CAE 208 Thermal-Fluids Engineering I
MMAE 320: Thermodynamics

Fall 2022

October 17, 2022

Mass and Energy Analysis of Control Volumes (I)

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ANNOUNCEMENTS

Announcements

- First mid-term will be graded by Thursday (Solution will be uploaded on Blackboard)

- Assignment 5 is graded (Please make sure to add table numbers and show your calculations)

- Assignment 6 will be posted on Thursday

OBJECTIVES

Objectives

- Develop the conservation of mass principle
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes

Objectives

- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy

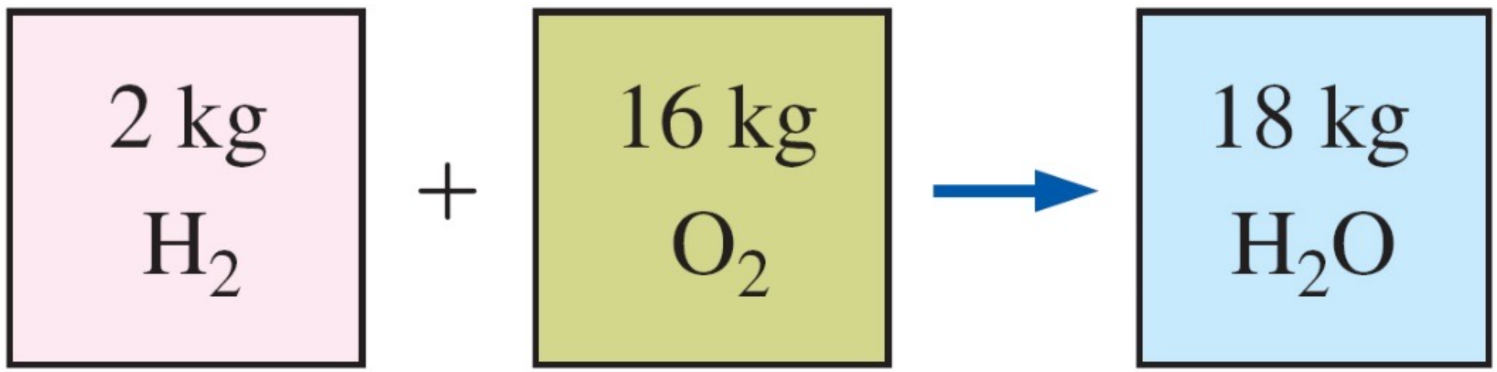
Objectives

- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes

CONSERVATION OF MASS

Conservation of Mass

- Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process

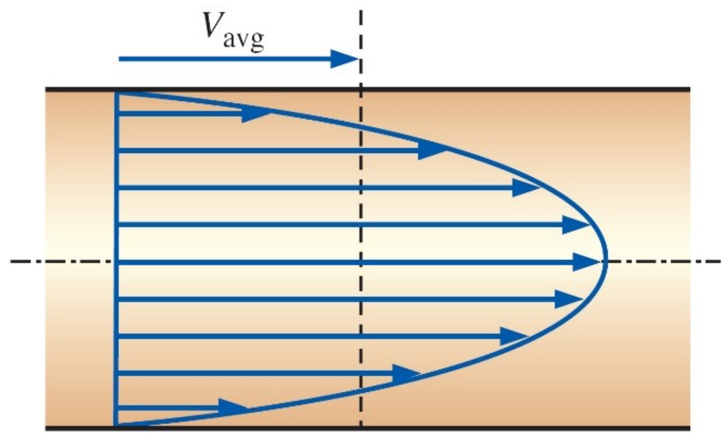


Conservation of Mass

- In using control volumes, mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume

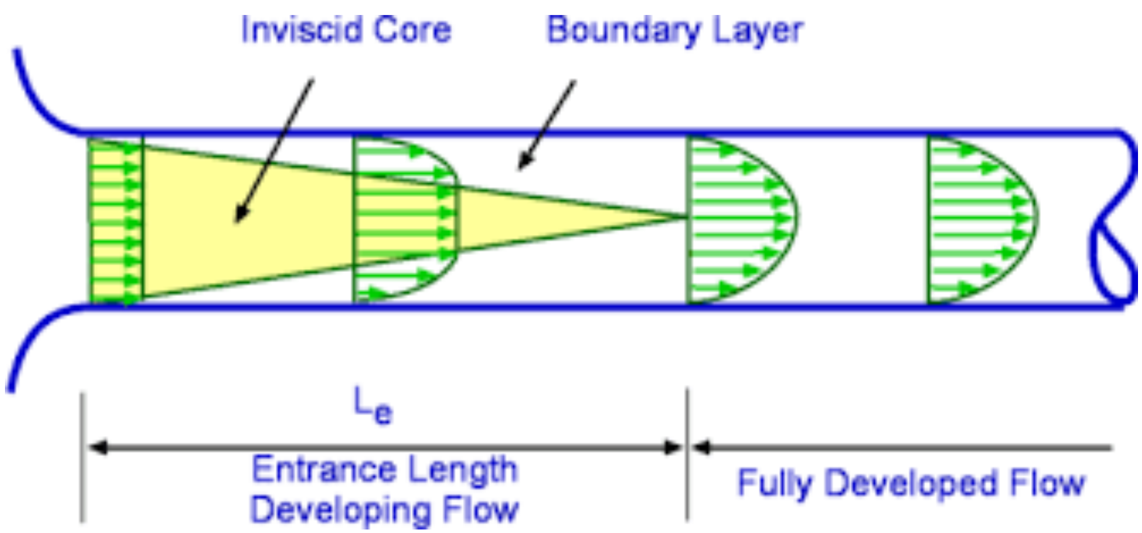
Conservation of Mass

- Let's look at a flow in a pipe:



Conservation of Mass

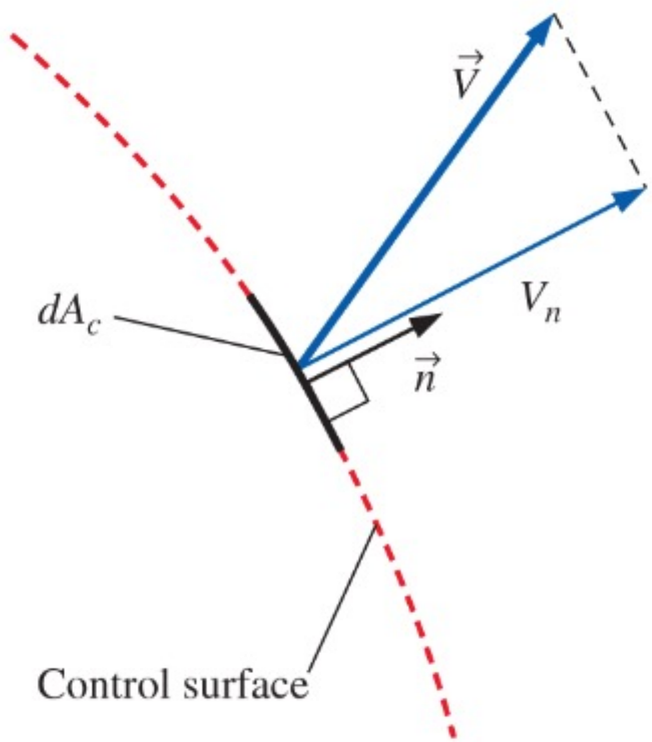
- Let's look at a flow in a pipe:



Conservation of Mass

- Mass and volume flow rates

$$\delta\dot{m} = \rho V_n dA_c$$



Conservation of Mass

- Mass and volume flow rates

$$\delta \dot{m} = \rho V_n dA_c$$

$$\dot{m} = \int \delta \dot{m} = \int \rho V_n dA_c$$

$$\dot{m} = \rho V_{avg} A_c$$

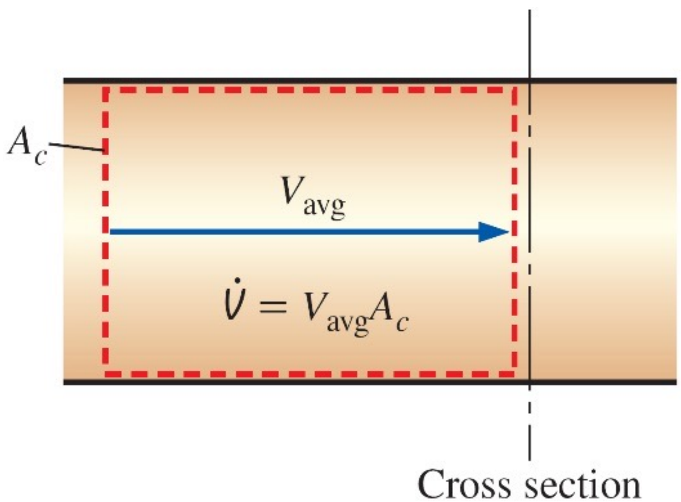
$$\dot{m} = \rho \dot{\forall} = \frac{\dot{\forall}}{v}$$

Conservation of Mass

- Mass and volume flow rates

$$V_{avg} = \frac{1}{A_c}\int V_n dA_c$$

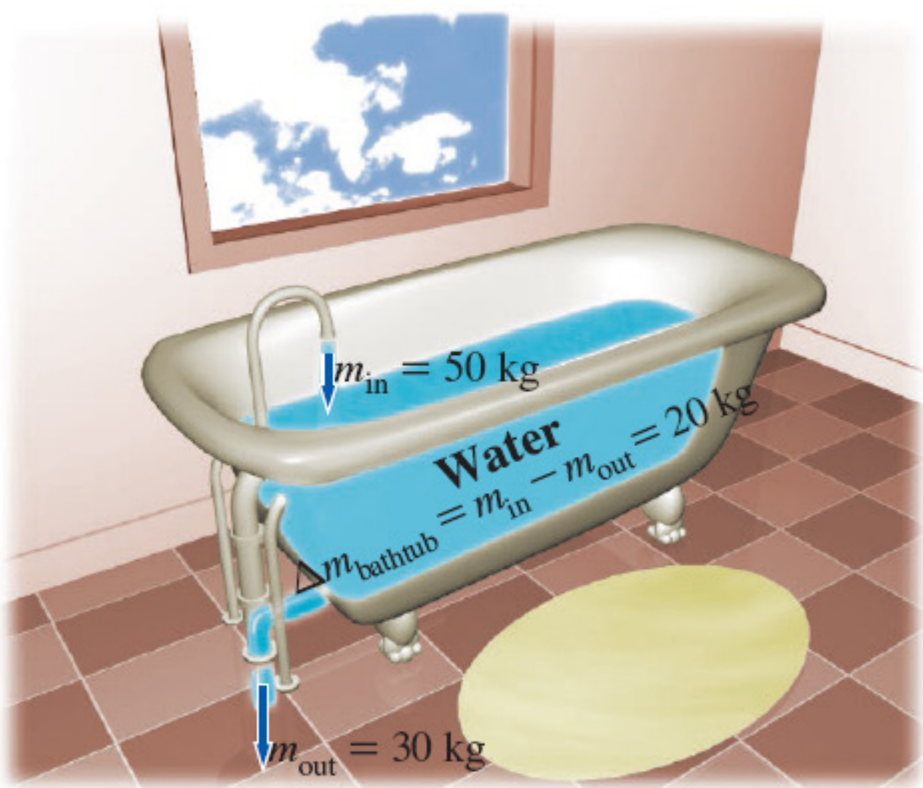
$$\dot{\forall} = \int V_n dA_c = V_{avg} A_c$$



Conservation of Mass

- The conservation of mass for a control volume:

(Total mass entering the CV during Δt) − (Total mass leaving the CV during Δt) = (Net change of mass within the CV during Δt)



Conservation of Mass

- The conservation of mass for a control volume:

$$m_{in} - m_{out} = \Delta m_{CV}$$

$$\Delta m_{CV} = m_{final} - m_{initial}$$

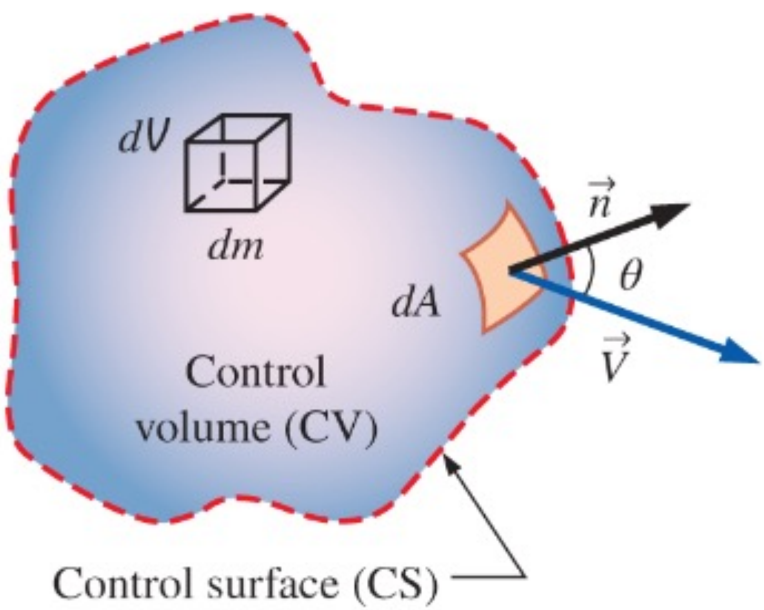
$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt}$$

These equations are often referred to as the ***mass balance*** *and are applicable to any control volume undergoing any kind of process.*

Conservation of Mass

- Total mass within the CV:

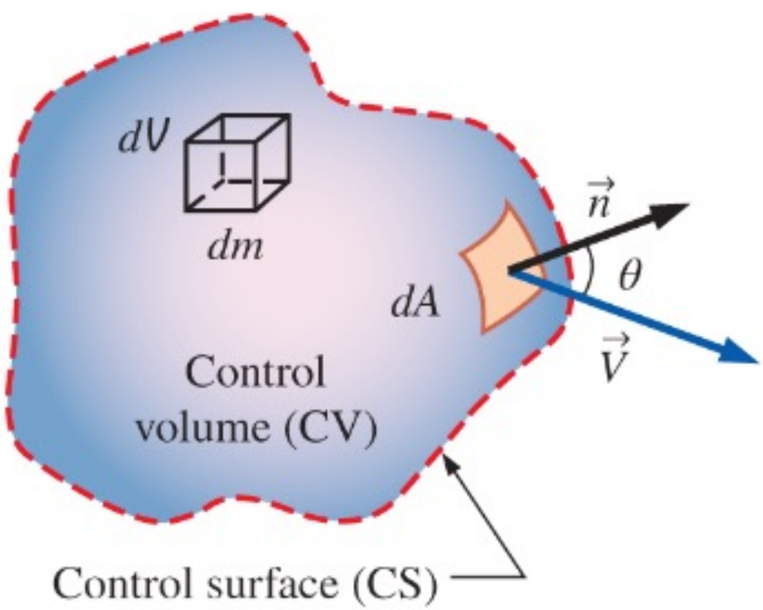
Total mass within the CV $$m_{CV} = \int_{CS} \rho d\forall$$



Conservation of Mass

- The time rate of change of the amount of mass within the control volume is expressed as:

Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CS} \rho d\forall$$



What does happen when $\frac{dm_{CV}}{dt} = 0$?

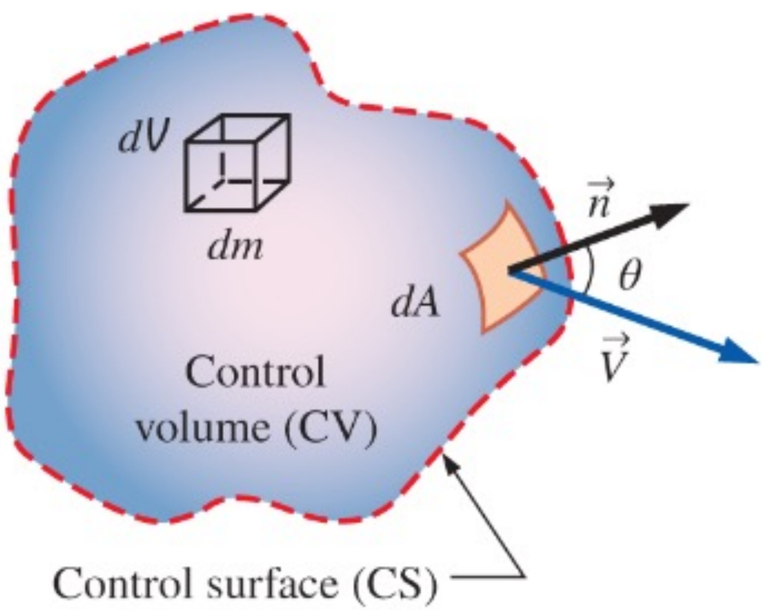
Conservation of Mass

- The mass flow rate through dA is proportional to the fluid density and normal velocity (V_n)

Normal component of velocity $$V_n = V cos(\theta) = \vec{V}.\vec{n}$$

Differential mass flow rate $$\delta \dot{m} = \rho V_n dA = \rho\big(V cos(\theta)\big)dA$$

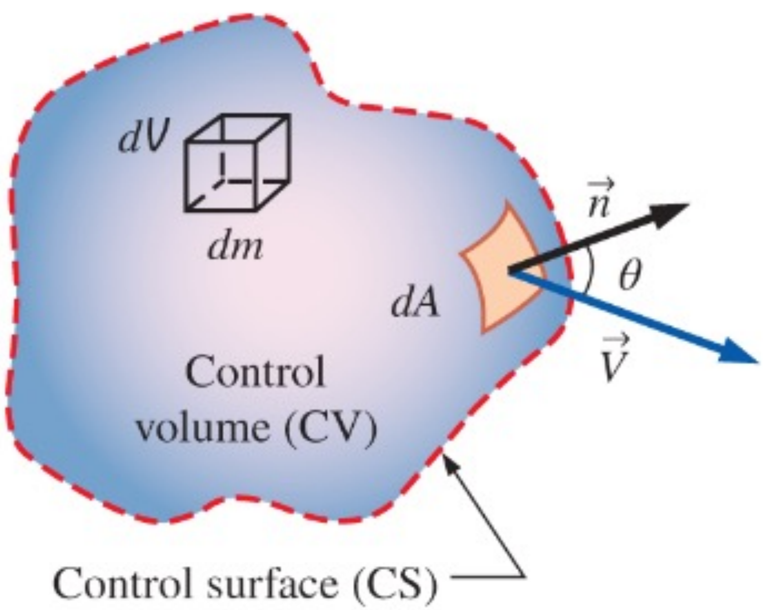
$$= \rho\big(\vec{V}.\vec{n}\big)dA$$



Conservation of Mass

- We can write the net flow rate:

Net mass flow rate: $\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho(\vec{V}.\vec{n})dA$



Conservation of Mass

- We can write the net flow rate:

$$\frac{dm_{CV}}{dt} + \dot{m}_{out} - \dot{m}_{in} = 0$$

$$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CS} \rho d\forall$$

$$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho(\vec{V}.\vec{n}) dA$$

$$\frac{d}{dt}\int_{CS} \rho d\forall + \int_{CS} \rho(\vec{V}.\vec{n}) dA = 0$$

Conservation of Mass

- We can write the net flow rate:

$$\frac{d}{dt}\int_{CS} \rho d\forall + \int_{CS} \rho(\vec{V}.\vec{n})dA = 0$$

$$\frac{d}{dt}\int_{CS} \rho d\forall + \sum_{out} \rho|V_n|A - \sum_{in} \rho|V_n|A = 0$$

Conservation of Mass

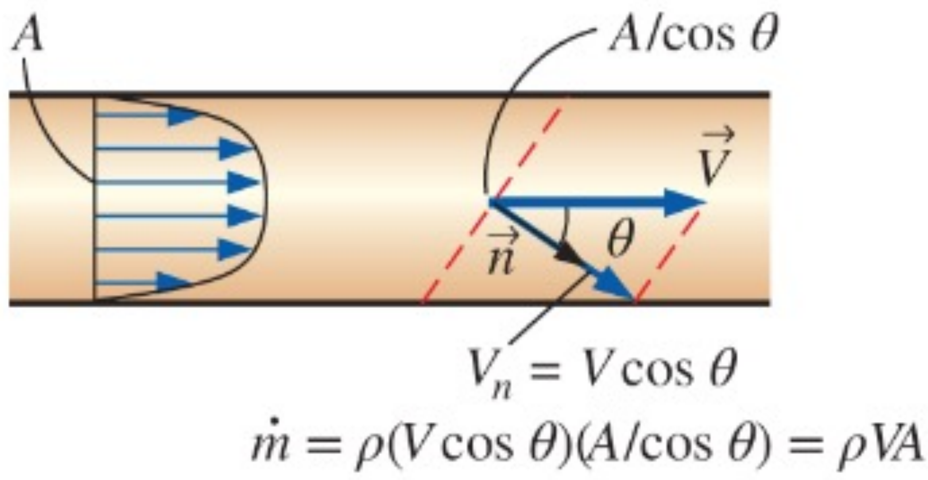
- We can write the net flow rate:

$$\frac{d}{dt}\int_{CS} \rho d\forall = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

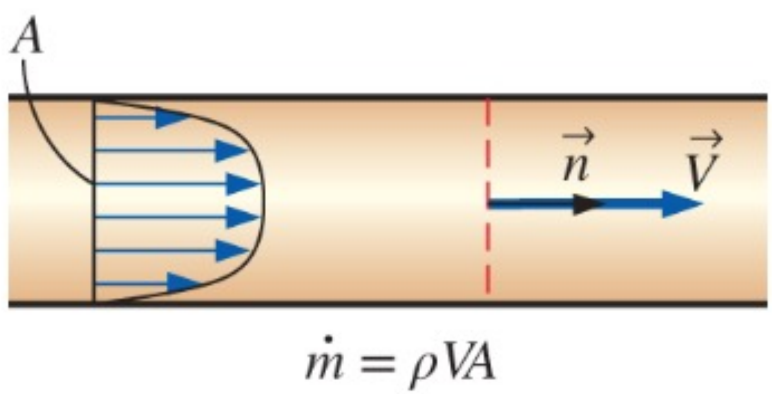
$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

Conservation of Mass

- A control surface



at an angle to the flow

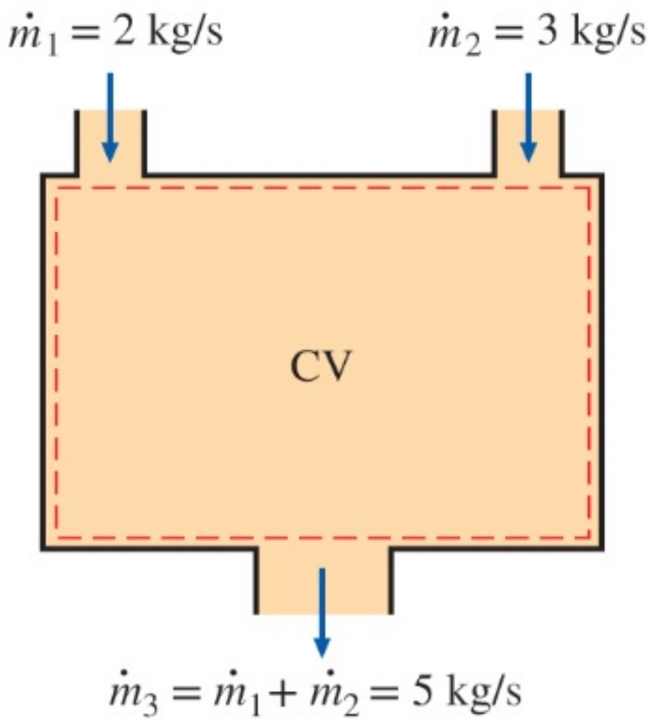


normal to the flow

Conservation of Mass

- Steady–flow:

$$\sum_{in} \dot{m} = \sum_{out} \dot{m}$$



Conservation of Mass

- Steady–flow for a single stream:

$$\dot{m}_1 = \dot{m}_2 \qquad \rightarrow \qquad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Conservation of Mass

- Special case: Incompressible flow:

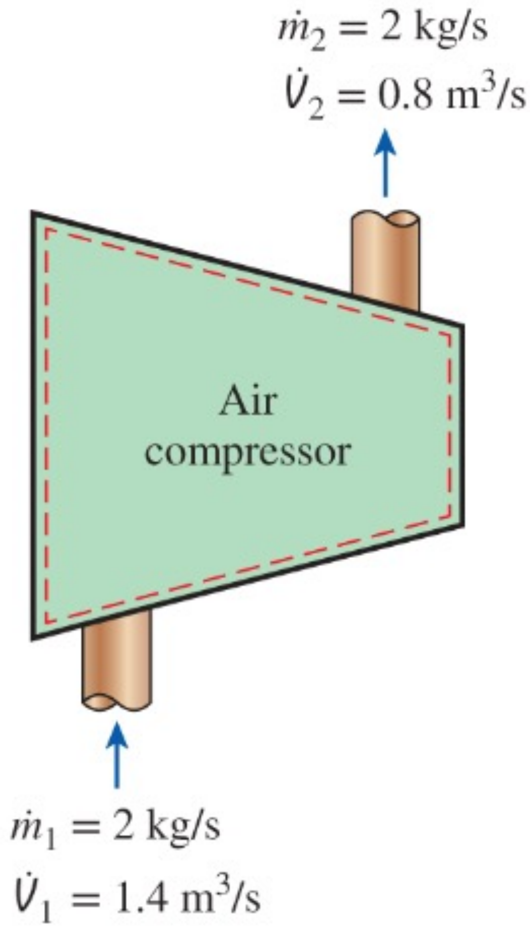
$$\sum_{in} \dot{\forall} = \sum_{out} \dot{\forall}$$

- Steady, incompressible flow (single stream):

$$\dot{\forall}_1 = \dot{\forall}_2 \quad \rightarrow \quad V_1 A_1 = V_2 A_2$$

Conservation of Mass

- Let's look at the following steady-state:



CLASS ACTIVITY

Class Activity

- A garden hose attached with a nozzle is used to fill a 10-gallon bucket. The inner diameter of the hose is 2 cm and it reduces to 0.8 cm at the nozzle exits. If it takes 50 s to fill the bucket with water, determine:
  a) The volume and mass flow rates of water through the hose
  b) The average velocity of water at the nozzle exit

Class Activity

- Solution (a):

$$\forall = 10\ gal \times \left(\frac{3.7854\ L}{1\ gal}\right) = 37.854\ L$$

$$\dot{\forall} = \frac{\forall}{\Delta t} = \frac{37.854\ L}{50\ s} = 0.757\ \frac{L}{s}$$

$$\dot{m} = \rho\dot{\forall} = \left(1000\ \frac{kg}{m^3}\right)\left(\frac{1000\ \frac{kg}{L}}{1\ \frac{kg}{m^3}}\right)\left(0.757\ \frac{L}{s}\right) = 0.757\ \frac{kg}{s}$$

Class Activity

- Solution (b):

$$A_e = \pi \times r_e^2 = \pi \times (0.4\ cm)^2 = 0.5027\ cm^2 = 0.5027 \times 10^{-4} m^2$$

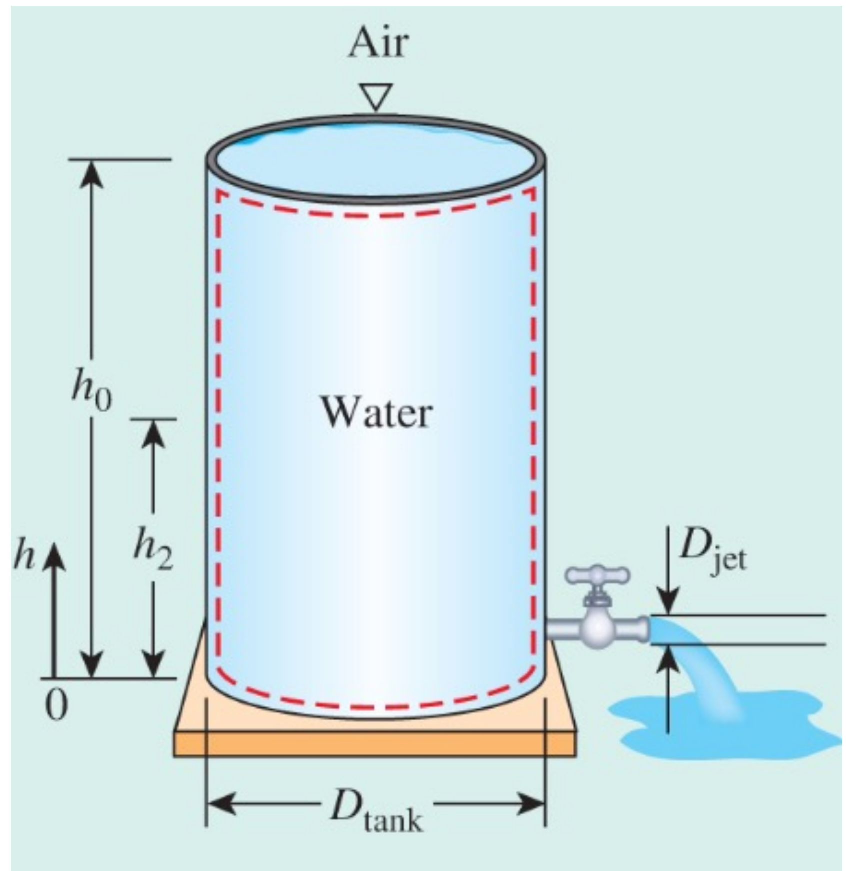
$$V_e = \frac{\dot{\forall}}{A_e} = \frac{0.757\ L/s}{0.5027 \times 10^{-4} m^2}\left(\frac{1\ m^3}{1000\ L}\right) = 15.1\ m/s$$

CLASS ACTIVITY

Class Activity

- A 4-ft high, 3-ft diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams.

The average velocity of the jet is approximated as $V = \sqrt{2gh}$ where h is the height of water in the tank measured from the center of the hole (a variable) and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop 2 ft from the bottom.



Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\dot{m}_{in} = 0$$

$$\dot{m}_{out} = (\rho VA)_{out} = \rho\sqrt{2gh}\, A_{jet}$$

$$m_{CV} = \rho\forall = \rho A_{tank} h$$

Class Activity

- Solution:

$$\frac{dm_{CV}}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \frac{d}{dt}(\rho A_{tank} h)$$

$$-\rho\sqrt{2gh}\left(\frac{D_{tank}^2}{4}\right) = \rho(\frac{\pi D_{tank}^2}{4})\frac{dh}{dt}$$

$$dt = \frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

Class Activity

- Solution:

$$dt = -\frac{D_{tank}^2}{D_{jet}^2}\frac{dh}{\sqrt{2gh}}$$

$$\begin{cases} t = 0 \rightarrow & h = h_0 \\ t = t \rightarrow & h = h_2 \end{cases}$$

$$\int_0^t dt = -\frac{D_{tank}^2}{D_{jet}^2\sqrt{2g}}\int_{h_1}^{h_2}\frac{dh}{\sqrt{h}}$$

$$t = \frac{\left(\sqrt{h_0} - \sqrt{h_2}\right)}{\sqrt{\frac{g}{2}}}\left(\frac{D_{tank}}{D_{jet}}\right)^2$$

Class Activity

- Solution:

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{2\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times12\ in}{0.5\ in}\right)^2 = 757\ s = 12.6\ min$$

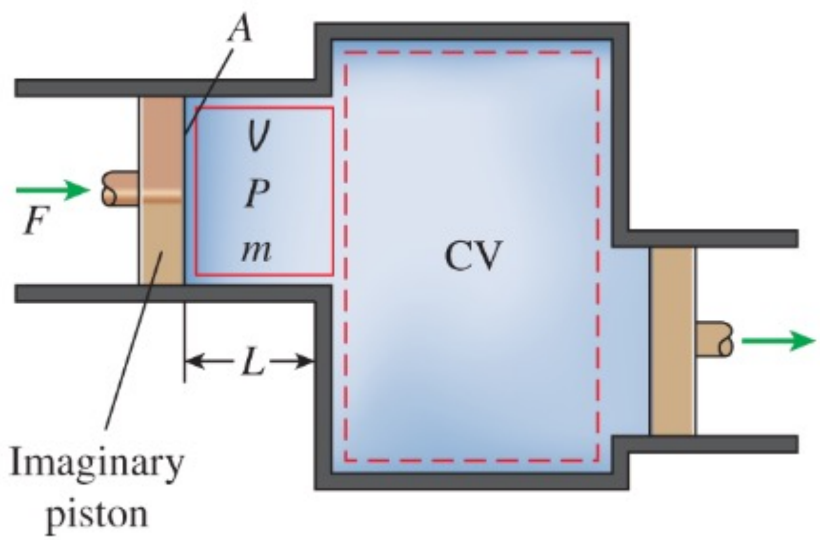
What does the entire flow exit the system?

$$t = \frac{\left(\sqrt{4\ ft} - \sqrt{0\ ft}\right)}{\sqrt{\frac{32.2}{2}}}\left(\frac{3\times12\ in}{0.5\ in}\right)^2 = 43.1\ min$$

FLOW WORK AND ENERGY OF FLOWING FLUID

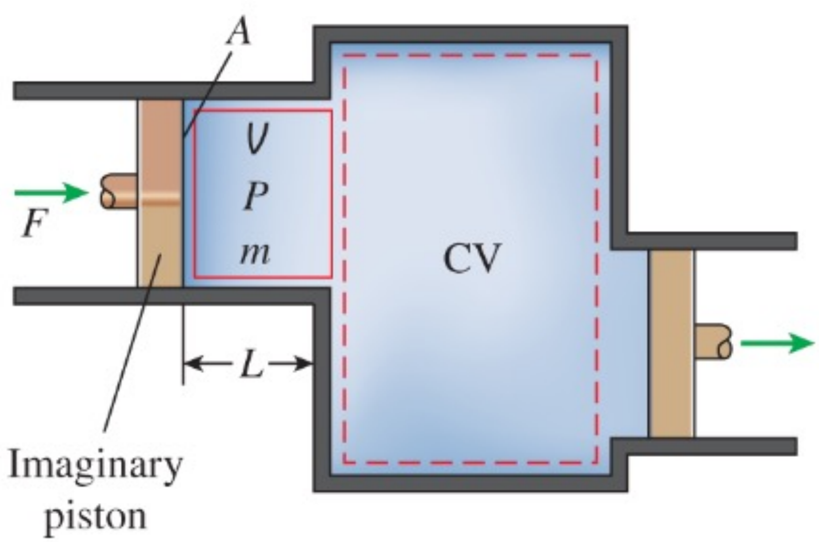
Flow Work and Energy of Flowing Fluid

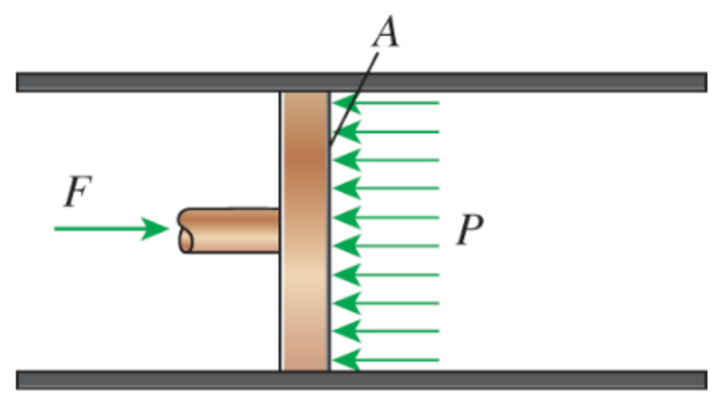
- Flow work (or flow energy): The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume



Flow Work and Energy of Flowing Fluid

- If the fluid pressure is P and the cross-sectional area of the fluid element is A, the force applied on the fluid by the imaginary piston is:





$$F = PA$$

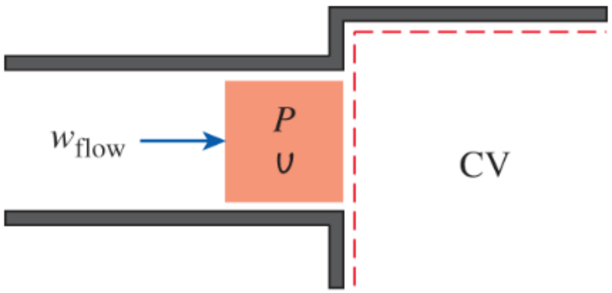
Flow Work and Energy of Flowing Fluid

- To push the entire fluid element into the control volume, this force must act through a distance L

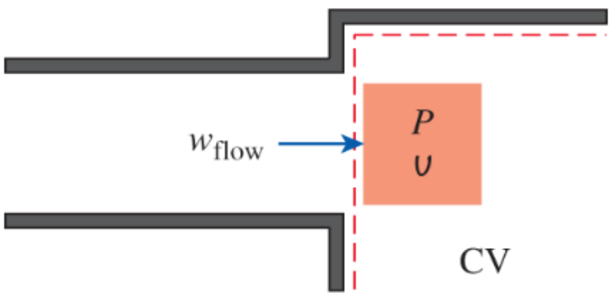
$$F = PA$$

$$W_{flow} = FL = PAL = P\forall$$

$$w_{flow} = Pv$$



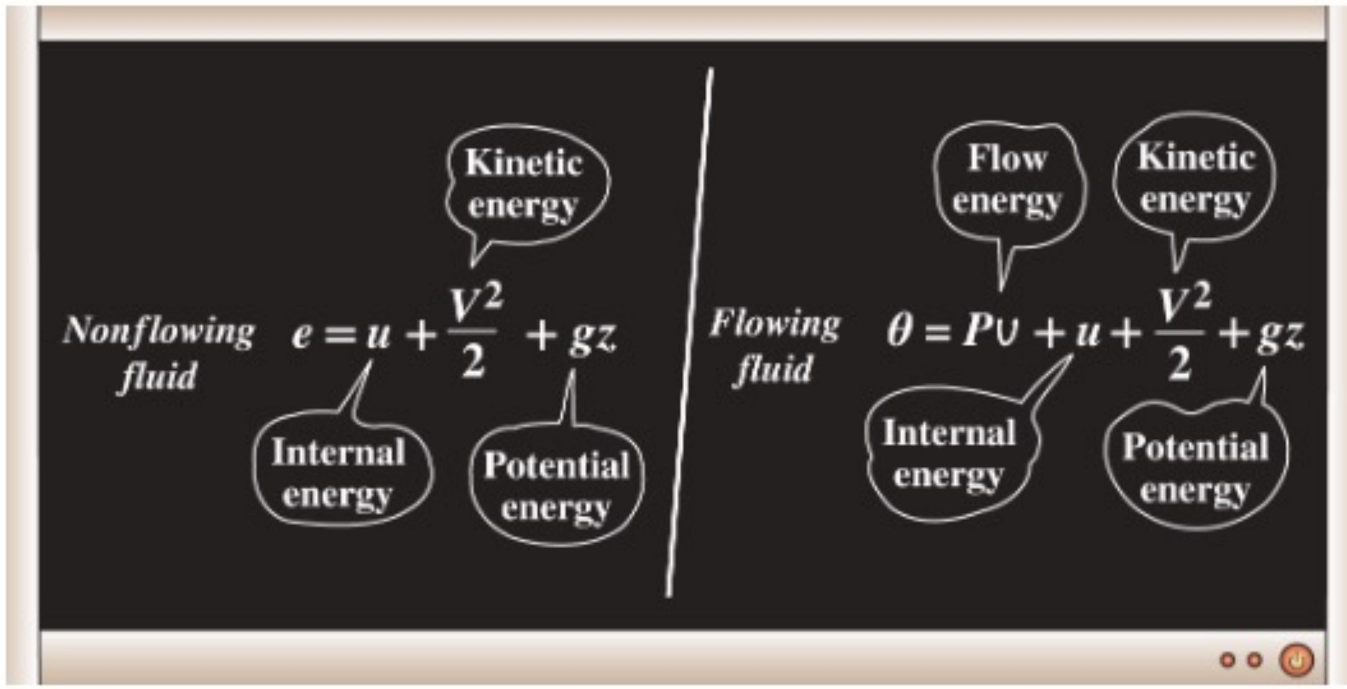
(a) Before entering



(b) After entering

Flow Work and Energy of Flowing Fluid

- Total energy of a flowing fluid is:



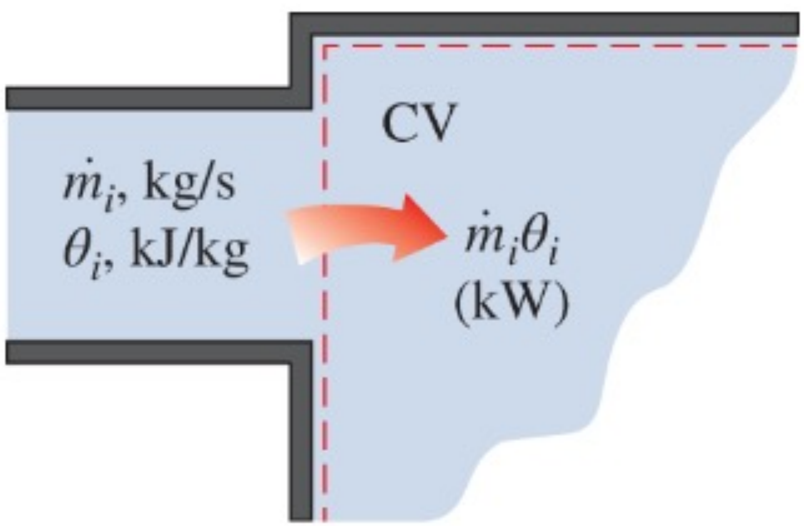
$$\theta = Pv + e = Pv + (u + ke + pe) = (u + Pv) + ke + pe$$

Flow Work and Energy of Flowing Fluid

- Energy transport by mass:

$$E_{mass} = m\theta = m(h + \frac{V^2}{2} + gz)$$

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}(h + \frac{V^2}{2} + gz)$$



Flow Work and Energy of Flowing Fluid

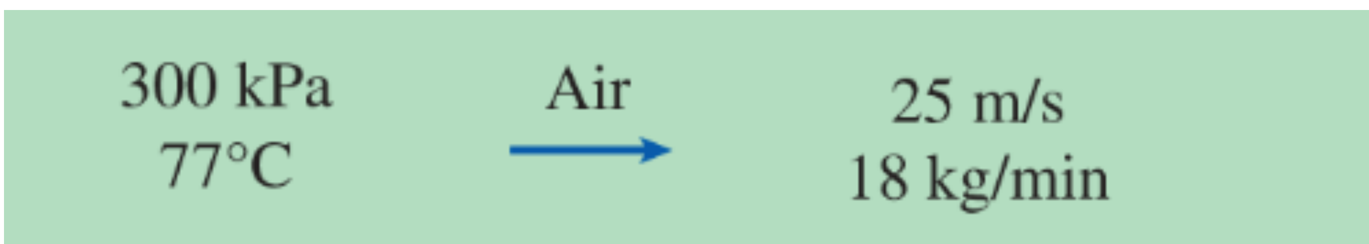
- What does happen when mass is not constant:

$$E_{in,mass} = \int_{m_i} \theta_i \delta m_i = \int_{m_i} (h_i + \frac{V_i^2}{2} + g z_i)\, \delta m_i$$

CLASS ACTIVITY

Class Activity

- Air flows steadily in a pipe at 300 kPa, 77 °C, and 25 m/s at a rate of 18 kg/min. Determine:
  a) The diameter of the pipe
  b) The rate of flow energy
  c) The rate of energy transport by mass
  d) The error involved in part c if the kinetic energy is neglected.



Class Activity

- Solution (assumptions):
  - ❑ The flow is steady
  - ❑ The potential energy is negligible
  - ❑ R = 0.287 kJ/kg-K
  - ❑ c_p=1.008 kJ/kg-K (at 350 K from Table A-2b)

Class Activity

- Solution (a):

$$v = \frac{RT}{P} = \frac{(0.287\,\frac{kJ}{kg-K})(77+273)}{300\ kPa} = 0.3349\,\frac{m^3}{kg}$$

$$A = \frac{\dot{m}v}{\forall} = \frac{\left(\frac{18}{60}\,\frac{kg}{s}\right)(0.3349\,\frac{m^3}{kg})}{25\,\frac{m}{s}} = 0.004018\ m^2$$

$$D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.004018\ m^2)}{\pi}} = 0.0715\ m$$

Class Activity

- Solution (b):

$$\dot{W}_{flow} = \dot{m}Pv = \left(\frac{18}{60}\frac{kg}{s}\right)(300\ kPa)\left(0.3349\,\frac{m^3}{kg}\right) = 30.14\ kW$$

Class Activity

- Solution (c):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}(c_p T + \frac{V^2}{2})$$

$$\dot{E}_{mass} = \left(\frac{18}{60}\frac{kg}{s}\right)\left[\left(1.008\,\frac{kJ}{kg-K}\right)(77 + 273\;K) + \left(\frac{1}{2}\right)\left(25\,\frac{m}{s}\right)^2 \left(\frac{1\,\frac{kJ}{kg}}{1000\,\frac{m^2}{s^2}}\right)\right]$$

$$\dot{E}_{mass} = 105.94\;kW$$

Class Activity

- Solution (d):

$$\dot{E}_{mass} = \dot{m}(h + ke) = \dot{m}h = \dot{m}c_pT$$

$$\dot{E}_{mass} = \dot{m}c_pT = \left(\frac{18}{60}\frac{kg}{s}\right)\left(1.005\,\frac{kJ}{kg-K}\right)(77 + 273K) = 105.84\;kW$$