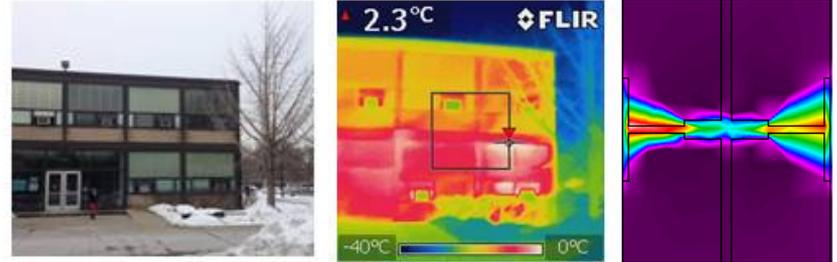


CAE 331/513

Building Science

Fall 2014



Week 5: September 23, 2014

Psychrometrics (chart and definitions)

Built
Environment
Research

@ IIT



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sustainability research within the built environment*

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Dr. Brent Stephens, Ph.D.

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Illinois Institute of Technology

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HW 2

- Was originally due today – now due Thursday
- Need any help?

Race to Zero: DOE student design competition

Announcement:



- Registration due December 15
- Submittals due March 22
- Competition held April 18-20

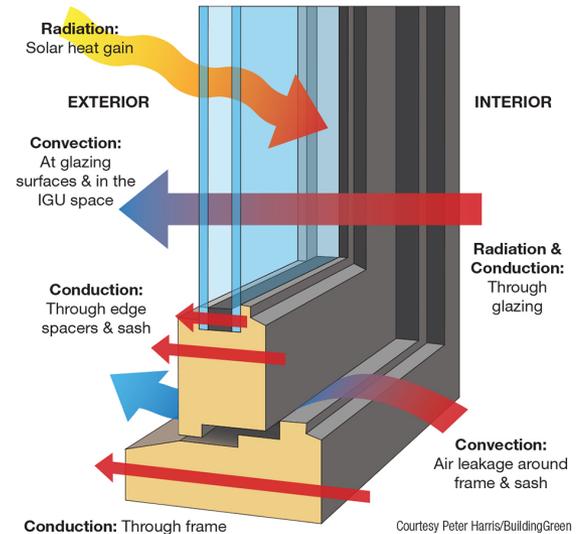
*We will have a CAE 497 Special Problems course in Spring 2015 devoted to this competition

<http://www.energy.gov/eere/buildings/us-department-energy-race-zero-student-design-competition>

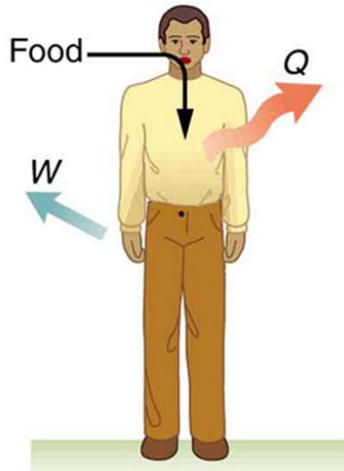
Last time

- Finished solar radiation and heat transfer through windows

$$Q_{window} = UA_{pf} (T_{out} - T_{in}) + I_{solar} A_{pf} SHGC$$



- Human thermal comfort



$$\dot{Q} = MA_{skin}$$

$$1 \text{ met} = 18.4 \frac{\text{Btu}}{\text{h} \cdot \text{ft}^2} = 58 \frac{\text{W}}{\text{m}^2}$$

$$A_D = 0.202m^{0.425}l^{0.725}$$

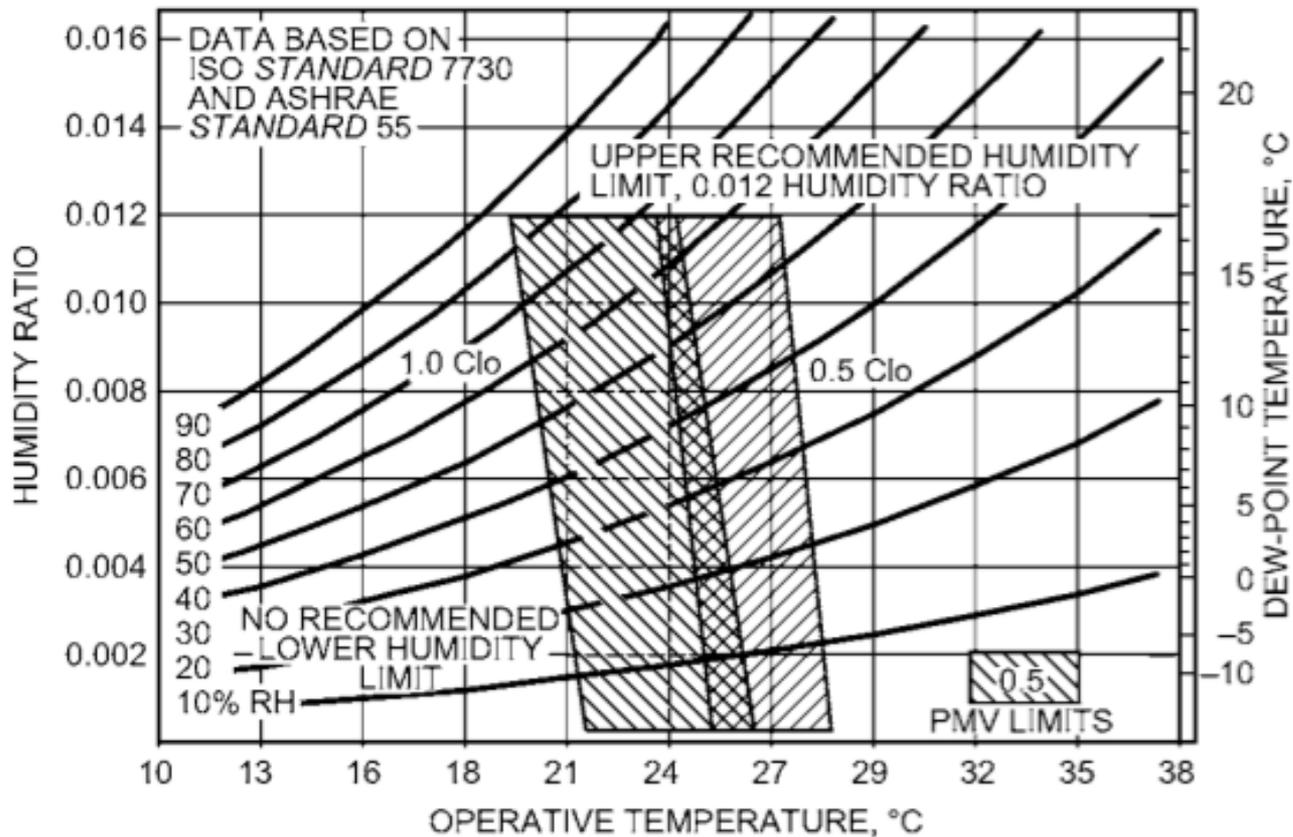
A_D = DuBois surface area, m^2

m = mass, kg

l = height, m

-3	-2	-1	0	+1	+2	+3
cold	cool	slightly cool	neutral	slightly warm	warm	hot

ASHRAE comfort zone



$$t_o = \frac{(h_r t_{mr} + h_c t_a)}{h_r + h_c}$$

$$t_o = \frac{(t_{mr} + (t_a \times \sqrt{10v}))}{1 + \sqrt{10v}}$$

$$t_o = \frac{(t_a + t_{mr})}{2}$$

Fig. 5 ASHRAE Summer and Winter Comfort Zones
 [Acceptable ranges of operative temperature and humidity with air speed ≤ 0.2 m/s for people wearing 1.0 and 0.5 clo clothing during primarily sedentary activity (≤ 1.1 met)].

ASHRAE comfort zone: CBE Thermal Comfort Tool

CBE Thermal Comfort Tool

ASHRAE-55

Compare

Ranges

Select method:

PMV method

Air temperature

24.6 °C

Use operative temperature

Mean radiant temperature

26 °C

Air speed

0.07

Humidity

50

Metabolic

1.3

Clothing

0.55

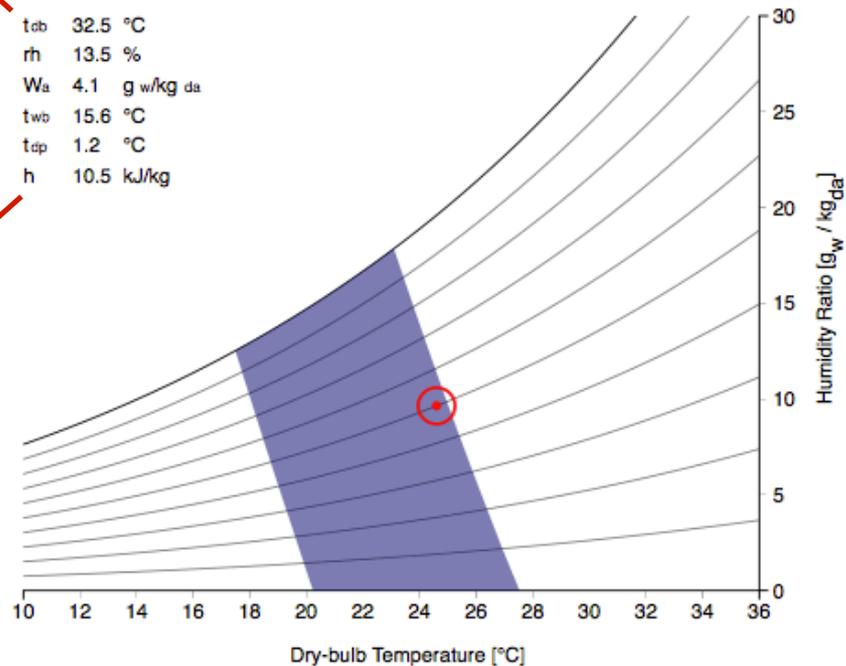
t_{db} 32.5 °C
 rh 13.5 %
 W_a 4.1 g w/kg da
 t_{wb} 15.6 °C
 t_{dp} 1.2 °C
 h 10.5 kJ/kg

✓ Complies with ASHRAE Standard 55-2010

PMV 0.44
PPD 9%
Sensation Neutral
SET 26.7°C

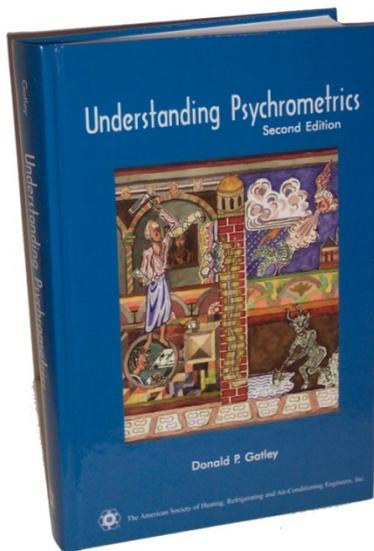
Psychrometric chart (air temperature)

t_{db} 32.5 °C
 rh 13.5 %
 W_a 4.1 g w/kg da
 t_{wb} 15.6 °C
 t_{dp} 1.2 °C
 h 10.5 kJ/kg



PSYCHROMETRICS

Psychrometrics



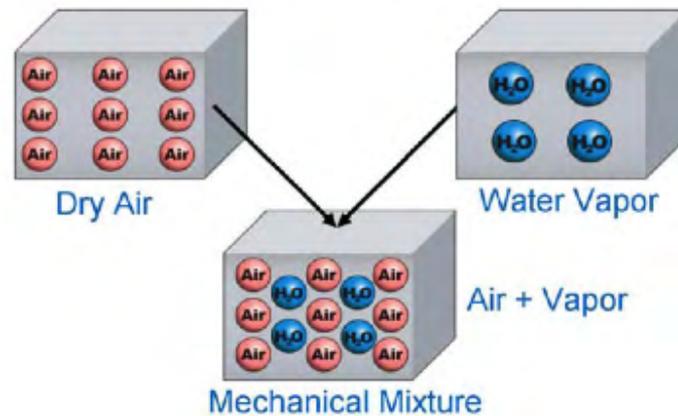
- Psychrometrics is the science and engineering of air/vapor mixtures
- For architectural engineers, the vapor is usually **water vapor**
 - The term psychrometry comes from the Greek *psuchron* meaning cold and *metron* meaning measure
 - In building engineering we use psychrometrics to relate the thermodynamic and physical properties of moist air

Applying psychrometrics

- We need to understand **air temperature** and **moisture content** to understand **human thermal comfort**
 - In **hot, humid** weather we design HVAC systems to **remove** moisture by dehumidification/cooling
 - In **dry, cold** weather, we **add** moisture by humidifiers
- We are also concerned about moisture for structural, aesthetic, and indoor air quality reasons
- Psychrometrics also involves learning how to use and combine a variety of moist air parameters

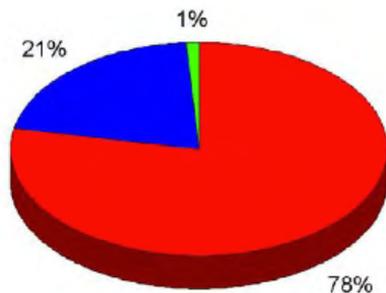
Composition of dry and moist air

- **Atmospheric air** contains:
 - Many gaseous components
 - Water vapor
 - Contaminants (smoke, pollen, gaseous pollutants)
- **Dry air** is atmospheric air with all of the water vapor removed
- **Moist air** is a two-component mixture of dry air and water vapor



Standard composition of dry air

Gas	Molecular weight (g/mol)	Volume %
Nitrogen (N ₂)	32.000	78.084
Oxygen (O ₂)	28.016	20.946
Argon (Ar)	39.444	0.9340
Carbon Dioxide (CO ₂)	44.010	0.03697
Neon (Ne)	20.179	0.00182
Helium (He)	4.002	0.00052
Methane (CH ₄)	16.042	0.00014
Krypton	83.800	0.00010



■ Nitrogen ■ Oxygen ■ Other Gases

Where does water fit in?

Standard composition of **moist** air

Gas	Molecular weight (g/mol)	Volume %
Nitrogen (N ₂)	32.000	78.084%
Oxygen (O ₂)	28.016	20.946%
Water (H₂O)	18.015	0 to 4%
Argon (Ar)	39.444	0.9340%
Carbon Dioxide (CO ₂)	44.010	0.03697%
Neon (Ne)	20.179	0.00182%
Helium (He)	4.002	0.00052%
Methane (CH ₄)	16.042	0.00014%
Krypton	83.800	0.00010%

Key terms for describing moist air

- To describe and deal with moist air, we need to be able to describe the fractions of dry air and water vapor
- There are several different equivalent measures
 - Which one you use depends on what data you have to start with and what quantity you are trying to find

Key terms to know:

- Dry bulb temperature
- Vapor pressure
- Saturation
- Relative humidity
- Absolute humidity (or humidity ratio)
- Dew point temperature
- Wet bulb temperature
- Enthalpy
- Density
- Specific volume

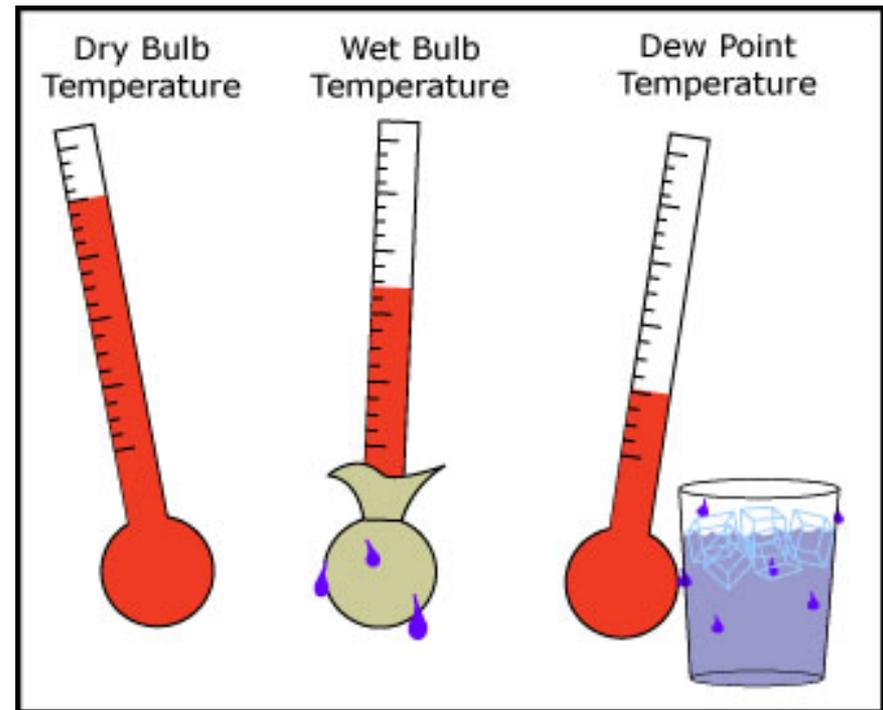
Three different temperatures: T , T_{dew} , and T_{wb}

The standard temperature, T , we are all familiar with is called the **dry-bulb** temperature, or T_d

- It is a measure of internal energy

We can also define:

- **Dew-point** temperature, T_{dew}
 - Temperature at which water vapor changes into liquid (condensation)
 - Air is maximally **saturated** with water vapor
- **Wet-bulb** temperature, T_{wb}
 - The temperature that a parcel of air would have if it were cooled to saturation (100% **relative humidity**) by the evaporation of water into it



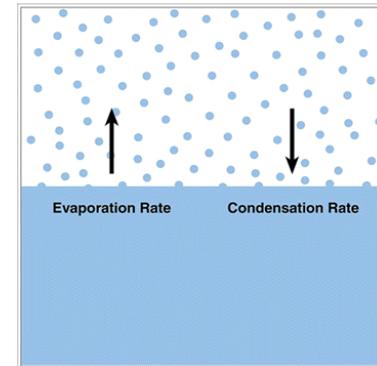
Units of Celsius, Fahrenheit, or Kelvin

Key concepts: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is a measurement of the amount of water vapor in a volume/parcel of air

$$p_w$$

*Units of pressure, Pa or kPa



- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
 - We call the limit **saturation**

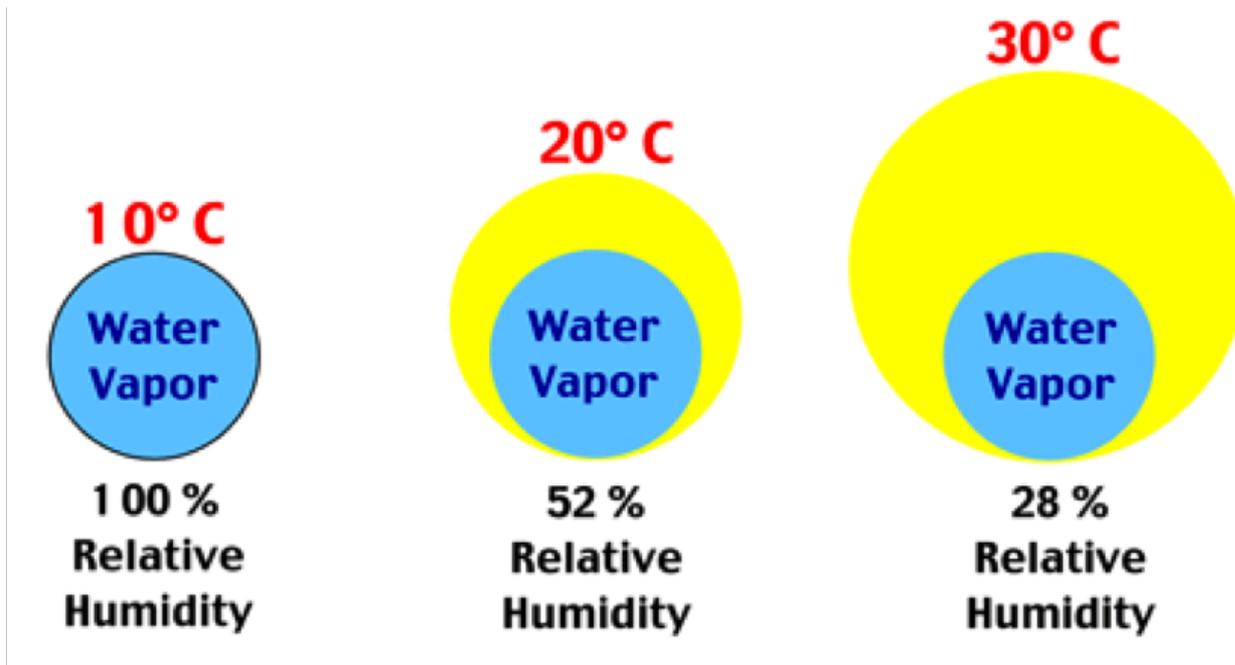
$$p_{ws}$$

*Units of pressure, Pa or kPa



Key concept: Relative humidity, ϕ

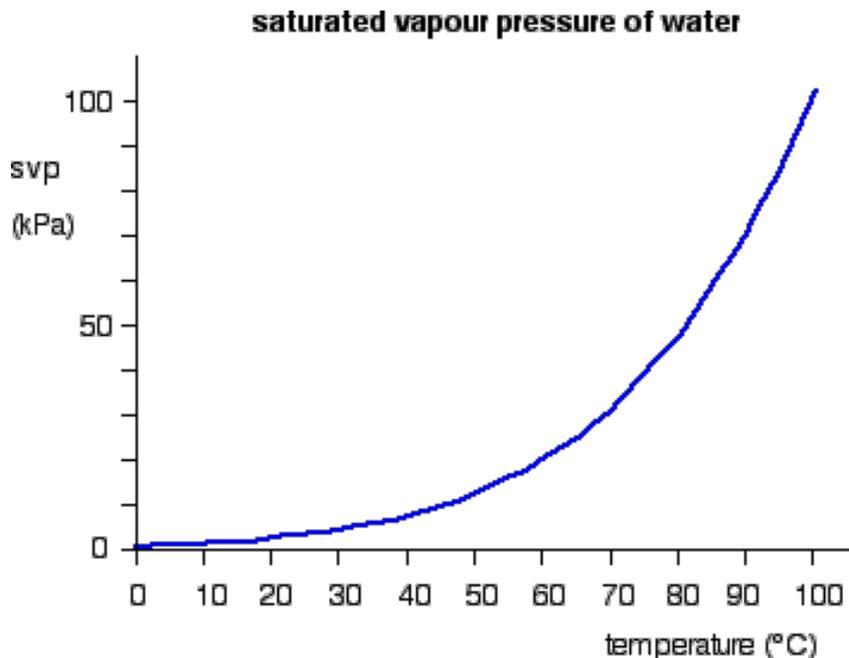
- **Relative humidity** (RH, or ϕ) is the ratio of the vapor pressure of moisture in a sample of air to the *saturation* vapor pressure at the dry bulb temperature of the sample
 - RH is therefore a function of temperature
- Relative humidity \neq absolute humidity (or humidity ratio)



$$\phi = \frac{p_w}{p_{ws}}$$

Key concept: Saturation vapor pressure, p_{ws}

- The **saturation vapor pressure** is the partial pressure of water vapor at saturation (p_{ws}) *Units of pressure, Pa or kPa
 - Cannot absorb any more moisture at that temperature
- We can look up p_{ws} in tables (as a function of T)
 - Table 3 in Ch.1 of 2013 ASHRAE Fundamentals
- We can also use empirical equations:



$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

where

$$C_8 = -5.800\ 220\ 6\ E+03$$

$$C_9 = 1.391\ 499\ 3\ E+00$$

$$C_{10} = -4.864\ 023\ 9\ E-02$$

$$C_{11} = 4.176\ 476\ 8\ E-05$$

$$C_{12} = -1.445\ 209\ 3\ E-08$$

$$C_{13} = 6.545\ 967\ 3\ E+00$$

p_{ws} = saturation pressure, Pa

T = absolute temperature, K = °C + 273.15

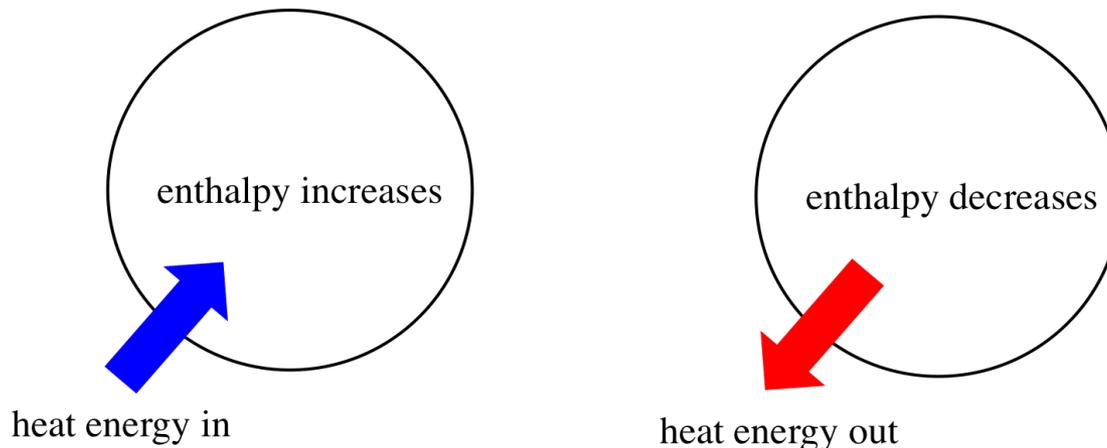
Key concept: Humidity ratio, W

- The **humidity ratio** (or absolute humidity) is a direct measure of the **moisture content** of a parcel of air
- Simply, the humidity ratio (or absolute humidity) is the mass quantity of water vapor that exists in a mass parcel of air
 - Units of mass of water vapor per mass of dry air
 - kg/kg ($\text{kg}_w/\text{kg}_{da}$)
 - g/kg (g_w/g_{da})

$$W = \frac{\text{mass of water vapor}}{\text{mass of dry air}} \left[\frac{\text{kg}_w}{\text{kg}_{da}} \right]$$

Key concept: **Enthalpy**

- **Enthalpy** is a measure of the amount of energy in a system
 - Units of Joules
- The enthalpy of moist air is the total enthalpy of the dry air and the water vapor mixture per mass of moist air
- Includes:
 - Enthalpy of dry air, or **sensible** heat
 - Enthalpy of evaporated water, or **latent** heat



Key concept: Density and specific volume

Specific volume

- Specific volume is the volume of unit mass of dry air at a given temperature, expressed as m³/kg (inverse of dry density)

$$v = \frac{\text{volume of dry air}}{\text{mass of dry air}} \quad \left[\frac{\text{m}^3}{\text{kg}_{\text{da}}} \right]$$

Air density

- Density is a measure of the mass of moist air per unit volume of air
- Includes mass of dry air + water vapor

$$\rho = \frac{\text{mass of moist air}}{\text{volume of moist air}} \quad \left[\frac{\text{kg}}{\text{m}^3} \right]$$

The Psychrometric Chart

- There are both simple and complex ways to estimate these properties
 - Equations and tables (more complex, save for next lecture)
 - Graphically using ...
- **The Psychrometric Chart**
 - Plots dry bulb temperature (T) on the x-axis and humidity ratio (W) on the y-axis
 - Shows relationships between T and W and relative humidity, wet-bulb temperature, vapor pressure, specific volume, and enthalpy
 - Charts are unique at each value of atmospheric pressure (p)
- Both SI and IP versions are on BB in the materials folder



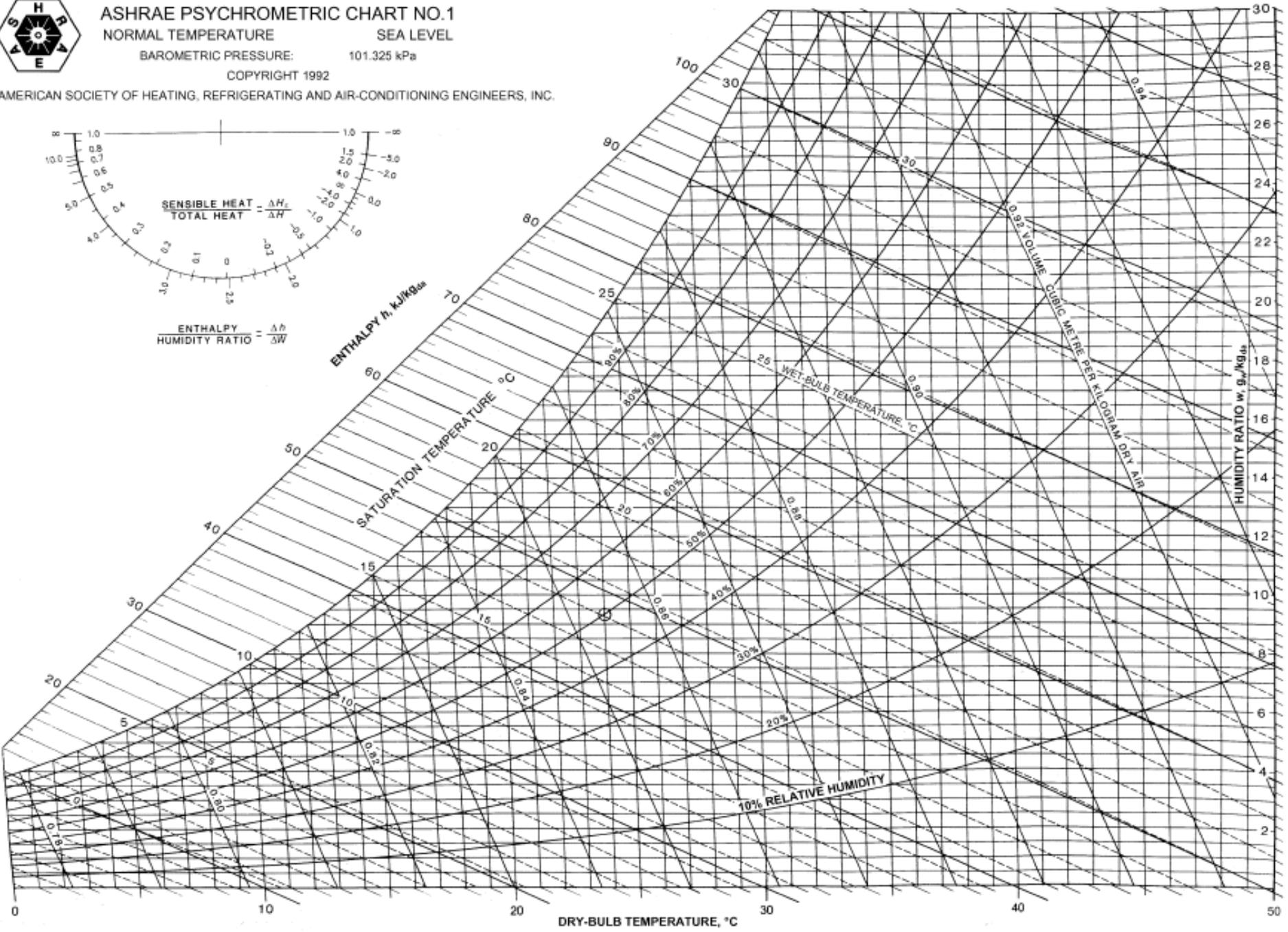
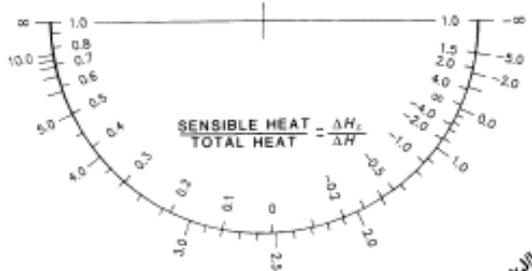
ASHRAE PSYCHROMETRIC CHART NO. 1

NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

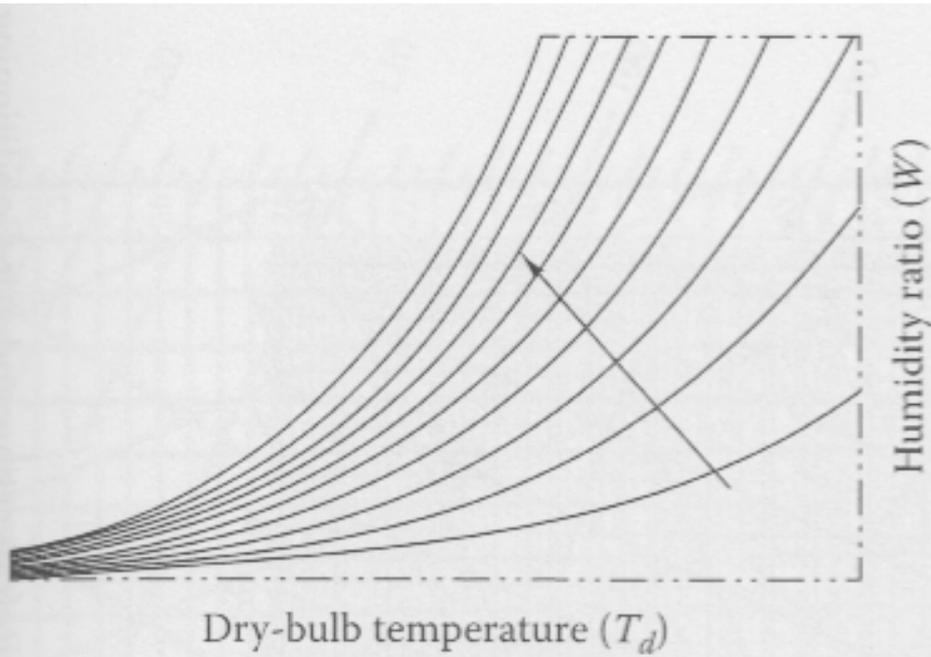
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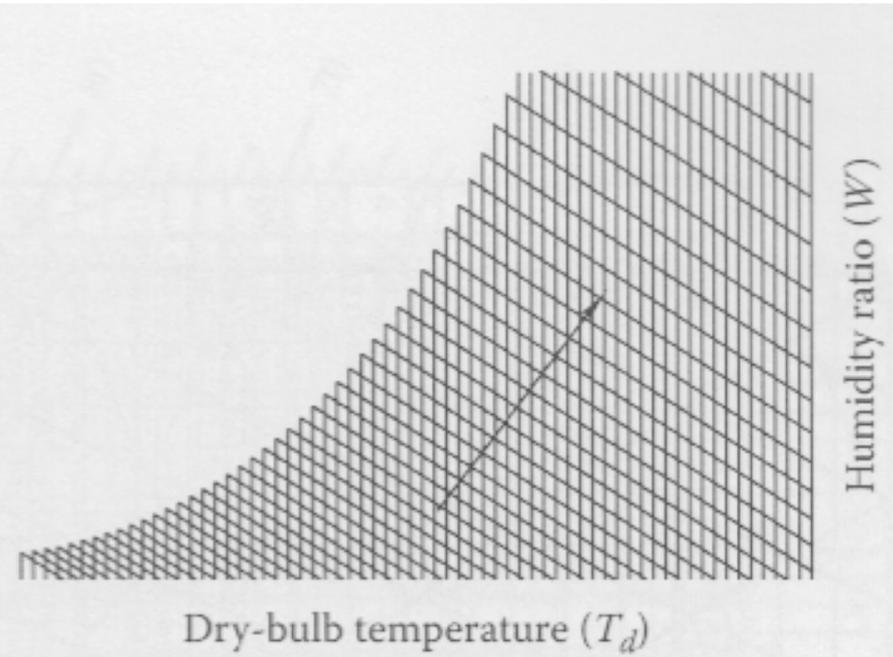


Deciphering the psychrometric chart

Lines of constant RH

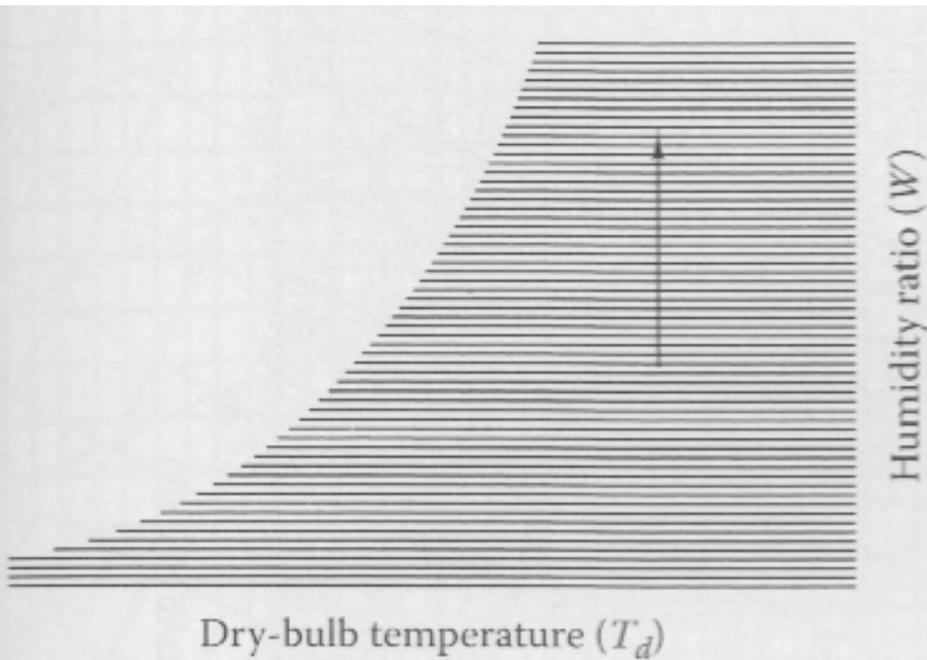


Lines of constant wet-bulb and dry-bulb

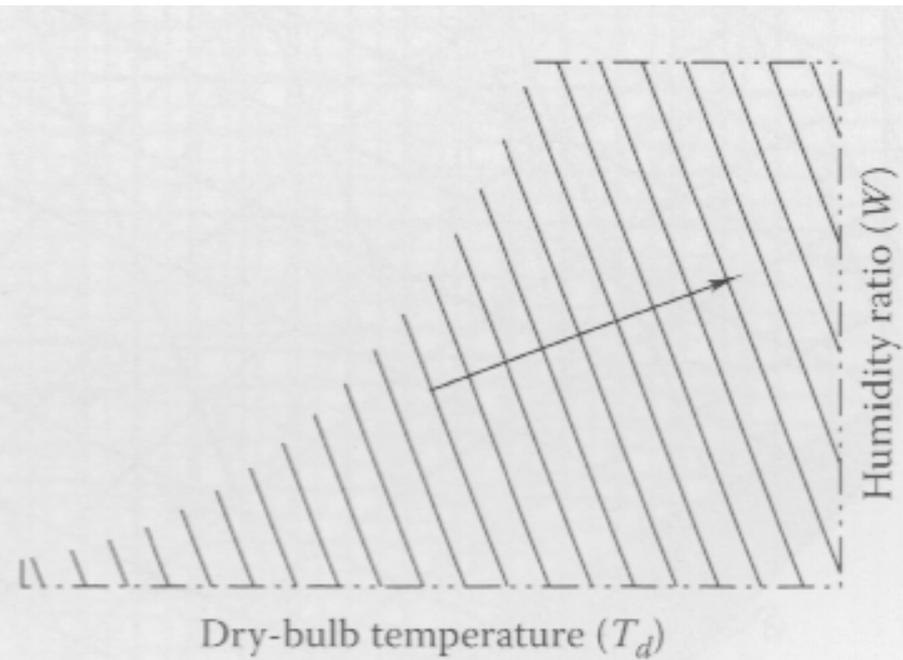


Deciphering the psychrometric chart

Lines of constant humidity ratio

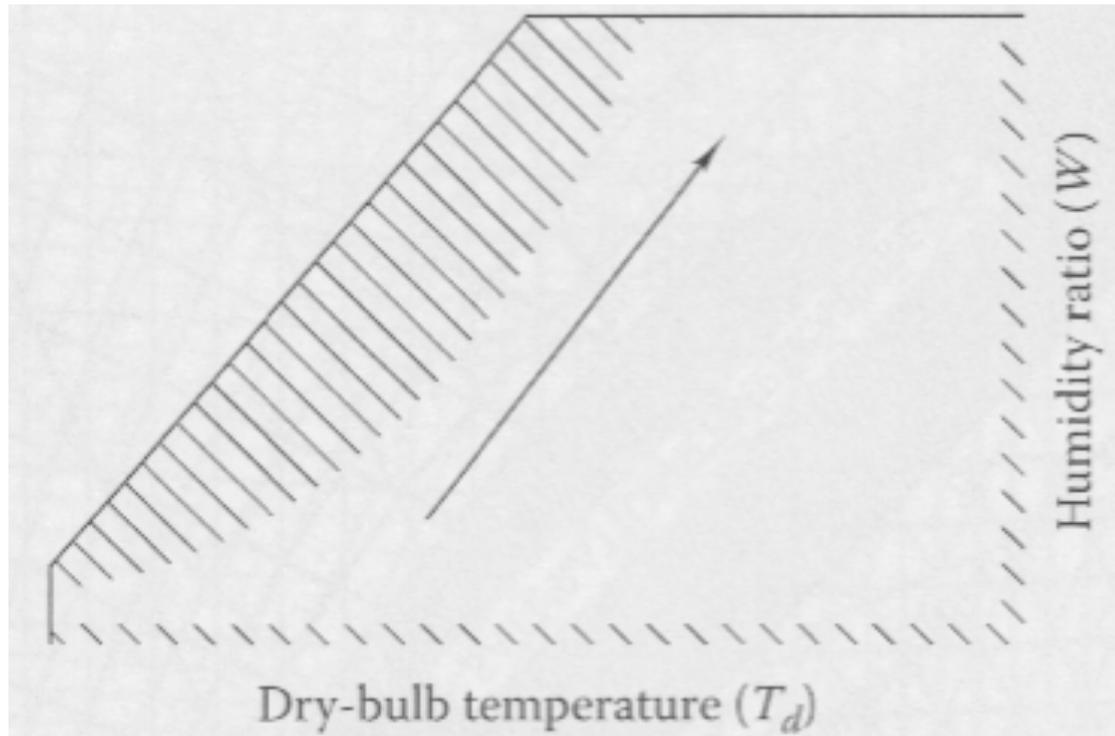


Lines of constant specific volume



Deciphering the psychrometric chart

Lines of constant enthalpy





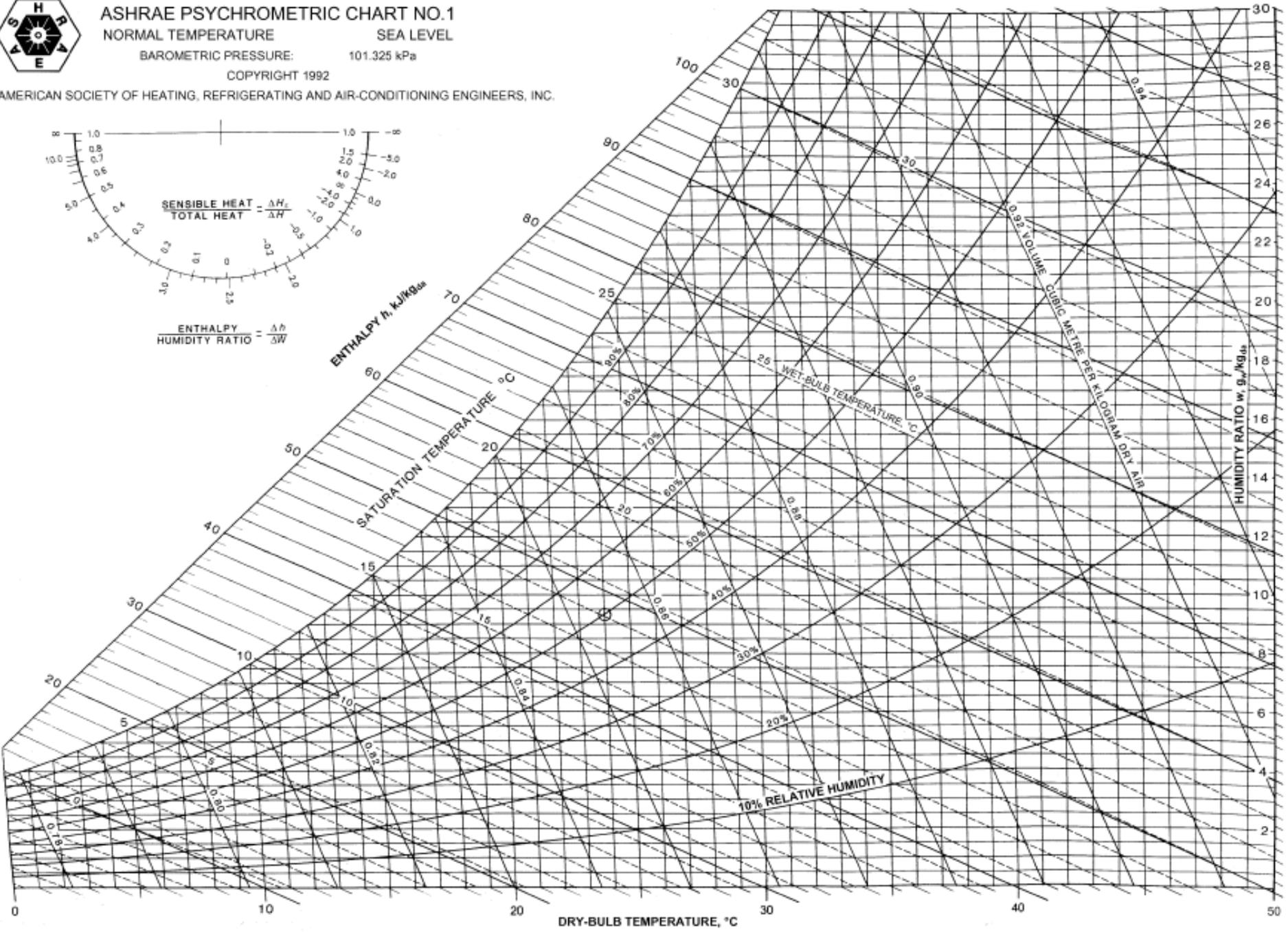
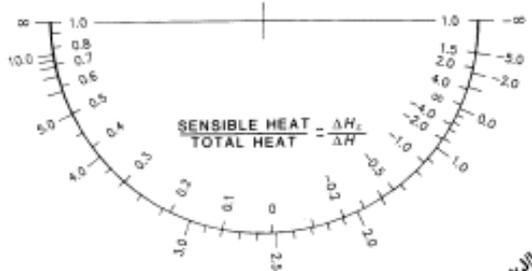
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NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

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Some psychrometric examples

Moist air exists at 22°C dry-bulb temperature with 50% RH

Find the following:

- (a) the humidity ratio, W
- (b) dew point temperature, T_{dew}
- (c) wet-bulb temperature, T_{wb}
- (d) enthalpy, h
- (e) specific volume, v
- (f) density, ρ



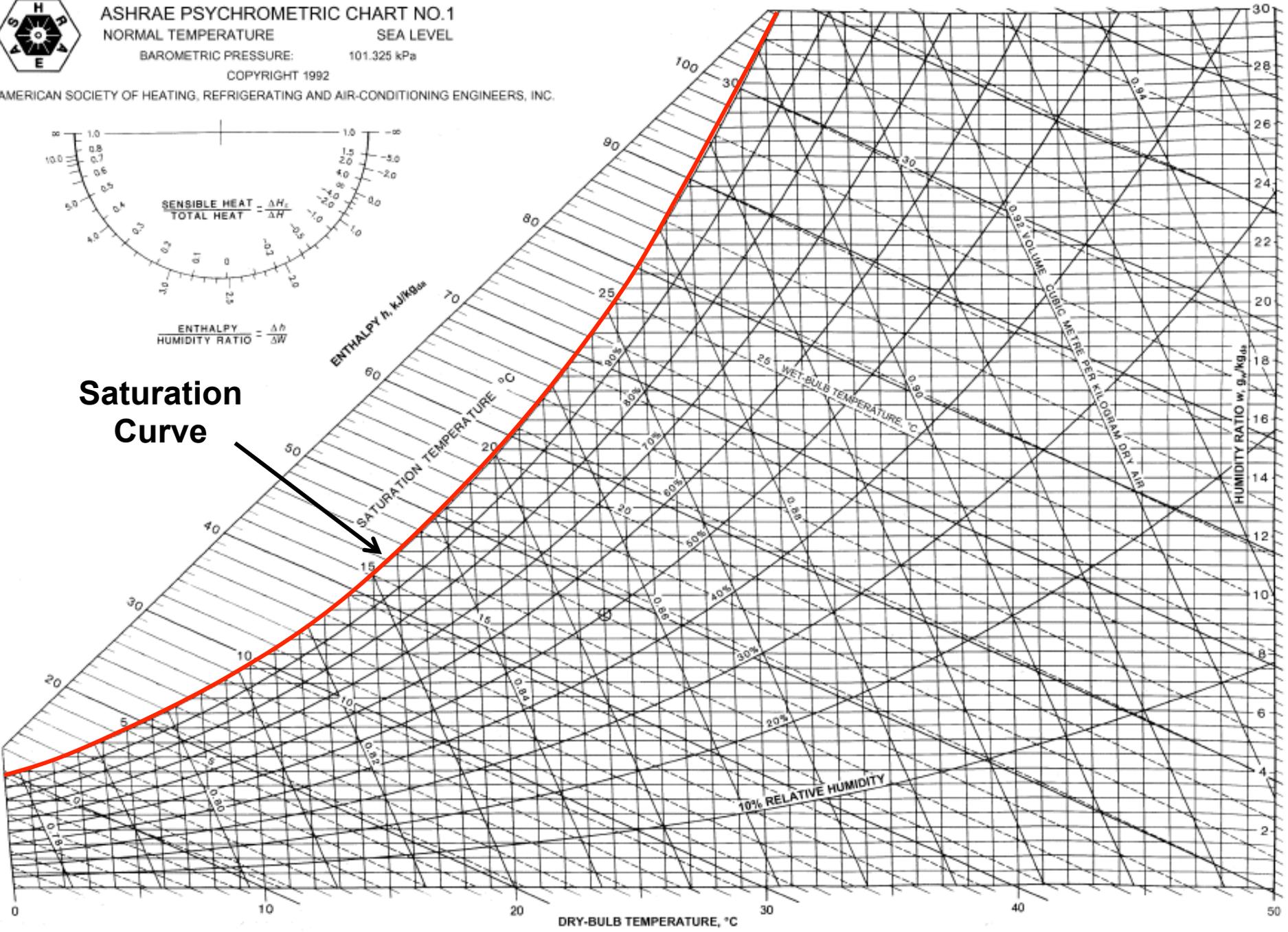
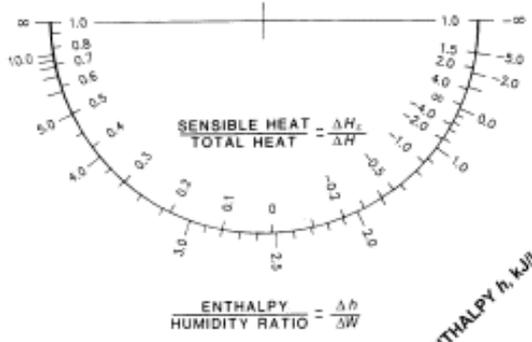
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Saturation Curve





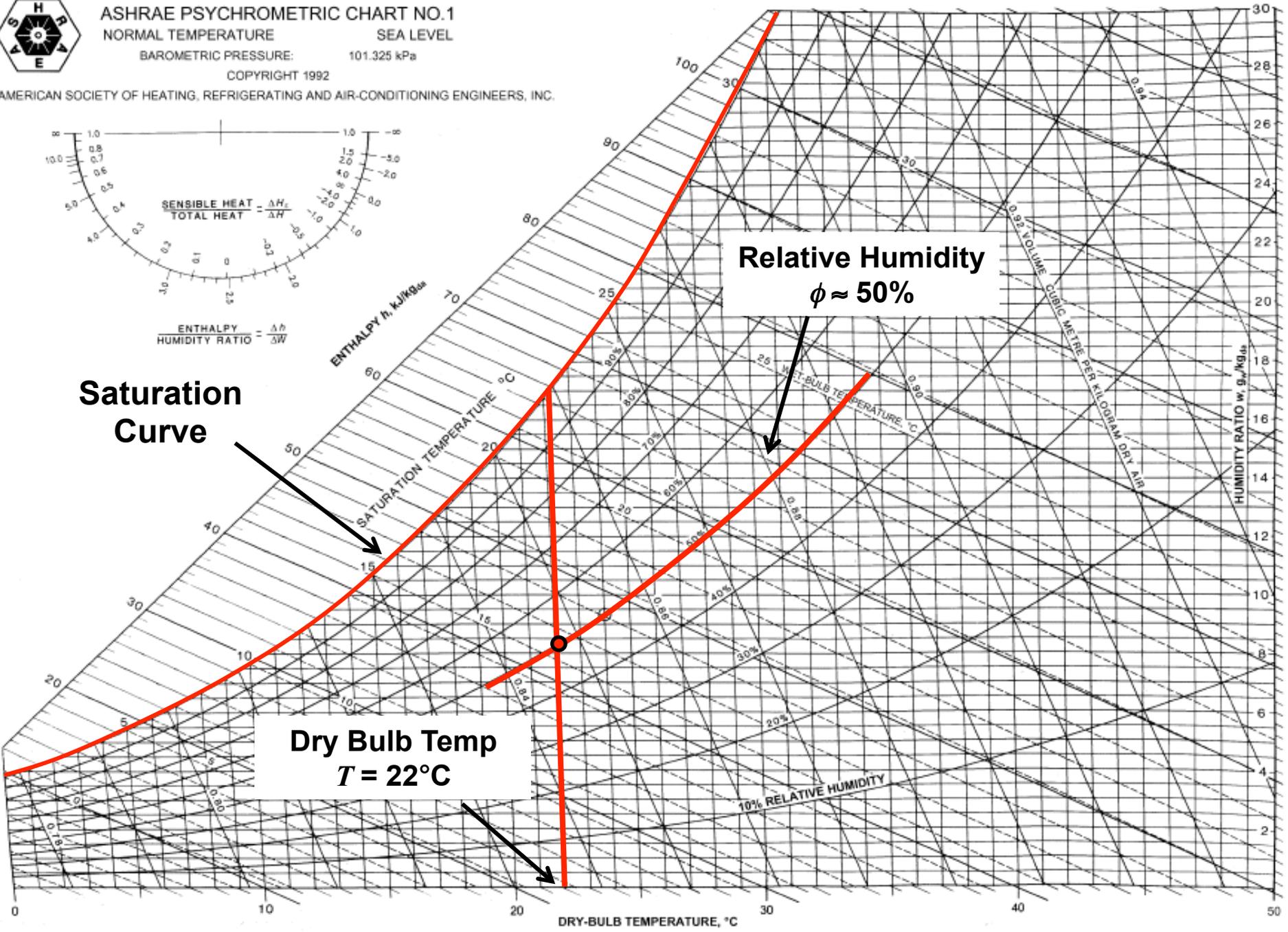
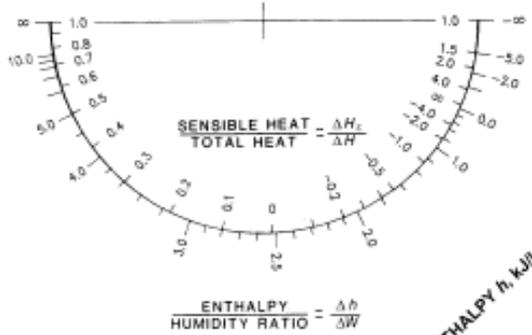
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Saturation Curve

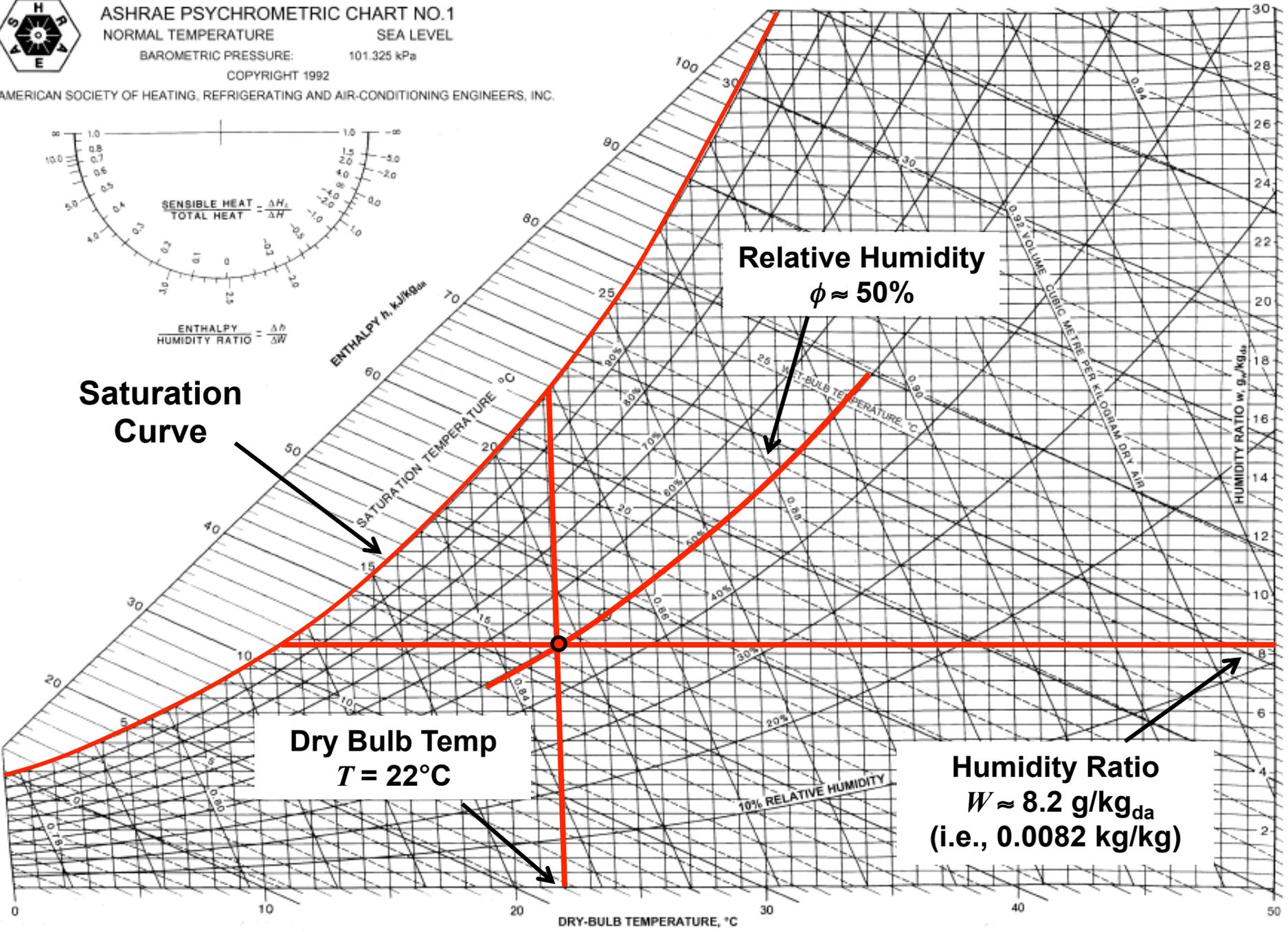
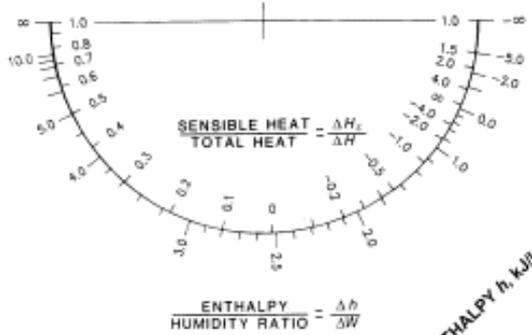
Relative Humidity $\phi \approx 50\%$

Dry Bulb Temp $T = 22^\circ\text{C}$



ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Relative Humidity
 $\phi \approx 50\%$

Saturation Curve

Dry Bulb Temp
 $T = 22^\circ\text{C}$

Humidity Ratio
 $W \approx 8.2 \text{ g/kg}_{da}$
 (i.e., 0.0082 kg/kg)



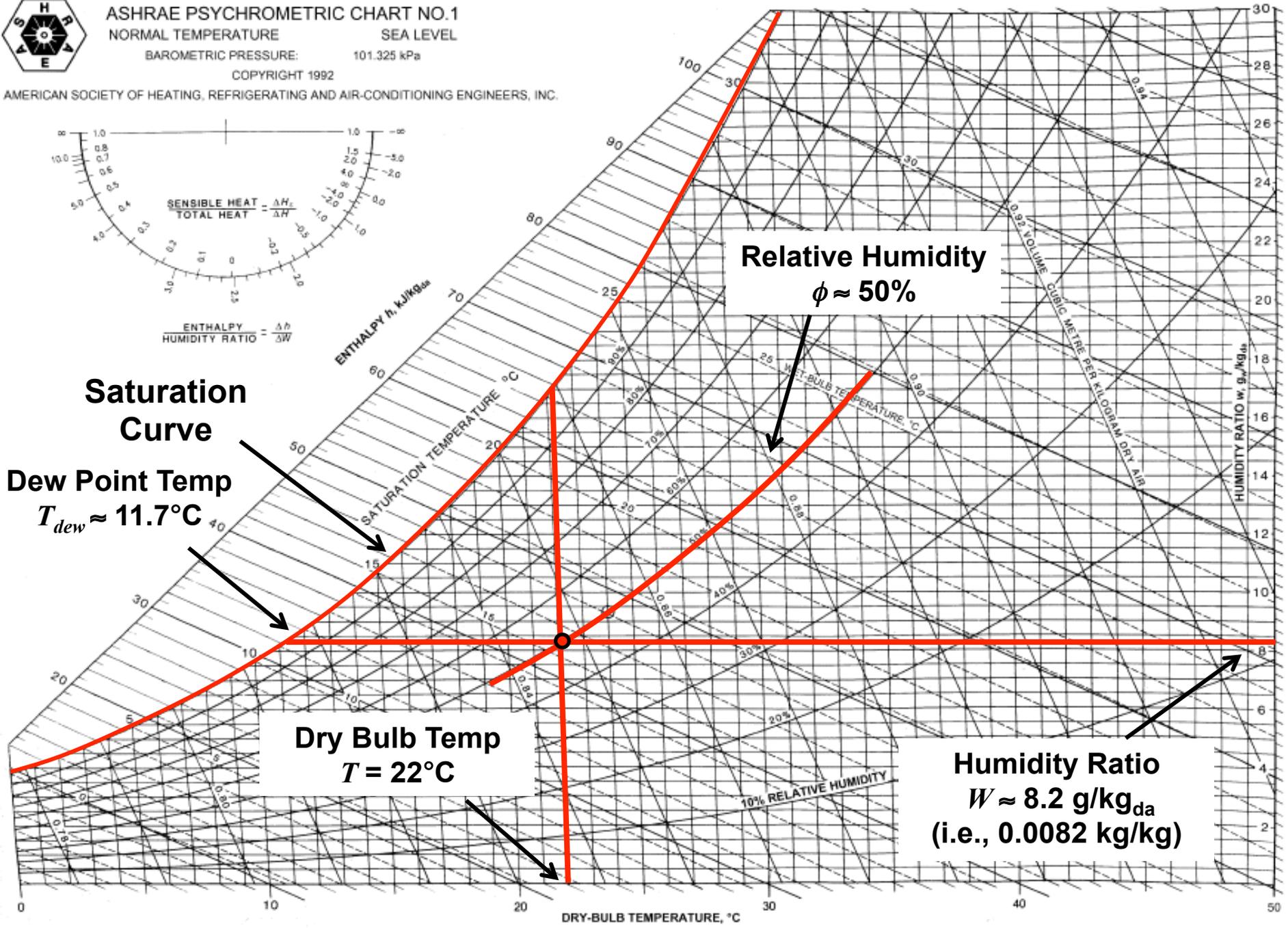
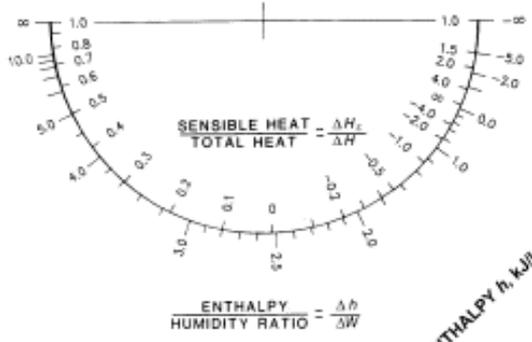
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Saturation Curve
Dew Point Temp
 $T_{dew} \approx 11.7^\circ\text{C}$

Dry Bulb Temp
 $T = 22^\circ\text{C}$

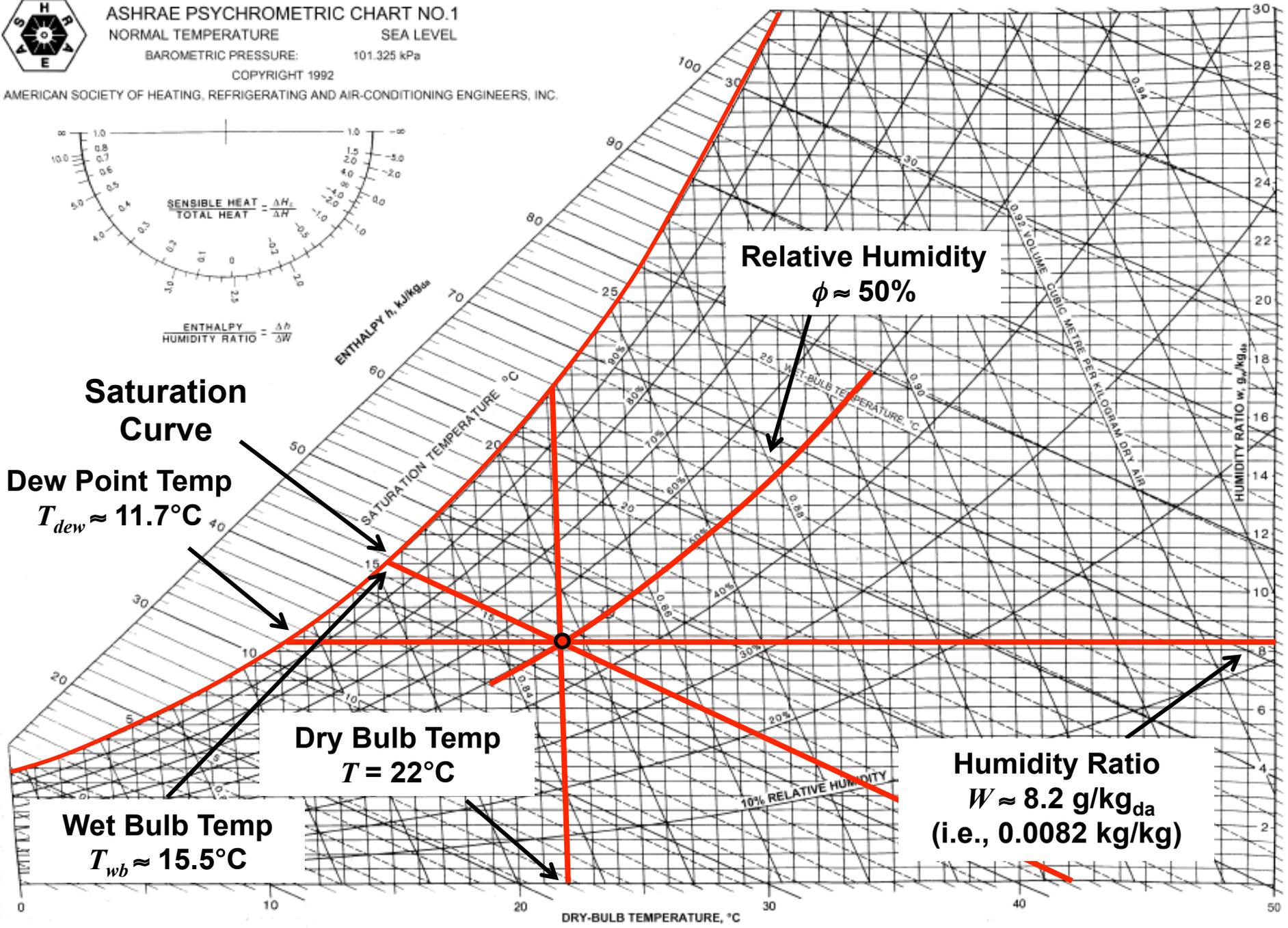
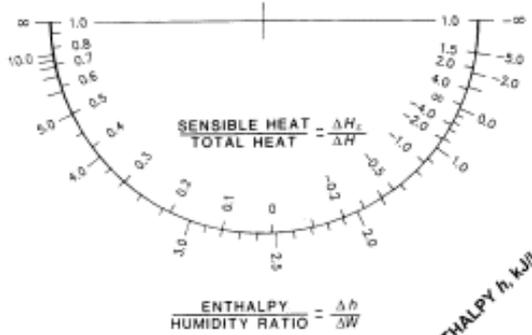
Relative Humidity
 $\phi \approx 50\%$

Humidity Ratio
 $W \approx 8.2 \text{ g/kg}_{da}$
(i.e., 0.0082 kg/kg)



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Relative Humidity
 $\phi \approx 50\%$

Saturation Curve
 Dew Point Temp
 $T_{dew} \approx 11.7^\circ\text{C}$

Dry Bulb Temp
 $T = 22^\circ\text{C}$

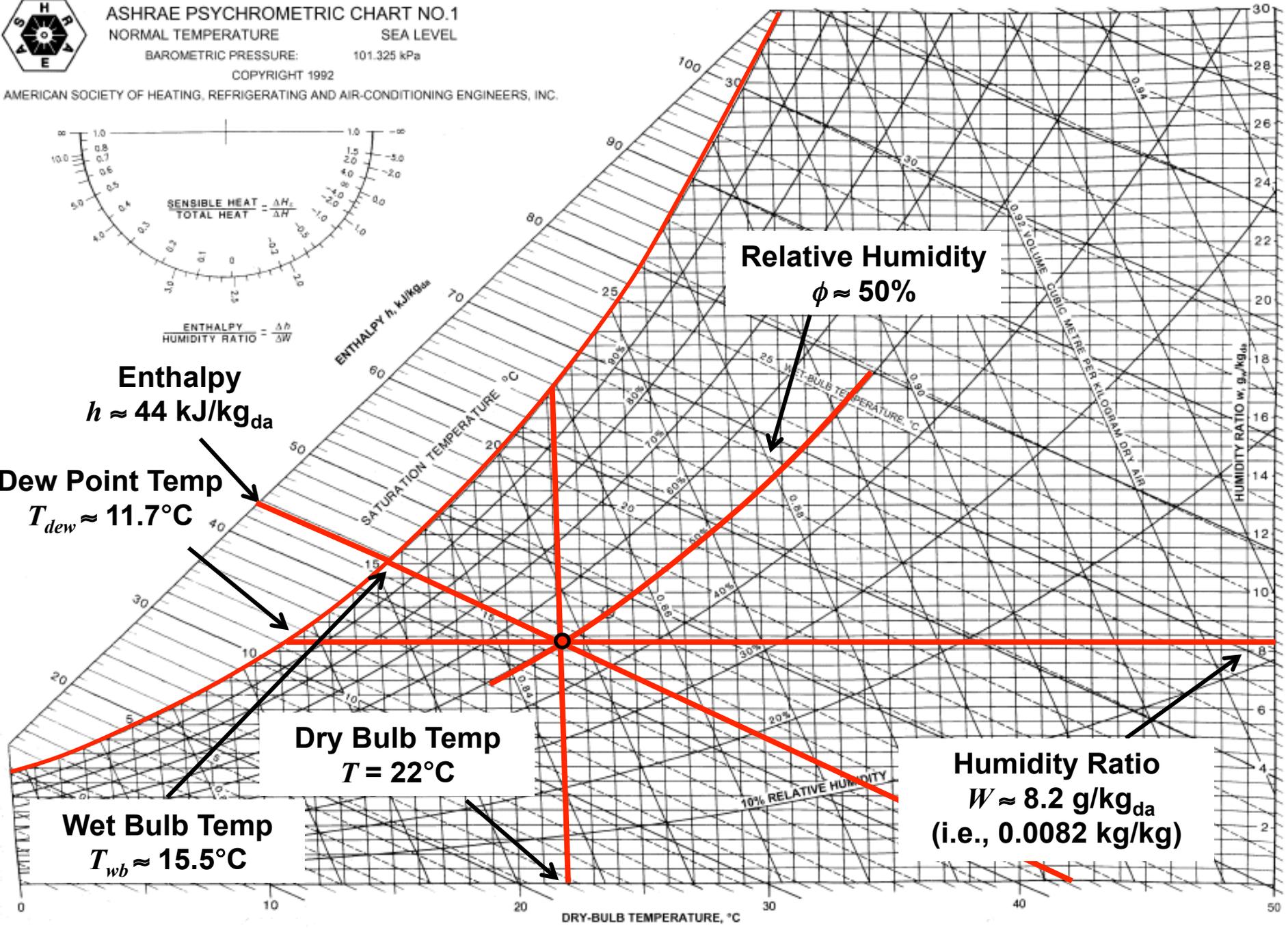
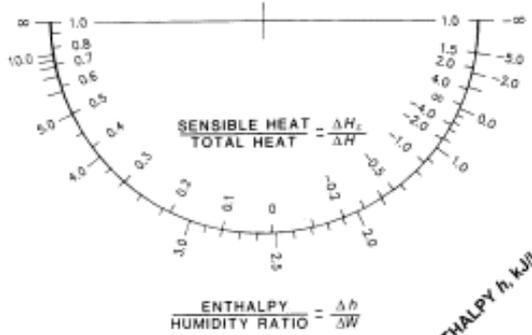
Wet Bulb Temp
 $T_{wb} \approx 15.5^\circ\text{C}$

Humidity Ratio
 $W \approx 8.2 \text{ g/kg}_{da}$
 (i.e., 0.0082 kg/kg)



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Relative Humidity
 $\phi \approx 50\%$

Enthalpy
 $h \approx 44 \text{ kJ/kg}_{da}$

Dew Point Temp
 $T_{dew} \approx 11.7^\circ\text{C}$

Dry Bulb Temp
 $T = 22^\circ\text{C}$

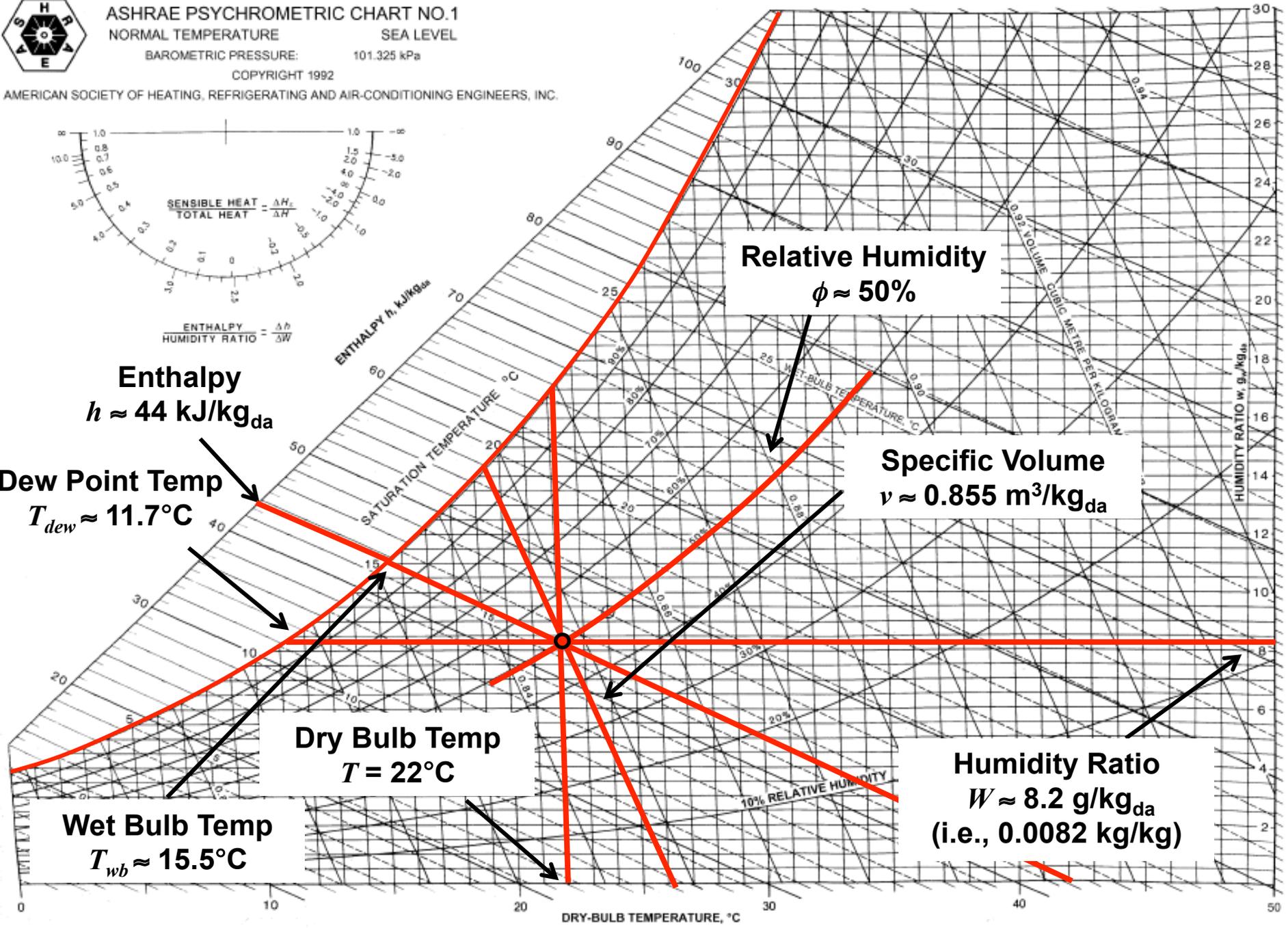
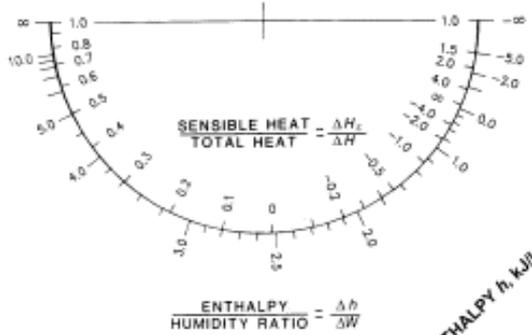
Wet Bulb Temp
 $T_{wb} \approx 15.5^\circ\text{C}$

Humidity Ratio
 $W \approx 8.2 \text{ g/kg}_{da}$
 (i.e., 0.0082 kg/kg)



ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Relative Humidity
 $\phi \approx 50\%$

Specific Volume
 $v \approx 0.855 \text{ m}^3/\text{kg}_{da}$

Humidity Ratio
 $W \approx 8.2 \text{ g}/\text{kg}_{da}$
 (i.e., 0.0082 kg/kg)

Dry Bulb Temp
 $T = 22^\circ\text{C}$

Wet Bulb Temp
 $T_{wb} \approx 15.5^\circ\text{C}$

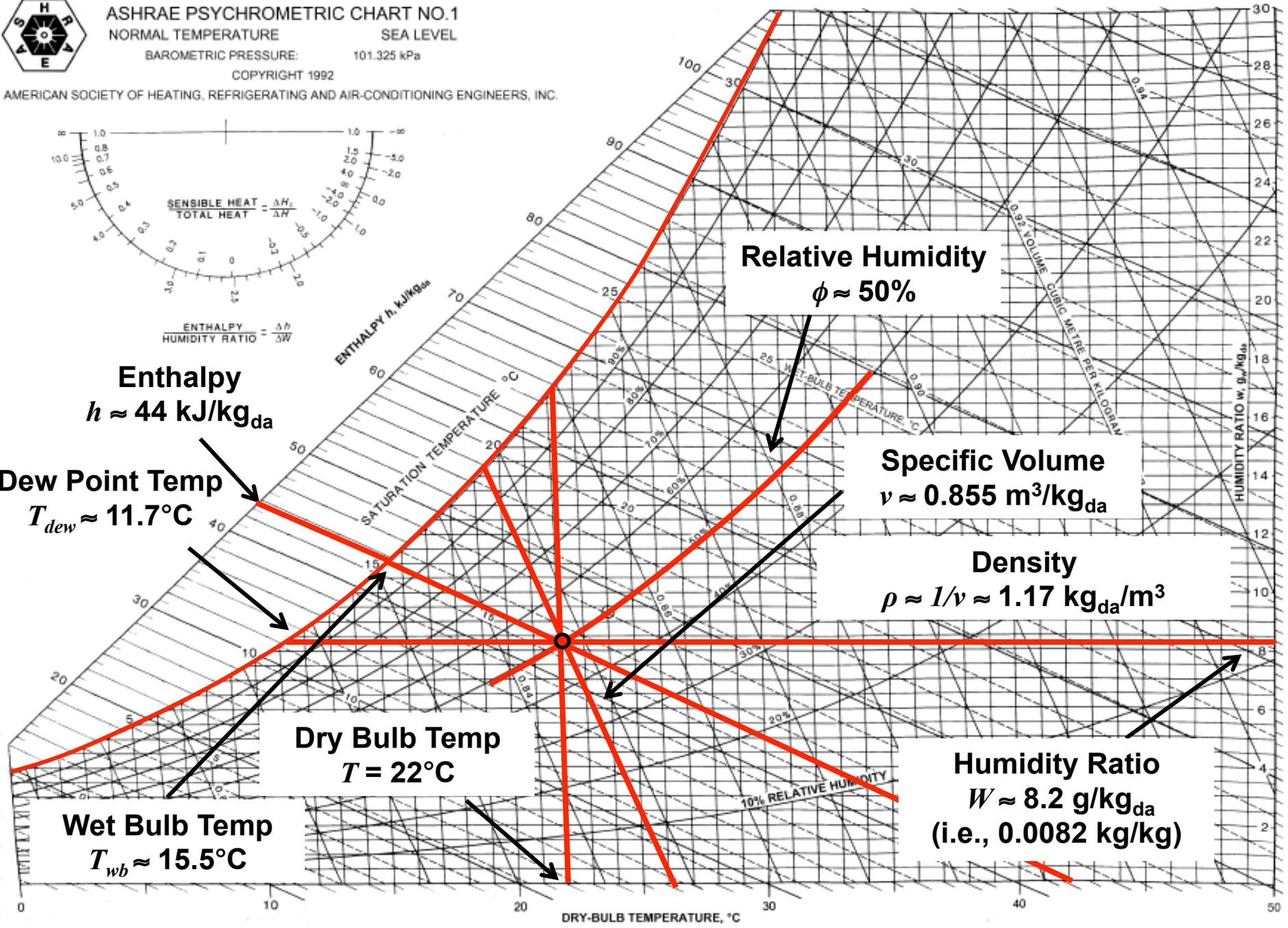
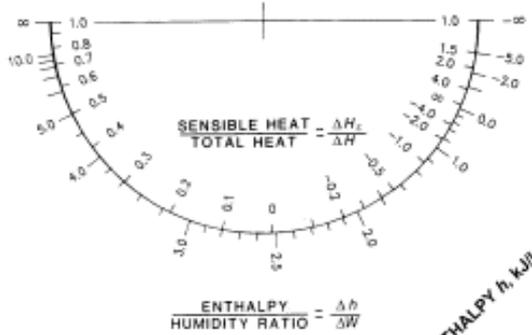
Dew Point Temp
 $T_{dew} \approx 11.7^\circ\text{C}$

Enthalpy
 $h \approx 44 \text{ kJ}/\text{kg}_{da}$



ASHRAE PSYCHROMETRIC CHART NO.1
 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Some psychrometric examples

Moist air exists at 30°C dry-bulb temperature with a 15°C dew point temperature

Find the following:

- (a) the humidity ratio, W
- (b) wet-bulb temperature, T_{wb}
- (c) enthalpy, h
- (d) specific volume, v
- (e) relative humidity, ϕ



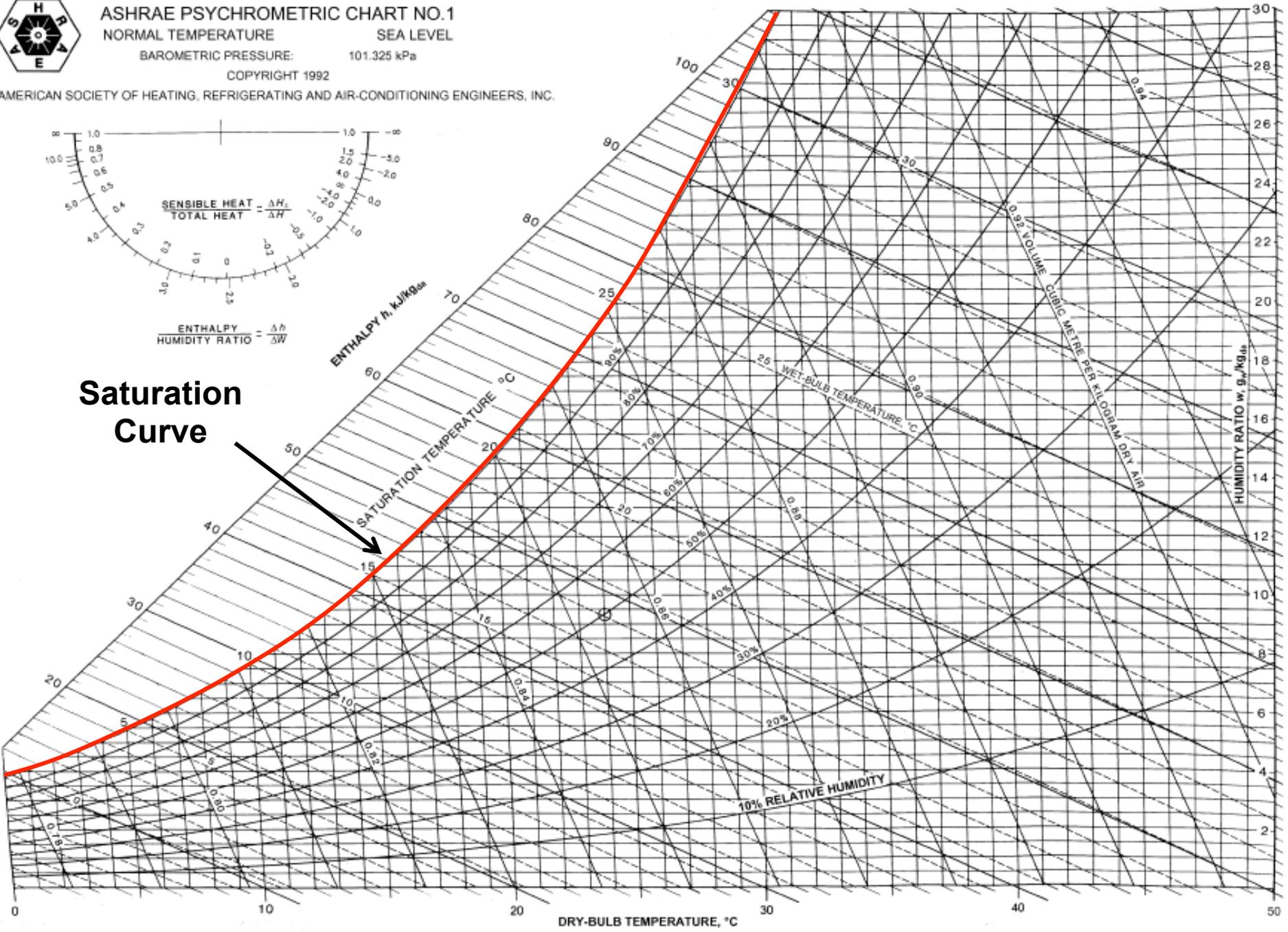
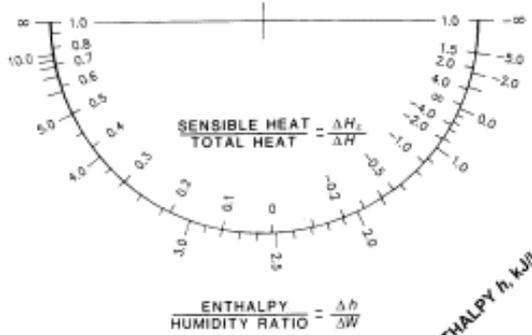
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NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

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Saturation Curve





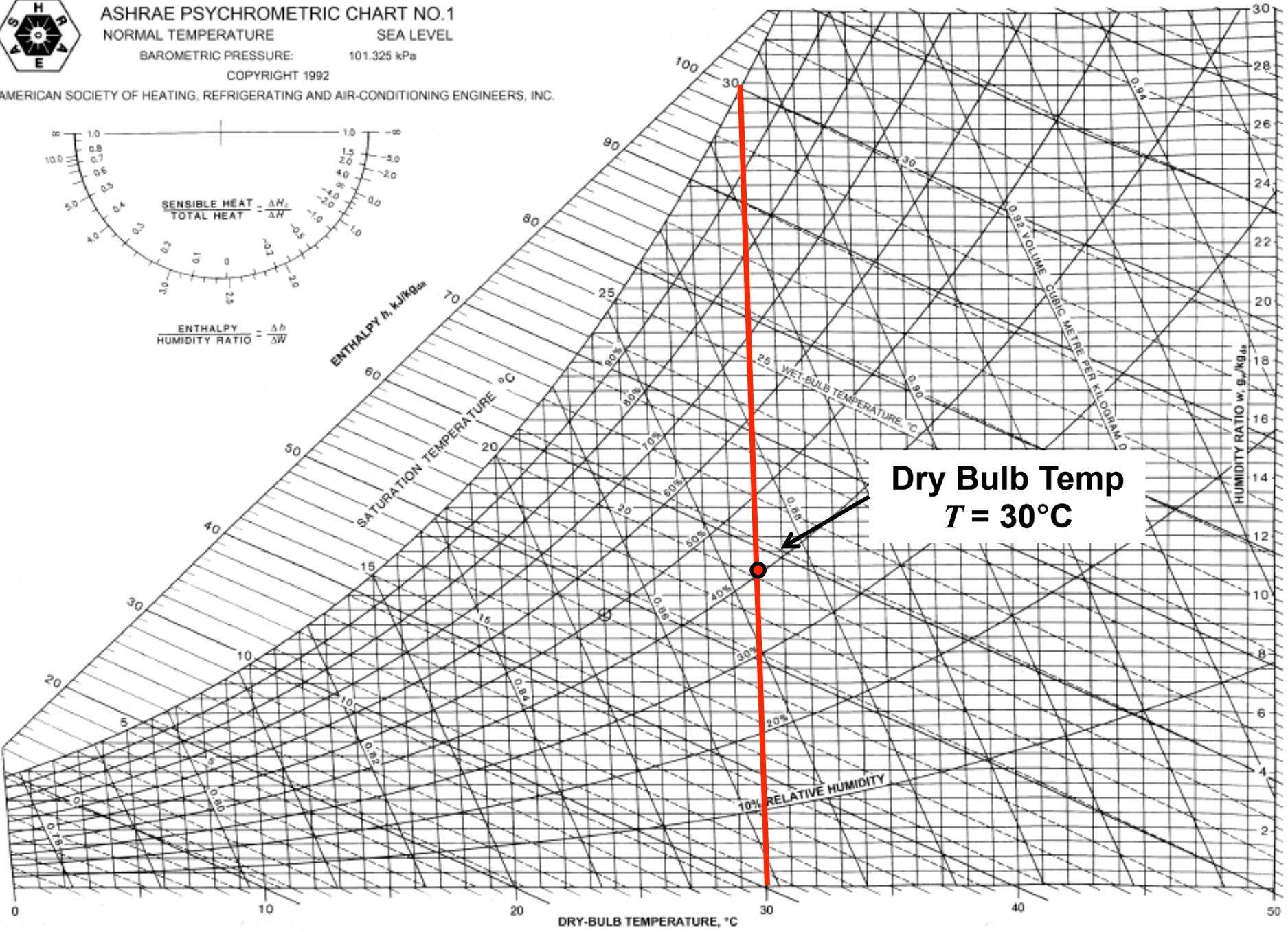
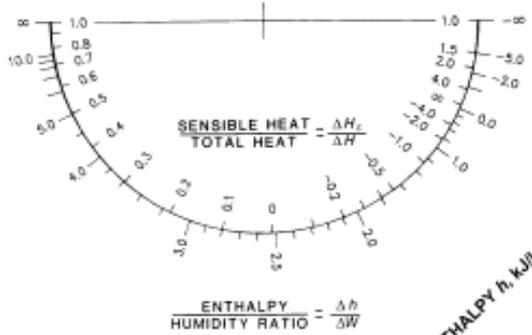
ASHRAE PSYCHROMETRIC CHART NO.1

NORMAL TEMPERATURE SEA LEVEL

BAROMETRIC PRESSURE: 101.325 kPa

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Dry Bulb Temp
 $T = 30^\circ\text{C}$





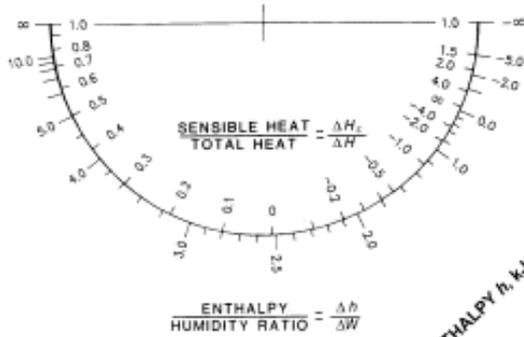
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Dew Point Temp

$T_{dew} \approx 15^\circ\text{C}$

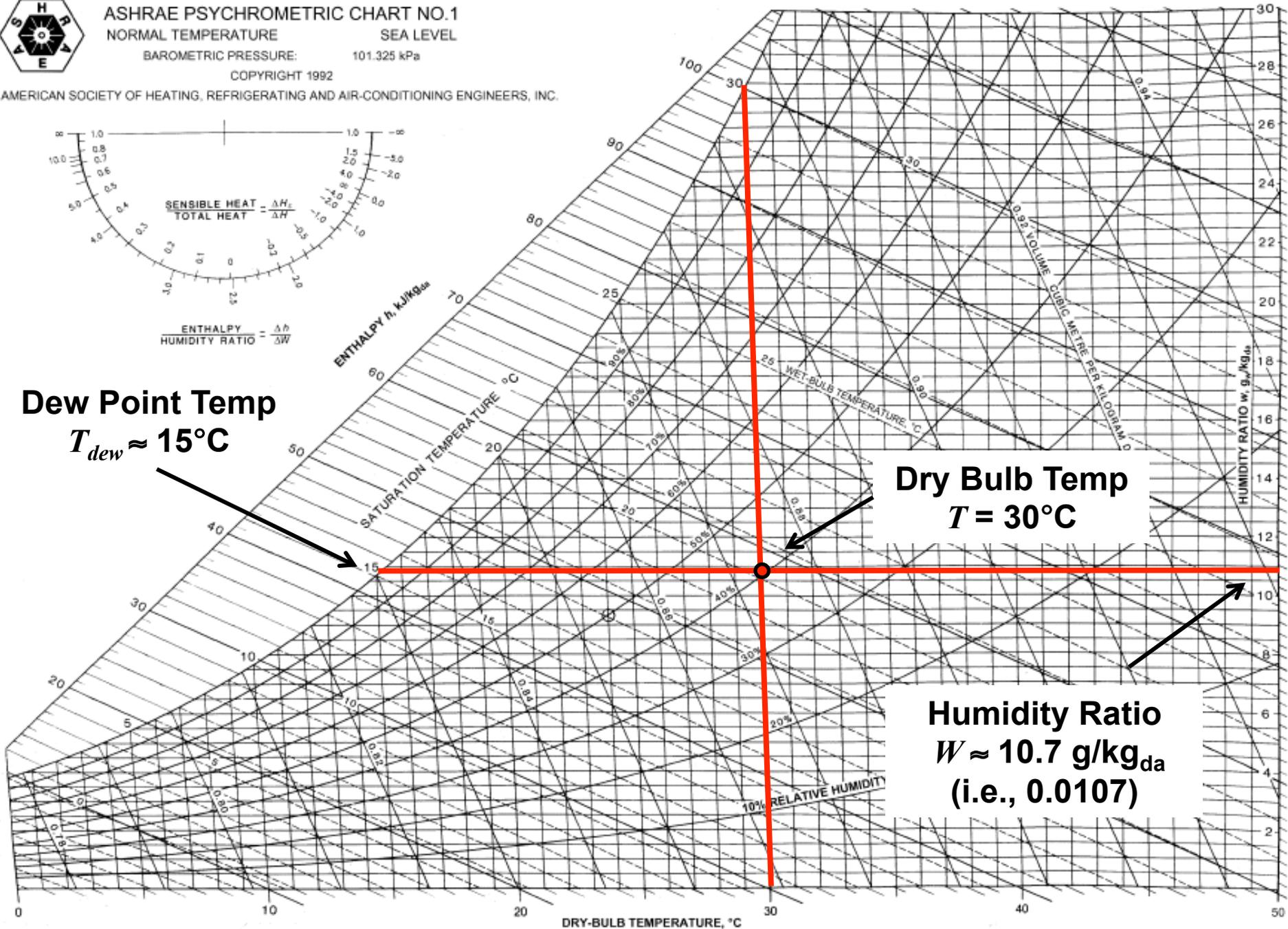
Dry Bulb Temp

$T = 30^\circ\text{C}$

Humidity Ratio

$W \approx 10.7 \text{ g/kg}_{da}$

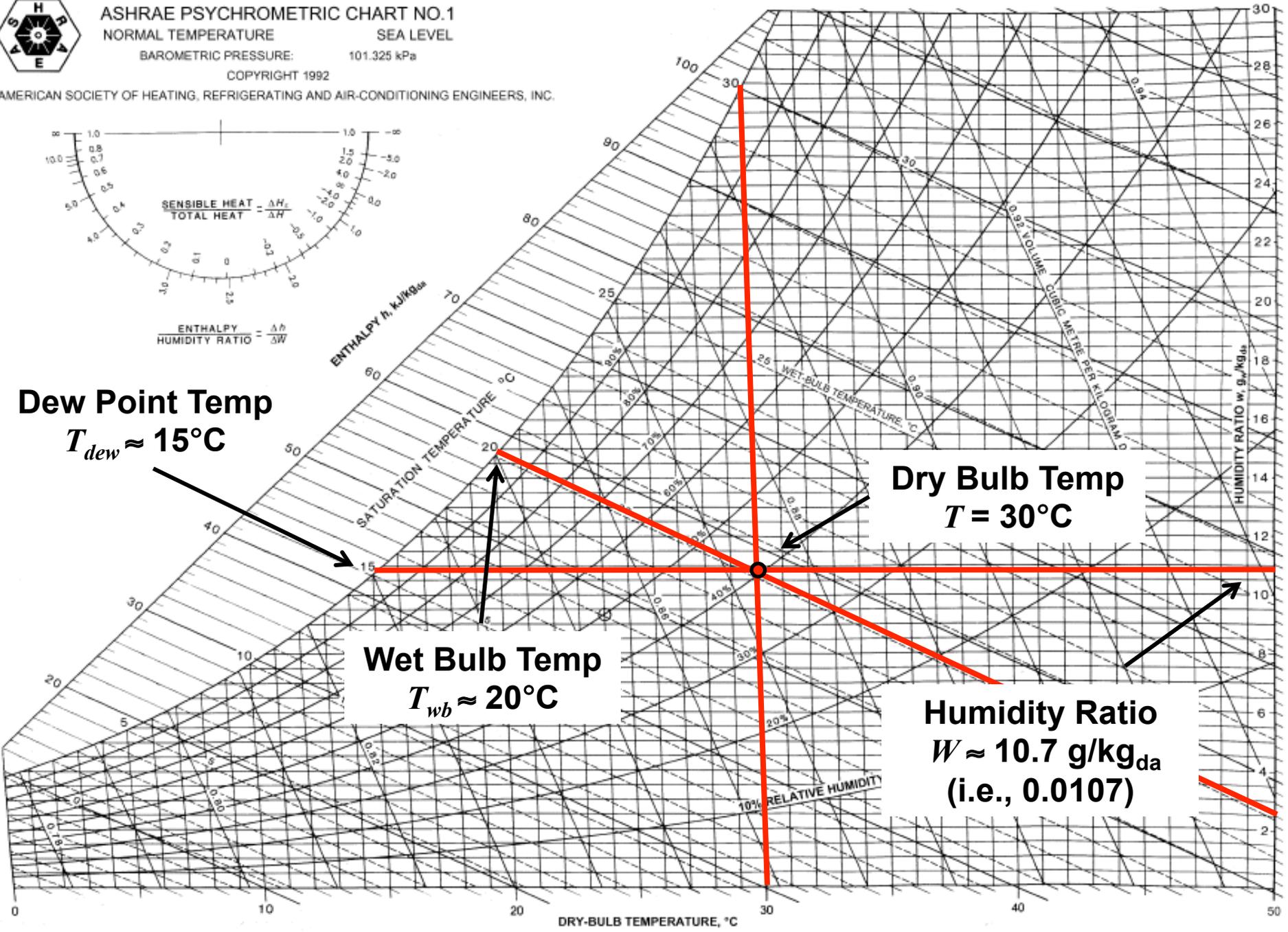
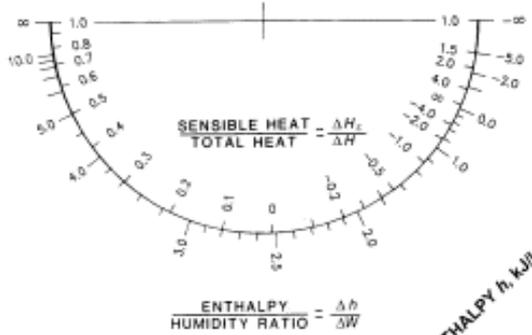
(i.e., 0.0107)





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Dew Point Temp
 $T_{dew} \approx 15^\circ\text{C}$

Dry Bulb Temp
 $T = 30^\circ\text{C}$

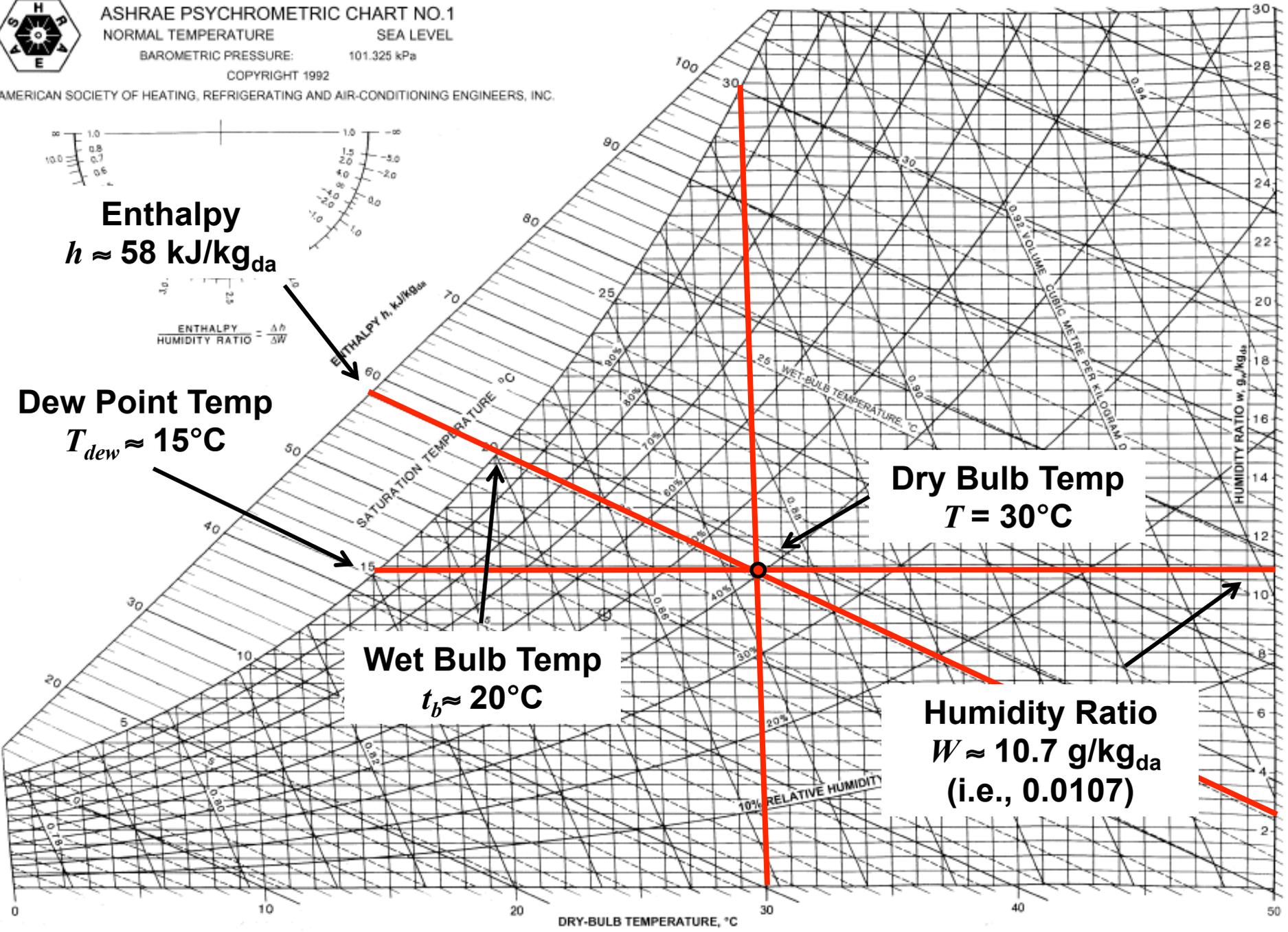
Wet Bulb Temp
 $T_{wb} \approx 20^\circ\text{C}$

Humidity Ratio
 $W \approx 10.7 \text{ g/kg}_{da}$
(i.e., 0.0107)



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Enthalpy
 $h \approx 58 \text{ kJ/kg}_{da}$

Dew Point Temp
 $T_{dew} \approx 15^\circ\text{C}$

Dry Bulb Temp
 $T = 30^\circ\text{C}$

Wet Bulb Temp
 $t_b \approx 20^\circ\text{C}$

Humidity Ratio
 $W \approx 10.7 \text{ g/kg}_{da}$
 (i.e., 0.0107)

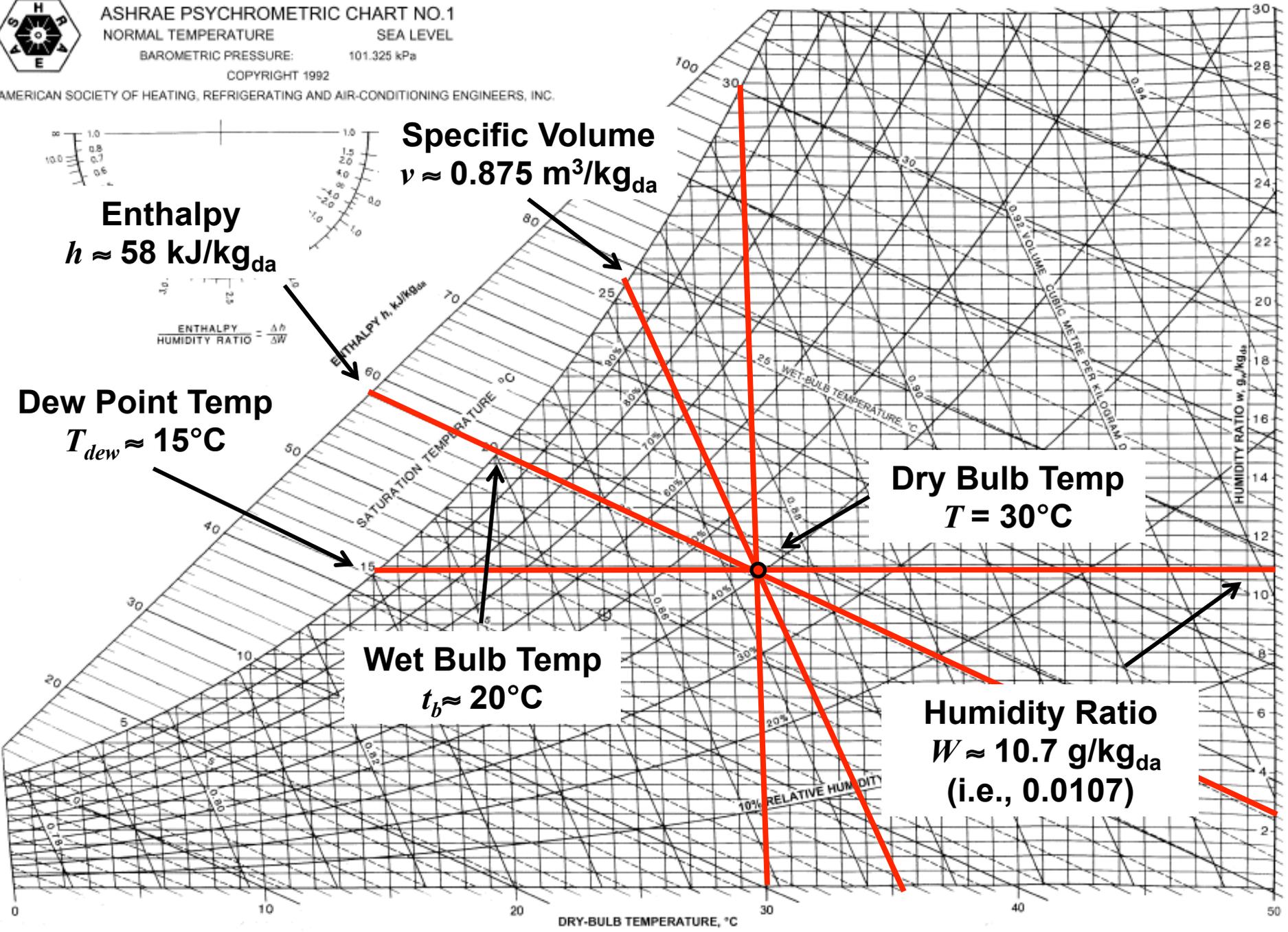
DRY-BULB TEMPERATURE, °C

HUMIDITY RATIO w , g_w/kg_{da}



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 NORMAL TEMPERATURE SEA LEVEL
 BAROMETRIC PRESSURE: 101.325 kPa
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Specific Volume
 $v \approx 0.875 \text{ m}^3/\text{kg}_{da}$

Enthalpy
 $h \approx 58 \text{ kJ/kg}_{da}$

Dew Point Temp
 $T_{dew} \approx 15^\circ\text{C}$

Dry Bulb Temp
 $T = 30^\circ\text{C}$

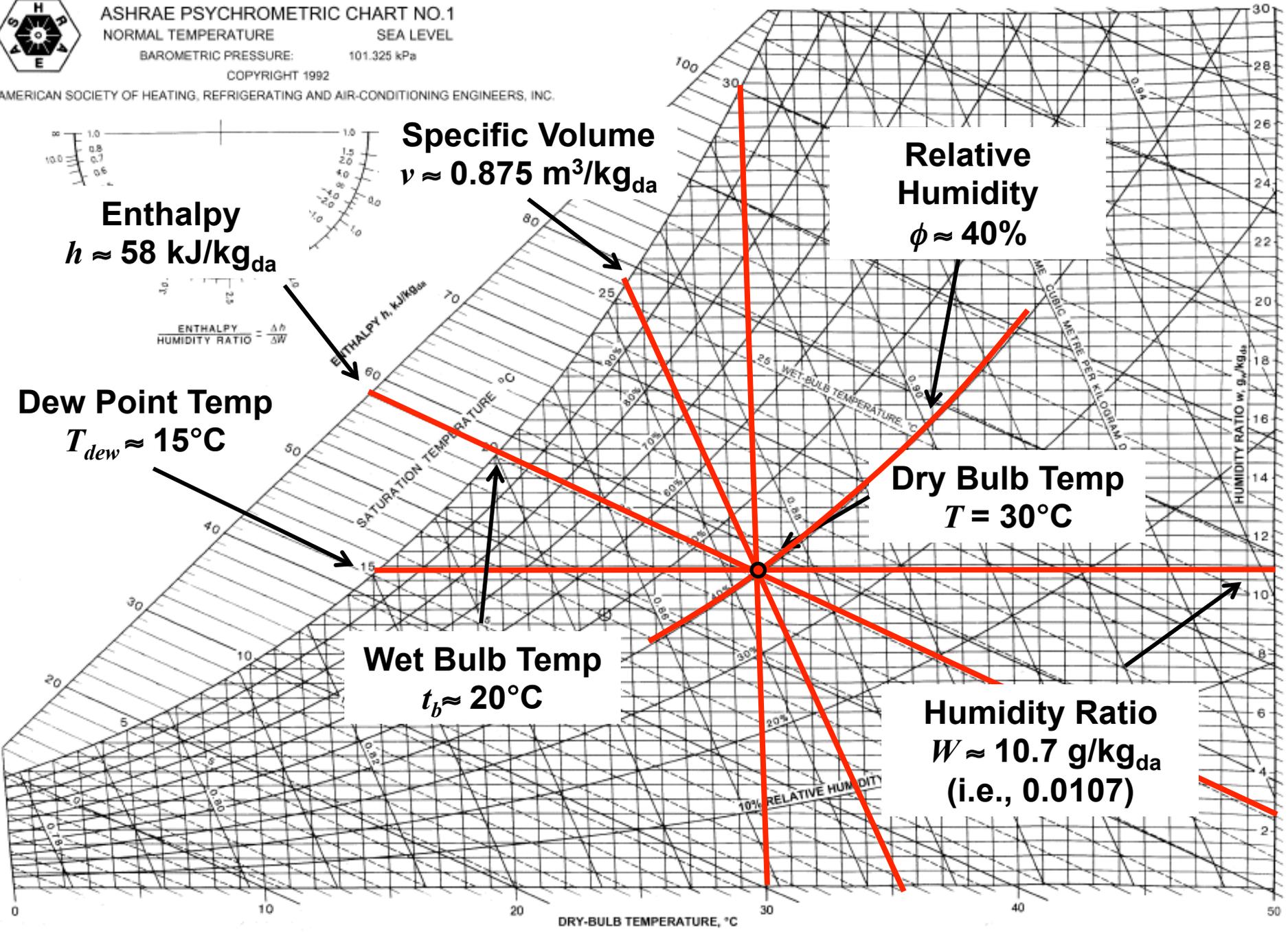
Wet Bulb Temp
 $t_b \approx 20^\circ\text{C}$

Humidity Ratio
 $W \approx 10.7 \text{ g/kg}_{da}$
 (i.e., 0.0107)

ENTHALPY HUMIDITY RATIO = $\frac{\Delta h}{\Delta W}$

DRY-BULB TEMPERATURE, °C

HUMIDITY RATIO w , g/kg_{da}



Dew Point Temp
 $T_{dew} \approx 15^\circ\text{C}$

Enthalpy
 $h \approx 58 \text{ kJ/kg}_{da}$

Specific Volume
 $\nu \approx 0.875 \text{ m}^3/\text{kg}_{da}$

Relative Humidity
 $\phi \approx 40\%$

Dry Bulb Temp
 $T = 30^\circ\text{C}$

Wet Bulb Temp
 $t_b \approx 20^\circ\text{C}$

Humidity Ratio
 $W \approx 10.7 \text{ g/kg}_{da}$
 (i.e., 0.0107)

ENTHALPY HUMIDITY RATIO = $\frac{\Delta h}{\Delta W}$

DRY-BULB TEMPERATURE, °C

HUMIDITY RATIO w , g_w/kg_{da}

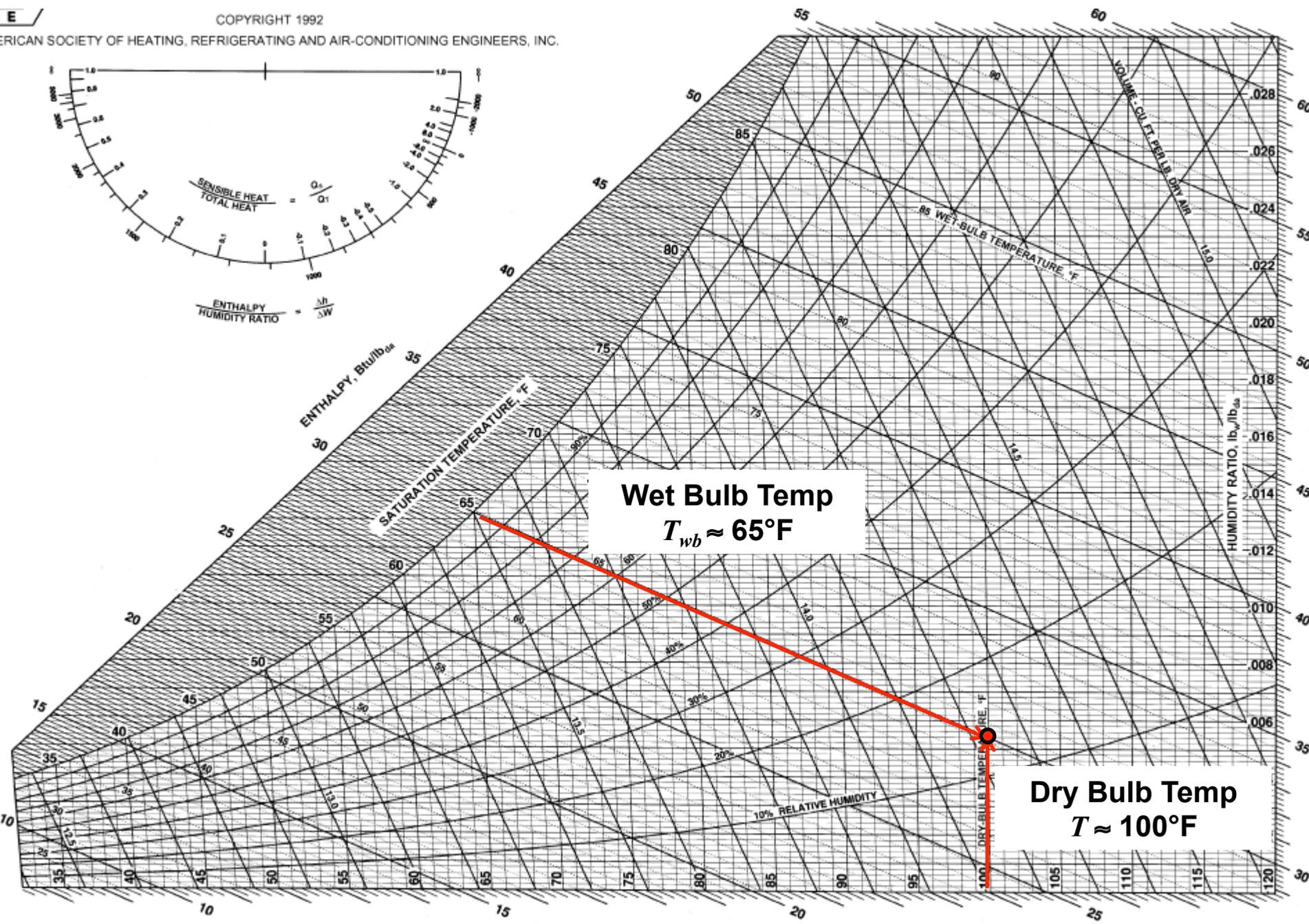
30
28
26
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16
14
12
10
8
6
4
2

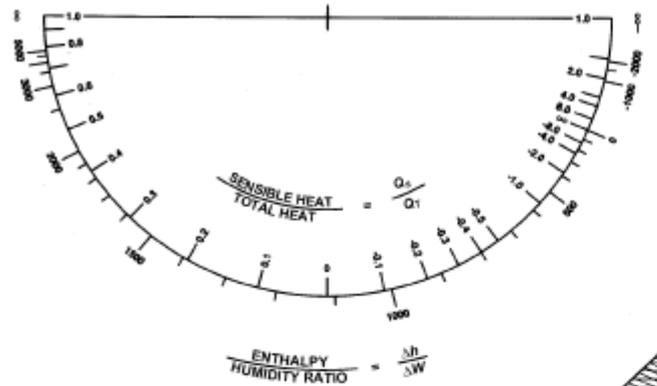
Psychrometrics: IP units example

- Moist air exists at 100°F dry bulb, 65°F wet bulb and 14.696 psia

Find:

- a) Humidity ratio
- b) Enthalpy
- c) Dew-point temperature
- d) Relative humidity
- e) Specific volume



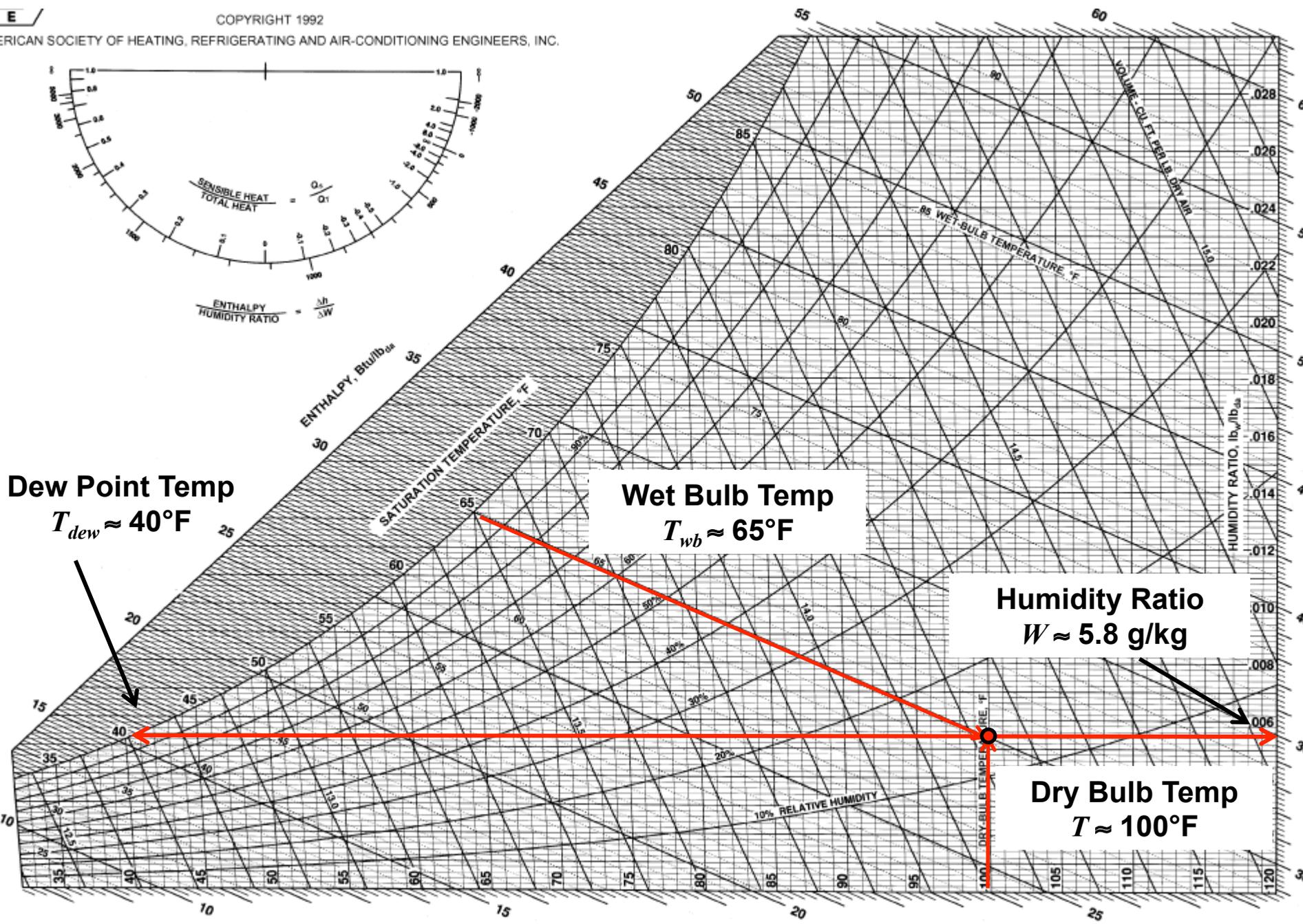


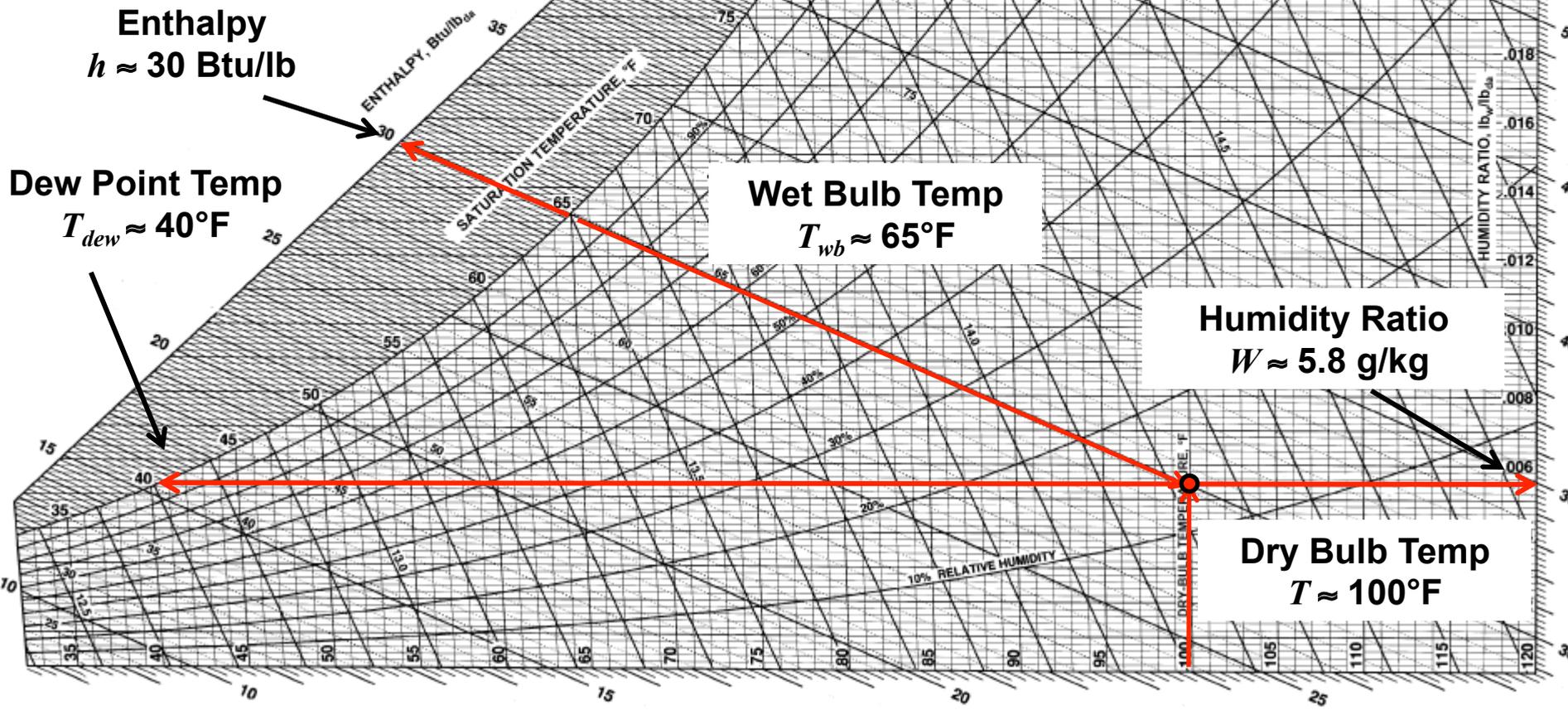
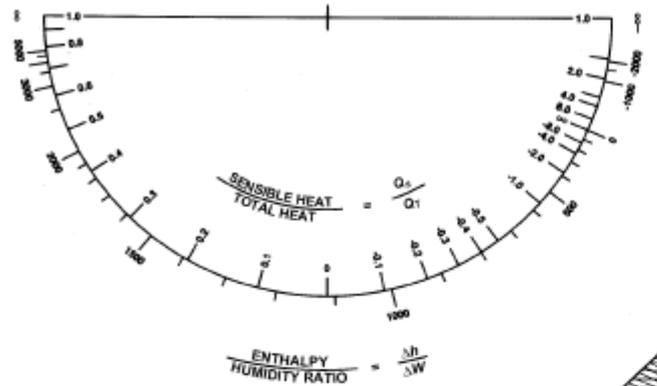
Dew Point Temp
 $T_{dew} \approx 40^\circ\text{F}$

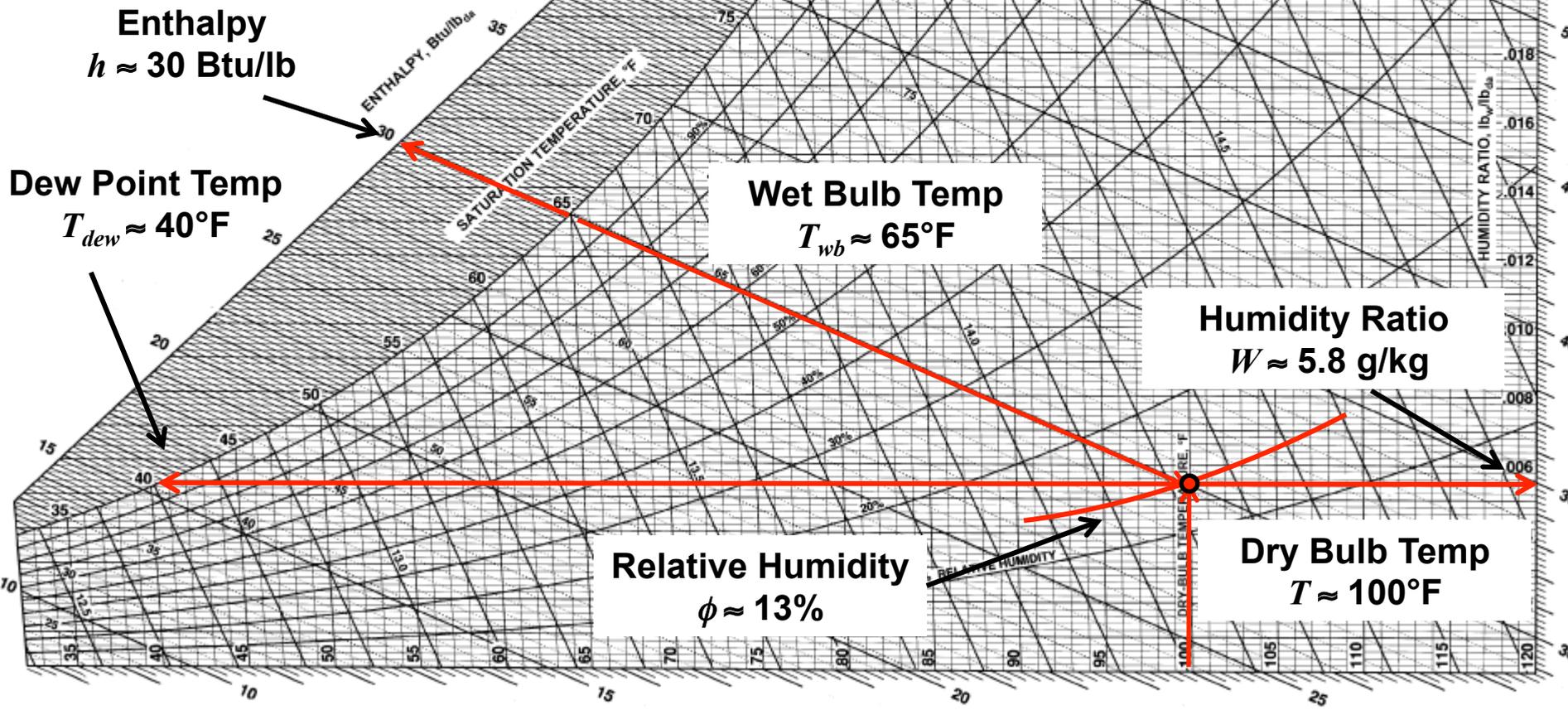
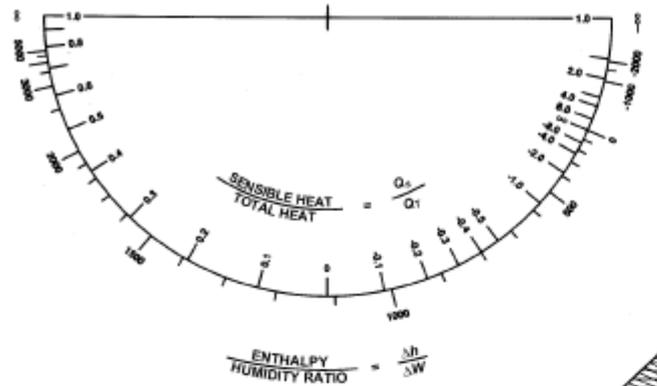
Wet Bulb Temp
 $T_{wb} \approx 65^\circ\text{F}$

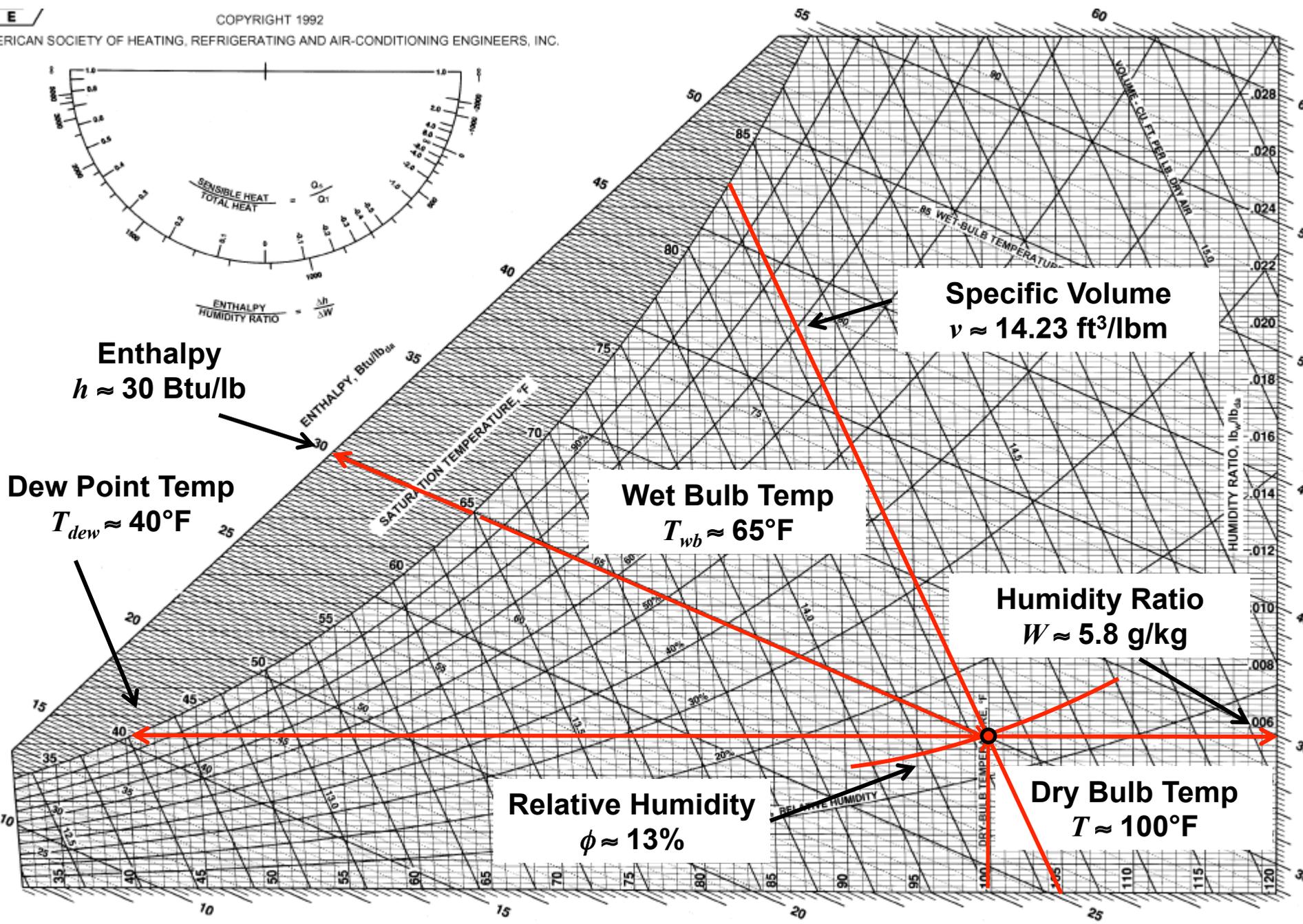
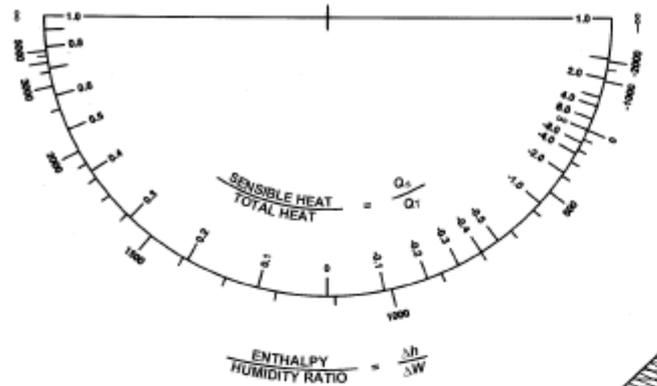
Humidity Ratio
 $W \approx 5.8 \text{ g/kg}$

Dry Bulb Temp
 $T \approx 100^\circ\text{F}$

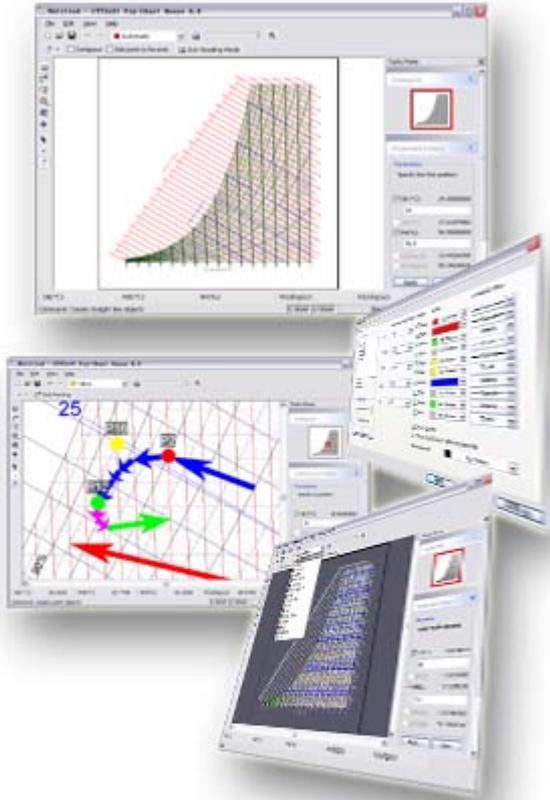








Applying psychrometrics



- We can also use psychrometric charts or software
 - Psych and Psychpro
 - Very popular psych chart and analysis software
 - I think at least one of these is in the AM 217 lab
- There are a bunch of online calculators as well
 - <http://www.psychrometric-calculator.com>
 - <http://www.sugartech.co.za/psychro/>
 - <http://www.wolframalpha.com/examples/Psychrometrics.html>
- And smart phone apps too
- You can also make your own (i.e., in Excel)
 - You will have a HW problem where you have to do this

Psychrometrics also involves learning how to use and combine those quantities to determine things like sensible and latent heating and cooling loads (i.e., **processes**) (covered in a future lecture)

PSYCHROMETRIC EQUATIONS

Treating air as an ideal gas

- At typical temperatures and pressures within buildings, air and its constituents act approximately as ideal gases
- Each gas i in the mixture, as well as the entire mixture, will follow the ideal gas law:

$$pV = nRT$$

or

$$pv = \frac{p}{\rho} = RT$$

p = pressure (Pa)

V = volume (m³)

n = number of moles (#)

R = gas constant (Pa·m³/(mol K))

T = absolute temperature (K)

v = specific volume (= 1/ ρ = m³/kg)

ρ = density (kg/m³)

Air as an ideal gas

- Every gas in air acts as an ideal gas

$$PV = nRT \quad \text{Ideal Gas Law (Boyle's law + Charles's law)}$$

- Air as a composition of ideal gases
 - A bunch of ideal gases acting as an ideal gas
- For individual gases (e.g., N₂, O₂, Ar, H₂O, CO₂, pollutant *i*):

$$P_i V = n_i RT$$

P_i = partial pressure exerted by gas *i*
 n_i = # of moles of gas *i*
 R, V, T = gas constant, volume, temperature

$$P_i = \frac{n_i}{V} RT$$

Rearrange so that n_i/V is the molar concentration

$$P_i = y_i P_{tot}$$

P_{tot} = total pressure of air (atm, Pa, etc.)
 y_i = mole fraction of gas *i* in air (moles *i* / moles air)

Air as an ideal gas

- Air as a composite mixture

$$P_i = y_i P_{tot}$$

$$P_{tot} = \sum P_i = \sum \frac{n_i}{V} RT = \frac{RT}{V} \sum n_i = \frac{RT}{V} n_{tot}$$

$$PV = nRT$$

Density of air

$$PV = nRT \longrightarrow \frac{n}{V} = \frac{P}{RT}$$

$$\frac{n}{V} = \frac{P}{RT} = \frac{1 \text{ atm}}{\left(8.205 \times 10^{-5} \frac{\text{atm} \cdot \text{m}^3}{\text{mol} \cdot \text{K}}\right) \times 293 \text{ K}}$$

20°C, 68°F

$$\frac{n}{V} = 41.6 \frac{\text{moles}}{\text{m}^3} = 0.0416 \frac{\text{moles}}{\text{L}}$$

$$\rho_{air} = MW_{air} \times 0.0416 \frac{\text{moles}}{\text{L}} @ 20 \text{ degrees C}$$

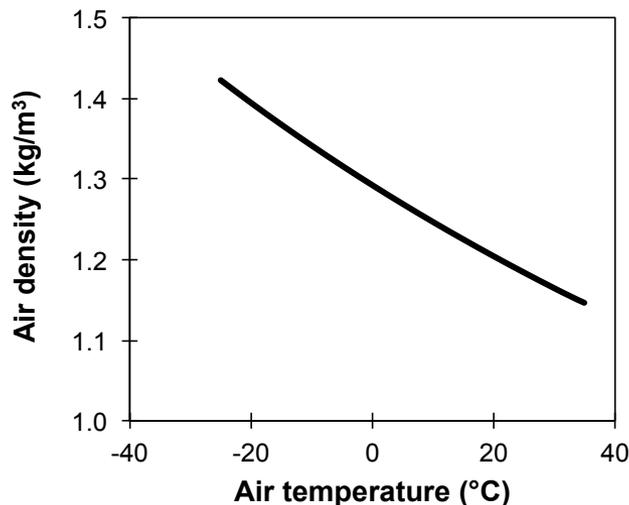
What is the molecular weight (MW) of air?

$$MW_{air} = \sum y_i MW_i = y_{N_2} MW_{N_2} + y_{O_2} MW_{O_2} + y_{H_2O} MW_{H_2O} + \dots$$

$$MW_{air} = 0.781(28 \text{ g/mol}) + 0.209(32 \text{ g/mol}) + \dots = 29 \text{ g/mol}$$

$$\rho_{air} = \left(29 \frac{\text{g}}{\text{mol}}\right) \times 0.0416 \frac{\text{mol}}{\text{L}} = 1.2 \frac{\text{g}}{\text{L}} = 1.2 \frac{\text{kg}}{\text{m}^3} \text{ @20 degrees C}$$

Hang on to this number: density of air is $\sim 1.2 \text{ kg/m}^3$ at 20°C



$$\rho_{air} \approx 1.3 - 0.0046(T_{air}) \text{ where } T_{air} \text{ is in degrees C}$$

Universal gas constant

- The universal gas constant relates energy and temperature
 - It takes many forms depending on units

$$pv = \frac{p}{\rho} = RT$$

Value of R	Units (V P T ⁻¹ n ⁻¹)
8.314	J/(K·mol)
8.314	m ³ ·Pa/(K·mol)
0.08206	L·atm/(K·mol)
8.206×10 ⁻⁵	m ³ ·atm/(K·mol)
10.731	ft ³ ·psi/(R·lb-mol)
1.986	Btu/(lb-mol·R)

Mass-specific gas constants

- To work with air and water vapor we need gas-specific gas constants (which are functions of molecular weight)
- Dry air (no water vapor): $MW_{da} = 28.965 \text{ g/mol}$

$$R_i = \frac{R}{MW_i}$$

$$R_{da} = \frac{R}{MW_{da}} = \frac{8.314 \frac{\text{J}}{\text{K} \cdot \text{mol}} \frac{1000 \text{g}}{\text{kg}}}{28.965 \frac{\text{g}}{\text{mol}}} = 287 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

- Water vapor alone: $MW_w = 18.015 \text{ g/mol}$

$$R_w = \frac{R}{MW_w} = \frac{8.314 \frac{\text{J}}{\text{K} \cdot \text{mol}} \frac{1000 \text{g}}{\text{kg}}}{18.015 \frac{\text{g}}{\text{mol}}} = 462 \frac{\text{J}}{\text{kg}_w \cdot \text{K}}$$

$$pv = \frac{p}{\rho} = RT$$

Air pressure variations

- The barometric (atmospheric) pressure and temperature of air vary with both altitude and local weather conditions
 - But there are standard values for pressure as a function of altitude that are normally used
- At sea level, the standard temperature is 15°C and the standard pressure is 101.325 kPa (1 atm)
 - Temperature is assumed to decrease linearly with altitude
 - Pressure is more complicated

$$T_{air} = 15 - 0.0065Z \quad p = 101.325 \left(1 - \left(2.25577 \times 10^{-5} \right) Z \right)^{5.2559}$$

$$pv = \frac{p}{\rho} = RT$$

T = temperature (°C)

Z = altitude (m)

p = barometric pressure (kPa)

Air pressure variations

Table 1 Standard Atmospheric Data for Altitudes to 10 000 m

Altitude, m	Temperature, °C	Pressure, kPa
-500	18.2	107.478
0	15.0	101.325
500	11.8	95.461
1000	8.5	89.875
1500	5.2	84.556
2000	2.0	79.495
2500	-1.2	74.682
3000	-4.5	70.108
4000	-11.0	61.640
5000	-17.5	54.020
6000	-24.0	47.181
7000	-30.5	41.061
8000	-37.0	35.600
9000	-43.5	30.742
10 000	-50	26.436

Source: Adapted from NASA (1976).

Dalton's law of partial pressures

- In an ideal gas, the total pressure can be considered to be the sum of the partial pressures of the constituent gases

$$p = p_{N_2} + p_{O_2} + p_{H_2O} + p_{CO_2} + p_{Ar} + \dots$$

- We can consider moist air as dry air combined with water vapor and break the pressure into only two partial pressures

$$p = p_{da} + p_w$$

Dalton's law of partial pressures

- We can analyze the dry air, the water vapor, and the mixture of each gas using the ideal gas law and assuming they are all at the same temperature

$$p_{da} v_{da} = R_{da} T \quad \& \quad p_w v_w = R_w T \quad \& \quad p v = R T$$

- For each individual gas, a mole fraction (Y_i) can be defined as the ratio of the partial pressure of gas i to the total pressure

$$\frac{n_i}{n} = \frac{p_i}{p} = Y_i$$

Specifying the state of moist air



In order to specify the state of moist air, we need total atmospheric pressure, p , the air temperature, T , and at least one other property

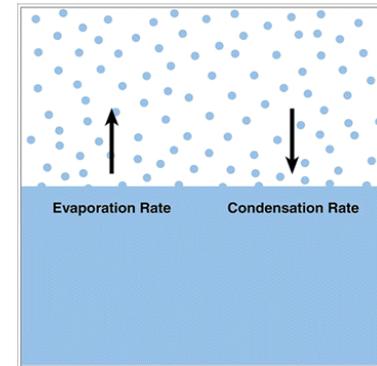
- W , ϕ , h , p_w , or T_{dew}

Key concepts: Vapor pressure and Saturation

- Air can hold moisture (i.e., **water vapor**)
- **Vapor pressure** is a measurement of the amount of water vapor in a volume/parcel of air

$$p_w$$

*Units of pressure, Pa or kPa



- The amount of moisture air can hold in vapor form before condensation occurs is dependent on temperature
 - We call the limit **saturation**

$$p_{ws}$$

*Units of pressure, Pa or kPa



Relative humidity, ϕ (RH)

- The relative humidity ratio, ϕ , is the mole fraction of water vapor (x_w) relative to the water vapor that would be in the mixture if it were saturated at the given T and P (x_{ws})
- Relative humidity is a common measure that relates well to how we perceive moisture in air



$$\phi = \left[\frac{x_w}{x_{ws}} \right]_{T,P} = \frac{p_w}{p_{ws}}$$

p_{ws} for $0^{\circ}\text{C} < T < 200^{\circ}\text{C}$

For p_{ws} , the saturation pressure over liquid water:

$$\ln p_{ws} = \frac{C_8}{T} + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

where

$$C_8 = -5.800\ 220\ 6\ \text{E}+03$$

$$C_9 = 1.391\ 499\ 3\ \text{E}+00$$

$$C_{10} = -4.864\ 023\ 9\ \text{E}-02$$

$$C_{11} = 4.176\ 476\ 8\ \text{E}-05$$

$$C_{12} = -1.445\ 209\ 3\ \text{E}-08$$

$$C_{13} = 6.545\ 967\ 3\ \text{E}+00$$

Unit

p_{ws} = saturation pressure, Pa

T = absolute temperature, K = $^{\circ}\text{C} + 273.15$

Note:

These constants are only for SI units
IP units are different

*We will use this equation for most conditions in building science

p_{ws} for $-100^{\circ}\text{C} < T < 0^{\circ}\text{C}$

For p_{ws} , the saturation pressure over **ice**:

$$\ln p_{ws} = \frac{C_1}{T} + C_2 + C_3 T + C_4 T^2 + C_5 T^3 + C_6 T^4 + C_7 \ln T$$

where

$$C_1 = -5.674\ 535\ 9\ \text{E}+03$$

$$C_2 = 6.392\ 524\ 7\ \text{E}+00$$

$$C_3 = -9.677\ 843\ 0\ \text{E}-03$$

$$C_4 = 6.221\ 570\ 1\ \text{E}-07$$

$$C_5 = 2.074\ 782\ 5\ \text{E}-09$$

$$C_6 = -9.484\ 024\ 0\ \text{E}-13$$

$$C_7 = 4.163\ 501\ 9\ \text{E}+00$$

Note:

These constants are only for SI units
IP units are different

Units:

p_{ws} = saturation pressure, Pa

T = absolute temperature, K = $^{\circ}\text{C} + 273.15$

Humidity ratio, W

- The humidity ratio, W , is ratio of the mass of water vapor to mass of dry air in a given volume
 - We use W when finding other mixture properties
 - Note 1: W is small ($W < 0.04$ for any real building conditions)
 - Note 2: W is sometimes expressed in grains/lb where 1 lb = 7000 grains (I don't use this but you might in CAE 464 HVAC Design)

$$W = \frac{m_w}{m_{da}} = \frac{MW_w p_w}{M_{da} P_{da}} = 0.622 \frac{p_w}{P_{da}} = 0.622 \frac{p_w}{p - p_w} \quad \left[\frac{\text{kg}_w}{\text{kg}_{da}} \right] \quad \text{UNITS}$$

Saturation humidity ratio, W_s

- At a given temperature T and pressure P there is a maximum W that can be obtained
- If we try to add any more moisture, it will just condense out
 - It is when the partial pressure of vapor has reached the saturation pressure
- This maximum humidity ratio is called the saturation humidity ratio, W_s
 - From our previous equation we can write:

$$W_s = 0.622 \frac{p_s}{p_{da}} = 0.622 \frac{p_s}{p - p_s}$$

Degree of saturation, μ

- The degree of saturation, μ (dimensionless), is the ratio of the humidity ratio W to that of a saturated mixture W_s at the same T and P
 - Note that μ and ϕ are not quite the same
 - Their values are very similar at lower temperatures but may differ a lot at higher temperatures

$$\mu = \left[\frac{W}{W_s} \right]_{T,P}$$

$$\mu = \frac{\phi}{1 + (1 - \phi)W_s / (0.6295)}$$

$$\phi = \frac{\mu}{1 - (1 - \mu)p_{ws} / p}$$

Specific volume, v

- The specific volume of moist air (or the volume per unit mass of air, m^3/kg) can be expressed as:

$$v = \frac{R_{da} T}{p - p_w} = \frac{R_{da} T (1 + 1.6078W)}{p}$$

where

v = specific volume, $\text{m}^3/\text{kg}_{da}$
 t = dry-bulb temperature, $^{\circ}\text{C}$
 W = humidity ratio, $\text{kg}_w/\text{kg}_{da}$
 p = total pressure, kPa

$$v \approx 0.287042(T + 273.15)(1 + 1.6078W) / p$$

- If we have v we can also find moist air density, ρ (kg/m^3):

$$\rho = \frac{m_{da} + m_w}{V} = \frac{1}{v} (1 + W)$$

Enthalpy, h

- The enthalpy of a mixture of perfect gases equals the sum of the individual partial enthalpies of the components
- Therefore, the enthalpy (h) for moist air is:
$$h = h_{da} + Wh_g$$

h = enthalpy for moist air [kJ/kg]

h_g = specific enthalpy for saturated water vapor (i.e., h_{ws}) [kJ/kg_w]

h_{da} = specific enthalpy for dry air (i.e., h_{ws}) [kJ/kg_{da}]

- Some approximations:
$$h_{da} \approx 1.006T \quad h_g \approx 2501 + 1.86T$$

$$h \approx 1.006T + W(2501 + 1.86T)$$

*where T is in °C

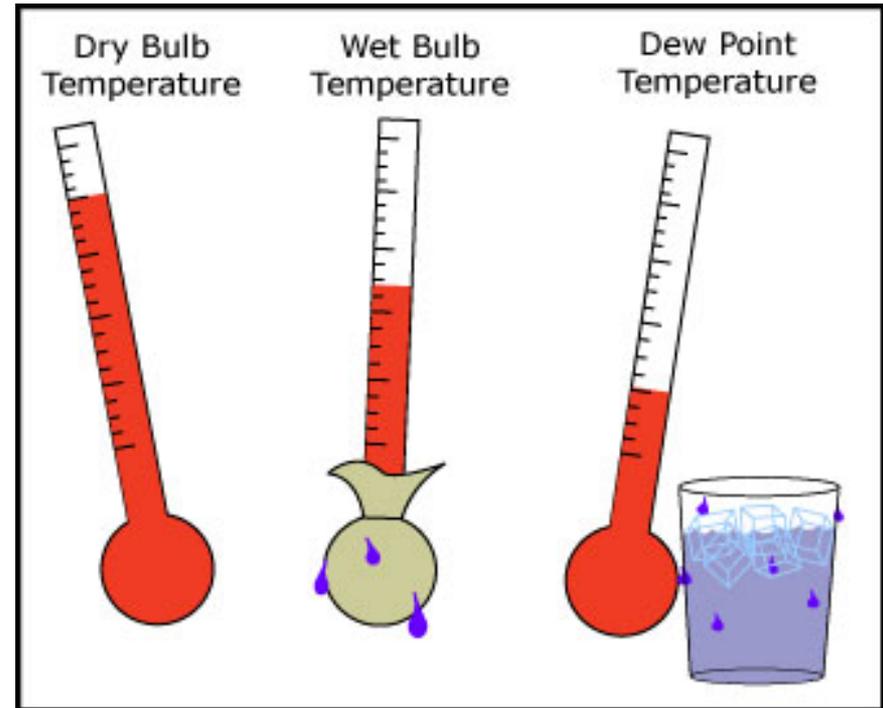
Three different temperatures: T , T_{dew} , and T_{wb}

The standard temperature, T , we are all familiar with is called the **dry-bulb** temperature, or T_d

- It is a measure of internal energy

We can also define:

- **Dew-point** temperature, T_{dew}
 - Temperature at which water vapor changes into liquid (condensation)
 - Air is maximally **saturated** with water vapor
- **Wet-bulb** temperature, T_{wb}
 - The temperature that a parcel of air would have if it were cooled to saturation (100% **relative humidity**) by the evaporation of water into it



Units of Celsius, Fahrenheit, or Kelvin

Dew-point temperature, T_{dew}



The dew point temperature, T_{dew} , is the air temperature at which the current humidity ratio W is equal to the saturation humidity ratio W_s at the same temperature

$$\text{i.e. } W_s(p, T_{dew}) = W$$

When the air temperature is lowered to the dew-point at constant pressure, the relative humidity rises to 100% and condensation occurs

T_{dew} is a direct measure of the humidity ratio W since $W = W_s$ at $T = T_{dew}$

Dew-point temperature, T_{dew}

- Dew-point temperature, T_{dew}

Between dew points of 0 and 93°C,

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}(p_w)^{0.1984} \quad (39)$$

Below 0°C,

$$t_d = 6.09 + 12.608\alpha + 0.4959\alpha^2 \quad (40)$$

where

t_d = dew-point temperature, °C

$\alpha = \ln p_w$

p_w = water vapor partial pressure, kPa

$C_{14} = 6.54$

$C_{15} = 14.526$

$C_{16} = 0.7389$

$C_{17} = 0.09486$

$C_{18} = 0.4569$

Note:

These constants are only for SI units
IP units are different

Wet-bulb temperature, T_{wb}

- Wet-bulb temperature, T_{wb}
- Requires iterative solver... find the T_{wb} that satisfies the following equation (above freezing):

$$W = \frac{(2501 - 2.326T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2501 + 1.86T - 4.186T_{wb}}$$

- And below freezing:

$$W = \frac{(2830 - 0.24T_{wb})W_{s@T_{wb}} - 1.006(T - T_{wb})}{2830 + 1.86T - 2.1T_{wb}}$$

Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 2 gives us W_s , v_{da} , and v_s directly at different temperatures:

Table 2 Thermodynamic Properties of Moist Air at Standard Atmospheric Pressure

Temp., °C <i>t</i>	Humidity Ratio W_s , kg _w /kg _{da}	Specific Volume, m ³ /kg _{da}			Specific Enthalpy, kJ/kg _{da}		
		v_{da}	v_{as}	v_s	h_{da}	h_{as}	h_s
15	0.010694	0.8159	0.0140	0.8299	15.087	27.028	42.115
16	0.011415	0.8188	0.0150	0.8338	16.093	28.873	44.966
17	0.012181	0.8216	0.0160	0.8377	17.099	30.830	47.929
18	0.012991	0.8245	0.0172	0.8416	18.105	32.906	51.011
19	0.013851	0.8273	0.0184	0.8457	19.111	35.107	54.219
20	0.014761	0.8301	0.0196	0.8498	20.117	37.441	57.558
21	0.015724	0.8330	0.0210	0.8540	21.124	39.914	61.037
22	0.016744	0.8358	0.0224	0.8583	22.130	42.533	64.663

Obtaining these data from ASHRAE Tables

ASHRAE HoF Ch. 1 (2013) Table 2 gives us W_s , v_{da} , and v_s directly at different temperatures:

Table 3 Thermodynamic Properties of Water at Saturation

Temp., °C <i>t</i>	Absolute Pressure <i>p_{ws}</i> , kPa	Specific Volume, m ³ /kg _w			Specific Enthalpy, kJ/kg _w		
		Sat. Liquid <i>v_i/v_f</i>	Evap. <i>v_{ig}/v_{fg}</i>	Sat. Vapor <i>v_g</i>	Sat. Liquid <i>h_i/h_f</i>	Evap. <i>h_{ig}/h_{fg}</i>	Sat. Vapor <i>h_g</i>
3	0.7581	0.001000	168.013	168.014	12.60	2493.80	2506.40
4	0.8135	0.001000	157.120	157.121	16.81	2491.42	2508.24
5	0.8726	0.001000	147.016	147.017	21.02	2489.05	2510.07
6	0.9354	0.001000	137.637	137.638	25.22	2486.68	2511.91
7	1.0021	0.001000	128.927	128.928	29.43	2484.31	2513.74
8	1.0730	0.001000	120.833	120.834	33.63	2481.94	2515.57
9	1.1483	0.001000	113.308	113.309	37.82	2479.58	2517.40
10	1.2282	0.001000	106.308	106.309	42.02	2477.21	2519.23