

CAE 463/524

Building Enclosure Design

Spring 2015

Lecture 4: February 10, 2015

Complex conduction in building enclosures

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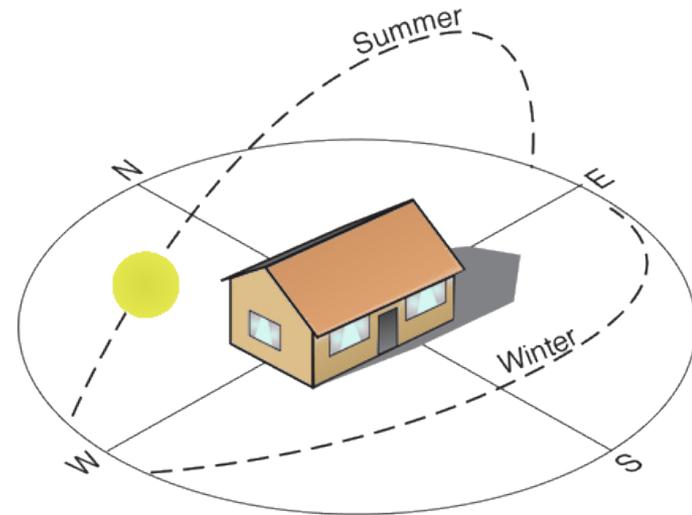
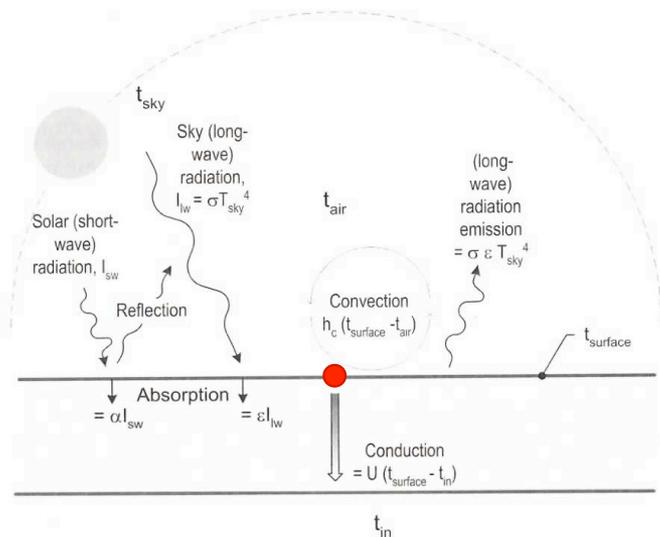
Campus projects

- Need to do thermal assessments by early March
 - 20 students in this class: 5 teams of 4

Building	Team members
Alumni	Maria, Yin Ling, Whitney, Liz
Crown	Henry, Yun Joon, Jose, Oleg
E1 - Rettaliata	Jinzhe, Julie, Roger, Rebecca
Hermann	Zack, Dilip, Allan, Dhaval
SSV	Thomas, Kim, Larry, Michelle

Last time

- Surface energy balances
- Solar orientation and enclosures



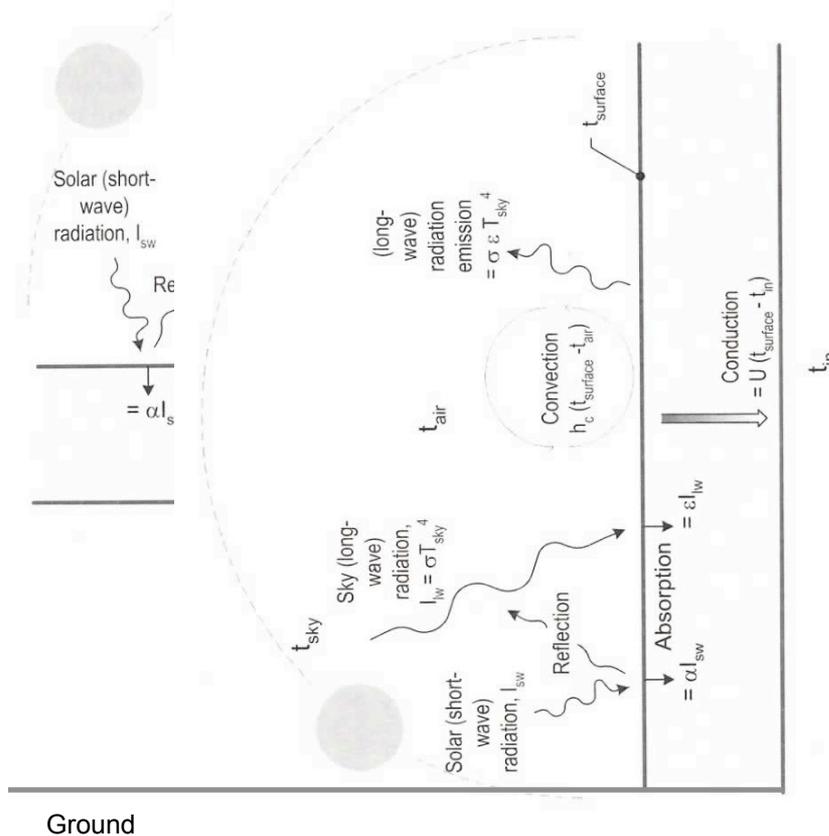
- Assigned HW #1
 - HW #1 due today

This time (and probably part of next time)

- **Complex conduction** in building enclosures
 - Temperature profiles through enclosures
 - Layers with different materials
 - Parallel path vs. isothermal methods
 - Thermal bridges
 - 2-D and 3-D conduction
 - THERM modeling
 - Floor-ground conduction

Last time: Bringing all the modes together

- Energy balance for a vertical surface:



$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$

$$\begin{aligned} & \alpha I_{solar} \\ & + \epsilon_{surface} \sigma F_{sky} (T_{sky}^4 - T_{surf}^4) \\ & + \epsilon_{surface} \sigma F_{air} (T_{air}^4 - T_{surface}^4) \\ & + \epsilon_{surface} \sigma F_{ground} (T_{air}^4 - T_{ground}^4) \\ & + h_{conv} (T_{air} - T_{surface}) \\ & - U (T_{surface} - T_{surface,interior}) = 0 \end{aligned}$$

We need to understand conduction through enclosures that are more complex than just single materials

COMPLEX CONDUCTION IN ENCLOSURES

Combining elements

Multiple layers and temperature distributions

Thermal bridges

Combining elements in an actual enclosure

- So far we have been exploring single assemblies
 - Just roofs or just walls without windows and doors
 - If you design a building without windows and doors, something probably went wrong!
- Concept of **combined thermal transmittance**: U_o
 - U_o is the combined thermal transmittance of the respective areas of a gross exterior wall, roof, or floor (elements in **parallel**)
 - It is basically an **area-weighted average U-value**

$$U_o = (U_{wall} A_{wall} + U_{window} A_{window} + U_{door} A_{door}) / A_o$$

where

U_o = average thermal transmittance of gross wall area

A_o = gross area of exterior walls

U_{wall} = thermal transmittance of all elements of opaque wall area

A_{wall} = opaque wall area

U_{window} = thermal transmittance of window area (including frame)

A_{window} = window area (including frame)

U_{door} = thermal transmittance of door area

A_{door} = door area (including frame)

Combined thermal transmittance example

- Calculate U_o for a 10 m x 2.4 m wall with two double-glazed windows with wood/vinyl frames and one solid core door
 - One window is 1.5 x 0.86 m; the other window is 0.9 x 0.76 m
 - $U_{\text{window}} = 2.90 \text{ W/m}^2\text{K}$
 - The door is 0.86 x 2 m
 - $U_{\text{door}} = 1.42 \text{ W/m}^2\text{K}$
 - The wall has a U value of $U_{\text{wall}} = 0.404 \text{ W/m}^2\text{K}$

$$A_{\text{window}} = (1.500 \times 0.860) + (0.900 \times 0.760) = 1.97 \text{ m}^2$$

$$A_{\text{door}} = (0.860 \times 2.000) = 1.72 \text{ m}^2$$

$$A_{\text{wall}} = (10 \times 2.4) - (1.97 + 1.72) = 20.31 \text{ m}^2$$

Therefore, the combined thermal transmittance for the wall is

$$\begin{aligned} U_o &= \frac{(0.404 \times 20.31) + (2.90 \times 1.97) + (1.42 \times 1.72)}{10 \times 2.4} \\ &= 0.68 \text{ W}/(\text{m}^2 \cdot \text{K}) \end{aligned}$$

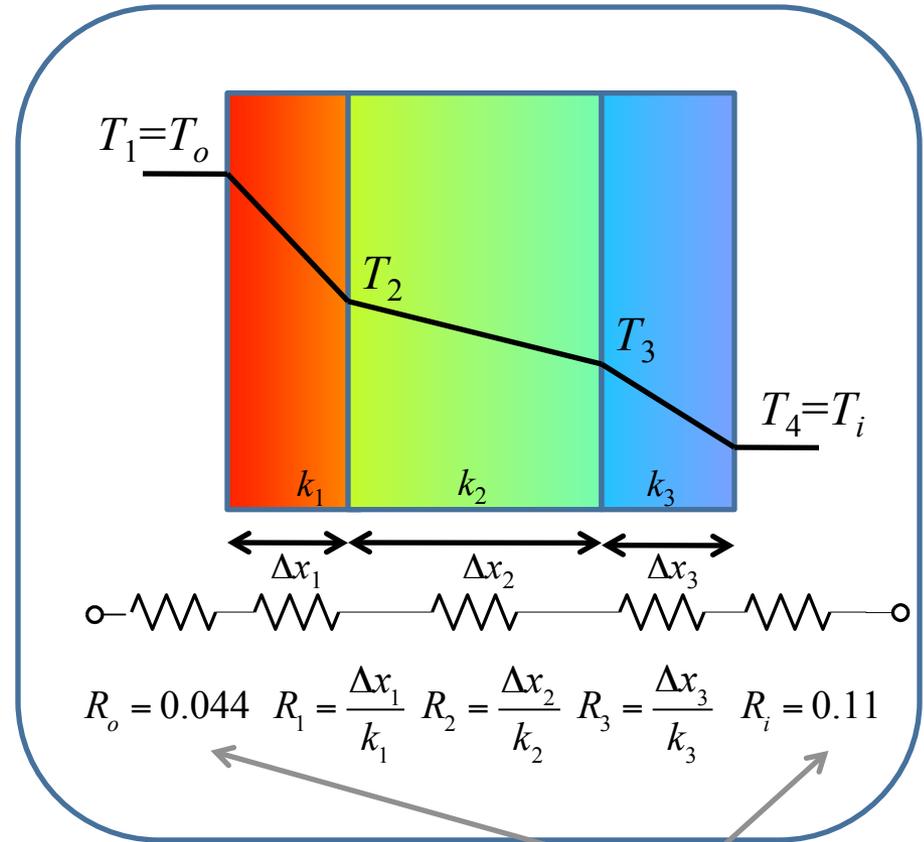
Conduction through multiple layers

- Just as in electrical circuits, the overall thermal resistance of a series of elements (layers) can be expressed as the sum of the resistances of each layer
 - Don't forget the interior and exterior convective resistances
- By continuity of energy we can write

$$q = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3}$$

so

$$q = \frac{T_1 - T_4}{R_{total}} \text{ where } R_{total} = R_o + R_1 + R_2 + R_3 + R_i$$



Can only add resistances (R) in series, not conductances (U)

Simple conduction through multiple layers

- Calculate the R-value of an enclosure assembly

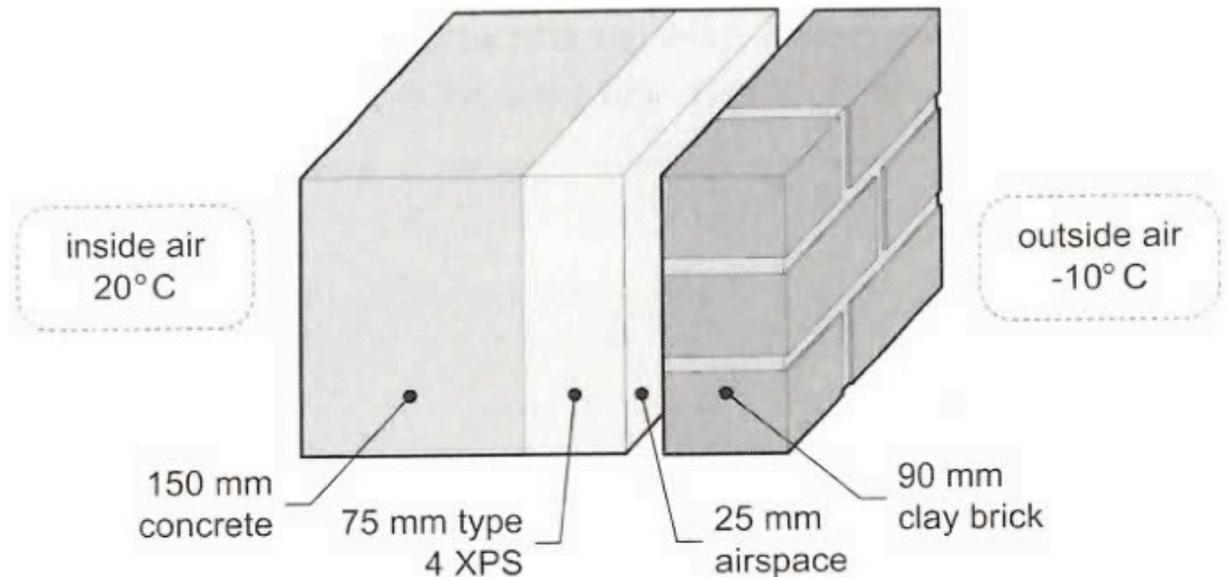
Steps:

1. List each material in the assembly
 - And its conductivity and thickness
2. Calculate conductance of each layer
 - $U = k/L$
3. Calculate thermal resistance of each layer
 - $R = 1/U$
4. Sum the individual thermal resistances to get R_{total}

Conduction through multiple layers

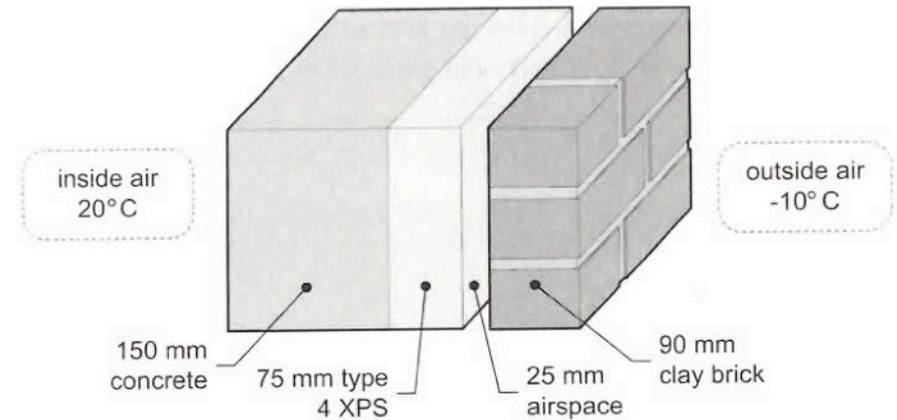
Example problem:

- Calculate the total thermal resistance, R_{total} , and the temperature distribution through the wall shown below



Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for material data



Layer material	Conductivity $k = [\text{W/mK}]$	Thickness $L = [\text{m}]$	Conductance $U = [\text{W/m}^2\text{K}]$	Resistance $R = [\text{m}^2\text{K/W}]$
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A note on R-values of air cavities

- ASHRAE has measured the combined convective + radiative R-values for thin planar cavities of various orientations and depths with various “ ϵ_{eff} ”
- These are the best data to use for air spaces in assemblies
 - If you do not know that the material in the cavity is reflective or “low e”, just assume that both walls of the cavity have $\epsilon=0.9$ for each surface, so that when combined, $\epsilon_{eff} = 0.82$

$$\epsilon_{eff} = \epsilon_1 \epsilon_2$$

2013 ASHRAE Handbook, Chapter 26 (small cavities)

R-values for different air gap characteristics

Table 3 Thermal Resistances of Plane Air Spaces^{a,b,c}, (m²·K)/W

Position of Air Space	Direction of Heat Flow	Air Space		13 mm Air Space ^c					20 mm Air Space ^c				
		Mean Temp. ^d , °C	Temp. Diff. ^d , °C	Effective Emittance $\epsilon_{eff}^{d,e}$					Effective Emittance $\epsilon_{eff}^{d,e}$				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
Horiz.	Up 	32.2	5.6	0.37	0.36	0.27	0.17	0.13	0.41	0.39	0.28	0.18	0.13
		10.0	16.7	0.29	0.28	0.23	0.17	0.13	0.30	0.29	0.24	0.17	0.14
		10.0	5.6	0.37	0.36	0.28	0.20	0.15	0.40	0.39	0.30	0.20	0.15
		-17.8	11.1	0.30	0.30	0.26	0.20	0.16	0.32	0.32	0.27	0.20	0.16
		-17.8	5.6	0.37	0.36	0.30	0.22	0.18	0.39	0.38	0.31	0.23	0.18
		-45.6	11.1	0.30	0.29	0.26	0.22	0.18	0.31	0.31	0.27	0.22	0.19
		-45.6	5.6	0.36	0.35	0.31	0.25	0.20	0.38	0.37	0.32	0.26	0.21
45° Slope	Up 	32.2	5.6	0.43	0.41	0.29	0.19	0.13	0.52	0.49	0.33	0.20	0.14
		10.0	16.7	0.36	0.35	0.27	0.19	0.15	0.35	0.34	0.27	0.19	0.14
		10.0	5.6	0.45	0.43	0.32	0.21	0.16	0.51	0.48	0.35	0.23	0.17
		-17.8	11.1	0.39	0.38	0.31	0.23	0.18	0.37	0.36	0.30	0.23	0.18
		-17.8	5.6	0.46	0.45	0.36	0.25	0.19	0.48	0.46	0.37	0.26	0.20
		-45.6	11.1	0.37	0.36	0.31	0.25	0.21	0.36	0.35	0.31	0.25	0.20
		-45.6	5.6	0.46	0.45	0.38	0.29	0.23	0.45	0.43	0.37	0.29	0.23
Vertical	Horiz. 	32.2	5.6	0.43	0.41	0.29	0.19	0.14	0.62	0.57	0.37	0.21	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.51	0.49	0.35	0.23	0.17
		10.0	5.6	0.47	0.45	0.33	0.22	0.16	0.65	0.61	0.41	0.25	0.18
		-17.8	11.1	0.50	0.48	0.38	0.26	0.20	0.55	0.53	0.41	0.28	0.21
		-17.8	5.6	0.52	0.50	0.39	0.27	0.20	0.66	0.63	0.46	0.30	0.22
		-45.6	11.1	0.51	0.50	0.41	0.31	0.24	0.51	0.50	0.42	0.31	0.24
		-45.6	5.6	0.56	0.55	0.45	0.33	0.26	0.65	0.63	0.51	0.36	0.27
32.2	5.6	0.44	0.41	0.29	0.19	0.14	0.62	0.58	0.37	0.21	0.15		

Usually we use values from the $\epsilon_{eff} = 0.82$ column unless one material is low-e

2013 ASHRAE Handbook, Chapter 26 (small cavities)

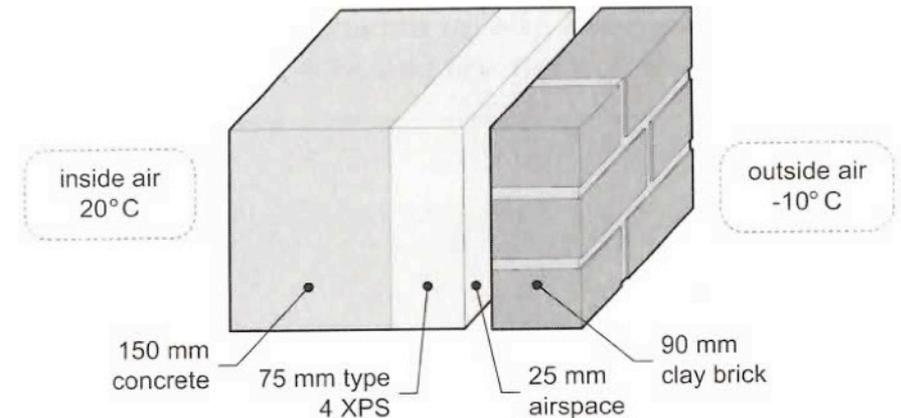
R-values for different air gap characteristics

Position of Air Space	Direction of Heat Flow	Mean Temp. ^d , °C	Temp. Diff. ^d , °C	Effective Emittance $\epsilon_{eff}^{d,e}$					Effective Emittance $\epsilon_{eff}^{d,e}$				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
		Air Space		40 mm Air Space ^c					90 mm Air Space ^c				
Horiz.	Up 	32.2	5.6	0.45	0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
		10.0	16.7	0.33	0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
		10.0	5.6	0.44	0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
		-17.8	11.1	0.35	0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
		-17.8	5.6	0.43	0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
		-45.6	11.1	0.34	0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
45° Slope	Up 	-45.6	5.6	0.42	0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
		32.2	5.6	0.51	0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
		10.0	16.7	0.38	0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
		10.0	5.6	0.51	0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
		-17.8	11.1	0.40	0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
		-17.8	5.6	0.49	0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
Vertical	Horiz. 	-45.6	11.1	0.39	0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
		-45.6	5.6	0.48	0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
		32.2	5.6	0.70	0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
		10.0	5.6	0.67	0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
		-17.8	11.1	0.49	0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
		-17.8	5.6	0.62	0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
		-45.6	11.1	0.46	0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
		-45.6	5.6	0.58	0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26
		32.2	5.6	0.89	0.80	0.45	0.24	0.16	0.85	0.76	0.44	0.24	0.16

Usually we use values from the $\epsilon_{eff} = 0.82$ column

Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for material data



Layer material	Conductivity $k = [\text{W/mK}]$	Thickness $L = [\text{m}]$	Conductance $U = [\text{W/m}^2\text{K}]$	Resistance $R = [\text{m}^2\text{K/W}]$
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17

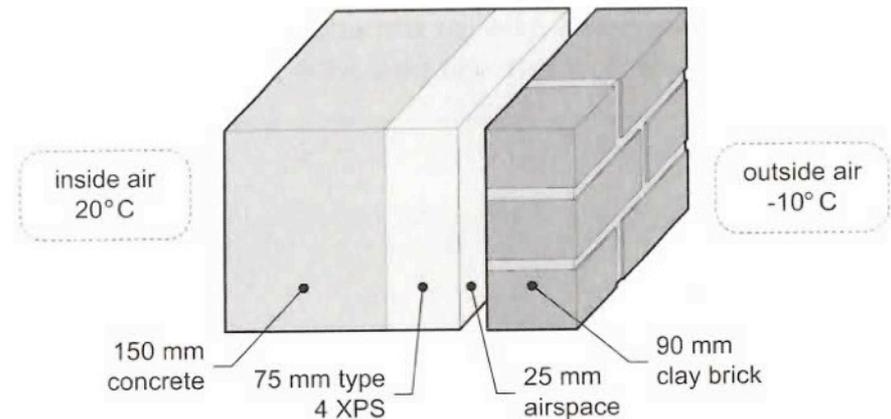
$$R_{\text{total}} (\text{IP}) = 17.3 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{F/Btu}$$

R-values of deeper cavities

- The R-value of cavities stops increasing much at 3 inches (75 mm) depth
 - Beyond 3 inches (75 mm), convection and radiation dominate
 - For a deep cavity, either compute R-values with more advanced methods or use the 3 inch (75 mm) value
- Do **NOT** take the R value of a 1 inch (25 mm) cavity and multiply by the thickness of the cavity for thick cavities
 - If you did that, you would guess that an 8 foot attic would have an R value of about $100 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{F}/\text{Btu}$, which is a factor of 20 too high!

Conduction through multiple layers

- $U_{\text{total}} = 0.33 \text{ W/m}^2\text{K}$
- Calculate steady-state heat flux through the enclosure



- $q = U\Delta T$
- $q = (0.33 \text{ W/m}^2\text{K}) * (T_{\text{inside}} - T_{\text{outside}})$
- $q = (0.33 \text{ W/m}^2\text{K}) * (30 \text{ K}) = 10 \text{ W/m}^2$
 - From inside to outside

Conduction through multiple layers

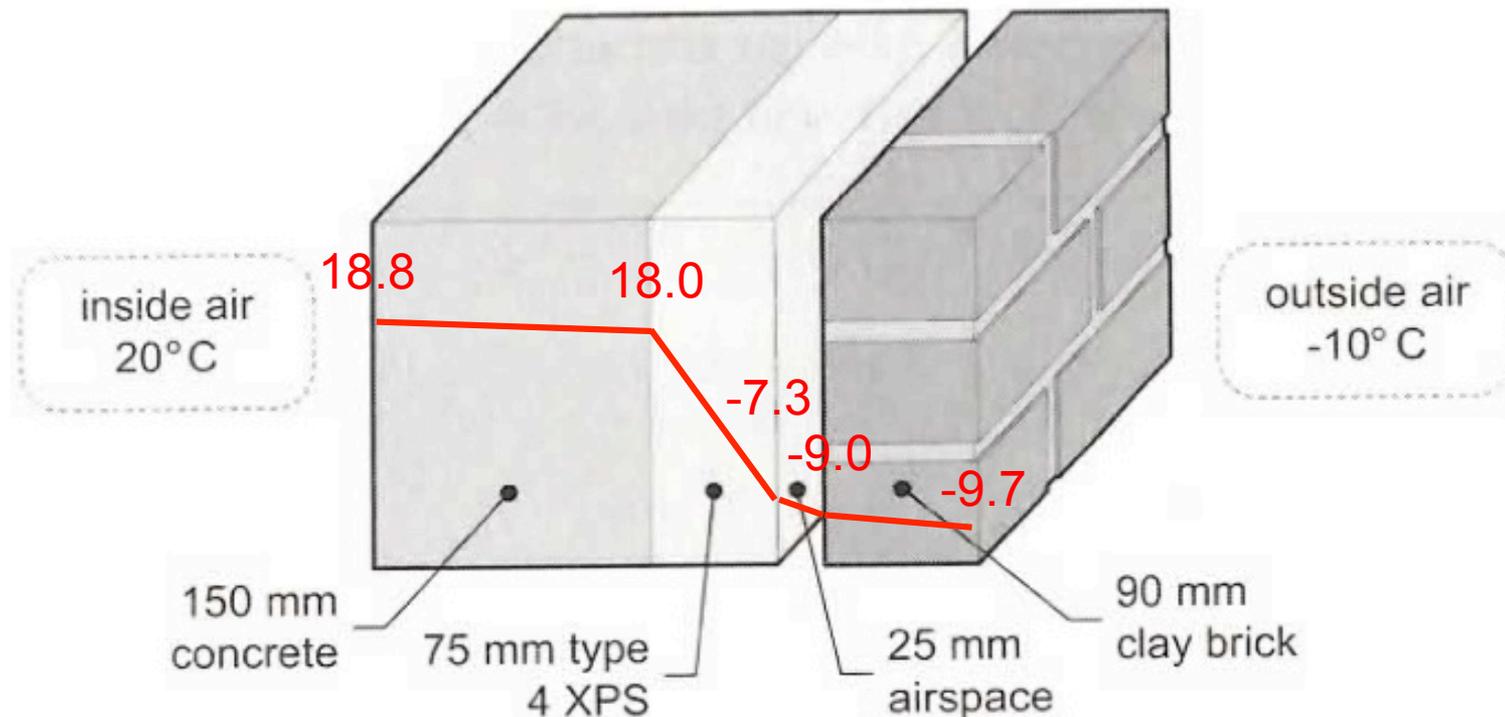
- Calculating the **temperature gradient** through an enclosure of i materials

$$\Delta T_i = \frac{T_{internal} - T_{external}}{\sum_{i=0}^n R_i} R_i$$

Layer	Conductivity W/mK	Thickness m	Conductance W/m ² K	Resistance m ² K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			R _{total} (m ² K/W)	3.04
			U _{total} (W/m ² K)	0.33

Conduction through multiple layers

- Calculating the **temperature gradient** through an enclosure of i materials



Limitations to the summation rule

The summation rule for finding R_{total} has several limitations:

- Only works for **layers**
- Layers must be **same area**
- Layers must be **uniform thickness**
- Layers must have **constant material properties**
 - This is the **biggest** limitation

What do we do with more realistic constructions?

- **Parallel path or ISO thermal equivalents**
- **Computer modeling**

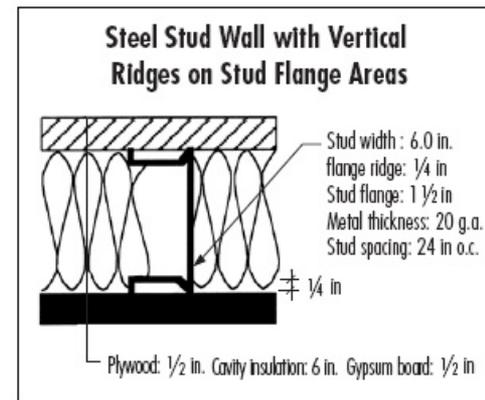


Figure 1. Vertical ridges on a steel stud reduce the contact area between the stud and the sheathing material and improve the whole wall R-value.

CONDUCTION IN MORE COMPLICATED ENCLOSURE ASSEMBLIES

Thermal networks

- We have already learned how to combine layers to get an overall thermal resistance (or U value) for an assembly made of homogenous layers
 - But we often have to find R (or U) of a *more complicated* assembly
- The first level of more complicated analysis is best done using thermal networks
 - We've already touched on networks a bit
- References:
 - ISO 6946 Building components and building elements thermal resistance and thermal transmittance calculation method
 - ASHRAE Handbook of Fundamentals

Developing a thermal network

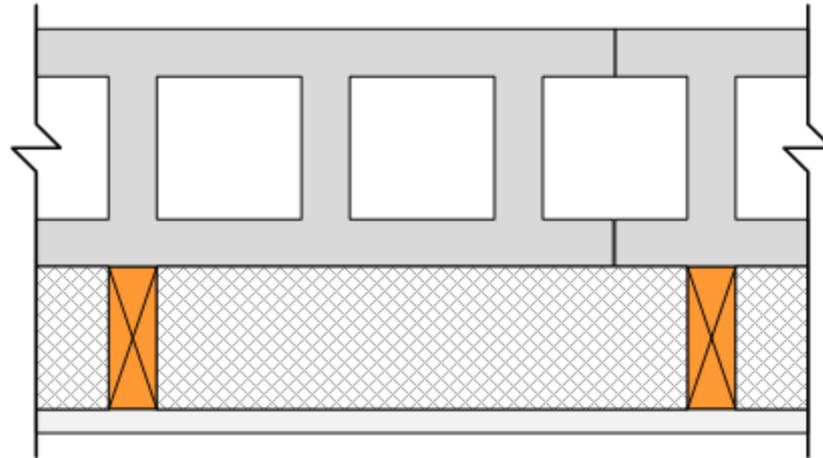
1. Identify the layers of the assembly
2. Identify all elements in the layers with differing thermal conductivity
3. Find the R value for each element
4. Draw a resistor for each element
 - Don't forget the internal and external convection resistance
5. Set the resistance to the R value divided by the fractional cross sectional area of the element
6. Connect resistors assuming **isothermal** or **parallel path** conditions

Isothermal vs. Parallel Path?

- In the **isothermal** assumption, the temperature at any layer interface is assumed to be constant, even if the layer is made of more than one material
 - This means that there is a network node that corresponds to the interface of each layer
- In the **parallel path** assumption, the heat transfer is assumed to be only normal to planes
 - This means that the network is several parallel branches
- This is probably best illustrated by example

Isothermal/parallel path example

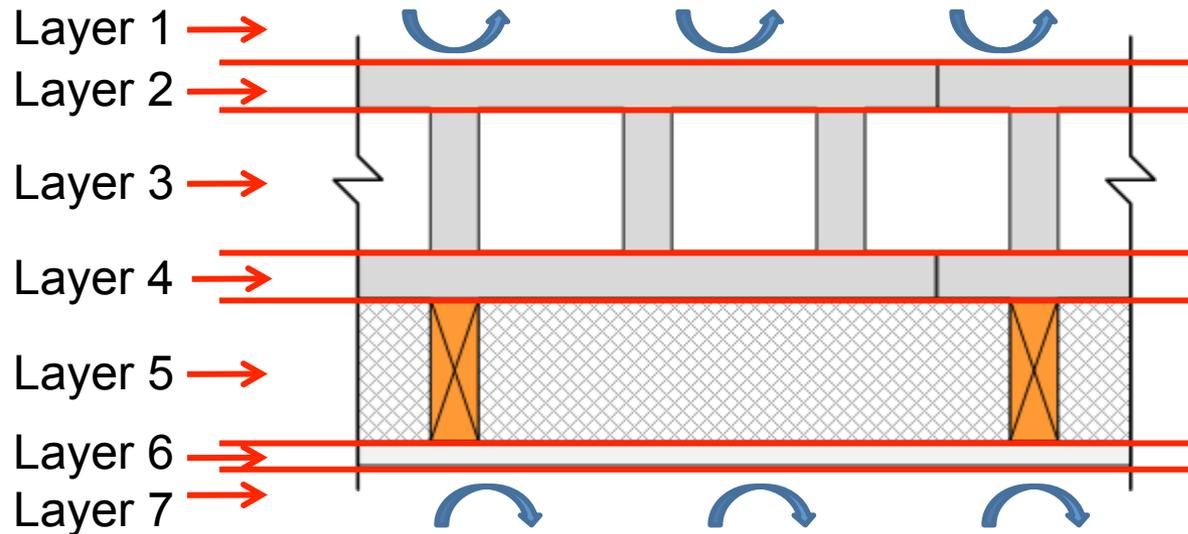
- Consider a CMU Block wall with an attached stud wall
 - A section of the wall is shown below in plan view
 - Draw the isothermal and parallel path thermal networks



Isothermal/parallel path example

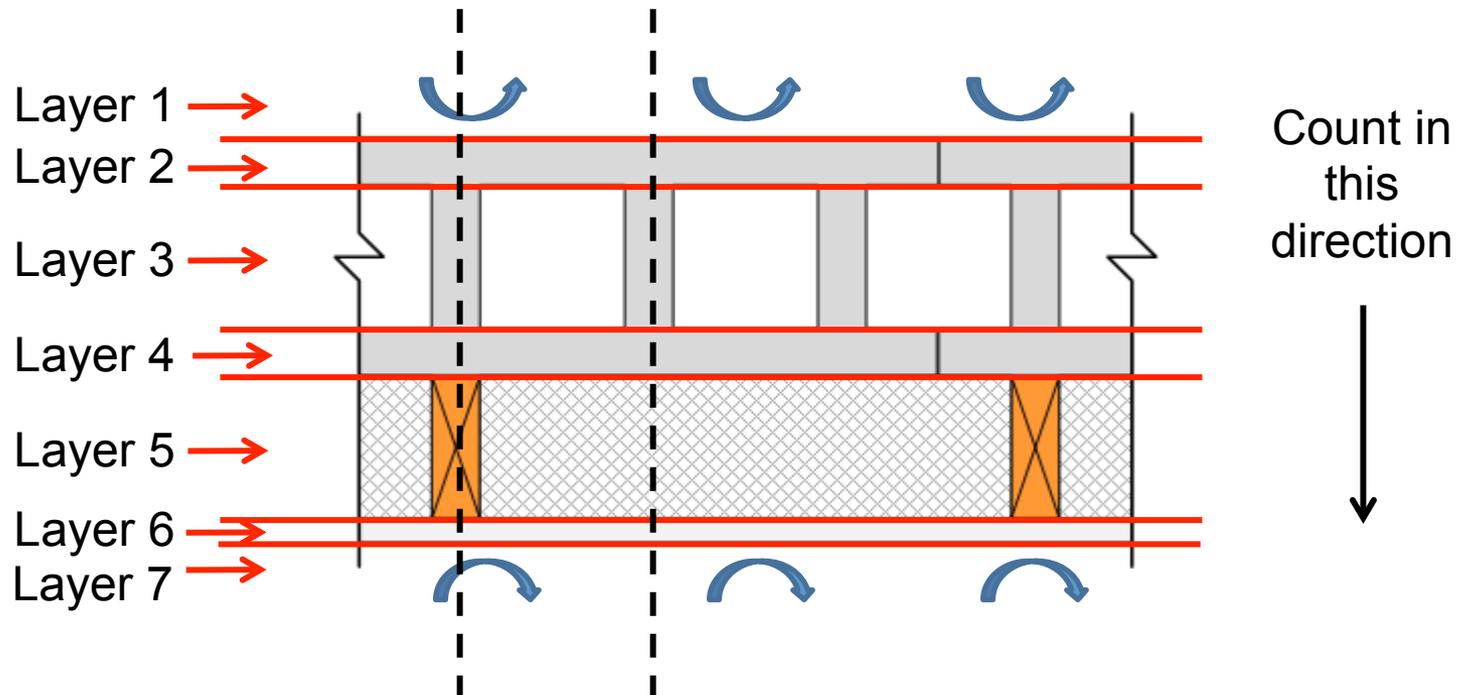
1. Identify the layers

- This example has seven layers
- Five are within the assembly
 - The other two are internal and external convection resistances



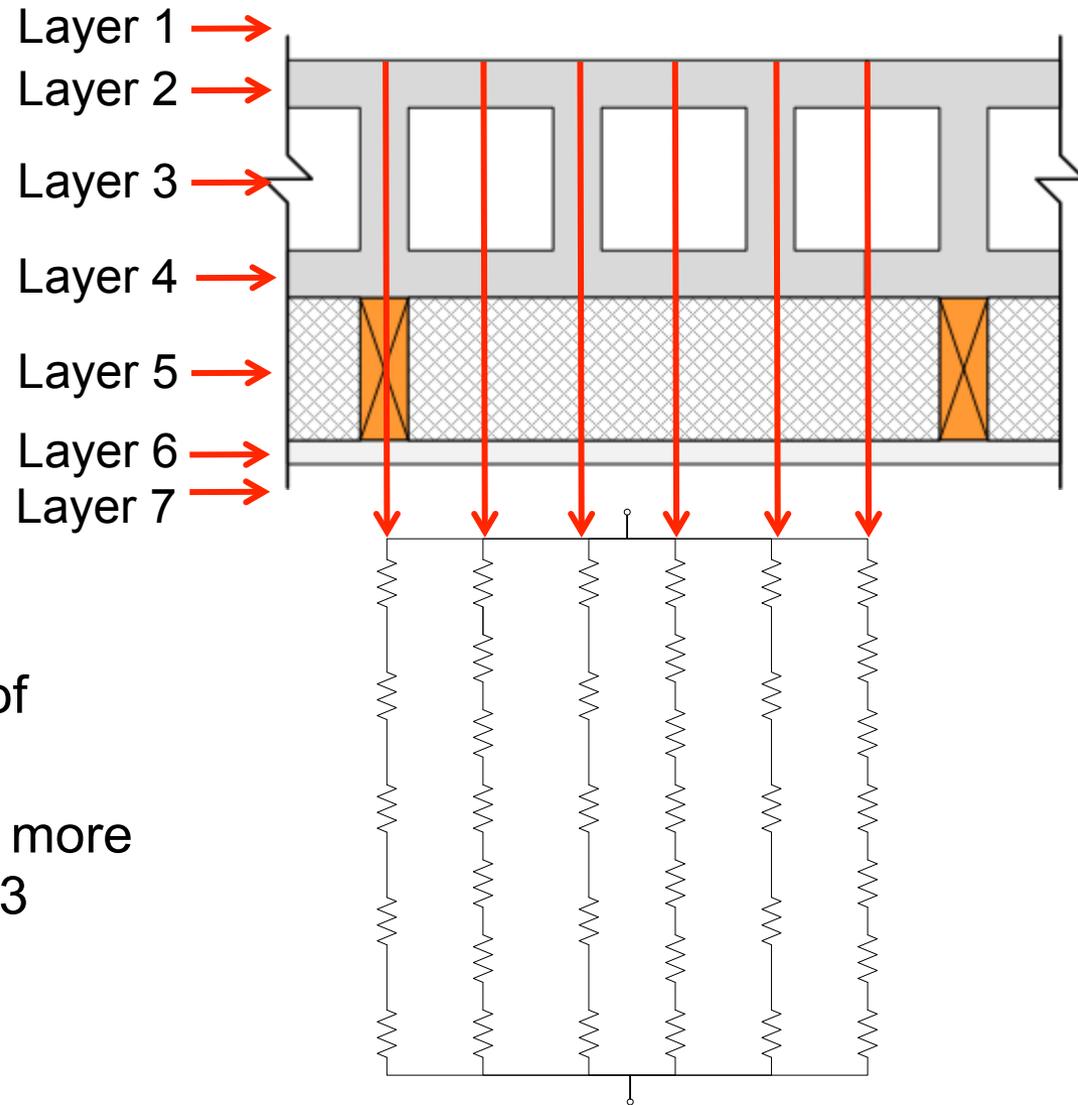
2. Identify the elements and 3. Find R-value for each element

- Layer 1 and 7 have 1 element (convection)
- Layer 2 and 4 have 1 element (1.5 inches of concrete)
- Layer 6 has 1 element (0.5 inches of gypsum wallboard)
- Layer 3 has 2 elements (3.5 inches air cavity and 3.5 inches concrete)
- Layer 5 has 2 elements (3.5 inches insulation and 3.5 inches wood stud)



Parallel Path method

- Identify the different paths and draw them in **parallel**

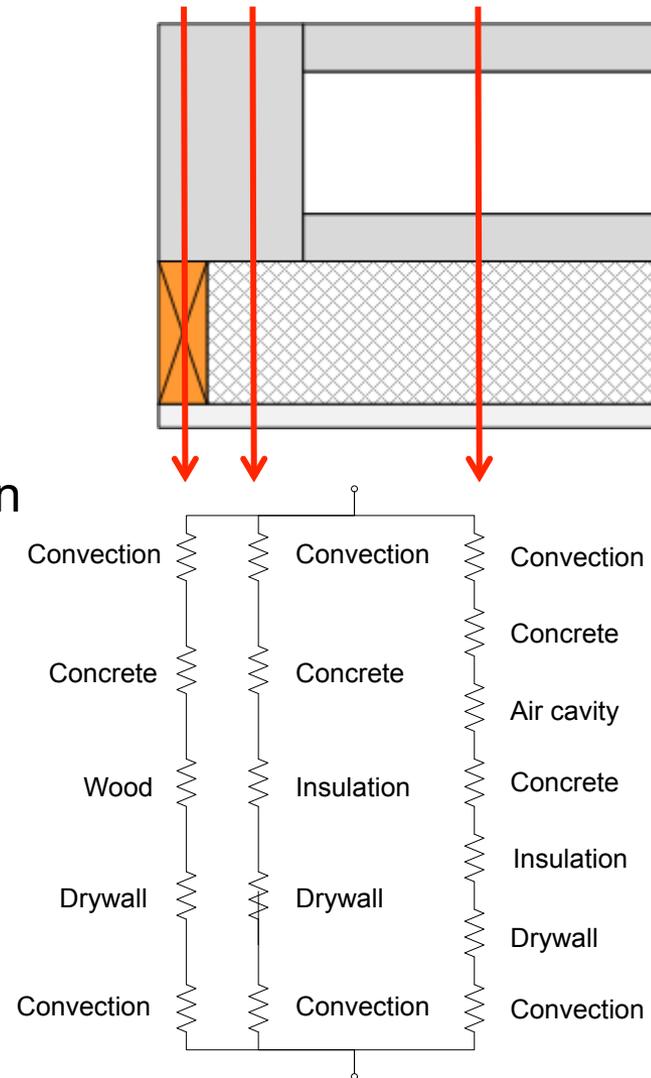


Note: In this example, several of these paths are identical

- So although there are many more components, there are only 3 unique ones

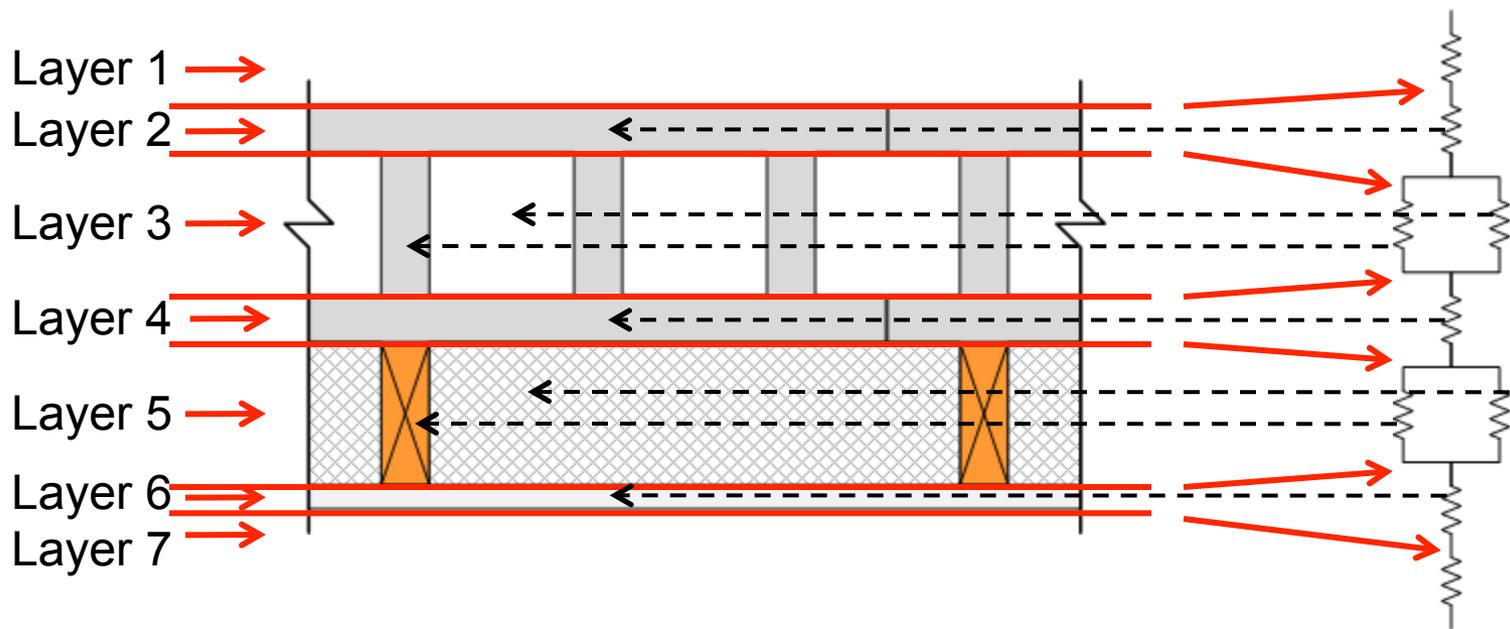
Alternate/Shortened Parallel Path

- We could combine like elements in each layer to reduce the number of paths to analyze without changing the answer for either isothermal or parallel path
 - This parallel network only has three paths, but will have the same temperatures at each interface location as the previous network
- Note the difference in the number of resistances between each path – varies according to # of elements/ layers involved in each path



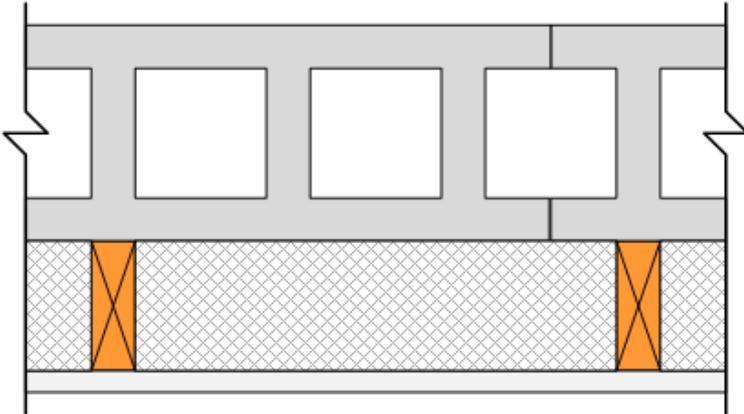
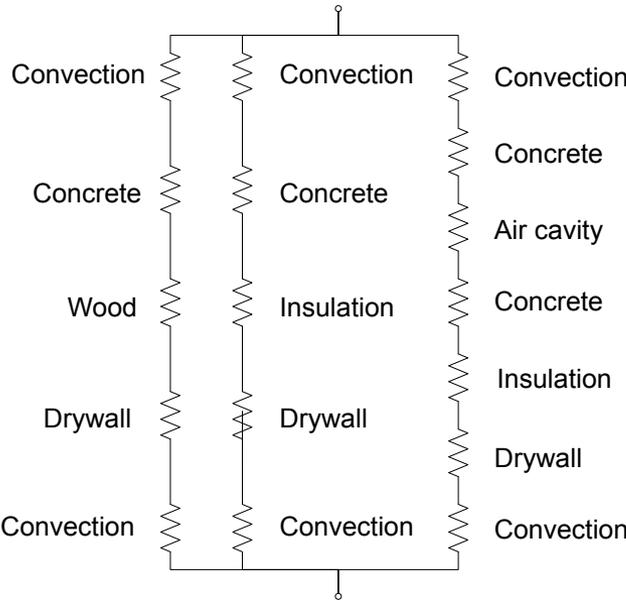
Isothermal method

- To apply the **isothermal** method, we put a node at each layer interface and add a resistor for each element in the layer:

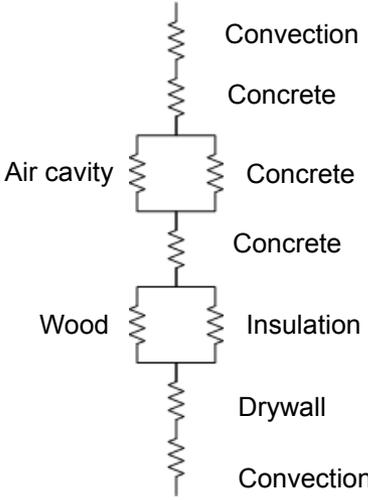


Isothermal vs. Parallel Path setup

Parallel Path



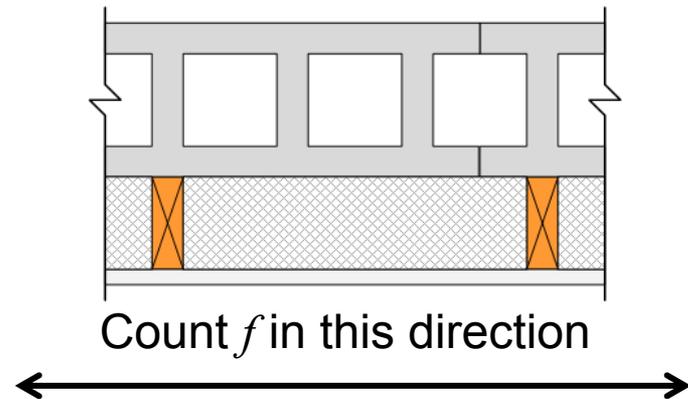
Isothermal



Assigning resistance values

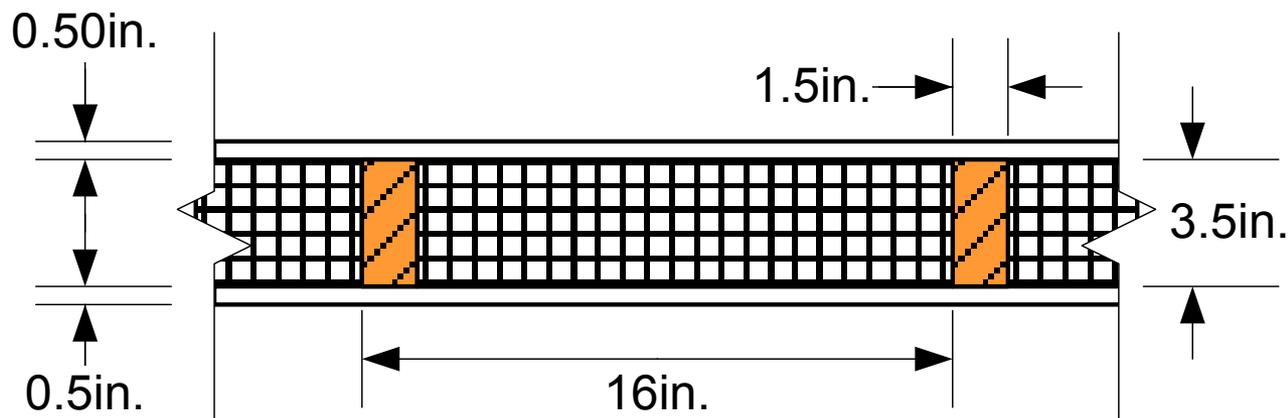
- The resistance value for the network elements will be the R-value of the element represented **divided by the fractional area (f)** of each element
 - Fractional area is the fraction of the entire cross section that the element takes up
 - This must be a number between 0 and 1

$$R_{element, network} = \frac{R_{element}}{f}$$



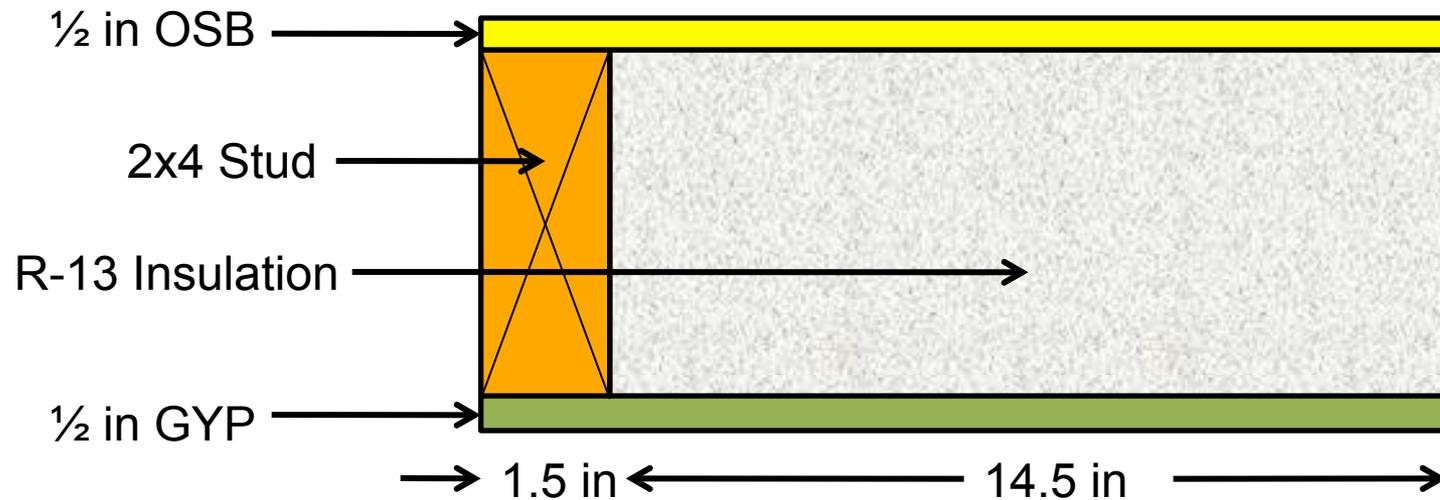
Example problem

- A wood frame wall has 2x4 studs spaced 16 inches OC (“on center”) with R-13 (IP) insulation in the cavity
- The interior wall is 0.5 inches of gypsum wallboard and the outside wall is 0.5 inches of OSB sheathing
- Draw the isothermal and parallel path networks for this wall
 - Assume winter outdoor conditions



Example problem continued

- Let's redraw the wall construction to make it easier to identify the layers
 - We can show only the unique part of the wall if we prefer
 - Note: This drawing is not to scale



Example problem continued

Identify the layers and elements:

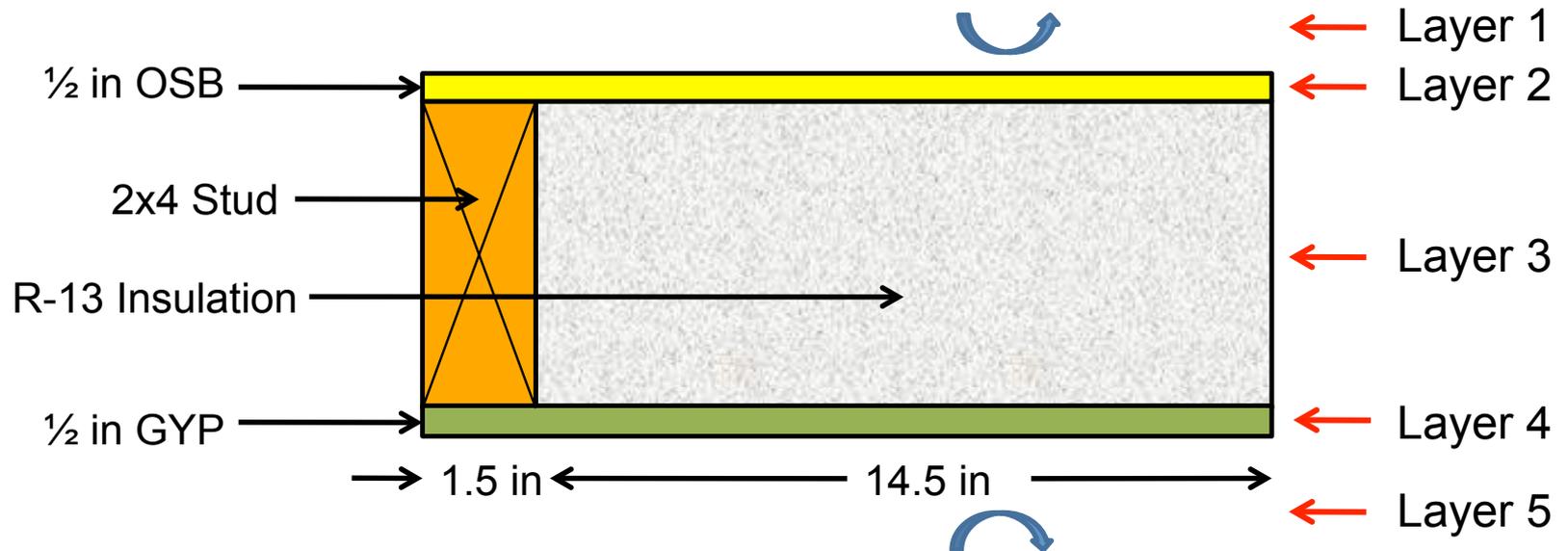
Layer 1: Exterior convection (winter conditions)

Layer 2: 0.5 inches of OSB

Layer 3: 3.5 inches of wood stud and 3.5 inches R-13 insulation

Layer 4: 0.5 inches of gypsum wallboard

Layer 5: Interior convection



Example problem continued

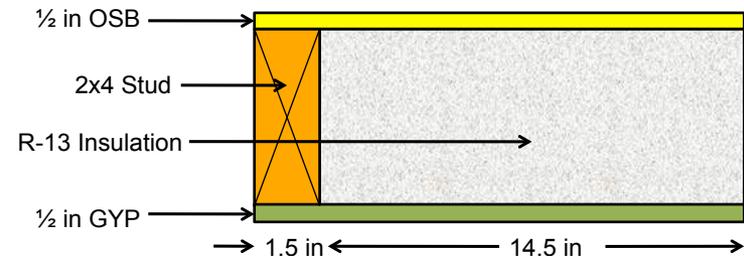
Look up R values for each element:

- Outdoor winter convection: $R_{ext} = 0.03 \text{ (m}^2\text{K)/W}$
- 1/2 in. OSB, $R_{osb} = 0.12 \text{ (m}^2\text{K)/W}$
- R-13 insulation: $R_{ins} = 2.29 \text{ (m}^2\text{K)/W}$
- 2x4 (3.5 in. thick) wood stud: $R_{2x4} = 0.96 \text{ (m}^2\text{K)/W}$
- 1/2 in. gypsum: $R_{gyp} = 0.079 \text{ (m}^2\text{K)/W}$
- Indoor convection: $R_{int} = 0.12 \text{ (m}^2\text{K)/W}$

Example problem continued

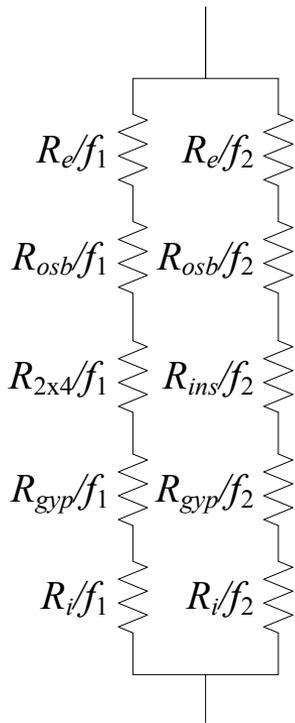
Find fractional areas of each element

- The full width of the assembly is 16 inches
- All elements are full height of the wall, so the fractional width = fractional area
- Layers 1, 2, 4 and 5 are all 16 inches
 - $f = 1.0$
- 2x4 stud is 1.5 inches
 - $f_1 = 1.5/16 = 0.094$
- R-13 insulation is 14.5 inches
 - $f_2 = 14.5/16 = 0.906$

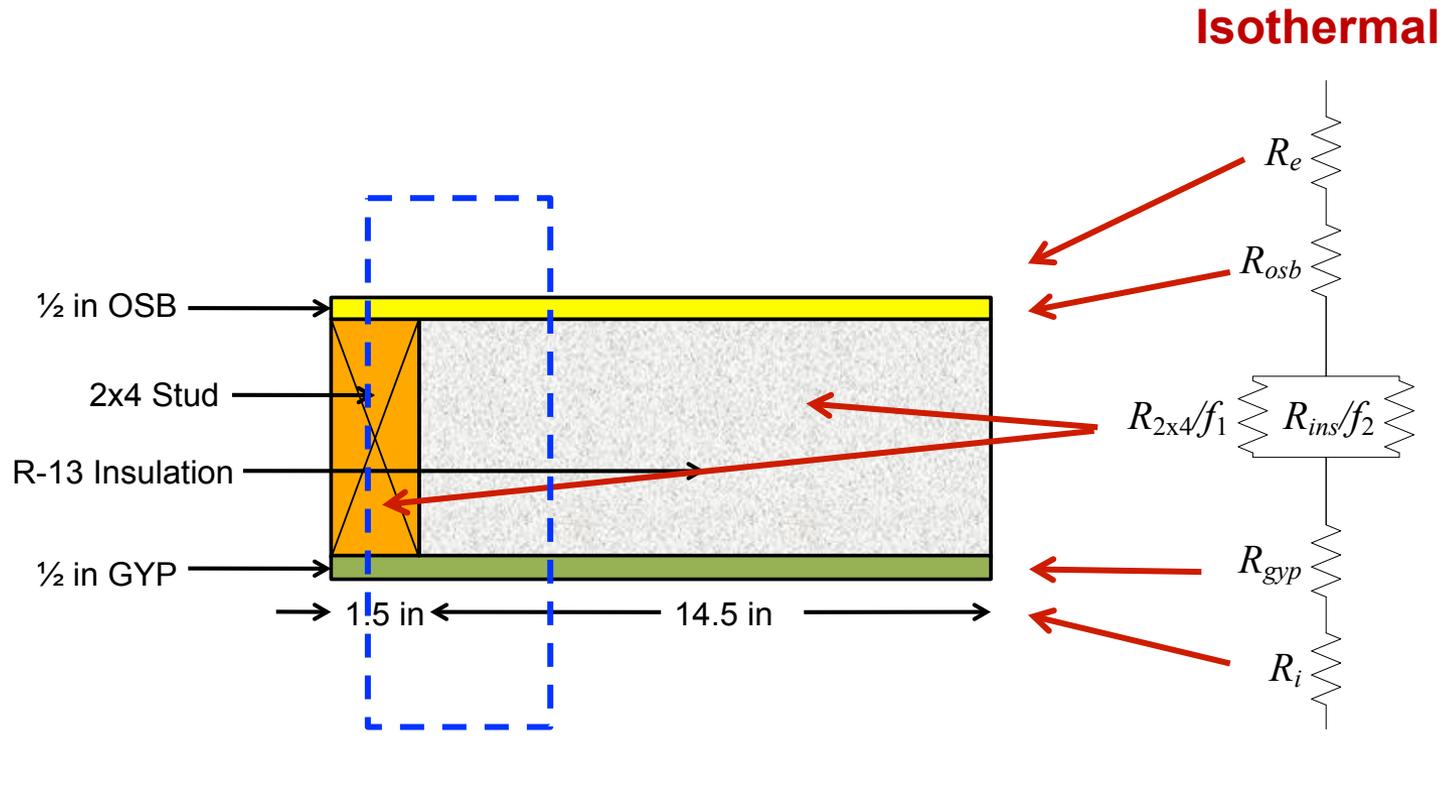


Example problem continued

- Draw the thermal networks



Parallel path



Example problem continued

Parallel path network resistor values:

$$\frac{R_{ext}}{f_1} = \frac{0.03}{0.094} = 0.32 \frac{m^2K}{W},$$

$$\frac{R_{int}}{f_1} = \frac{0.12}{0.094} = 1.27 \frac{m^2K}{W},$$

$$\frac{R_{osb}}{f_1} = \frac{0.12}{0.094} = 1.27 \frac{m^2K}{W},$$

$$\frac{R_{gyp}}{f_1} = \frac{0.08}{0.094} = 0.84 \frac{m^2K}{W},$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25 \frac{m^2K}{W},$$

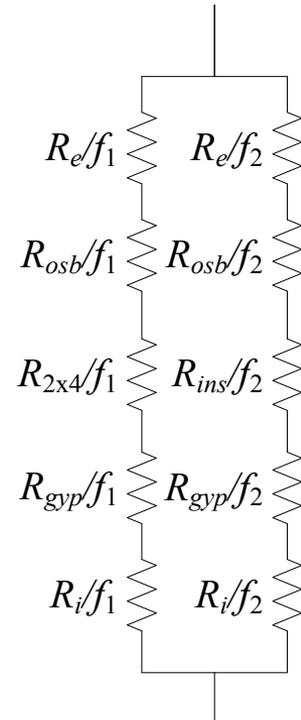
$$\frac{R_{ext}}{f_2} = \frac{0.03}{0.906} = 0.03 \frac{m^2K}{W}$$

$$\frac{R_{int}}{f_2} = \frac{0.12}{0.906} = 0.13 \frac{m^2K}{W}$$

$$\frac{R_{osb}}{f_2} = \frac{0.12}{0.906} = 0.13 \frac{m^2K}{W}$$

$$\frac{R_{gyp}}{f_2} = \frac{0.08}{0.906} = 0.09 \frac{m^2K}{W}$$

$$\frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53 \frac{m^2K}{W}$$



Example problem continued

Parallel path network resistor values

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 = R_{ext}/f_1 + R_{osb}/f_1 + R_{gyp}/f_1 + R_{int}/f_1 + R_{2x4}/f_1$$

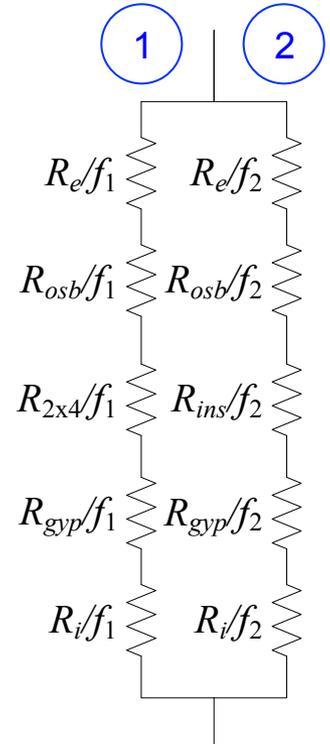
$$R_2 = R_{ext}/f_2 + R_{osb}/f_2 + R_{gyp}/f_2 + R_{int}/f_2 + R_{insulation}/f_2$$

$$R_1 = 0.32 + 1.27 + 10.25 + 1.27 + 0.84 = 13.96$$

$$R_2 = 0.03 + 0.13 + 2.53 + 0.13 + 0.09 = 2.91$$

$$\frac{1}{R_{total}} = \frac{1}{13.96} + \frac{1}{2.91} = 0.415 \frac{W}{m^2K}$$

$$R_{total} = 2.41 \frac{m^2K}{W} = R-13.68 \text{ (IP)}$$



Example problem continued

We can now find the network resistor values for the **isothermal** network:

$$R_{ext} = \frac{0.03}{1.0} = 0.03 \frac{m^2K}{W}$$

$$R_{int} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W}$$

$$R_{osb} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W}$$

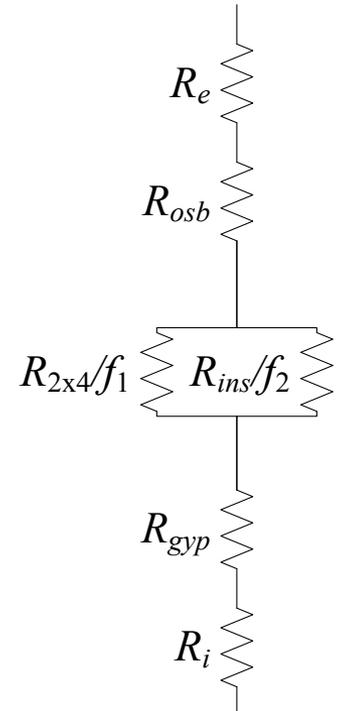
$$R_{gyp} = \frac{0.08}{1.0} = 0.08 \frac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25 \frac{m^2K}{W}$$

$$\frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53 \frac{m^2K}{W}$$

$$R_{total} = R_e + R_{osb} + \frac{1}{\frac{1}{R_{2x4}/f_1} + \frac{1}{R_{insulation}/f_2}} + R_{gyp} + R_i$$

$$R_{total} = 0.03 + 0.12 + \frac{1}{1/10.25 + 1/2.53} + 0.08 + 0.12 = 2.38 \frac{m^2K}{W} = R-13.49 \text{ (IP)}$$



Utility of thermal networks

By developing the full thermal network and then combining elements, we can better see the heat transfer paths

In particular:

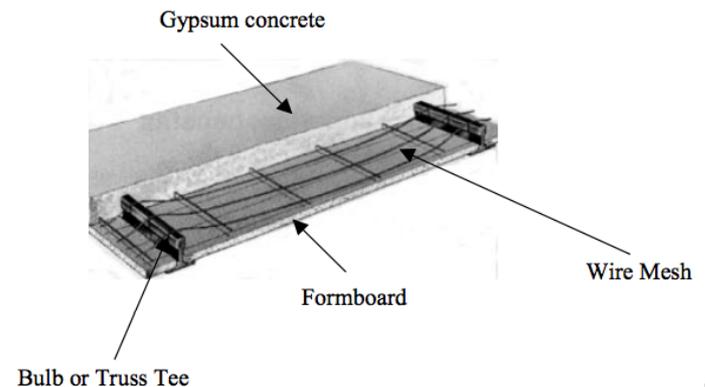
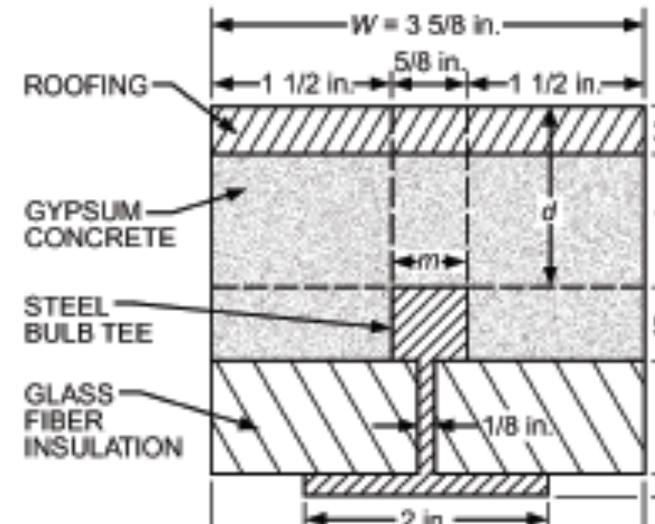
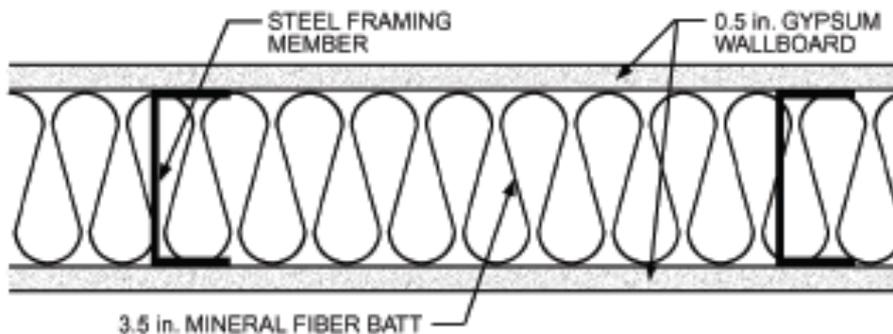
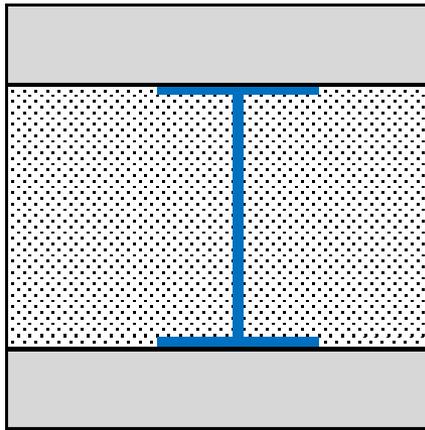
- We can identify thermal bridges more easily
 - Areas of particularly low resistance
- We can identify the relative contribution of the elements to heat transfer more easily
- We can use nodal analysis techniques to find the temperature everywhere quickly and easily

Limitations to these methods

- Previous methods do not work well when the assembly has **metal thermal bridges**
 - A thermal bridge is an element of high thermal conductivity that spans through enclosure layers
 - For example:
 - Embedded steel beams can create thermal bridges
- Any flanges on metal beams draw heat into metal areas that cross through layers
 - Increases heat transfer
 - Often odd geometries and narrow dimensions
- We need more accurate methods of estimating U-values

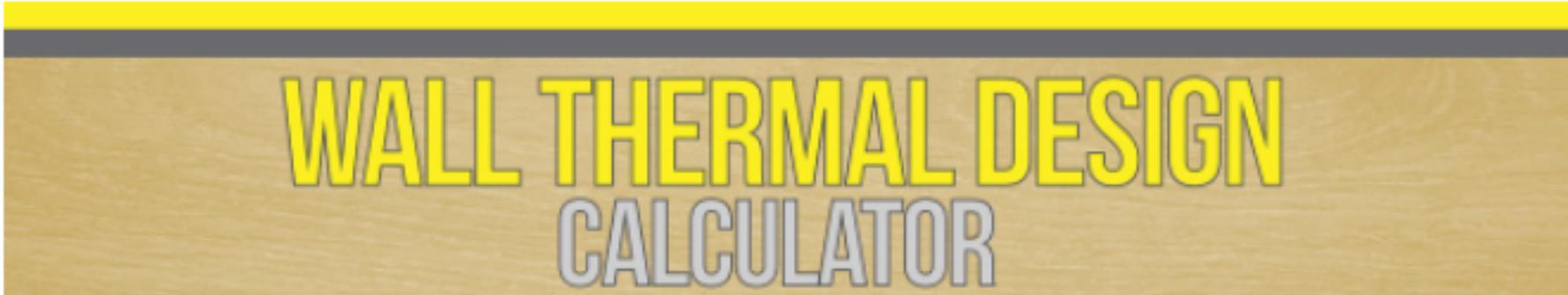
Metal thermal bridge examples

- I-beams sandwiched between sheathing
- Metal stud walls
- Metal embedded in concrete



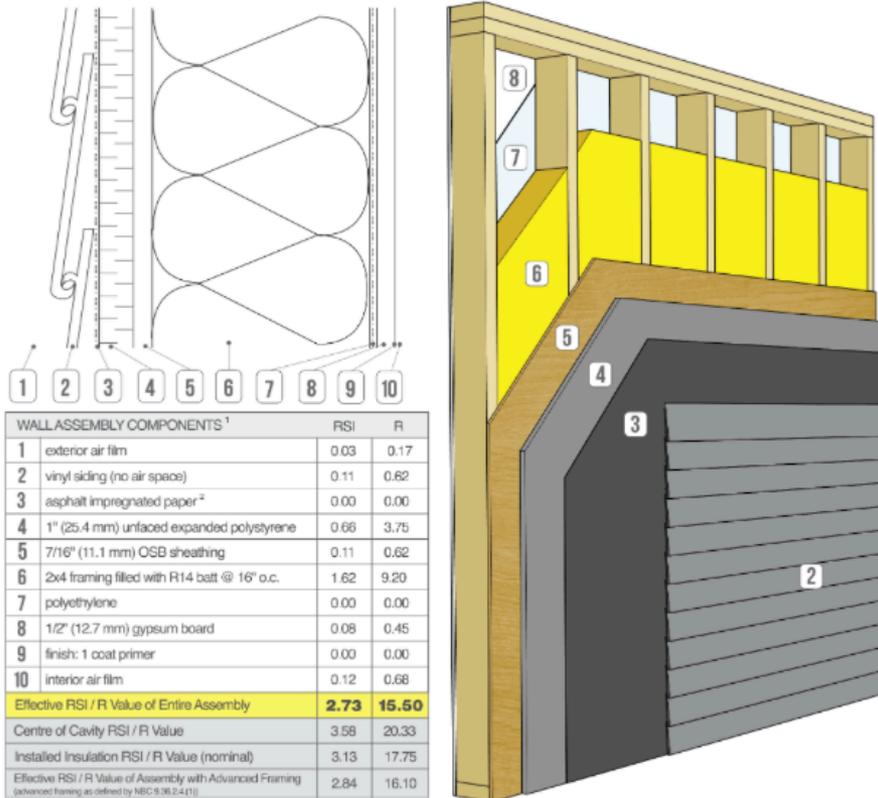
Online tools to help us out with thermal bridges

- There are some online “modified zone” calculators developed by Oakridge National Laboratory
 - <http://www.ornl.gov/sci/roofs+walls/calculators/modzone/index.html>
 - <http://www.ornl.gov/sci/roofs+walls/calculators/modzone/modzone2.html>
- Building Science Corp:
 - <http://www.buildingscience.com/documents/information-sheets/high-r-value-wall-assemblies/?topic=/doctype/information-sheets>
- Canadian Wood Council:
 - <http://www.cwc.ca/index.php/en/resources/wallthermaldesignmainpage>

A banner with a yellow top bar and a light wood-grain background. The text "WALL THERMAL DESIGN" is in large, bold, yellow, sans-serif font, and "CALCULATOR" is in a smaller, bold, grey, sans-serif font below it.

WALL THERMAL DESIGN
CALCULATOR

Wall thermal design calculator



Note: ¹Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. ²Sheathing membrane materials must comply with CAN/CSG-91.30, "Sheathing Membrane, Gypsum Type".

LEGEND Pass Proceed with caution

SIMULATED DURABILITY ANALYSIS

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WIFI HYDROTHERMAL MODELING	<input checked="" type="checkbox"/>				

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 50 ng/(Pa·s·m²) must comply to NBC 9.25.5.2.

15.5
R_{eff}



Note: ¹Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. ²The thermal resistance of mortar was not considered. ³Sheathing membrane material must comply with CAN/CSG-91.30, "Sheathing Membrane, Gypsum Type".

LEGEND Pass Proceed with caution Check permeance of material

SIMULATED DURABILITY ANALYSIS

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WIFI HYDROTHERMAL MODELING	<input checked="" type="checkbox"/>				
OUTBOARD TO INBOARD RATIO COMPLIANCE	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.3	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.2

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 50 ng/(Pa·s·m²) must comply to NBC 9.25.5.2.

OUTBOARD TO INBOARD RATIO **0.39**

33.0
R_{eff}

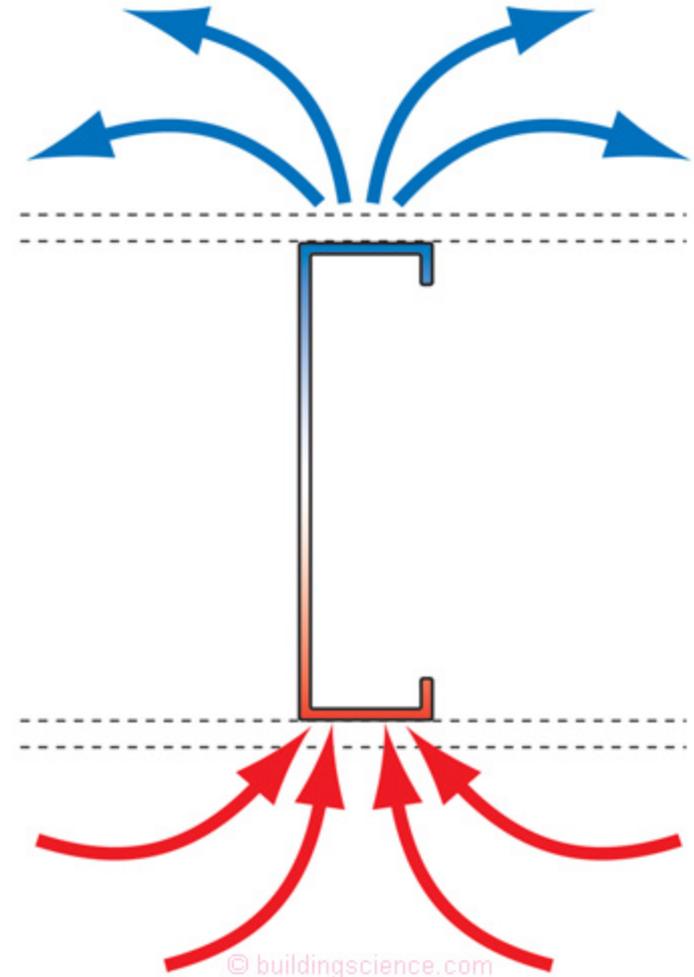
THERMAL BRIDGES

Examples

Even more complex (and more accurate) analysis

Thermal bridges

- A thermal bridge is a high conductivity portion of an assembly that penetrates insulating layers and significantly reduces the overall R value of the assembly
- These bridges act as thermal “short circuits”



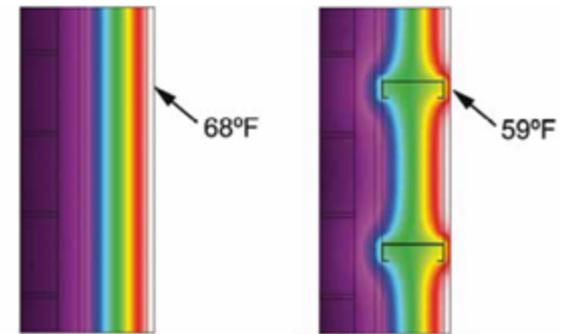
A note on materials and k , U , and R values

- We've already looked up material properties
 - Many are listed in ASHRAE HOF
 - It's important to have a good working knowledge of these values for common building materials (particularly **high conductivity** materials)
 - Pay special attention to **metals**, in addition to those values we already know
 - Thermal conductivity (k) of common materials
 - Insulation <0.05 W/mK
 - Wood ~0.1-0.2 W/mK
 - Rubber ~0.1 W/mK
 - Brick/stone/concrete ~0.5-2.0 W/mK
 - **Steel ~45 W/mK**
 - **Cast iron ~50 W/mK**
 - **Aluminum ~220 W/mK**

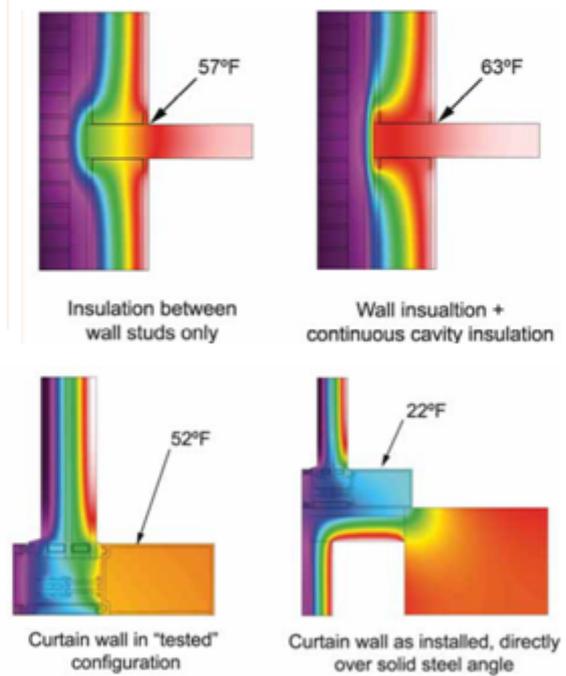
↓
Increasing conductivity
Decreasing resistance
(also depends on thickness)

Common thermal bridges

- Metal studs in insulated walls without external insulation
- Floor planks without continuous external insulation
- Curtain walls in contact with steel framing



Thermal Bridging through Light Gauge Steel Studs



Thermal Bridging at Curtain Wall Connection

Identifying thermal bridges

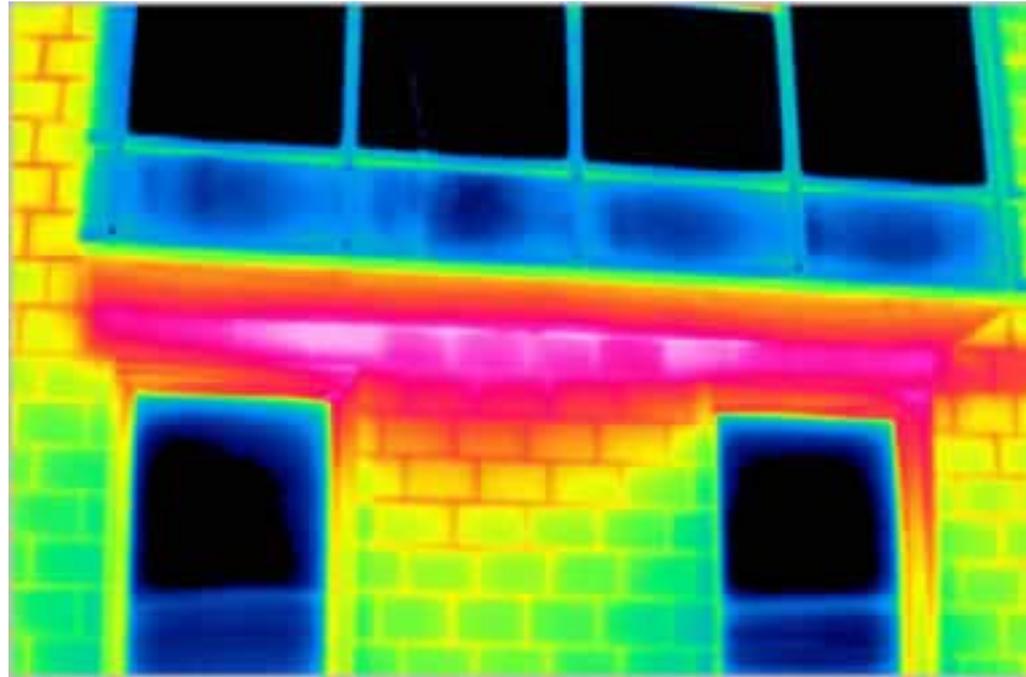
We can identify thermal bridges in many ways

- Inspection of detail drawings
 - Use this to identify potential problems
- Thermal analysis of detail drawings
 - Use to quantify the extent of thermal bridging
- Thermal imaging
 - Useful way to find bridges in existing construction
- Visually without tools

Thermal imaging

- One way to find existing thermal bridges in buildings is to use thermal imaging
 - Particularly when the inside-outside temperature difference is large
- Regions around thermal bridges will have very different temperatures than the rest of the wall
 - This will be visible to IR

$$q = \varepsilon T^4$$

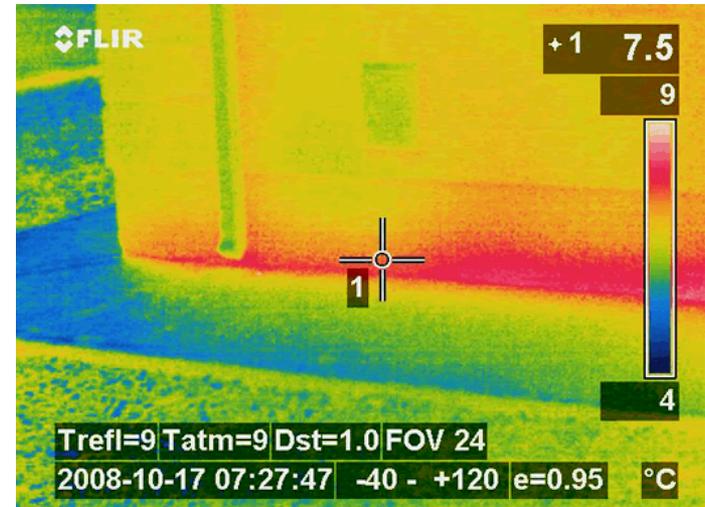
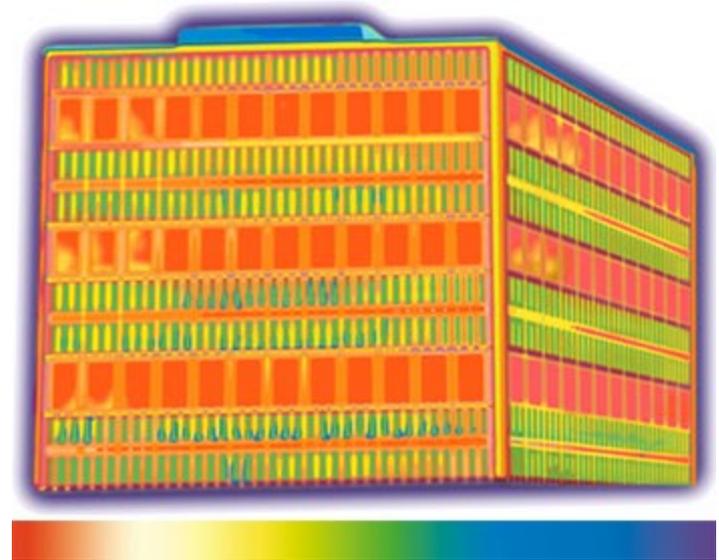


The red here indicates a thermal bridge created by a structural beam below the window

Cold outside, hot inside

More thermal imaging

- These figures show hot spots in red
- The windows, frames, and locations where the floor meets the walls are clearly visible bridges in the top figure
- The floor/foundation junction in the bottom has clear thermal bridges

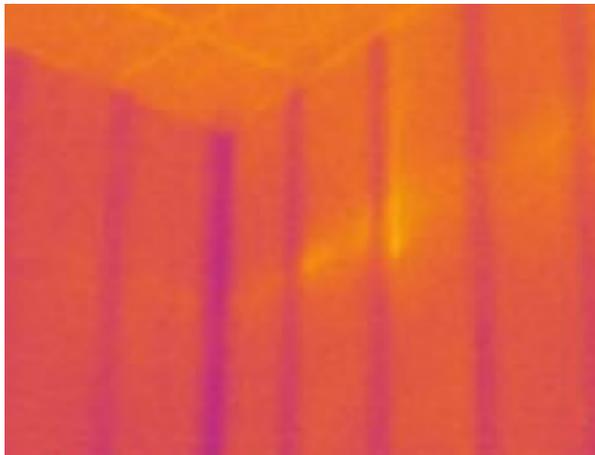
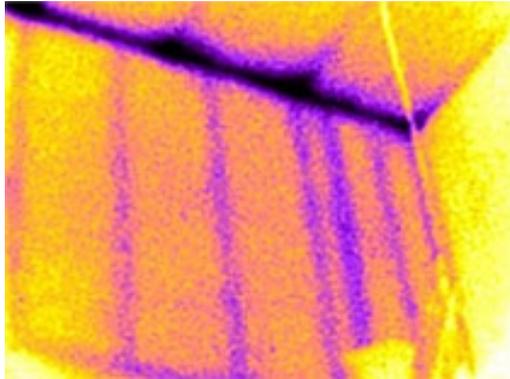


Framing effect

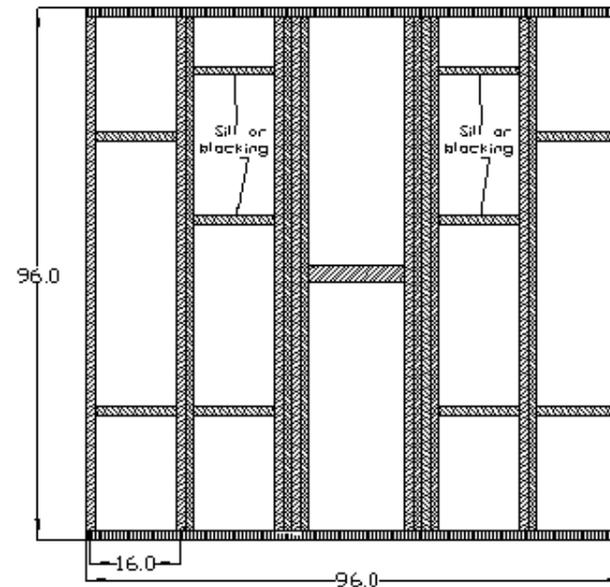
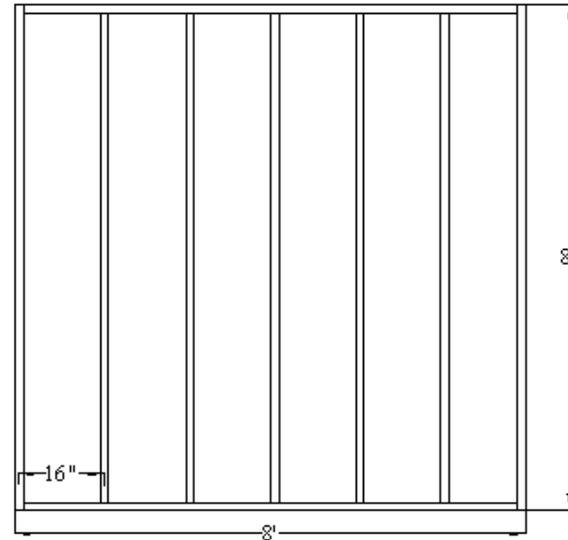


- The “framing effect” is a term used to denote the reduction in R-value from thermal bridging of the framing
 - Steel stud walls commonly have a framing effect of 40-50%
- A reduction in the framing effect means an increase in the effective R-value of a wall

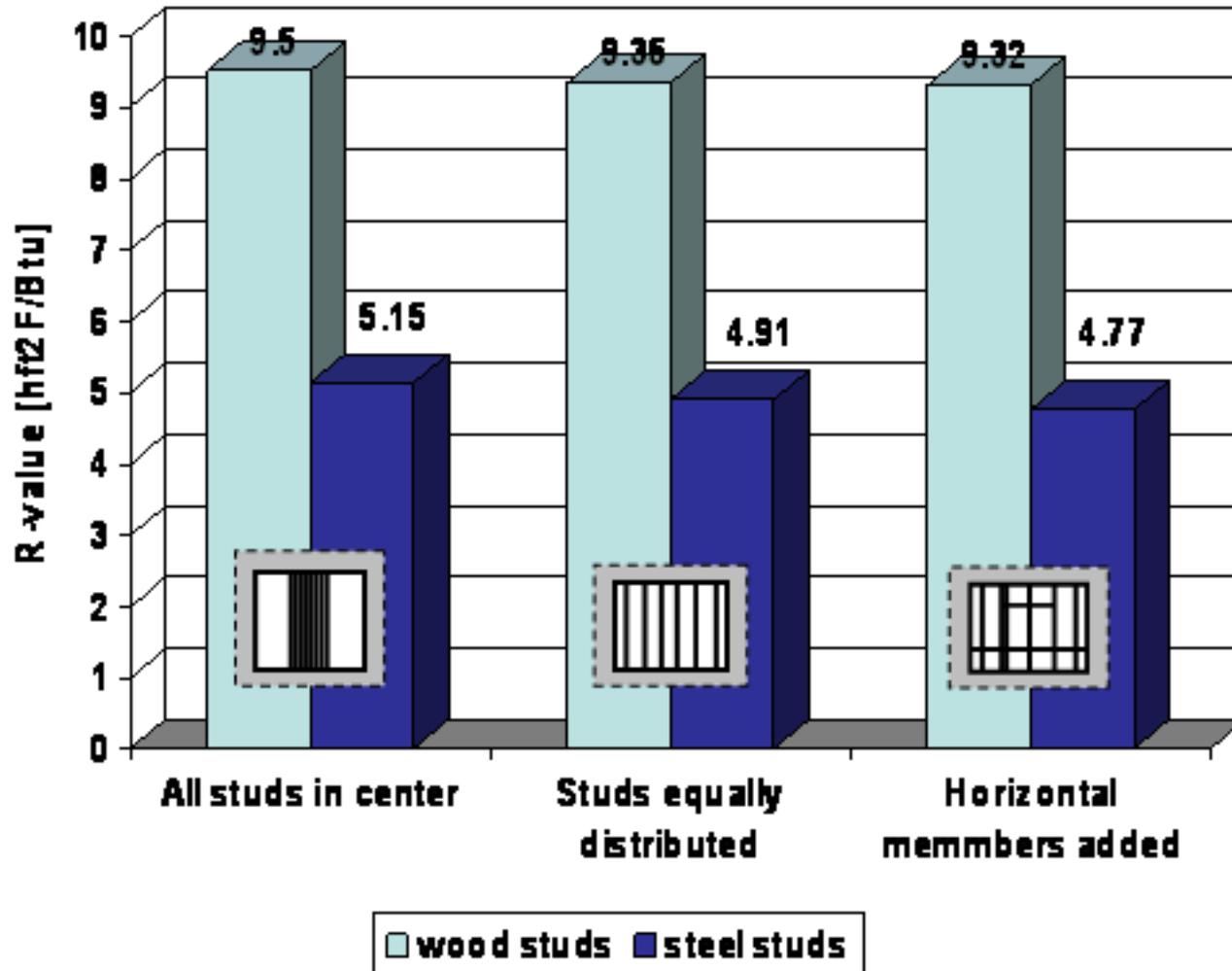
Framing effect



Thermal bridges show up in blue on these figures. Notice that corners seem to have the worst thermal bridges



Framing effect



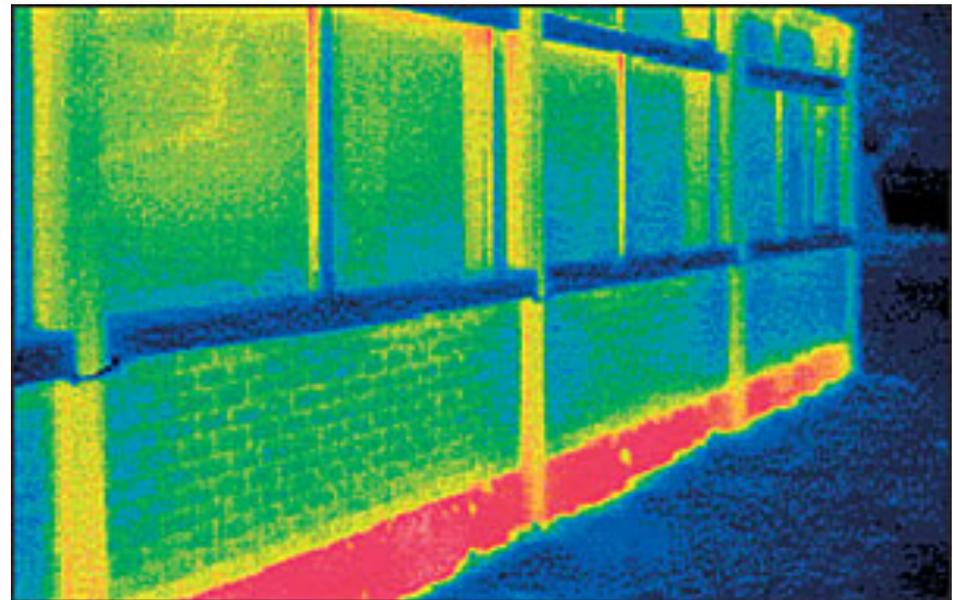
Framing effect



Thermal bridging in the uninsulated roof framed with steel (R-value of around 0.04-IP per inch) means bare roof at the rafter lines

Foundation thermal bridges

Here are some thermal images showing thermal bridges where a slab floor or foundation meets a wall



Wall-floor thermal bridges

Here are some examples of thermal bridges where interior floors and supports meet with exterior walls



Thermal bridge for support under the window

Thermal bridge from lintel over the door



Reducing thermal bridging

Basic concept:

- To reduce thermal bridging we need to **provide a thermal break** in the high-conductivity heat transfer path

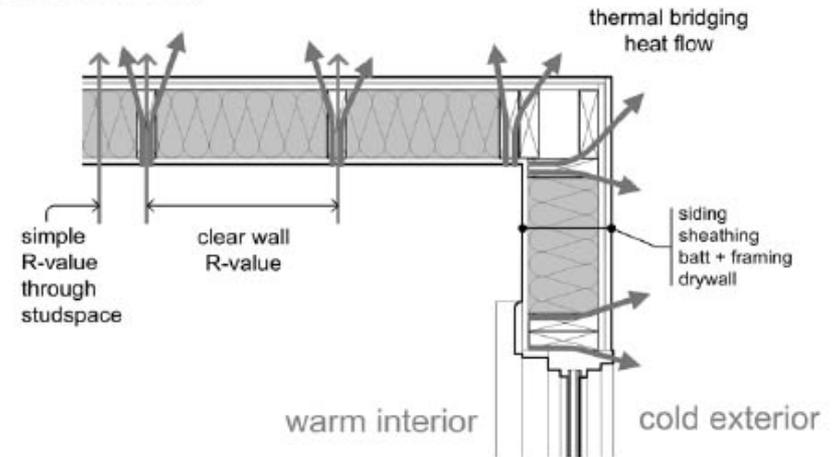
Basic solution:

- Add insulation to the path to reduce direct heat flow

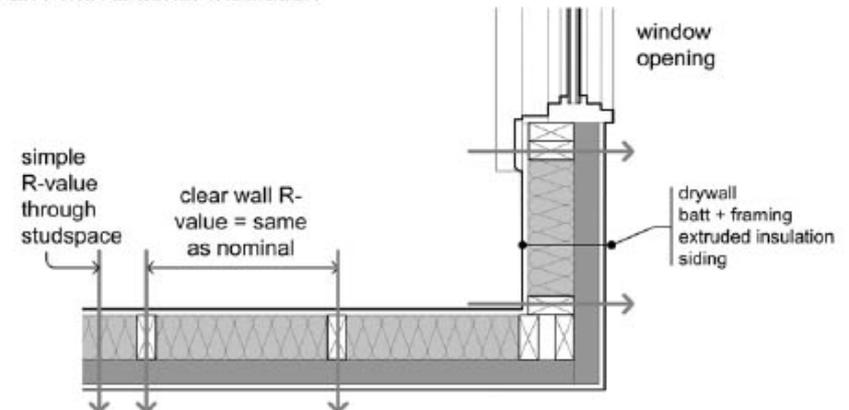
An important solution: **External insulation**

- The use of external insulation is a very effective way of reducing the bridging effects
 - It also increases the R value of the non-bridge areas
 - A 2x4 wall with 2 inches external insulation is much better than a 2x6 wall with 6 inches interior insulation
 - Need a continuous (or near continuous) layer of external insulation to break the path of high-conductivity material
 - Add to the exterior to ensure as much continuity as possible

2x6 Framed Wall

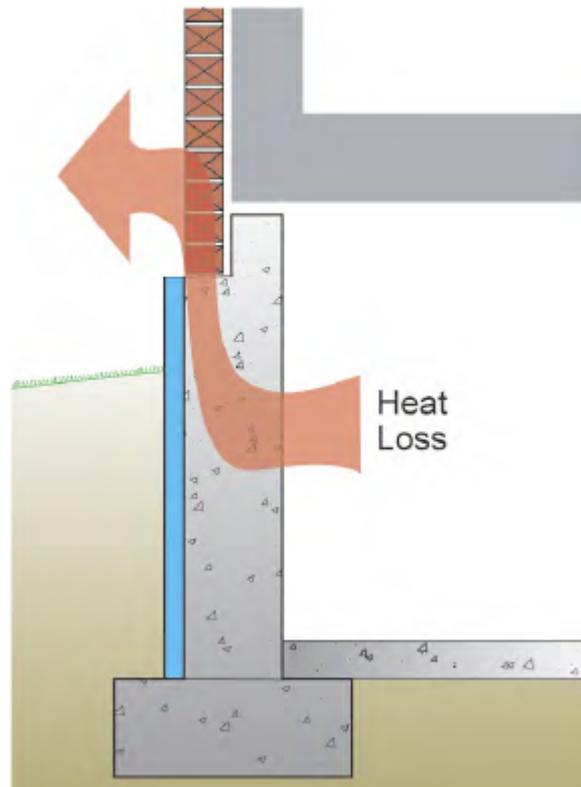


2x4 with Exterior Insulation



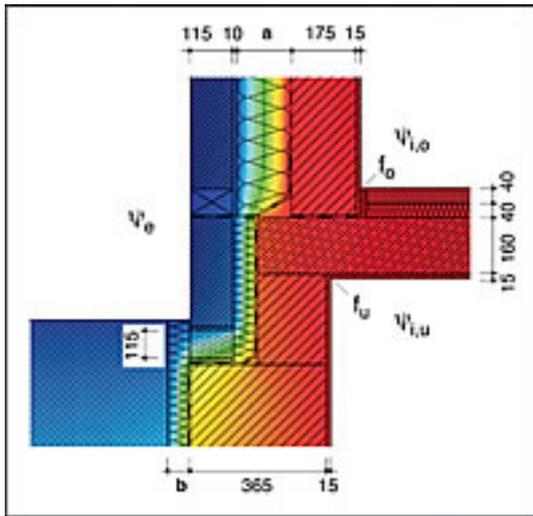
Foundation thermal bridges

Thermal bridges between brick veneer and foundation are common

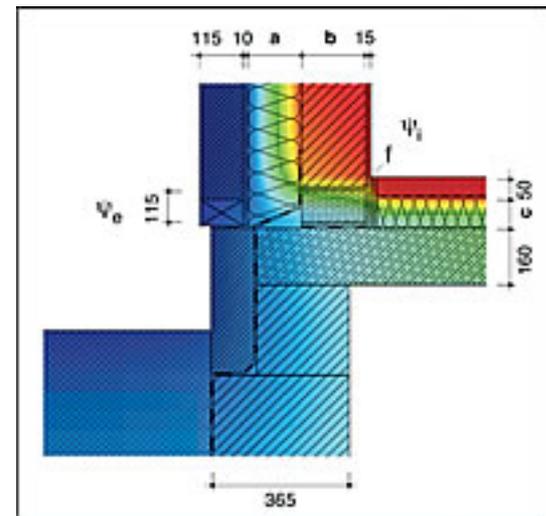


Fixing wall-floor bridges

To correct this problem we need to break the heat flow path:



External insulation added down along the foundation breaks the thermal bridge



Insulation added to the floor also breaks the thermal bridge

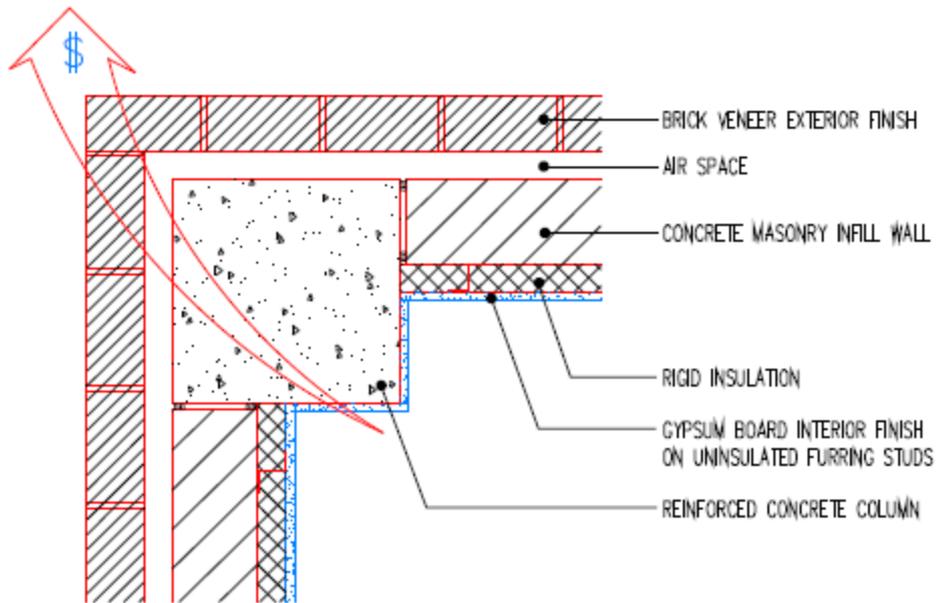
In both of these two designs the continuous insulation breaks the thermal bridge

Corner columns

PROBLEM

THERMAL BRIDGE:

AT CORNER
THERMAL MASS OUTSIDE OF
INSULATION.



Corner columns

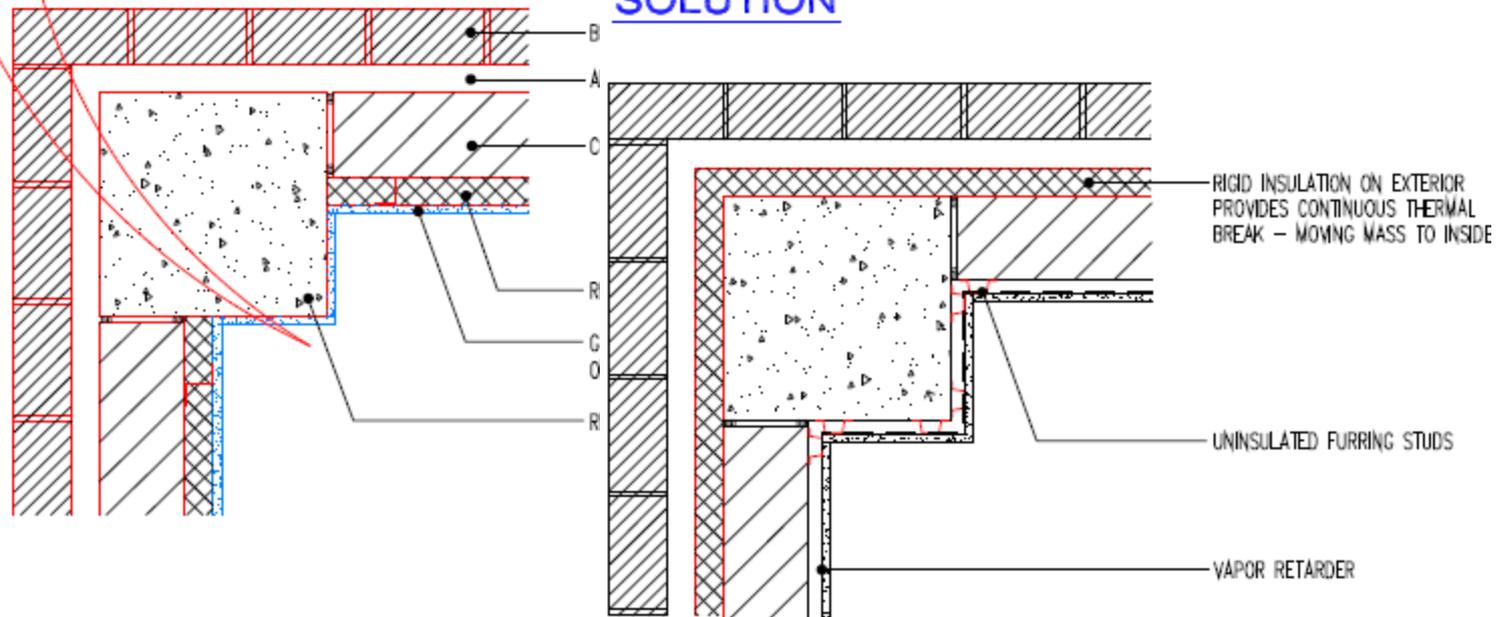
PROBLEM

THERMAL BRIDGE:

AT CORNER
THERMAL MASS OUTSIDE OF
INSULATION.



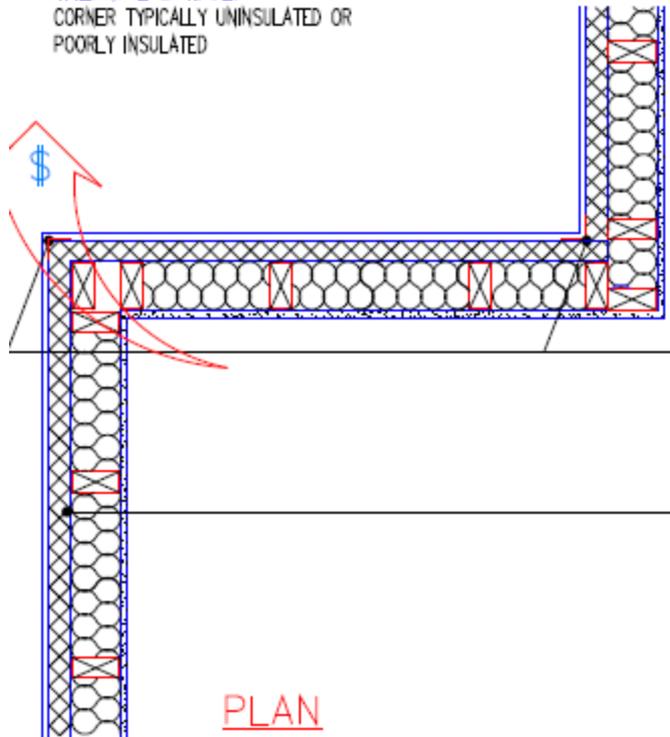
SOLUTION



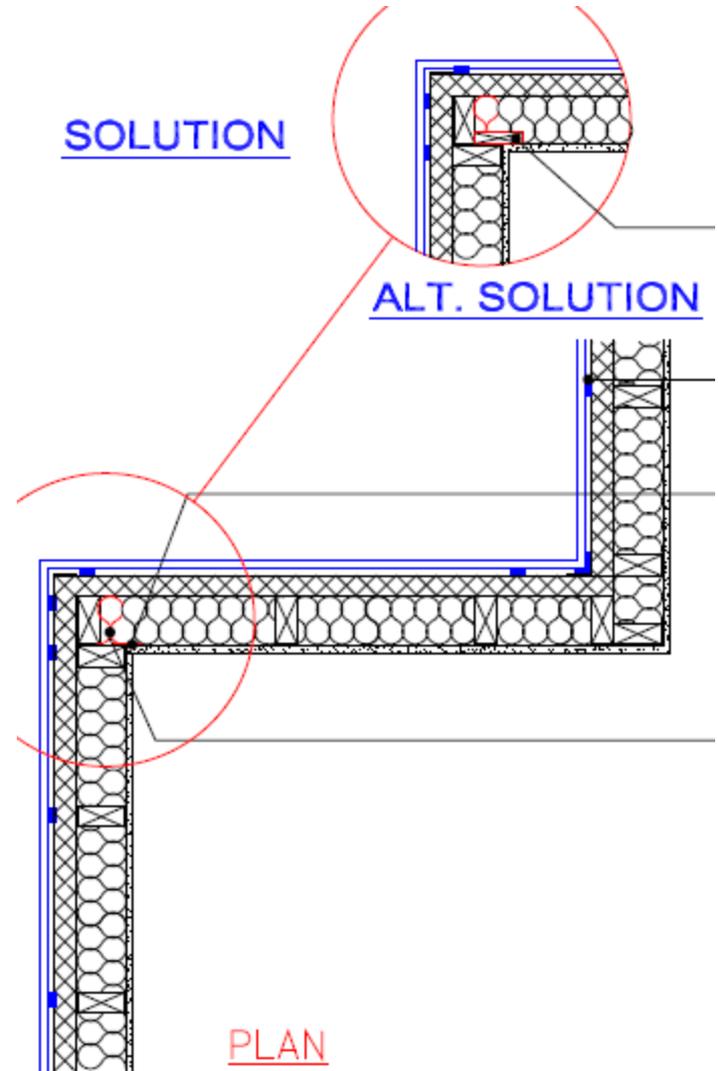
Wood stud corners

PROBLEM

THERMAL BRIDGE:
CORNER TYPICALLY UNINSULATED OR
POORLY INSULATED



SOLUTION

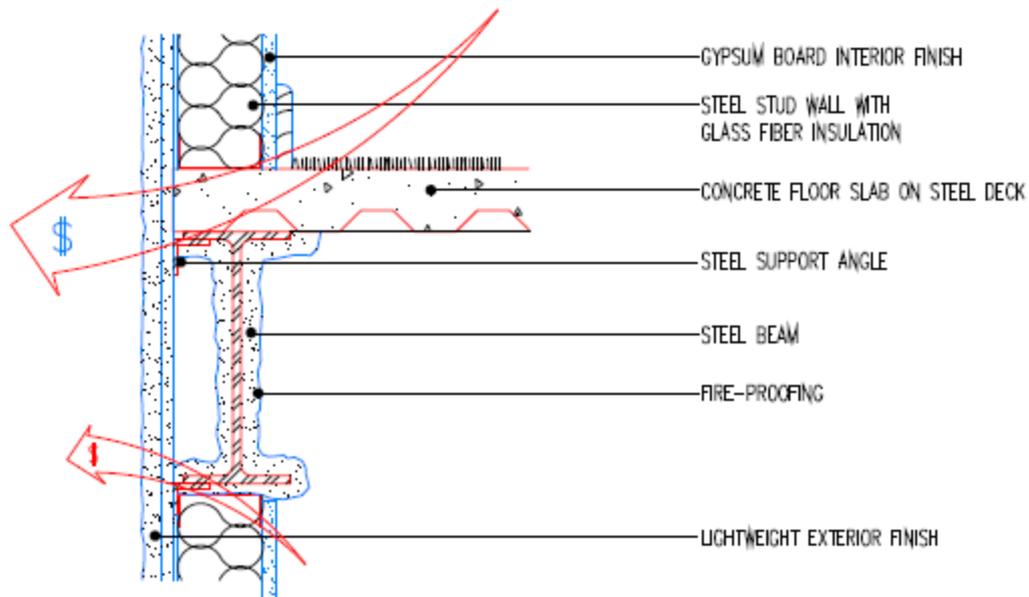


Slab edge stud wall

PROBLEM

THERMAL BRIDGE:

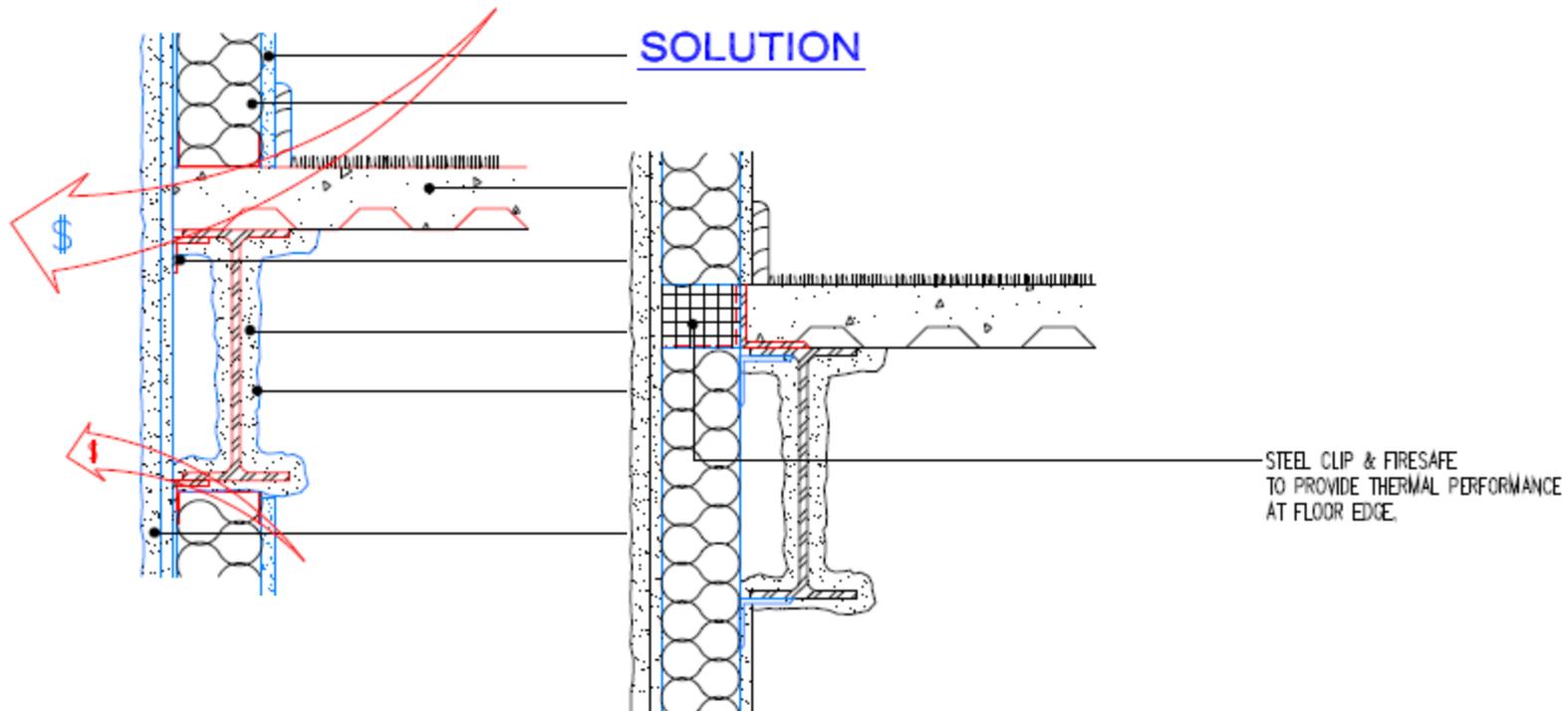
AT EDGE OF CONCRETE FLOOR SLAB.



Slab edge stud wall

PROBLEM

THERMAL BRIDGE:
AT EDGE OF CONCRETE FLOOR
SLAB.

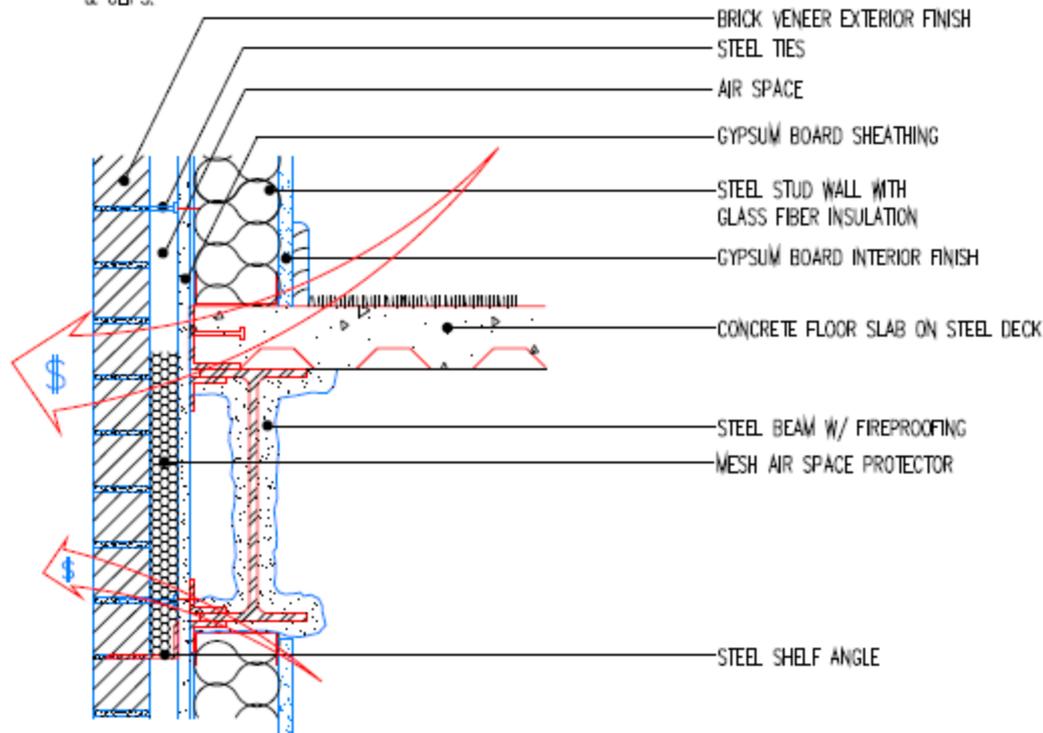


Slab edge brick/stud wall

PROBLEM

THERMAL BRIDGE:

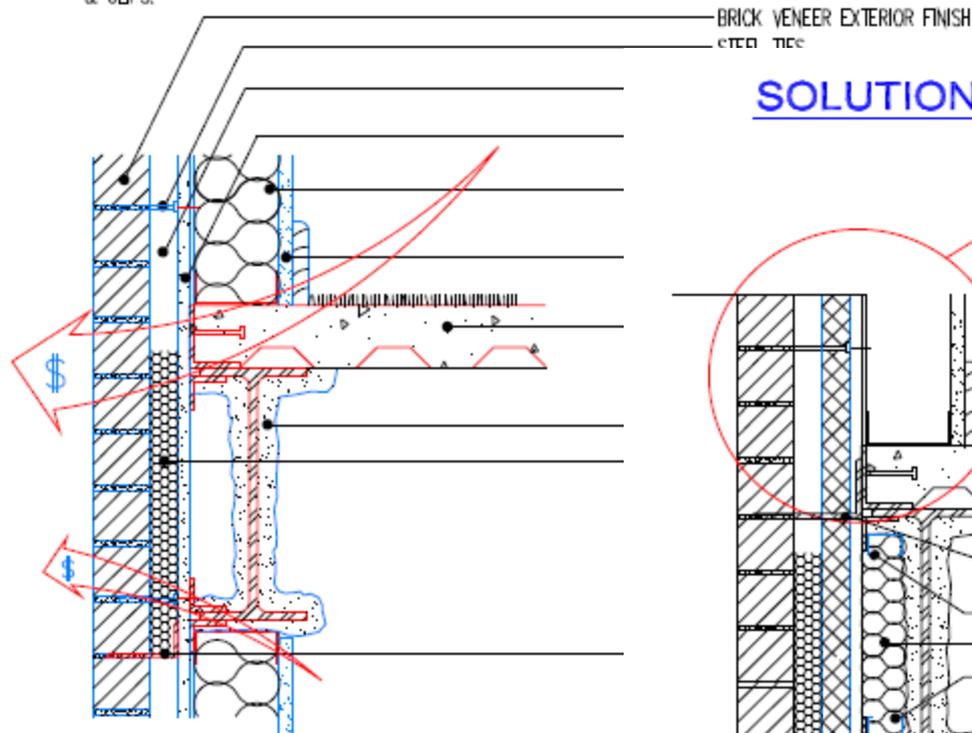
AT EDGE OF CONCRETE FLOOR
& CLIPS.



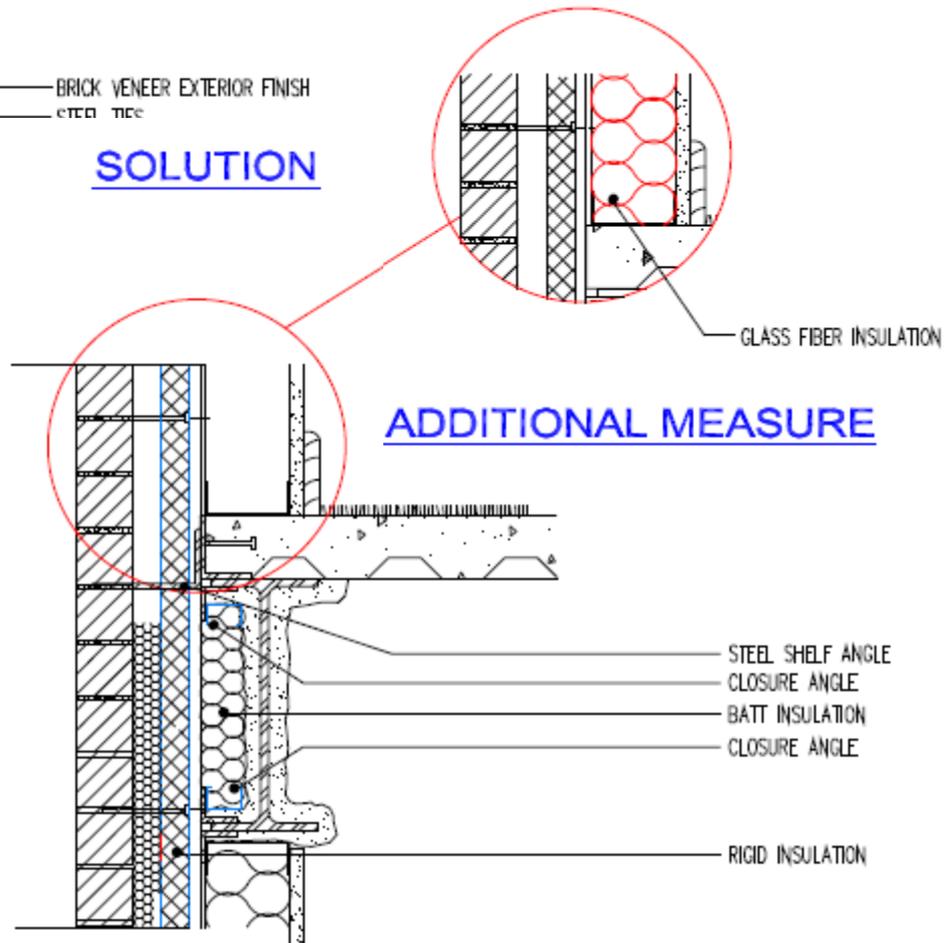
Slab edge brick/stud wall

PROBLEM

THERMAL BRIDGE:
AT EDGE OF CONCRETE FLOOR
& CLIPS.



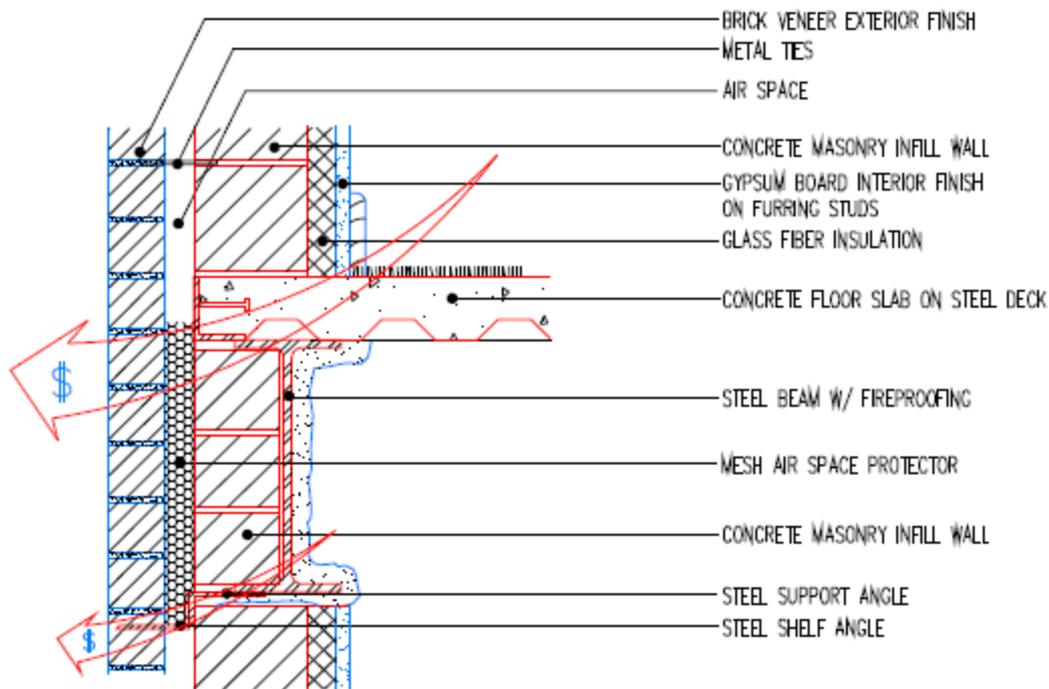
SOLUTION



Slab edge masonry wall

PROBLEM

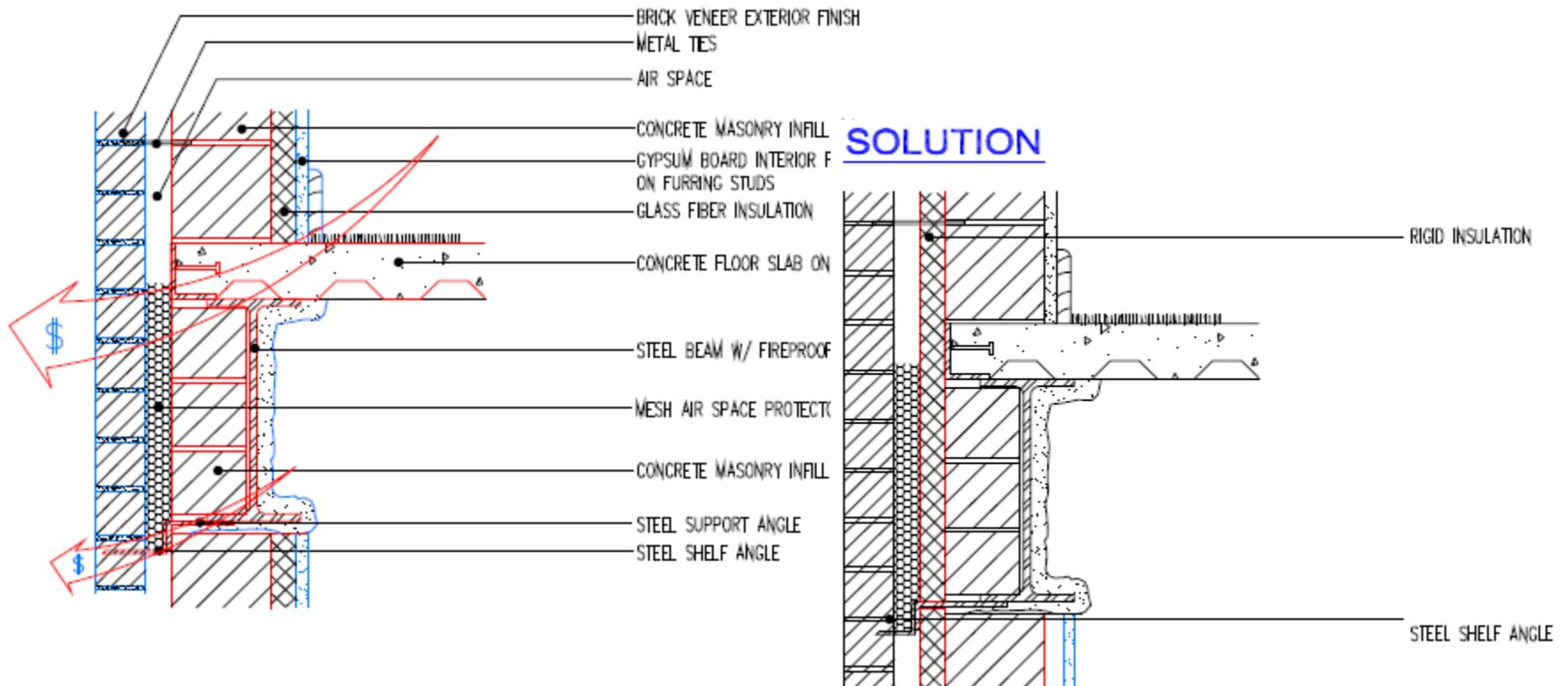
THERMAL BRIDGE:
AT FLOOR SLAB EDGE &
AND MASONRY INFILL



Slab edge masonry wall

PROBLEM

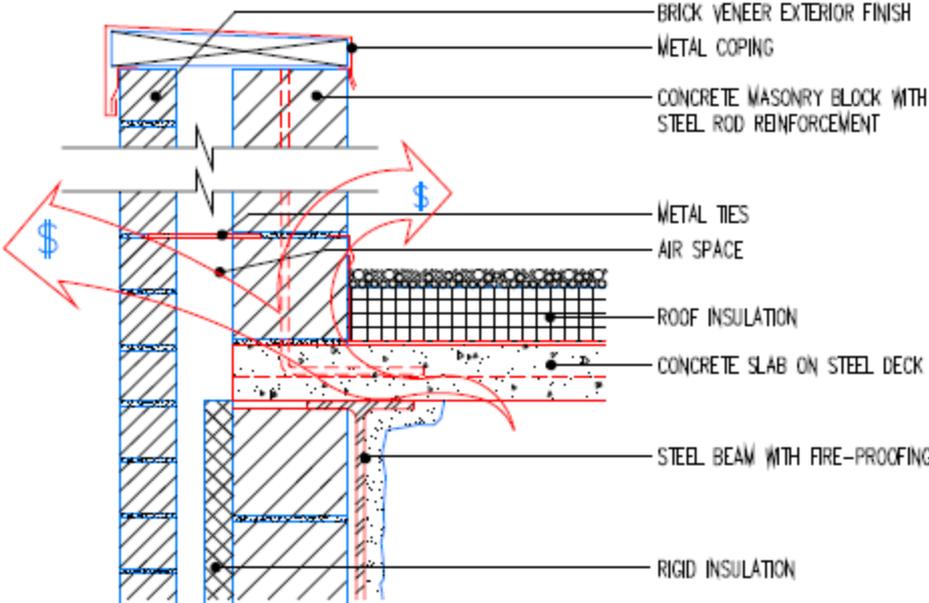
THERMAL BRIDGE:
AT FLOOR SLAB EDGE &
AND MASONRY INFILL



Roof parapet

PROBLEM

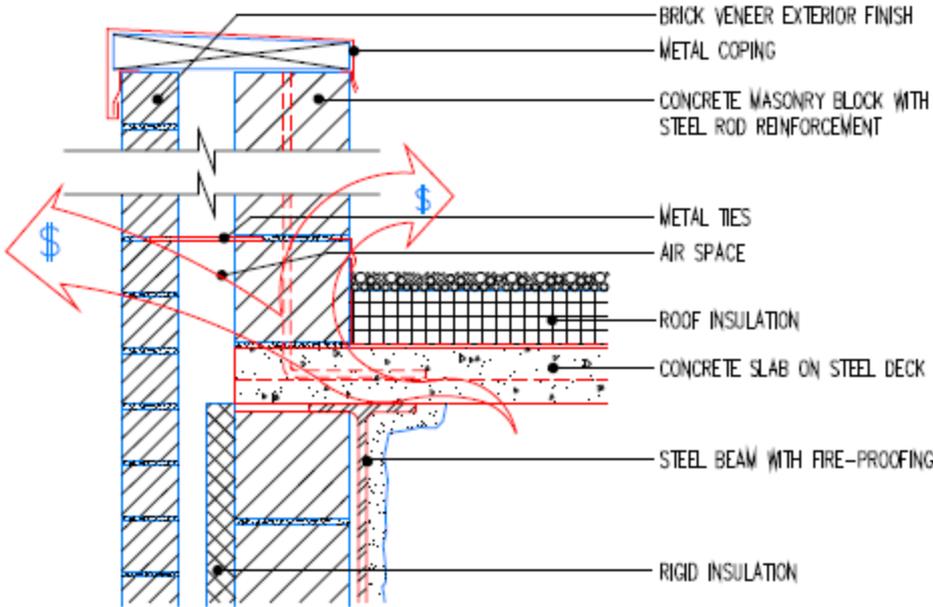
THERMAL BRIDGE:
AT EDGE OF CONCRETE ROOF DECK



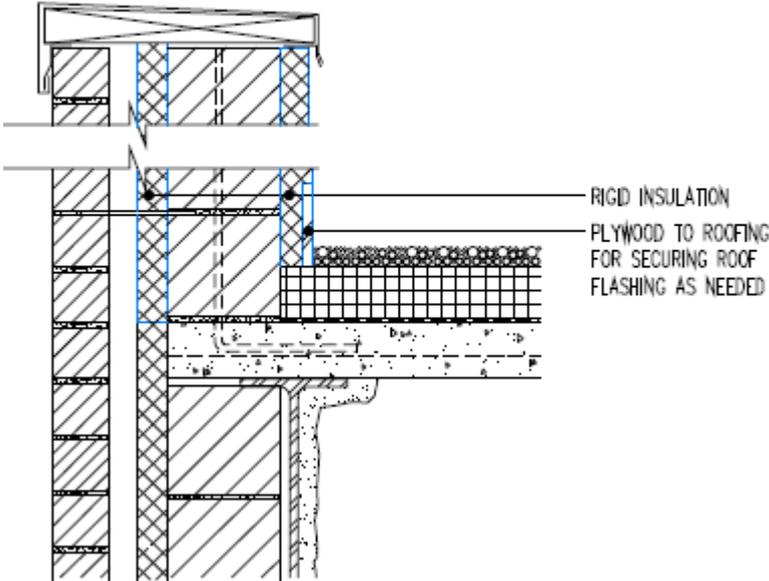
Roof parapet

PROBLEM

THERMAL BRIDGE:
AT EDGE OF CONCRETE ROOF DECK



SOLUTION



Roof parapet

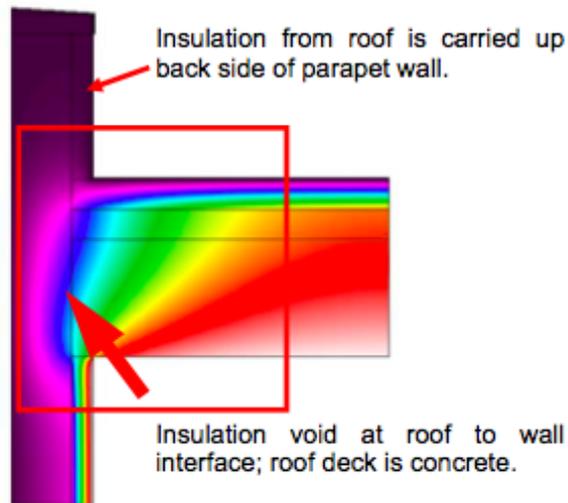


Figure 2

Results of a THERM model showing a thermal bridge at the roof to wall interface of a precast wall where roof insulation is not tied to wall insulation; although insulation now carries up the backside of the parapet wall, there is still no major effect on the bridge.

$$T_{\text{interior}} = 72^{\circ}\text{F}$$
$$T_{\text{exterior}} = 7^{\circ}\text{F}$$

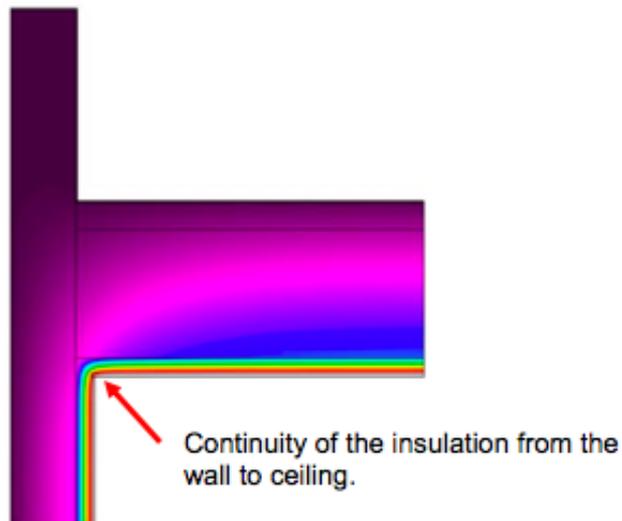


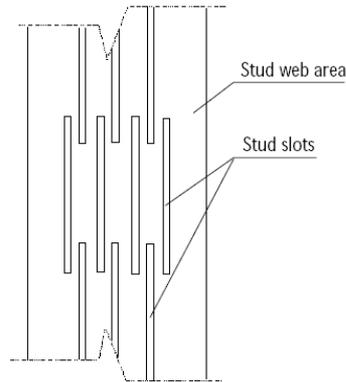
Figure 3

Results of a THERM model shows continuity of insulation below the roof deck; another option not shown here is to carry the wall insulation up by the roof deck and tie it into the roof insulation; structural considerations must be evaluated for this option. Window head below either option also requires careful evaluation.

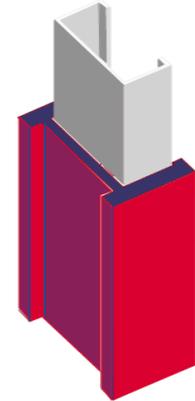
$$T_{\text{interior}} = 72^{\circ}\text{F}$$
$$T_{\text{exterior}} = 7^{\circ}\text{F}$$

Reducing framing effects: Modified steel studs

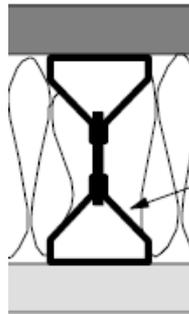
Slotted Studs



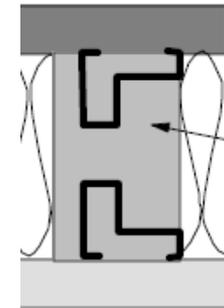
Insulated Studs



Triangular Studs



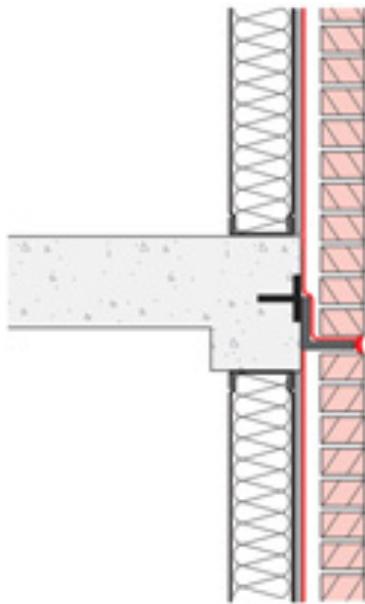
Small Studs + Foam Binder



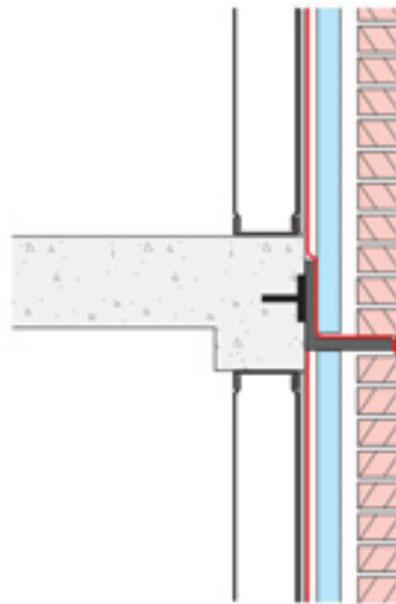
AISI/DOE report from 2003 on alternative steel options with better insulation:
<http://steeltrp.com/finalreports/finalreports/9703NonPropFinalReport.pdf>

Slab edges and exterior wall connections

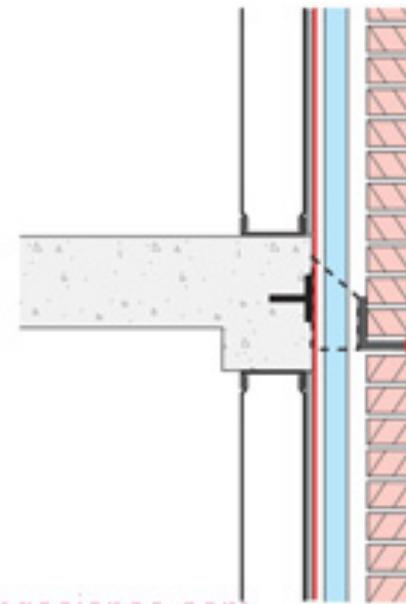
- Exterior veneer connections can be significant thermal bridges
 - Here are three different design examples:



“The Ugly”



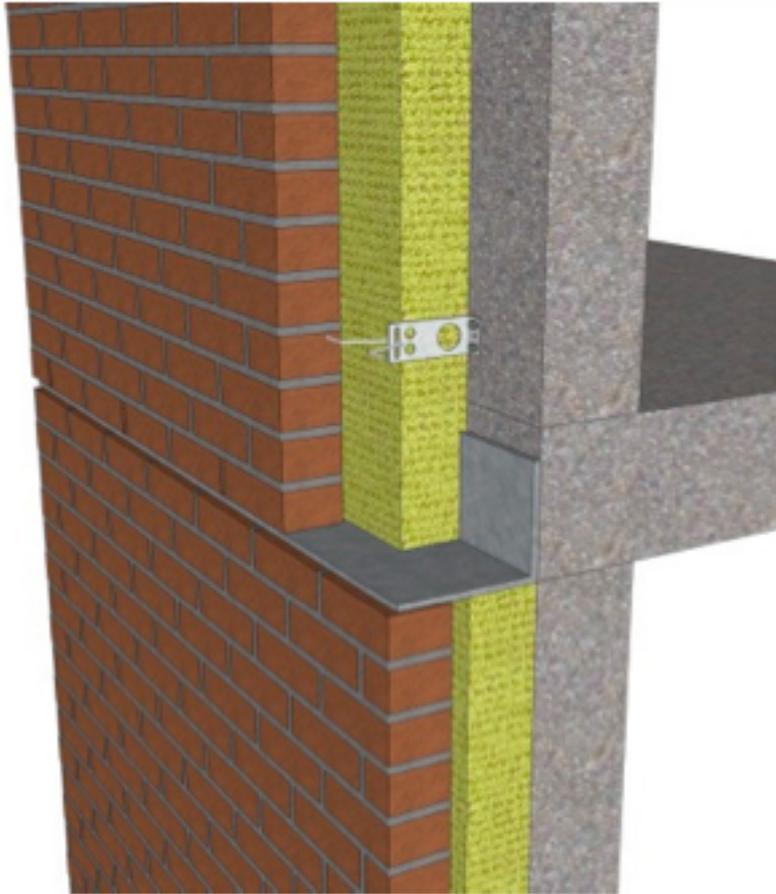
“The Bad”



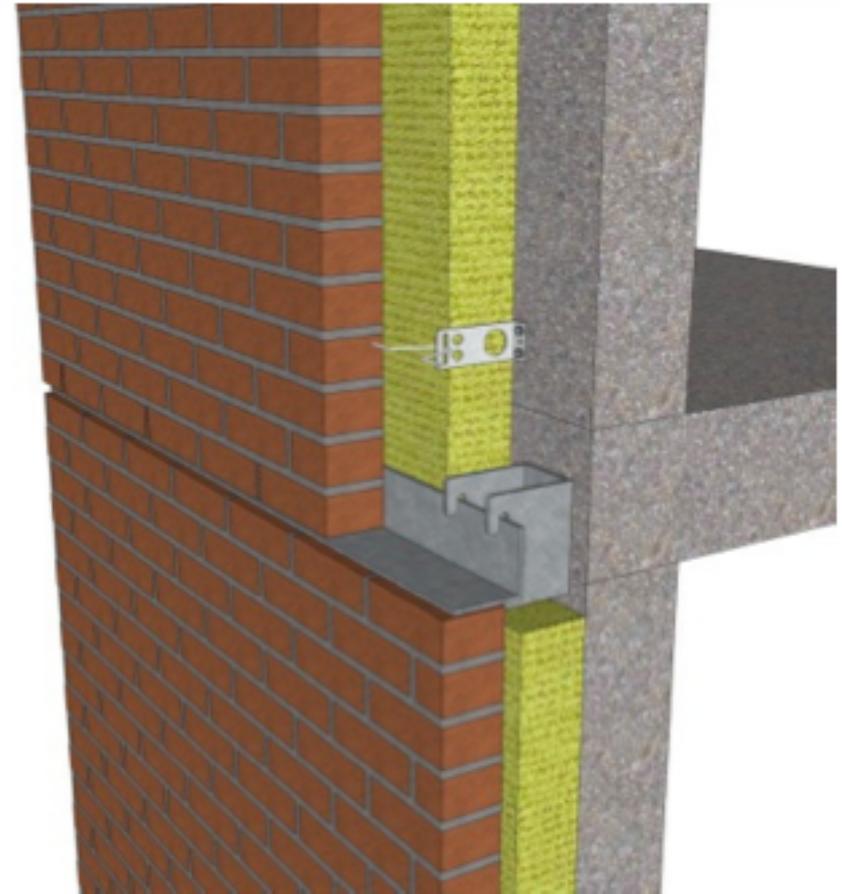
“The Good”

© buildingscience.com

Exterior wall connections



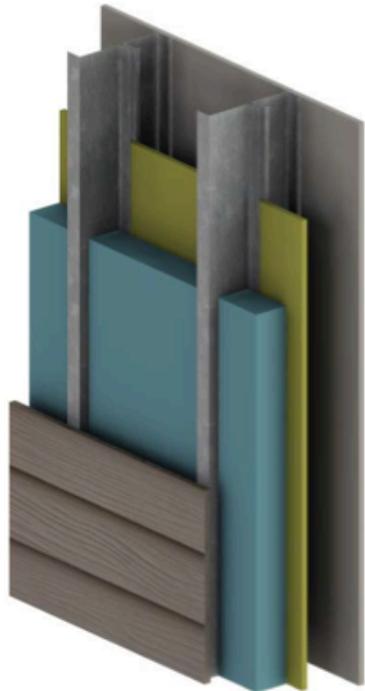
Standard slab attached shelf angle



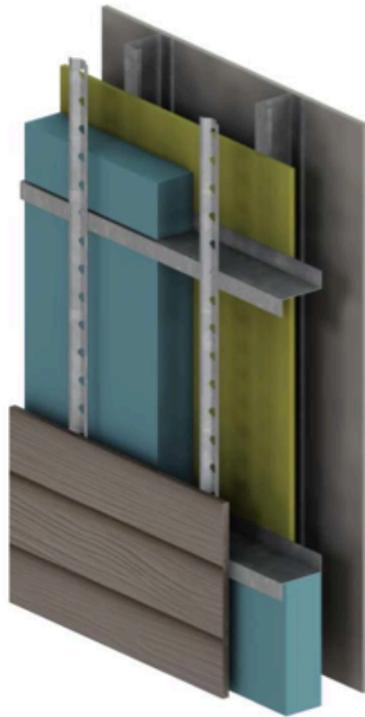
Intermittent shelf angle support

Improved thermal performance

Exterior wall connections: Diminishing R value of external insulation



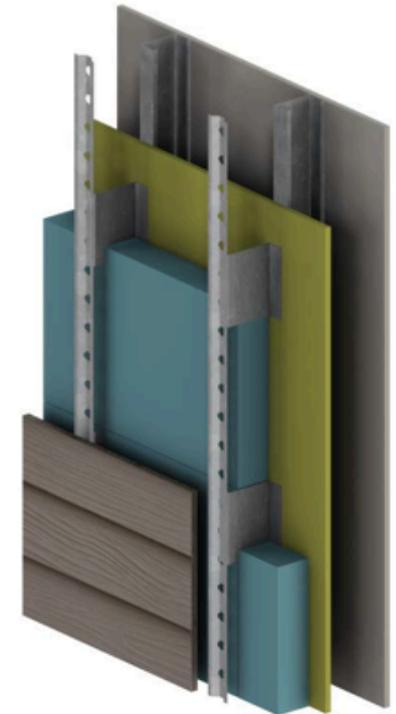
Vertical Z-Girts



Horizontal Z-Girts



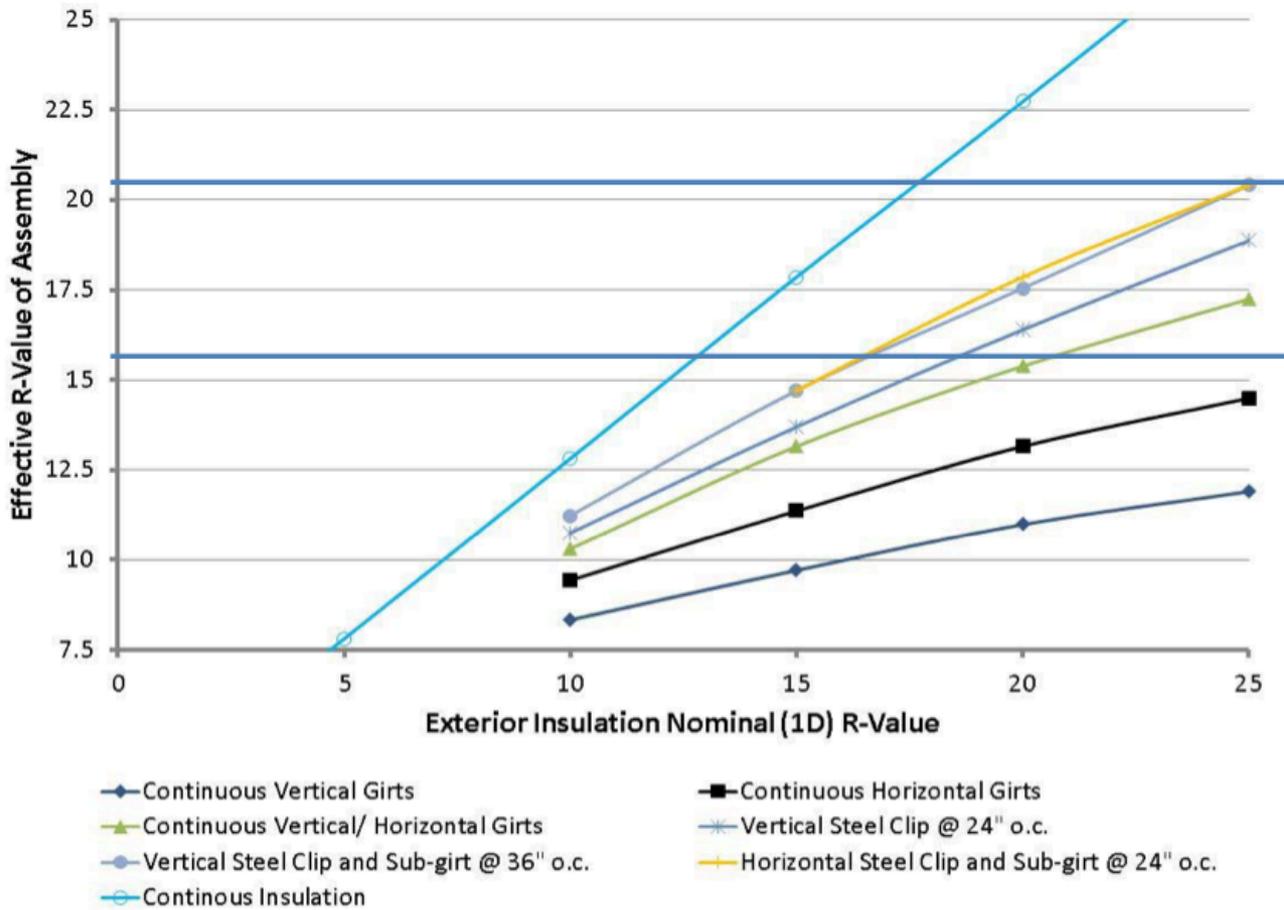
Mixed Z-Girts



Intermittent Z-Girts



Exterior wall connections: Diminishing R value of external insulation



Ultimate thermal bridge?

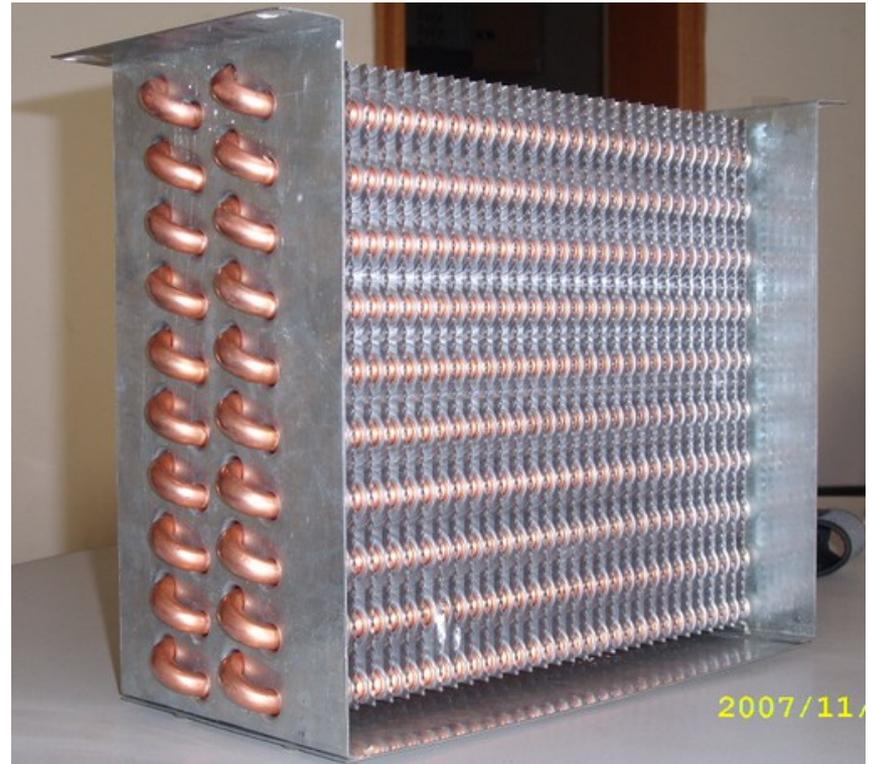


- This **cantilevered balcony** design provides a thermal connection from the interior to an exterior heating/cooling fin
- The balcony makes a fantastic heat exchanger!

Ultimate thermal bridge?

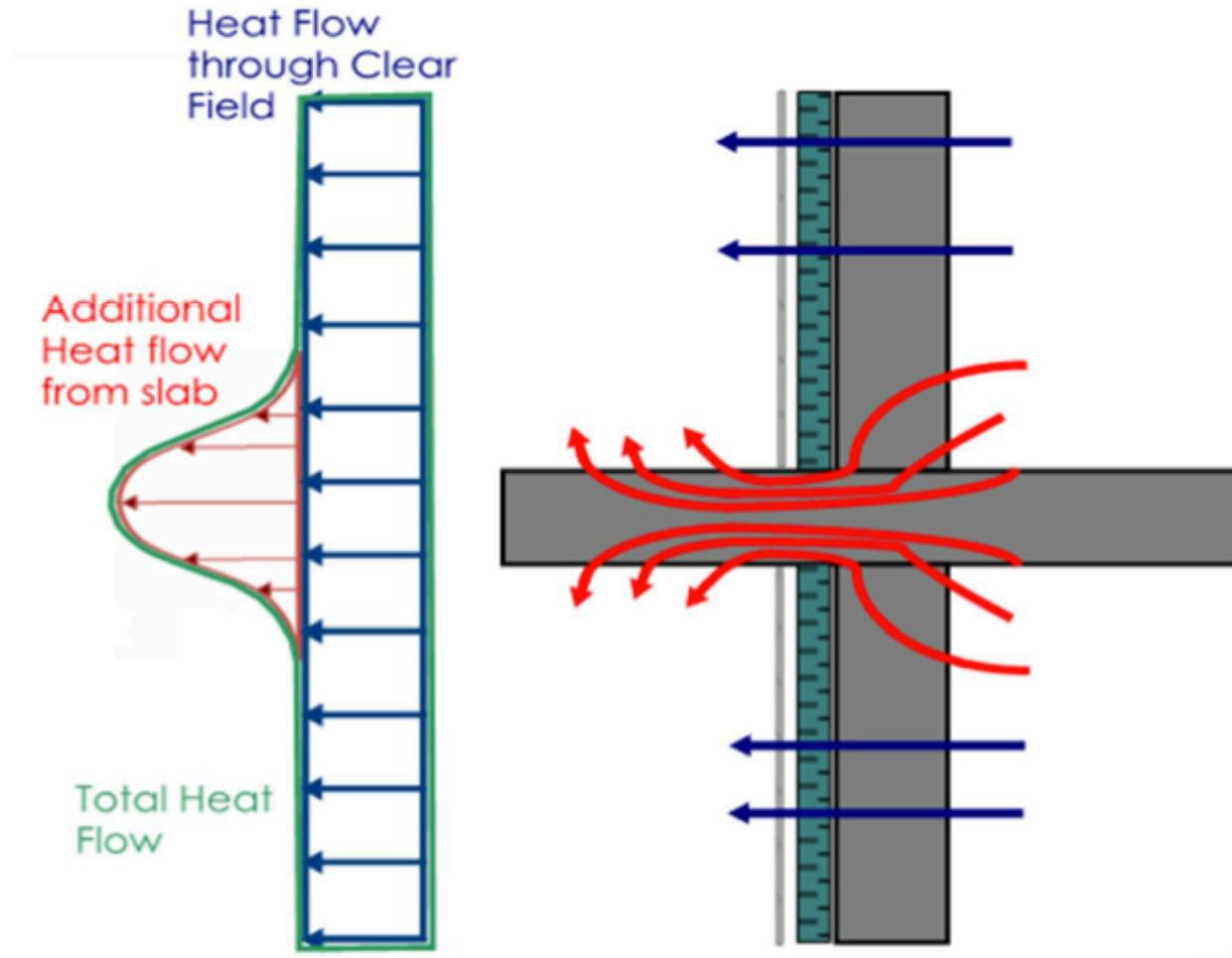


==



Balconies can act as a heat exchanger – heat exchangers are specifically designed to use fins to increase surface area for heat transfer (increasing UA)

Ultimate thermal bridge: Balconies



What about this building?



Aqua Tower

- Designed by Jeanne Gang, of Studio Gang
- This design has won many awards but it consists of many cantilevered concrete slabs
- They did NOT use insulated cantilevers
 - Used “insulating paint” which isn’t insulating – it just changes the reflectivity/emissivity/absorptivity
 - May help solar gains in summer but does nothing for conduction, particularly in the winter



“Thermal Bridge Redux” by J. Lstiburek, ASHRAE Journal July 2012

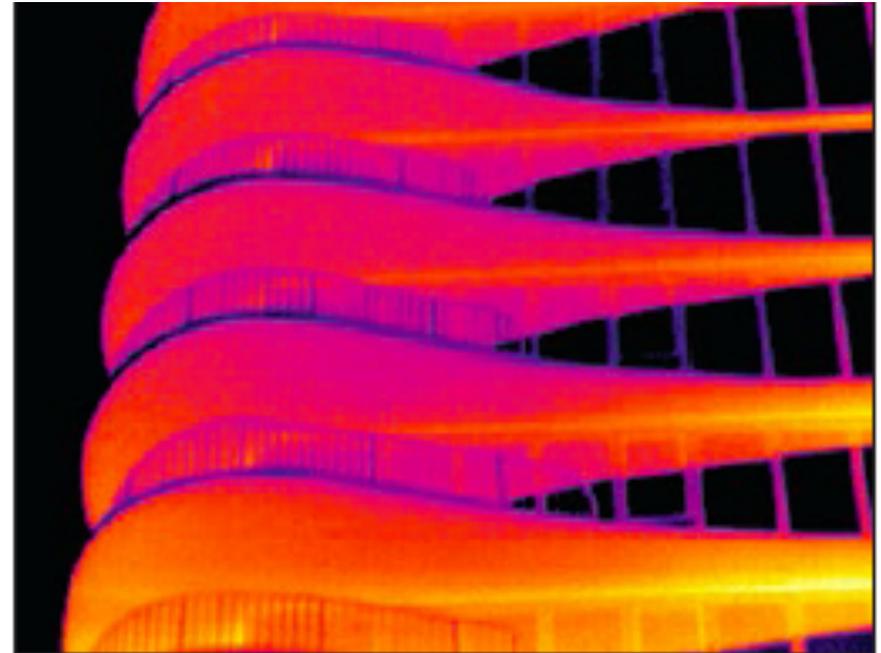
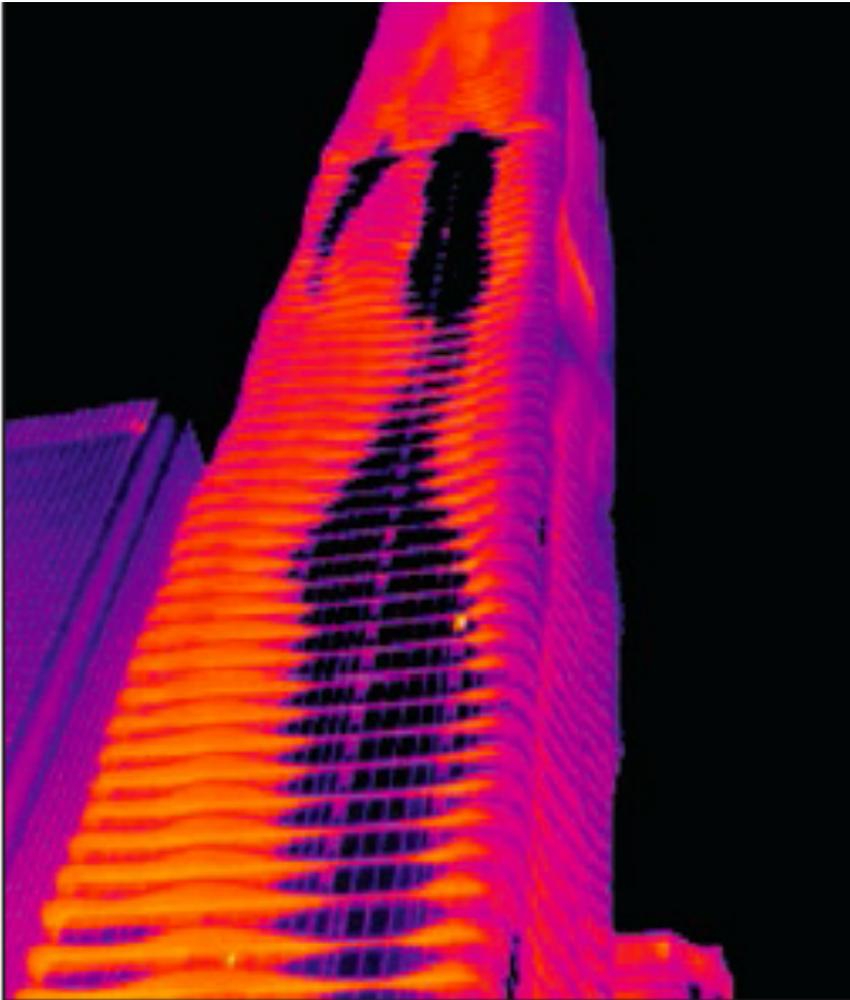


Photo 1f (left): Infrared of Aqua Tower.
Photo 1g (right): Infrared of Aqua Tower Balcony.

Thermographic images courtesy of Dave Robley, Thermographer, Fluke

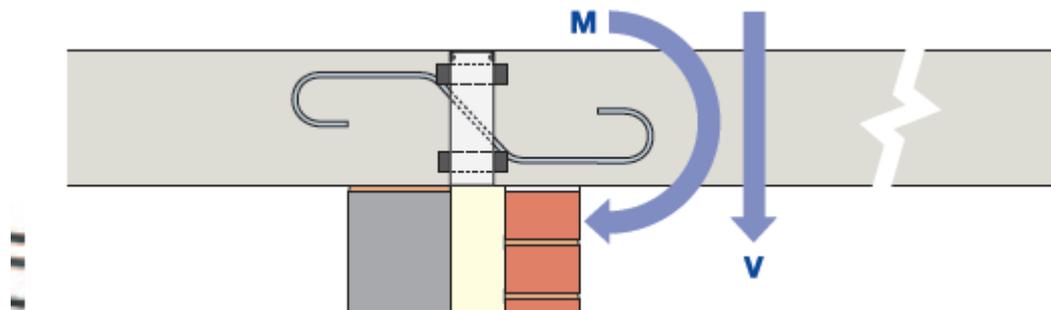
Solution 1: Hanging balconies



- These balconies are precast concrete that is connected with offset point supports and tie rods
- A foam thermal break is easily incorporated into this design
- Of course, this wouldn't work for the Aqua aesthetic

Solution 2: Insulated cantilever

- Insulated connectors for balcony cantilevers are now available on the market and used throughout Europe
- Search for: Ancon, Egccobox, Halfen



Solution 2: Insulated cantilever

k (carbon steel) ~36-54 W/mK

k (stainless steel) ~16-24 W/mK

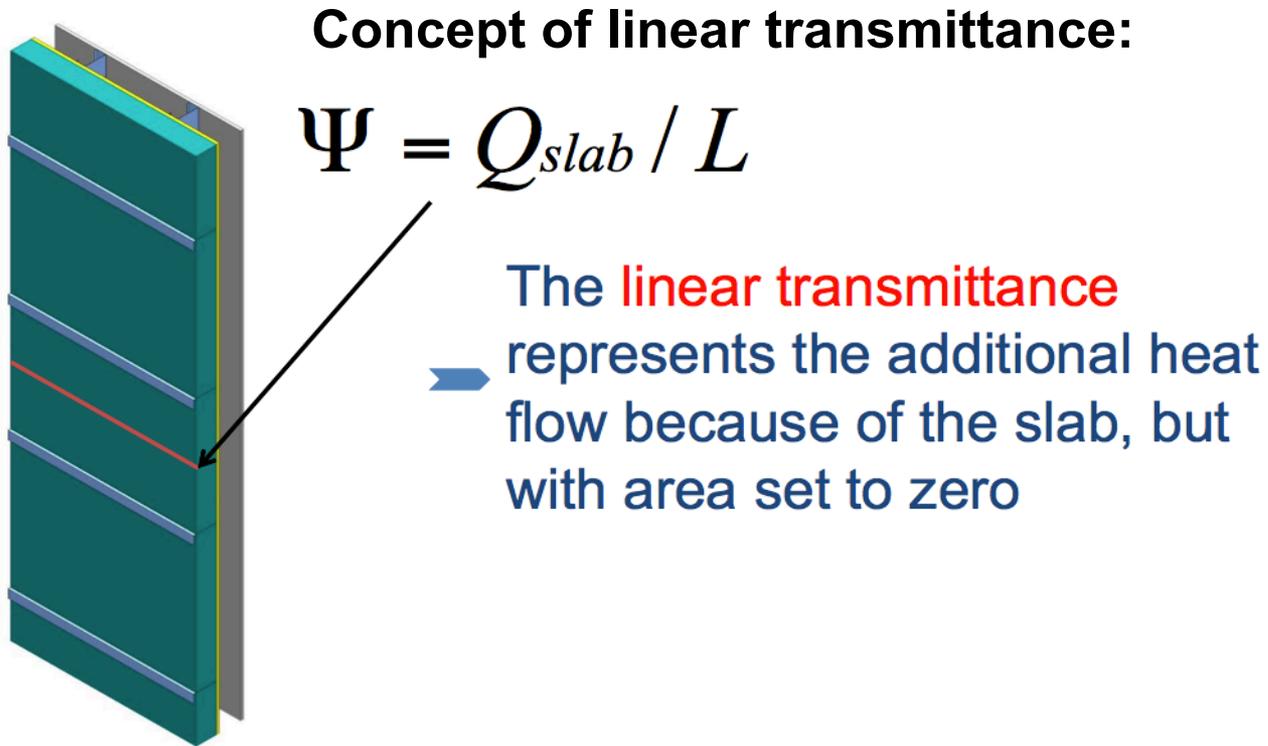


Photo 2: Premanufactured Thermal Break. High density graphite enhanced expanded polystyrene. Note the reinforcing rods penetrating the foam are stainless steel not carbon steel. Stainless steel has less than half the thermal conductivity of carbon steel. Neat, eh? Image courtesy of Schoeck Canada, Inc.

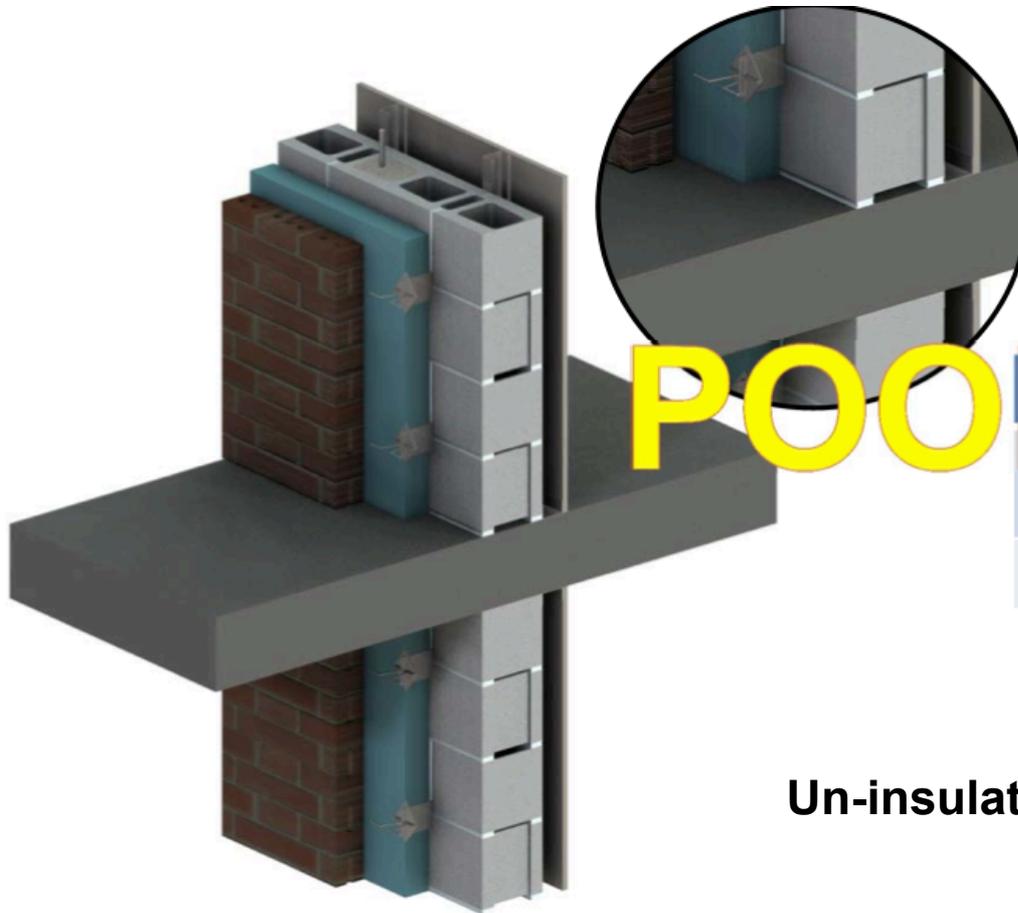


Photo 3: Belgrade Balcony. Nice view of the Danube River from a thermally broken balcony. OK, the river is out there somewhere. I have not been to Serbia recently, but apparently they can afford more efficiency than Chicago. Image courtesy of Beodom, Inc., Belgrade, Serbia.

BC Hydro: Building Envelope Thermal Bridging Guide



BC Hydro: Building Envelope Thermal Bridging Guide



POOR

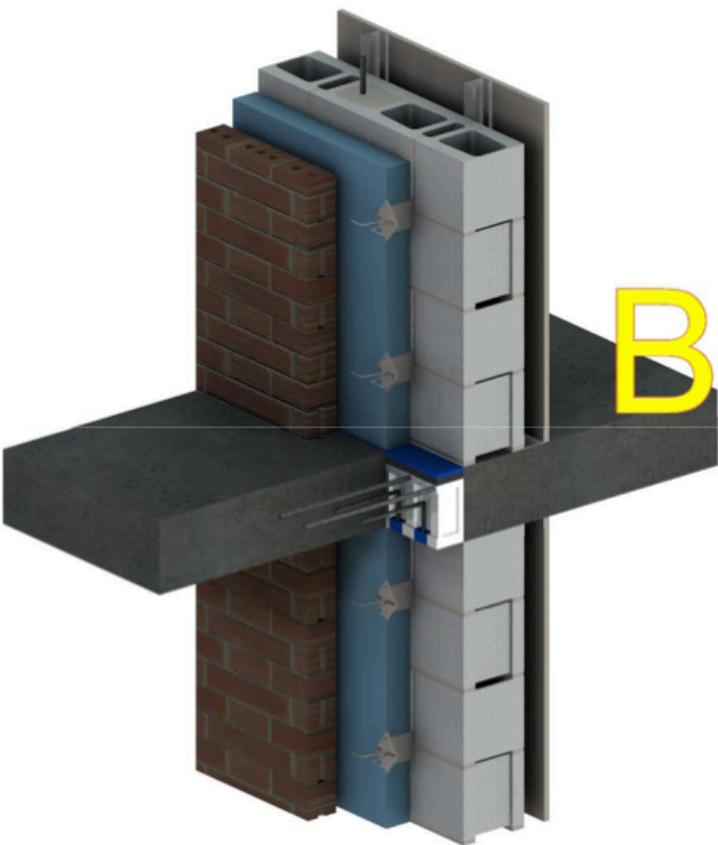
Un-Insulated Balcony Slab

	SI (W/m·K)	IP (BTU/hr·ft°F)
Ψ	0.59	0.34

Un-insulated balcony



BC Hydro: Building Envelope Thermal Bridging Guide



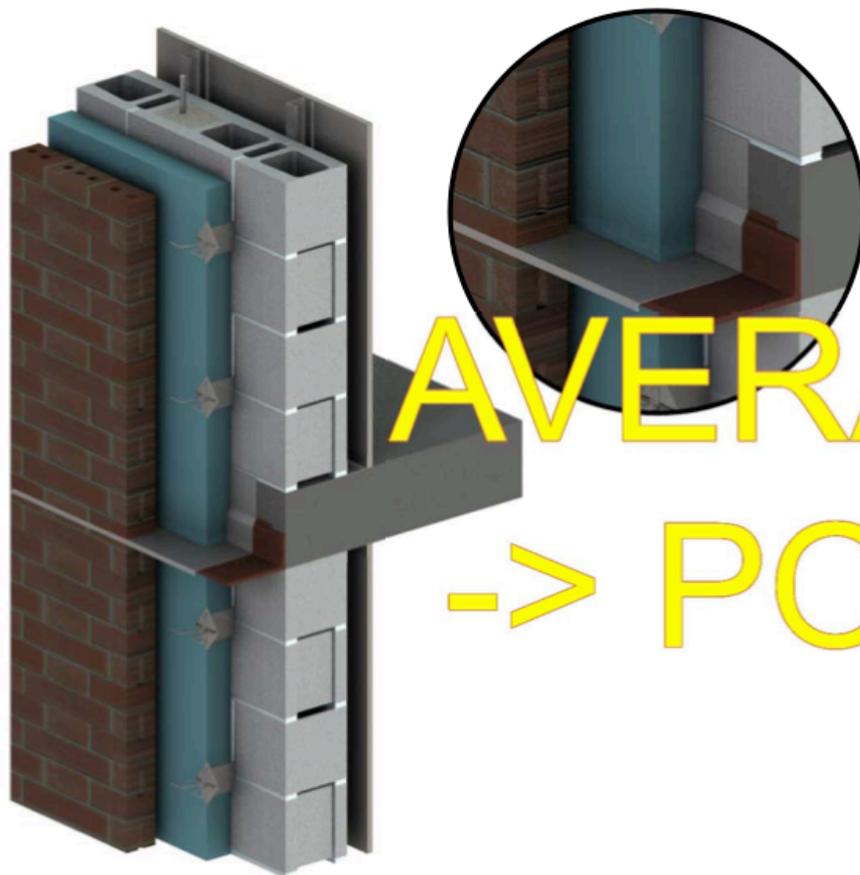
Structural thermal breaks

BEST

Thermally Broken Balcony Slab		
	SI (W/m·K)	IP (BTU/hr·ft ² ·F)
Ψ	0.21	0.12



BC Hydro: Building Envelope Thermal Bridging Guide



AVERAGE
-> POOR

Shelf Angle	
SI (W/m·K)	IP (BTU/hr·ft°F)
0.47	0.27

Basic shelf angle



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GOOD

Spaced Shelf Angle

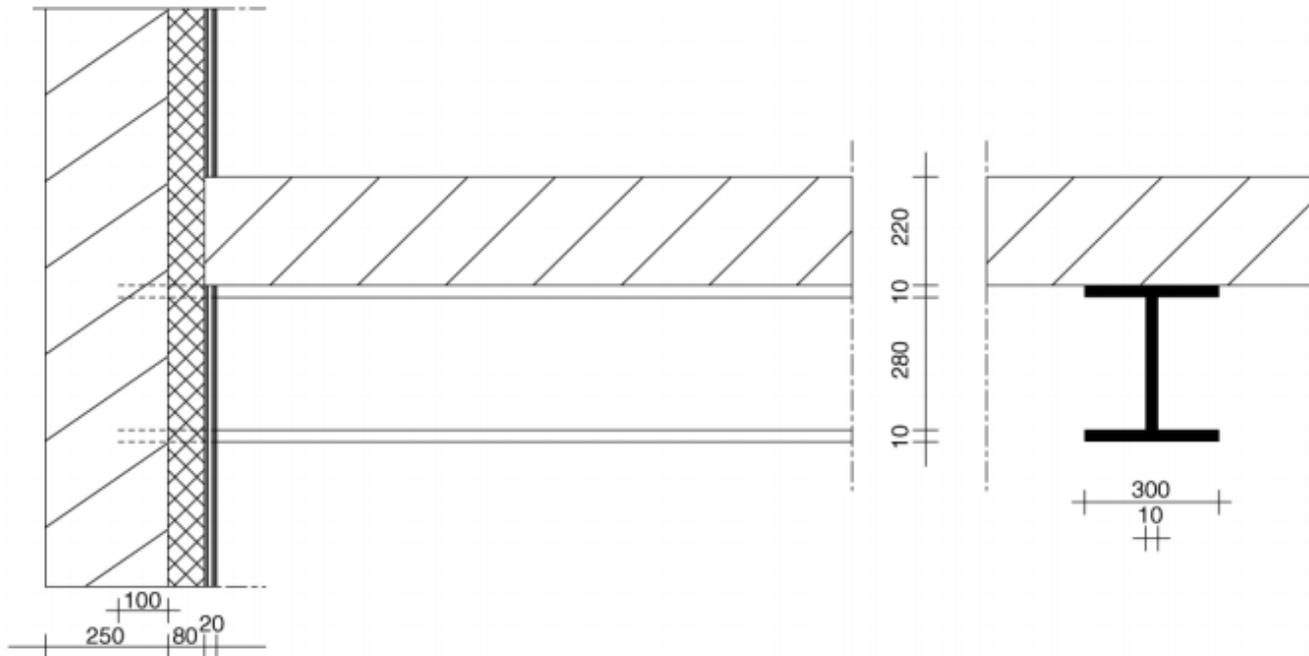
	SI (W/m·K)	IP (BTU/hr·ft ² ·°F)
Ψ	0.31	0.18

Alternative shelf angle



Thermal analysis of these even more complex geometries

- How do we estimate U and R values for complex geometries and combinations of materials?



- Simple 1-D calculations can have significant errors
 - Hard (or impossible) to capture all phenomena
- Need to model 2-D or 3-D heat transfer using computer simulations

2-D AND 3-D HEAT TRANSFER USING NUMERICAL SOLVERS

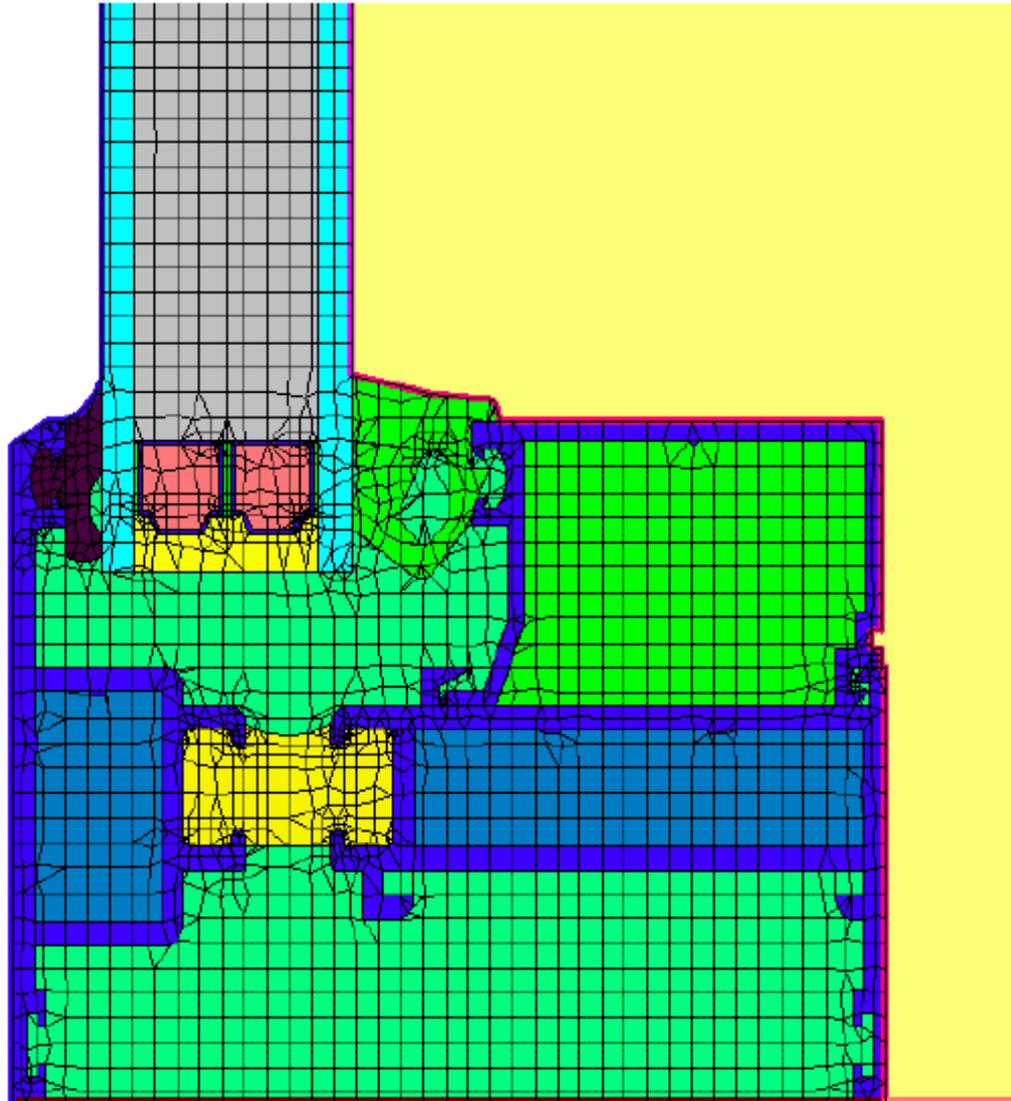
Numerical solutions

- In addition to our analytical solution methods, there is another way to solve these problems:
 - Numerical analysis using computers
- For assemblies with thermal bridges, it is probably a better solution to utilize finite-element or similar heat transfer software to estimate the U-value, R-value, and temperatures in and around the assembly

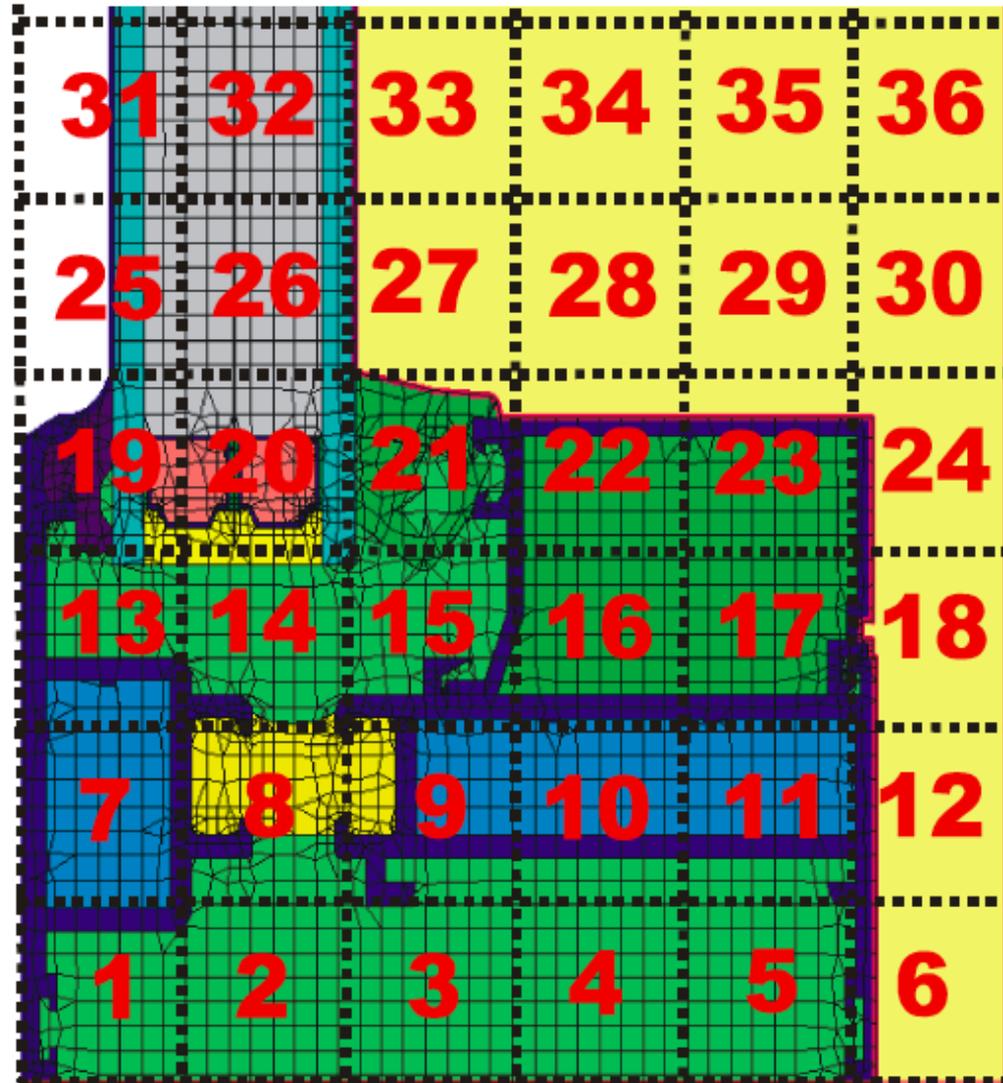
Basic idea

- Break assembly into a hundreds or thousands of homogenous elements
- Use the basic equations of heat transfer and write heat balances on each element to create a huge set of simultaneous equations
- Solve the simultaneous equations numerically to find heat flow and temperatures throughout the system

Grid for numerical solver

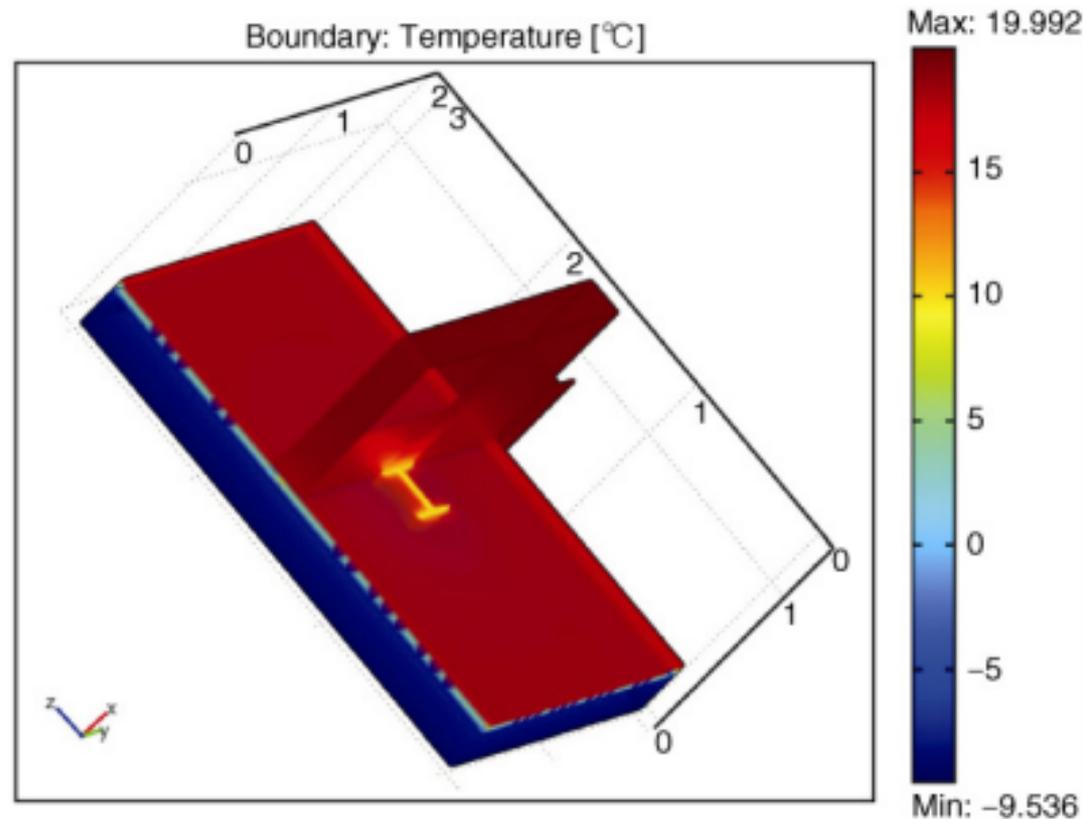
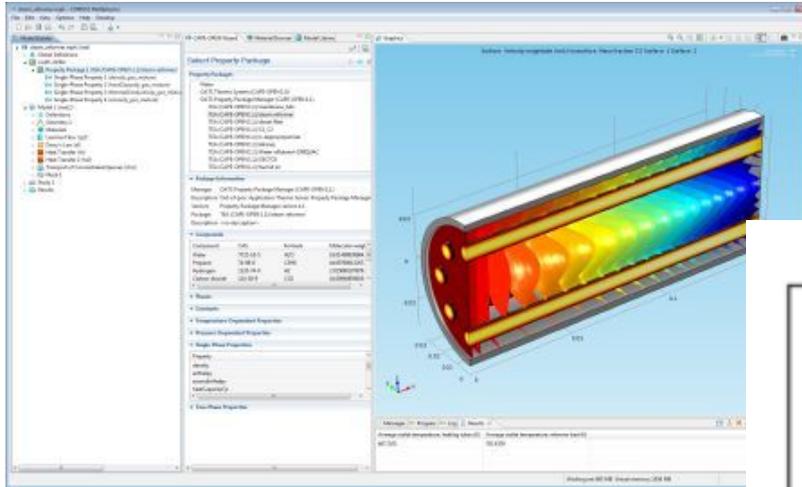


Grid for numerical solver



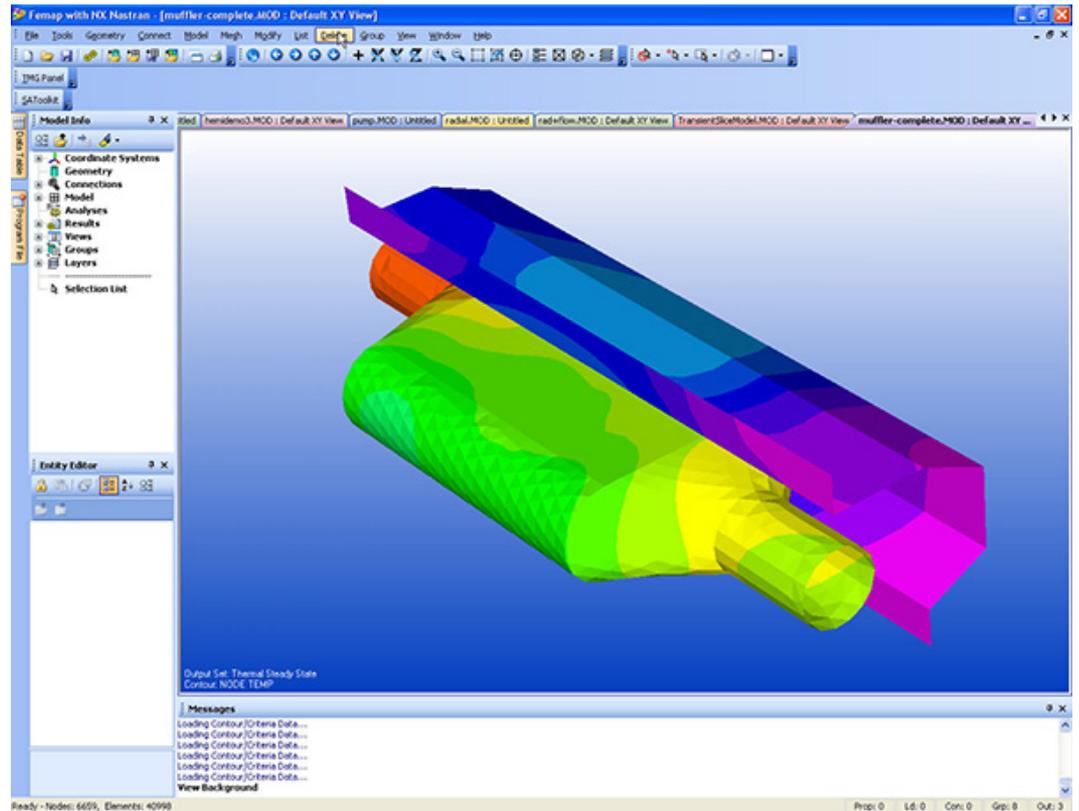
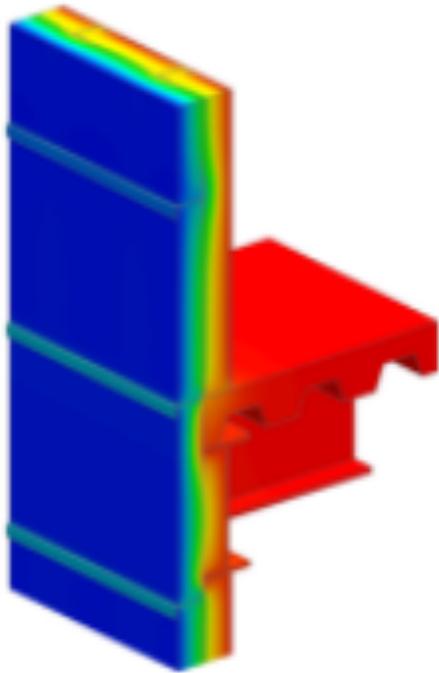
3-D solvers

- COMSOL finite element solver



3-D solvers

- Femap and Nx finite element analysis



THERM – popular 2-D solver

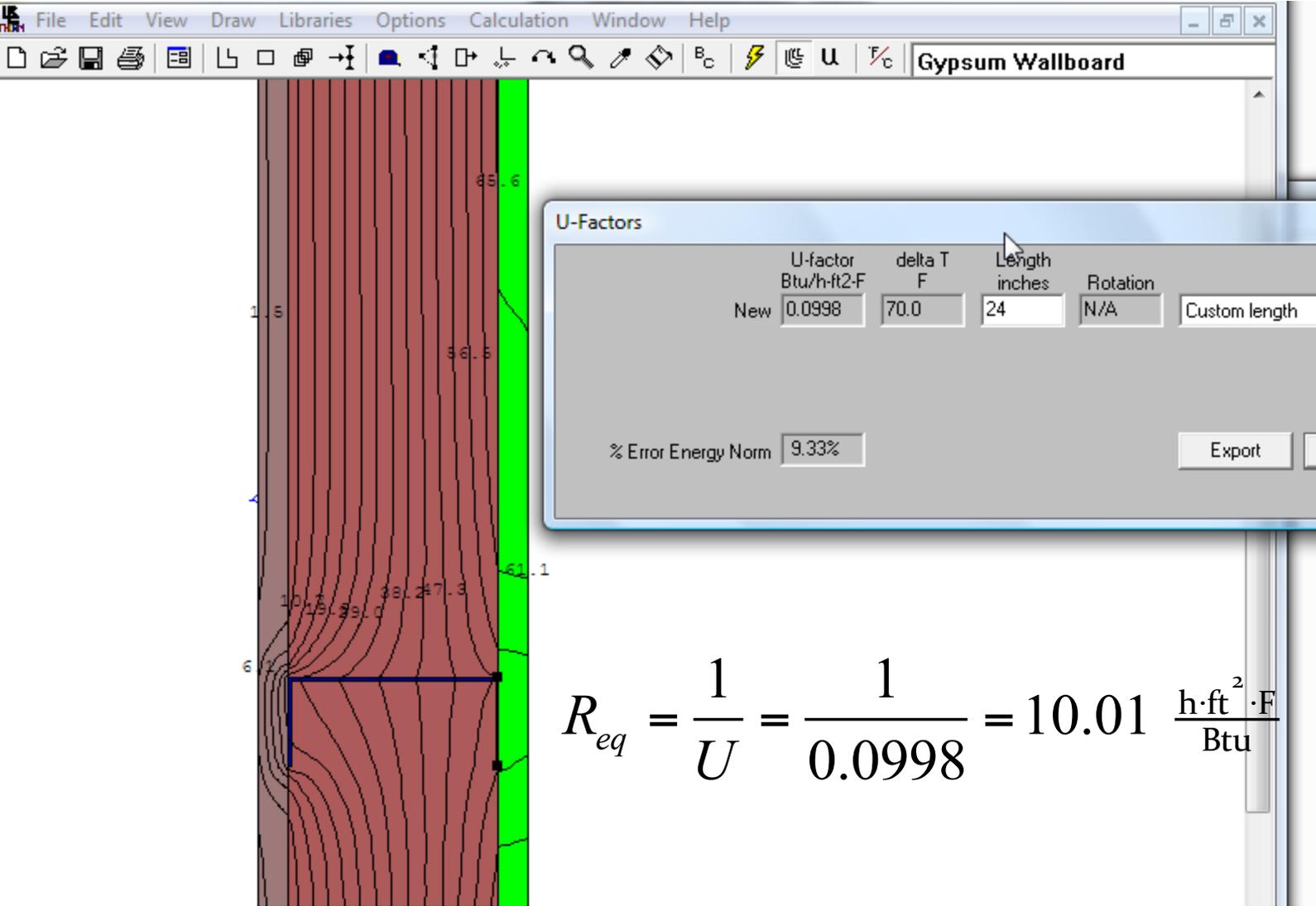
- A very popular 2-D heat transfer program used in the US – especially for window analysis - is called THERM
 - URL to download: <http://windows.lbl.gov/software/therm/therm.html>
 - Requires registration but is FREE
- THERM is used with WINDOW for designing windows
 - But can also be used as a stand alone program for any assembly
 - Very similar 2-D and 3-D programs often used in Europe are called heat2 and heat3
- You will use THERM on a HW
 - You can download from my website:
<http://built-envi.com/wp-content/uploads/2012/09/THERM63Setup.exe>
 - Only runs on Windows ☹
 - It is also installed in the AM 218 computer lab

Using THERM

- In THERM you “draw” out an assembly
 - Assign materials
 - Assign boundary conditions
 - And let the program solve for the temperatures and heat flow throughout the assembly

- The program then can analyze that output to calculate a U value for the entire 2-D assembly
 - You can then calculate $R = 1/U$ for the assembly

Example screenshot: THERM analysis of steel stud



$$R_{eq} = \frac{1}{U} = \frac{1}{0.0998} = 10.01 \frac{\text{h}\cdot\text{ft}^2\cdot\text{F}}{\text{Btu}}$$

Governing equations for THERM

1-D Conduction

$$q_{cond} = -k \frac{dT}{dx} = -\frac{k}{L} dT$$

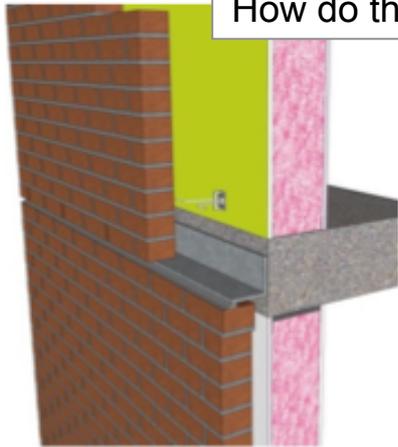
2-D Conduction

$$q_{cond} = -k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

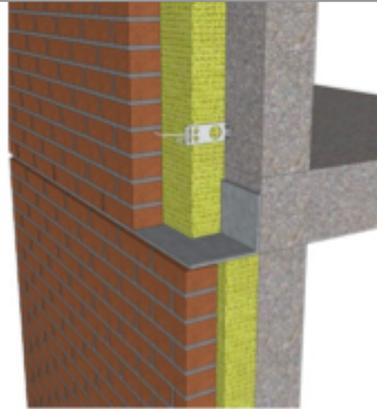
- Energy balance at a surface of each discretized element is the same as previously, except the conduction term contains X and Y components
- Set boundary conditions on interior and exterior and the solver will compute temperature throughout assembly
 - Then it can calculate a U-value for the whole assembly

What you can do with THERM

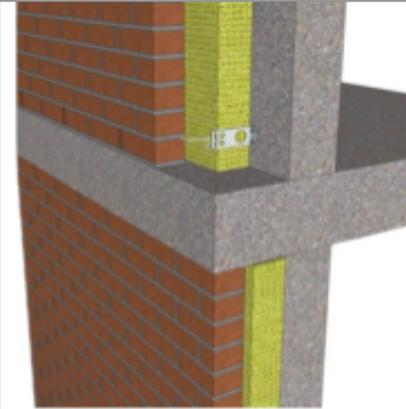
How do thermal bridges affect R-values of typical exterior wall connections?



Steel Stud Backup Wall

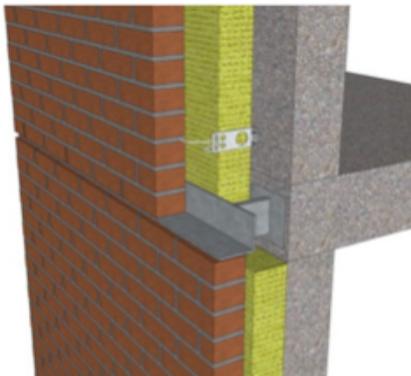


Concrete Backup Wall

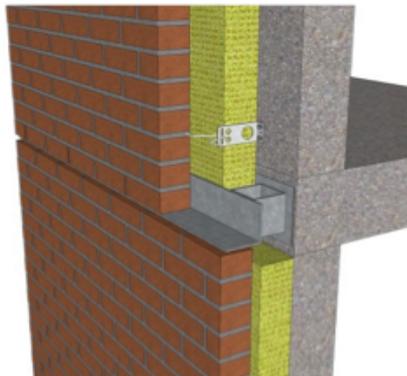


Exposed Concrete Slab

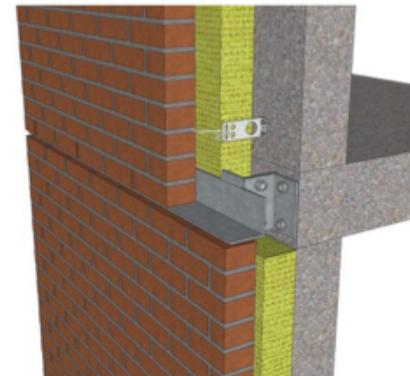
Knife Plate



HSS Structural Section



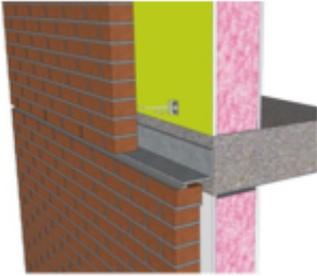
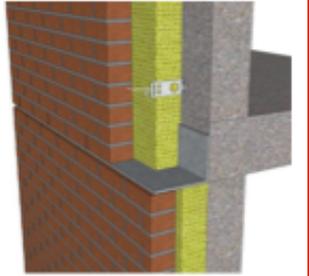
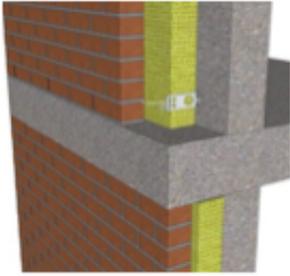
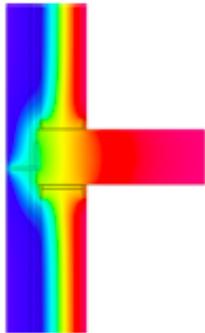
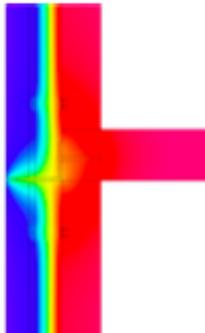
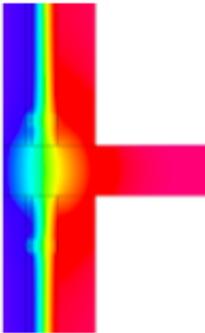
Overlapping Angles



And how do improved connections improve thermal performance?

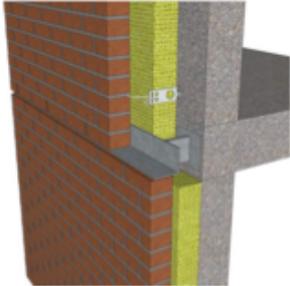
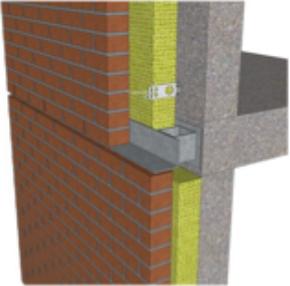
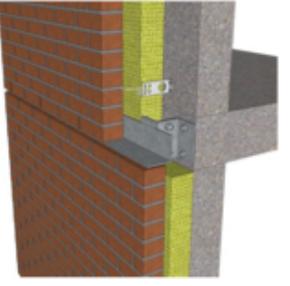
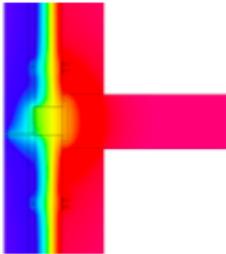
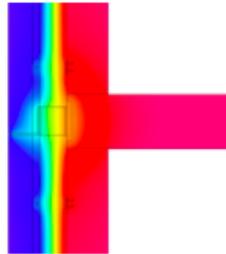
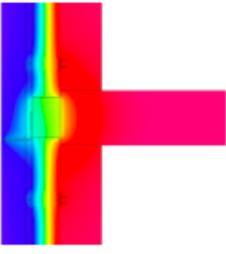
What you can do with THERM

Table 1: Summary of Nominal and Effective R-Values and U-Values for Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Steel Stud Backup	Poured Concrete Backup	Exposed Slab Edge
			
			
Nominal Insulation R-Value/U-Value	R-20 (RSI 3.52) U-0.05 (USI 0.284)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-7.3 (RSI 1.29) U-0.137 (USI 0.777)	R-10.5 (RSI 1.84) U-0.096 (USI 0.543)	R-9 (RSI 1.58) U-0.112 (USI 0.634)
Effective Reduction	63.5%	37.5%	46.4%
Linear Transmission	-	$\psi = 0.339$ IP (0.586 SI)	$\psi = 0.478$ IP (0.827 SI)

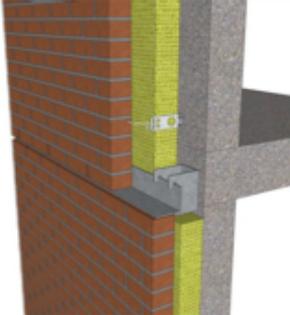
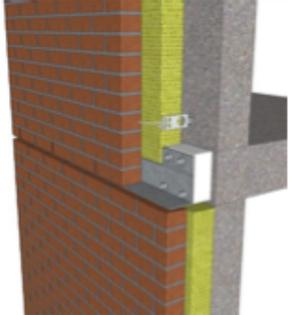
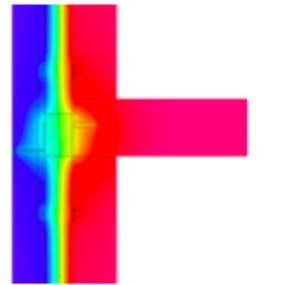
What you can do with THERM

Table 2: Summary of Nominal and Effective R-Values and U-Values for Typical Stand-Off Modifications to the Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Knife Plate	HSS Structural Section	Overlapping Angles
			
			
	shelf angle: 4"x4"x1/4" outside of insulation. 4"x4"x3/4" stand-off knife plates welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 4"x4"x1/4" HSS tube welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 2-6"x4"x5/16" angles bolted to slab edge at 48" o.c.
Nominal Insulation R-Value/U-Value	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-14.8 (RSI 2.6) U-0.068 (USI 0.384)	R-14.8 (RSI 2.6) U-0.068 (USI 0.385)	R-15.0 (RSI 2.64) U-0.067 (USI 0.379)
Effective Reduction	16.4%	16.5%	15.3%
Linear Transmission	$\psi = 0.096$ IP (0.166 SI)	$\psi = 0.097$ IP (0.168 SI)	$\psi = 0.089$ IP (0.153 SI)

What you can do with THERM

Table 3: Summary of Nominal and Effective R-Values and U-values for Proprietary Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging

	Standoff Bracket	4-Bolt Cast-In
		
		
	shelf angle 4"x4"x1/4" outside insulation. Proprietary clip is 1/4" thick steel, 4"x4"x1/4" 6 "lg C-section. Non-welded connection. .	Shelf angle 4"x4"x1/4" outside insulation. Pre-manufactured cast-in place thermal break connection with 4 stainless steel bolts attached to 7"x7"x 3/8" plate.
Nominal Insulation R-Value/U-Value	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-14.9 (RSI 2.62) U-0.067 (USI 0.381)	R-16.4 (RSI 2.9) U-0.061 (USI 0.345)
Effective Reduction	16.4%	7%
Linear Transmission	$\psi = 0.091$ IP (0.158 SI)	$\psi = 0.037$ IP (0.064 SI)

THERM demonstration

WRAPPING UP COMPLEX CONDUCTION

- (1) Below-grade walls and floors
- (2) On-grade heat transfer

Below-grade heat flow

- Where does heat flow?
 - Depends on surface and **ground temperature** distributions

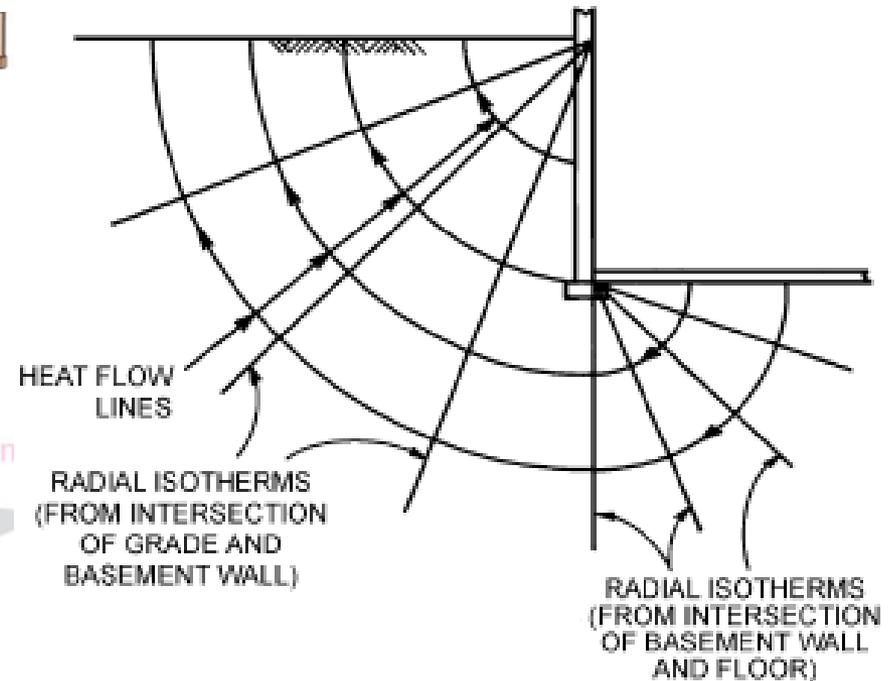
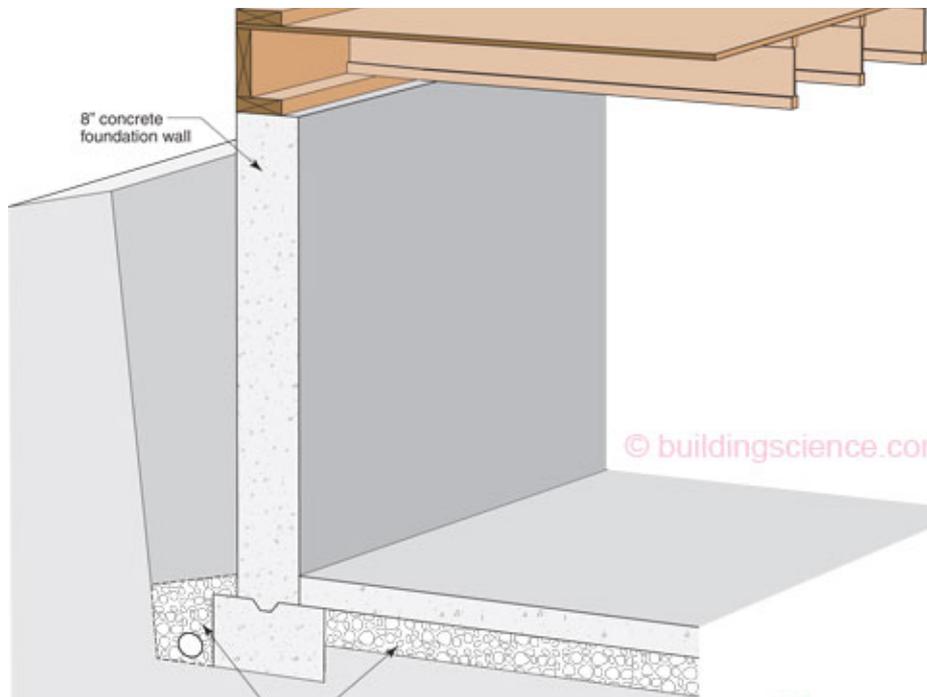
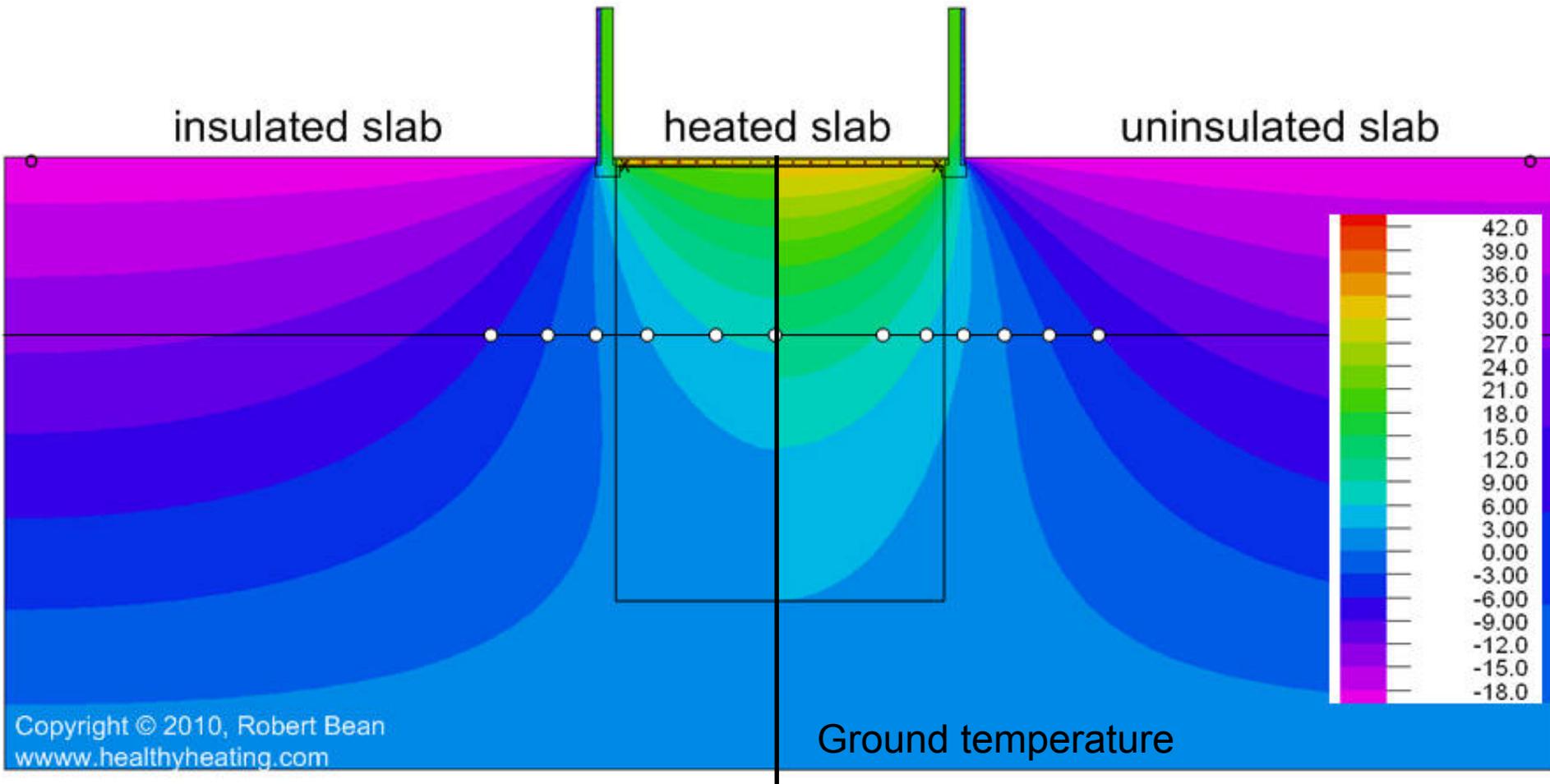


Fig. 4 Heat Flow from Basement

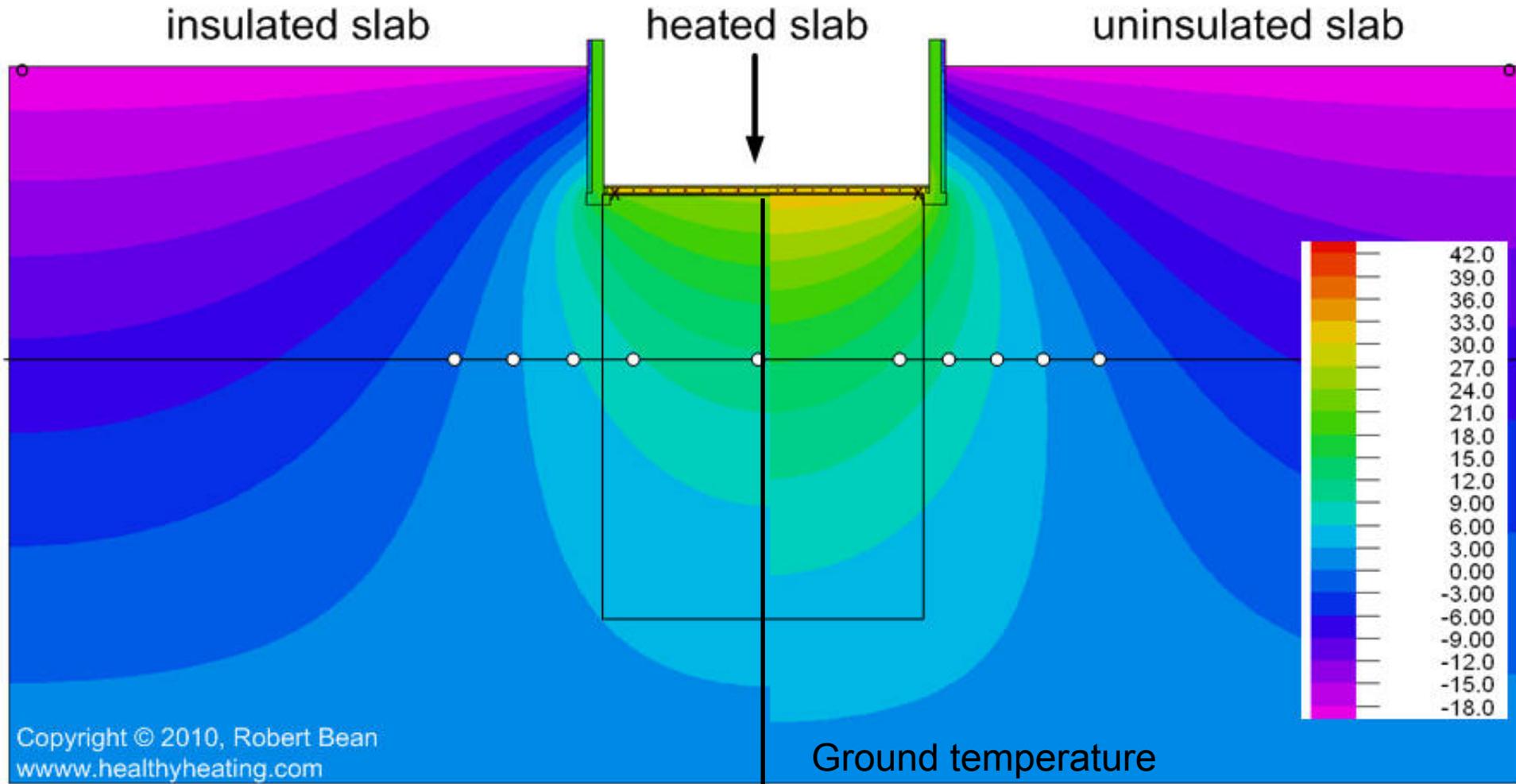
On-grade and below-grade heat transfer

- Often we have floors built directly on grade, in contact with the ground



On-grade and below-grade heat transfer

- Or we have walls and floors built below-grade, or “submerged” within the soil



Mean ground temperatures

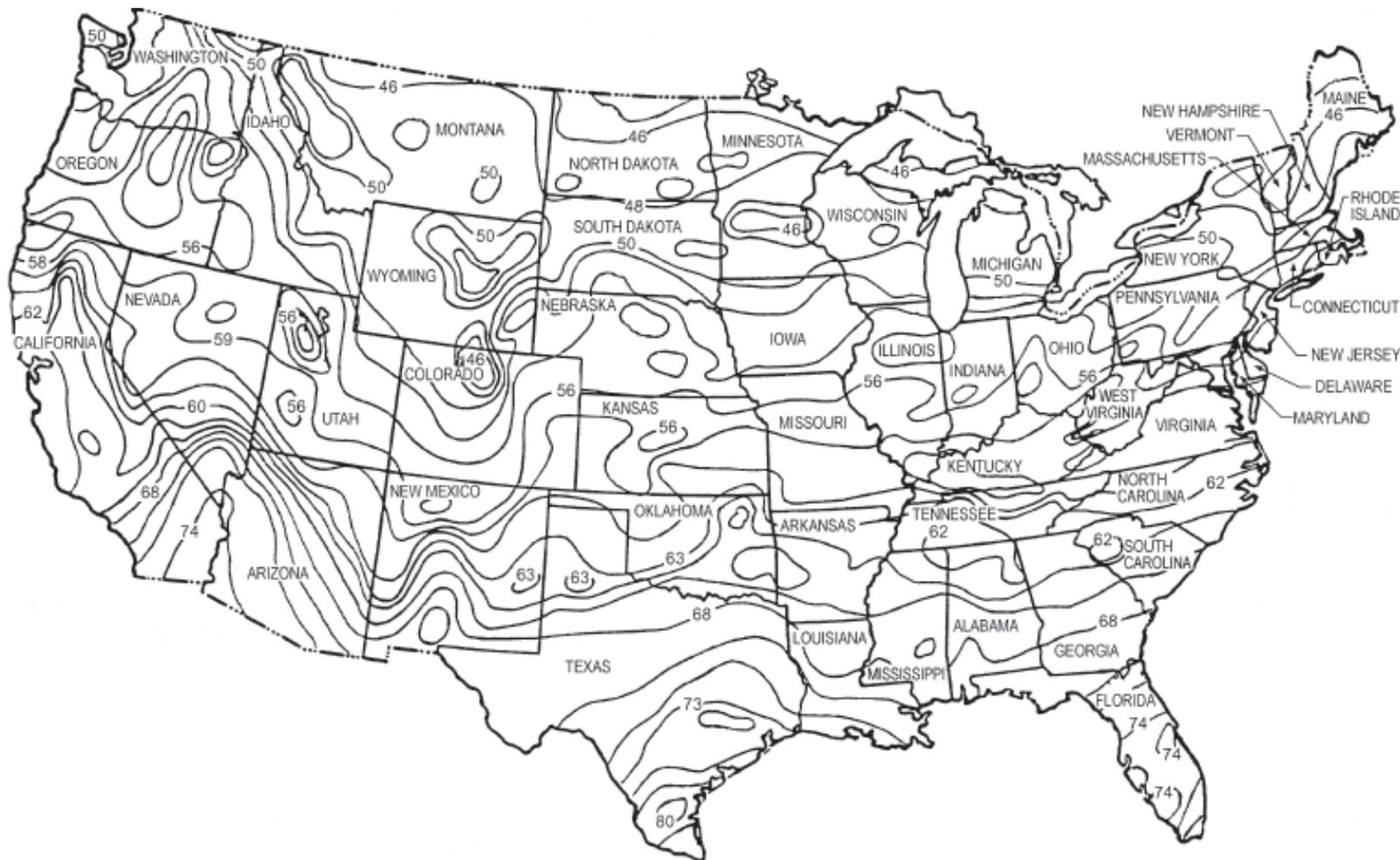
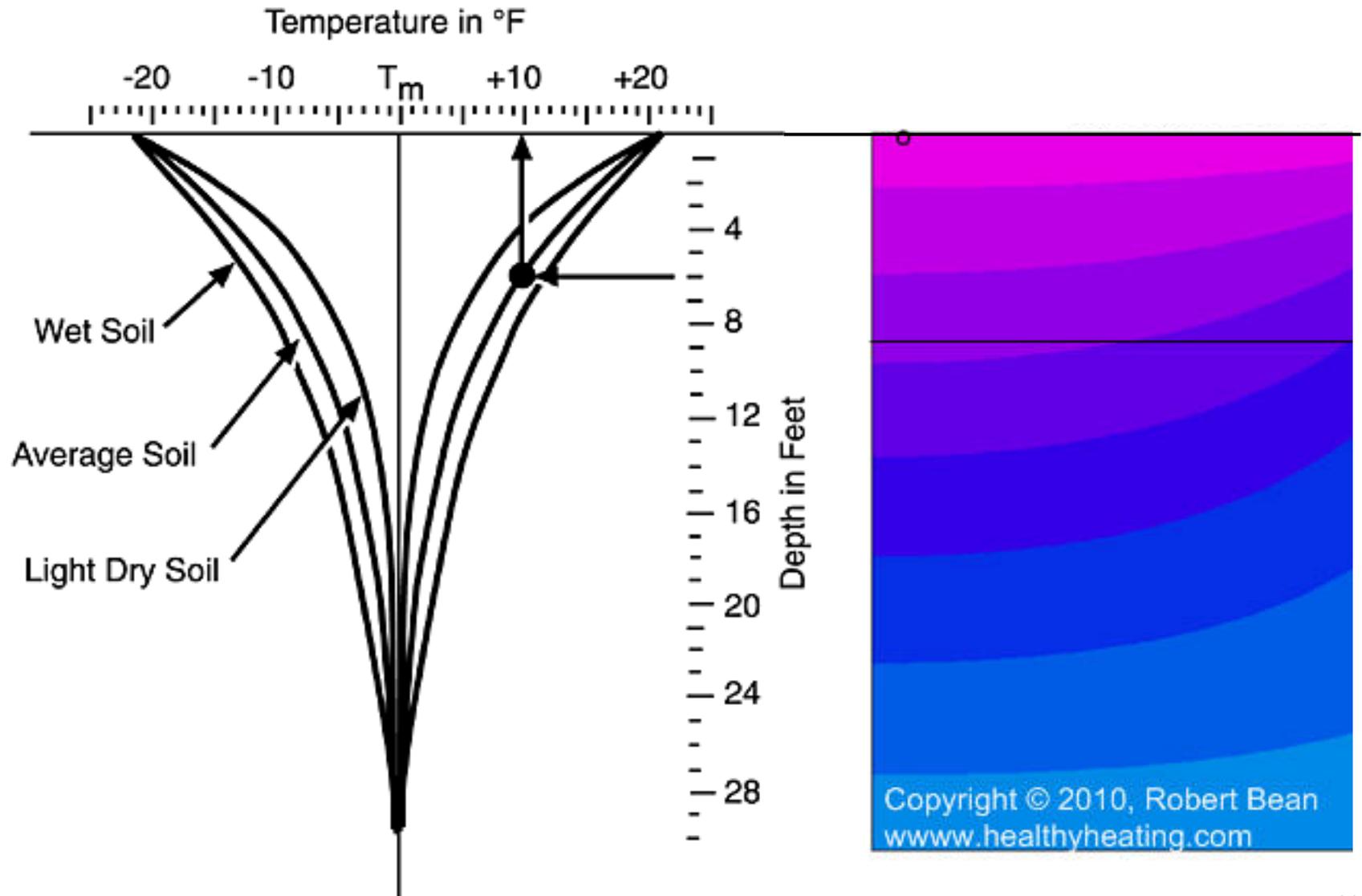
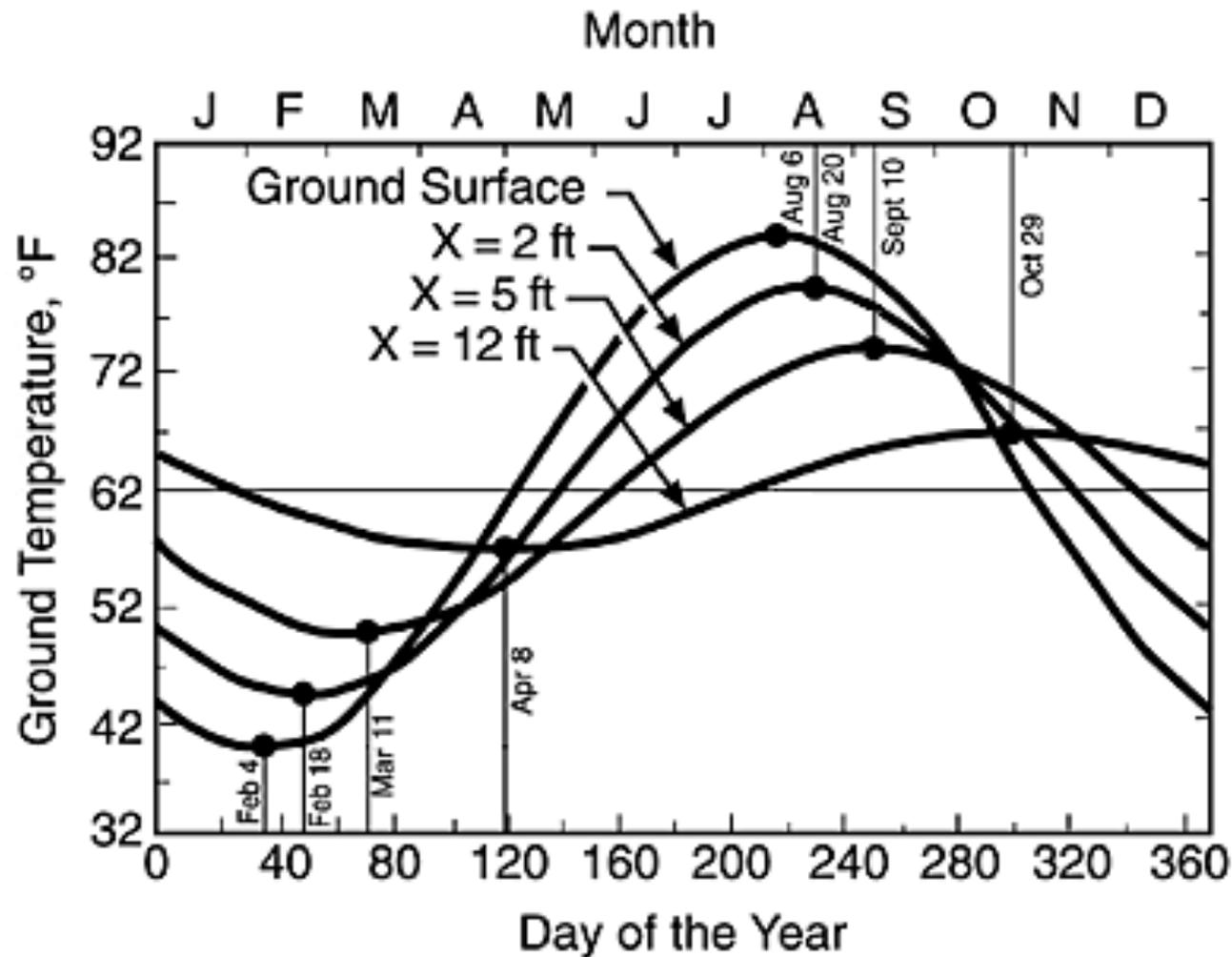


Fig. 17 Approximate Groundwater Temperatures (°F) in the Continental United States

Ground temperatures vary with depth and soil moisture



Ground temperatures also vary with **season**



Design ground temperatures

- Design (worst-case) ground temperatures
 - Adjust mean ground temperature by a peak seasonal amplitude:

$$T_{gr} = T_{gm} - T_A$$

where

T_A = the ground temperature variation amplitude (right)

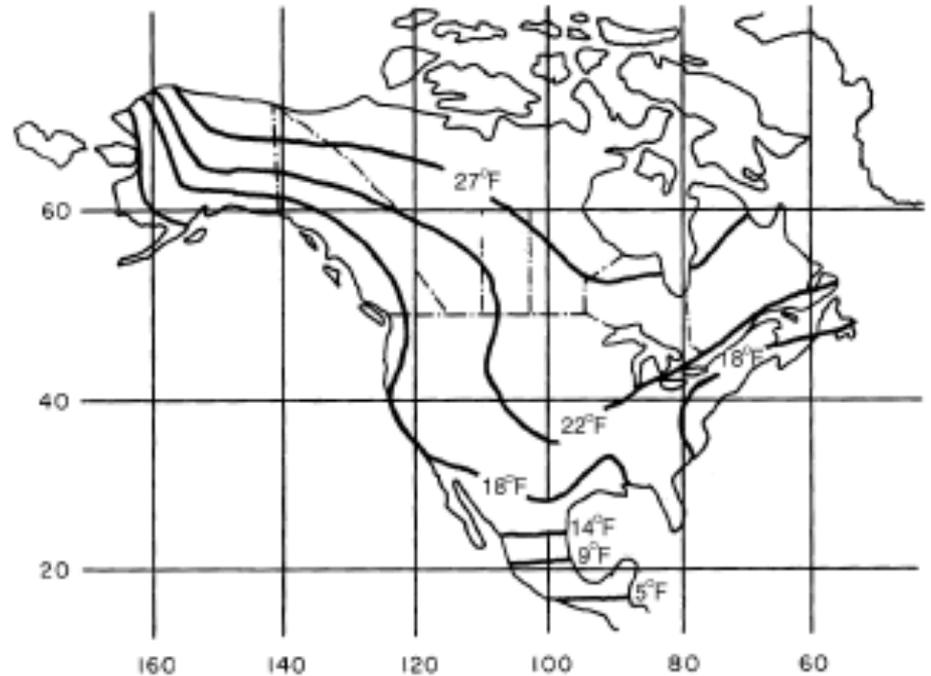
T_{gm} = mean ground temperature

*Note for Chicago:

$$T_A \approx 22^\circ\text{F} \quad T_{gm} \approx 54^\circ\text{F}$$

$$T_{gr} \approx 54 - 22 = 32^\circ\text{F}$$

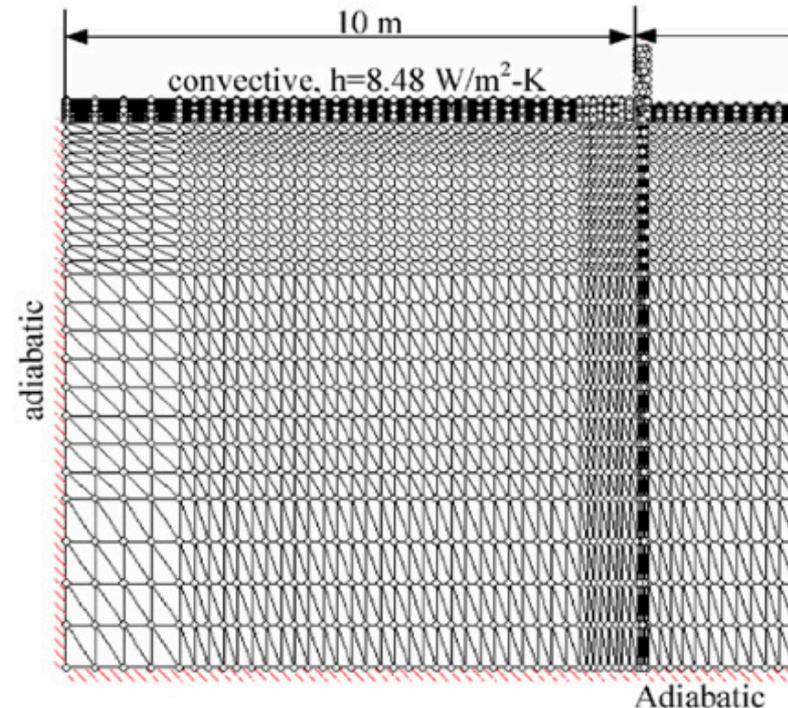
Alternatively, T_{gr} can be estimated as the mean air temperature in the coldest month



Ground Temp **Amplitude**, T_A

Below-grade heat transfer

- Heat transfer through below-grade walls and floors
 - Conduction is truly 2-D or 3-D
 - 1-D modeling is not appropriate
- Heat transfer through walls
 - Between inside and surrounding soil
 - (not exterior air)
 - Depends on the wall area
- Heat transfer through the floor
 - Between inside and the soil below
 - Depends on the floor area
- ASHRAE HOF has some guidelines for transforming 2-D into 1-D



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Below-grade heat transfer

$$Q = AU_{avg} (T_i - T_{gr}) \text{ [W]}$$

$$q = U_{avg} (T_i - T_{gr}) \text{ [W/m}^2\text{]}$$

where

A is the wall or floor area below grade [m^2] (analyze any wall portion above-grade in the normal way)

T_i is the below grade inside temp [K]

T_{gr} is the **design** ground surface temp [K]

U_{avg} is the average U factor for the below grade surface [$\text{W}/(\text{m}^2\text{K})$]
(see following slides)

Below grade depth parameters for estimating U value

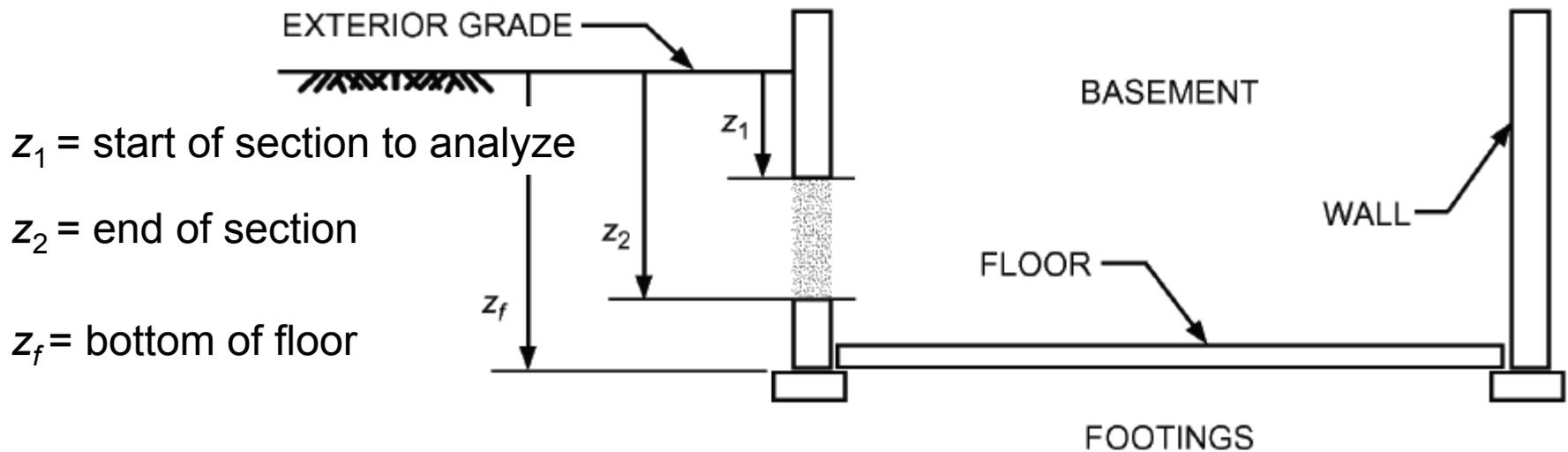


Fig. 14 Below-Grade Parameters

$U_{avg,bf}$ for below-grade floors

- For **average below-grade floor** value with a floor depth of height z_f from ground (“grade”)

$$U_{avg,bf} = \frac{2k_{soil}}{\pi w_b} \times \left[\ln \left(\frac{w_b}{2} + \frac{z_f}{2} + \frac{k_{soil} R_{other}}{\pi} \right) - \ln \left(\frac{k_{soil} R_{other}}{\pi} \right) \right]$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK

R_{other} = R value of floor + insulation + convection [m²K/W]

w_b = shortest dimension of basement width [m]

z_f = floor depth below grade [m]

Pre-computed tables for $U_{avg,bf}$

Table 17 Average U-Factor for Basement Floors

z_f (depth of floor below grade), m	$U_{avg,bf}$, W/(m ² ·K)			
	w_b (shortest width of basement), m			
	6	7	8	9
0.3	0.370	0.335	0.307	0.283
0.6	0.310	0.283	0.261	0.242
0.9	0.271	0.249	0.230	0.215
1.2	0.242	0.224	0.208	0.195
1.5	0.220	0.204	0.190	0.179
1.8	0.202	0.188	0.176	0.166
2.1	0.187	0.175	0.164	0.155

Soil conductivity is 1.4 W/(m·K); floor is uninsulated. For other soil conductivities and insulation, use Equation (38).

- Assuming **un-insulated concrete** floor

$U_{avg,bw}$ for below-grade walls

$$U_{avg,bw} = \frac{2k_{soil}}{\pi(z_1 - z_2)} \times \left[\ln\left(z_2 + \frac{2k_{soil}R_{other}}{\pi}\right) - \ln\left(z_1 + \frac{2k_{soil}R_{other}}{\pi}\right) \right]$$

- k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
- R_{other} = R value of wall, insulation and inside surface resistance [m²K/W]
- z_1, z_2 = depths of top and bottom of wall segment under consideration [m]

Pre-computed tables for $U_{avg,bw}$

Table 16 Average U-Factor for Basement Walls with Uniform Insulation

Depth, m	$U_{avg,bw}$ from grade to depth, W/(m ² ·K)			
	Uninsulated	R-0.88	R-1.76	R-2.64
0.3	2.468	0.769	0.458	0.326
0.6	1.898	0.689	0.427	0.310
0.9	1.571	0.628	0.401	0.296
1.2	1.353	0.579	0.379	0.283
1.5	1.195	0.539	0.360	0.272
1.8	1.075	0.505	0.343	0.262
2.1	0.980	0.476	0.328	0.252
2.4	0.902	0.450	0.315	0.244

Soil conductivity = 1.4 W/(m·K); insulation is over entire depth. For other soil conductivities and partial insulation, use Equation (37).

- Assuming **concrete** walls with **uniform insulation**

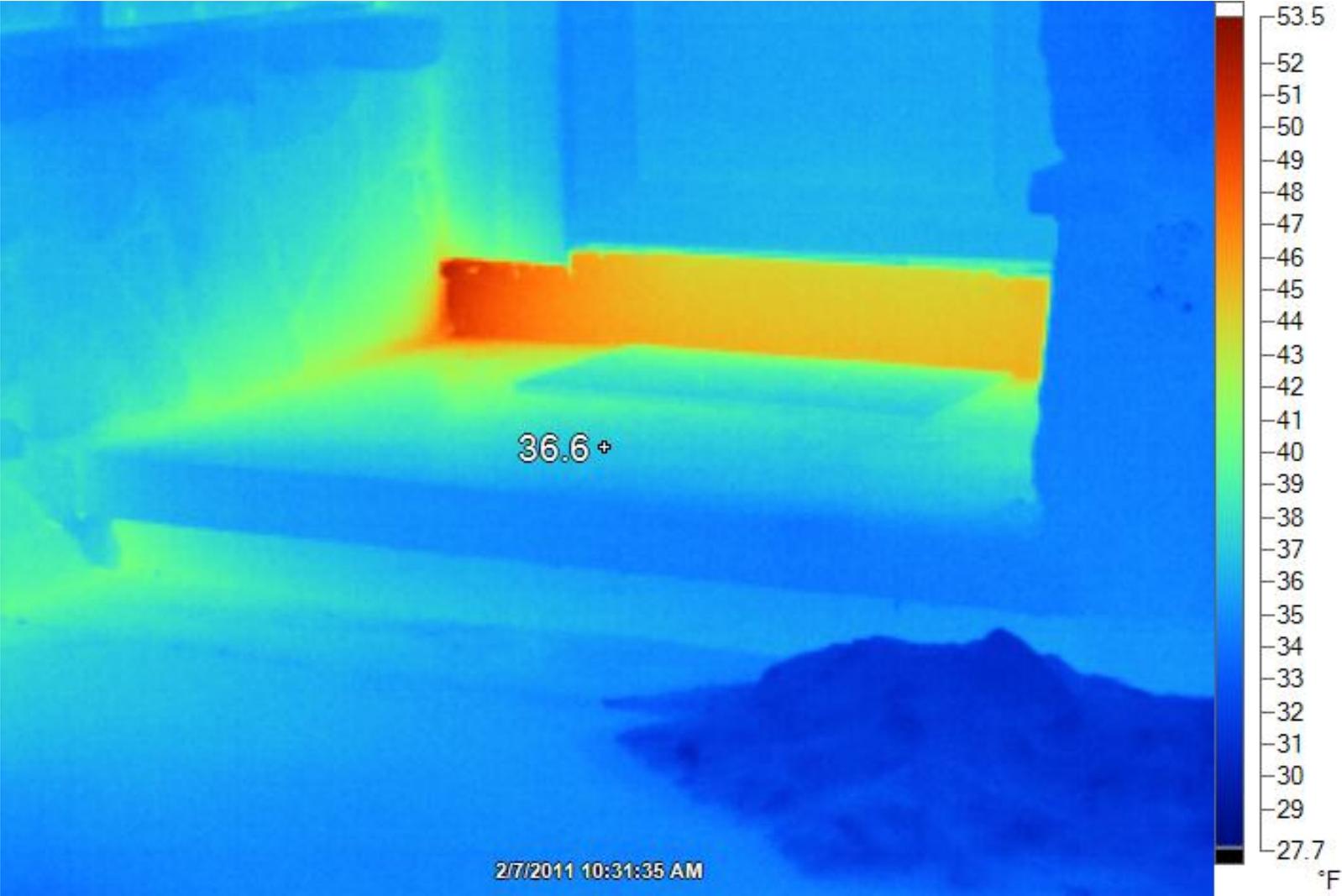
On-grade heat transfer



On-grade heat transfer

- Heat transfer for slab-on-grade floors
 - Concrete slabs can be heated or unheated
 - In either case:
 - Ground is often at a lower temperature than indoor air
 - Soil and concrete are fairly conductive
 - Perimeter can be exposed directly to outdoor air
 - It turns out that the **perimeter** is often most important for both energy and comfort
 - Need to insulate the perimeter

Slab-on-grade floors



Slab-on-grade floors

- Heat transfer through slab-on-grade floors
 - Function of perimeter of slab (not area)

$$Q = pF_p (T_i - T_o)$$



where T_i and T_o are the inside and outside temps [K]
 p is the perimeter of the exposed floor surface [m]
 F_p is the heat loss coefficient per unit length of
perimeter [W/mK]

Design considerations

- To reduce heat transfer through slab on grade floors, we obviously need to:
 - Reduce the perimeter length, and/or
 - Decrease the heat loss coefficient, F_p
- Decreasing F_p is as simple as adding insulation to the foundation exterior
 - Typically no need to exceed $R = 8$ (IP)

Figure 3. Insulated Form Board Field Installation



Heat loss coefficient: F_p

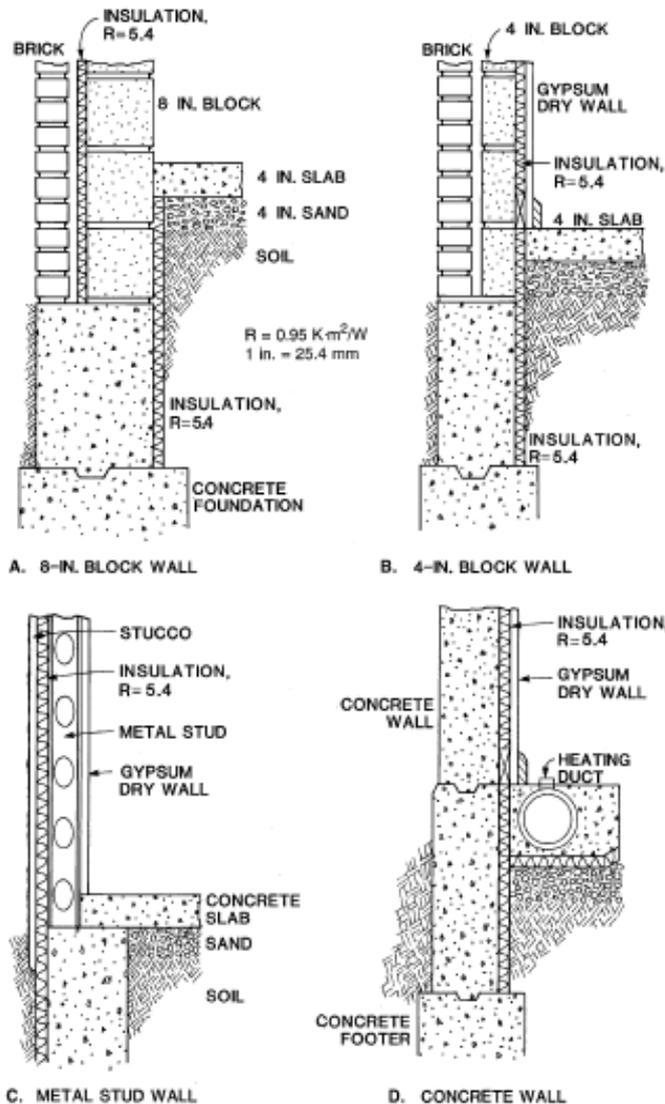


Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

Construction	Insulation	F_p , W/(m·K)
200 mm block wall, brick facing	Uninsulated	1.17
	R-0.95 (m ² ·K)/W from edge to footer	0.86
4 in. block wall, brick facing	Uninsulated	1.45
	R-0.95 (m ² ·K)/W from edge to footer	0.85
Metal stud wall, stucco	Uninsulated	2.07
	R-0.95 (m ² ·K)/W from edge to footer	0.92
Poured concrete wall with duct near perimeter*	Uninsulated	3.67
	R-0.95 (m ² ·K)/W from edge to footer	1.24

*Weighted average temperature of the heating duct was assumed at 43°C during heating season (outdoor air temperature less than 18°C).

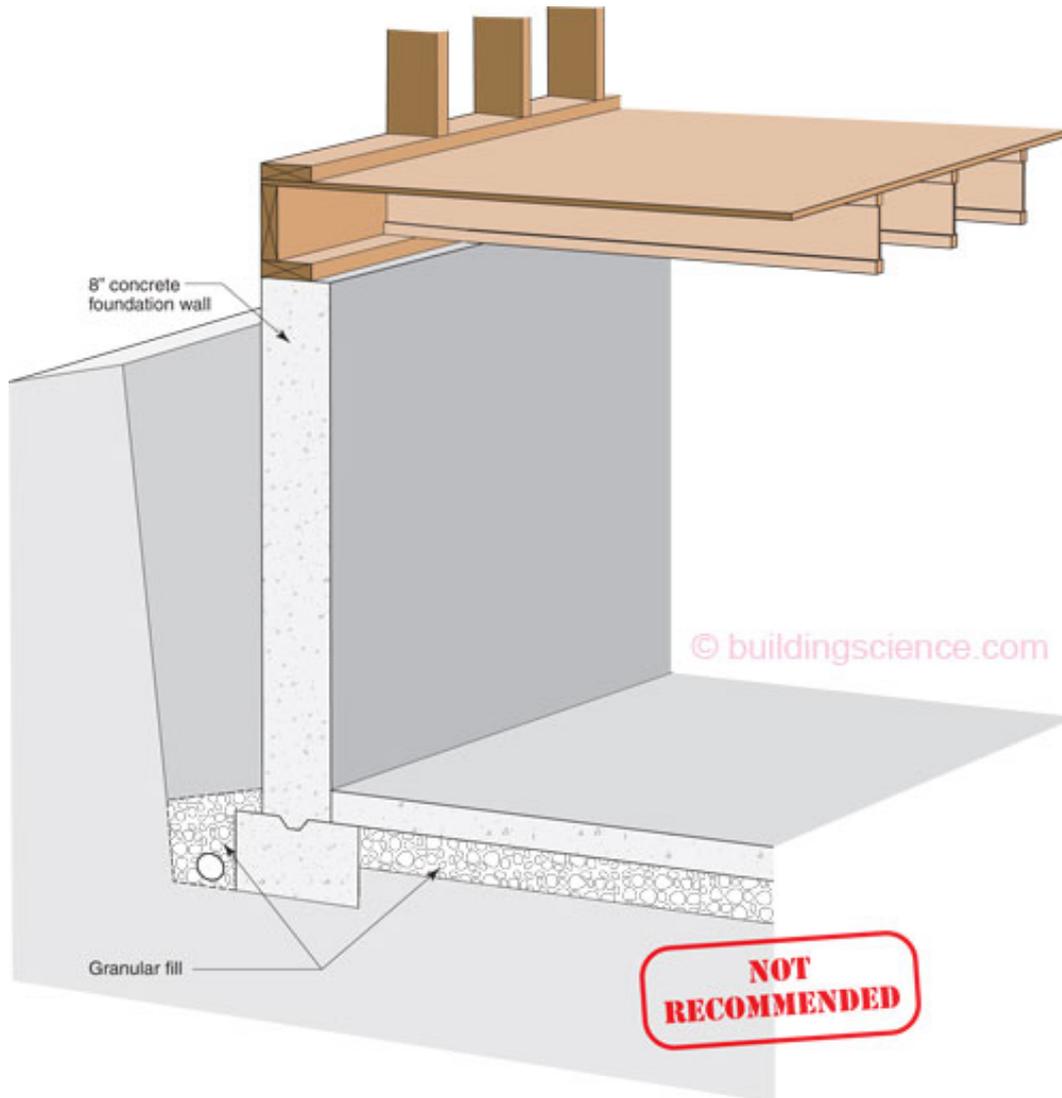
Fig. 8 Slab-on-Grade Foundation Insulation

Using THERM for finding U_{avg}

- You can use THERM to model 2-D heat transfer but be careful ...
 - Must create new convection boundary conditions for interior convection to/from floor
 - Must model a large area of soil around the foundation as a solid with adiabatic boundary conditions
 - Must model outside soil/air interface with new exterior convection
- Basement floor is a 3-D problem and so it cannot be easily modeled in THERM

Slab-on-grade and below-grade enclosures

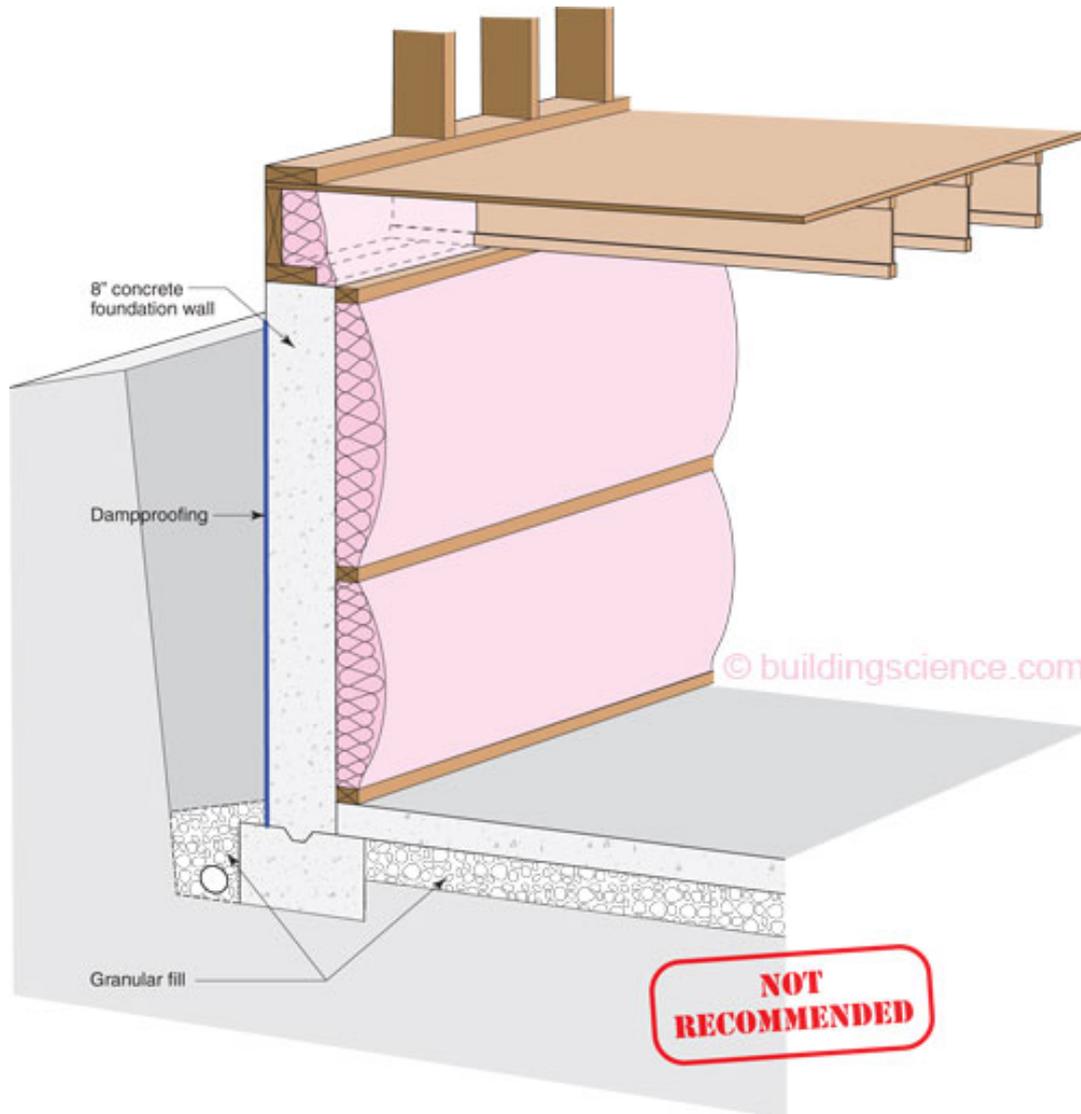
Uninsulated concrete foundation wall and slab



- No thermal control
- Not even allowed by code if basement is conditioned
- No moisture control
- Water vapor diffusion and capillary action are near-constant moisture sources

Slab-on-grade and below-grade enclosures

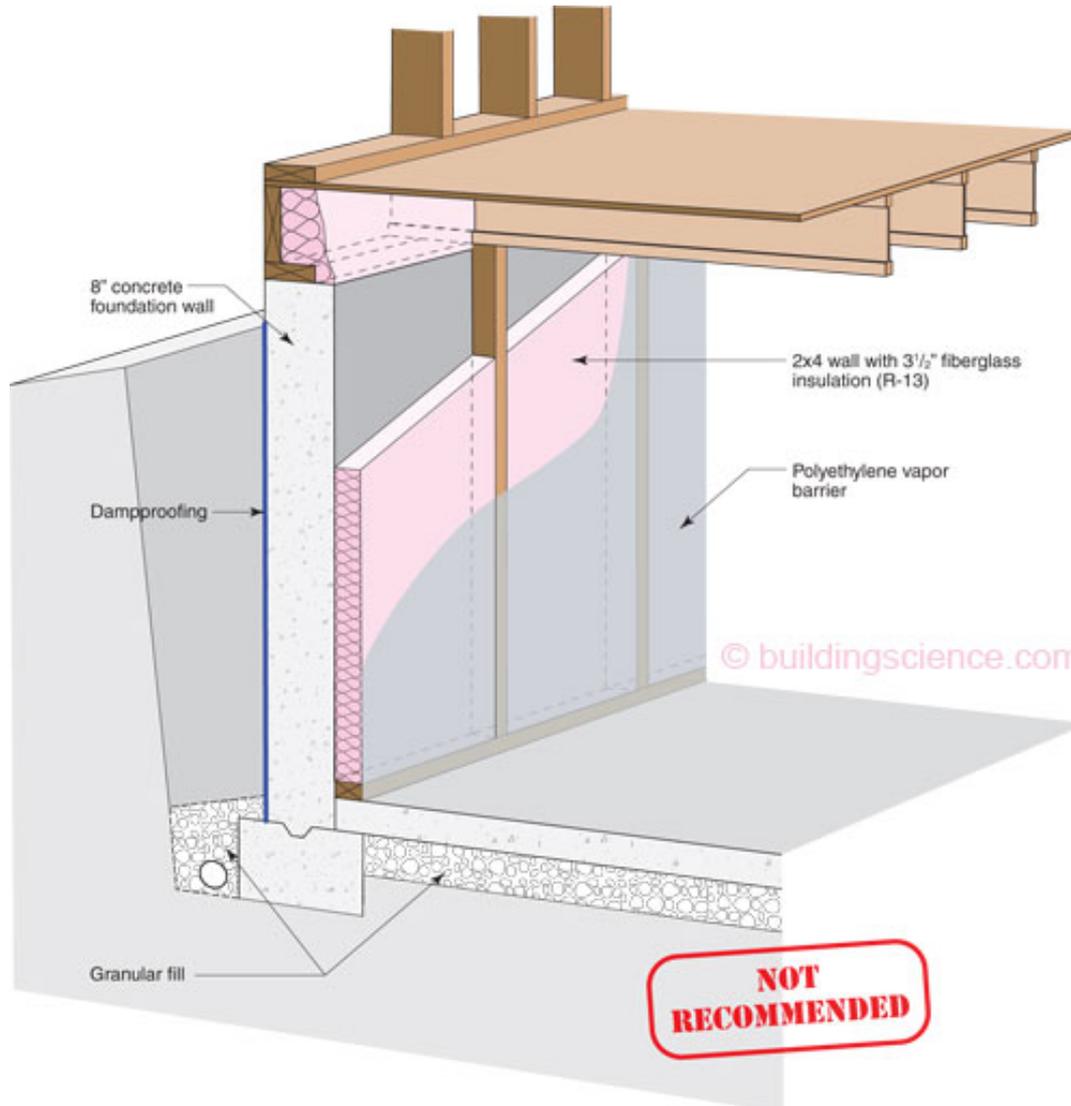
Code minimum R-10 (IP) continuous insulation in a framed wall



- Slab not insulated
- Better thermal control
- Inexpensive
- Sometimes wall insulation batt is covered with vapor barrier
- Moisture issues (batt is air and vapor permeable)
- High RH at concrete wall most of the year

Slab-on-grade and below-grade enclosures

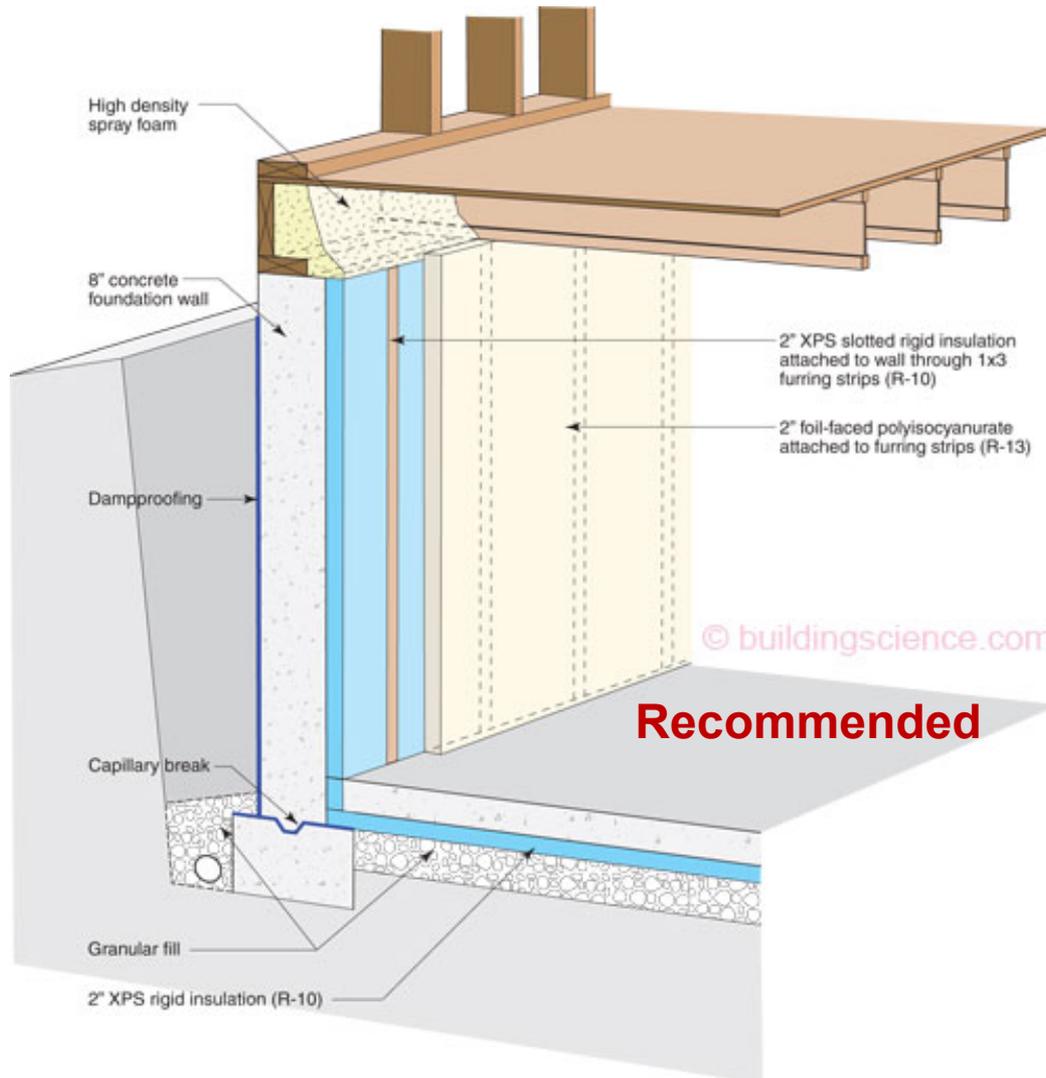
R-13 (IP) insulation in a 2x4 framed wall



- Similar to last construction
- Moisture issues
- High RH at concrete wall most of the year
- Particularly a problem if there is any air leakage

Slab-on-grade and below-grade enclosures

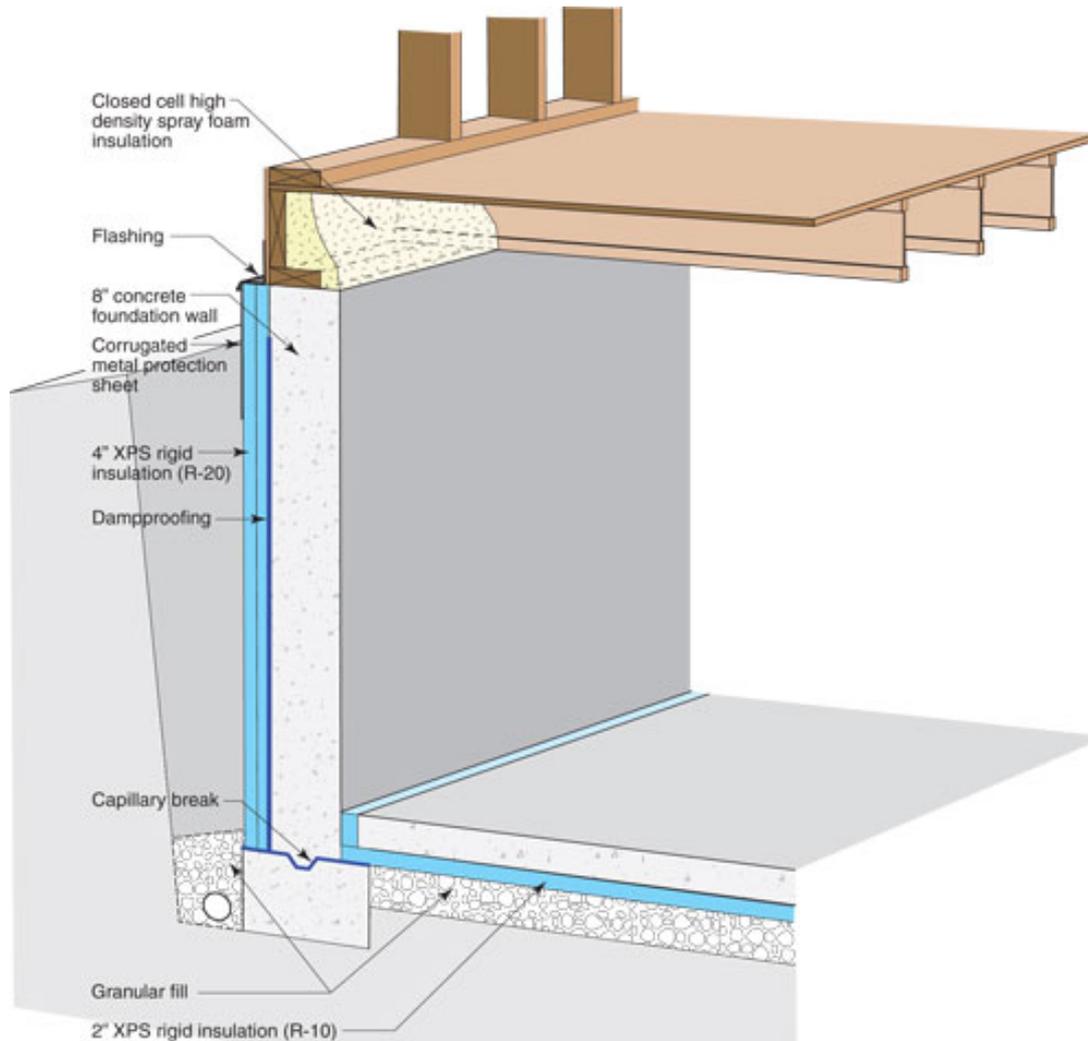
2" XPS rigid insulation + 2" foil-face polyisocyanurate foam board



- Very good thermal control (R-23 walls)
- Water vapor diffusion is prevented
- Capillary action is prevented by the thermal/capillary break at the edge of the slab and top of footing

Slab-on-grade and below-grade enclosures

Rigid XPS exterior insulation



- Very good thermal control (R-20 walls)
- Exterior insulation can be joined with first floor insulation
- Excellent resistance to vapor diffusion
- Capillary action is a potential problem (through the footing)
 - Need a break
- Exposed concrete provides moisture buffer after it dries
- May be hard to construct 139