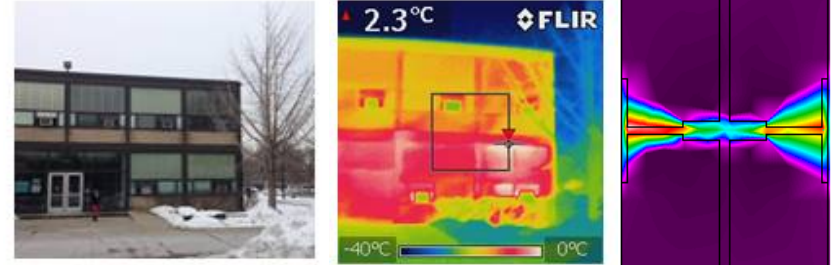


# CAE 331/513

## Building Science

Fall 2014



## Week 5: September 25, 2013

### Psychrometrics (equations)

Built  
Environment  
Research

@ IIT



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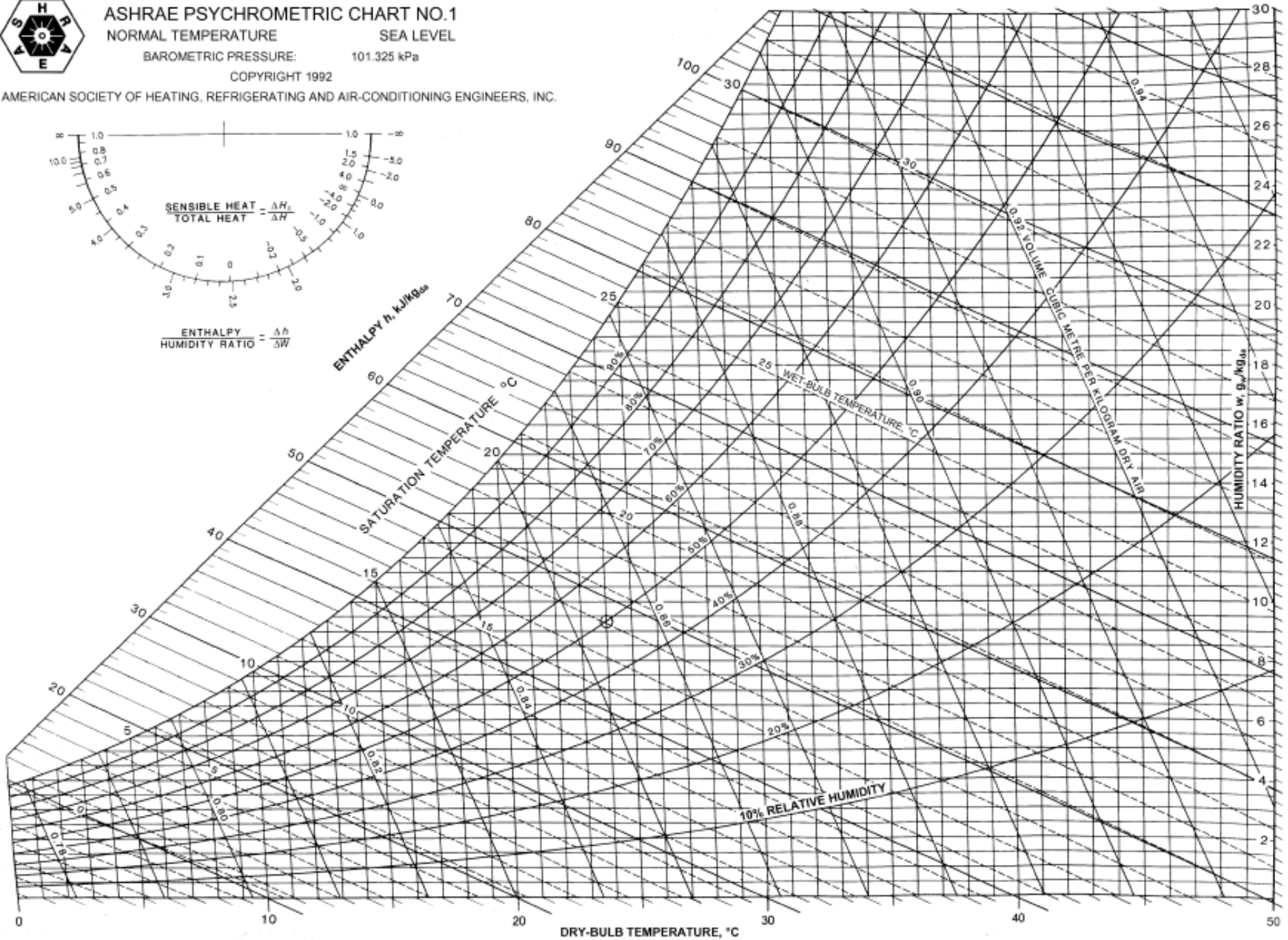
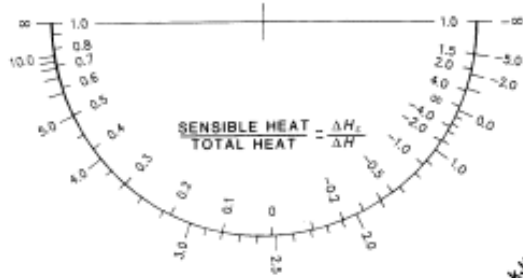
# ASHRAE PSYCHROMETRIC CHART NO. 1

NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

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# ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE

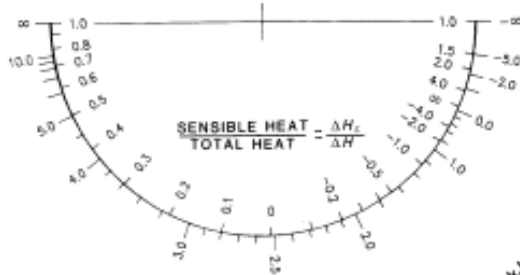
SEA LEVEL

BAROMETRIC PRESSURE:

101.325 kPa

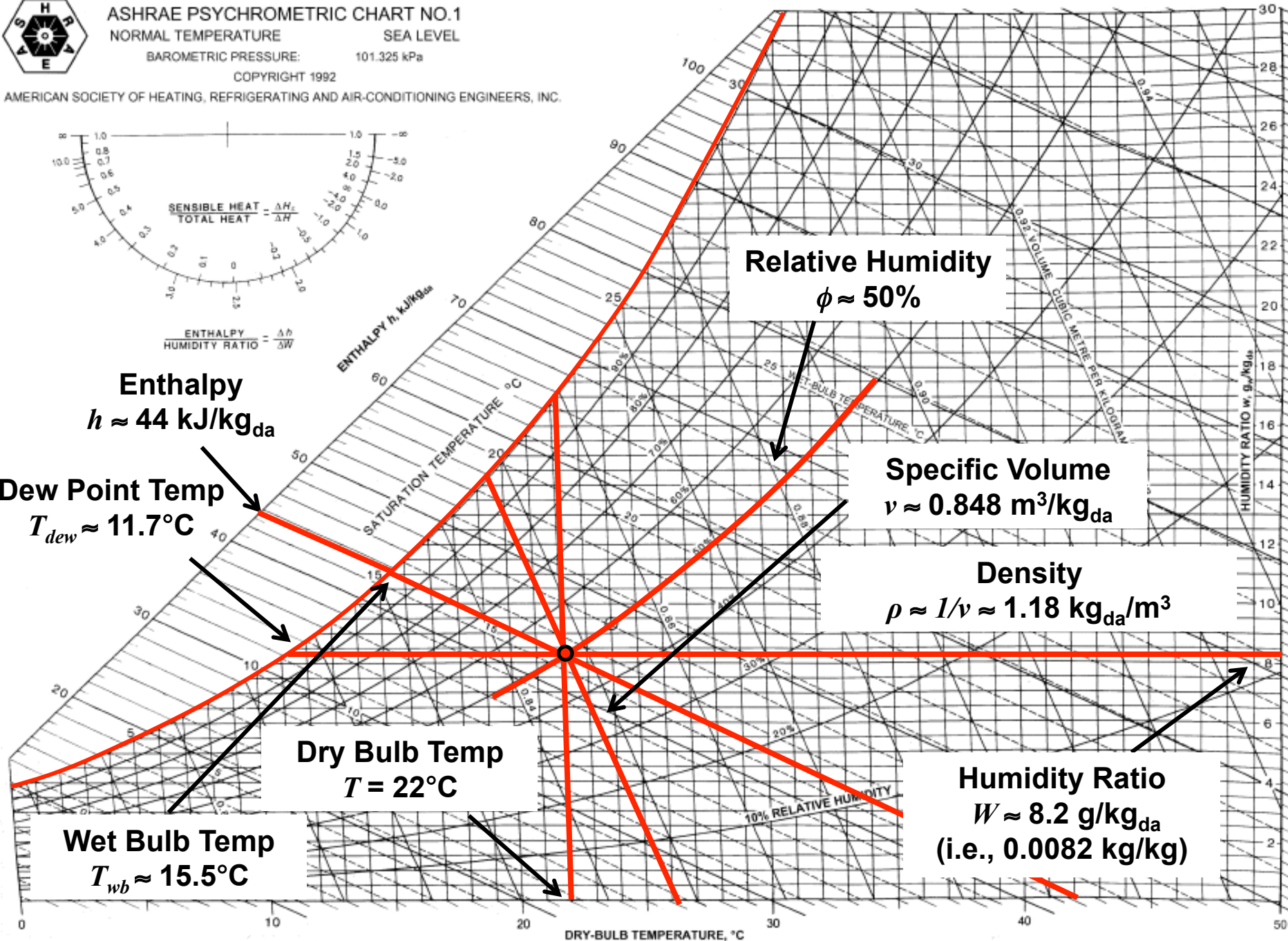
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$\frac{\text{SENSIBLE HEAT}}{\text{TOTAL HEAT}} = \frac{\Delta H_s}{\Delta H}$

$\frac{\text{ENTHALPY}}{\text{HUMIDITY RATIO}} = \frac{\Delta h}{\Delta W}$



**Relative Humidity**  
 $\phi \approx 50\%$

**Specific Volume**  
 $v \approx 0.848 \text{ m}^3/\text{kg}_{da}$

**Density**  
 $\rho \approx 1/v \approx 1.18 \text{ kg}_{da}/\text{m}^3$

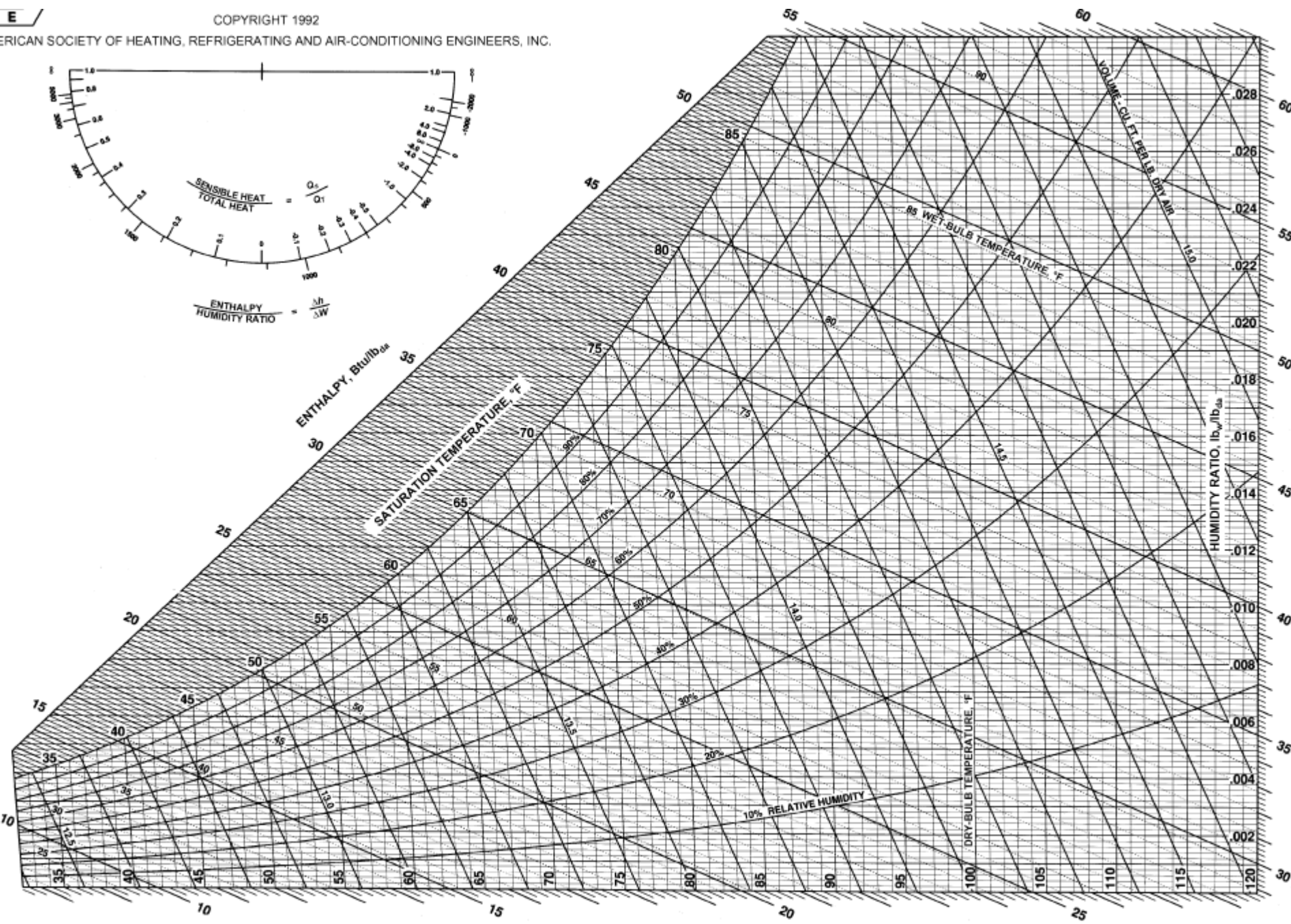
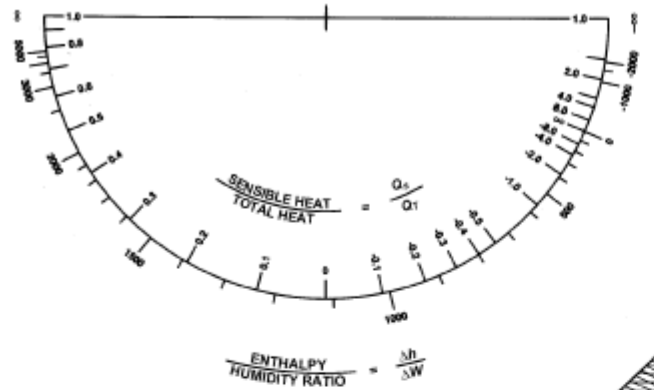
**Humidity Ratio**  
 $W \approx 8.2 \text{ g}/\text{kg}_{da}$   
(i.e., 0.0082 kg/kg)

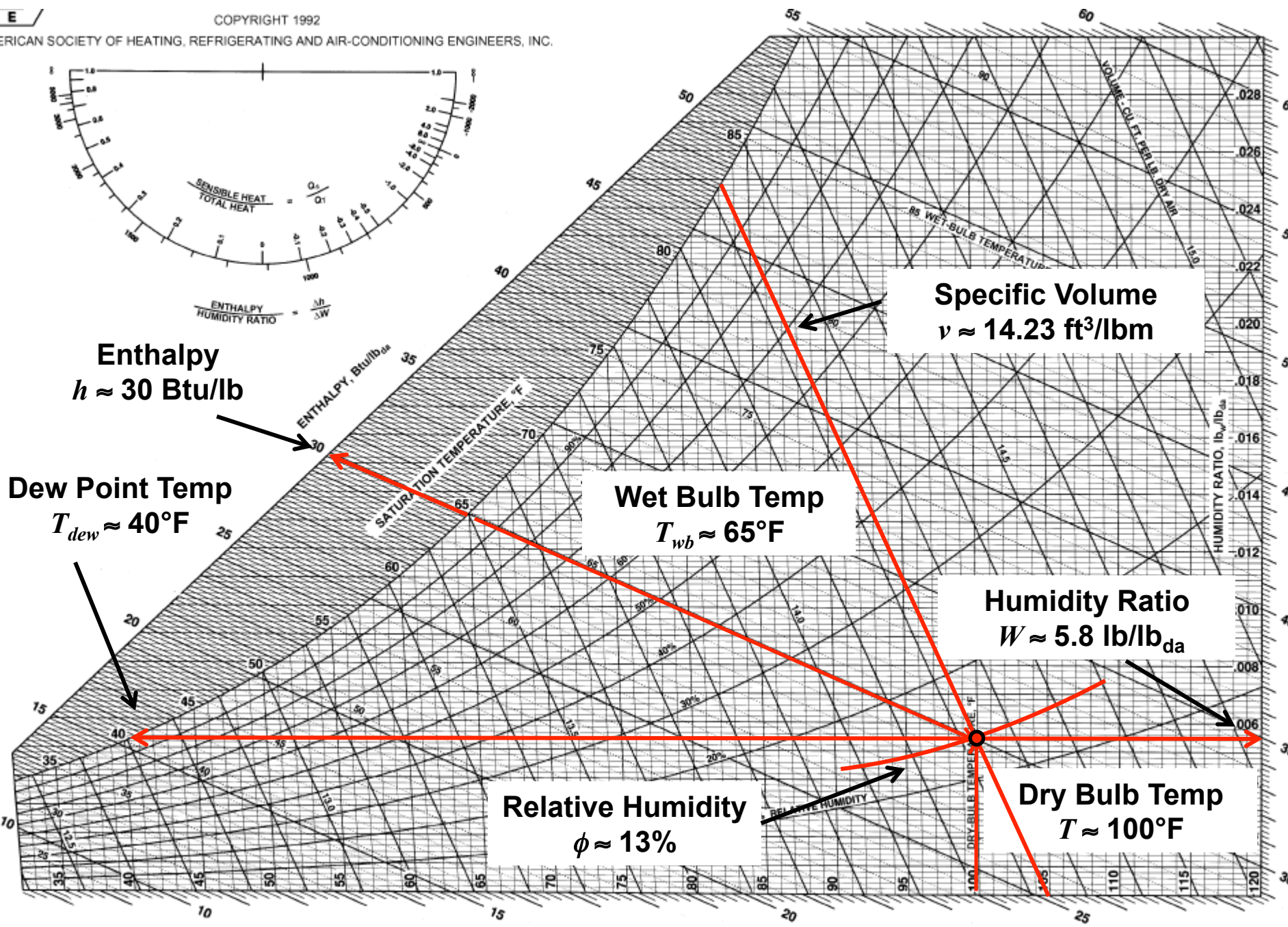
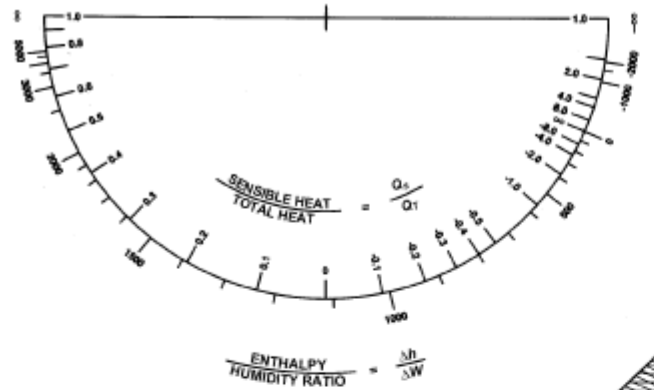
**Dry Bulb Temp**  
 $T = 22^\circ\text{C}$

**Wet Bulb Temp**  
 $T_{wb} \approx 15.5^\circ\text{C}$

**Dew Point Temp**  
 $T_{dew} \approx 11.7^\circ\text{C}$

**Enthalpy**  
 $h \approx 44 \text{ kJ}/\text{kg}_{da}$





# Using these parameters

---

- What is the mass of water vapor in this classroom right now?

# PSYCHROMETRIC EQUATIONS

# Treating air as an ideal gas

---

- At typical temperatures and pressures within buildings, air and its constituents act approximately as ideal gases
- Each gas  $i$  in the mixture, as well as the entire mixture, will follow the ideal gas law:

$$pV = nRT$$

$p$  = pressure (Pa)

$V$  = volume (m<sup>3</sup>)

$n$  = number of moles (#)

$R$  = gas constant (Pa·m<sup>3</sup>/(mol K))

$T$  = absolute temperature (K)



# Air as an ideal gas

---

$$PV = nRT \quad \text{Ideal Gas Law (Boyle's law + Charles's law)}$$

- Air as a composition of ideal gases
  - A bunch of ideal gases acting as an ideal gas
- For individual gases (e.g., N<sub>2</sub>, O<sub>2</sub>, Ar, H<sub>2</sub>O, CO<sub>2</sub>, pollutant *i*):

$$P_i V = n_i RT$$

$P_i$  = partial pressure exerted by gas *i*  
 $n_i$  = # of moles of gas *i*  
 $R, V, T$  = gas constant, volume, temperature

$$P_i = \frac{n_i}{V} RT$$

Rearrange so that  $n_i/V$  is the molar concentration

$$P_i = y_i P_{tot}$$

$P_{tot}$  = total pressure of air (atm, Pa, etc.)  
 $y_i$  = mole fraction of gas *i* in air (moles *i* / moles air)

# Air as an ideal gas

---

- Air as a composite mixture

$$P_i = y_i P_{tot}$$

$$P_{tot} = \sum P_i = \sum \frac{n_i}{V} RT = \frac{RT}{V} \sum n_i = \frac{RT}{V} n_{tot}$$

$$PV = nRT$$

# Density of air

---

$$PV = nRT \longrightarrow \frac{n}{V} = \frac{P}{RT}$$

$$\frac{n}{V} = \frac{P}{RT} = \frac{1 \text{ atm}}{\left(8.205 \times 10^{-5} \frac{\text{atm} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right) \times 293 \text{ K}}$$

↖ 20°C, 68°F

$$\frac{n}{V} = 41.6 \frac{\text{moles}}{\text{m}^3} = 0.0416 \frac{\text{moles}}{\text{L}}$$

$$\rho_{air} = \frac{n}{V} MW_{air} = MW_{air} \times 0.0416 \frac{\text{moles}}{\text{L}} @ 20 \text{ degrees C}$$

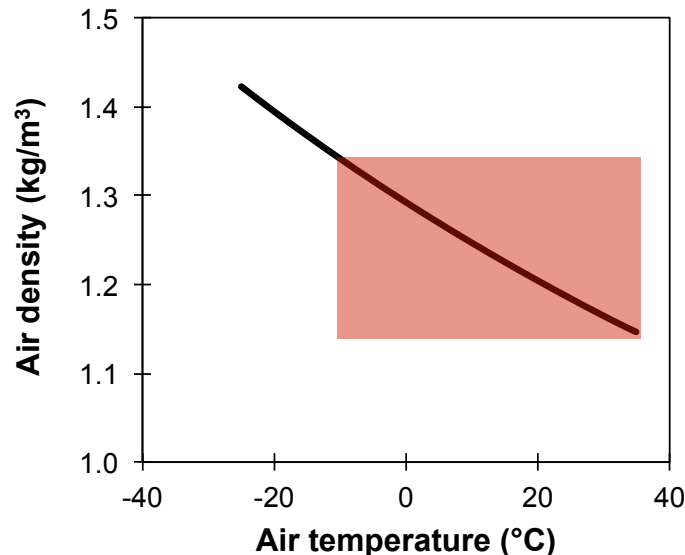
# What is the molecular weight (MW) of air?

$$MW_{air} = \sum y_i MW_i = y_{N_2} MW_{N_2} + y_{O_2} MW_{O_2} + y_{H_2O} MW_{H_2O} + \dots$$

$$MW_{air} = 0.781(28 \text{ g/mol}) + 0.209(32 \text{ g/mol}) + \dots \approx 29 \text{ g/mol}$$

$$\rho_{air} = \left(29 \frac{\text{g}}{\text{mol}}\right) \times 0.0416 \frac{\text{mol}}{\text{L}} = 1.2 \frac{\text{g}}{\text{L}} = 1.2 \frac{\text{kg}}{\text{m}^3} \text{ @20 degrees C}$$

Hang on to this number: density of air is  $\sim 1.2 \text{ kg/m}^3$  at  $20^\circ\text{C}$



$$\rho_{air} \approx 1.3 - 0.0046(T_{air}) \text{ where } T_{air} \text{ is in degrees C}$$

In building applications, where:

$$-15^\circ\text{C} < T < 40^\circ\text{C}$$

$$1.15 \text{ kg/m}^3 < \rho_{air} < 1.3 \text{ kg/m}^3$$

# Universal gas constant

- The universal gas constant relates energy and temperature
  - It takes many forms depending on units

$$pv = \frac{p}{\rho} = RT$$

Value of R	Units (V P T <sup>-1</sup> n <sup>-1</sup> )
8.314	J/(K·mol)
8.314	m <sup>3</sup> ·Pa/(K·mol)
0.08206	L·atm/(K·mol)
8.206×10 <sup>-5</sup>	m <sup>3</sup> ·atm/(K·mol)
10.731	ft <sup>3</sup> ·psi/(R·lb-mol)
1.986	Btu/(lb-mol·R)

# Mass-specific gas constants

- To work with air and water vapor we need gas-specific gas constants (which are functions of molecular weight)
- Dry air (no water vapor):  $MW_{da} = 28.965 \text{ g/mol}$

$$R_i = \frac{R}{MW_i}$$

$$R_{da} = \frac{R}{MW_{da}} = \frac{8.314 \frac{\text{J}}{\text{K} \cdot \text{mol}} \frac{1000 \text{g}}{\text{kg}}}{28.965 \frac{\text{g}}{\text{mol}}} = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

- Water vapor alone:  $MW_w = 18.015 \text{ g/mol}$

$$pv = \frac{p}{\rho} = RT \quad R_w = \frac{R}{MW_w} = \frac{8.314 \frac{\text{J}}{\text{K} \cdot \text{mol}} \frac{1000 \text{g}}{\text{kg}}}{18.015 \frac{\text{g}}{\text{mol}}} = 462 \frac{\text{J}}{\text{kg}_w \cdot \text{K}}$$

# Air pressure variations

---

- The barometric (atmospheric) pressure and temperature of air vary with both altitude and local weather conditions
  - But there are standard values for pressure as a function of altitude that are normally used
- At sea level, the standard temperature is 15°C and the standard pressure is 101.325 kPa (1 atm)
  - Temperature is assumed to decrease linearly with altitude
    - Pressure is more complicated

$$T_{air} = 15 - 0.0065Z$$

$$p = 101.325 \left( 1 - \left( 2.25577 \times 10^{-5} \right) Z \right)^{5.2559}$$

$$pv = \frac{p}{\rho} = RT$$

T = temperature (°C)

Z = altitude (m)

p = barometric pressure (kPa)

# Air pressure variations

---

**Table 1 Standard Atmospheric Data for Altitudes to 10 000 m**

Altitude, m	Temperature, °C	Pressure, kPa	
-500	18.2	107.478	
0	15.0	101.325	<i>Chicago, IL</i>
500	11.8	95.461	
1000	8.5	89.875	
1500	5.2	84.556	<i>Denver, CO</i>
2000	2.0	79.495	<i>Big Sky, MT</i>
2500	-1.2	74.682	
3000	-4.5	70.108	<i>Breckenridge, CO</i>
4000	-11.0	61.640	
5000	-17.5	54.020	
6000	-24.0	47.181	
7000	-30.5	41.061	
8000	-37.0	35.600	
9000	-43.5	30.742	
10 000	-50	26.436	

*Source:* Adapted from NASA (1976).



# Dalton's law of partial pressures for psychrometrics

---

- In an ideal gas, the total pressure can be considered to be the sum of the partial pressures of the constituent gases

$$p = p_{N_2} + p_{O_2} + p_{H_2O} + p_{CO_2} + p_{Ar} + \dots$$

- We can consider moist air as dry air combined with water vapor and break the pressure into only two partial pressures

$$p = p_{da} + p_w$$

# Dalton's law of partial pressures for psychrometrics

---

- We can analyze the dry air, the water vapor, and the mixture of each gas using the ideal gas law and assuming they are all at the same temperature

$$p_{da} v_{da} = R_{da} T \quad \& \quad p_w v_w = R_w T \quad \& \quad p v = R T$$

- For each individual gas, a mole fraction ( $Y_i$ ) can be defined as the ratio of the partial pressure of gas  $i$  to the total pressure

$$\frac{n_i}{n} = \frac{p_i}{p} = Y_i$$

# Specifying the state of moist air

---



In order to specify the state of moist air, we need total atmospheric pressure,  $p$ , the air temperature,  $T$ , and at least one other property

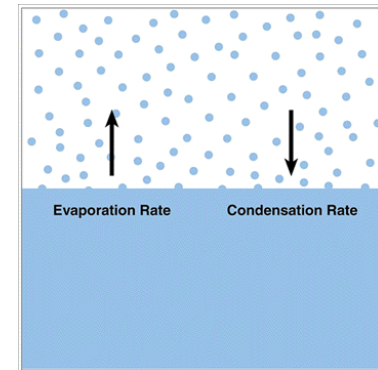
- $W$ ,  $\phi$ ,  $h$ ,  $p_w$ , or  $T_{dew}$
- We can use the psychrometric chart
- We can also use the underlying equations for greater accuracy and automation

# Key concepts: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is a measurement of the amount of water vapor in a volume/parcel of air

$$p_w$$

\*Units of pressure, Pa or kPa



- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
  - We call the limit **saturation**

$$p_{ws}$$

\*Units of pressure, Pa or kPa



# Relative humidity, $\phi$ (RH)

- The relative humidity ratio,  $\phi$ , is the mole fraction of water vapor ( $x_w$ ) relative to the water vapor that would be in the mixture if it were saturated at the given  $T$  and  $P$  ( $x_{ws}$ )
- Relative humidity is a common measure that relates well to how we perceive moisture in air



$$\phi = \left[ \frac{x_w}{x_{ws}} \right]_{T,P} = \frac{p_w}{p_{ws}}$$

## $p_{ws}$ for $0^{\circ}\text{C} < T < 200^{\circ}\text{C}$

---

For  $p_{ws}$ , the saturation pressure over **liquid water**:

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

where

$$C_8 = -5.800\ 220\ 6\ \text{E}+03$$

$$C_9 = 1.391\ 499\ 3\ \text{E}+00$$

$$C_{10} = -4.864\ 023\ 9\ \text{E}-02$$

$$C_{11} = 4.176\ 476\ 8\ \text{E}-05$$

$$C_{12} = -1.445\ 209\ 3\ \text{E}-08$$

$$C_{13} = 6.545\ 967\ 3\ \text{E}+00$$

**Unit**

$p_{ws}$  = saturation pressure, Pa

$T$  = absolute temperature, K =  $^{\circ}\text{C} + 273.15$

Note:

These constants are only for SI units  
IP units are different

\*We will use this equation for most conditions in building science

## $p_{ws}$ for $-100^{\circ}\text{C} < T < 0^{\circ}\text{C}$

---

For  $p_{ws}$ , the saturation pressure over **ice**:

$$\ln p_{ws} = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T$$

where

$$C_1 = -5.674\ 535\ 9\ \text{E}+03$$

$$C_2 = 6.392\ 524\ 7\ \text{E}+00$$

$$C_3 = -9.677\ 843\ 0\ \text{E}-03$$

$$C_4 = 6.221\ 570\ 1\ \text{E}-07$$

$$C_5 = 2.074\ 782\ 5\ \text{E}-09$$

$$C_6 = -9.484\ 024\ 0\ \text{E}-13$$

$$C_7 = 4.163\ 501\ 9\ \text{E}+00$$

Note:

These constants are only for SI units  
IP units are different

**Units:**

$p_{ws}$  = saturation pressure, Pa

$T$  = absolute temperature, K =  $^{\circ}\text{C} + 273.15$

# Humidity ratio, $W$

---

- The humidity ratio,  $W$ , is ratio of the mass of water vapor to mass of dry air in a given volume
  - We use  $W$  when finding other mixture properties
  - Note 1:  $W$  is small ( $W < 0.03$  for most real building conditions)
  - Note 2:  $W$  is sometimes expressed in grains/lb where 1 lb = 7000 grains (I don't use this but you might in CAE 464 HVAC Design)

$$W = \frac{m_w}{m_{da}} = \frac{MW_w p_w}{MW_{da} p_{da}} = 0.622 \frac{p_w}{p_{da}} = 0.622 \frac{p_w}{p - p_w} \quad \begin{matrix} \text{UNITS} \\ \left[ \frac{\text{kg}_w}{\text{kg}_{da}} \right] \end{matrix}$$



# Saturation humidity ratio, $W_s$

---

- At a given temperature  $T$  and pressure  $P$  there is a maximum  $W$  that can be obtained
- If we try to add any more moisture, it will just condense out
  - It is when the partial pressure of vapor has reached the saturation pressure
- This maximum humidity ratio is called the saturation humidity ratio,  $W_s$ 
  - From our previous equation we can write:

$$W_s = 0.622 \frac{p_{ws}}{p_{da}} = 0.622 \frac{p_{ws}}{p - p_{ws}} \quad \text{UNITS} \quad \left[ \frac{\text{kg}_w}{\text{kg}_{da}} \right]$$

# Degree of saturation, $\mu$

---

- The degree of saturation,  $\mu$  (dimensionless), is the ratio of the humidity ratio  $W$  to that of a saturated mixture  $W_s$  at the same  $T$  and  $P$ 
  - Note that  $\mu$  and  $\phi$  are not quite the same
  - Their values are very similar at lower temperatures but may differ a lot at higher temperatures

$$\mu = \left[ \frac{W}{W_s} \right]_{T,P}$$

$$\mu = \frac{\phi}{1 + (1 - \phi)W_s / (0.6295)}$$

$$\phi = \frac{\mu}{1 - (1 - \mu)p_{ws} / p}$$

# Specific volume, $v$ , and density, $\rho$

---

- The specific volume of moist air (or the volume per unit mass of air,  $\text{m}^3/\text{kg}$ ) can be expressed as:

$$v = \frac{R_{da} T}{p - p_w} = \frac{R_{da} T (1 + 1.6078W)}{p}$$

where

$v$  = specific volume,  $\text{m}^3/\text{kg}_{da}$   
 $t$  = dry-bulb temperature,  $^{\circ}\text{C}$   
 $W$  = humidity ratio,  $\text{kg}_w/\text{kg}_{da}$   
 $p$  = total pressure,  $\text{kPa}$

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

- If we have  $v$  we can also find moist air density,  $\rho$  ( $\text{kg}/\text{m}^3$ ):

$$\rho = \frac{m_{da} + m_w}{V} = \frac{1}{v} (1 + W)$$

# Enthalpy, $h$

---

- The enthalpy of a mixture of perfect gases equals the sum of the individual partial enthalpies of the components
- Therefore, the enthalpy ( $h$ ) for moist air is:  $h = h_{da} + Wh_g$

$h$  = enthalpy for moist air [kJ/kg]

$h_g$  = specific enthalpy for saturated water vapor (i.e.,  $h_{ws}$ ) [kJ/kg<sub>w</sub>]

$h_{da}$  = specific enthalpy for dry air (i.e.,  $h_{ws}$ ) [kJ/kg<sub>da</sub>]

- Some approximations:  $h_{da} \approx 1.006T$      $h_g \approx 2501 + 1.86T$

$$h \approx 1.006T + W(2501 + 1.86T)$$

\*where  $T$  is in °C

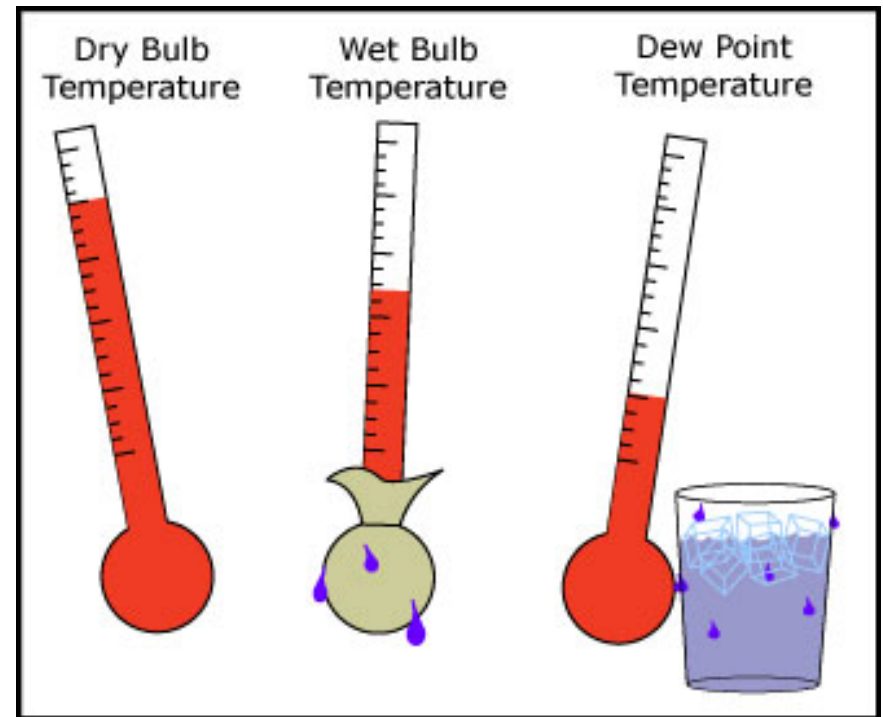
# Remember: 3 different temperatures $T$ , $T_{dew}$ , and $T_{wb}$

The standard temperature,  $T$ , we are all familiar with is called the **dry-bulb** temperature, or  $T_d$

- It is a measure of internal energy

We can also define:

- **Dew-point** temperature,  $T_{dew}$ 
  - Temperature at which water vapor changes into liquid (condensation)
  - Air is maximally **saturated** with water vapor
- **Wet-bulb** temperature,  $T_{wb}$ 
  - The temperature that a parcel of air would have if it were cooled to saturation (100% **relative humidity**) by the evaporation of water into it



Units of Celsius, Fahrenheit, or Kelvin

# Dew-point temperature, $T_{dew}$

---



The dew point temperature,  $T_{dew}$ , is the air temperature at which the current humidity ratio  $W$  is equal to the saturation humidity ratio  $W_s$  at the same temperature

$$\text{i.e. } W_s(p, T_{dew}) = W$$

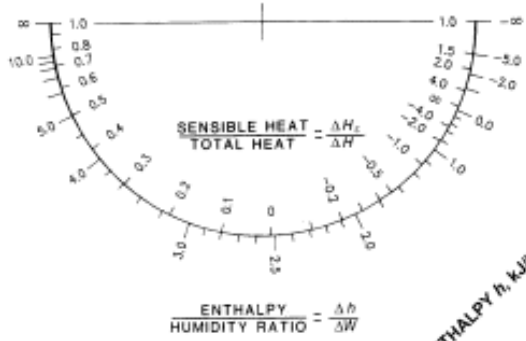
When the air temperature is lowered to the dew-point at constant pressure, the relative humidity rises to 100% and condensation occurs

$T_{dew}$  is a direct measure of the humidity ratio  $W$  since  $W = W_s$  at  $T = T_{dew}$



ASHRAE PSYCHROMETRIC CHART NO.1  
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SEA LEVEL  
BAROMETRIC PRESSURE: 101.325 kPa  
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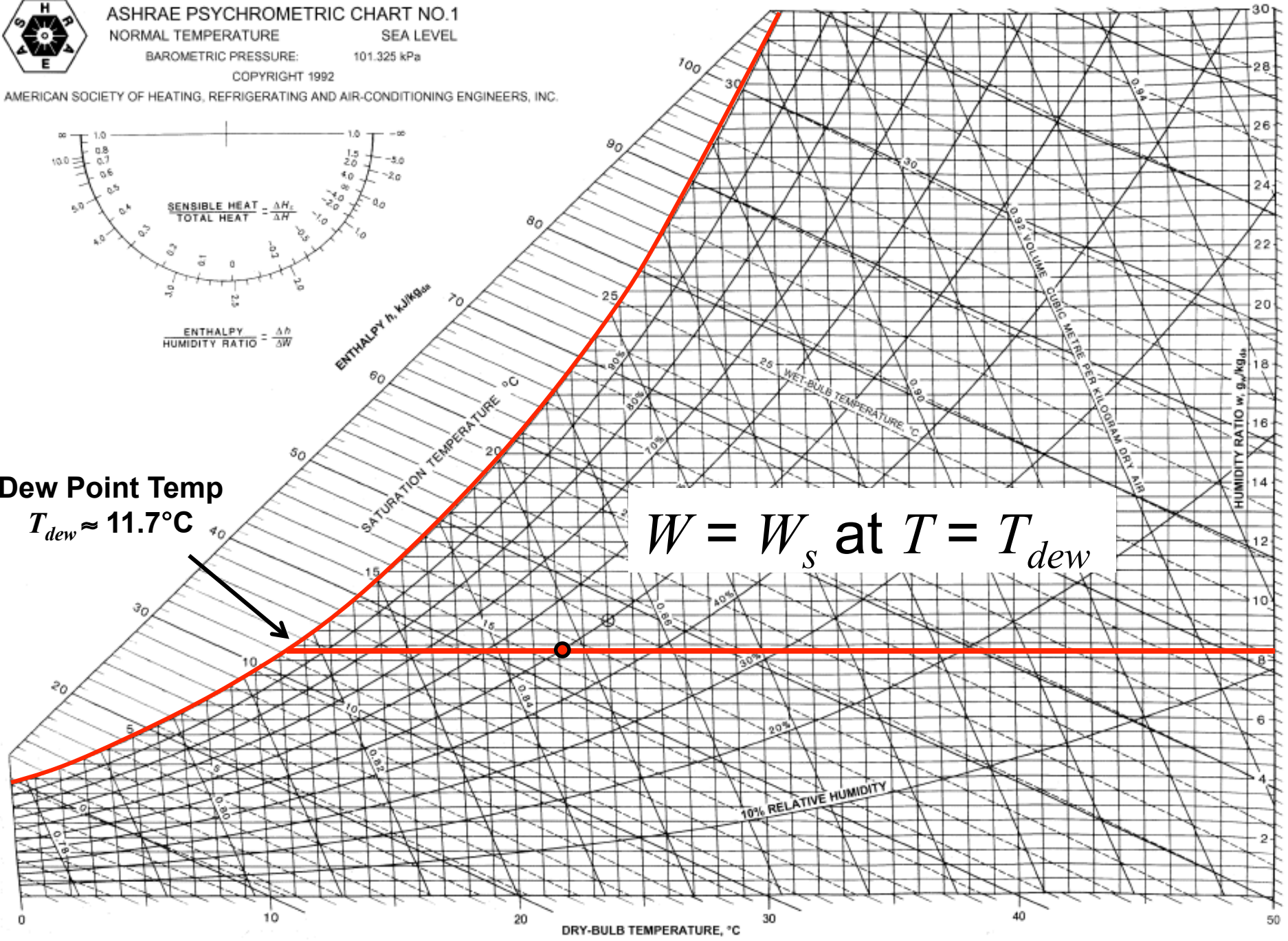
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Dew Point Temp  
 $T_{dew} \approx 11.7^\circ\text{C}$



$$W = W_s \text{ at } T = T_{dew}$$



# Dew-point temperature, $T_{dew}$

---

- Dew-point temperature,  $T_{dew}$

Between dew points of 0 and 93°C,

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}(p_w)^{0.1984}$$

Below 0°C,

$$t_d = 6.09 + 12.608\alpha + 0.4959\alpha^2$$

*where*

$t_d$  = dew-point temperature, °C

$\alpha = \ln p_w$

$p_w$  = water vapor partial pressure, kPa

$C_{14} = 6.54$

$C_{15} = 14.526$

$C_{16} = 0.7389$

$C_{17} = 0.09486$

$C_{18} = 0.4569$

Note:

These constants are only for SI units  
IP units are different



# Wet-bulb temperature, $T_{wb}$

---

- Wet-bulb temperature,  $T_{wb}$
- Requires **iterative solver**... find the  $T_{wb}$  that satisfies the following equation (above freezing):

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = \text{actual } W$$

- And for  $T$  below freezing:

$$W = \frac{(2830 - 0.24T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2830 + 1.86T - 2.1T_{wb}} = \text{actual } W$$

\*Where  $T_{wb}$  and  $T$  are in Kelvin

# Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 2 gives us  $W_s$ ,  $v_{da}$ , and  $v_s$  directly at different temperatures:

**Table 2 Thermodynamic Properties of Moist Air at Standard Atmospheric Pressure**

Temp., °C <i>t</i>	Humidity Ratio $W_s$ , kg <sub>w</sub> /kg <sub>da</sub>	Specific Volume, m <sup>3</sup> /kg <sub>da</sub>			Specific Enthalpy, kJ/kg <sub>da</sub>		
		$v_{da}$	$v_{as}$	$v_s$	$h_{da}$	$h_{as}$	$h_s$
15	0.010694	0.8159	0.0140	0.8299	15.087	27.028	42.115
16	0.011415	0.8188	0.0150	0.8338	16.093	28.873	44.966
17	0.012181	0.8216	0.0160	0.8377	17.099	30.830	47.929
18	0.012991	0.8245	0.0172	0.8416	18.105	32.906	51.011
19	0.013851	0.8273	0.0184	0.8457	19.111	35.107	54.219
20	0.014761	0.8301	0.0196	0.8498	20.117	37.441	57.558
21	0.015724	0.8330	0.0210	0.8540	21.124	39.914	61.037
22	0.016744	0.8358	0.0224	0.8583	22.130	42.533	64.663

# Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 3 gives us  $p_{ws}$  at different temperatures:

**Table 3 Thermodynamic Properties of Water at Saturation**

Temp., °C <i>t</i>	Absolute Pressure $p_{ws}$ , kPa	Specific Volume, m <sup>3</sup> /kg <sub>w</sub>			Specific Enthalpy, kJ/kg <sub>w</sub>		
		Sat. Liquid $v_i/v_f$	Evap. $v_{ig}/v_{fg}$	Sat. Vapor $v_g$	Sat. Liquid $h_i/h_f$	Evap. $h_{ig}/h_{fg}$	Sat. Vapor $h_g$
3	0.7581	0.001000	168.013	168.014	12.60	2493.80	2506.40
4	0.8135	0.001000	157.120	157.121	16.81	2491.42	2508.24
5	0.8726	0.001000	147.016	147.017	21.02	2489.05	2510.07
6	0.9354	0.001000	137.637	137.638	25.22	2486.68	2511.91
7	1.0021	0.001000	128.927	128.928	29.43	2484.31	2513.74
8	1.0730	0.001000	120.833	120.834	33.63	2481.94	2515.57
9	1.1483	0.001000	113.308	113.309	37.82	2479.58	2517.40
10	1.2282	0.001000	106.308	106.309	42.02	2477.21	2519.23

# Revisit 1<sup>st</sup> example from last class

---

Moist air exists at 22°C dry-bulb temperature with 50% RH

Find the following:

- (a) the humidity ratio,  $W$
- (b) dew point temperature,  $T_{dew}$
- (c) wet-bulb temperature,  $T_{wb}$
- (d) enthalpy,  $h$
- (e) specific volume,  $v$
- (f) density,  $\rho$

Also:

- (g) degree of saturation,  $\mu$

# Psychrometric equations

$$\phi = \frac{p_w}{p_{ws}} \quad W = 0.622 \frac{p_w}{p - p_w} \quad \mu = \frac{W}{W_s}$$

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

where

$$C_8 = -5.800\ 220\ 6\ \text{E}+03$$

$$C_9 = 1.391\ 499\ 3\ \text{E}+00$$

$$C_{10} = -4.864\ 023\ 9\ \text{E}-02$$

$$C_{11} = 4.176\ 476\ 8\ \text{E}-05$$

$$C_{12} = -1.445\ 209\ 3\ \text{E}-08$$

$$C_{13} = 6.545\ 967\ 3\ \text{E}+00$$

$p_{ws}$  = saturation pressure, Pa

$T$  = absolute temperature, K = °C + 273.15

**Dew point temperature:**

Between dew points of 0 and 93°C,

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}(p_w)^{0.1984}$$

Below 0°C,

$$t_d = 6.09 + 12.608\alpha + 0.4959\alpha^2$$

where

$t_d$  = dew-point temperature, °C

$\alpha$  =  $\ln p_w$

$p_w$  = water vapor partial pressure, kPa

$C_{14} = 6.54$

$C_{15} = 14.526$

$C_{16} = 0.7389$

$C_{17} = 0.09486$

$C_{18} = 0.4569$

# Psychrometric equations

**Wet bulb temperature (iterative solver):**

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = \text{actual } W$$

$$v = \frac{R_{da}T}{p - p_w} = \frac{R_{da}T(1 + 1.6078W)}{p}$$

where

$v$  = specific volume,  $\text{m}^3/\text{kg}_{da}$

$t$  = dry-bulb temperature,  $^{\circ}\text{C}$

$W$  = humidity ratio,  $\text{kg}_w/\text{kg}_{da}$

$p$  = total pressure, kPa

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

$$\rho = \frac{m_{da} + m_w}{V} = \frac{1}{v}(1 + W)$$

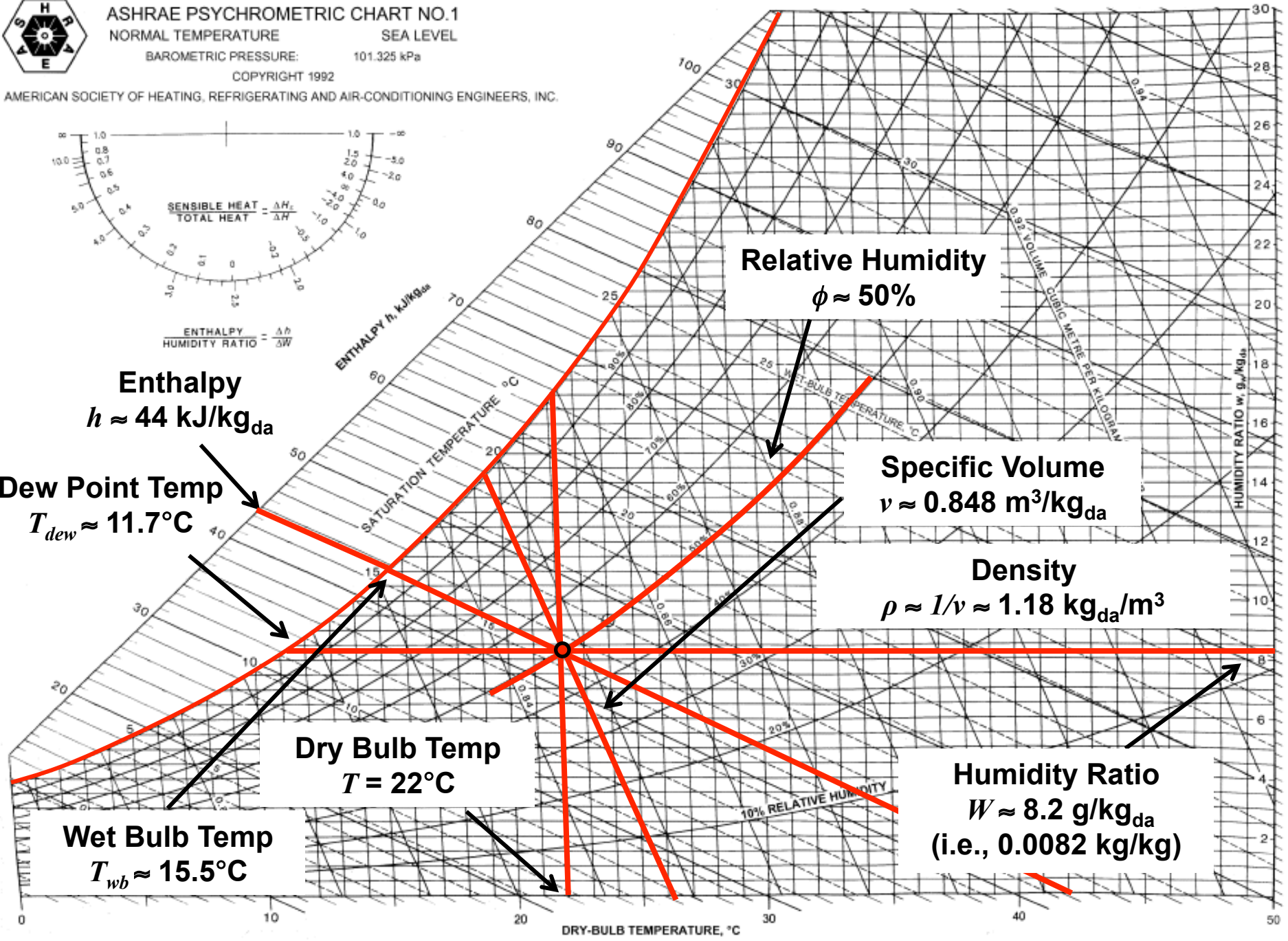
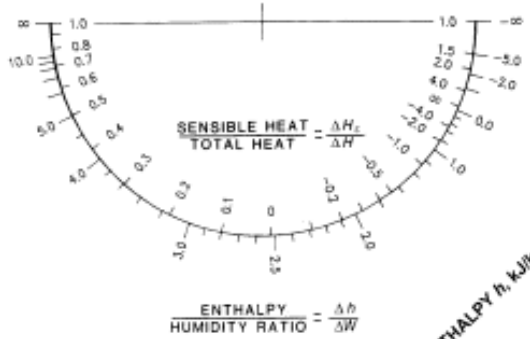
$$h \approx 1.006T + W(2501 + 1.86T)$$

\*where  $T$  is in  $^{\circ}\text{C}$



ASHRAE PSYCHROMETRIC CHART NO.1  
 NORMAL TEMPERATURE SEA LEVEL  
 BAROMETRIC PRESSURE: 101.325 kPa  
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Relative Humidity  
 $\phi \approx 50\%$

Specific Volume  
 $v \approx 0.848 \text{ m}^3/\text{kg}_{da}$

Density  
 $\rho \approx 1/v \approx 1.18 \text{ kg}_{da}/\text{m}^3$

Humidity Ratio  
 $W \approx 8.2 \text{ g}/\text{kg}_{da}$   
 (i.e., 0.0082 kg/kg)

Enthalpy  
 $h \approx 44 \text{ kJ}/\text{kg}_{da}$

Dew Point Temp  
 $T_{dew} \approx 11.7^\circ\text{C}$

Dry Bulb Temp  
 $T = 22^\circ\text{C}$

Wet Bulb Temp  
 $T_{wb} \approx 15.5^\circ\text{C}$





## Revisit 2<sup>nd</sup> example from last class

---

Moist air exists at 30°C dry-bulb temperature with a 15°C dew point temperature

Find the following:

- (a) the humidity ratio,  $W$
- (b) wet-bulb temperature,  $T_{wb}$
- (c) enthalpy,  $h$
- (d) specific volume,  $v$
- (e) relative humidity,  $\phi$

Also:

- (f) degree of saturation,  $\mu$
- (g) density,  $\rho$

# Humidity ratio

$$W = 0.622 \frac{p_w}{p - p_w} \Big|_{@T=30^\circ C}$$

- Assume  $p = 101.325$  kPa (sea level)
- For a known  $T_{dew} = 15^\circ C$ , we know that the actual humidity ratio in the air,  $W$ , is by definition the same as the saturation humidity ratio,  $W_s$ , at an air temperature of  $15^\circ C$

$$W_{@T=30^\circ C} = W_{s@T=15^\circ C} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T=15^\circ C}$$

Temp., °C <i>t</i>	Absolute Pressure $p_{ws}$ kPa
14	1.5989
15	1.7057

$$p_{ws@15C} = 1.7057 \text{ kPa}$$

$$W_{@T=30^\circ C} = W_{s@T=15^\circ C} = 0.622 \frac{1.7057}{101.325 - 1.7057} = 0.01065 \frac{\text{kg}_w}{\text{kg}_{da}}$$

# Degree of saturation

$$\mu = \left[ \frac{W}{W_s} \right]_{@T=30^\circ C}$$

- Need the saturation humidity ratio @  $T = 30^\circ C$ :

$$W_{s@T=30^\circ C} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T=30^\circ C}$$

Temp., °C <i>t</i>	Absolute Pressure <i>p<sub>ws</sub></i> , kPa
30	4.2467
31	4.4966

$p_{ws@15C} = 4.2467 \text{ kPa}$  ←

$$W_{s@T=30^\circ C} = 0.622 \frac{4.2467}{101.325 - 4.2467} = 0.02720 \frac{\text{kg}_w}{\text{kg}_{da}}$$

$$\mu = \frac{W}{W_s} = \frac{0.01065}{0.02720} = 0.39$$

# Relative humidity

---

$$\phi = \frac{p_w}{p_{ws}}$$

- From previous:

$$p_{@T=30^\circ C} = p_{ws@T=15^\circ C} = 1.7057 \text{ kPa}$$

$$p_{ws@T=30^\circ C} = 4.2467 \text{ kPa}$$

$$\phi = \frac{1.7057}{4.2467} = 0.40 = 40\%$$

# Enthalpy

---

$$h \approx 1.006T + W(2501 + 1.86T)$$

\*where  $T$  is in °C

$$h \approx 1.006(30) + (0.01065)(2501 + 1.86(30)) = 57.4 \frac{\text{kJ}}{\text{kg}}$$

# Specific volume and density

---

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

$$v \approx 0.287042(30 + 273.15)(1 + 1.6078(0.01065)) / (101.325)$$

$$v \approx 0.873 \frac{\text{m}^3}{\text{kg}_{\text{da}}}$$

$$\rho = \frac{1}{v}(1 + W) = \frac{1}{0.873}(1 + 0.01065) = 1.157 \frac{\text{kg}}{\text{m}^3}$$

# Wet-bulb temperature

---

- Wet-bulb temperature is the  $T_{wb}$  that fits this equation:

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}} = 0.01065$$

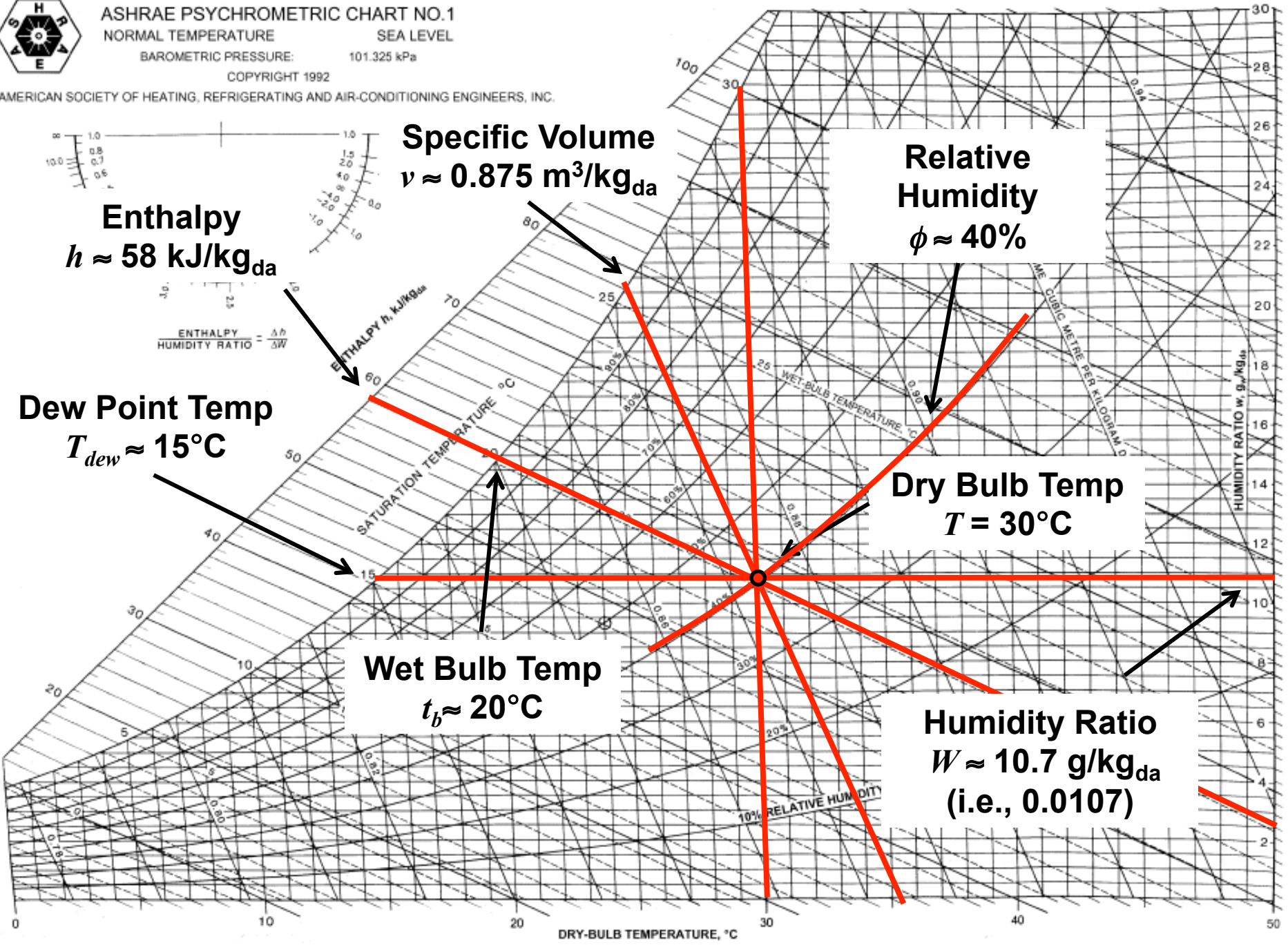
where:  $T = 30^\circ\text{C}$   
 $T_{wb} = ?^\circ\text{C}$

$$W_{s@T_{wb}=?} = 0.622 \frac{p_{ws}}{p - p_{ws}} \Big|_{@T_{wb}=?}$$

## Procedure:

- Guess  $T_{wb}$ , calculate  $p_{ws}$  for that  $T$ , calculate  $W_s$  for that  $T$ 
  - Repeat until  $W$  calculated based on those values (and original  $T$ ) in equation above is equal to actual  $W$  (0.01065 in our case)

$$T_{wb} = 20.1^\circ\text{C}$$



**Specific Volume**  
 $\nu \approx 0.875 \text{ m}^3/\text{kg}_{\text{da}}$

**Relative Humidity**  
 $\phi \approx 40\%$

**Enthalpy**  
 $h \approx 58 \text{ kJ/kg}_{\text{da}}$

**Dew Point Temp**  
 $T_{\text{dew}} \approx 15^\circ\text{C}$

**Dry Bulb Temp**  
 $T = 30^\circ\text{C}$

**Wet Bulb Temp**  
 $t_b \approx 20^\circ\text{C}$

**Humidity Ratio**  
 $W \approx 10.7 \text{ g/kg}_{\text{da}}$   
(i.e., 0.0107)

$$\frac{\text{ENTHALPY}}{\text{HUMIDITY RATIO}} = \frac{\Delta h}{\Delta W}$$

DRY-BULB TEMPERATURE, °C

HUMIDITY RATIO  $w$ , g<sub>w</sub>/kg<sub>da</sub>



# HW 3

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- Released today, due on Thursday October 2
- Involves creating a psychrometric spreadsheet