CAE 463/524
Building Enclosure Design
Spring 2015

Lecture 3: February 3, 2015
Introduce surface energy balances
Solar orientation and radiation modeling
Begin complex conduction

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Campus projects: Expectations uploaded to BB

- Need to do thermal assessments by early March
  - 20 students in this class: 5 teams of 4

Course	Name	Major	Level	Campus building
CAE463-01	Behrens, Maria C.	ARCE	U4	Alumni
CAE463-01	Geoghegan, Thomas	ARC2	GR	SSV
CAE463-01	Irazabal, Carlos H.	ARCE	U4	Crown
CAE463-01	Jung, Yun Joon	ARCE	U4	Crown
CAE463-01	Lis, Kimberly A.	ARCE	U5	SSV
CAE463-01	Ng, Yin Ling	ARCE	U5	Alumni
CAE463-01	Theisen, Whitney A.	ARCE/EMGT	U5	Alumni
CAE463-01	Zanzinger, Zachary D.	ARCE	U5	
CAE524-01	Carrillo Garcia, Jose	ARCE	GR	Crown
CAE524-01	Dorn, Lawrence E.	CM	GR	SSV
CAE524-01	Erukulla, Dilip Kumar	ARCE	GR	
CAE524-01	Liang, Jinzhe	CE	GD	E1 – Rettaliata Eng. Center
CAE524-01	Mullin, Elizabeth M.	ARCE/ARCE	U5	Alumni
CAE524-01	Tuz, Oleg	CM	GR	Crown
CAE524-02	Chandler, Julie A.	ARCE	GR	E1 – Rettaliata Eng. Center
CAE524-02	Chung, Allan	CM	GR	
CAE524-02	Fortune, Roger G.	ARCE	GR	E1 – Rettaliata Eng. Center
CAE524-02	Gadani, Dhaval S.	ARCH/CM	U5	
CAE524-02	Jarosz, Michelle M.	STE	GR	SSV
CAE524-02	Linn, Rebecca C.	ARCE	GR	E1 – Rettaliata Eng. Center

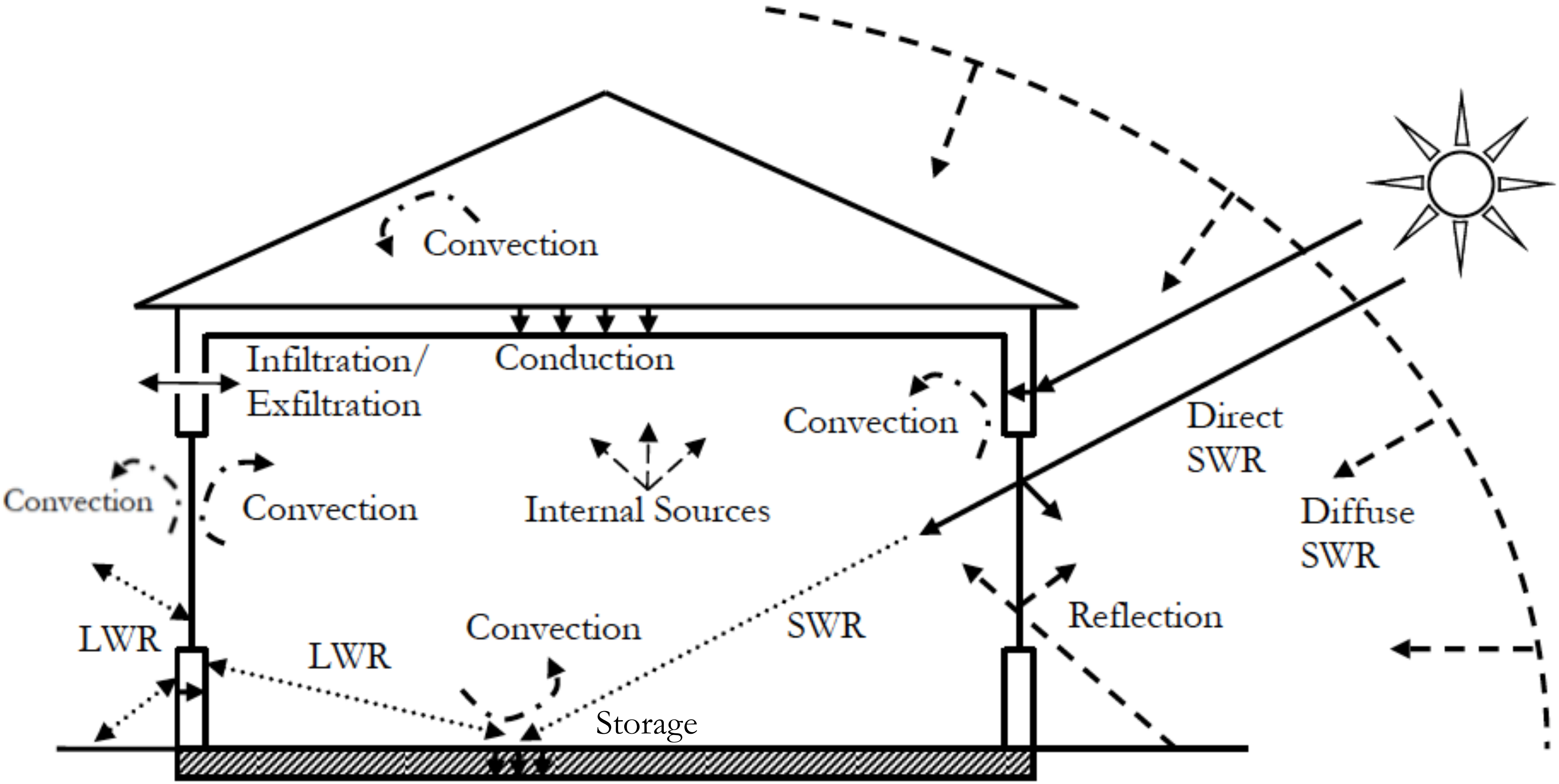
Last time (Jan 20th)

- Review of building science
  - Psychrometrics
  - Individual modes of heat transfer
  - 4 example problems of individual modes of heat transfer

Today's objectives

- Bring all the heat transfer modes together to introduce surface energy balances
- Solar orientation and enclosures
- Begin more complex conduction in building enclosures
- Assign HW #1

Building enclosures and heat transfer, visualized



Heat transfer in building science: Summary

Conduction

$$q = \frac{k}{L}\left(T_{surf,1} - T_{surf,2}\right)$$

$$\frac{k}{L} = U = \frac{1}{R}$$

$$R_{total} = \frac{1}{U_{total}}$$

$$R_{total} = R_1 + R_2 + R_3 + \ldots$$

For thermal bridges and combined elements:

$$U_{total} = \frac{A_1}{A_{total}} U_1 + \frac{A_2}{A_{total}} U_2 + \ldots$$

Convection

$$q_{conv} = h_{conv}\left(T_{fluid} - T_{surf}\right)$$

$$R_{conv} = \frac{1}{h_{conv}}$$

Radiation

Long-wave

$$q_{1\rightarrow 2} = \frac{\sigma\left(T^4_{surf,1} - T^4_{surf,2}\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{12}}}$$

$$q_{rad,1\rightarrow 2} = h_{rad}\left(T_{surf,1} - T_{surf,2}\right)$$

$$h_{rad} = \frac{4\sigma T^3_{avg}}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad R_{rad} = \frac{1}{h_{rad}}$$

Or more simply: $q_{1\rightarrow 2} = \varepsilon_{surf}\sigma F_{12}\left(T^4_{surf,1} - T^4_{surf,2}\right)$

Solar radiation: (opaque surface) $q_{solar} = \alpha I_{solar}$

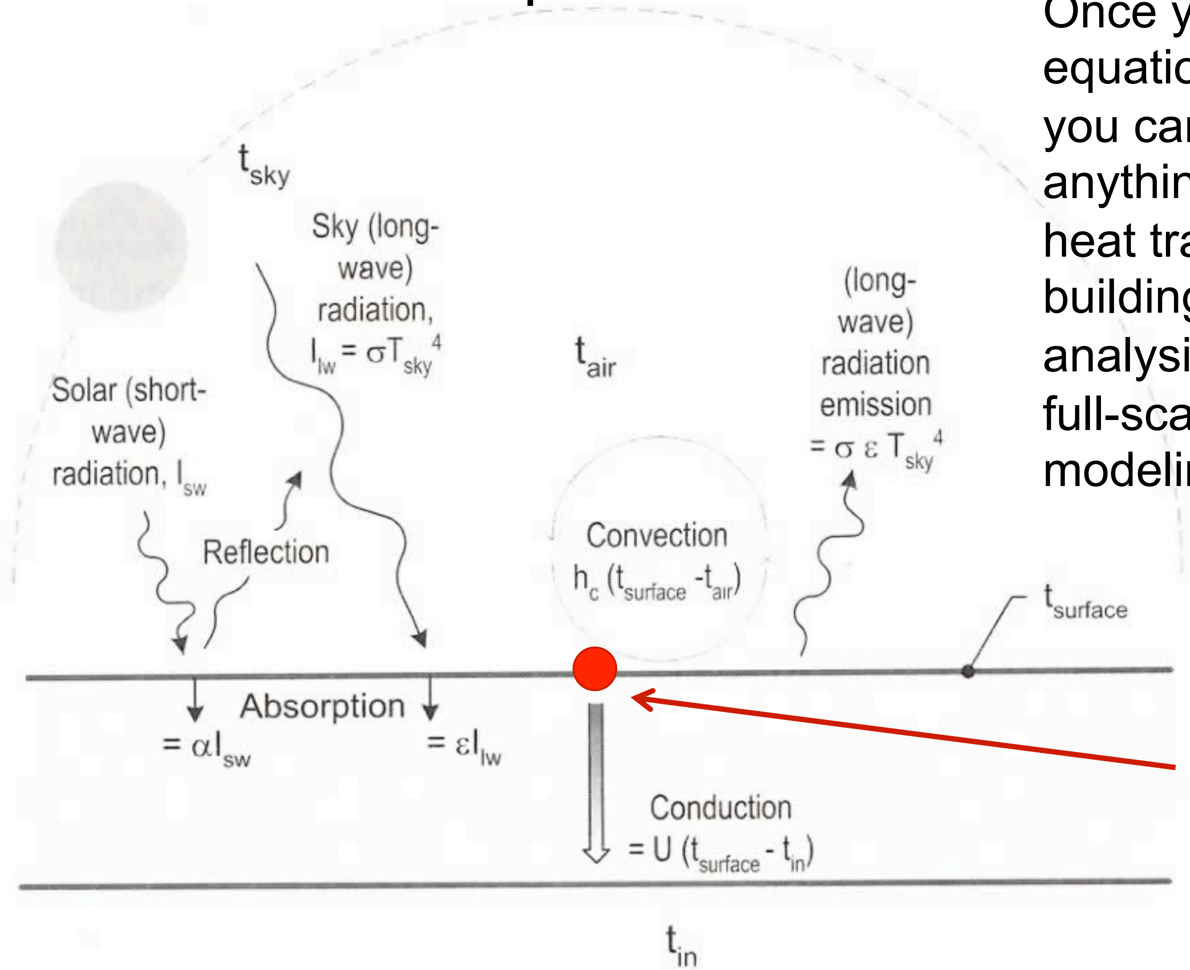
Transmitted solar radiation: (transparent surface) $q_{solar} = \tau I_{solar}$

Combined heat transfer

- Heat transfer to/from a surface is dominated by one or more modes of heat transfer
- In cavities (window spaces, wall cavities, crawl spaces), convection and radiation may be of similar magnitudes
  - So, heat transfer is fairly complicated
- We need to be able to describe all heat transfer mechanisms acting on each surface of an enclosure to understand how the enclosure affects heat, air, and moisture performance

Surface energy balance: Bringing all the modes together

- Exterior surface example: **Roof**



Once you have this equation described, you can do just about anything regarding heat transfer in building enclosure analysis, leading into full-scale energy modeling

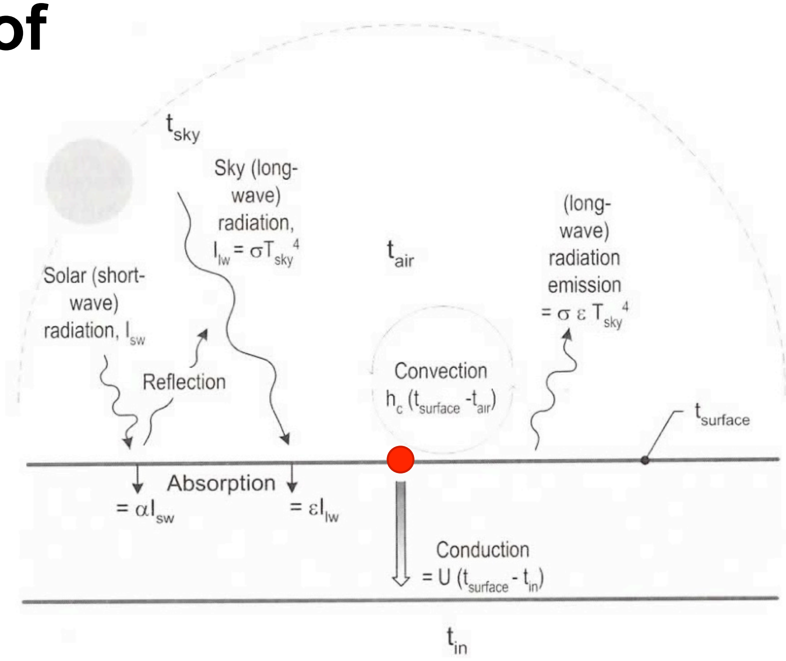
Steady-state energy balance at this exterior surface:
What enters must also leave (no storage)

$$\boxed{q_{solar} + q_{longwaveradiation} + q_{convection} - q_{conduction} = 0}$$

Surface energy balance: Bringing all the modes together

- Exterior surface example: **Roof**

$$\boxed{\sum q = 0}$$



We can use this equation to estimate indoor and outdoor surface temperatures

At steady state, net energy balance is zero

- Because of T^4 term, often requires iteration

Solar gain	αI_{solar}	$q_{sw,solar}$
Surface-sky radiation	$+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surface}^4)$	$+q_{lw,surface-sky}$
Convection on external wall	$+h_{conv}(T_{air} - T_{surface})$	$+q_{convection}$
Conduction through wall	$-U(T_{surface} - T_{surface,interior}) = 0$	$-q_{conduction} = 0$

A note on sign conventions

- Move from left to right (or top to bottom)
- Assume that the temperature to the left (or upstream) is higher than the temperature to the right (or downstream)
  - The signs will work themselves out and let you know if that is not the case
  - Just be consistent!

A note on sky temperatures

A note on sky temperatures

- There are many ways to get "apparent sky temperatures"
  - Varying levels of detail and accuracy
- For a partly cloudy night sky: $T_{sky} = T_{air}\left[0.8 + \frac{(T_{dewpoint} - 273)}{250}\right]^{1/4}$
  - For 50% cloud cover
- For daytime: $T_{sky} = \left(\varepsilon_{sky} T_{air}^{4}\right)^{0.25}$

$$\varepsilon_{sky} = \left[0.787 + 0.764\ln\left(\frac{T_{dewpoint}}{273}\right)\right]\left(1 + 0.0224N - 0.0035N^2 + 0.00028N^3\right)$$

  - For a clear sky: $N = 0$
  - For 50% cloud cover, $N = 0.5$

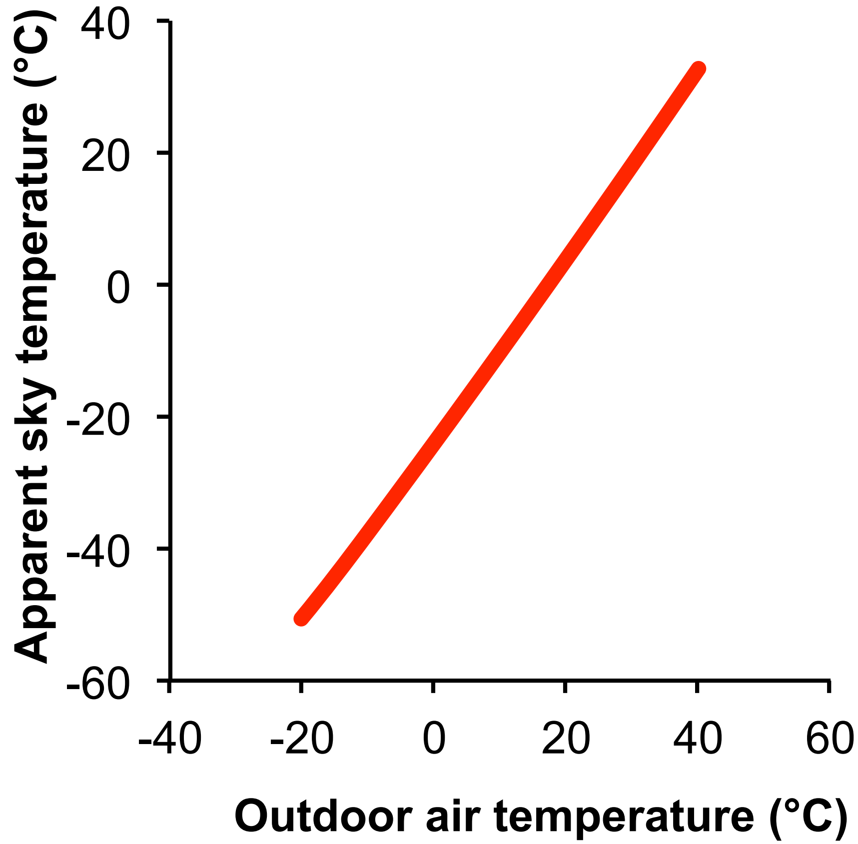
Where N = cloud cover (tenths)

Simpler sky temperatures

- Other models estimate apparent sky temperatures ignoring differences in water vapor:

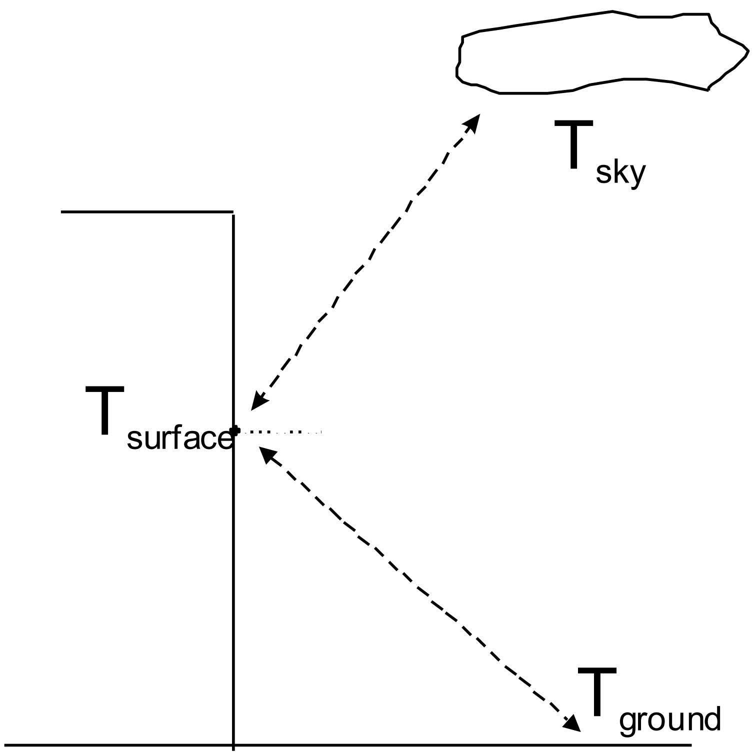
$$T_{\mathrm{sky}} = 0.0552T_{\mathrm{at}}^{1.5}$$

Where T_{sky} is in K and T_{at} is ambient air temperature $[K]$



Typical view factors, F_{1-2}

- Some typical view factors from surfaces to ground or sky



View ("shape") factors for:

Vertical surfaces:

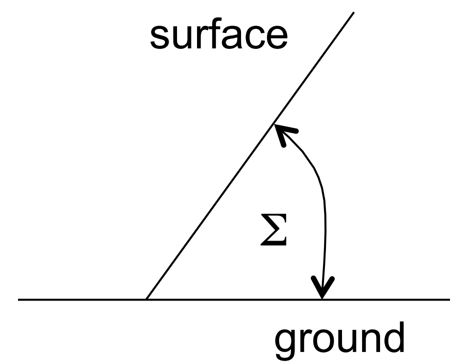
- To sky ($F_{surface-sky}$) 0.5
- To ground ($F_{surface-ground}$) 0.5

Horizontal surfaces:

- To sky ($F_{surface-sky}$) 1
- To ground ($F_{surface-ground}$) 0

3) Tilted surfaces

- To sky (1+cosΣ)/2
- To ground (1-cosΣ)/2



Typically assume:

$$T_{ground} = T_{air}$$

*Note that other surrounding buildings complicate view factors, but their net temperature differences probably aren't that different so long-wave radiation can be negligible

Example: Roof surface temperature

- Estimate the surface temperature that might be reached by a bituminous roof (absorptivity of 0.9) installed over a highly insulating substrate (R-20 IP) exposed to intense sun (q_{solar} = 1000 W/m^2) on a calm, cloudless day with an ambient temperature of 20°C, RH = 30%, and wind speed of 2 m/s
  - Indoor surface temperature is 22°C

- What happens if surface absorptivity is reduced to 0.3?
- What happens if wind speed increases to 6 m/s?
- What happens if insulation value decreases to R-3?

Example: Solution

Surface energy balance	*Add*	*Subtract*
	W/m^2	W/m^2
Solar (short-wave)	900	
Surface-sky long-wave radiation	-338	
Convection on roof	-551	
Conduction through roof		11
SUM	0	

Given	alpha	0.9	bituminous membrane		
Given	Itotal, W/m2	1000			
Assume	Fsurface-sky	1			
Assume	e,surface	0.9			
Given	Tair,out, K	293.15	20	degC	
Assume	Tair,out,dewpoint, K	275.06	1.91	degC	*psych chart*
Calculate	e,sky	0.79	N = 0		
Calculate	Tsky, K	276.61	Tsky equation for clear day		
Guess	**Tsurface, K**	**334.25**	**61.1**	**degC**	
Given	Tsurf,in, K	295.15	22.0	degC	
Constant	stef-boltz, W/(m2K4)	5.6704E-08			
Calculate	hconv, Wm2K	13.4			
Given	R-value IP, h-ft2-F/Btu	20			
Given	R-value, SI	3.52			
Given	U-value, W/m2K	0.28			

Adjust $T_{surface}$ until sum of all heat transfer modes equals zero

Example: Solution (low absorptivity)

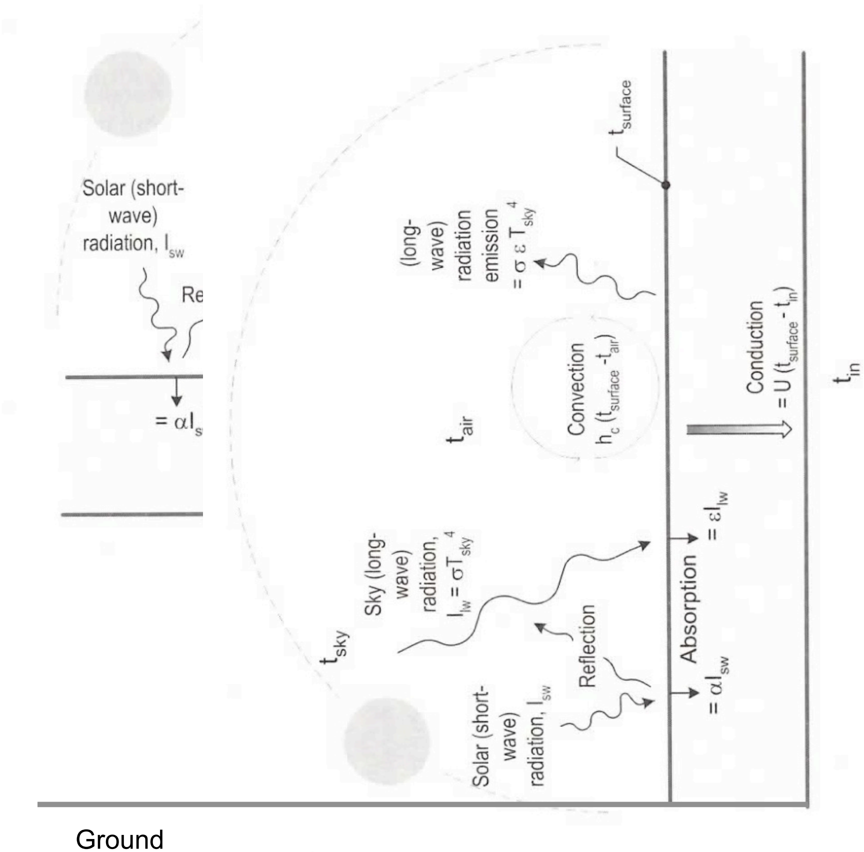
Surface energy balance	Add	Subtract
	W/m^2	W/m^2
Solar (short-wave)	300	
Surface-sky long-wave radiation	-141	
Convection on roof	-155	
Conduction through roof		3
SUM	0	

Given	alpha	0.3	bituminous membrane		
Given	Itotal, W/m2	1000			
Assume	Fsurface-sky	1			
Assume	e,surface	0.9			
Given	Tair,out, K	293.15	20	degC	
Assume	Tair,out,dewpoint, K	275.06	1.91	degC	*psych chart*
Calculate	e,sky	0.79	N = 0		
Calculate	Tsky, K	276.61	Tsky equation for clear day		
Guess	**Tsurface, K**	**304.75**	**31.6**	**degC**	
Given	Tsurf,in, K	295.15	22.0	degC	
Constant	stef-boltz, W/(m2K4)	5.6704E-08			
Calculate	hconv, Wm2K	13.4			
Given	R-value IP, h-ft2-F/Btu	20			
Given	R-value, SI	3.52			
Given	U-value, W/m2K	0.28			

Surface energy balance: Bringing all the modes together

- Similarly, for a vertical surface:

$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$



$$\alpha I_{solar}$$
$$+\varepsilon_{surface} \sigma F_{sky} (T_{sky}^4 - T_{surface}^4)$$
$$+\varepsilon_{surface} \sigma F_{ground} (T_{ground}^4 - T_{surface}^4)$$
$$+h_{conv} (T_{air} - T_{surface})$$
$$-U (T_{surface} - T_{surface,interior}) = 0$$

Bringing all modes (and nodes) together

- For an example room like this, you would setup a system of equations where the temperature at each node (either a surface or within a material) is unknown
  - 12 material nodes + 1 indoor air node

Heat Xfer @ external surfaces:
Radiation and convection

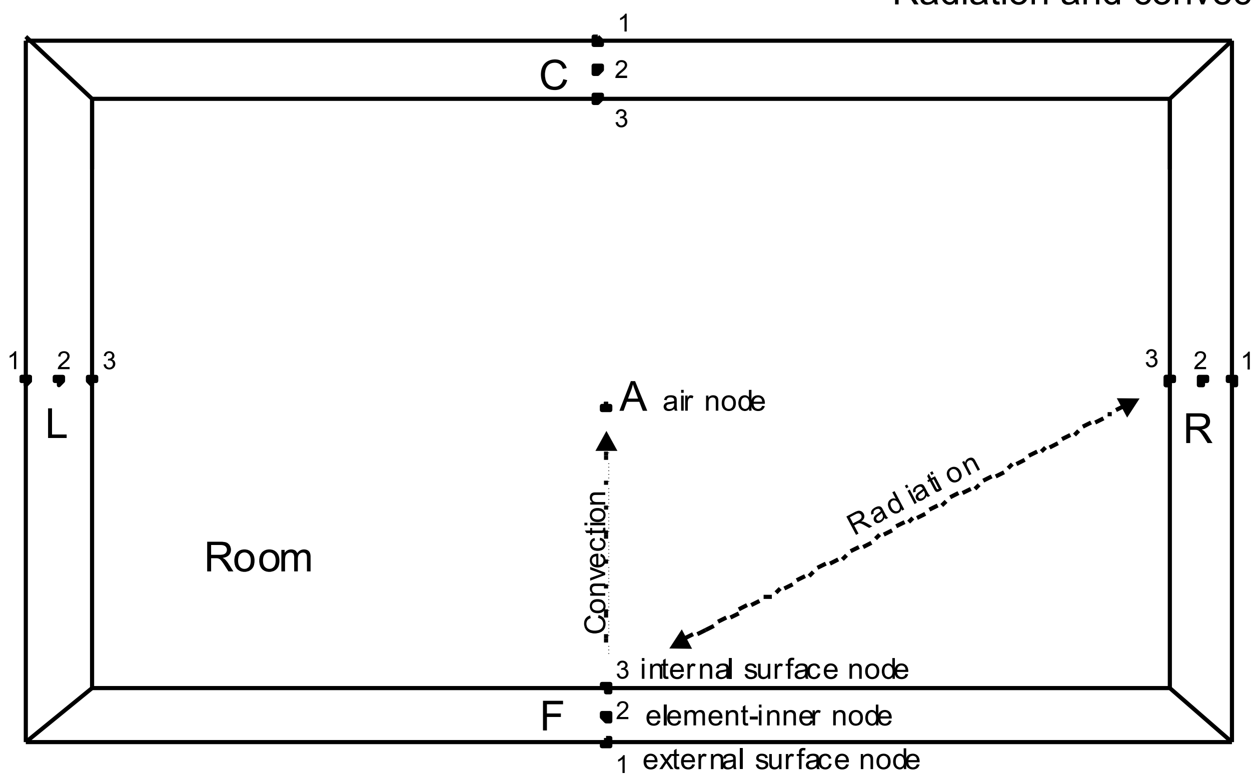
At surface nodes:

$$\sum q = 0$$

At nodes inside materials:

$$mc_p \frac{dT}{dt} = \sum q_{at\ boundaries}$$

Based on density and heat capacity of material (later)

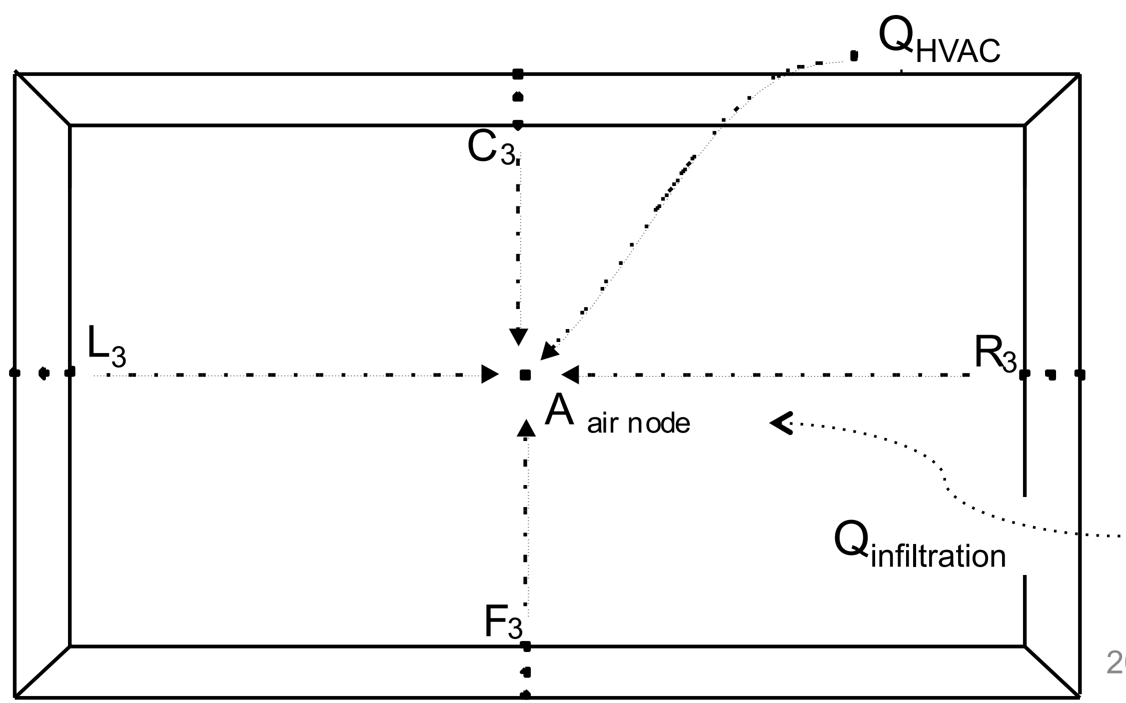


Bringing all modes (and nodes) together

- To get the impact on indoor air temperature (and close the system of equations)
  - Write an energy balance on the indoor air node
  - Air impacted directly only by convection (bulk and surface)

$$(V_{room}\rho_{air}c_{p,air})\frac{dT_{air,in}}{dt} = \sum_{i=1}^{n} h_i A_i \left(T_{i,surf} - T_{air,in}\right) + \dot{m}c_p\left(T_{out} - T_{air,in}\right) + Q_{HVAC}$$

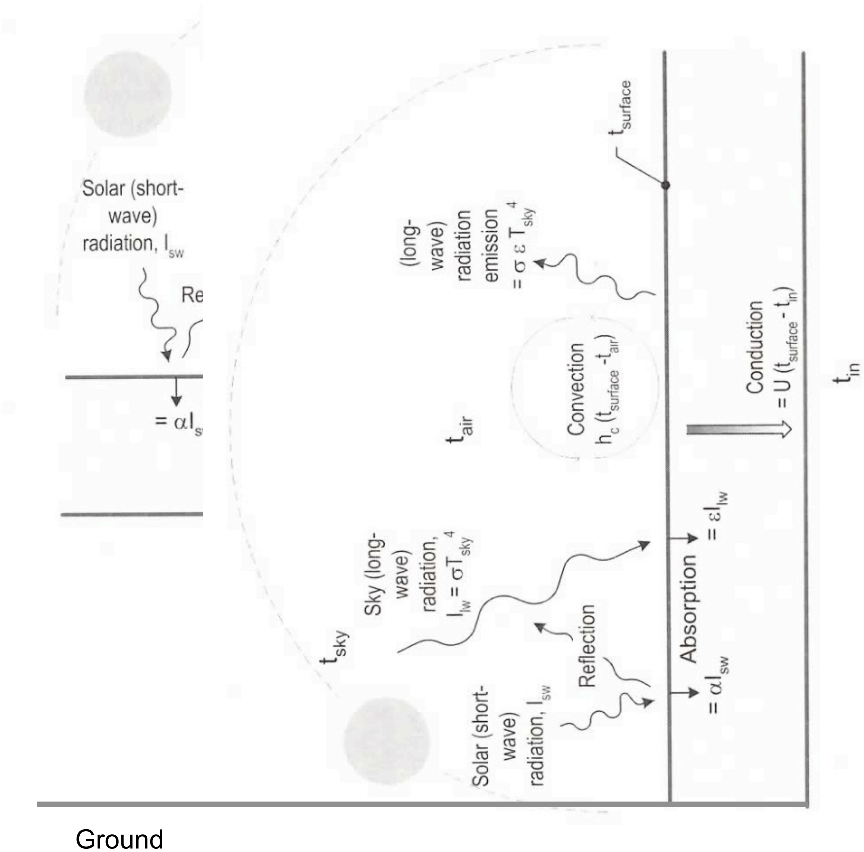
In plain English:
The change in indoor air temperature is equal to the sum of convection from each interior surface plus outdoor air delivery (by infiltration or dedicated outdoor air supply), plus the bulk convective heat transfer delivered by the HVAC system



Bringing all the modes together

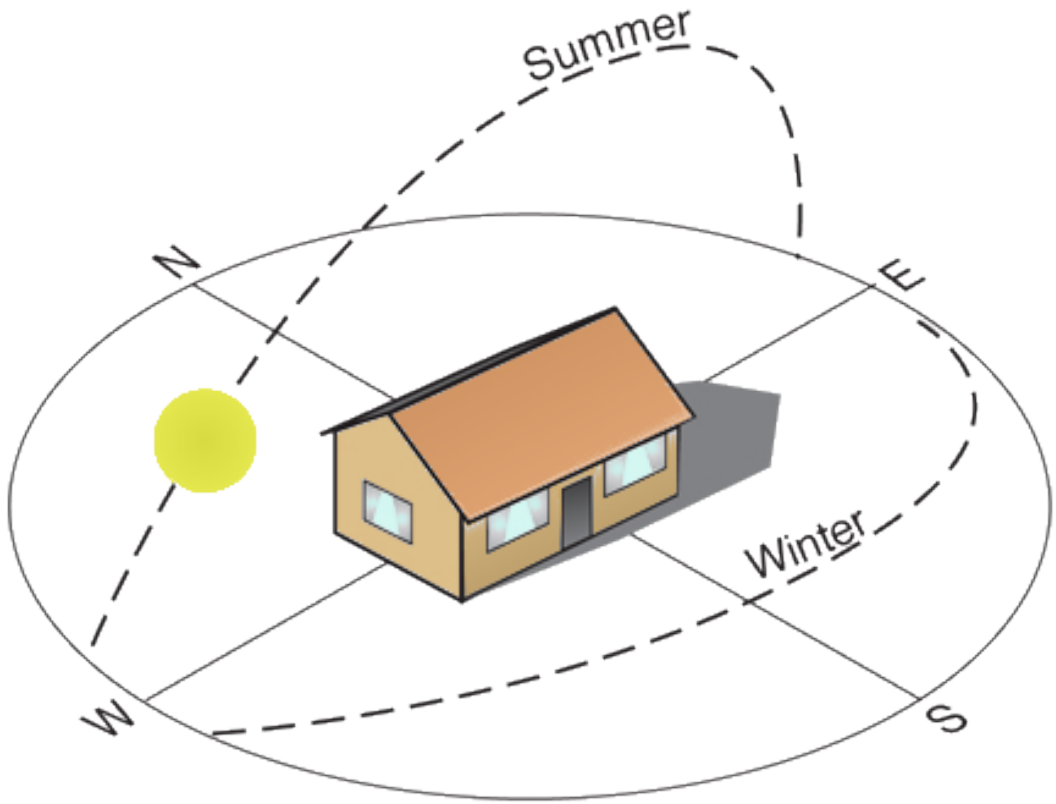
- Similarly, for a vertical surface:

$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$



$$
\begin{aligned}
&\alpha I_{solar} \\
&+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surface}^4) \\
&+\varepsilon_{surface}\sigma F_{ground}(T_{ground}^4 - T_{surface}^4) \\
&+h_{conv}(T_{air} - T_{surface}) \\
&\boxed{-U(T_{surface} - T_{surface,interior})} = 0
\end{aligned}
$$

Next ~2 lectures



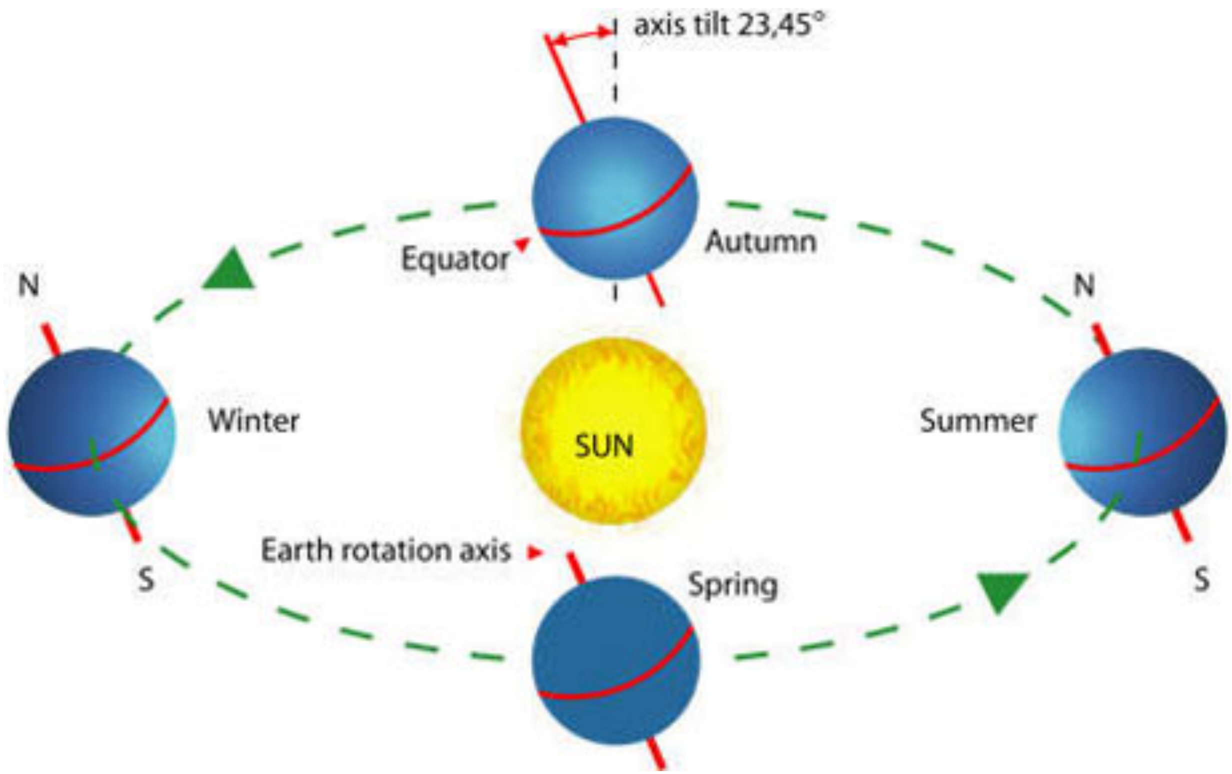
SOLAR ORIENTATION

Solar radiation

- The sun is the source of most energy on the earth
- We need to have a working knowledge of earth's relationship to the sun
- We should be able to estimate solar radiation intensity
  - To understand thermal effects of solar radiation and how to control or utilize them,
  - We need to estimate solar gains on a building, and
  - We need to predict intensity of solar radiation and the direction at which it strikes building surfaces
    - It starts with relationships between the sun and the earth

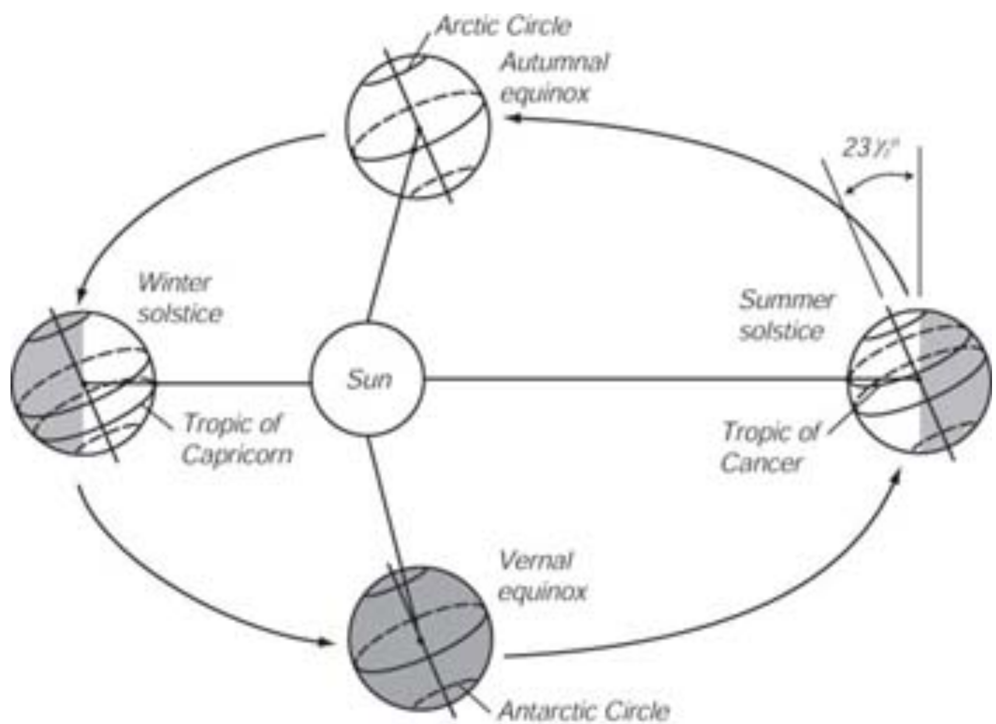
Solar radiation: earth-sun relationship

- Earth rotates about its axis every 24 hours
- Earth revolves around sun every 365.2425 days
- Earth is titled at an angle of 23.45°



Solar radiation: earth-sun relationship

- Therefore, different locations on earth receive different levels of solar radiation during different times of the year (and different times of the day)
  - The greatest amount of solar radiation is delivered to northern hemisphere on **June 21**
  - Least amount of solar energy delivered on **December 21**
- There are methods of determining the amount of flux of solar radiation to surfaces on the earth



Earth-sun relationships

- The position of a point P on the earth's surface with respect to the sun's rays can be calculated if we know:
  - Latitude of point on earth, l (degrees)
  - Hour angle of the point on earth, h (degrees)
  - Sun's declination, d (degrees)

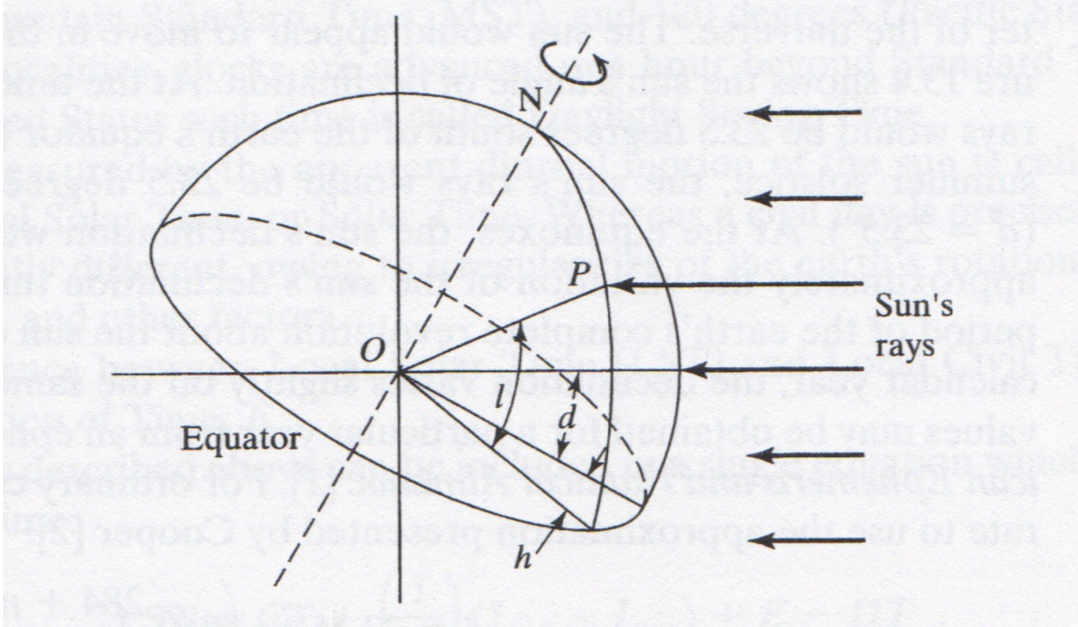


Figure 13.3 Latitude, hour angle, and sun's declination.

Earth-sun relationships

- Sun's declination, d, can be estimated by:

$$d = 23.45 \sin\left(360 \frac{284 + n}{365}\right)$$

Where n is the day of the year, which you can determine by counting on your hands, looking up online, or using this table:

TABLE 13.1 Variation in *n* throughout the Year for Eq. (13.1)

Month	*n* for the Day of the Month, *D*	Month	*n* for the Day of the Month, *D*
January	*D*	July	181 + *D*
February	31 + *D*	August	212 + *D*
March	59 + *D*	September	243 + *D*
April	90 + *D*	October	273 + *D*
May	120 + *D*	November	304 + *D*
June	151 + *D*	December	334 + *D*

Where *D* is the day of the month

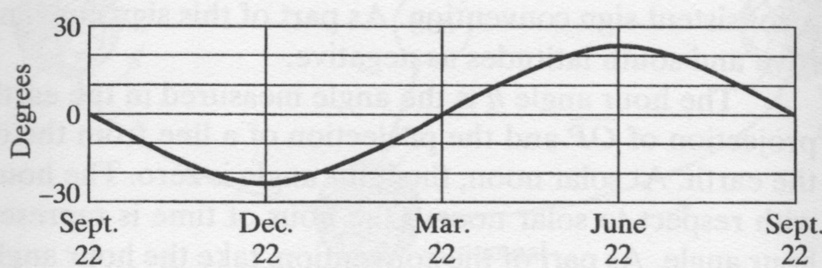
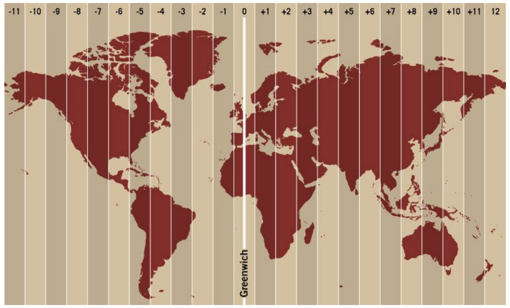


Figure 13.5 Variation of sun's declination.

d is **positive** when sun's rays are **north** of the equator

Earth-sun relationships



- Now we have latitude (l) and sun's declination (d)
  - Need hour angle (h)

It's all about **time**:

- Greenwich Civil Time = time at line of zero longitude
- Local Civil Time (CT) is governed by your longitude
  - 1/15th of an hour (4 mins) of time for each degree difference in longitude
    - Central Standard Time is 90 degrees from 0
    - 4 min per degree * 90 degrees = 360 minutes = 6 hours
- Time is also measured by apparent diurnal motion of the sun
  - Apparent Solar Time (AST), Local Solar Time (LST), or Solar Time (ST)
    - Interchangeable
  - Slightly different than a civil day because of irregularities of the earth's rotation and shape of earth's orbit
  - The difference between solar time (LST) and civil time (CT) is called the

    Equation of Time (E)

Calculating solar time (LST)

- Local **solar** time (LST):

$$\mathrm{LST} = \mathrm{CT} + \left(\frac{1}{15}\right)(L_{\mathrm{std}} - L_{\mathrm{loc}}) + E - DT$$

Where:
LST = local solar time (hour)
CT = clock time (hour)
L_{std} = standard meridian longitude for local time zone (degrees west)
L_{loc} = longitude of actual location (degrees west)
E = Equation of Time (hour)
DT = Daylight savings time correction (hour)
*DT = 1 if on DST; otherwise 0

**Note that all times should be converted to decimal format from 0 to 24. For example, 3:45 PM = 15.75 hours

- Equation of Time: $E = 0.165 \sin 2B - 0.126 \cos B - 0.025 \sin B$

where $B = \dfrac{360(n - 81)}{364}$ and n is the day of the year. *B* is in degrees

Calculating solar time (LST)

- Finally, the solar hour angle, h, can be calculated:

$$h = 15(\text{LST} - 12) \text{ degrees}$$

h is **positive *after*** solar noon and **negative *before***

LST is in 24 hour format

- Again, you can either calculate these values, use a website*, or look them up in a table like this:

TABLE 13.2 The Sun's Declination and Equation of Time, Calculated

	Day 7		Day 14		Day 21		Day 28	
Month	Declination, Degrees	Eq. of Time, Hours	Declination, Degrees	Eq. of Time, Hours	Declination, Degrees	Eq. of Time, Hours	Declination, Degrees	Eq. of Time, Hours
January	−22.4	−0.10	−21.4	−0.15	−20.1	−0.19	−18.5	−0.22
February	−15.8	−0.24	−13.6	−0.24	−11.2	−0.24	−8.7	−0.22
March	−6.0	−0.20	−3.2	−0.17	−0.4	−0.13	2.4	−0.09
April	6.4	−0.04	9.0	−0.01	11.6	0.02	13.9	0.04
May	16.7	0.06	18.5	0.06	20.1	0.06	21.4	0.05
June	22.7	0.02	23.3	0.00	23.45	−0.03	23.3	−0.05
July	22.6	−0.08	21.7	−0.09	20.4	−0.10	18.9	−0.10
August	16.3	−0.09	14.1	−0.07	11.8	−0.04	9.2	−0.01
September	5.4	0.05	2.6	0.09	−0.2	0.13	−3.0	0.17
October	−6.6	0.22	−9.2	0.25	−11.8	0.27	−14.1	0.27
November	−17.1	0.27	−18.9	0.25	−20.4	0.22	−21.7	0.18
December	−22.8	0.12	−23.3	0.07	−23.45	0.02	−23.3	−0.04

*NOAA has website for this: http://www.esrl.noaa.gov/gmd/grad/solcalc/

Calculating solar time (LST) and hour angle (h)

- Example problem:
- Determine the local solar time and sun’s hour angle in Minneapolis, MN (44.9° N, 93.3° W) at 2:25 PM Central Daylight Savings Time on July 21

Earth-sun relationships

- Once we have our local latitude l, the sun's declination angle d, and the hour angle h, we can move on to other important relationships:

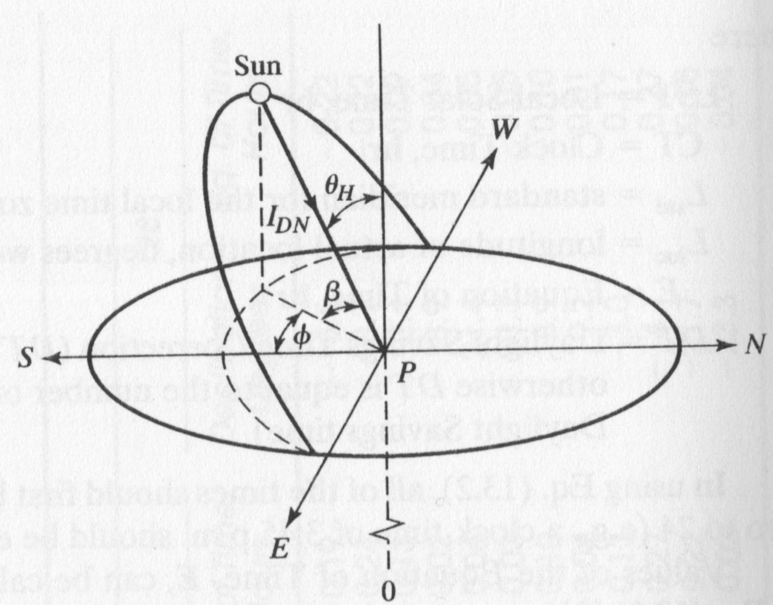


Figure 13.6 Definition of sun's zenith, altitude, and azimuth angles

<u>**Three important angles (°)**</u>

***θ_H* = sun's zenith angle**
angle between the sun's rays and the local vertical

***β* = altitude angle**
angle in a vertical plane between the sun's rays and the projection of the earth's horizontal plane

***ϕ* = solar azimuth angle**
angle in the horizontal plane measured from south to the horizontal projection of the sun's rays

*Note that I_{DN} represents the sun's rays

Earth-sun relationships

- Relationships between l, h, d, and θ_H, β, and ϕ can all be described in this figure:

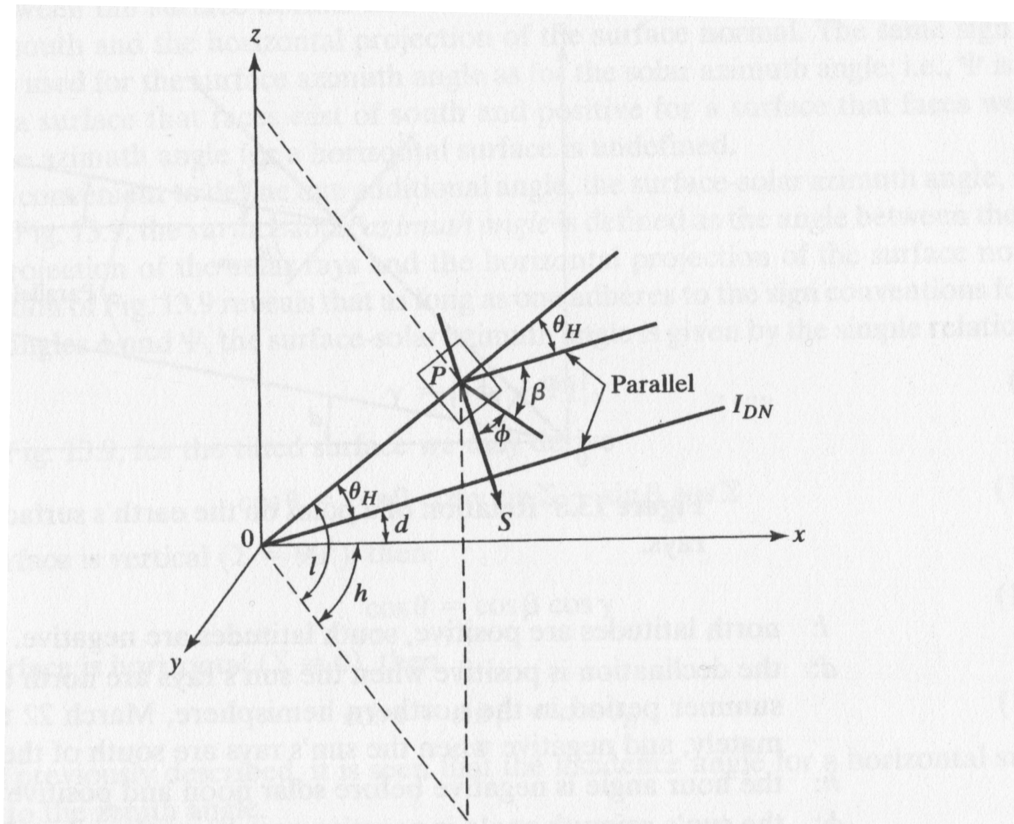


Figure 13.7 Relation of a point on the earth's surface to sun's rays.

Don't worry if this doesn't all make sense; there are formulas!

Determining solar angles

- After a lot of complex geometry/trigonometry…

$$\cos \theta_H = \cos l \cos h \cos d + \sin l \sin d$$

$$\sin \beta = \cos l \cos h \cos d + \sin l \sin d$$

$$\cos \phi = (\cos d \sin l \cos h - \sin d \cos l) / \cos \beta$$

A note on sign conventions for all of these relationships:
North latitudes (l) are positive, south latitudes are negative
Declination (d) is positive when sun's rays are north of equator
Hour angle (h) is negative before solar noon, positive after
Azimuth angle (ϕ) is negative east of south and positive west of south

Note that β for solar noon = 90 degrees - $| l - d |$
Also note that $\beta + \theta_H$ = 90 degrees

**Keep units consistent

Earth-sun relationships

- Last but not least…
- The previous relationships identify a point on the earth's surface in relation to the sun
  - All valid for horizontal surfaces
  - Buildings are not horizontal surfaces!
- Need to describe surface-sun relationships:

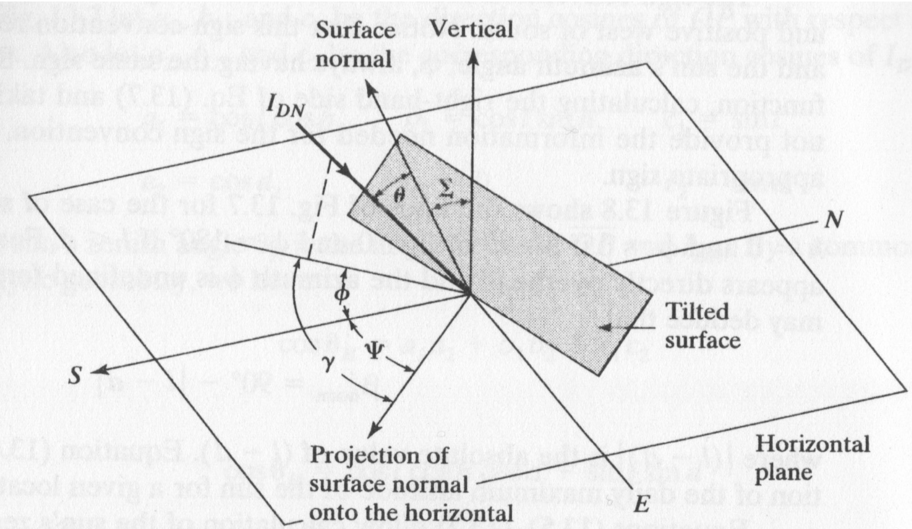


Figure 13.9 Definitions of surface azimuth, surface tilt, and surface-solar azimuth angles and the relation of sun's rays to a tilted surface.

Surface-sun relationships

<u>More important angles (°)</u>
θ = incidence angle
angle between the solar rays and the surface normal

Σ = surface tilt angle
angle between surface normal and the vertical
Vertical surface: Σ = 90°
Horizontal surface: Σ = 0°

Ψ = surface azimuth angle
angle between south and the horizontal projection of the surface normal

γ = surface-solar azimuth angle
angle between horizontal projection of solar rays and the horizontal projection of the surface normal
$\gamma = | \phi - \Psi |$

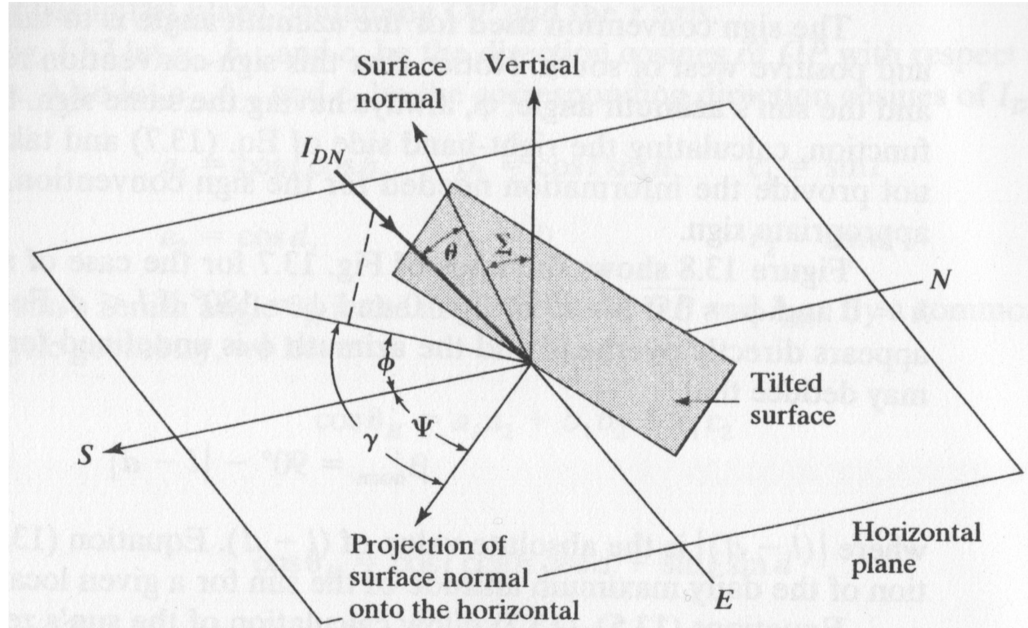


Figure 13.9 Definitions of surface azimuth, surface tilt, and surface-solar azimuth angles and the relation of sun's rays to a tilted surface.

*Sign convention: Ψ is negative for a surface that faces east of south and positive for a surface that faces west of south

Tilted surface:
$\cos\theta = \cos\beta \cos\gamma \sin\Sigma + \sin\beta \cos\Sigma$

Vertical surface (Σ = 90°):
$\cos\theta = \cos\beta \cos\gamma$

Surface-sun relationships: Example problem

- Calculate sun's altitude (β) and azimuth (ϕ) angles at 7:30 am local solar time (LST) on August 7 for a location at 40 degrees north latitude

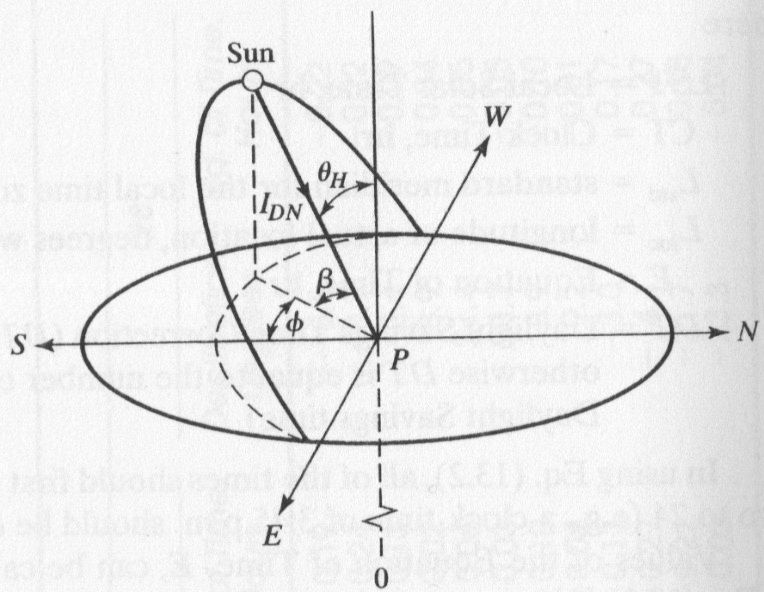


Figure 13.6 Definition of sun's zenith, altitude, and azimuth angles

Surface-sun relationships: Example problem

- Calculate sun's incidence angle for a vertical surface that faces 25 degrees east of south and has a tilt angle of 60 degrees at 3:00 pm local solar time on June 7 for a location at 36 degrees north latitude

Translation:

Find θ

Given:

Ψ, Σ, l, h, β, ϕ

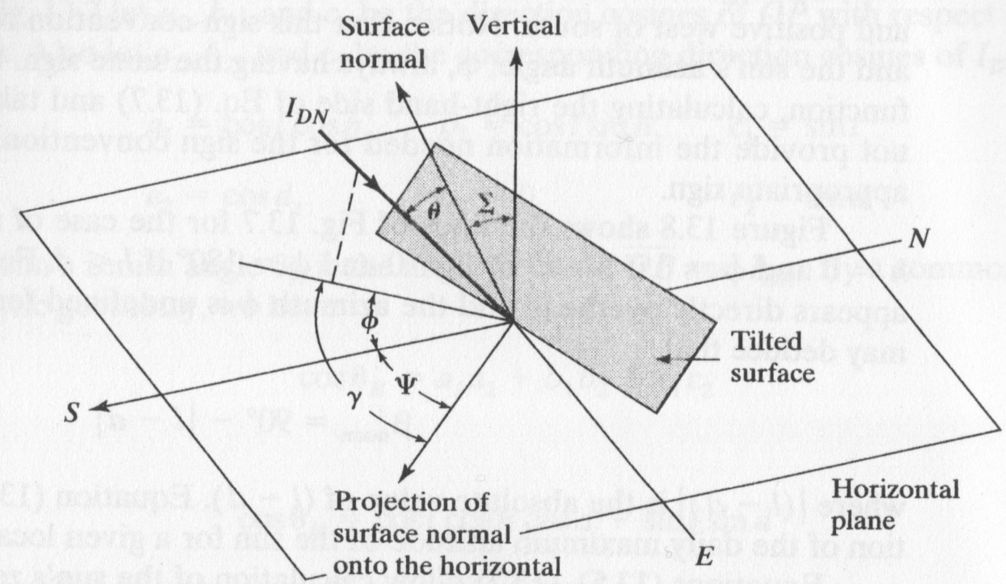


Figure 13.9 Definitions of surface azimuth, surface tilt, and surface-solar azimuth angles and the relation of sun's rays to a tilted surface.

What is this all about? … Solar flux

- Once we know earth-surface-sun relationships, we can eventually get to the effects of those relationships on actual solar radiation

- Solar radiation intensity is roughly constant at the outer layer of the atmosphere
  - 1367 W/m^2 – varying a few percent depending on time of year

- The earth's atmosphere depletes some direct solar radiation
  - Intercepted by other air molecules, water molecules, dust particles
  - Remaining reaches earth's surface unchanged in wavelength
    - Direct radiation
  - The deflected radiation turns aside from the direct beam
    - Diffuse radiation

Estimating solar flux

- Estimating intensity of direct normal solar radiation:
  - There are many, *many* ways to estimate this
  - ASHRAE uses a model for “average clear days” that works well for most of our purposes

$$I_{DN} = Ae^{-B/\sin\beta}$$

Where:

I_{DN} = direct normal irradiance, or amount of solar radiation per unit area on a surface that is always held perpendicular to the sun’s rays (W/m^2)

A = apparent direct normal solar flux at outer edge of earth’s atmosphere (W/m^2)

B = empirically determined atmospheric extinction coefficient (dimensionless)

β = altitude angle

- Estimating intensity of diffuse horizontal radiation: $I_{dH} = CI_{DN}$

Where:

I_{dH} = diffuse horizontal irradiance, or that which is scattered (W/m^2)

C = empirically determined coefficient for typical “clear days” (dimensionless)

Typical clear day values for solar radiation

TABLE 13.3 Coefficients for Average Clear Day Solar Radiation Calculations for the Twenty-First Day of Each Month, Base Year 1964

	A		B	C		
	$\frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2}$	$\frac{\text{W}}{\text{m}^2}$	*Dimensionless Ratios*		*Declination, deg*	*Equation of Time, hr*
January	390	1230	0.142	0.058	−20.0	−0.19
February	385	1215	0.144	0.060	−10.8	−0.23
March	376	1186	0.156	0.071	0.0	−0.13
April	360	1136	0.180	0.097	11.6	0.02
May	350	1104	0.196	0.121	20.0	0.06
June	345	1088	0.205	0.134	23.45	−0.02
July	344	1085	0.207	0.136	20.6	−0.10
August	351	1107	0.201	0.122	12.3	−0.04
September	365	1151	0.177	0.092	0	0.13
October	378	1192	0.160	0.073	−10.5	0.26
November	387	1221	0.149	0.063	−19.8	0.23
December	391	1233	0.142	0.057	−23.45	0.03

SOURCE: Adapted by permission from *ASHRAE Handbook, Fundamentals Edition*, 1993.

$$I_{DN} = Ae^{-B/\sin\beta}$$

Solar flux to building surfaces (finally!)

- Solar radiation striking a surface: $I_{solar} = I_D + I_d + I_R$
  - Direct + diffuse + reflected

- Direct (I_D): $I_D = I_{DN} \cos\theta$

Where:
θ = incidence angle, or the angle between the solar rays and the surface normal
I_{DN} = direct normal irradiance (W/m^2)

- Diffuse (I_d): $I_d = I_{dH} \dfrac{1 + \cos\Sigma}{2}$

Where:
Σ = surface tilt angle, or the angle between surface normal and surface vertical
I_{dH} = diffuse horizontal solar radiation (W/m^2)

Solar flux to building surfaces (finally!)

- Reflected (I_R)
  - Radiation striking a surface after reflecting off surrounding surfaces
  - Similar to diffuse
  - Usually concerned with reflection from the ground

$$I_R = \frac{\rho_g I_H (1 - \cos\Sigma)}{2}$$

Where:
ρ_g = solar reflectance of the ground (depends on surface, usually 0.1-0.4)
I_H = total solar flux striking the horizontal ground (W/m^2)

$$\mathrm{I_H} = I_{DN} \cos\theta_H + I_{dH}$$

Solar flux to building surfaces

- Reflected (I_R)
  - Values of reflectance (ρ_g) for common ground surfaces

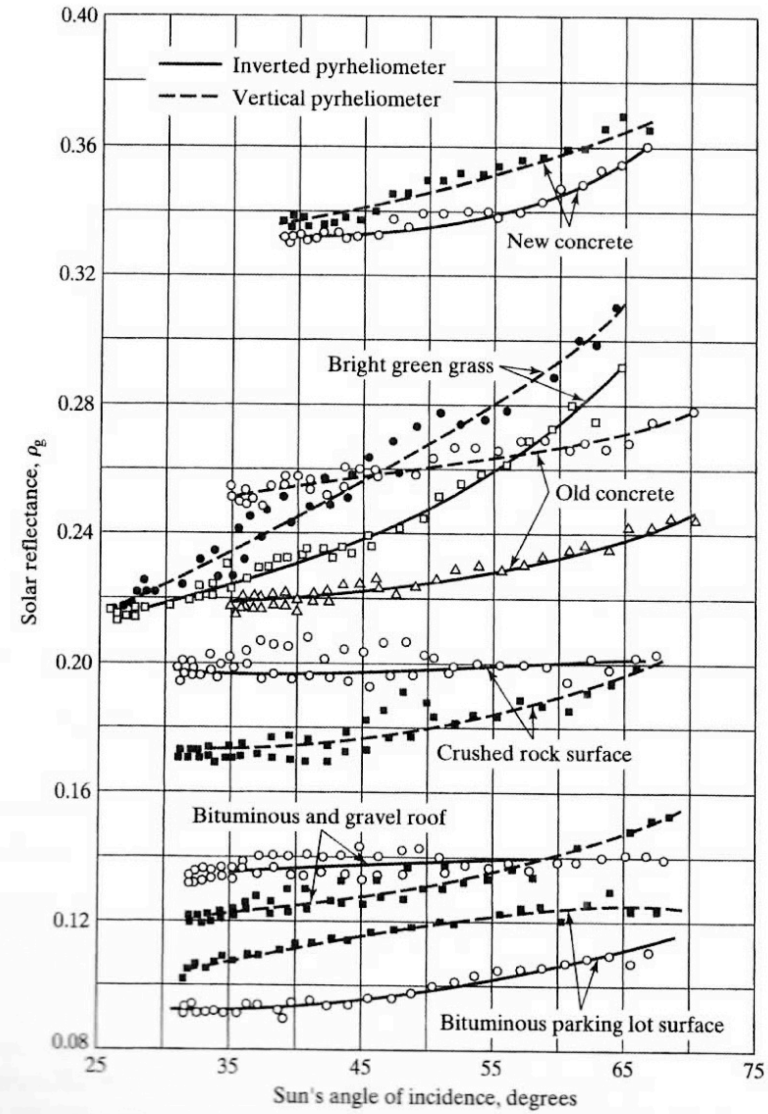


Figure 13.21 Solar reflectance for various ground surfaces. [Reprinted by permission from *ASHRAE Trans.*, 69 (1963), 31.]

Solar flux to building surfaces: Example problem

- Find the solar flux incident on the tilted surface used in the previous problem
  - Assume a ground reflectance of 0.15

Refined solar data

- Now, you could make all of these calculations by hand for every hour of the day…

OR

- You can build calculators or download data
- For hourly sun positions, you can build a calculator or use one from the internet
  - http://www.susdesign.com/sunposition/index.php
- For hourly solar data (direct + diffuse in W/m^2)
  - http://rredc.nrel.gov/solar/old_data/nsrdb/
  - You may be familiar with "typical meteorological years"
    - These data inform those databases
- For visualizing geometry, using something like IES-VE
  - Show videos (videos can be downloaded on course website)

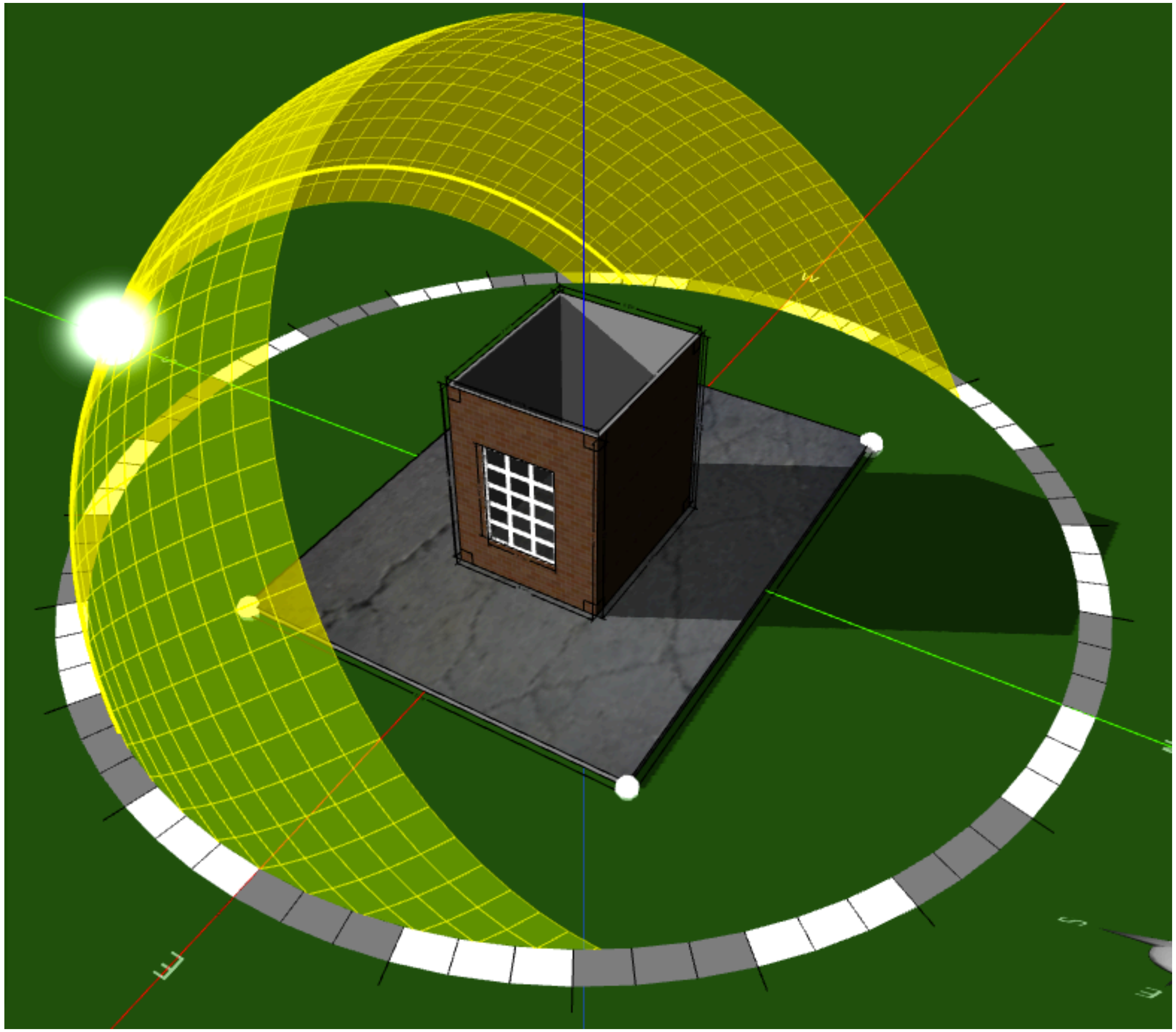
Solar orientation videos/software

- http://built-envi.com/wp-content/uploads/2013/07/solar_position_ies.zip
  - 56 mb zip file of several videos

- January, April, July, November 1st
  - Just one day (24 hours)

- 6 am, 9 am, 12 pm, and 4pm for an entire year

Solar orientation videos/software

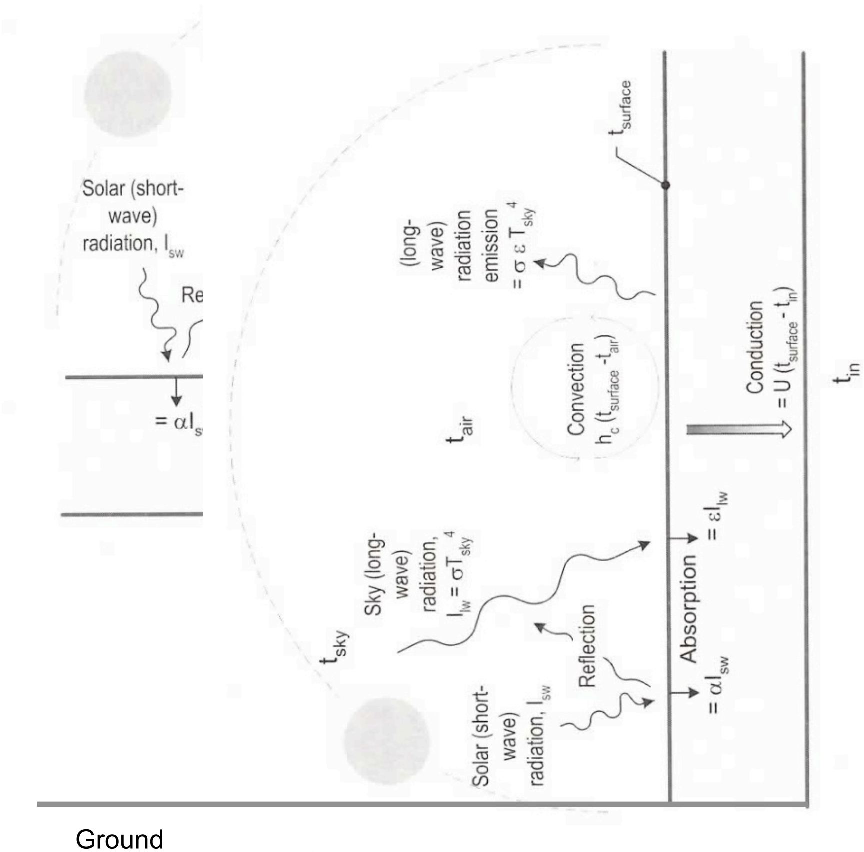


http://energy.concord.org/energy3d/

The Concord Consortium

Bringing all the modes together

- Back to our energy balance for a vertical surface:



$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$

$$\begin{aligned}
&\alpha I_{solar} \\
&+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surf}^4) \\
&+\varepsilon_{surface}\sigma F_{air}(T_{air}^4 - T_{surface}^4) \\
&+\varepsilon_{surface}\sigma F_{ground}(T_{air}^4 - T_{ground}^4) \\
&+h_{conv}(T_{air} - T_{surface}) \\
&-U(T_{surface} - T_{surface,interior}) = 0
\end{aligned}$$

We need to understand conduction through enclosures that are more complex than just single materials

COMPLEX CONDUCTION IN ENCLOSURES

Combining elements

Multiple layers and temperature distributions

Thermal bridges

Combining elements in an actual enclosure

- So far we have been exploring single assemblies
  - Just roofs or just walls without windows and doors
  - If you design a building without windows and doors, something probably went wrong!
- Concept of **combined thermal transmittance**: U_o
  - U_o is the combined thermal transmittance of the respective areas of a gross exterior wall, roof, or floor
    - It is basically an area-weighted average U-value

$$U_o = (U_{wall} A_{wall} + U_{window} A_{window} + U_{door} A_{door}) / A_o$$

where

U_o = average thermal transmittance of gross wall area
A_o = gross area of exterior walls
U_{wall} = thermal transmittance of all elements of opaque wall area
A_{wall} = opaque wall area
U_{window} = thermal transmittance of window area (including frame)
A_{window} = window area (including frame)
U_{door} = thermal transmittance of door area
A_{door} = door area (including frame)

Combined thermal transmittance example

- Calculate U_o for a 10 m x 2.4 m wall with two double-glazed windows with wood/vinyl frames and one solid core door
  - One window is 1.5 x 0.86 m; the other window is 0.9 x 0.76 m
    - Let's say we looked up window U-value in a table
    - U_{window} = 2.90 W/m^{2}K
  - The door is 0.86 x 2 m
    - Let's say we also looked up its U-value in a table
    - U_{door} = 1.42 W/m^{2}K
  - The wall has a U value of U_{wall} = 0.404 W/m^{2}K

$$A_{window} = (1.500 \times 0.860) + (0.900 \times 0.760) = 1.97\ \text{m}^2$$

$$A_{door} = (0.860 \times 2.000) = 1.72\ \text{m}^2$$

$$A_{wall} = (10 \times 2.4) - (1.97 + 1.72) = 20.31\ \text{m}^2$$

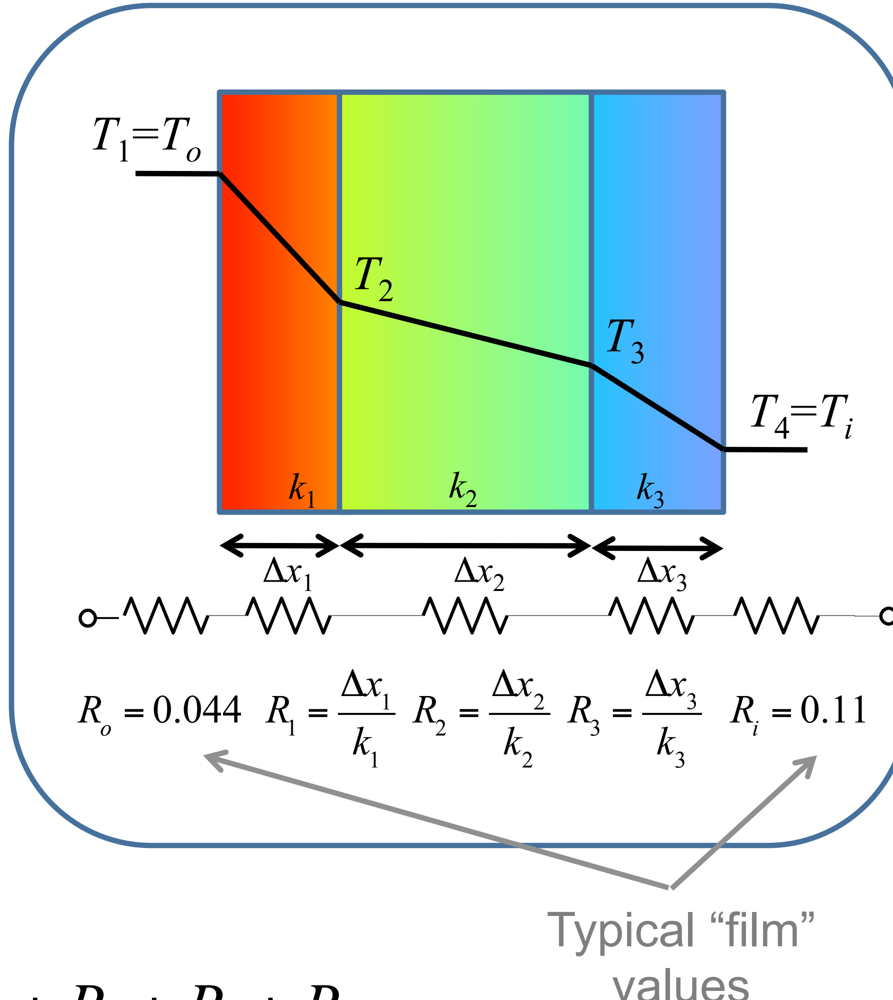
Therefore, the combined thermal transmittance for the wall is

$$U_o = \frac{(0.404 \times 20.31) + (2.90 \times 1.97) + (1.42 \times 1.72)}{10 \times 2.4}$$

$$= 0.68\ \text{W}/(\text{m}^2 \cdot \text{K})$$

Conduction through multiple layers

- Just as in electrical circuits, the overall thermal resistance of a series of elements (layers) can be expressed as the sum of the resistances of each layer
  - Don't forget the interior and exterior convective resistances



- By continuity of energy we can write

$$q = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3}$$

so

$$q = \frac{T_1 - T_4}{R_{total}} \quad \text{where} \quad R_{total} = R_o + R_1 + R_2 + R_3 + R_i$$

Can only add resistances (R) in series, not conductances (U)

Simple conduction through multiple layers

- Calculate the R-value of an enclosure assembly

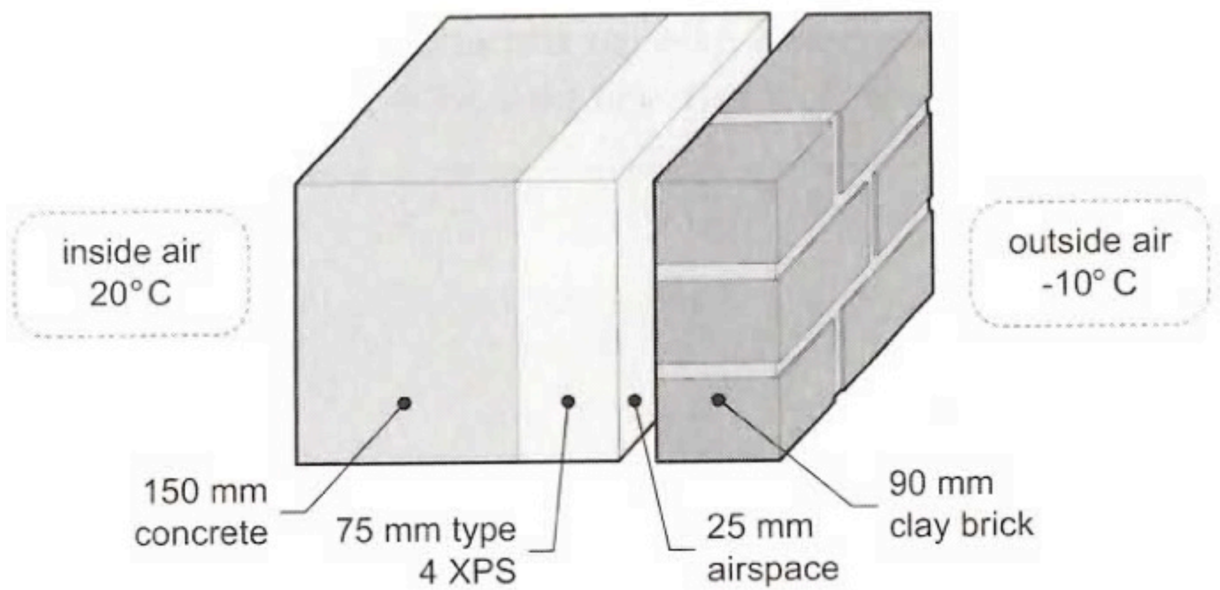
Steps:

1. List each material in the assembly
   - And its conductivity and thickness
2. Calculate conductance of each layer
   - $U = k/L$
3. Calculate thermal resistance of each layer
   - $R = 1/U$
4. Sum the individual thermal resistances to get R_{total}

Conduction through multiple layers

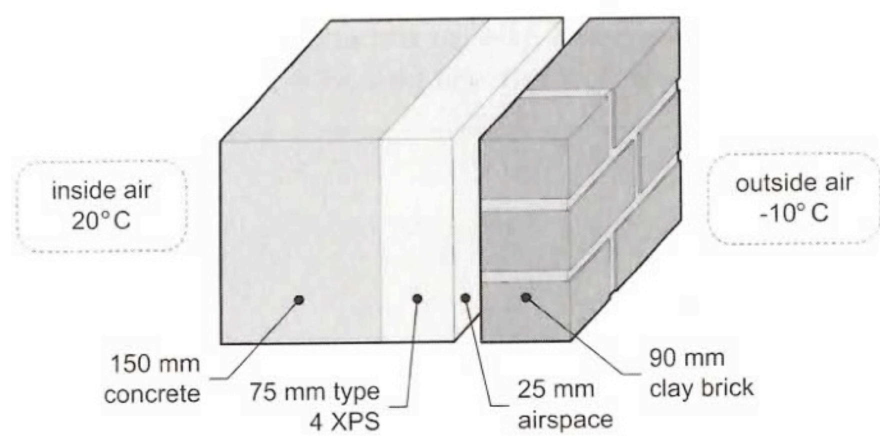
Example problem:

- Calculate the total thermal resistance, R_{total}, and the temperature distribution through the wall shown below



Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for data



Layer material	Conductivity k = [W/mK]	Thickness L = [m]	Conductance U = [W/m^{2}K]	Resistance R = [m^{2}K/W]

A note on R-values of air cavities

- ASHRAE has measured the combined convective + radiative R-values for thin planar cavities of various orientations and depths with various "ε_{eff}"

- These are the best data to use for air spaces in assemblies
  - If you do not know that the material in the cavity is reflective or "low e", just assume that both walls of the cavity have ε=0.9 for each surface, so that when combined, ε_{eff}= 0.82

$$\varepsilon_{eff} = \varepsilon_1 \varepsilon_2$$

2013 ASHRAE Handbook, Chapter 26 (small cavities)

R-values for different air gap characteristics

Table 3 Thermal Resistances of Plane Air Spaces[a,b,c], $(m^2 \cdot K)/W$

Position of Air Space	Direction of Heat Flow	Air Space: Mean Temp.[d], °C	Air Space: Temp. Diff.[d], °C	13 mm Air Space[c]: Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82	20 mm Air Space[c]: Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82
		32.2	5.6	0.37	0.36	0.27	0.17	0.13	0.41	0.39	0.28	0.18	0.13
		10.0	16.7	0.29	0.28	0.23	0.17	0.13	0.30	0.29	0.24	0.17	0.14
		10.0	5.6	0.37	0.36	0.28	0.20	0.15	0.40	0.39	0.30	0.20	0.15
Horiz.	Up	−17.8	11.1	0.30	0.30	0.26	0.20	0.16	0.32	0.32	0.27	0.20	0.16
		−17.8	5.6	0.37	0.36	0.30	0.22	0.18	0.39	0.38	0.31	0.23	0.18
		−45.6	11.1	0.30	0.29	0.26	0.22	0.18	0.31	0.31	0.27	0.22	0.19
		−45.6	5.6	0.36	0.35	0.31	0.25	0.20	0.38	0.37	0.32	0.26	0.21
		32.2	5.6	0.43	0.41	0.29	0.19	0.13	0.52	0.49	0.33	0.20	0.14
		10.0	16.7	0.36	0.35	0.27	0.19	0.15	0.35	0.34	0.27	0.19	0.14
		10.0	5.6	0.45	0.43	0.32	0.21	0.16	0.51	0.48	0.35	0.23	0.17
45° Slope	Up	−17.8	11.1	0.39	0.38	0.31	0.23	0.18	0.37	0.36	0.30	0.23	0.18
		−17.8	5.6	0.46	0.45	0.36	0.25	0.19	0.48	0.46	0.37	0.26	0.20
		−45.6	11.1	0.37	0.36	0.31	0.25	0.21	0.36	0.35	0.31	0.25	0.20
		−45.6	5.6	0.46	0.45	0.38	0.29	0.23	0.45	0.43	0.37	0.29	0.23
		32.2	5.6	0.43	0.41	0.29	0.19	0.14	0.62	0.57	0.37	0.21	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.51	0.49	0.35	0.23	0.17
		10.0	5.6	0.47	0.45	0.33	0.22	0.16	0.65	0.61	0.41	0.25	0.18
Vertical	Horiz.	−17.8	11.1	0.50	0.48	0.38	0.26	0.20	0.55	0.53	0.41	0.28	0.21
		−17.8	5.6	0.52	0.50	0.39	0.27	0.20	0.66	0.63	0.46	0.30	0.22
		−45.6	11.1	0.51	0.50	0.41	0.31	0.24	0.51	0.50	0.42	0.31	0.24
		−45.6	5.6	0.56	0.55	0.45	0.33	0.26	0.65	0.63	0.51	0.36	0.27
		32.2	5.6	0.44	0.41	0.29	0.19	0.14	0.62	0.58	0.37	0.21	0.15

Usually we use values from the $\varepsilon_{eff} = 0.82$ column unless one material is low-e

2013 ASHRAE Handbook, Chapter 26 (small cavities)

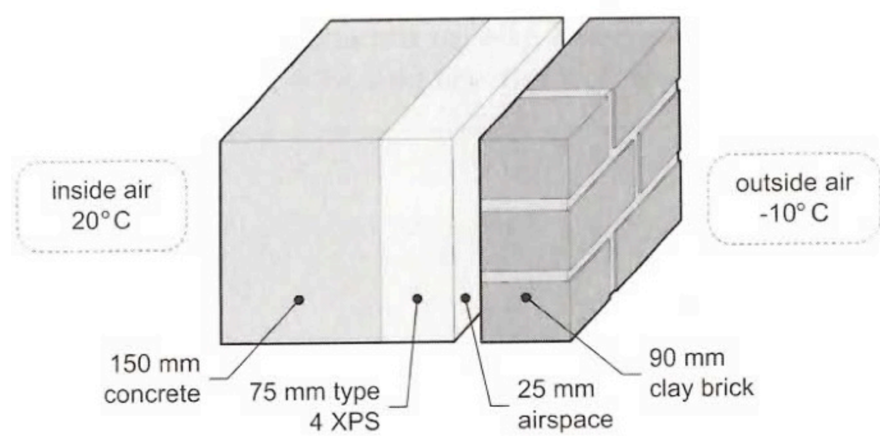
R-values for different air gap characteristics

Position of Air Space	Direction of Heat Flow	Mean Temp.[d], °C	Temp. Diff.[d], °C	Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82	Effective Emittance ε_{eff}[d,e] 0.03	0.05	0.2	0.5	0.82
		Air Space		40 mm Air Space[c]					90 mm Air Space[c]				
		32.2	5.6	0.45	0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
		10.0	16.7	0.33	0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
		10.0	5.6	0.44	0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
Horiz.	Up	−17.8	11.1	0.35	0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
		−17.8	5.6	0.43	0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
		−45.6	11.1	0.34	0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
		−45.6	5.6	0.42	0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
		32.2	5.6	0.51	0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
		10.0	16.7	0.38	0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
		10.0	5.6	0.51	0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
45° Slope	Up	−17.8	11.1	0.40	0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
		−17.8	5.6	0.49	0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
		−45.6	11.1	0.39	0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
		−45.6	5.6	0.48	0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
		32.2	5.6	0.70	0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
		10.0	5.6	0.67	0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
Vertical	Horiz.	−17.8	11.1	0.49	0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
		−17.8	5.6	0.62	0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
		−45.6	11.1	0.46	0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
		−45.6	5.6	0.58	0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26
		32.2	5.6	0.89	0.80	0.45	0.24	0.16	0.85	0.76	0.44	0.24	0.16

Usually we use values from the $\varepsilon_{eff} = 0.82$ column

Conduction through multiple layers

- Refer to 2013 ASHRAE Handbook Ch. 26 for data



Layer material	Conductivity k = [W/mK]	Thickness L = [m]	Conductance U = [W/m^{2}K]	Resistance R = [m^{2}K/W]
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17

R_{total} (IP) = 17.3 hr·ft^2·°F/Btu

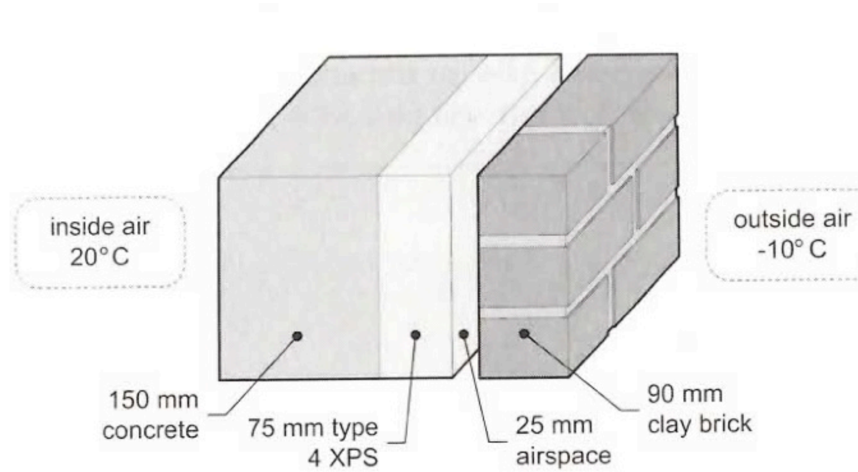
R-values of deeper cavities

- The R-value of cavities stops increasing much at 3 inches (75 mm) depth
  - Beyond 3 inches (75 mm), convection and radiation dominate
  - For a deep cavity, either compute R-values with more advanced methods or use the 3 inch (75 mm) value
- Do **NOT** take the R value of a 1 inch (25 mm) cavity and multiply by the thickness of the cavity for thick cavities
  - If you did that, you would guess that an 8 foot attic would have an R value of about 100 $hr \cdot ft^2 \cdot °F/Btu$, which is a factor of 20 too high!

Conduction through multiple layers

- U_{total} = 0.33 W/m^{2}K

- Calculate steady-state heat flow through the enclosure



- q = UΔT
- q = (0.33 W/m^{2}K)*(T_{inside} - $T_{outside}$)
- q = (0.33 W/m^{2}K)*(30 K) = 10 W/m^2
  - From inside to outside

Conduction through multiple layers

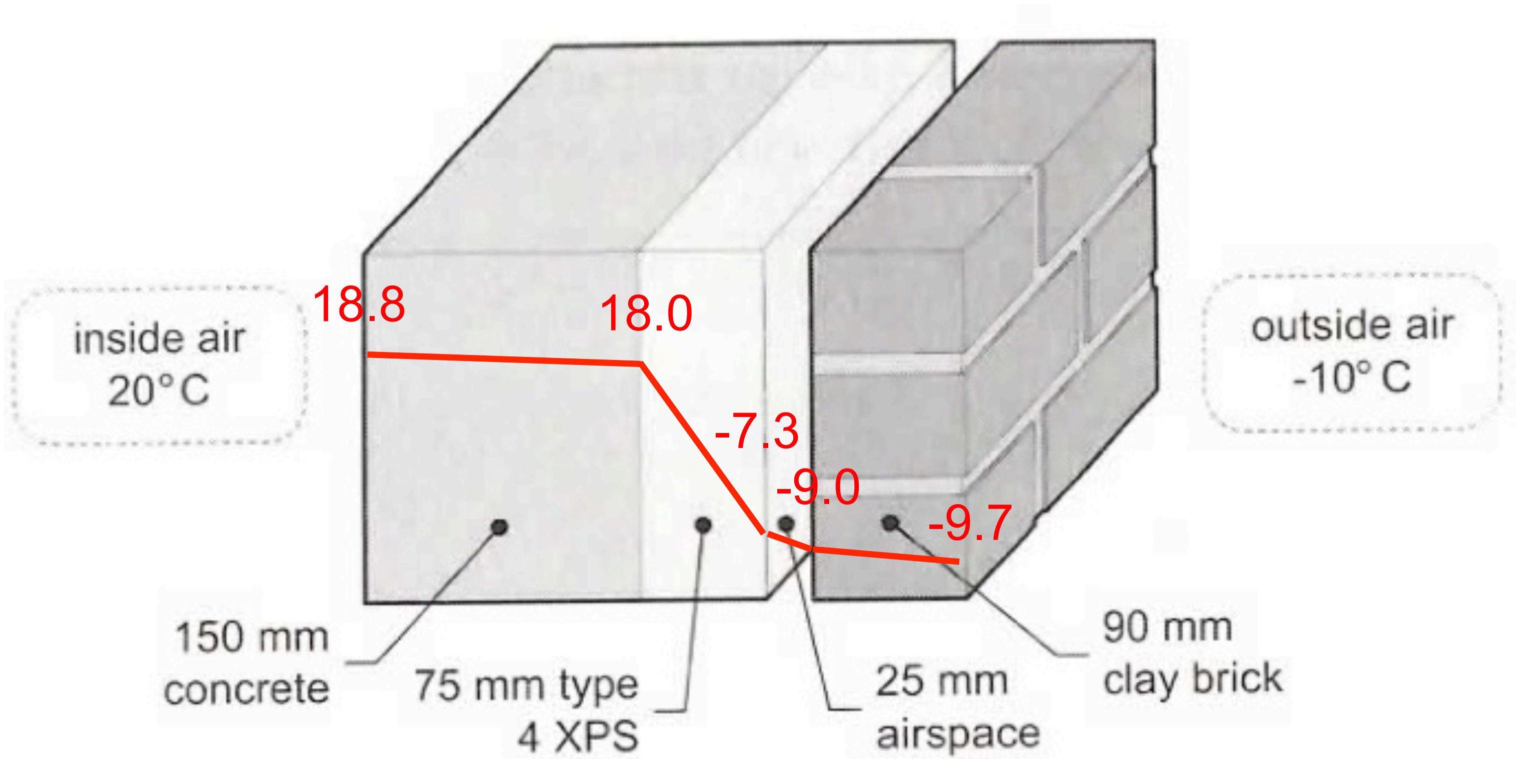
- Calculating the temperature gradient through an enclosure of i materials

$$\Delta T_i = \frac{T_{internal} - T_{external}}{\sum_{i=0}^{n} R_i} R_i$$

Layer	Conductivity W/mK	Thickness m	Conductance W/m^{2}K	Resistance m^{2}K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			R_{total} (m^{2}K/W)	3.04
			U_{total} (W/m^{2}K)	0.33

Conduction through multiple layers

- Calculating the temperature gradient through an enclosure



Total heat transfer through multiple layers

- We can continue to use the electrical resistance analogy

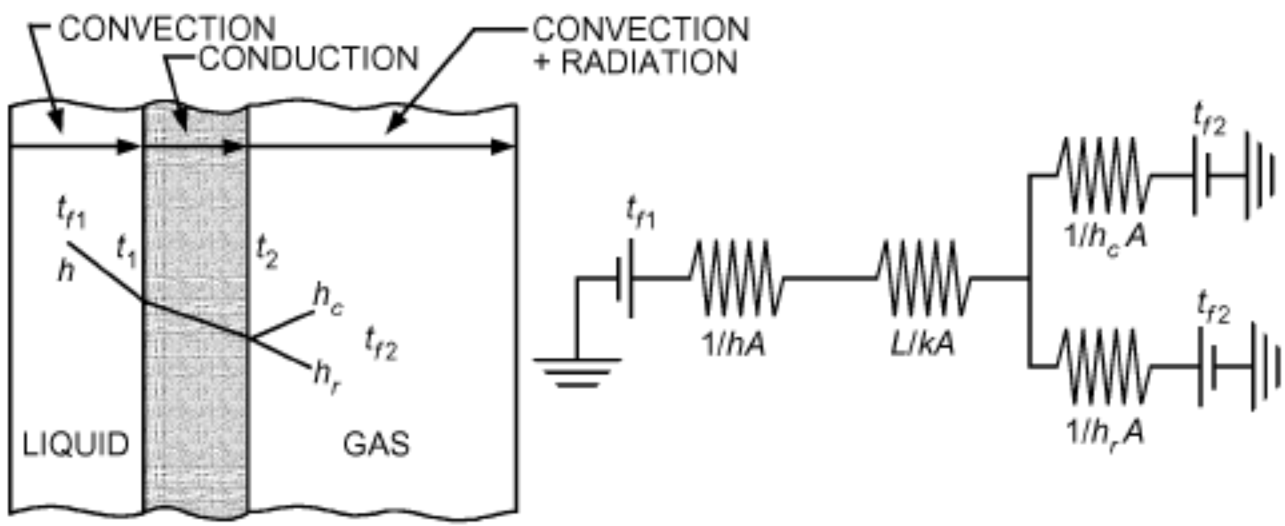


Fig. 1 Thermal Circuit

Limitations to the summation rule

The summation rule for finding R_{total} has several limitations:

- Only works for **layers**
- Layers must be **same area**
- Layers must be **uniform thickness**
- Layers must have **constant material properties**
  - This is the **biggest** limitation

What do we do with more realistic constructions?

- Parallel path or ISO thermal equivalents
- Computer modeling

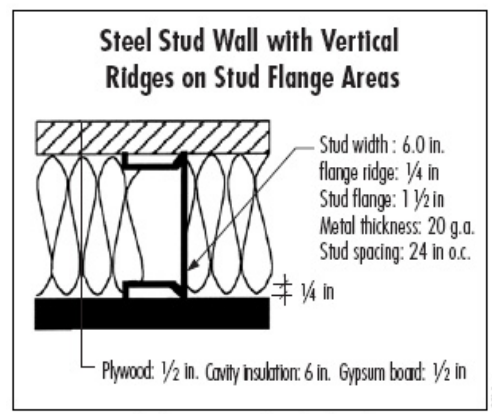


Figure 1. Vertical ridges on a steel stud reduce the contact area between the stud and the sheathing material and improve the whole wall R-value.