CAE 208 Thermal-Fluids Engineering I
MMAE 320: Thermodynamics

Fall 2022

October 25, 2022

Mass and Energy Analysis of Control Volumes (II)

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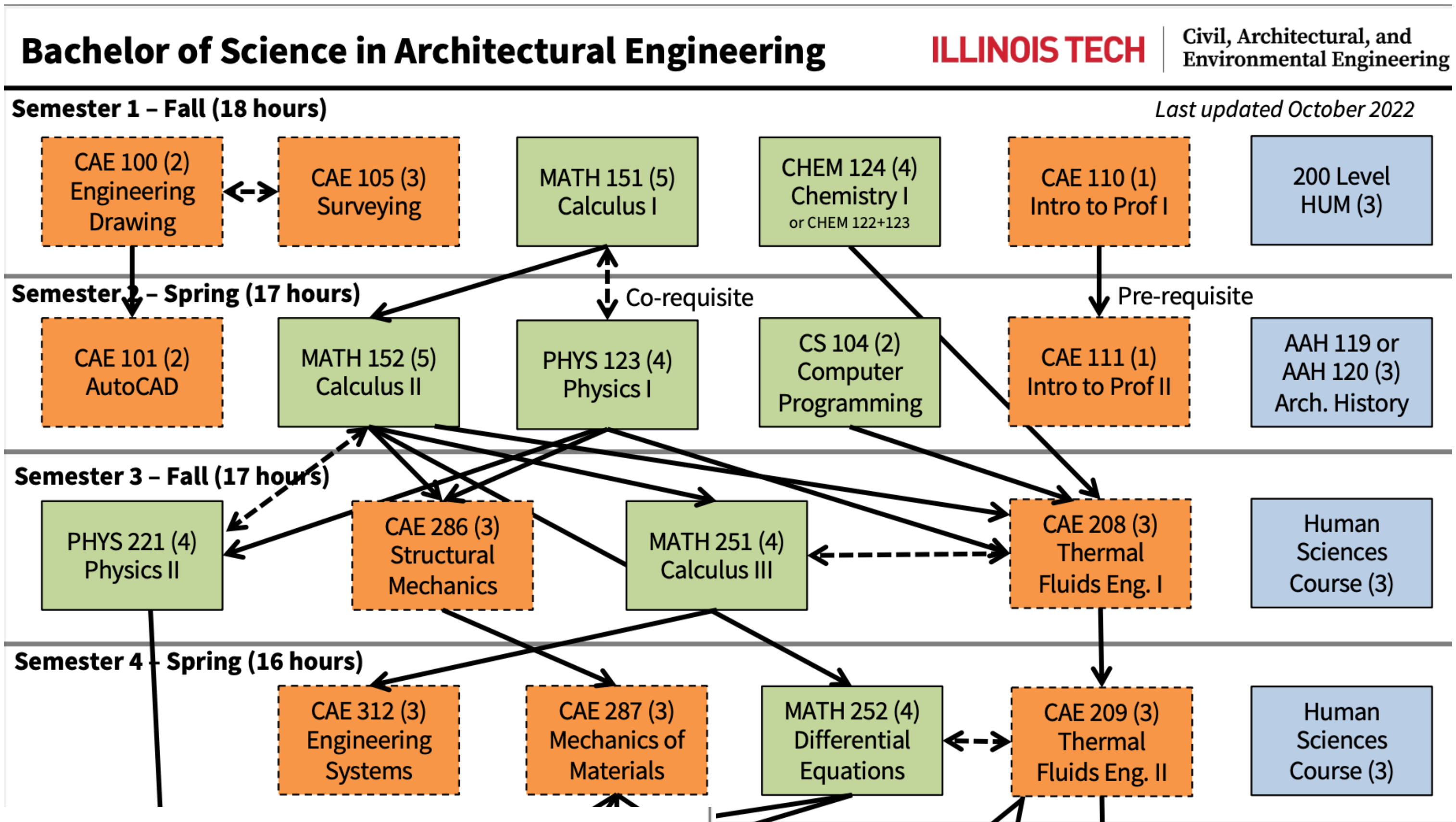
ANNOUNCEMENTS

Announcements

- Assignment 6 is posted
- Next midterm exam is November 10

Announcements

- CAE 209



- MMAE 320 can count for CAE 208
- MMAE 313 or CAE 302 can count for CAE 209
- Technical electives must be CAE, ENVE, or EG 400- or 500-level
  - Maximum of 1 EG elective

Human Sciences requirements:

- 3 hrs of HUM 200 level
- 6 hrs of 300/400 level (H)umanities (AAH, COM, HIST, HUM, LIT or PHIL)
- 9 hrs of (S)ocial Sciences (ECON, PS, PSYC, SOC or SSCI):
  - 6 hrs @ 300/400 level & courses from at least 2 different fields

Announcements

- CAE 464

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CAE 464/517: HVAC Systems Design (Spring 2021)

Course Description

This course introduce students to both theory and applied design procedures for HVAC equipment and systems. By taking this course students will be able to:

1. Understand fundamentals of fluid and energy flows for HVAC equipment and systems
2. Design and size air distribution systems, hydronic systems, and refrigeration systems
3. Design, draw, and read mechanical drawings
4. Design, review, and assess different HVAC designs
5. Propose recommendations to revise HVAC designs and retrofit existing HVAC systems
6. Utilize both hand calculations and computer modeling (graduate students) for sizing air distribution systems, hydronic systems, and refrigeration systems

Course Syllabus (updated as we go; includes current schedule)

- Most recent syllabus, updated January 19, 2021

Lecture

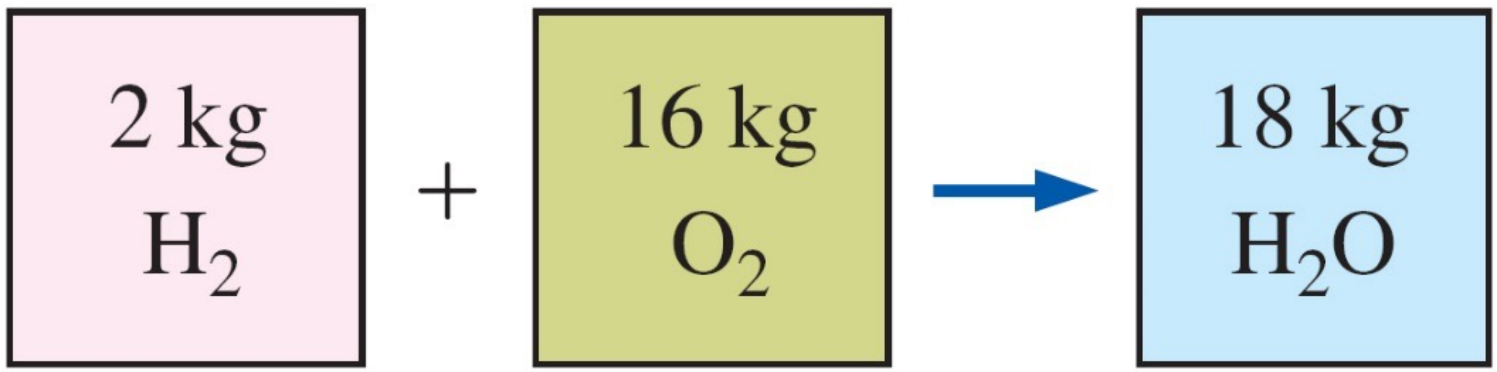
- Lecture 01: Course overview and introduction
- Lecture 02: Review of HVAC system drawings
- Lecture03: Installation
- Lecture 04: Psychrometrics processes and space conditioning
- Lecture 05: Design conditions and heating loads

http://built-envi.com/courses/cae-464-517-hvac-systems-design-spring-2021/

RECAP

Recap

- Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process

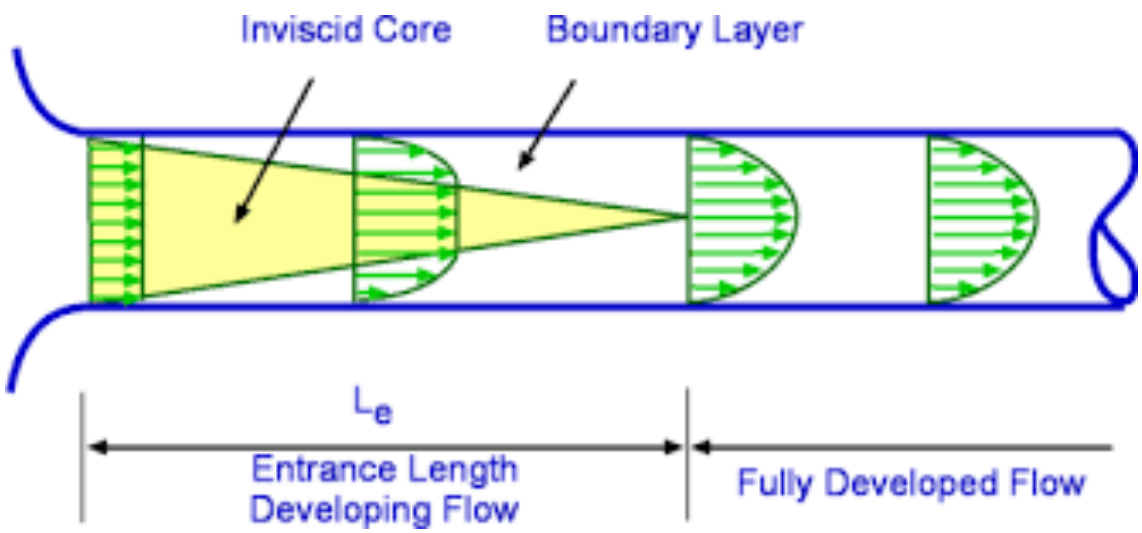


Recap

- In using control volumes, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume

Recap

- Let's look at a flow in a pipe:



Recap

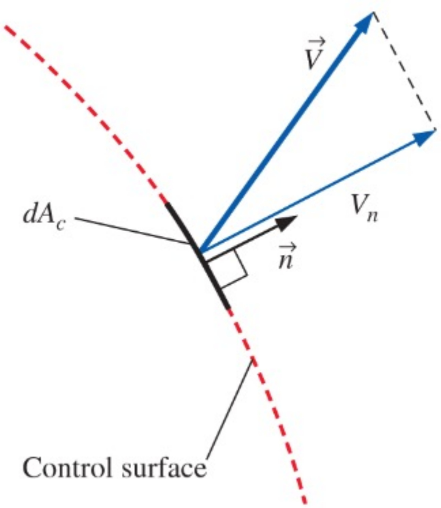
- Mass and volume flow rates

$$\delta \dot{m} = \rho V_n dA_c$$

$$\dot{m} = \int \delta \dot{m} = \int \rho V_n dA_c$$

$$\dot{m} = \rho V_{avg} A_c$$

$$\dot{m} = \rho \dot{\forall} = \frac{\dot{\forall}}{v}$$



Recap

- The conservation of mass for a control volume:

$$m_{in} - m_{out} = \Delta m_{CV}$$

$$\Delta m_{CV} = m_{final} - m_{initial}$$

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

These equations are often referred to as the ***mass balance*** *and are applicable to any control volume undergoing any kind of process.*

Recap

- We can write the net flow rate:

$$\frac{d}{dt}\int_{CS} \rho d\forall = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

Recap

- Special case: Incompressible flow:

$$\sum_{in} \dot{\forall} = \sum_{out} \dot{\forall}$$

- Steady, incompressible flow (single stream):

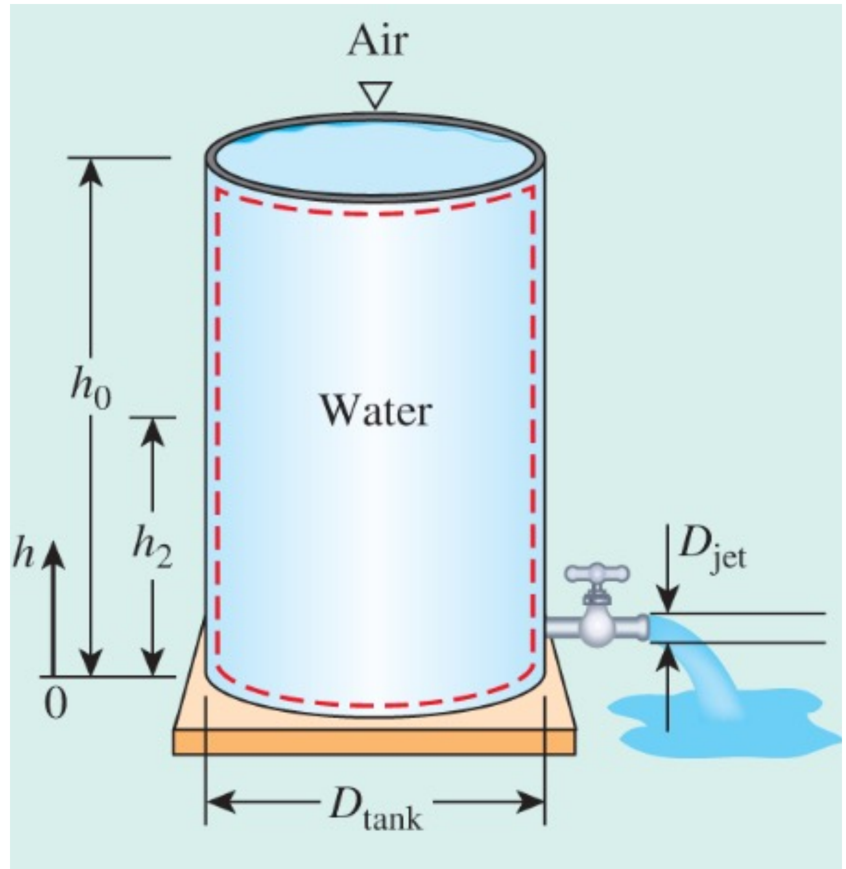
$$\dot{\forall}_1 = \dot{\forall}_2 \quad \rightarrow \quad V_1 A_1 = V_2 A_2$$

CLASS ACTIVITY

Class Activity

- A 4-ft high, 3-ft diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams.

The average velocity of the jet is approximated as $V = \sqrt{2gh}$ where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop 2 ft from the bottom.



Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m}_{in} = 0$$

$$\dot{m}_{out} = (\rho V A)_{out} = \rho\sqrt{2gh}\,A_{jet}$$

$$m_{CV} = \rho\forall = \rho A_{tank} h$$

Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \frac{d}{dt}(\rho A_{tank} h)$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \rho(\frac{\pi D_{tank}^2}{4})\frac{dh}{dt}$$

$$dt = \frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

Class Activity

- Solution:

$$dt = -\frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

$$\begin{cases} t = 0 \;\rightarrow \qquad h = h_0 \\ t = t \;\rightarrow \qquad h = h_2 \end{cases}$$

$$\int_0^t dt = -\frac{D_{tank}^2}{D_{jet}^2\sqrt{2g}}\int_{h_1}^{h_2}\frac{dh}{\sqrt{h}}$$

$$t = \frac{\left(\sqrt{h_0} - \sqrt{h_2}\right)}{\sqrt{\frac{g}{2}}}\left(\frac{D_{tank}}{D_{jet}}\right)^2$$

Class Activity

- Solution:

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{2\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times 12\ in}{0.5\ in}\right)^2 = 757\ s = 12.6\ min$$

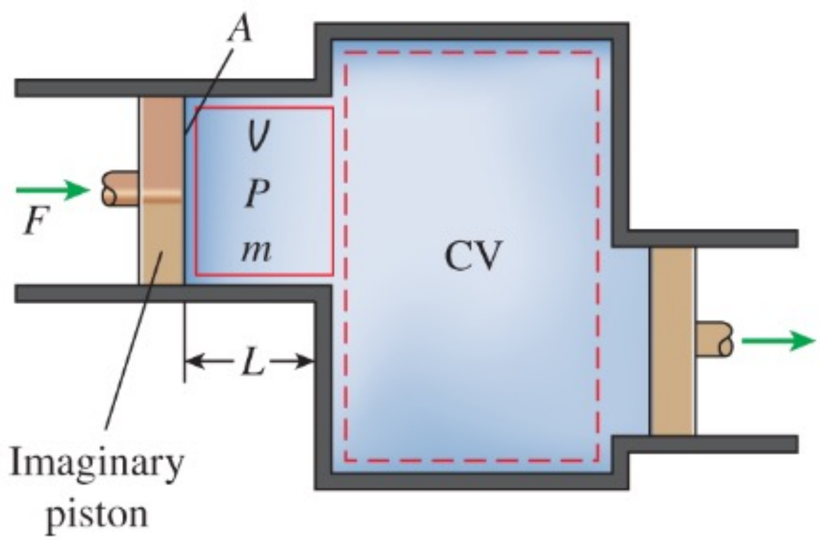
What does the entire flow exit the system?

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{0\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times 12\ in}{0.5\ in}\right)^2 = 43.1\ min$$

FLOW WORK AND ENERGY OF FLOWING FLUID

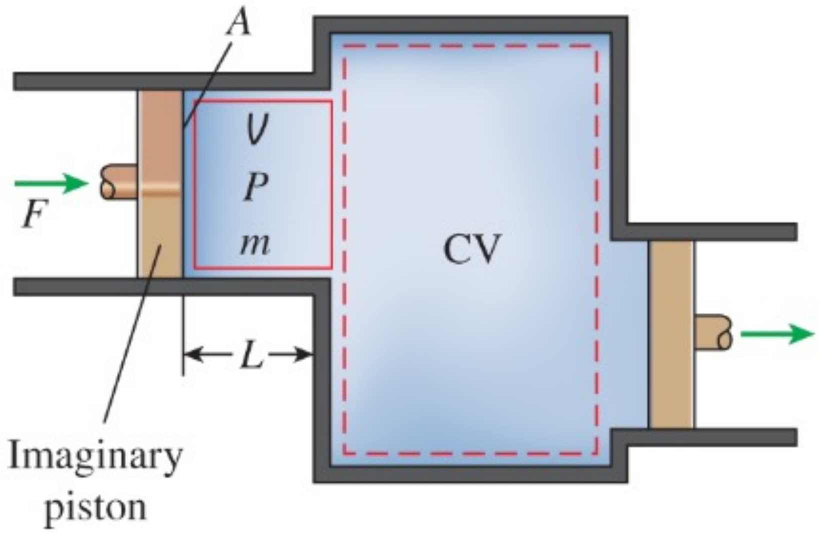
Flow Work and Energy of Flowing Fluid

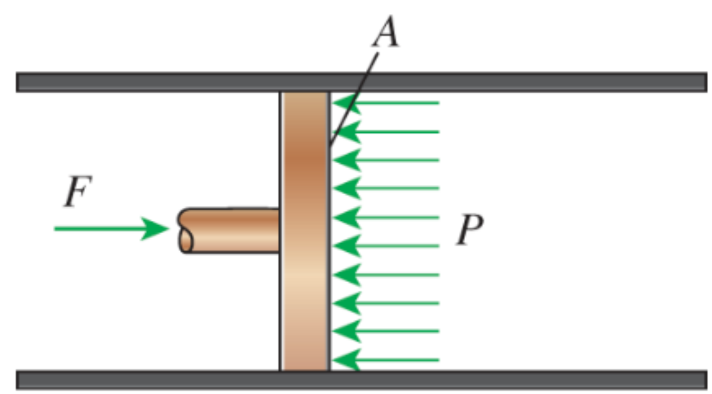
- Flow work (or flow energy): The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume



Flow Work and Energy of Flowing Fluid

- If the fluid pressure is P and the cross-sectional area of the fluid element is A, the force applied on the fluid by the imaginary piston is:





$$F = PA$$

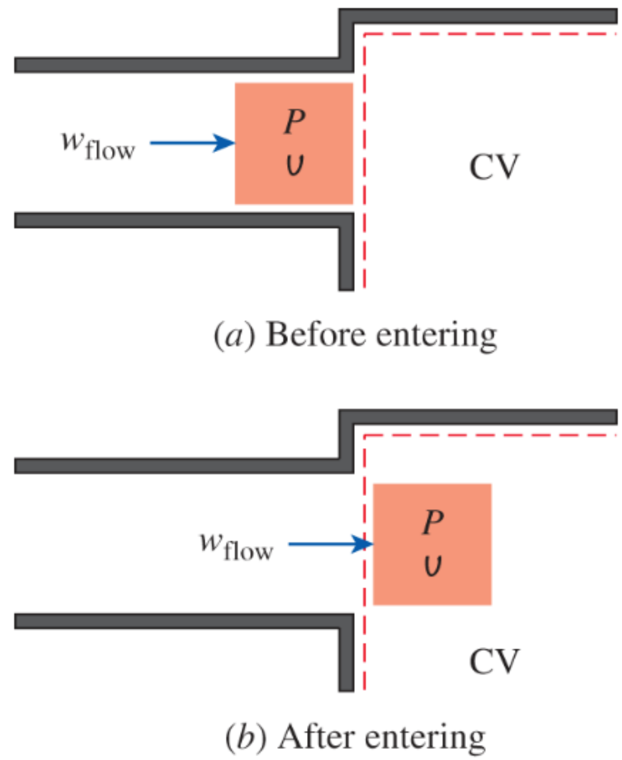
Flow Work and Energy of Flowing Fluid

- To push the entire fluid element into the control volume, this force must act through a distance L

$$F = PA$$

$$W_{flow} = FL = PAL = P\forall$$

$$w_{flow} = Pv$$

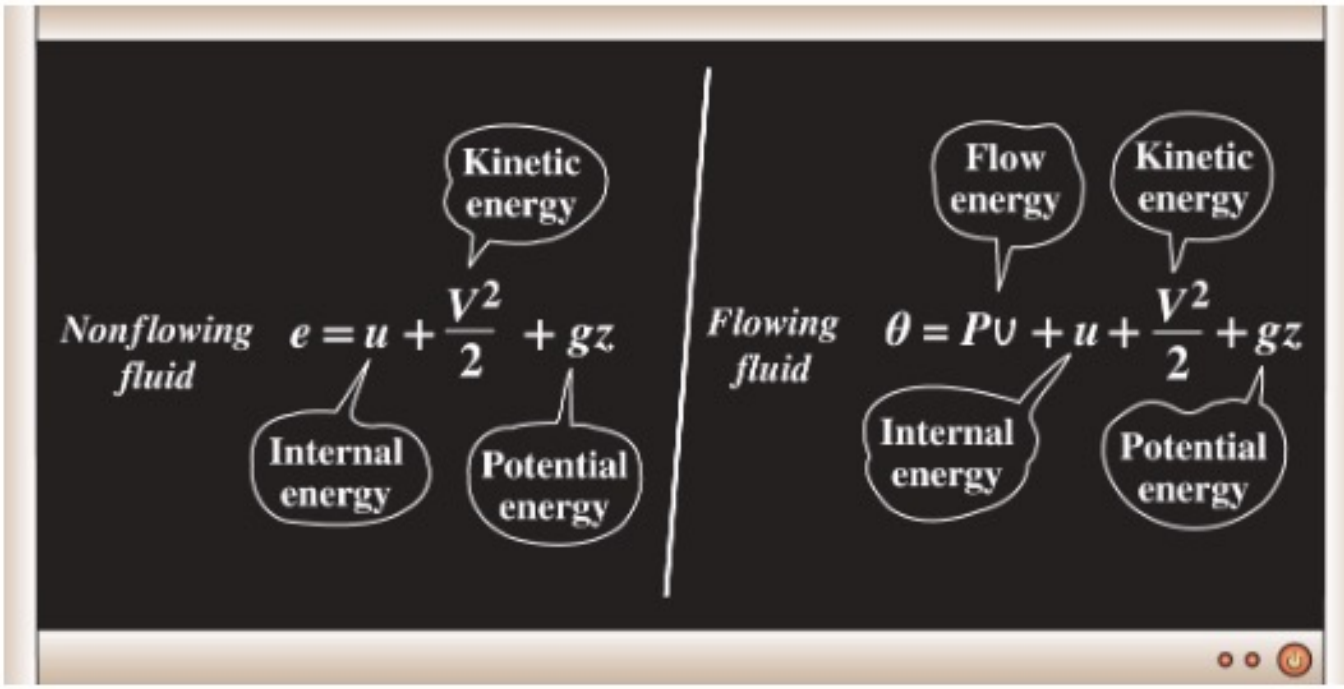


(*a*) Before entering

(*b*) After entering

Flow Work and Energy of Flowing Fluid

- Total energy of a flowing fluid is:



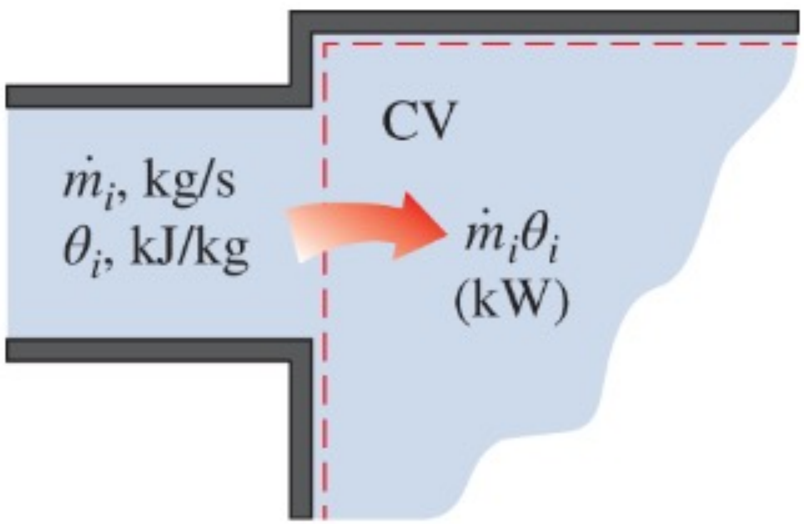
$$\theta = Pv + e = Pv + (u + ke + pe) = (u + Pv) + ke + pe$$

Flow Work and Energy of Flowing Fluid

- Energy transport by mass:

$$E_{mass} = m\theta = m(h + \frac{V^2}{2} + gz)$$

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}(h + \frac{V^2}{2} + gz)$$



Flow Work and Energy of Flowing Fluid

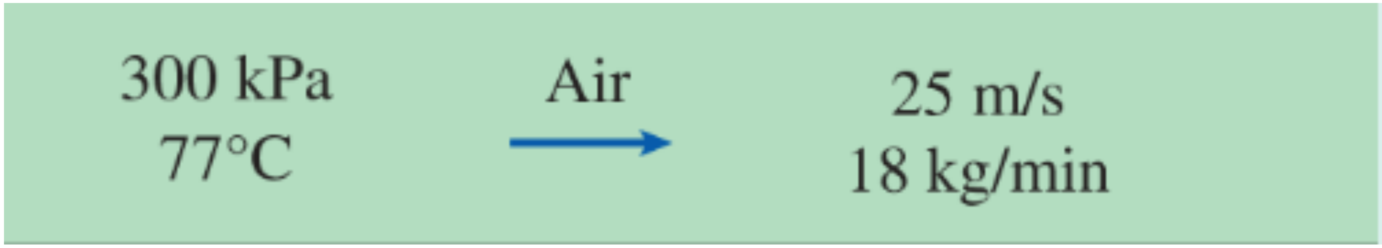
- What does happen when mass is not constant:

$$E_{in,mass} = \int_{m_i} \theta_i \delta m_i = \int_{m_i} (h_i + \frac{V_i^2}{2} + gz_i)\,\delta m_i$$

CLASS ACTIVITY

Class Activity

- Air flows steadily in a pipe at 300 kPa, 77 °C, and 25 m/s at a rate of 18 kg/min. Determine:
  a) The diameter of the pipe
  b) The rate of flow energy
  c) The rate of energy transport by mass
  d) The error involved in part c if the kinetic energy is neglected.



Class Activity

- Solution (assumptions):
  - ❑ The flow is steady
  - ❑ The potential energy is negligible
  - ❑ R = 0.287 kJ/kg-K
  - ❑ c_p=1.008 kJ/kg-K (at 350 K from Table A-2b)

Class Activity

- Solution (a):

$$v = \frac{RT}{P} = \frac{(0.287\,\frac{kJ}{kg-K})(77+273)}{300\ kPa} = 0.3349\,\frac{m^3}{kg}$$

$$A = \frac{\dot{m}v}{\forall} = \frac{\left(\frac{18}{60}\frac{kg}{s}\right)(0.3349\,\frac{m^3}{kg})}{25\,\frac{m}{s}} = 0.004018\,m^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.004018\,m^2)}{\pi}} = 0.0715\,m$$

Class Activity

- Solution (b):

$$\dot{W}_{flow} = \dot{m}Pv = \left(\frac{18}{60}\frac{kg}{s}\right)(300\ kPa)\left(0.3349\,\frac{m^3}{kg}\right) = 30.14\ kW$$

Class Activity

- Solution (c):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}(c_p T + \frac{V^2}{2})$$

$$\dot{E}_{mass} = \left(\frac{18}{60}\frac{kg}{s}\right)\left[\left(1.008\,\frac{kJ}{kg-K}\right)(77 + 273\;K) + \left(\frac{1}{2}\right)\left(25\,\frac{m}{s}\right)^2\left(\frac{1\,\frac{kJ}{kg}}{1000\,\frac{m^2}{s^2}}\right)\right]$$

$$\dot{E}_{mass} = 105.94\;kW$$

Class Activity

- Solution (d):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}h = \dot{m}c_pT$$

$$\dot{E}_{mass} = \dot{m}c_pT = \left(\frac{18}{60}\frac{kg}{s}\right)\left(1.005\,\frac{kJ}{kg - K}\right)(77 + 273K) = 105.84\,kW$$

ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS

Energy Analysis of Steady-Flow Systems

- A large number of engineering devices such as turbines, compressors, and nozzles operate for long periods of time under the same conditions once the transient start-up period is completed and steady operation is established, and they are classified as *steady-flow devices*.

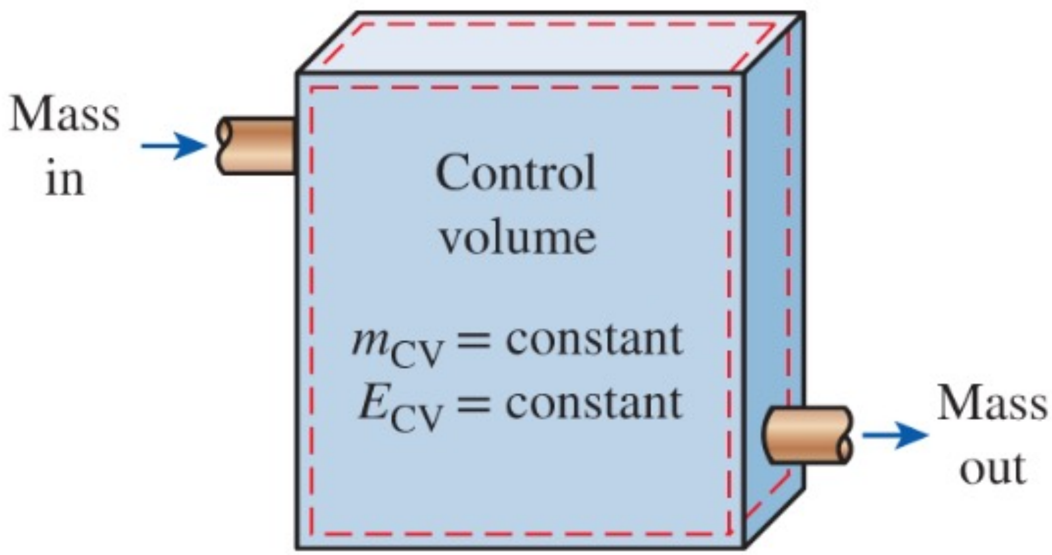
Energy Analysis of Steady-Flow Systems

- Process involving such devices can be represented reasonably well by a somewhat idealized process, called steady-flow process which was defined as a process during which a fluid flows through a control volume steadily

What do you think about a spatial and temporal change in a tank with a steady-flow?

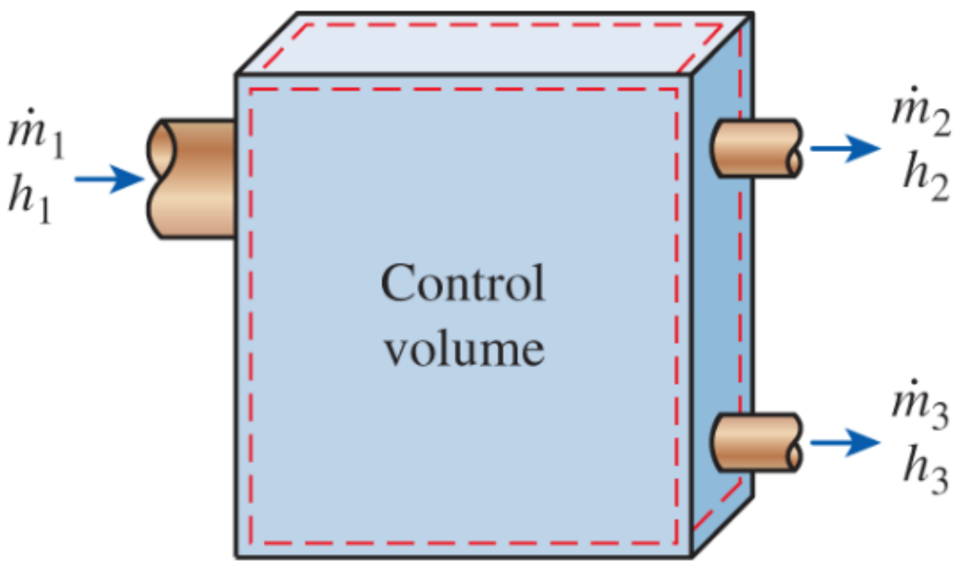
Energy Analysis of Steady-Flow Systems

- Steady-flow process:
  - No intensive or extensive properties within the control volume change with time
  - Boundary work is zero (why?)
  - $\Delta E_{CV} = 0$ (why?)



Energy Analysis of Steady-Flow Systems

- Steady-flow process:
  - ❑ Power remain constant why?



Energy Analysis of Steady-Flow Systems

- Steady-flow process:

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$

$$\dot{m}_1 = \dot{m}_2 \;\rightarrow\; \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Energy Analysis of Steady-Flow Systems

- Steady-flow process:

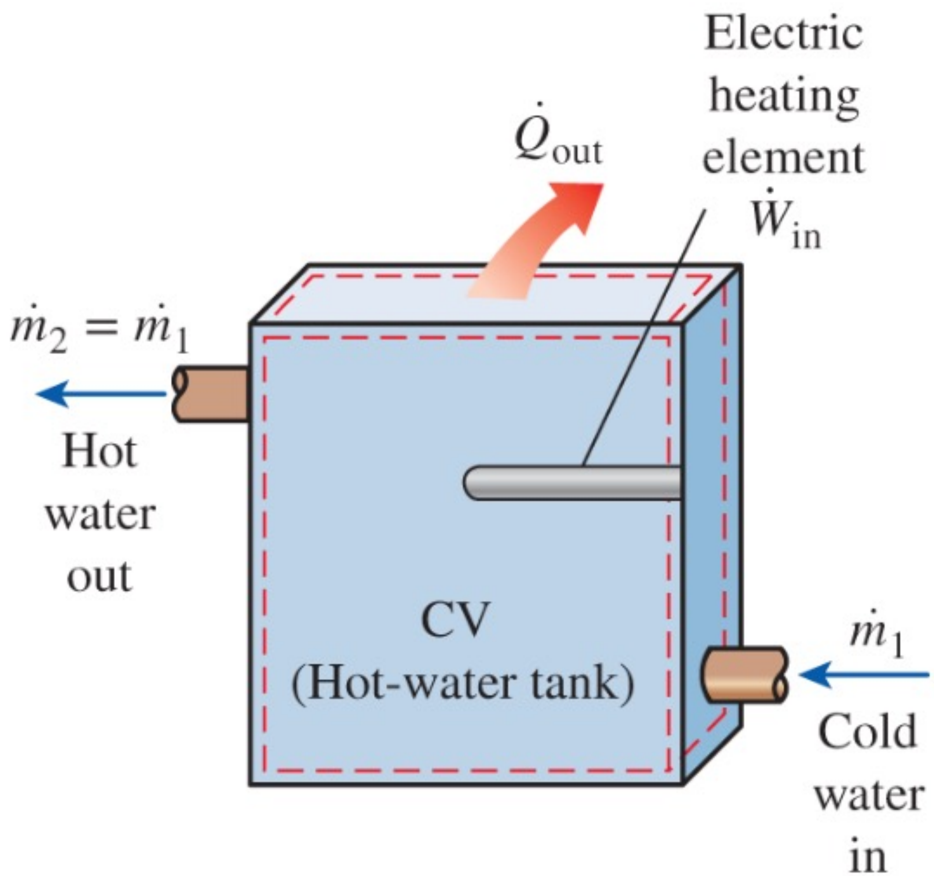
$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} = 0 \qquad \rightarrow \qquad \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{in} + W_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + W_{out} + \sum_{out} \dot{m}\theta$$

$$\dot{Q}_{in} + W_{in} + \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz) = \dot{Q}_{out} + W_{out} + \sum_{out} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Consider an electric hot water heater under steady condition



Energy Analysis of Steady-Flow Systems

- Consider an electric hot water heater under steady condition

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

$$\dot{Q} - \dot{W} = \dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$q - w = h_2 - h_1$$

Energy Analysis of Steady-Flow Systems

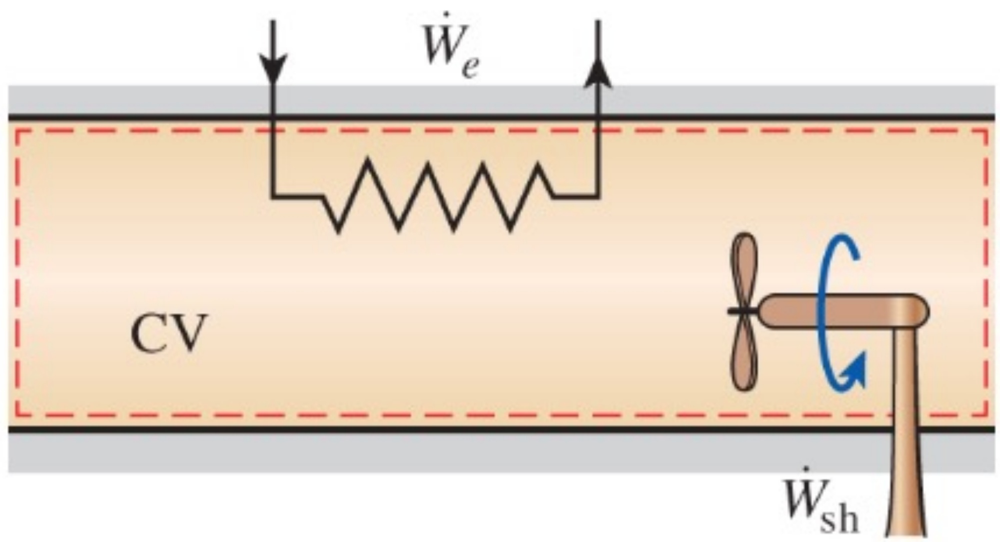
- Let's look at $\dot{Q}$

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Let's look at $\dot{W}$:

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$



Energy Analysis of Steady-Flow Systems

- Let's look at $\Delta h = h_2 - h_1 = c_{p,avg}(T_2 - T_1)$

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

Energy Analysis of Steady-Flow Systems

- Let's look at Δke

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

V_1 m/s	V_2 m/s	Δke kJ/kg
0	45	1
50	67	1
100	110	1
200	205	1
500	502	1

$$\frac{\mathrm{J}}{\mathrm{kg}} \equiv \frac{\mathrm{N \cdot m}}{\mathrm{kg}} \equiv \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\frac{\mathrm{m}}{\mathrm{kg}} \equiv \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

$$\left(\text{Also, } \frac{\mathrm{Btu}}{\mathrm{lbm}} \equiv 25{,}037\ \frac{\mathrm{ft}^2}{\mathrm{s}^2}\right)$$

Energy Analysis of Steady-Flow Systems

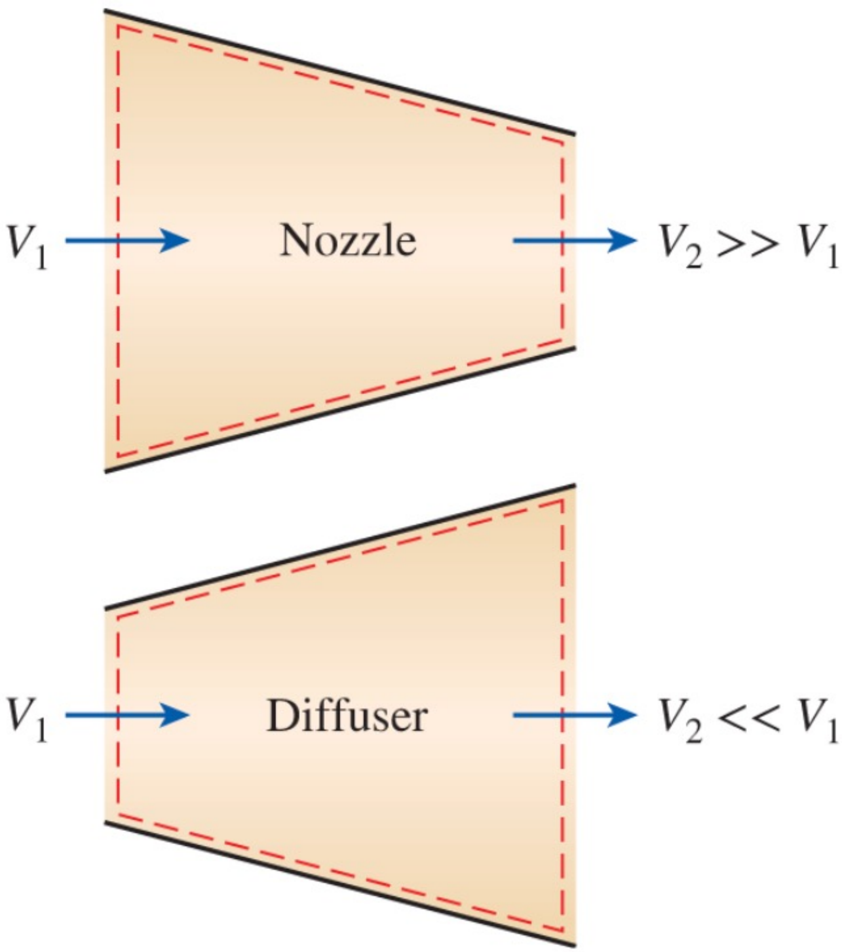
- Let's look at Δpe (when do you think it becomes important?)

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{in} \dot{m}(h + \frac{V^2}{2} + gz)$$

SOME STEADY-FLOW ENGINEERING DEVICES

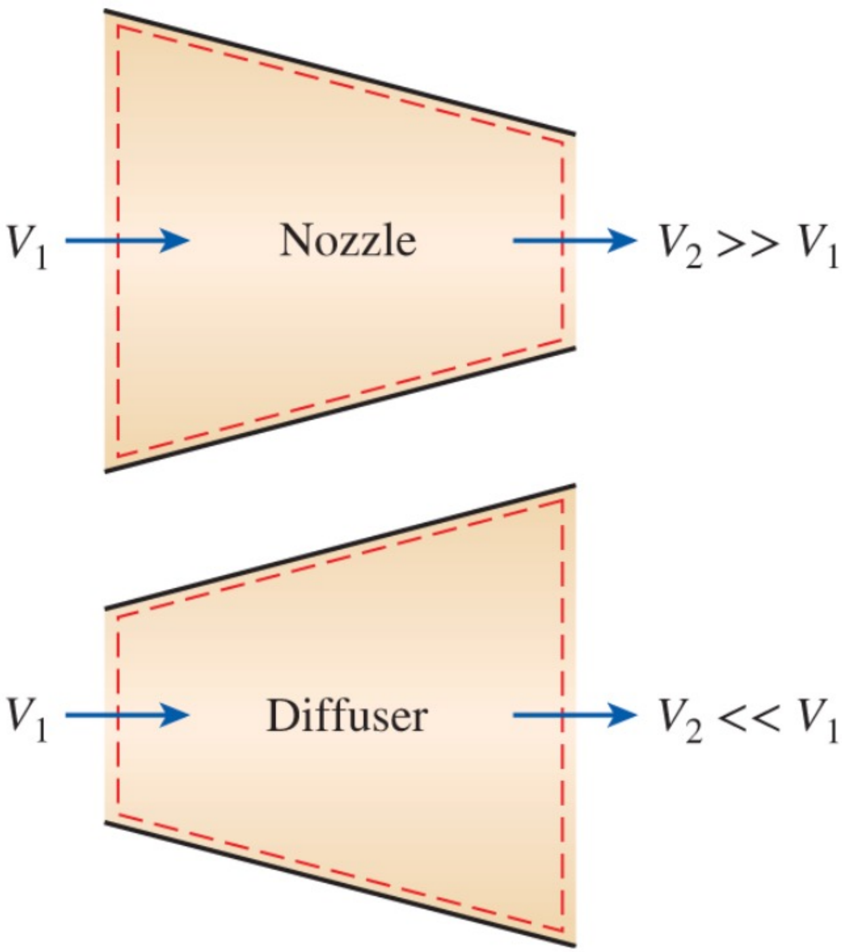
Some Steady-Flow Energy Devices

- **Nozzles and Diffusers**



Some Steady-Flow Energy Devices

- **Nozzles and Diffusers**



$$\dot{Q} \cong 0$$

$$\dot{W} = 0 \; (most\ times)$$

$$\Delta pe \cong 0$$

$$\Delta ke \neq 0$$

CLASS ACTIVITY

Class Activity

- Air at 10 °C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m^2. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine:
  a) The mass flow rate of the air
  b) The temperature of the air leaving the diffuser

Class Activity

- Solution (assumptions):
  1. This is a steady-flow process ($\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$)
  2. Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values
  3. The potential energy balance change is zero
  4. $\Delta pe = 0$
  5. Heat transfer is negligible
  6. Kinetic energy at the diffuser exit is negligible
  7. There are no work interactions

Class Activity

- Solution (a):

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287\ kPa - \frac{m^3}{kg - K}\right)(283\ K)}{80\ kPa} = 1.015\ \frac{m^3}{kg}$$

Class Activity

- Solution (b):

$$\dot{E}_{in} = \dot{E}_{out} = \frac{dE_{system}}{dt} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}(h_2 + V_2^2)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2} \qquad (V_2^2 \ll V_1^2)$$

Class Activity

- Solution (b):

$$Using\ Table\ A-21\ \rightarrow\ h_1 = h_{@\ 283\ K} = 283.14\,\frac{kJ}{kg}$$

$$h_2 = 283.14\,\frac{kJ}{kg} - \frac{0-\left(200\,\frac{m}{s}\right)^2}{2}\left(\frac{1\,\frac{kJ}{kg}}{1000\,\frac{m^2}{s^2}}\right) = 303.14\,\frac{kJ}{kg}$$

$$From\ Table\ A-21\ \rightarrow T_2 = 303\ K$$