.0%

** If the final exam grade is greater than the best of two exams, the entire final exam grade will be counted as 70.*

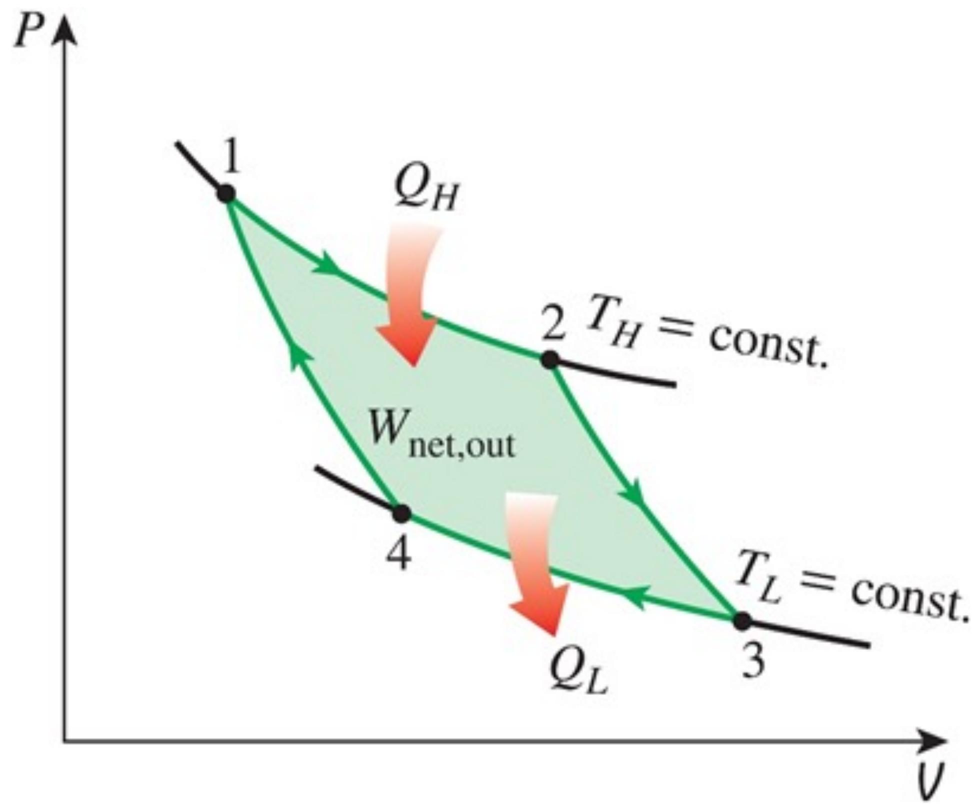
Announcements

- Assignment 9 is posted (extra assignment – if you have good assignment grades, no need to submit this one)

RECAP

Recap

- The Reversed Carnot Cycle
  - ❑ The Carnot heat-engine cycle is a totally reversible cycle



P-V diagram of the Carnot cycle

Recap

- The equality in the *Clausius inequality* holds for totally or just internally reversible cycles and the inequality for the irreversible ones

$$\left(\oint \frac{\delta Q}{T}\right)_{int,rev} = 0$$

Recap

- Let's try to find out more about entropy with looking into a cycle:

$$dS = \left(\oint \frac{\delta Q}{T} \right)_{rev} \qquad \left(\frac{kJ}{K}\right)$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{int,rev}$$

Recap

- A special case: *Internally reversible isothermal* heat transfer processes:

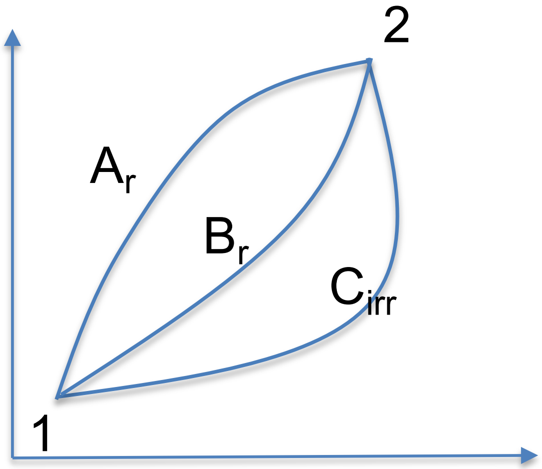
$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{int,rev} = \int_1^2 \left(\frac{\delta Q}{T_0}\right)_{int,rev} = \frac{1}{T_0}\int_1^2 \delta Q_{int,rev}$$

$$\Delta S_{isothermal} = \frac{Q}{T_0} \qquad \left(\frac{kJ}{K}\right)$$

(A reservoir can absorb or supply heat indefinitely at a constant temperature)

Recap

- The Increase of Entropy Principle



$$\oint_1^2 \left(\frac{\delta Q}{T}\right)_A + \oint_2^1 \left(\frac{\delta Q}{T}\right)_B = 0$$

$$\oint_1^2 \left(\frac{\delta Q}{T}\right)_B + \oint_2^1 \left(\frac{\delta Q}{T}\right)_C < 0$$

$$\longrightarrow$$

$$\oint_1^2 \left(\frac{\delta Q}{T}\right)_A > \oint_2^1 \left(\frac{\delta Q}{T}\right)_C$$

$$\oint_1^2 \left(\frac{\delta Q}{T}\right)_A = \oint_1^2 (dS)_A = \oint_1^2 (dS)_C$$

$$\oint_1^2 (dS)_C > \oint_2^1 \left(\frac{\delta Q}{T}\right)_C$$

$$S_2 - S_1 > \oint_2^1 \frac{\delta Q}{T}$$

Recap

- For entropy, we can say

$$dS = \left(\frac{\delta Q}{T}\right)_{rev}$$

$$dS > \left(\frac{\delta Q}{T}\right)_{irr}$$

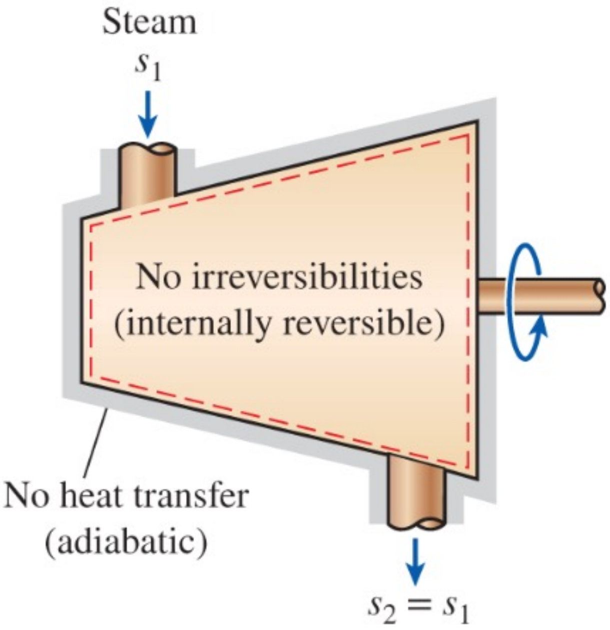
$$dS = \frac{\delta Q}{T} + \delta S_{gen}$$

Recap

- The entropy of a fixed mass can be changed by:
  - ❑ Heat Transfer
  - ❑ Irreversibilities

- Entropy of a fixed mass does not change during a process that is internally reversible and adiabatic. During this process entropy remains constant and we call it *isentropic* process

$$\Delta s = 0 \quad or \quad s_2 = s_1 \quad (\frac{kJ}{kg-K})$$



PROPERTY DIAGRAMS INVOLVING ENTROPY

Property Diagrams Involving Entropy

- Property diagrams serve as great visual aids in the thermodynamic analysis of processes

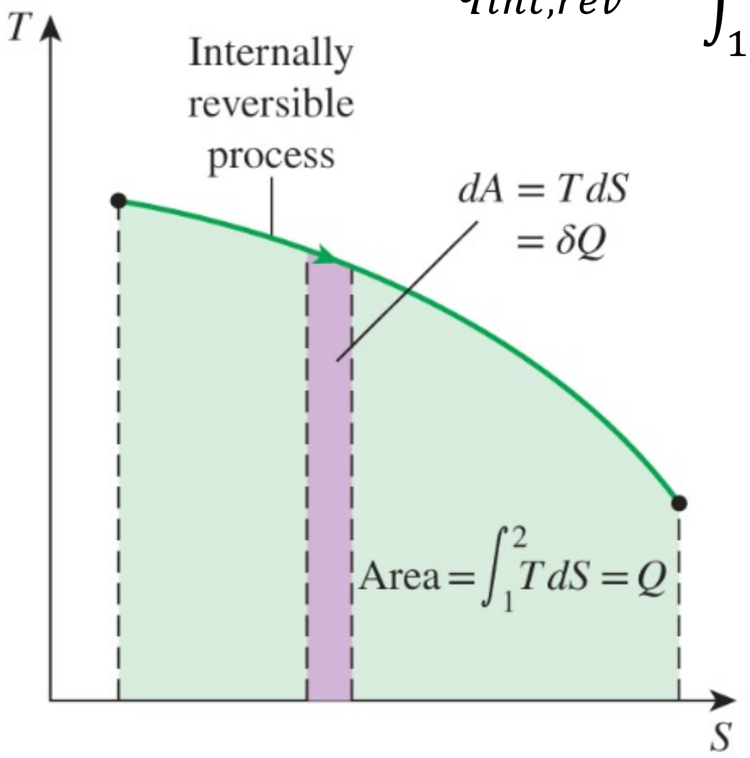
- Based on the 2nd law, we can plot new diagrams that involve entropy:
  - ☐ Temperature-entropy
  - ☐ Enthalpy-entropy

Property Diagrams Involving Entropy

- We can rearrange our entropy equation:

$$\delta Q_{int,rev} = T\,dS \qquad Q_{int,rev} = \int_1^2 T\,dS \quad (kJ)$$

$$\delta q_{int,rev} = T\,ds \qquad q_{int,rev} = \int_1^2 T\,ds \quad \left(\frac{kJ}{kg}\right)$$



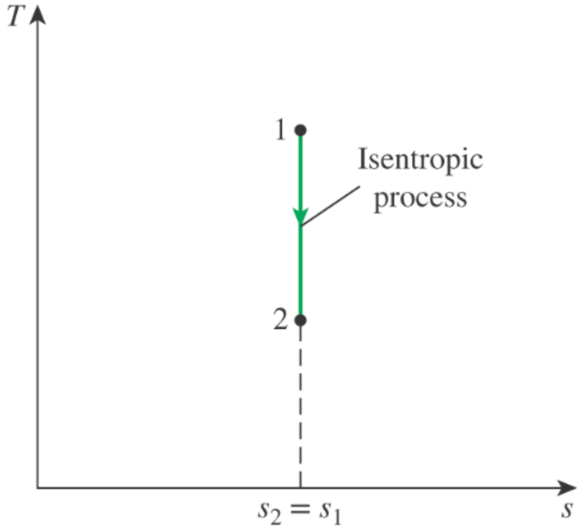
(The area under the process curve on a T-S diagram represents heat transfer during an internally reversible process)

Property Diagrams Involving Entropy

- Special cases:
  - ❑ Internally reversible isothermal process):

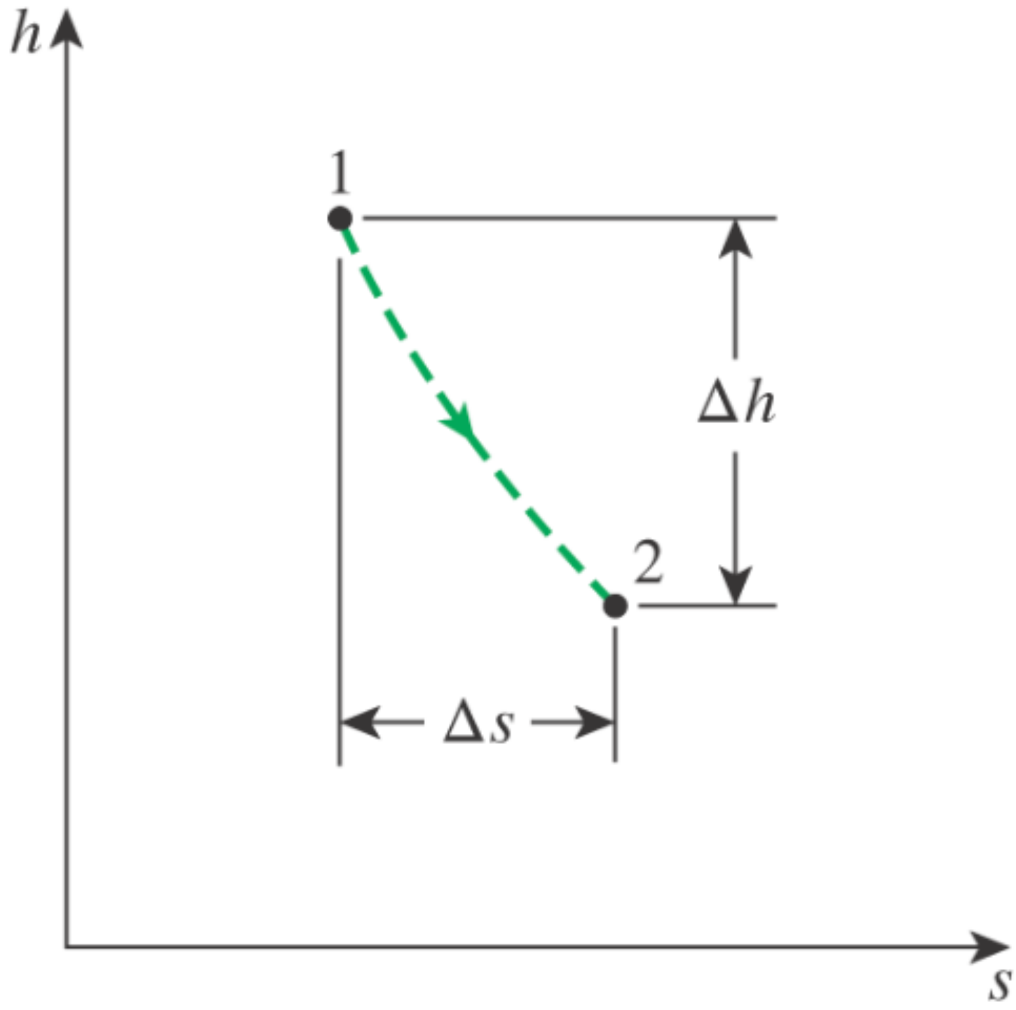
$Q_{int,rev} = T_0 \Delta S$ $\qquad$ $q_{int,rev} = T_0 \Delta s$

  - ❑ T-s diagram for an isentropic process:



Property Diagrams Involving Entropy

- h-s diagram (could be helpful for steady flow of devices such as nozzles, compressors, turbines):



Property Diagrams Involving Entropy

- T-s diagram of water is given in the appendix:

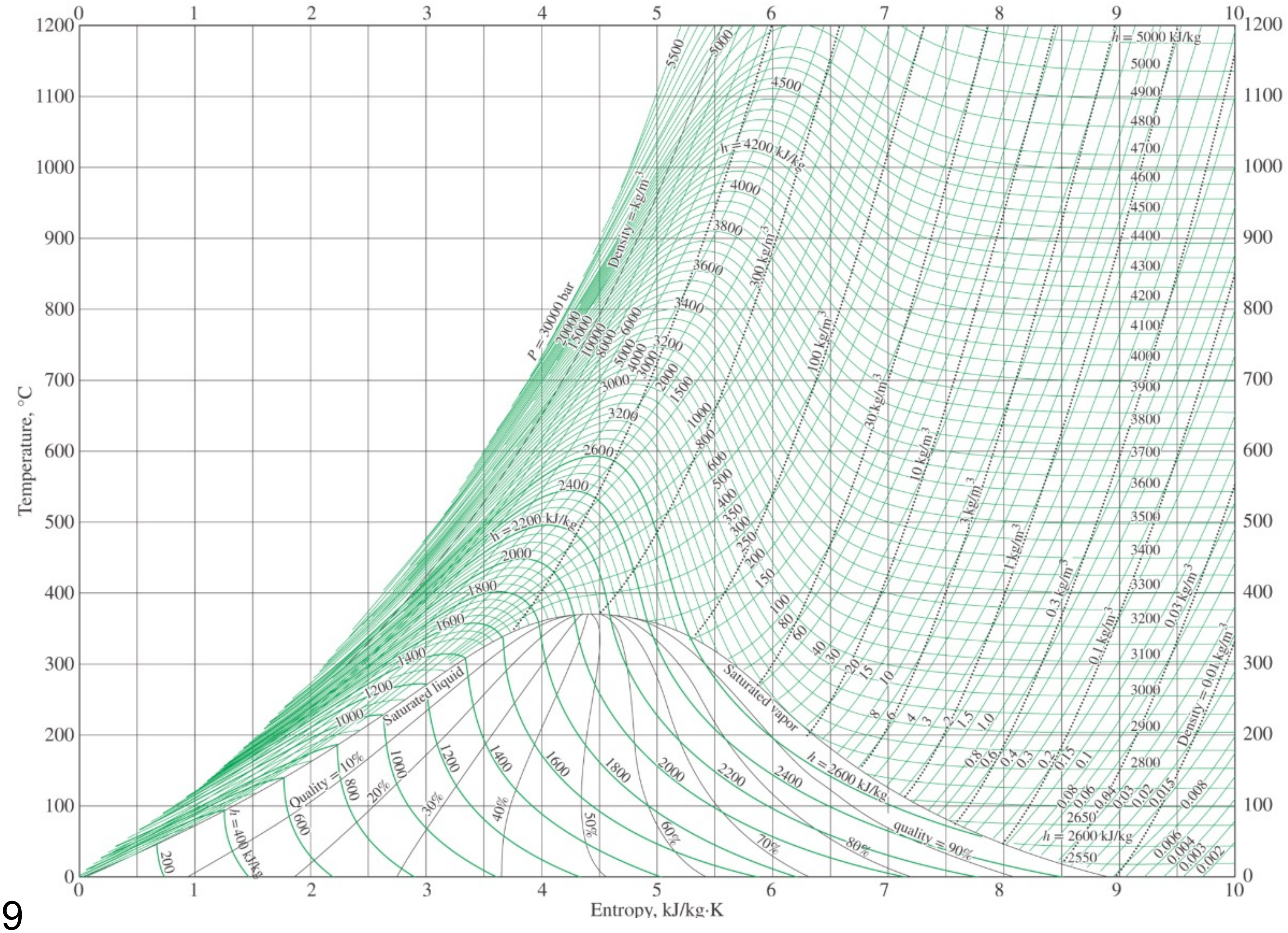


Figure A-9

Property Diagrams Involving Entropy

- h-s diagram of steam is given in the appendix:

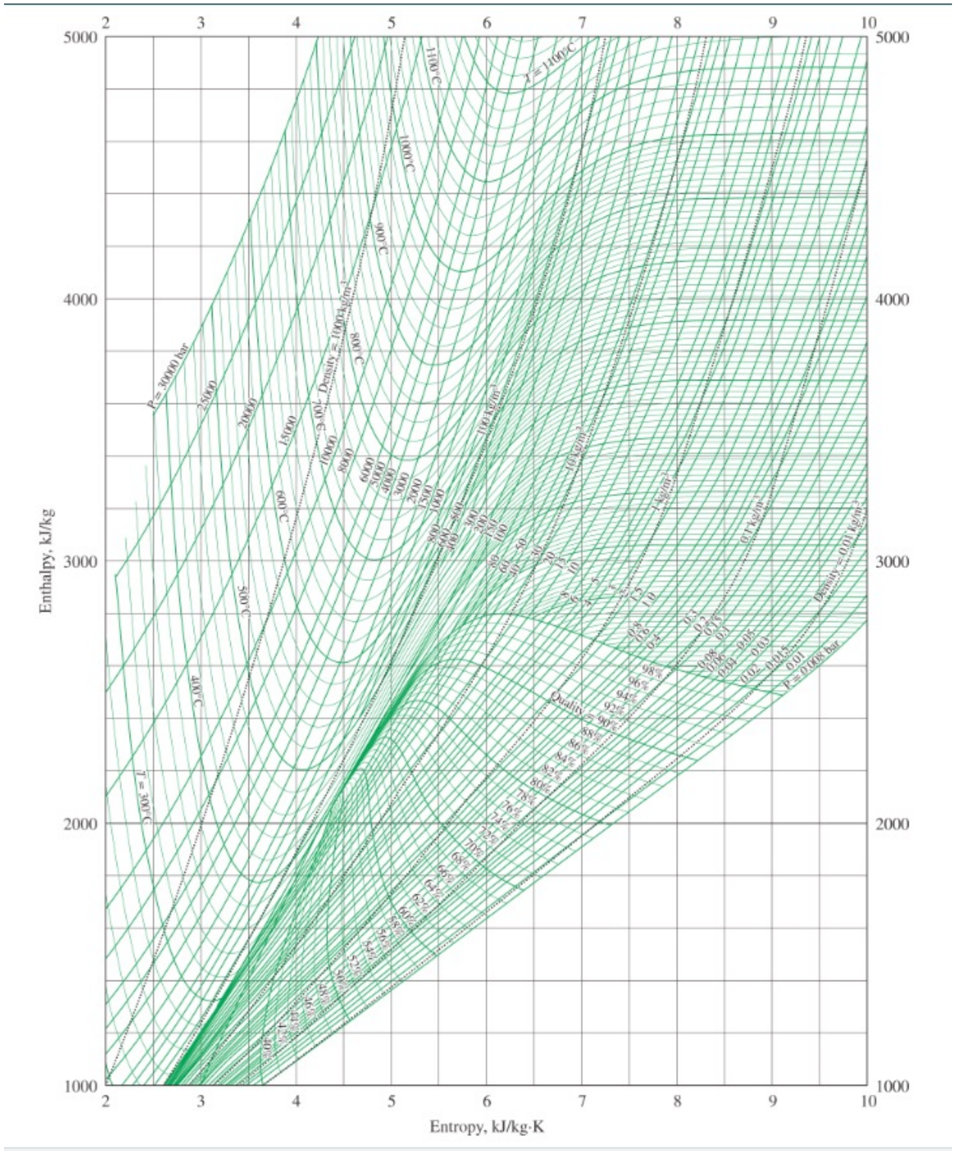


Figure A-10

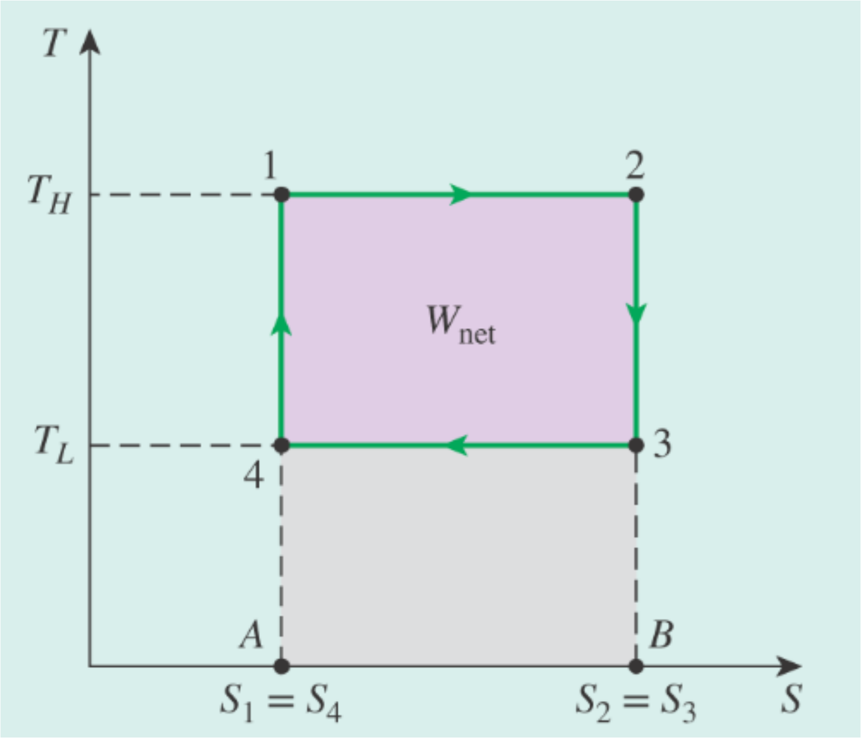
CLASS ACTIVITY

Class Activity

- Show the Carnot cycle on a T-S diagram and indicate the areas that represent the heat supplied and rejected and the network in the diagram.

Class Activity

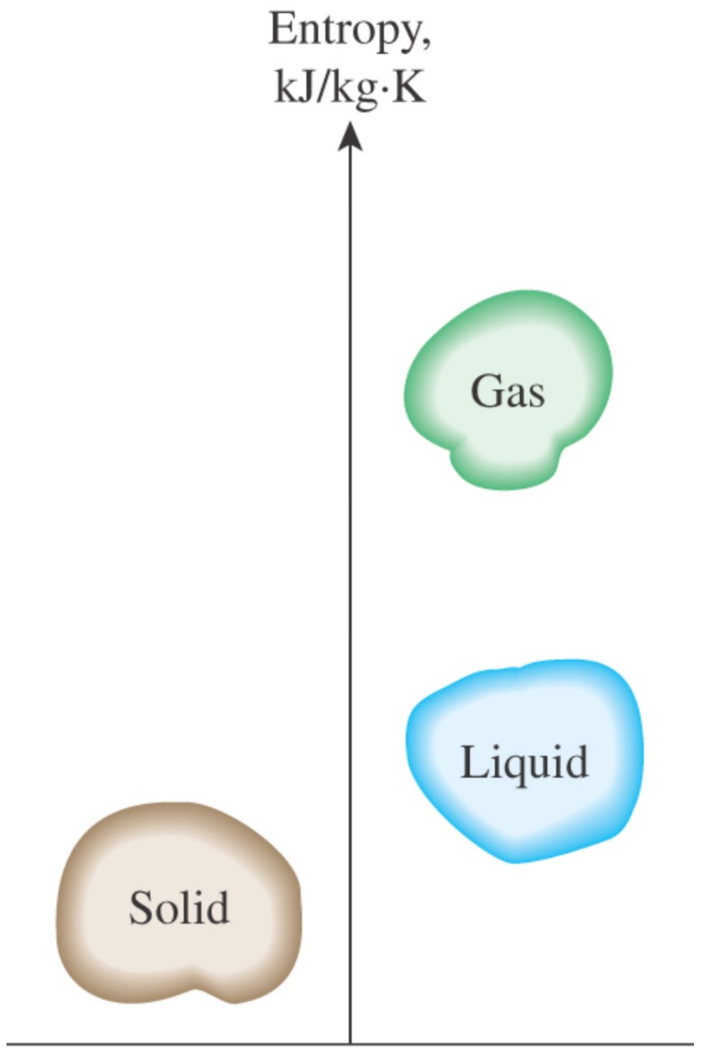
- Solution:



WHAT IS ENTROPY

What Is Entropy

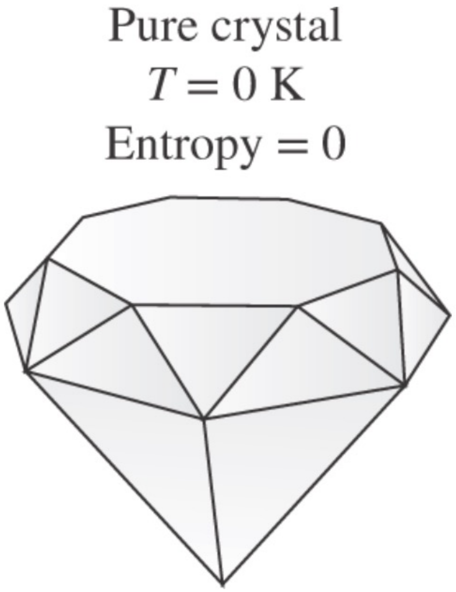
- Entropy can be viewed as a measure of molecular disorder or molecular randomness (lowest in solid phase and highest in the gas phase)



What Is Entropy

- Oscillations in solid phase fade as temperature is decreased, and the molecules supposedly become motionless at absolute zero. This represents a state of ultimate molecular order (and *minimum energy*).

- The entropy of a pure crystalline substance at absolute zero temperature is zero since there is no uncertainty about the state of the molecules at that instant (3rd law of thermodynamics)



What Is Entropy

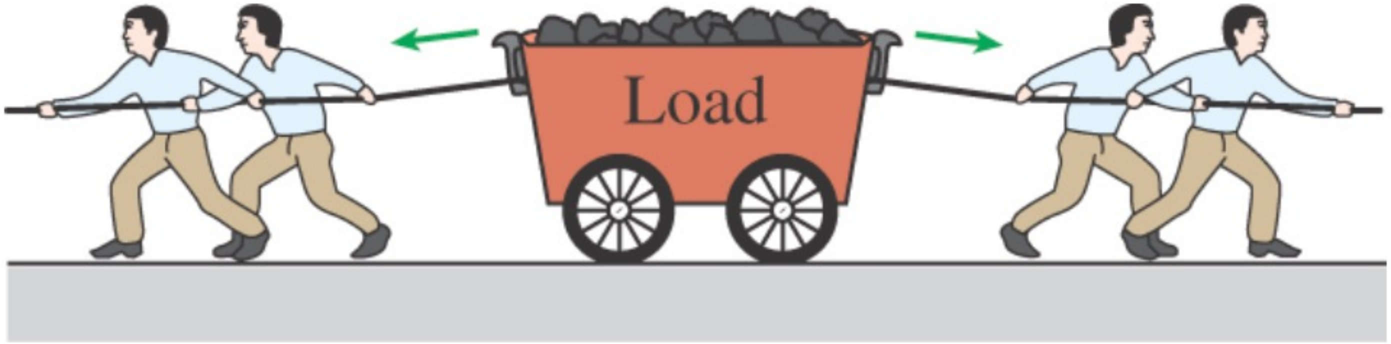
- The third law of thermodynamics provide an absolute reference point for the determination of entropy

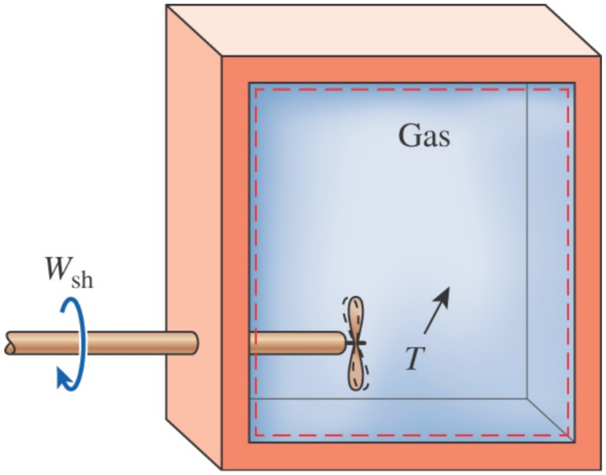
- The entropy determined relative to this point is called absolute entropy and it is extremely useful in the thermodynamics analysis of entropy

- The entropy of a substance that is not pure crystalline (such as a solid solution) is not zero at absolute zero temperature

What Is Entropy

- A few examples:





THE T DS RELATIONS

The T ds Relations

- The first T ds (or Gibbs) equation (closed stationary system):

$$\delta Q_{int,rev} - \delta W_{int,rev,out} = dU$$

$$TdS = dU + PdV \qquad (kJ)$$

$$\delta Q_{int,rev} = Tds$$

$$Tds = du + Pdv \qquad (\frac{kJ}{kg})$$

$$\delta W_{int,rev,out} = PdV$$

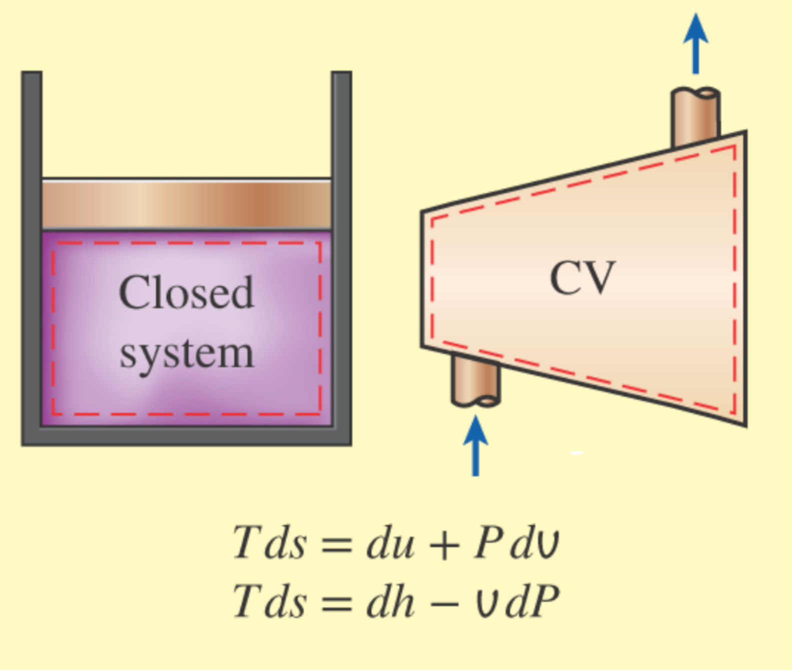
$$\left.\begin{array}{l} Tds = du + Pdv \\ h = u + Pv \quad \rightarrow dh = du + Pdv + vdP \end{array}\right\} \rightarrow Tds = dh - vdP$$

The T ds Relations

- Use the first T ds (or Gibbs) equation to solve for entropy changes

$$ds = \frac{du}{T} + \frac{Pdv}{T}$$

$$ds = \frac{du}{T} - \frac{vdP}{T}$$



ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Entropy Change of Liquids and Solids

- Liquids and solids can be approximated as incompressible substances ($dv \cong 0$ & $c_p = c_v = c_p = c$):

$$ds = \frac{du}{T} - \frac{vdP}{T}$$

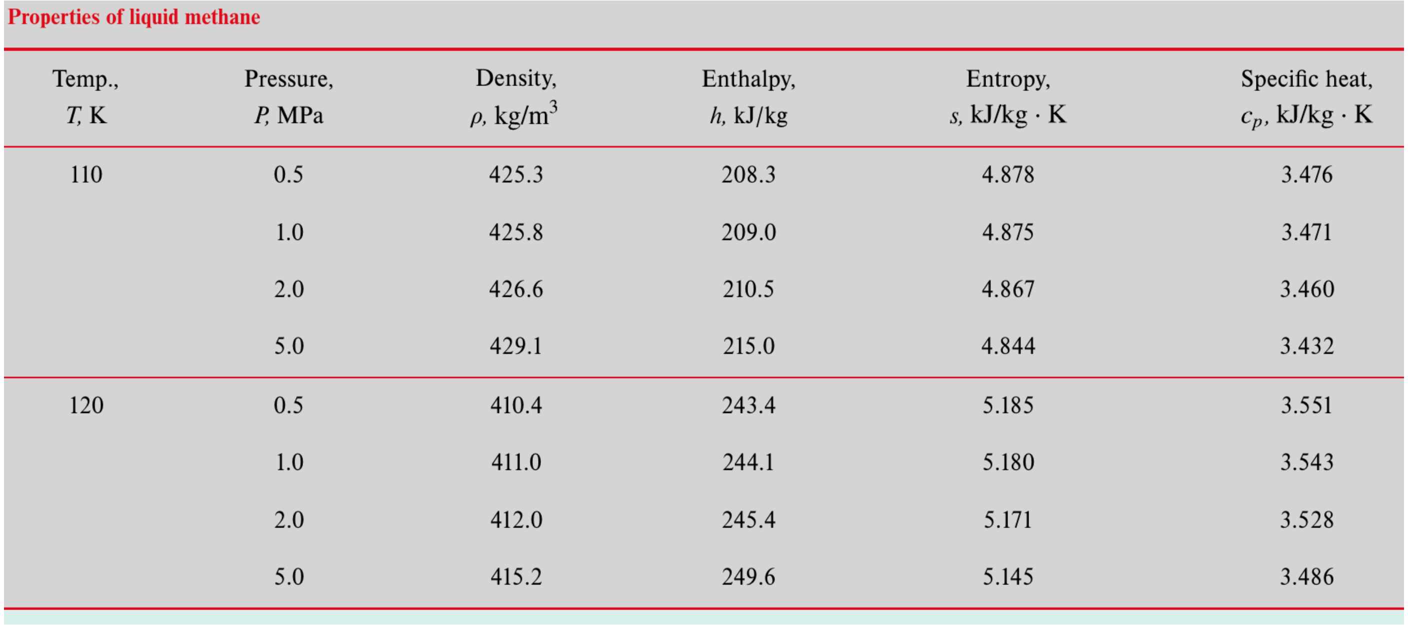
$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg}\ln(\frac{T_2}{T_1})$$

$$s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{avg}\ln\left(\frac{T_2}{T_1}\right) = 0 \quad \rightarrow \quad T_2 = T_1$$ (For isentropic)

CLASS ACTIVITY

Class Activity

- Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or ~ -82 °C) and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperature and pressure are given in Table below. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa.
  a) Using tabulated properties
  b) Approximating liquid methane as an incompressible substance?

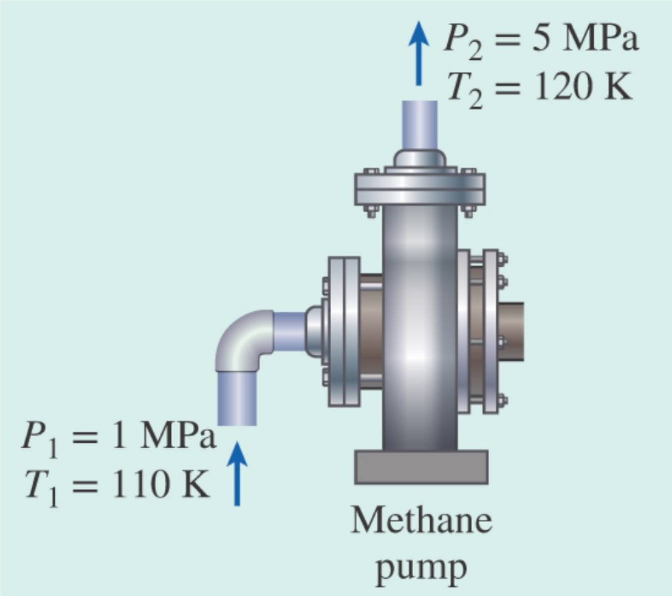


Properties of liquid methane

Temp., T, K	Pressure, P, MPa	Density, ρ, kg/m^3	Enthalpy, h, kJ/kg	Entropy, s, kJ/kg · K	Specific heat, c_p, kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

Class Activity

- Solutions (a):



$$\left.\begin{matrix} P_1 = 1\ MPa \\ T_1 = 110\ K \end{matrix}\right\} \rightarrow \begin{matrix} s_1 = 4.875\,\frac{kJ}{kg-K} \\ c_{p1} = 3.471\,\frac{kJ}{kg-K} \end{matrix}$$

$$\left.\begin{matrix} P_2 = 5MPa \\ T_2 = 120\ K \end{matrix}\right\} \rightarrow \begin{matrix} s_1 = 5.145\,\frac{kJ}{kg-K} \\ c_{p1} = 3.486\,\frac{kJ}{kg-K} \end{matrix}$$

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = 0.270\,\frac{kJ}{kg - K}$$

Class Activity

- Solutions (b):

$$c_{avg} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785\,\frac{kJ}{kg - K}$$

$$\Delta s = c_{avg}\ln\left(\frac{T_2}{T_1}\right) = (3.4785\,\frac{kJ}{kg - K})Ln\left(\frac{120K}{110K}\right) = 0.303\,\frac{kJ}{kg - K}$$

Class Activity

- Solutions (error):

$$Error = \frac{\Delta s_{actual} - \Delta_{ideal}}{\Delta s_{actual}} = \frac{|0.27 - 0.303|}{0.270} = 0.122\ (or\ 12.2\%)$$

THE ENTROPY CHANGE OF IDEAL GASES

The Entropy Change of Ideal Gases

- For gases, we can write:

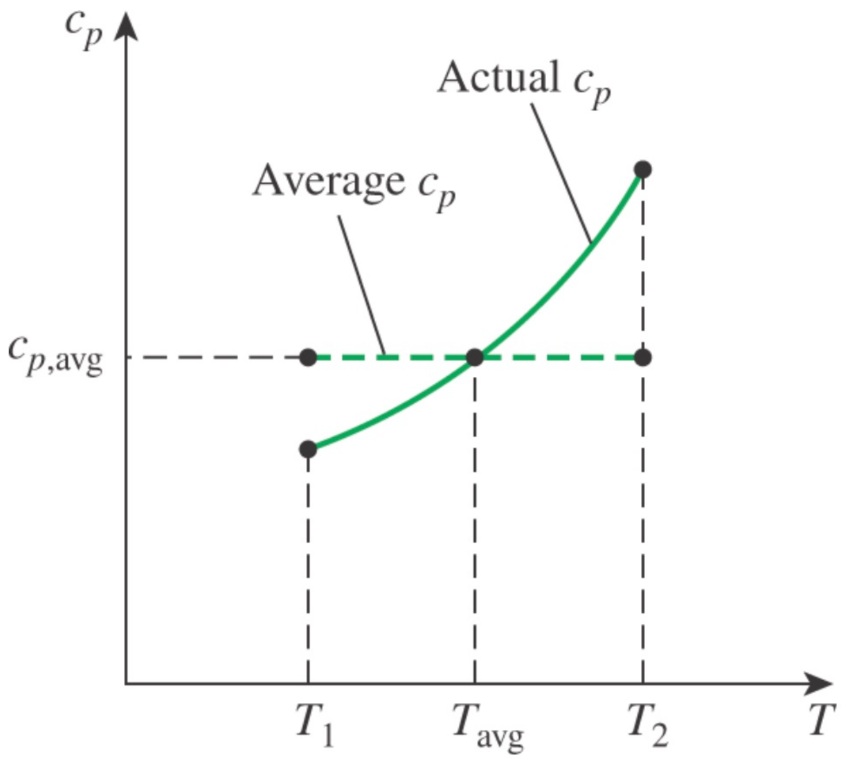
$$ds = \frac{du}{T} - \frac{vdP}{T} = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln(\frac{v_2}{v_1})$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} + R \ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis):



$$s_2 - s_1 = \int_1^2 c_v(T)\,\frac{dT}{T} + R\ln(\frac{v_2}{v_1})$$

$$s_2 - s_1 = c_{v,avg}\ln\left(\frac{T_2}{T_1}\right) + R\times ln\left(\frac{v_2}{v_1}\right)$$

$$\begin{cases} dh = c_p dT \\ v = \dfrac{RT}{P} \end{cases} \rightarrow s_2 - s_1 = c_{p,avg}\times\ln\left(\frac{T_2}{T_1}\right) - R\times\ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis): We can express in unit-mole basis:

$$\overline{s_2} - \overline{s_1} = \overline{c_{v,avg}} \ln\left(\frac{T_2}{T_1}\right) + R_u \times ln\left(\frac{v_2}{v_1}\right)$$

$$\overline{s_2} - \overline{s_1} = \overline{c_{p,avg}} \ln\left(\frac{T_2}{T_1}\right) - R_u \times ln\left(\frac{P_2}{P_1}\right)$$

The Entropy Change of Ideal Gases

- Approach 2: Variable Specific Heats (Exact Analysis):

$$s^0 = \int_0^T c_p(T)\frac{dT}{T}$$

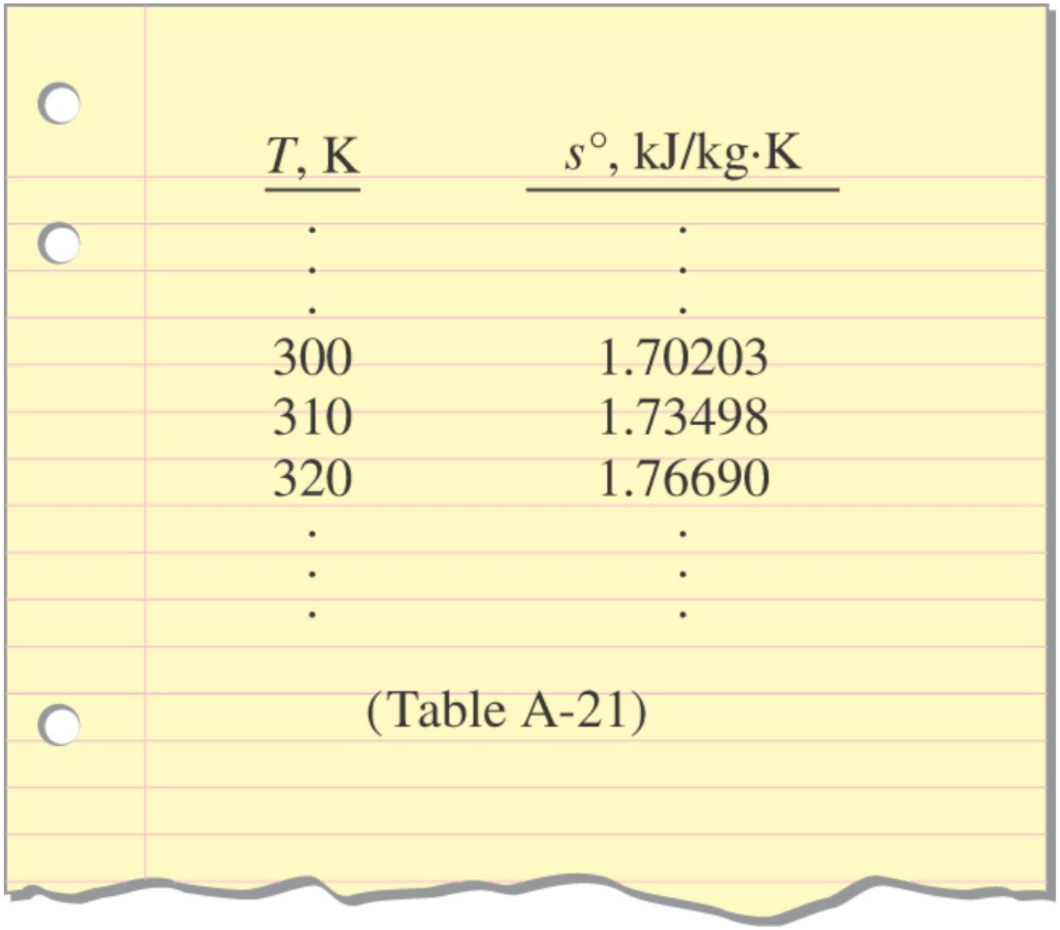
$$\int_0^T c_p(T)\frac{dT}{T} = s_2^0 - s_1^0$$

$$s_2 - s_1 = s_2^0 - s_1^0 - R\times\ln(\frac{P_2}{P_1})$$

$$\overline{s_2} - \overline{s_1} = \overline{s_2^0} - \overline{s_1^0} - R_u\times\ln(\frac{P_2}{P_1})$$

The Entropy Change of Ideal Gases

- Approach 2: Variable Specific Heats (Exact Analysis):



T, K	s°, kJ/kg·K
⋮	⋮
300	1.70203
310	1.73498
320	1.76690
⋮	⋮

(Table A-21)

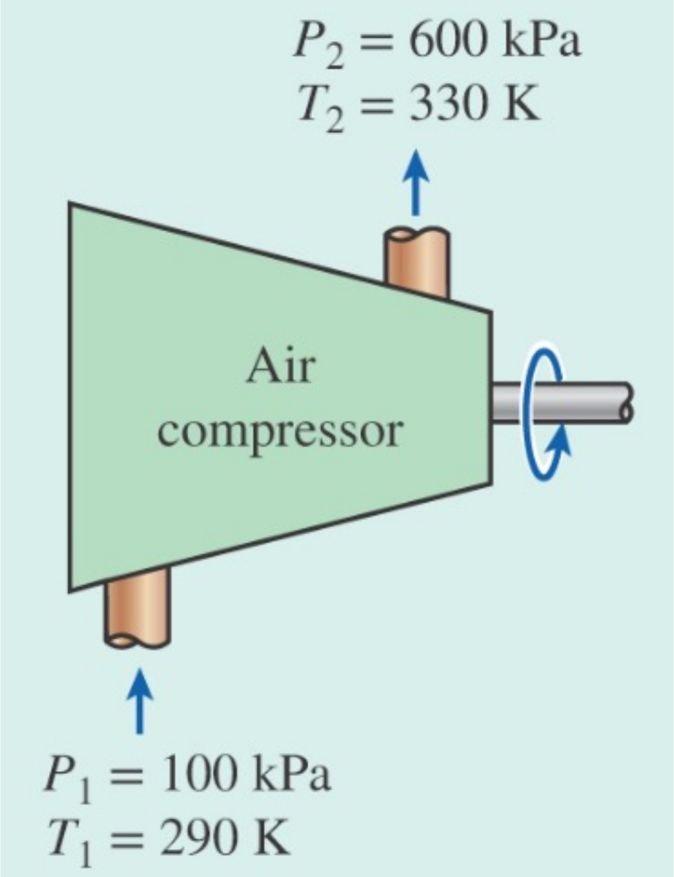
CLASS ACTIVITY

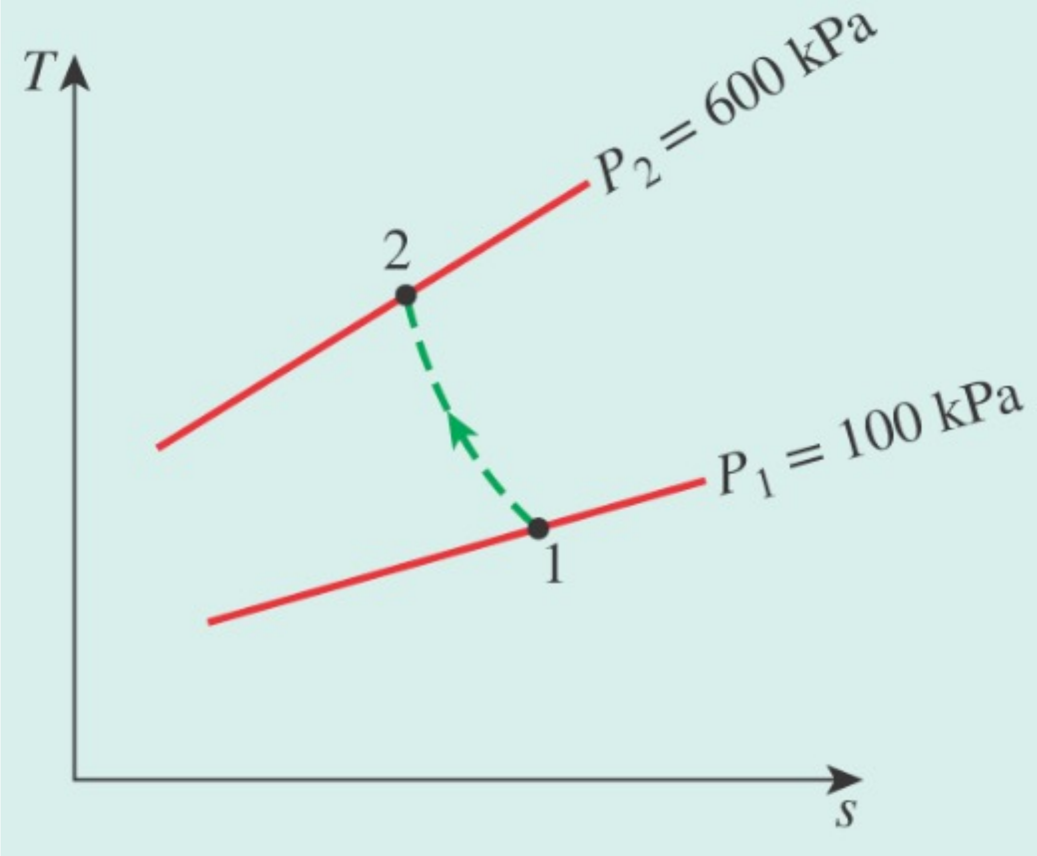
Class Activity

- Air is compressed from an initial state of 100 kPa and 17 °C to a final state of 600 kPa and 57 °C. Determine the entropy change of air during this compression process by using:
  a) Property values from the air table
  b) Average specific heats

Class Activity

- Solution (assumptions):
  - ☐ Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values





Class Activity

- Solution (calculations):
  - ❑ Part (a): Table A-21

$$s_2 - s_1 = s_2^0 - s_1^0 - R\times \ln\left(\frac{P_2}{P_1}\right) = \left[(1.79783 - 1.66802)\,\frac{kJ}{kg-K}\right]\times \ln(\frac{600\ kPa}{100\ kPa})$$

$$s_2 - s_1 = -0.3844\,\frac{kJ}{kg-K}$$

  - ❑ Part (b): Using a c_p value at the average temperature of 37 °C (Table A-2b)

$$s_2 - s_1 = c_{p,avg}\ln\left(\frac{T_2}{T_1}\right) - R\times \ln\left(\frac{P_2}{P_1}\right)$$

$$= \left(1.006\,\frac{kJ}{kg-K}\right)\ln\left(\frac{330\ K}{290\ K}\right) - \left(0.287\,\frac{kJ}{kg-K}\right)\ln\left(\frac{600\ kPa}{100\ kPa}\right) = -0.3842\,\frac{kJ}{kg-K}$$

The Entropy Change of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$\ln\left(\frac{T_2}{T_1}\right) = -\frac{R}{c_v}\ln\left(\frac{v_2}{v_1}\right)$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{v_1}{v_2}\right)^{\frac{R}{c_v}}$$

$$\begin{cases} c_p - c_v = R \\ k = \dfrac{c_p}{c_v} \end{cases} \rightarrow \frac{R}{c_v} = k - 1$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

The Entropy Change of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$\left(\frac{T_2}{T_1}\right)_{s=constant} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=constant} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\left(\frac{P_2}{P_1}\right)_{s=constant} = \left(\frac{v_1}{v_2}\right)^{k}$$

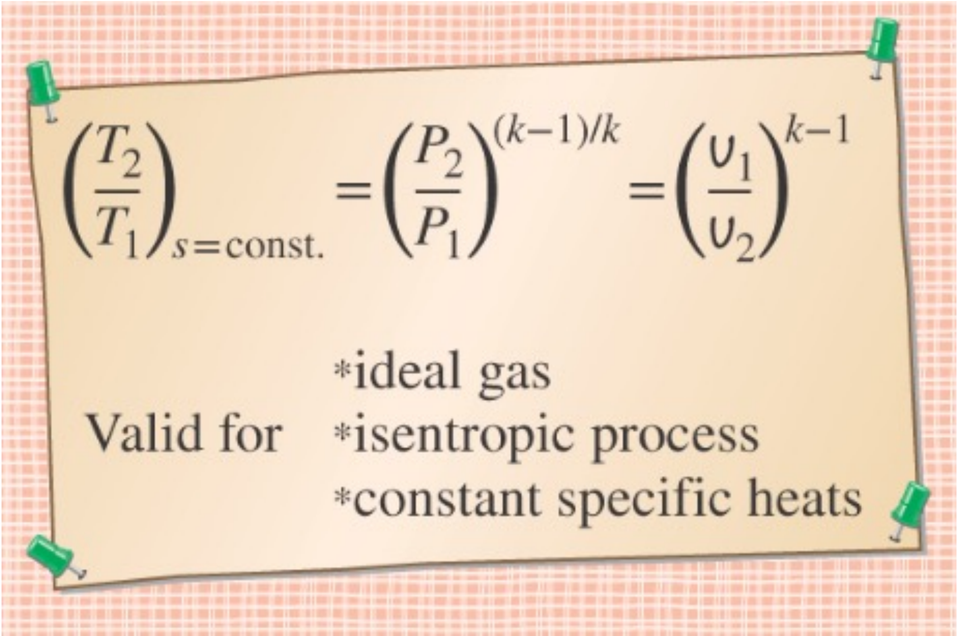
The Entropy Change of Ideal Gases

- Approach 1: Constant Specific Heats (Approximate Analysis) for Isentropic Processes of Ideal Gases

$$Tv^{k-1} = Constant$$

$$TP^{\frac{1-k}{k}} = Constant$$

$$Pv^{k} = Constant$$



$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

Valid for
*ideal gas
*isentropic process
*constant specific heats

The Entropy Change of Ideal Gases

- Approach 2: Variable Specific Heats (Exact Analysis) for Isentropic Processes of Ideal Gases

$$0 = s_2^0 - s_1^0 - R\times\ln(\frac{P_2}{P_1})$$

$$s_2^0 = s_1^0 + R\times\ln(\frac{P_2}{P_1})$$

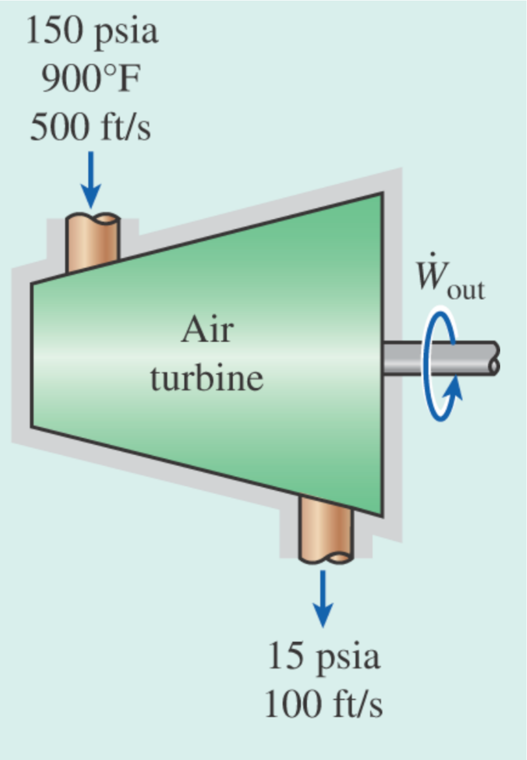
$$s_2^0 = s_1^0 + R\times\ln\left(\frac{P_2}{P_1}\right) \quad \rightarrow \quad \frac{P_2}{P_1} = \exp(\frac{s_2^0 - s_1^0}{R})$$

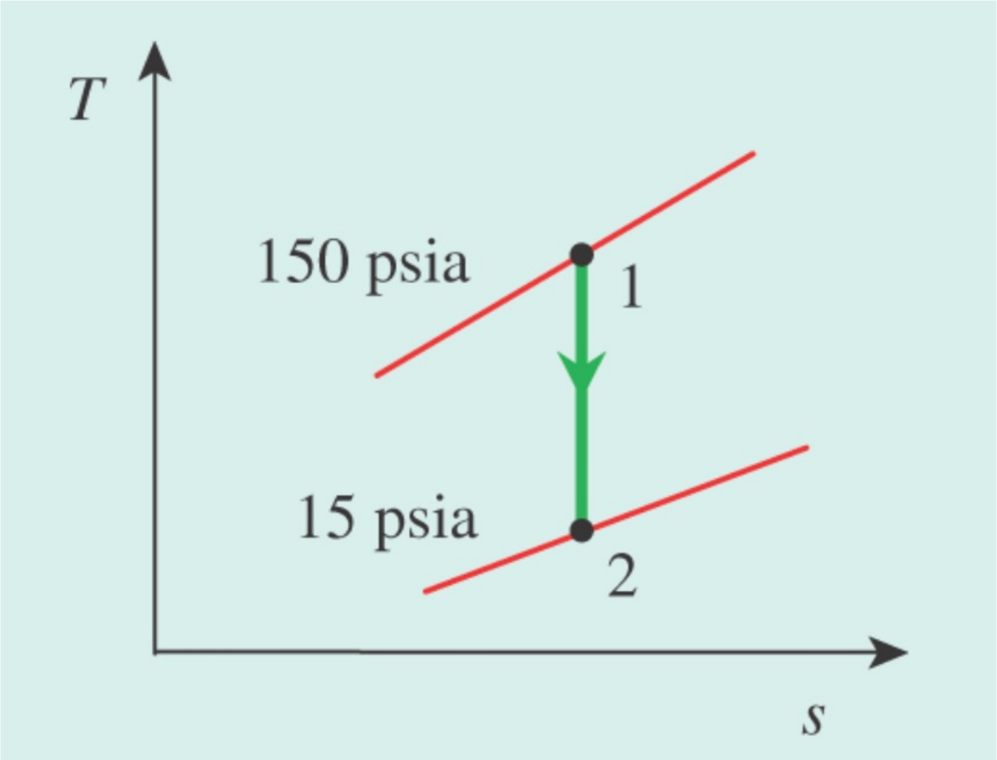
Class Activity

- Air enters an isentropic turbine at 150 psia and 900 °F through a 0.5 ft^2 inlet section with a velocity of 500 ft/s. It leaves at 15 psia with a velocity of 100 ft/s. Calculate the air temperature at the turbine exit and the power produced, in hp, by this turbine.

Class Activity

- Solution (assumptions):
  - ☐ Steady flow
  - ☐ The process is isentropic (both reversible and adiabatic)
  - ☐ Ideal gas with a constant specific heat





Class Activity

- Solution (Problem solving):
  - ❑ Table A-2Eb: $@600\ °F \rightarrow c_p = 0.250\,\frac{Btu}{lbm{-}R}\ and\ k = 1.3777$
  - ❑ Table A-1E: $R = 0.3704\,\frac{psia{-}ft^3}{lbm{-}R}$

$$\dot{m} = \dot{m}_1 = \dot{m}_2$$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{d\dot{E}_{system}}{dt} = 0$$

$$\dot{m}(h_1 + V_1^2) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_1 + \frac{V_1^2 - V_2}{2}\right)$$

Class Activity

- Solution (Calculations):

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \rightarrow T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = (900 + 460\ R)\left(\frac{15\ psia}{150\ psia}\right)^{\frac{0.3777}{1.377}} = 724\ R$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.3704\,\frac{psia - ft^3}{lbm - R}\right)(900 + 460\ R)}{150\ psia} = 3.358\,\frac{ft^3}{lbm}$$

$$m = \frac{A_1 V_1}{v_1} = \frac{(0.5\ ft^2)\left(500\,\frac{ft}{s}\right)}{3.358\,\frac{ft^3}{lbm}} = 74.45\,\frac{lbm}{s}$$

Class Activity

- Solution (Calculations):

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_1 + \frac{V_1^2 - V_2}{2}\right)$$

$$\dot{W}_{out} = \left(74.45\,\frac{lbm}{s}\right)\left[\left(0.250\,\frac{Btu}{lbm-R}\right)(1360 - 724R) + \left(\frac{\left(500\,\frac{ft}{s}\right)^2}{2} - \frac{\left(100\,\frac{ft}{s}\right)^2}{2}\right)\left(\frac{1\,\frac{Btu}{lbm}}{25.037}ft^2}{s^2}\right)\right.$$

$$\dot{W}_{out} = 12{,}194\,\frac{Btu}{s}\left(\frac{1\ hp}{0.7068\,\frac{Btu}{s}}\right) = 17{,}250\ hp$$

REVERSIBLE STEADY-FLOW WORK

Reversible Steady-Flow Work

- Recall we had

$$W_b = \int_1^2 P\, dV$$

- For steady flow:

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

$$-\delta w_{rev} = vdP + dke + dpe$$

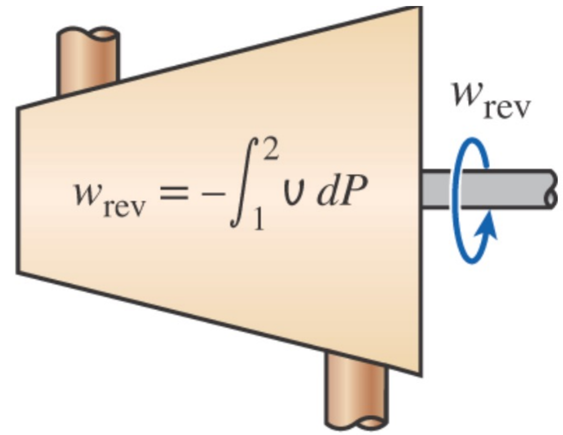
$$\left.\begin{array}{l} \delta q_{rev} = Td \\ Tds = dh - vdP \end{array}\right\} \rightarrow \delta q_{rev} = dh - vdP$$

$$w_{rev} = -\int_1^2 vdP - \Delta ke - \Delta pe$$

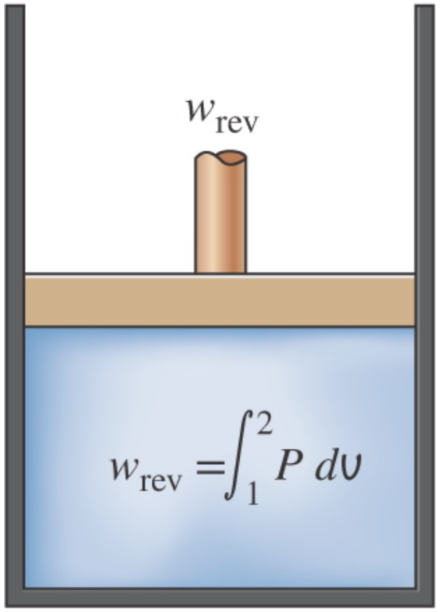
Reversible Steady-Flow Work

- For steady flow:

$$w_{rev} = -\int_1^2 v dP - \Delta ke - \Delta pe$$



(*a*) Steady-flow system



(*b*) Closed system

Reversible Steady-Flow Work

- We write the *Bernoulli equation*:

$$w_{rev} = 0 = v(P_2 - P_1) + \left(\frac{V_2^2 - V_1^2}{2}\right) + g(z_2 - z_1)$$

Reversible Steady-Flow Work

- Steady-flow devices deliver the most and consume the least work when the process is reversible: