

# CAE 463/524

## Building Enclosure Design

Spring 2014

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### Lecture 4: February 11, 2014

Complex conduction in building enclosures

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# Last time

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- Finished surface energy balances
- Solar orientation and enclosures
  
- Assigned HW #1
  - HW #1 due today

# This time

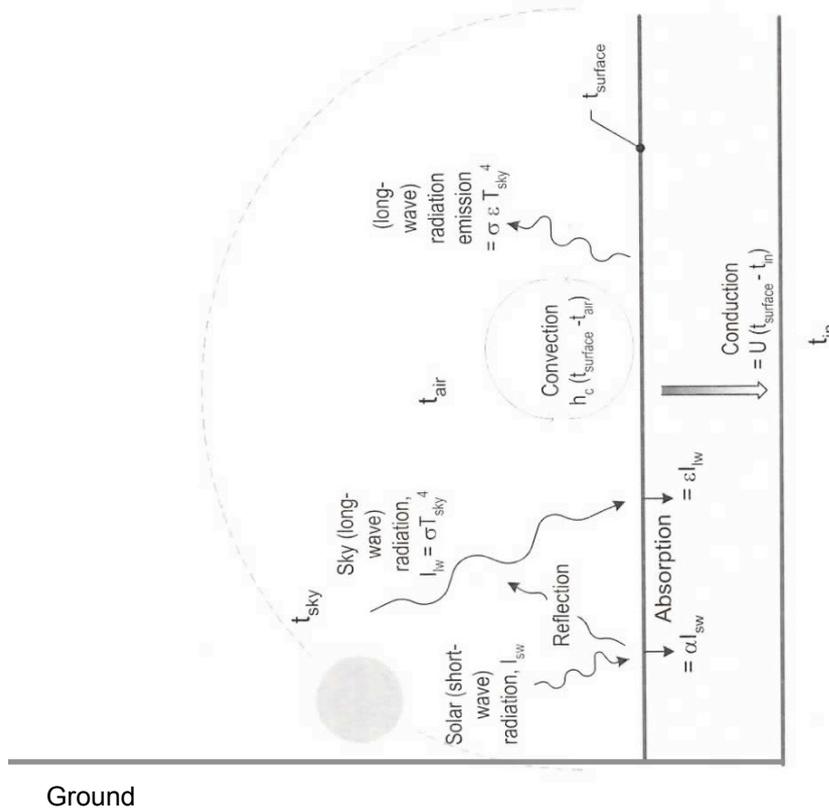
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- Complex conduction in building enclosures
  - Temperature profiles
  - Layers with different materials
    - Simple weighted average
    - Parallel paths vs. isothermal
  - Thermal bridges
  - 2-D and 3-D conduction
  - THERM
  - Thermal mass

# Bringing all the modes together

- For a vertical surface:

$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$



$$\begin{aligned} & \alpha I_{solar} \\ & + \epsilon_{surface} \sigma F_{sky} (T_{sky}^4 - T_{surface}^4) \\ & + \epsilon_{surface} \sigma F_{ground} (T_{ground}^4 - T_{surface}^4) \\ & + h_{conv} (T_{air} - T_{surface}) \\ & - U (T_{surface} - T_{surface,interior}) = 0 \end{aligned}$$

# **COMPLEX CONDUCTION IN ENCLOSURES**

Combining elements

Multiple layers and temperature distributions

Thermal bridges

# Combining elements in an actual enclosure

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- So far we have been exploring single assemblies
  - Just roofs or just walls without windows and doors
  - If you design a building without windows and doors, something probably went wrong!
- Concept of **combined thermal transmittance**:  $U_o$ 
  - $U_o$  is the combined thermal transmittance of the respective areas of a gross exterior wall, roof, or floor
    - It is basically an **area-weighted average U-value**

$$U_o = (U_{wall} A_{wall} + U_{window} A_{window} + U_{door} A_{door}) / A_o$$

where

$U_o$  = average thermal transmittance of gross wall area

$A_o$  = gross area of exterior walls

$U_{wall}$  = thermal transmittance of all elements of opaque wall area

$A_{wall}$  = opaque wall area

$U_{window}$  = thermal transmittance of window area (including frame)

$A_{window}$  = window area (including frame)

$U_{door}$  = thermal transmittance of door area

$A_{door}$  = door area (including frame)

# Combined thermal transmittance example

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- Calculate  $U_o$  for a 10 m x 2.4 m wall with two double-glazed windows with wood/vinyl frames and one solid core door
  - One window is 1.5 x 0.86 m; the other window is 0.9 x 0.76 m
    - Let's say we looked up window U-value in a table
    - $U_{\text{window}} = 2.90 \text{ W/m}^2\text{K}$
  - The door is 0.86 x 2 m
    - Let's say we also looked up its U-value in a table
    - $U_{\text{door}} = 1.42 \text{ W/m}^2\text{K}$
  - The wall has a U value of  $U_{\text{wall}} = 0.404 \text{ W/m}^2\text{K}$

$$A_{\text{window}} = (1.500 \times 0.860) + (0.900 \times 0.760) = 1.97 \text{ m}^2$$

$$A_{\text{door}} = (0.860 \times 2.000) = 1.72 \text{ m}^2$$

$$A_{\text{wall}} = (10 \times 2.4) - (1.97 + 1.72) = 20.31 \text{ m}^2$$

Therefore, the combined thermal transmittance for the wall is

$$U_o = \frac{(0.404 \times 20.31) + (2.90 \times 1.97) + (1.42 \times 1.72)}{10 \times 2.4}$$
$$= 0.68 \text{ W}/(\text{m}^2 \cdot \text{K})$$

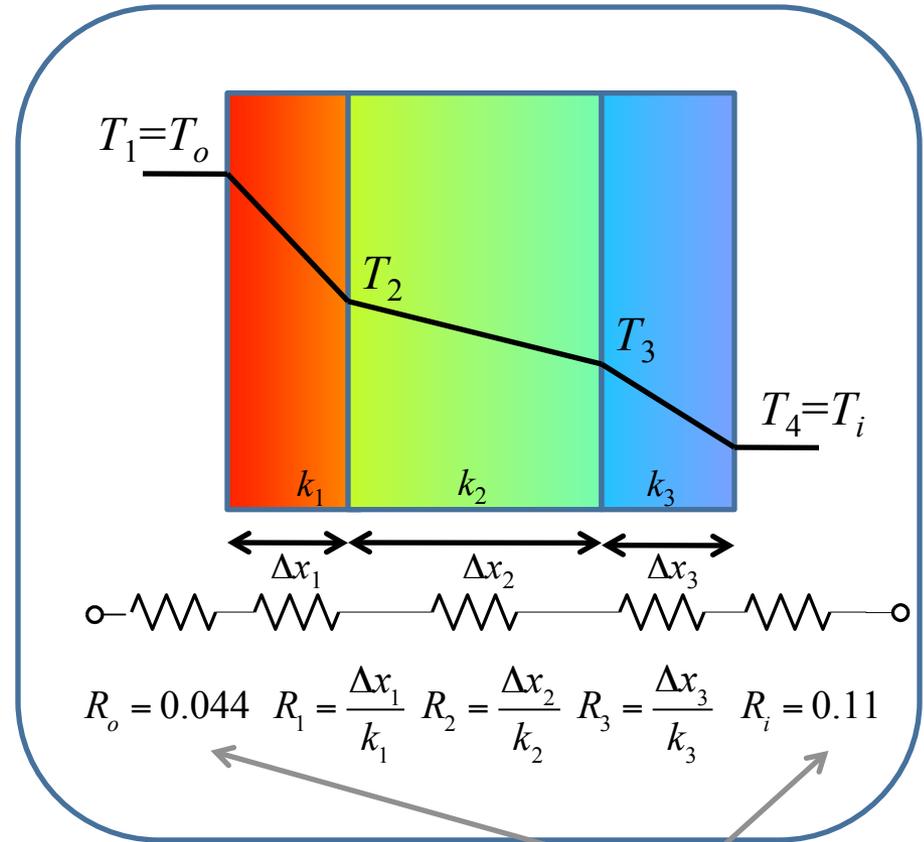
# Conduction through multiple layers

- Just as in electrical circuits, the overall thermal resistance of a series of elements (layers) can be expressed as the sum of the resistances of each layer
  - Don't forget the interior and exterior convective resistances
- By continuity of energy we can write

$$q = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3}$$

so

$$q = \frac{T_1 - T_4}{R_{total}} \text{ where } R_{total} = R_o + R_1 + R_2 + R_3 + R_i$$



Typical "film" values

Can only add resistances (R) in series, not conductances (U)

# Simple conduction through multiple layers

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- Calculate the R-value of an enclosure assembly

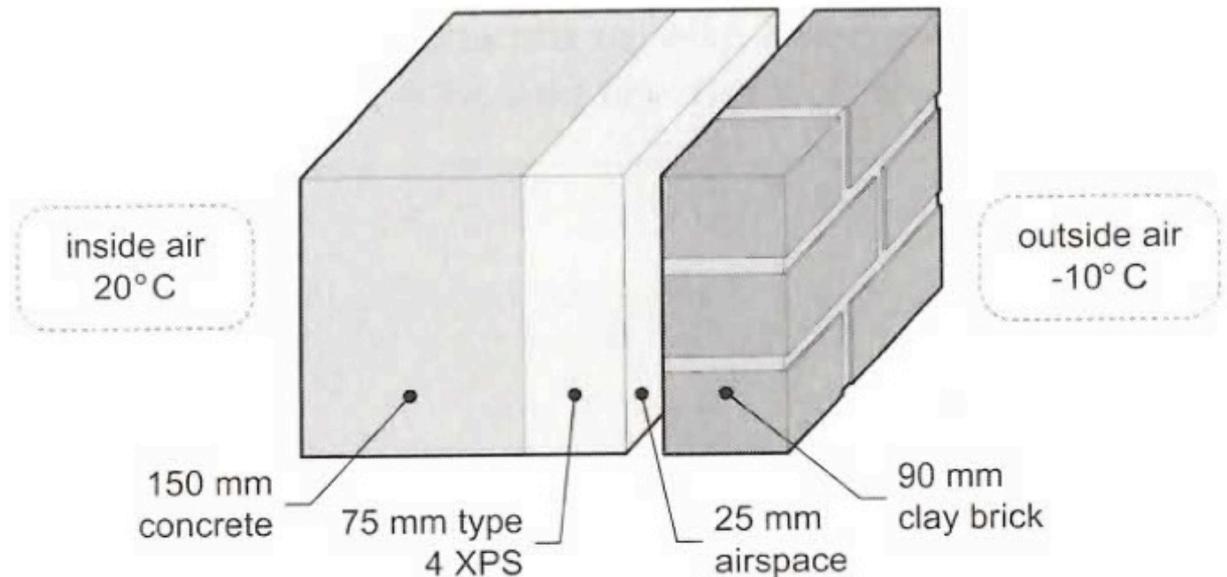
## Steps:

1. List each material in the assembly
  - And its conductivity and thickness
2. Calculate conductance of each layer
  - $U = k/L$
3. Calculate thermal resistance of each layer
  - $R = 1/U$
4. Sum the individual thermal resistances to get  $R_{\text{total}}$

# Conduction through multiple layers

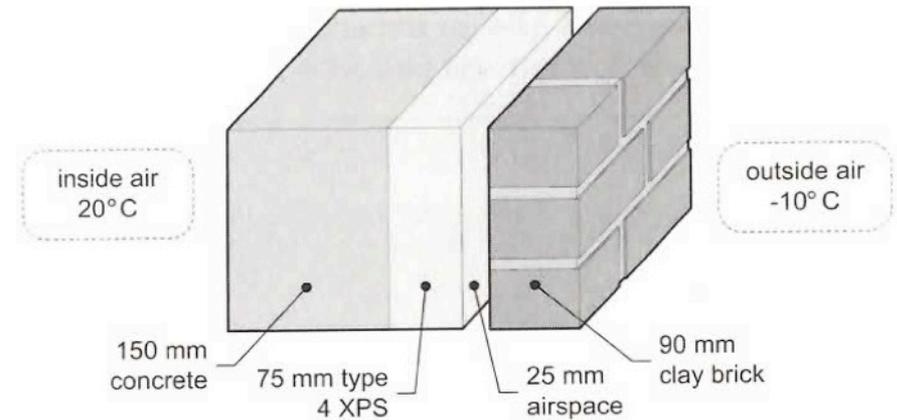
Example problem:

- Calculate the total thermal resistance,  $R_{\text{total}}$ , and the temperature distribution through the wall shown below



# Conduction through multiple layers

- Refer to ASHRAE 2005 HOF Ch. 25 for data



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Layer material	Conductivity $k = [\text{W/mK}]$	Thickness $L = [\text{m}]$	Conductance $U = [\text{W/m}^2\text{K}]$	Resistance $R = [\text{m}^2\text{K/W}]$
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# A note on R-values of air cavities

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- ASHRAE has measured the combined convective + radiative R-values for thin planar cavities of various orientations and depths with various “ $\epsilon_{eff}$ ”
- These are the best data to use for air spaces in assemblies
  - If you do not know that the material in the cavity is reflective or “low e”, just assume that both walls of the cavity have  $\epsilon=0.9$  for each surface, so that when combined,  $\epsilon_{eff} = 0.82$

$$\epsilon_{eff} = \epsilon_1 \epsilon_2$$

# ASHRAE HOF 2005, Chapter 25 (small cavities)

## R-values for different air gap characteristics

Table 3 Thermal Resistances of Plane Air Spaces<sup>a,b,c</sup>, (m<sup>2</sup>·K)/W

Position of Air Space	Direction of Heat Flow	Air Space		13 mm Air Space <sup>c</sup>					20 mm Air Space <sup>c</sup>				
		Mean Temp. <sup>d</sup> , °C	Temp. Diff. <sup>d</sup> , °C	Effective Emittance $\epsilon_{eff}^{d,e}$					Effective Emittance $\epsilon_{eff}^{d,e}$				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
Horiz.	Up 	32.2	5.6	0.37	0.36	0.27	0.17	0.13	0.41	0.39	0.28	0.18	0.13
		10.0	16.7	0.29	0.28	0.23	0.17	0.13	0.30	0.29	0.24	0.17	0.14
		10.0	5.6	0.37	0.36	0.28	0.20	0.15	0.40	0.39	0.30	0.20	0.15
		-17.8	11.1	0.30	0.30	0.26	0.20	0.16	0.32	0.32	0.27	0.20	0.16
		-17.8	5.6	0.37	0.36	0.30	0.22	0.18	0.39	0.38	0.31	0.23	0.18
		-45.6	11.1	0.30	0.29	0.26	0.22	0.18	0.31	0.31	0.27	0.22	0.19
		-45.6	5.6	0.36	0.35	0.31	0.25	0.20	0.38	0.37	0.32	0.26	0.21
45° Slope	Up 	32.2	5.6	0.43	0.41	0.29	0.19	0.13	0.52	0.49	0.33	0.20	0.14
		10.0	16.7	0.36	0.35	0.27	0.19	0.15	0.35	0.34	0.27	0.19	0.14
		10.0	5.6	0.45	0.43	0.32	0.21	0.16	0.51	0.48	0.35	0.23	0.17
		-17.8	11.1	0.39	0.38	0.31	0.23	0.18	0.37	0.36	0.30	0.23	0.18
		-17.8	5.6	0.46	0.45	0.36	0.25	0.19	0.48	0.46	0.37	0.26	0.20
		-45.6	11.1	0.37	0.36	0.31	0.25	0.21	0.36	0.35	0.31	0.25	0.20
		-45.6	5.6	0.46	0.45	0.38	0.29	0.23	0.45	0.43	0.37	0.29	0.23
Vertical	Horiz. 	32.2	5.6	0.43	0.41	0.29	0.19	0.14	0.62	0.57	0.37	0.21	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.51	0.49	0.35	0.23	0.17
		10.0	5.6	0.47	0.45	0.33	0.22	0.16	0.65	0.61	0.41	0.25	0.18
		-17.8	11.1	0.50	0.48	0.38	0.26	0.20	0.55	0.53	0.41	0.28	0.21
		-17.8	5.6	0.52	0.50	0.39	0.27	0.20	0.66	0.63	0.46	0.30	0.22
		-45.6	11.1	0.51	0.50	0.41	0.31	0.24	0.51	0.50	0.42	0.31	0.24
		-45.6	5.6	0.56	0.55	0.45	0.33	0.26	0.65	0.63	0.51	0.36	0.27
32.2	5.6	0.44	0.41	0.29	0.19	0.14	0.62	0.58	0.37	0.21	0.15		

Usually we use values from the  $\epsilon_{eff} = 0.82$  column unless one material is low-e

# ASHRAE HOF 2005, Chapter 25 (larger cavities)

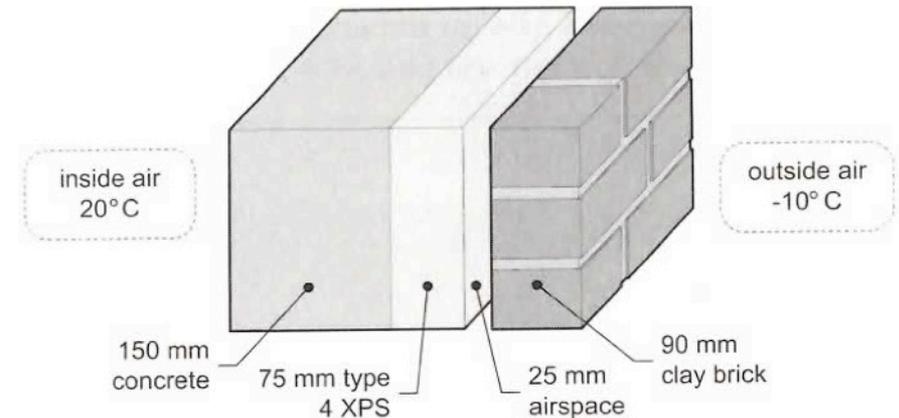
## R-values for different air gap characteristics

Position of Air Space	Direction of Heat Flow	Mean Temp. <sup>d</sup> , °C	Temp. Diff. <sup>d</sup> , °C	Effective Emittance $\epsilon_{eff}^{d,e}$					Effective Emittance $\epsilon_{eff}^{d,e}$				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
		Air Space		40 mm Air Space <sup>c</sup>					90 mm Air Space <sup>c</sup>				
Horiz.	Up 	32.2	5.6	0.45	0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
		10.0	16.7	0.33	0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
		10.0	5.6	0.44	0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
		-17.8	11.1	0.35	0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
		-17.8	5.6	0.43	0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
		-45.6	11.1	0.34	0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
45° Slope	Up 	-45.6	5.6	0.42	0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
		32.2	5.6	0.51	0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
		10.0	16.7	0.38	0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
		10.0	5.6	0.51	0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
		-17.8	11.1	0.40	0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
		-17.8	5.6	0.49	0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
Vertical	Horiz. 	-45.6	11.1	0.39	0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
		-45.6	5.6	0.48	0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
		32.2	5.6	0.70	0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
		10.0	5.6	0.67	0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
		-17.8	11.1	0.49	0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
		-17.8	5.6	0.62	0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
		-45.6	11.1	0.46	0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
		-45.6	5.6	0.58	0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26
		32.2	5.6	0.89	0.80	0.45	0.24	0.16	0.85	0.76	0.44	0.24	0.16

Usually we use values from the  $\epsilon_{eff} = 0.82$  column

# Conduction through multiple layers

- Refer to ASHRAE 2005 HOF Ch. 25 for data



Layer material	Conductivity $k = [\text{W/mK}]$	Thickness $L = [\text{m}]$	Conductance $U = [\text{W/m}^2\text{K}]$	Resistance $R = [\text{m}^2\text{K/W}]$
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17

$$R_{\text{total}} (\text{IP}) = 17.3 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{F/Btu}$$

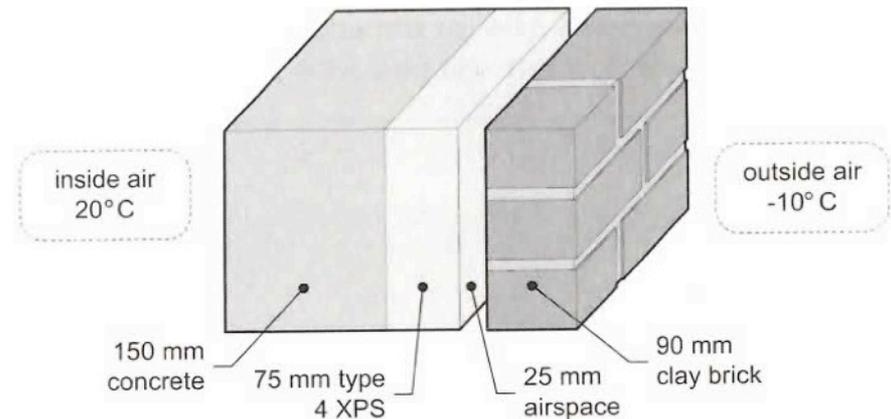
# R-values of deeper cavities

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- The R-value of cavities stops increasing much at 3 inches (75 mm) depth
  - Beyond 3 inches (75 mm), convection and radiation dominate
  - For a deep cavity, either compute R-values with more advanced methods or use the 3 inch (75 mm) value
- Do **NOT** take the R value of a 1 inch (25 mm) cavity and multiply by the thickness of the cavity for thick cavities
  - If you did that, you would guess that an 8 foot attic would have an R value of about  $100 \text{ hr}\cdot\text{ft}^2\cdot^\circ\text{F}/\text{Btu}$ , which is a factor of 20 too high!

# Conduction through multiple layers

- $U_{\text{total}} = 0.33 \text{ W/m}^2\text{K}$
- Calculate steady-state heat flow through the enclosure



- $q = U\Delta T$
- $q = (0.33 \text{ W/m}^2\text{K}) * (T_{\text{inside}} - T_{\text{outside}})$
- $q = (0.33 \text{ W/m}^2\text{K}) * (30 \text{ K}) = 10 \text{ W/m}^2$ 
  - From inside to outside

# Conduction through multiple layers

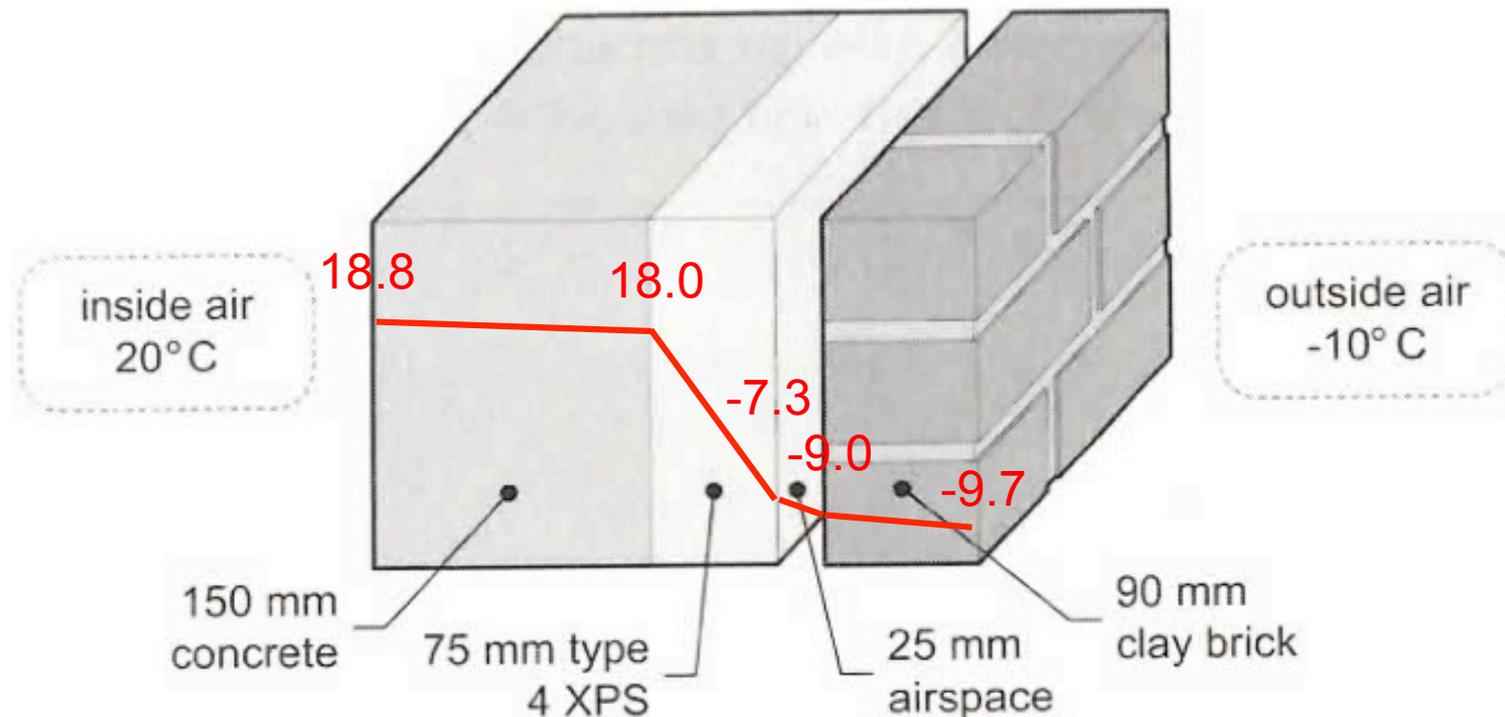
- Calculating the temperature gradient through an enclosure of  $i$  materials

$$\Delta T_i = \frac{T_{internal} - T_{external}}{\sum_{i=0}^n R_i} R_i$$

Layer	Conductivity W/mK	Thickness m	Conductance W/m <sup>2</sup> K	Resistance m <sup>2</sup> K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			$R_{total}$ (m <sup>2</sup> K/W)	3.04
			$U_{total}$ (W/m <sup>2</sup> K)	0.33

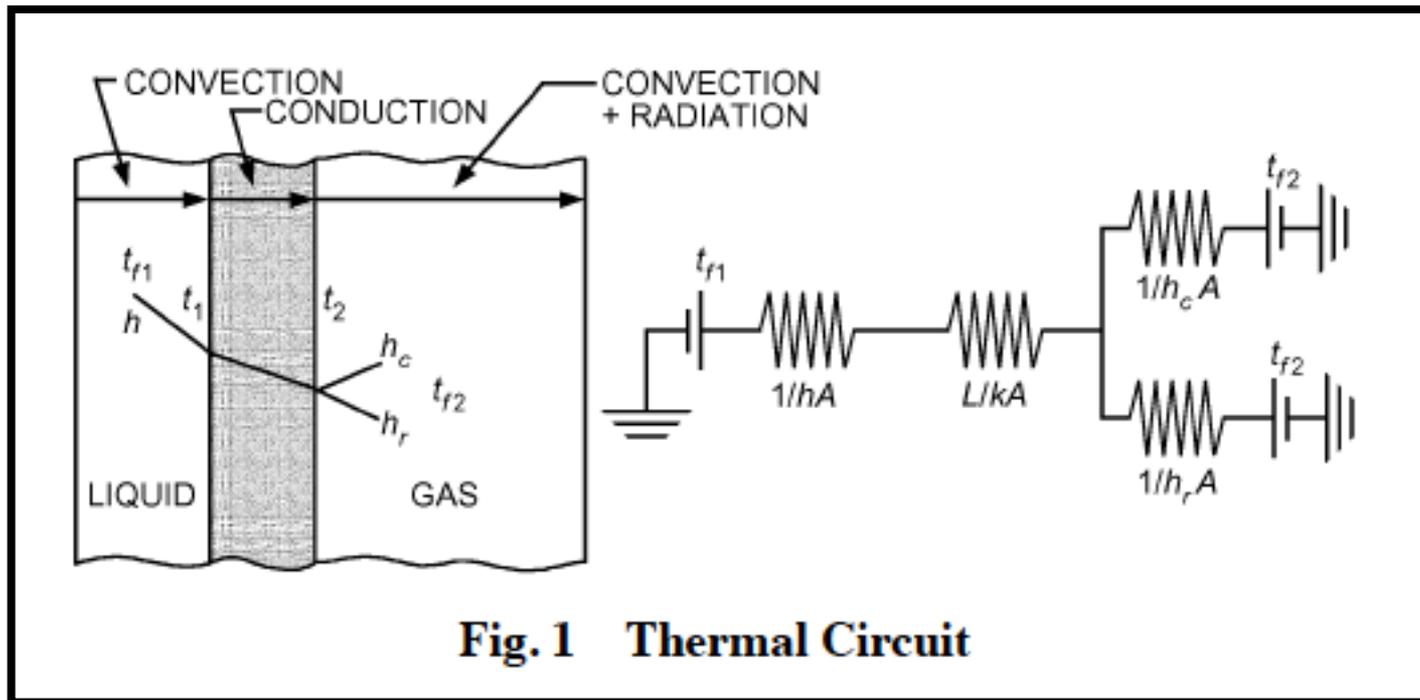
# Conduction through multiple layers

- Calculating the temperature gradient through an enclosure



# Total heat transfer through multiple layers

- We can continue to use the electrical resistance analogy



# Limitations to the summation rule

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The summation rule for finding  $R_{\text{total}}$  has several limitations:

- Only works for **layers**
- Layers must be **same area**
- Layers must be **uniform thickness**
- Layers must have **constant material properties**
  - This is the **biggest** limitation

What do we do with more realistic constructions?

- Parallel path or ISO thermal equivalents
- Computer modeling

# **CONDUCTION IN MORE COMPLICATED ENCLOSURE ASSEMBLIES**

# Thermal networks

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- We have learned how to combine layers to get an overall thermal resistance (or U value) for an assembly made of homogenous layers
  - But we often have to find R (or U) of a more complicated assembly
- The first level of more complicated analysis is best done using thermal networks
  - We've already touched on networks a bit
- References:
  - ISO 6946 Building components and building elements thermal resistance and thermal transmittance calculation method
  - ASHRAE Handbook of Fundamentals

# Developing a thermal network

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1. Identify the layers of the assembly
2. Identify all elements in the layers with differing thermal conductivity
3. Find the R value for each element
4. Draw a resistor for each element
  - Don't forget the internal and external convection resistance
5. Set the resistance to the R value divided by the fractional cross sectional area of the element
6. Connect resistors assuming **isothermal** or **parallel path** conditions

# Isothermal vs. Parallel Path?

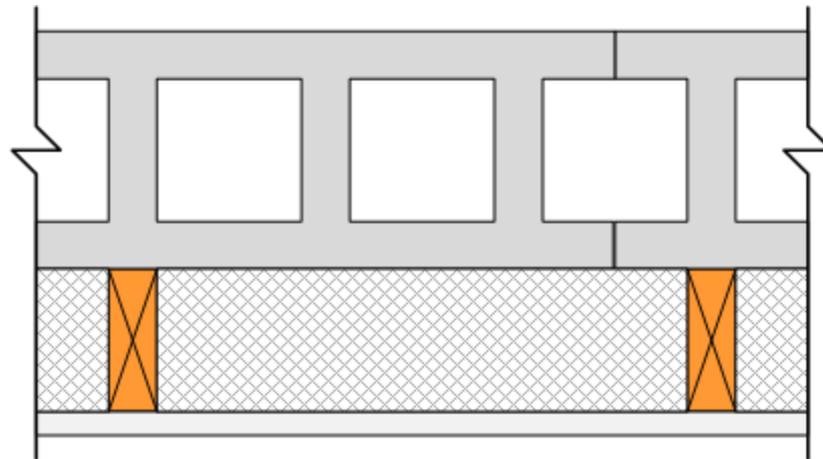
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- In the isothermal assumption, the temperature at any layer interface is assumed to be constant, even if the layer is more than one material
  - This means that there is a network node that corresponds to the interface of each layer
- In the parallel path assumption, the heat transfer is assumed to be only normal to planes
  - This means that the network is several parallel branches
- This is probably best illustrated by example

# Isothermal/parallel path example

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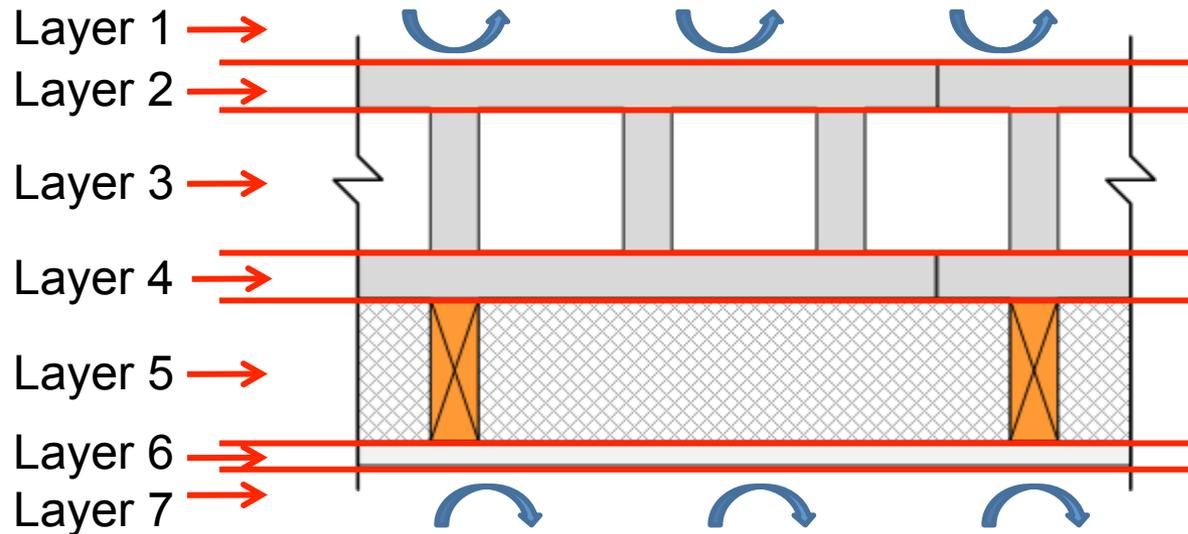
- Consider a CMU Block wall with an attached stud wall
  - A section of the wall is shown below in plan view
  - Draw the isothermal and parallel path thermal networks



# Isothermal/parallel path example

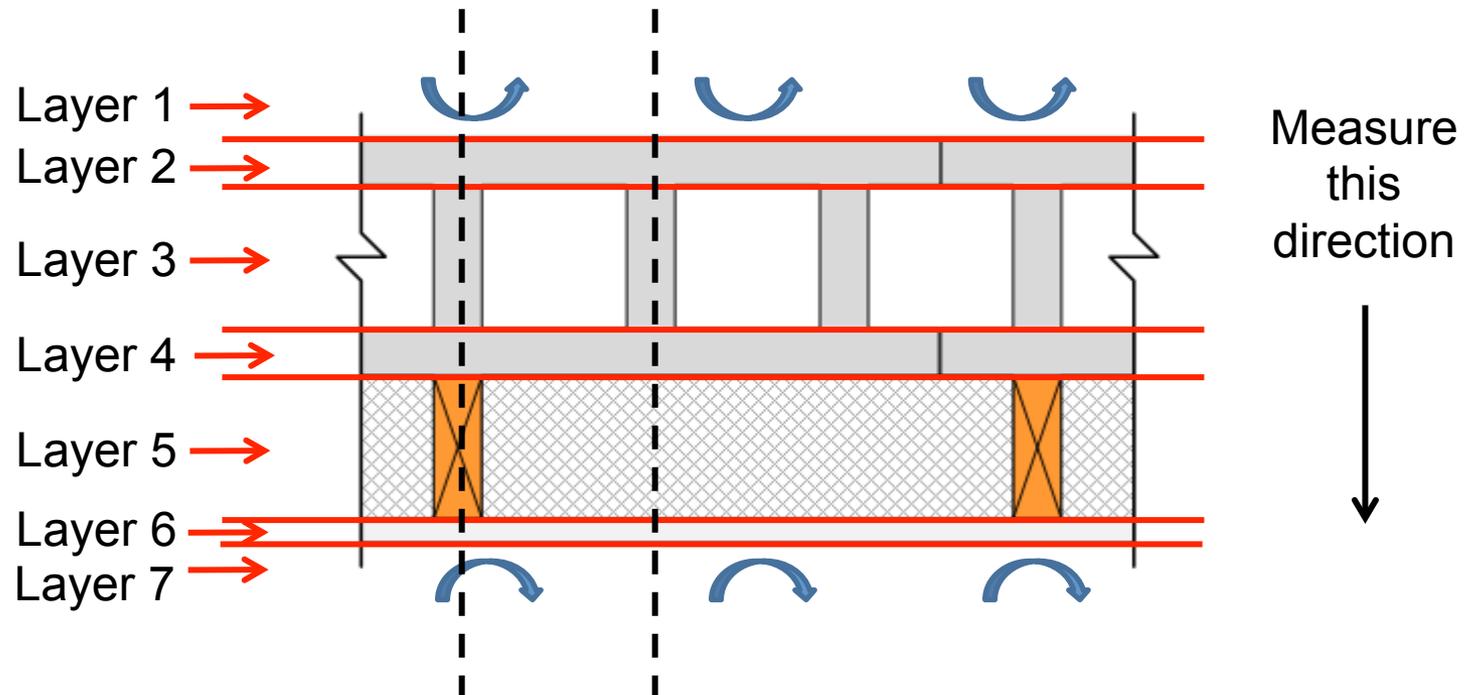
## 1. Identify the layers

- This example has seven layers
- Five are within the assembly
  - The other two are internal and external convection resistances



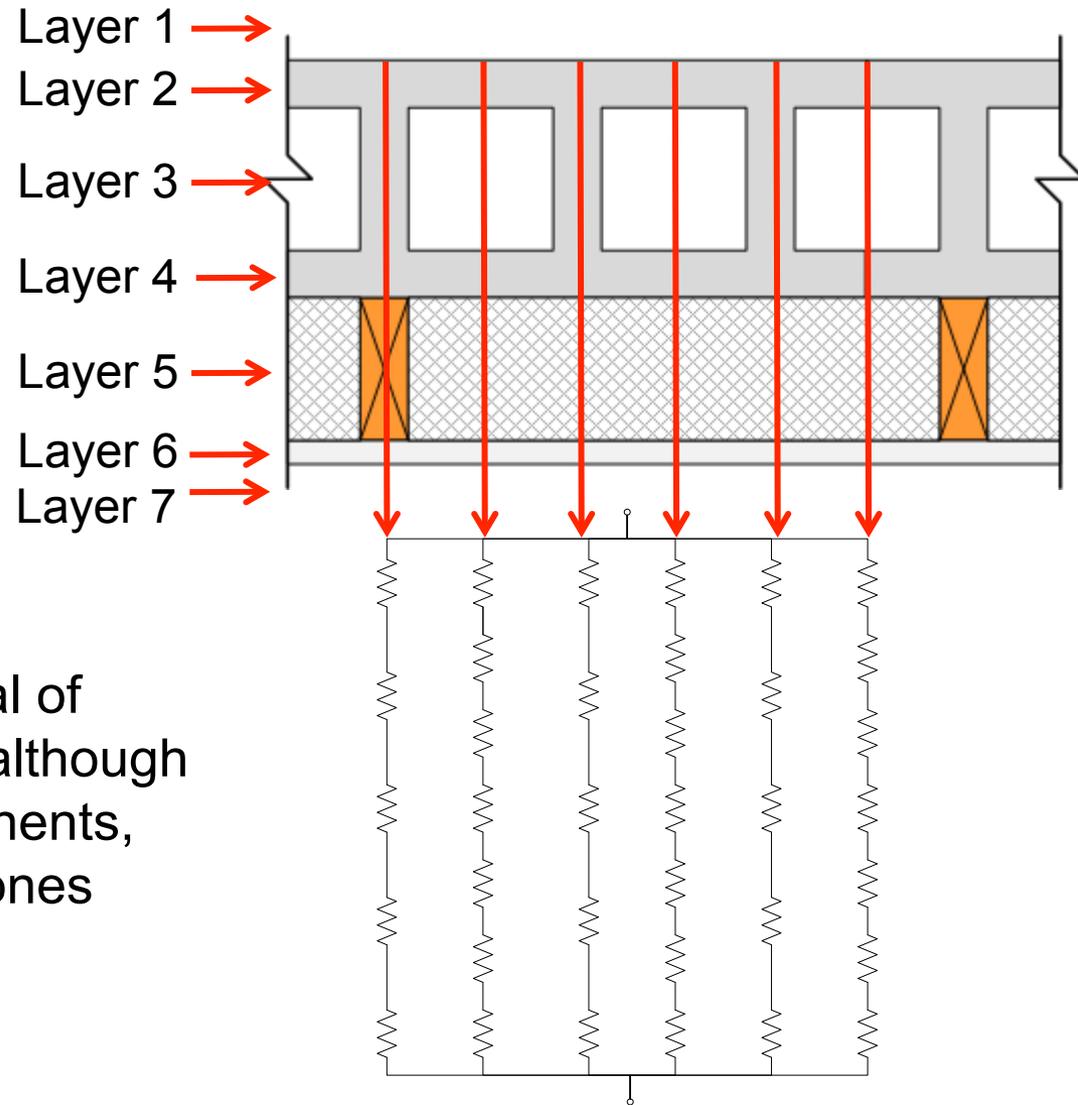
## 2. Identify the elements and 3. Find R-value for each element

- Layer 1 and 7 have 1 element (convection)
- Layer 2 and 4 have 1 element (1.5 inches of concrete)
- Layer 6 has 1 element (0.5 inches of gypsum wallboard)
- Layer 3 has 2 elements (3.5 inches air cavity and 3.5 inches concrete)
- Layer 5 has 2 elements (3.5 inches insulation and 3.5 inches wood stud)



# Parallel Path method

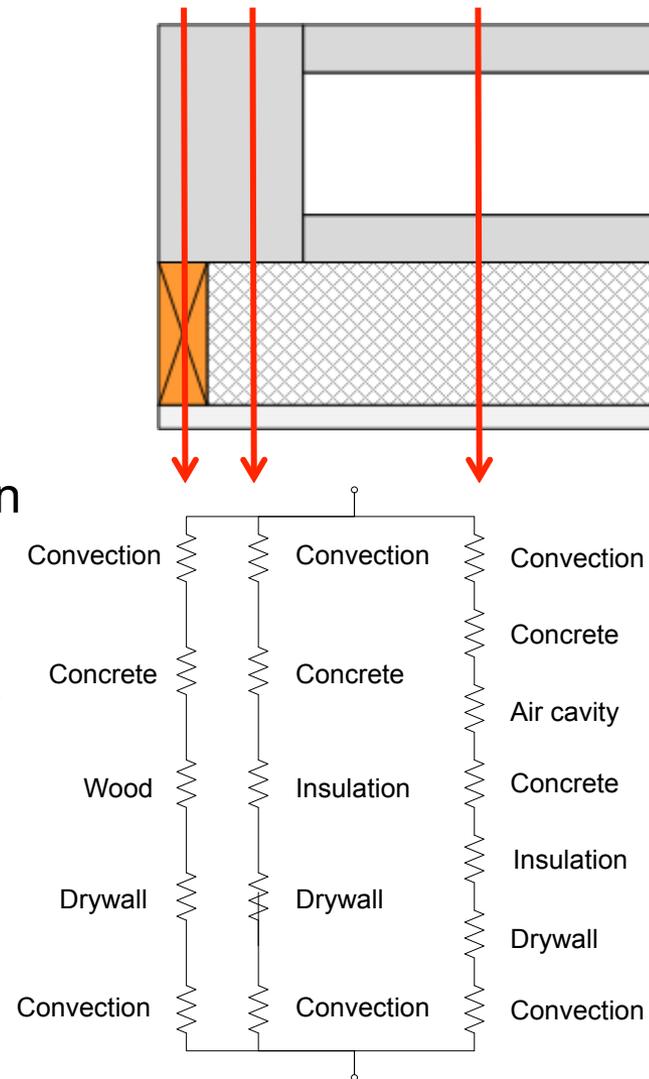
- Identify the different paths and draw them in parallel



Note: In this example, several of these paths are identical so although there are many more components, there are only a few unique ones

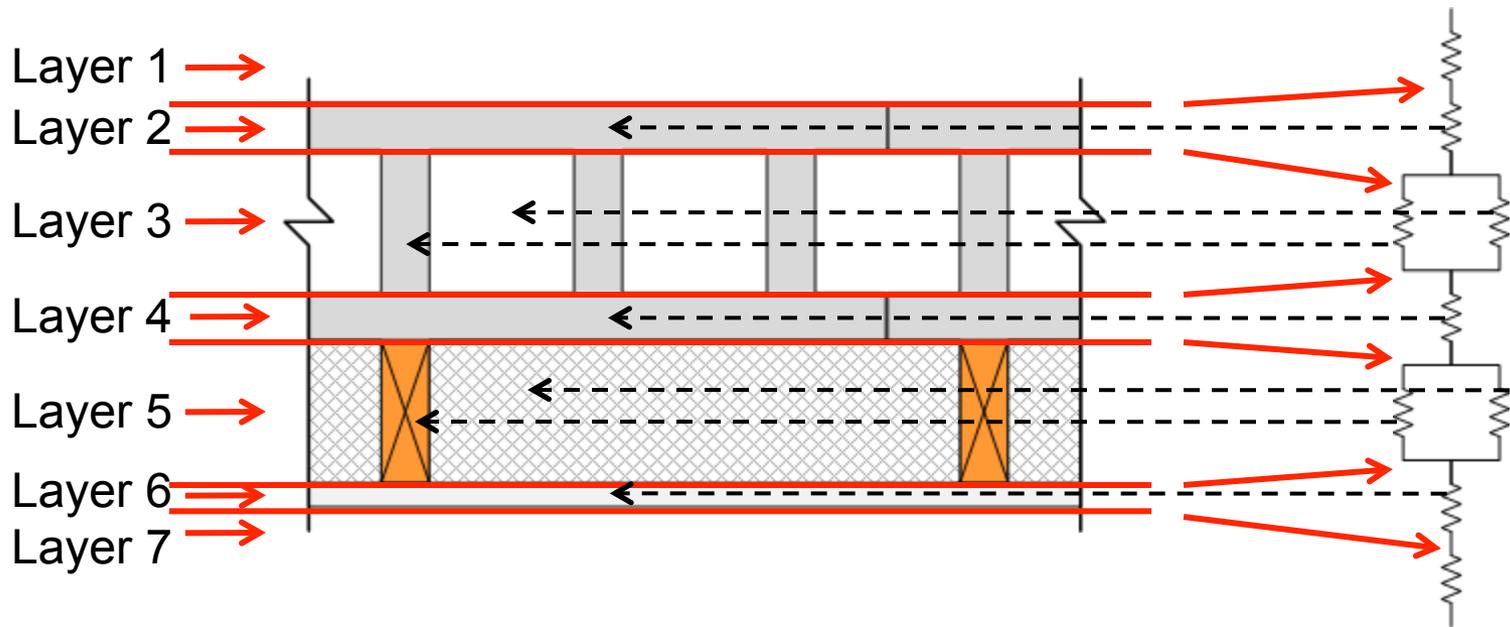
# Alternate/Shortened Parallel Path

- We could combine like elements in each layer to reduce the number of paths to analyze without changing the answer for either isothermal or parallel path
  - This parallel network only has three paths, but will have the same temperatures at each interface location as the previous network
- Note the difference in the number of resistances between each path – varies according to # of elements/layers involved in each path



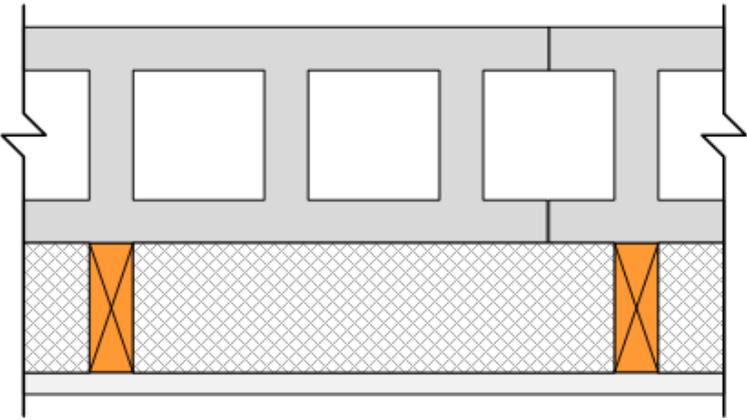
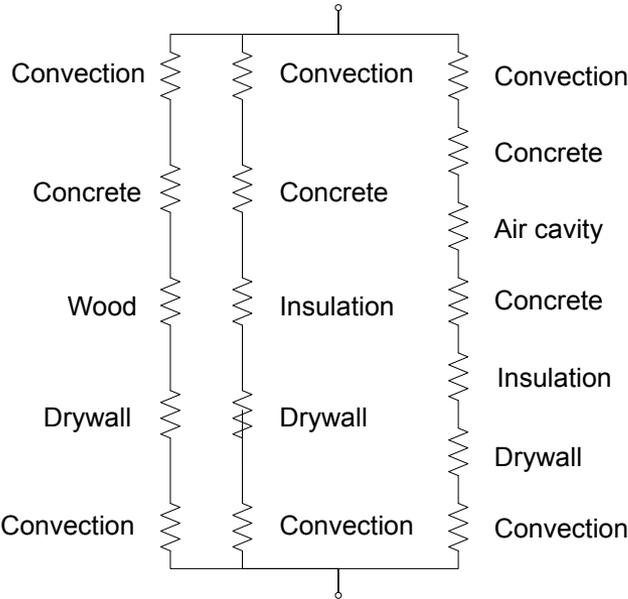
# Isothermal method

- To apply the isothermal method, we put a node at each layer interface and add a resistor for each element in the layer:

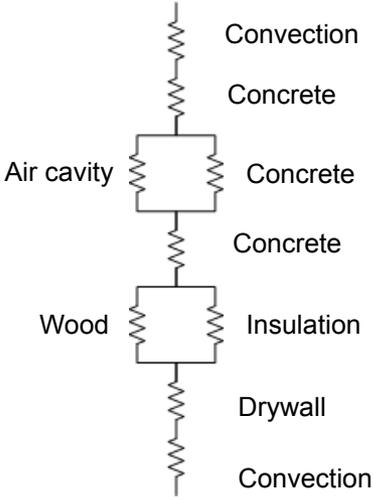


# Isothermal vs. Parallel Path setup

## Parallel Path



## Isothermal



# Assigning resistance values

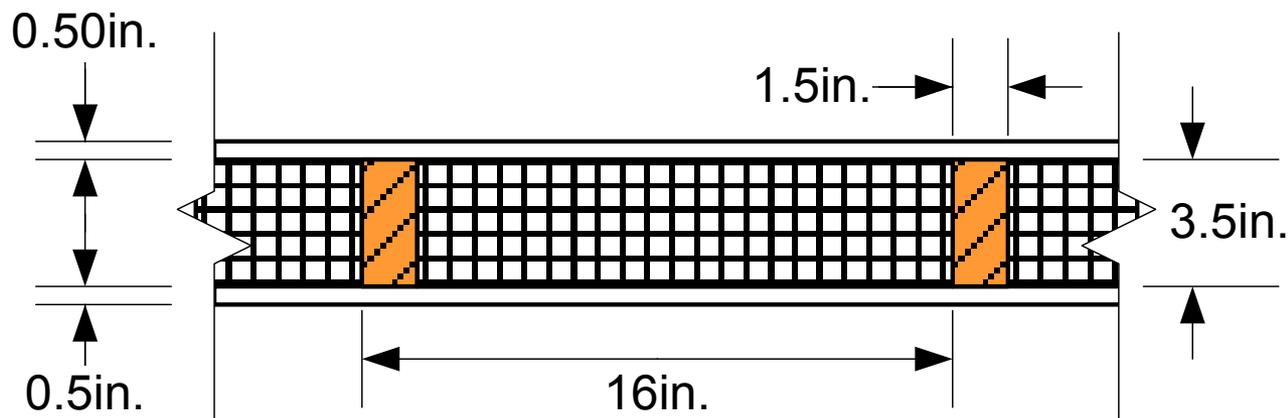
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- The resistance value for the network elements will be the R-value of the element represented **divided by the fractional area,  $f$** , of each element
  - Fractional area is the fraction of the entire cross section that the element takes up
  - This must be a number between 0 and 1

$$R_{network} = \frac{R_{element}}{f}$$

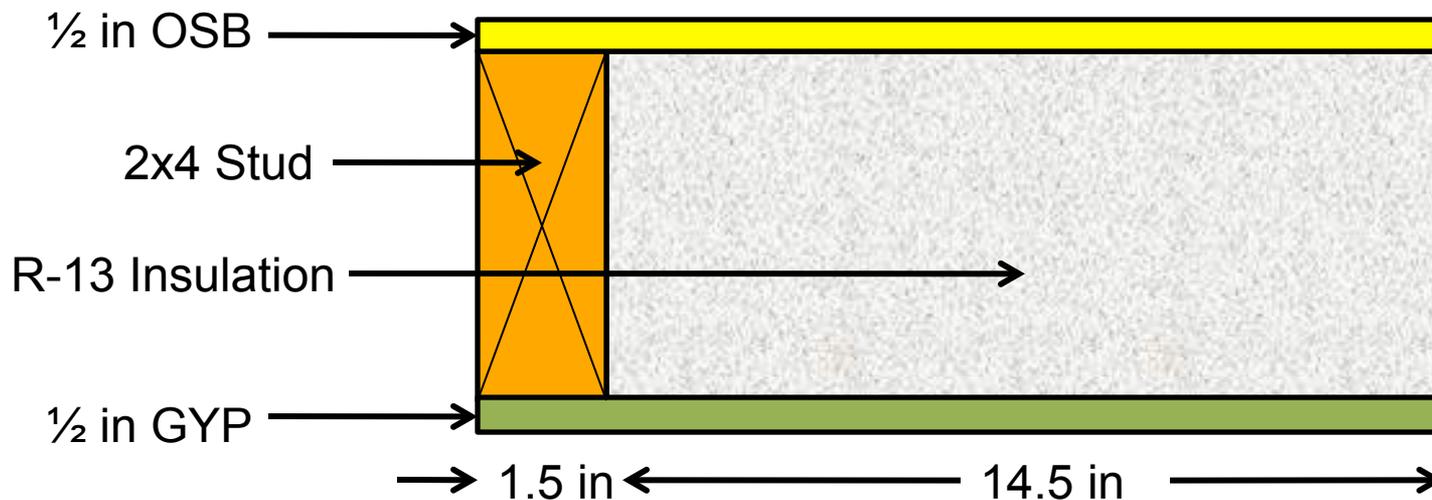
# Example problem

- A wood frame wall has 2x4 studs spaced 16 inches OC (“on center”) with R-13 (IP) insulation in the cavity
- The interior wall is 0.5 inches of gypsum wallboard and the outside wall is 0.5 inches of OSB sheathing
- Draw the isothermal and parallel path networks for this wall
  - Assume winter outdoor conditions



# Example problem continued

- Let's redraw the wall construction to make it easier to identify the layers
  - We can show only the unique part of the wall if we prefer
    - Note: This drawing is not to scale



# Example problem continued

## Identify the layers and elements:

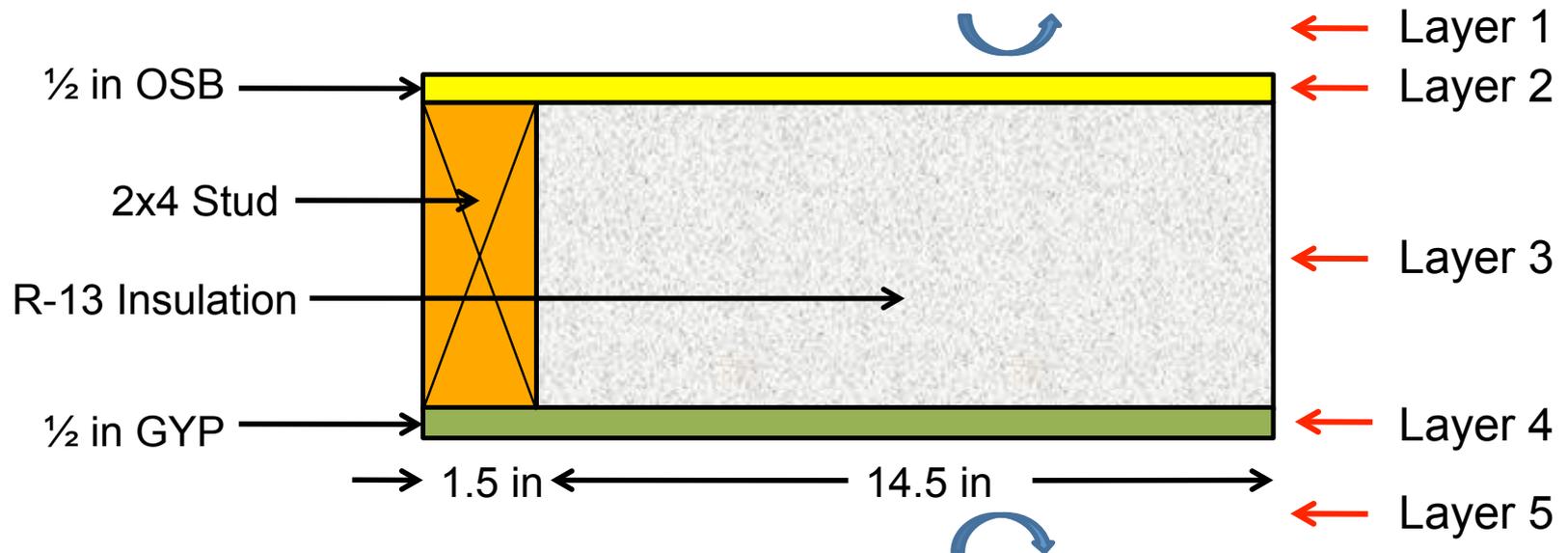
Layer 1: Exterior convection (winter conditions)

Layer 2: 0.5 inches of OSB

Layer 3: 3.5 inches of wood stud and 3.5 inches R-13 insulation

Layer 4: 0.5 inches of gypsum wallboard

Layer 5: Interior convection



## Example problem continued

---

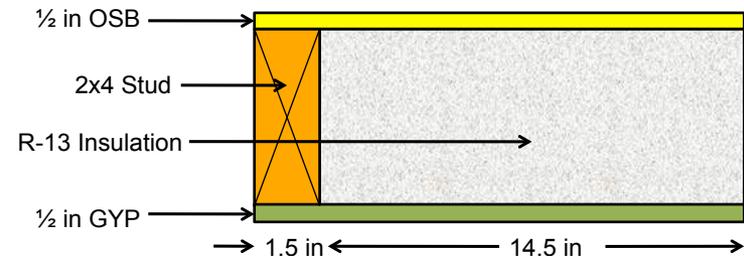
### Look up R values for each element:

- Outdoor winter convection:  $R_{ext} = 0.03 \text{ (m}^2\text{K)/W}$
- ½ in. OSB,  $R_{osb} = 0.12 \text{ (m}^2\text{K)/W}$
- R-13 insulation:  $R_{ins} = 2.29 \text{ (m}^2\text{K)/W}$
- 2x4 (3.5 in. thick) wood stud:  $R_{2x4} = 0.96 \text{ (m}^2\text{K)/W}$
- ½ in. gypsum:  $R_{gyp} = 0.079 \text{ (m}^2\text{K)/W}$
- Indoor convection:  $R_{int} = 0.12 \text{ (m}^2\text{K)/W}$

# Example problem continued

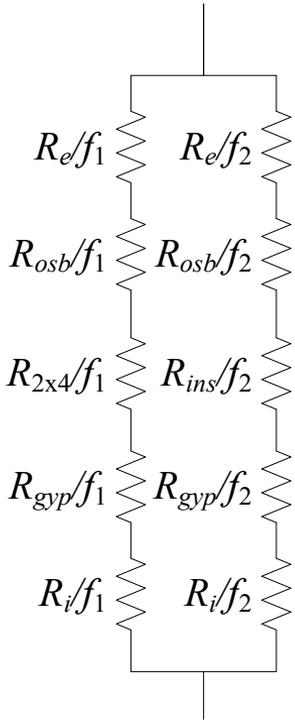
## Find fractional areas of each element

- The full width of the assembly is 16 inches
- All elements are full height of the wall, so the fractional width = fractional area
- Layers 1, 2, 4 and 5 are all 16 inches
  - $f = 1.0$
- 2x4 stud is 1.5 inches
  - $f_1 = 1.5/16 = 0.094$
- R-13 insulation is 14.5 inches
  - $f_2 = 14.5/16 = 0.906$

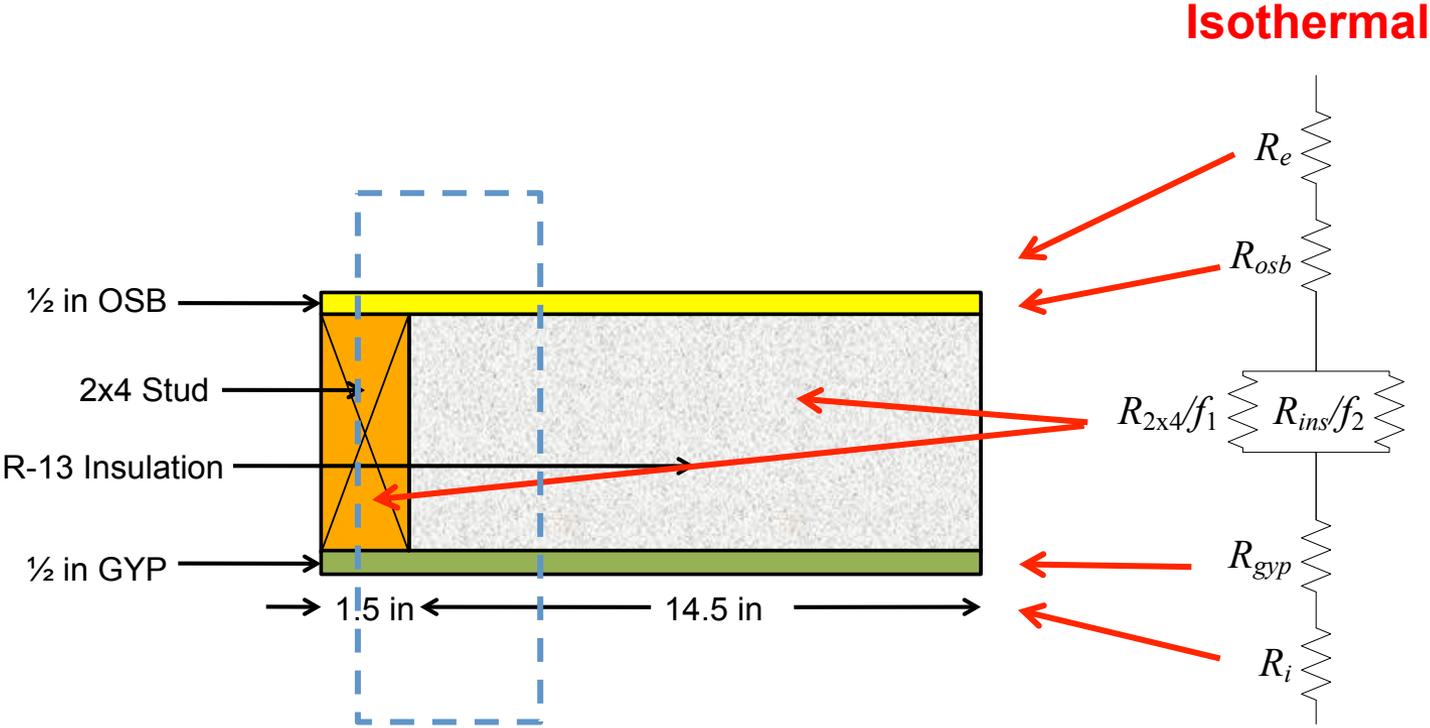


# Example problem continued

- Draw the thermal networks



Parallel path



# Example problem continued

**Parallel path network resistor values:**

$$\frac{R_{ext}}{f_1} = \frac{0.03}{0.094} = 0.32 \frac{m^2K}{W},$$

$$\frac{R_{int}}{f_1} = \frac{0.12}{0.094} = 1.27 \frac{m^2K}{W},$$

$$\frac{R_{osb}}{f_1} = \frac{0.12}{0.094} = 1.27 \frac{m^2K}{W},$$

$$\frac{R_{gyp}}{f_1} = \frac{0.08}{0.094} = 0.84 \frac{m^2K}{W},$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25 \frac{m^2K}{W},$$

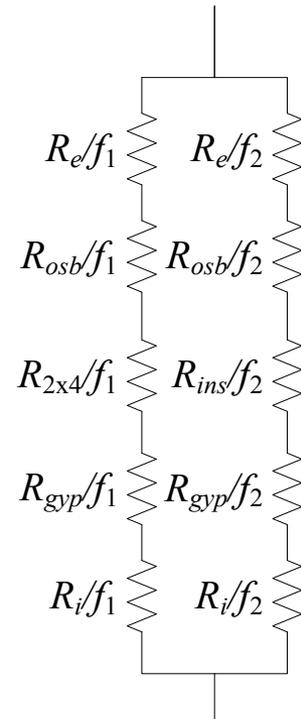
$$\frac{R_{ext}}{f_2} = \frac{0.03}{0.906} = 0.03 \frac{m^2K}{W}$$

$$\frac{R_{int}}{f_2} = \frac{0.12}{0.906} = 0.13 \frac{m^2K}{W}$$

$$\frac{R_{osb}}{f_2} = \frac{0.12}{0.906} = 0.13 \frac{m^2K}{W}$$

$$\frac{R_{gyp}}{f_2} = \frac{0.08}{0.906} = 0.09 \frac{m^2K}{W}$$

$$\frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53 \frac{m^2K}{W}$$



# Example problem continued

## Parallel path network resistor values

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 = R_{ext}/f_1 + R_{osb}/f_1 + R_{gyp}/f_1 + R_{int}/f_1 + R_{2x4}/f_1$$

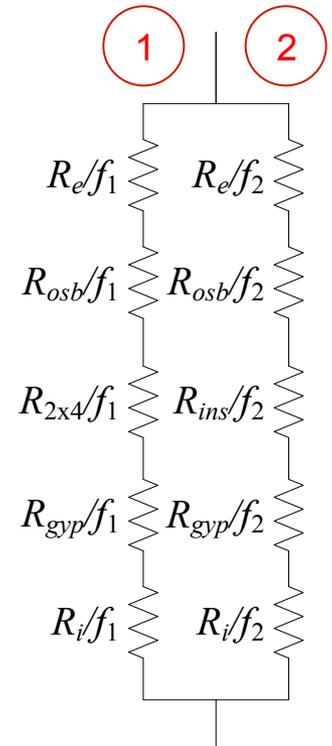
$$R_2 = R_{ext}/f_2 + R_{osb}/f_2 + R_{gyp}/f_2 + R_{int}/f_2 + R_{insulation}/f_2$$

$$R_1 = 0.32 + 1.27 + 10.25 + 1.27 + 0.84 = 13.96$$

$$R_2 = 0.03 + 0.13 + 2.53 + 0.13 + 0.09 = 2.91$$

$$\frac{1}{R_{total}} = \frac{1}{13.96} + \frac{1}{2.91} = 0.415 \frac{W}{m^2K}$$

$$R_{total} = 2.41 \frac{m^2K}{W} = R - 13.68 \text{ (IP)}$$



# Example problem continued

We can now find the network resistor values for the **isothermal** network:

$$R_{ext} = \frac{0.03}{1.0} = 0.03 \frac{m^2K}{W}$$

$$R_{int} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W}$$

$$R_{osb} = \frac{0.12}{1.0} = 0.12 \frac{m^2K}{W}$$

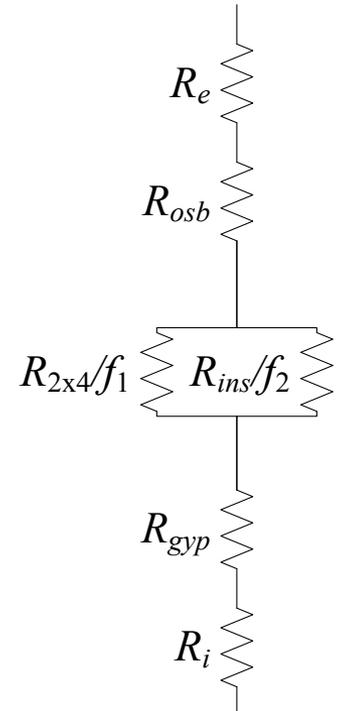
$$R_{gyp} = \frac{0.08}{1.0} = 0.08 \frac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25 \frac{m^2K}{W}$$

$$\frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53 \frac{m^2K}{W}$$

$$R_{total} = R_e + R_{osb} + \frac{1}{\frac{1}{R_{2x4}/f_1} + \frac{1}{R_{insulation}/f_2}} + R_{gyp} + R_i$$

$$R_{total} = 0.03 + 0.12 + \frac{1}{1/10.25 + 1/2.53} + 0.08 + 0.12 = 2.38 \frac{m^2K}{W} = R-13.49 \text{ (IP)}$$



# Utility of thermal networks

---

By developing the full thermal network and then combining elements, we can better see the heat transfer paths

In particular:

- We can identify thermal bridges more easily
  - Areas of particularly low resistance
- We can identify the relative contribution of the elements to heat transfer more easily
- We can use nodal analysis techniques to find the temperature everywhere quickly and easily

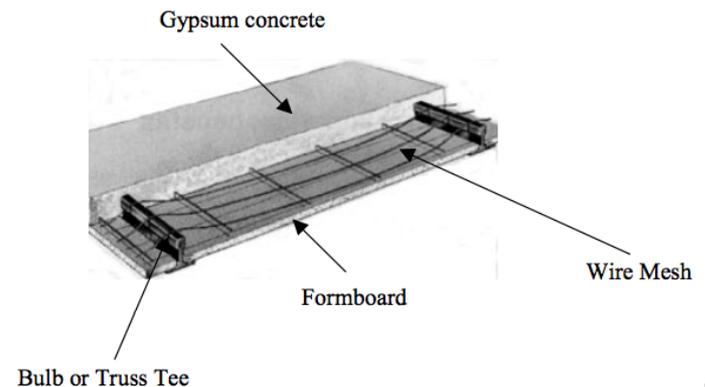
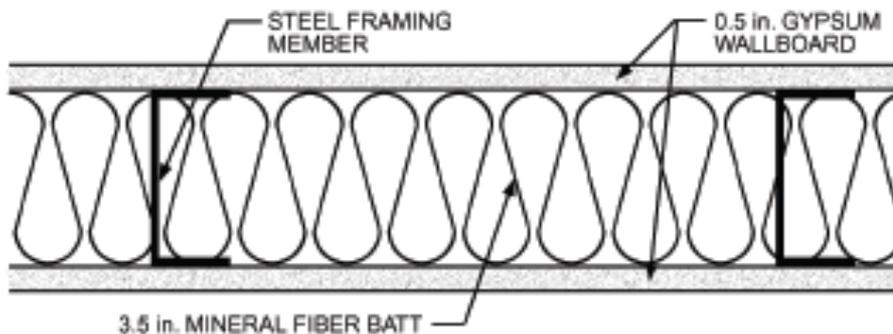
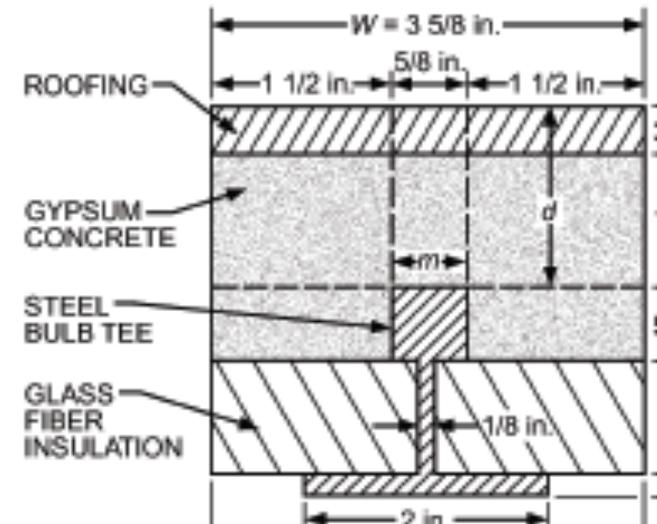
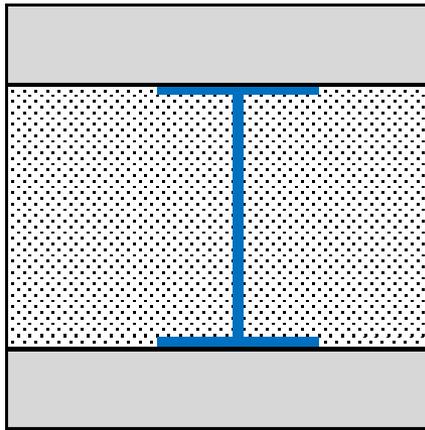
# Limitations to these methods

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- Previous methods do not work well when the assembly has **metal thermal bridges**
  - A thermal bridge is an element of high thermal conductivity that spans through enclosure layers
  - For example:
    - Embedded steel beams can create thermal bridges
- Any flanges on metal beams draw heat into metal areas that cross through layers
  - Increases heat transfer
  - Often odd geometries
    - We need more accurate methods of estimating U-values

# Metal thermal bridge examples

- I-beams sandwiched between sheathing
- Metal stud walls
- Metal embedded in concrete



# Online tools to help us out with thermal bridges

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- There are some online “modified zone” calculators developed by Oakridge National Laboratory

<http://www.ornl.gov/sci/roofs+walls/calculators/modzone/index.html>

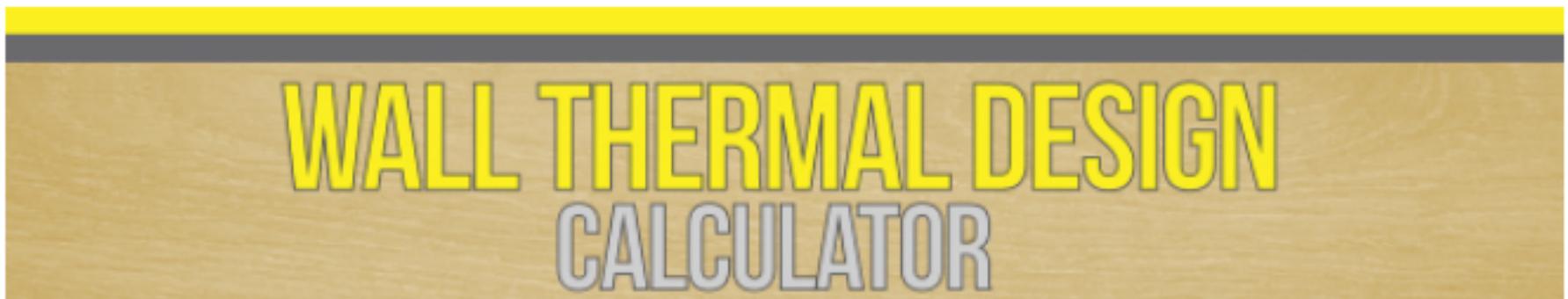
<http://www.ornl.gov/sci/roofs+walls/calculators/modzone/modzone2.html>

- Building Science Corp:

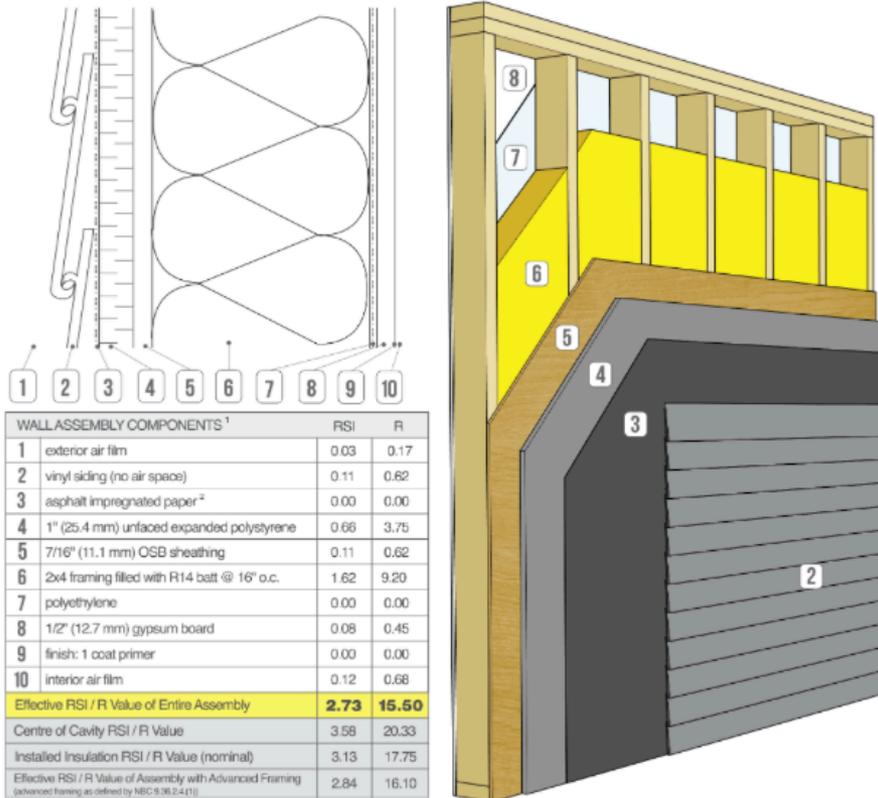
- <http://www.buildingscience.com/documents/information-sheets/high-r-value-wall-assemblies/?topic=/doctypes/information-sheets>

- Canadian Wood Council:

- <http://www.cwc.ca/index.php/en/resources/wallthermaldesignmainpage>



# Wall thermal design calculator



Note: <sup>1</sup>Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. <sup>2</sup>Sheathing membrane materials must comply with CAN/CSG-91.30, "Sheathing Membrane, Gypsum Type".

LEGEND  Pass  Proceed with caution

**SIMULATED DURABILITY ANALYSIS**

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WUFI HYDROTHERMAL MODELING	<input checked="" type="checkbox"/>				

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 50 ng/(Pa·s·m<sup>2</sup>) must comply to NBC 9.25.5.2.

**15.5**  
R<sub>eff</sub>



Note: <sup>1</sup>Values are for generic insulation products. Where a specific insulation product is used in the assembly the thermal resistance value, or long term thermal resistance value, where applicable, of that product is permitted to be used as reported by the Canadian Construction Materials Centre (CCMC) in the evaluation of such a product. <sup>2</sup>The thermal resistance of mortar was not considered. <sup>3</sup>Sheathing membrane material must comply with CAN/CSG-91.30, "Sheathing Membrane, Gypsum Type".

LEGEND  Pass  Proceed with caution  Check permeance of material

**SIMULATED DURABILITY ANALYSIS**

LOCATION:	Vancouver	Edmonton	Toronto	Montreal	St. John's
WUFI HYDROTHERMAL MODELING	<input checked="" type="checkbox"/>				
OUTBOARD TO INBOARD RATIO COMPLIANCE	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.3	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.2	<input checked="" type="checkbox"/> 0.2

Note: See WUFI Assumptions. Non-wood based exterior sheathing material that has a water vapour permeance less than 50 ng/(Pa·s·m<sup>2</sup>) must comply to NBC 9.25.5.2.

**OUTBOARD TO INBOARD RATIO 0.39**

**33.0**  
R<sub>eff</sub>

# **THERMAL BRIDGES**

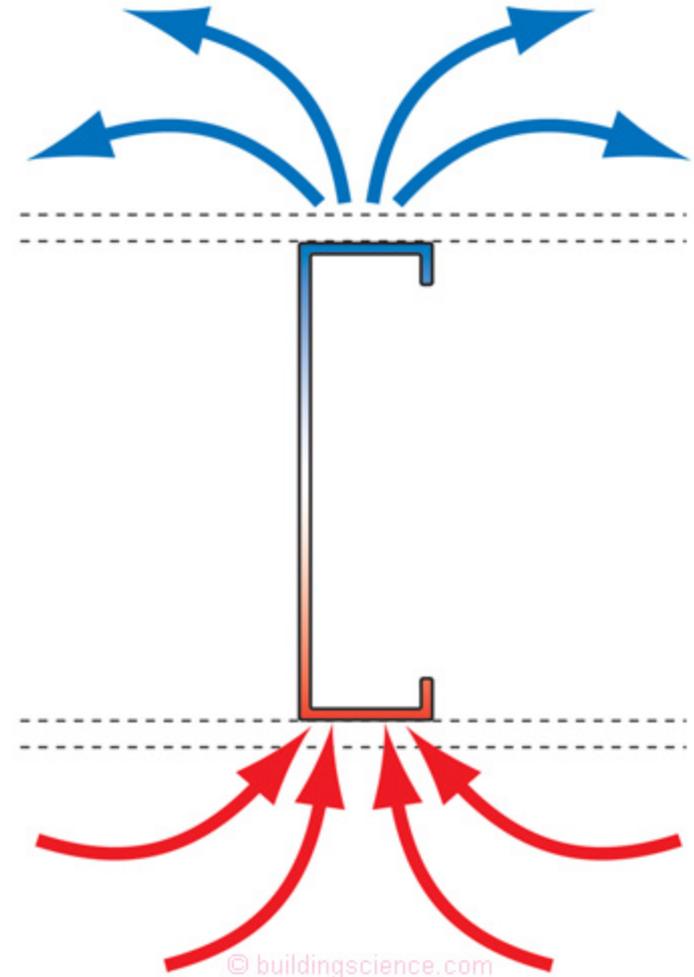
Examples

Even more complex (and more accurate) analysis

# Thermal bridges

---

- A thermal bridge is a high conductivity portion of an assembly that penetrates insulating layers and significantly reduces the overall R value of the assembly
- These bridges act as thermal “short circuits”



# A note on materials and $k$ , $U$ , and $R$ values

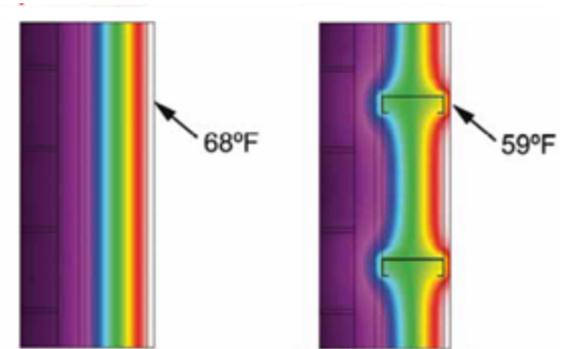
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- We've already looked up material properties several times
  - Many are listed in ASHRAE HOF
  - It's important to have a good working knowledge of these values for common building materials (particularly **high conductivity** materials)
  - One relevant ASHRAE HOF chapter on Blackboard:
    - From 2005: Ch. 39 - Physical properties of materials
  - Pay special attention to **metals**, in addition to those values we already know
  - Thermal conductivity ( $k$ ) of common materials
    - Insulation  $\sim 0.03$  W/mK
    - Wood  $\sim 0.1$ - $0.2$  W/mK
    - Rubber  $\sim 0.1$  W/mK
    - Brick/stone/concrete  $\sim 0.5$ - $2.0$  W/mK
    - **Steel  $\sim 45$  W/mK**
    - **Cast iron  $\sim 50$  W/mK**
    - **Aluminum  $\sim 220$  W/mK**

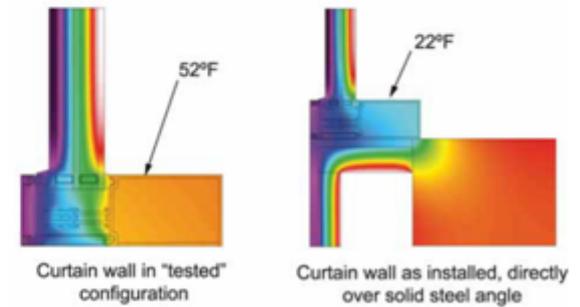
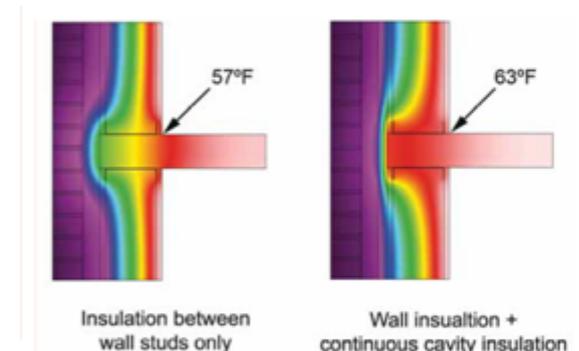
↓  
Increasing conductivity  
Decreasing resistance  
(also depends on thickness)

# Common thermal bridges

- Metal studs in insulated walls without external insulation
- Floor planks without continuous external insulation
- Curtain walls in contact with steel framing



Thermal Bridging through Light Gauge Steel Studs



Thermal Bridging at Curtain Wall Connection

# Identifying thermal bridges

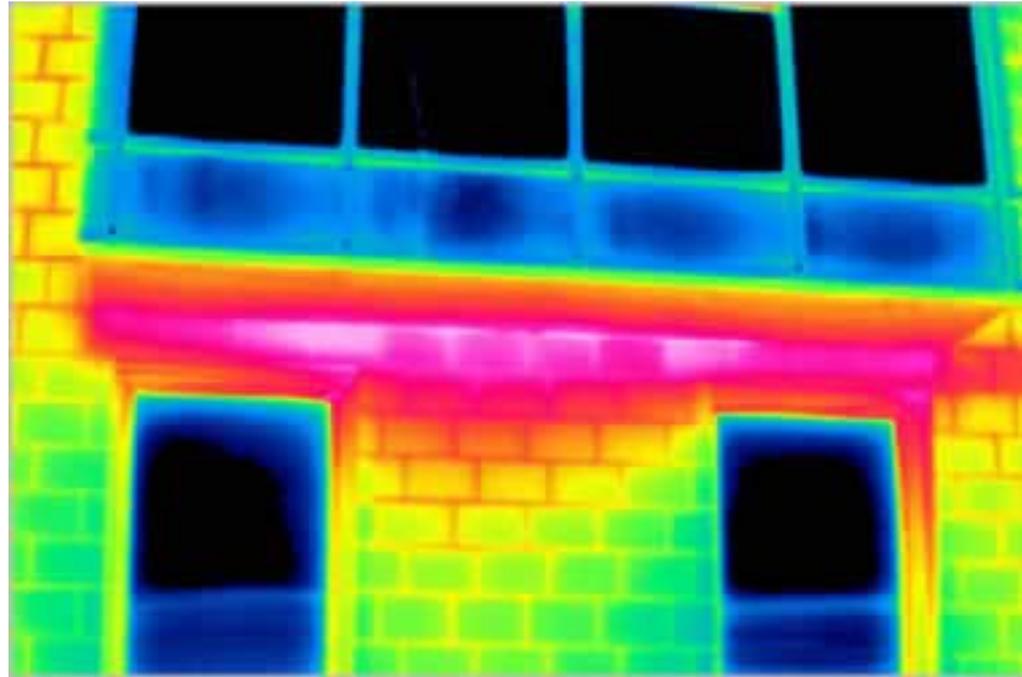
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We can identify thermal bridges in many ways

- Inspection of detail drawings
  - Use this to identify potential problems
- Thermal analysis of detail drawings
  - Use to quantify the extent of thermal bridging
- Thermal imaging
  - Useful way to find bridges in existing construction
- Visually without tools

# Thermal imaging

- One way to find existing thermal bridges in the built environment is to use thermal imaging
  - Particularly when the inside-outside temp difference is large
- Regions around thermal bridges will have vastly different temperatures than the rest of the wall
  - This will be visible to IR



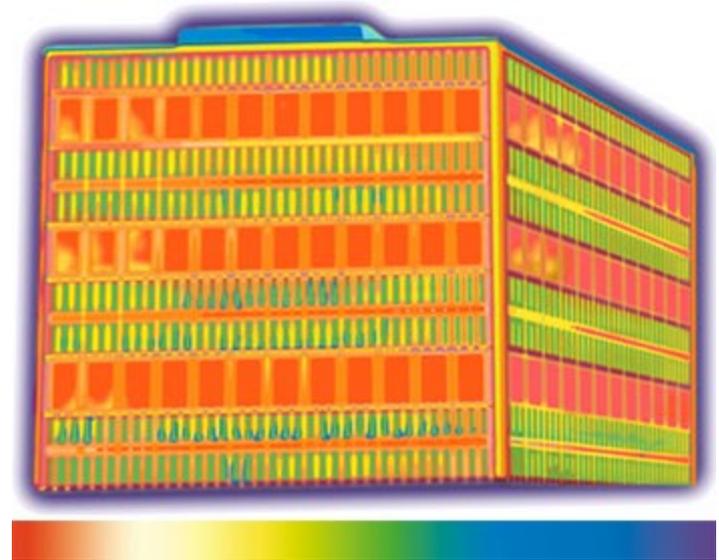
The red here indicates a thermal bridge created by a structural beam below the window

Cold outside, hot inside

$$q = \varepsilon T^4$$

# More thermal imaging

- These figures show hot spots in red
- The windows, frames, and locations where the floor meets the walls are clearly visible bridges in the top figure
- The floor/foundation junction in the bottom has clear thermal bridges



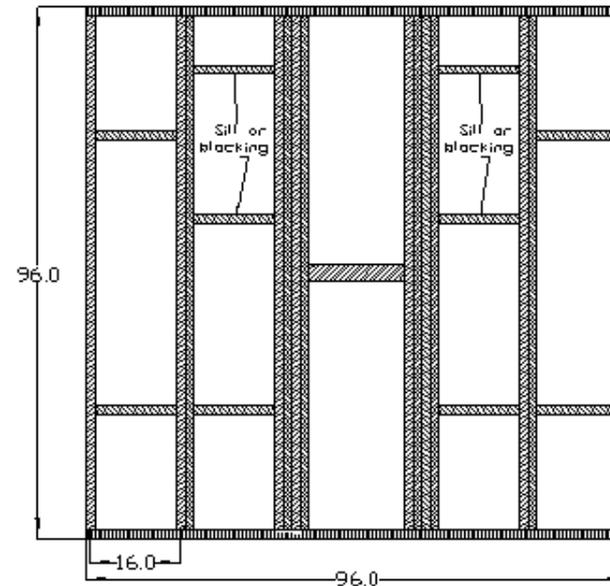
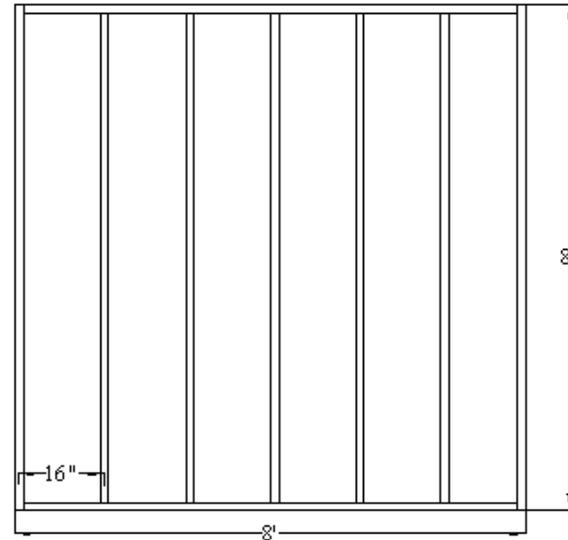
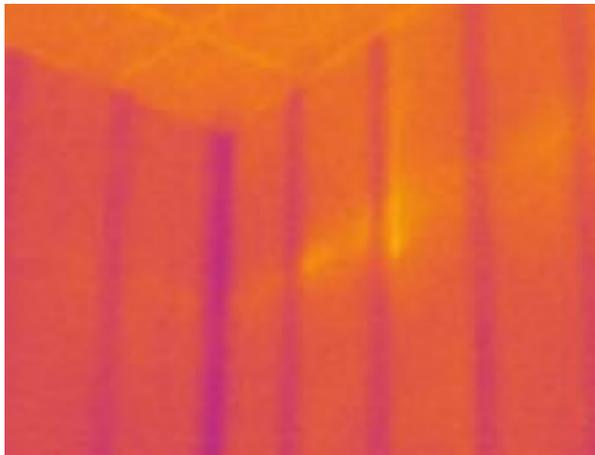
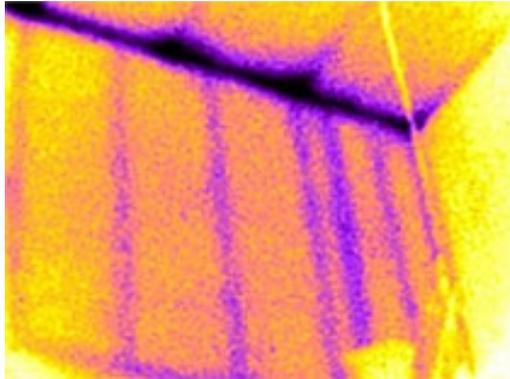
# Framing effect

---



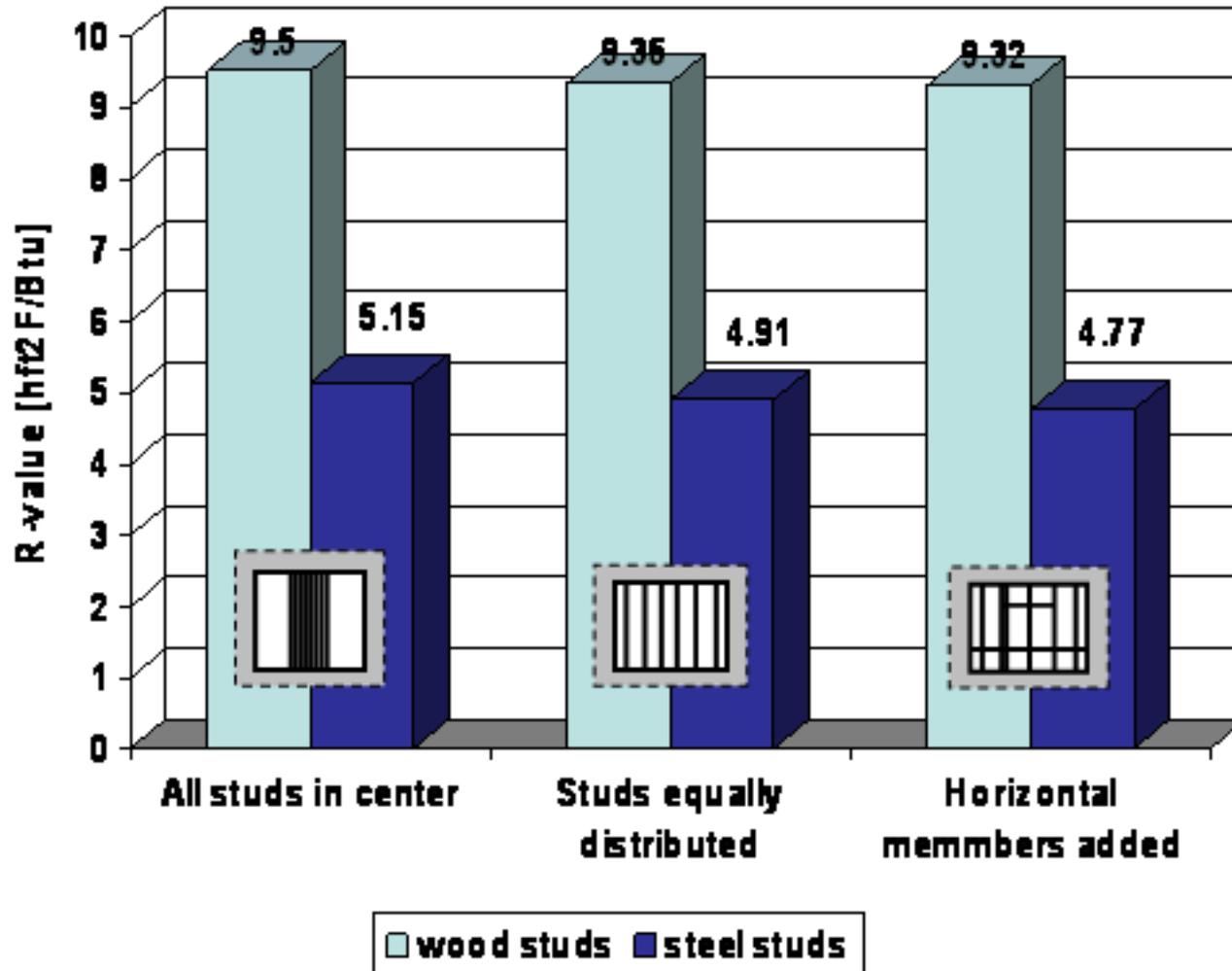
- The “framing effect” is a term used to denote the reduction in R-value from thermal bridging of the framing
  - Steel stud walls commonly have a framing effect of 40%-50%
- A reduction in the framing effect means an increase in the effective R-value of a wall

# Framing effects



Thermal bridges show up in blue on these figures. Notice that corners seem to have the worst thermal bridges

# Framing effects



# Framing effect

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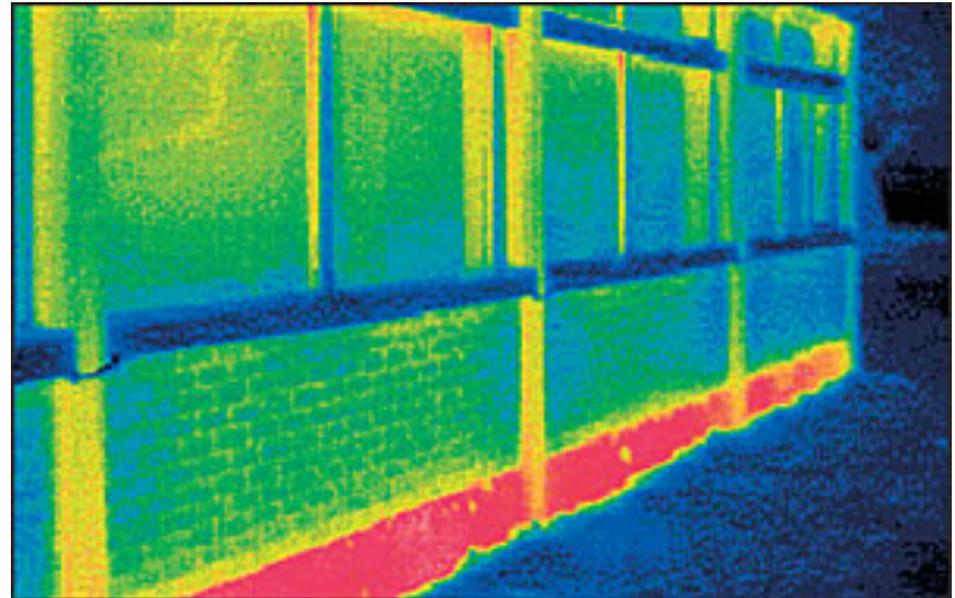


Thermal bridging in the uninsulated roof framed with steel (R-value of around 0.04-IP per inch) means bare roof at the rafter lines

# Foundation thermal bridges

---

Here are some thermal images showing thermal bridges where a slab floor or foundation meets a wall



# Wall-floor thermal bridges

Here are some examples of thermal bridges where interior floors and supports meet with exterior walls



Thermal bridge for support under the window

Thermal bridge from lintel over the door



# Reducing thermal bridging

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## Basic concept:

- To reduce thermal bridging we need to **provide a thermal break** in the high-conductivity heat transfer path

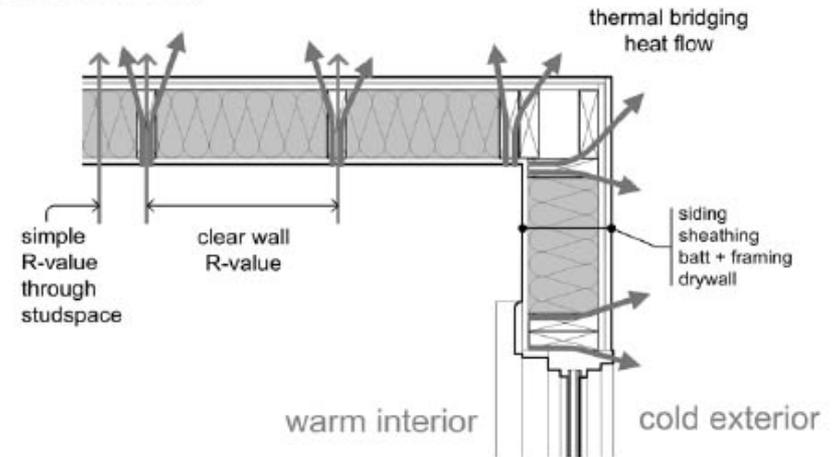
## Basic solution:

- Add insulation to the path to reduce direct heat flow

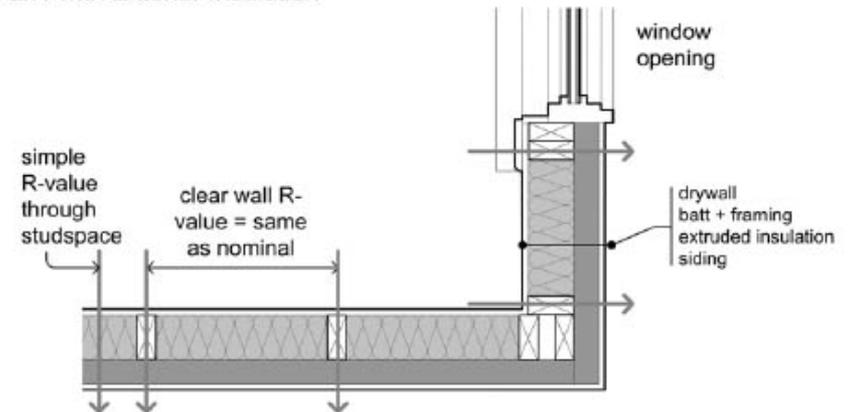
# An important solution: external insulation

- The use of external insulation is a very effective way of reducing the bridging effects
  - It also increases the R value of the non-bridge areas
    - A 2x4 wall with 2 inches external insulation is much better than a 2x6 wall with 6 inches interior insulation
  - Need a continuous (or near continuous) layer of external insulation to break the path of high-conductivity material
    - Add to the exterior to ensure as much continuity as possible

2x6 Framed Wall



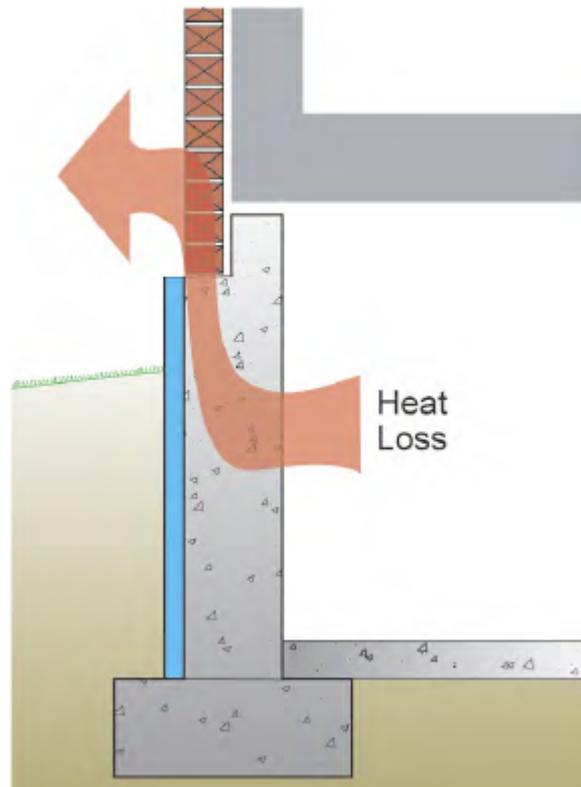
2x4 with Exterior Insulation



# Foundation thermal bridges

---

Thermal bridges between brick veneer and foundation are common



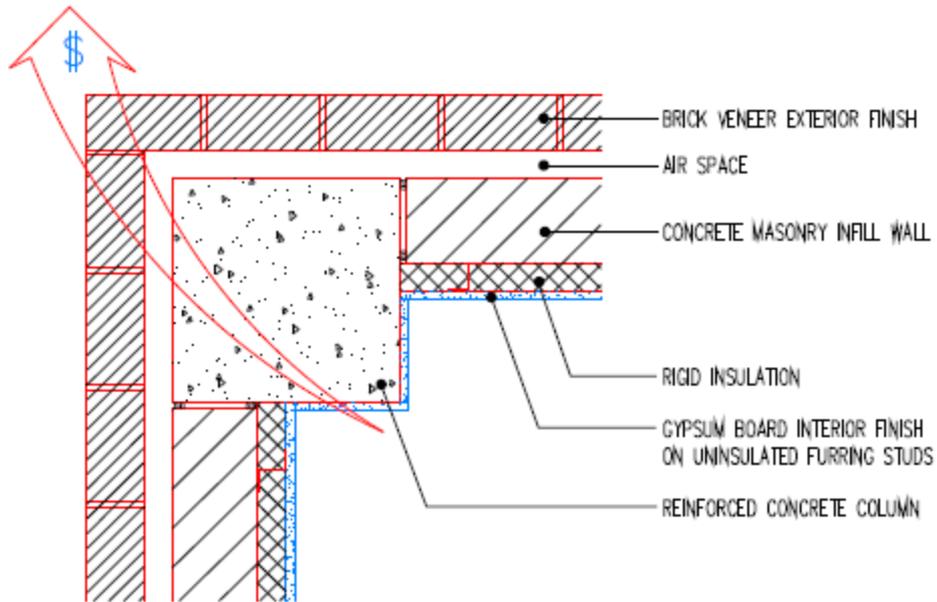


# Corner columns

## PROBLEM

### THERMAL BRIDGE:

AT CORNER  
THERMAL MASS OUTSIDE OF  
INSULATION.

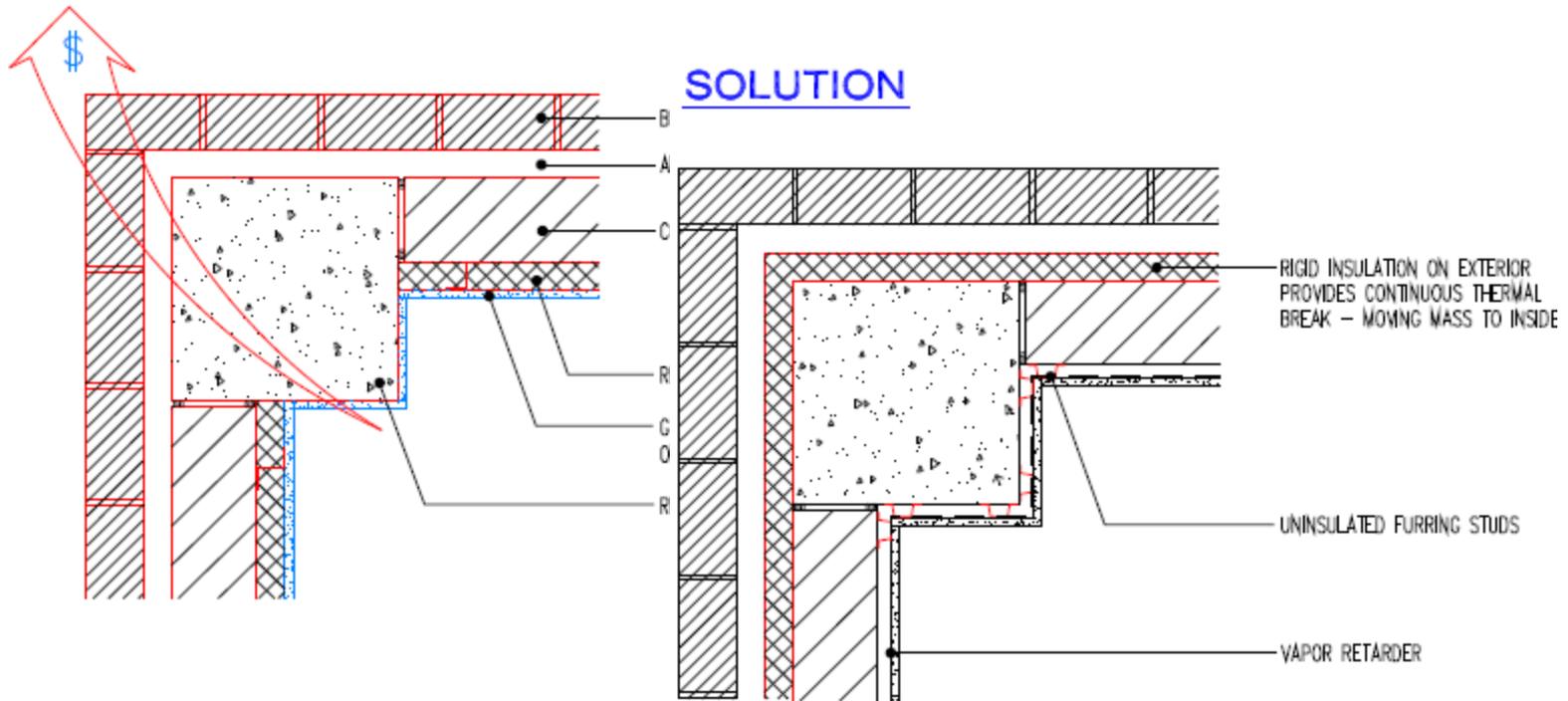


# Corner columns

## PROBLEM

### THERMAL BRIDGE:

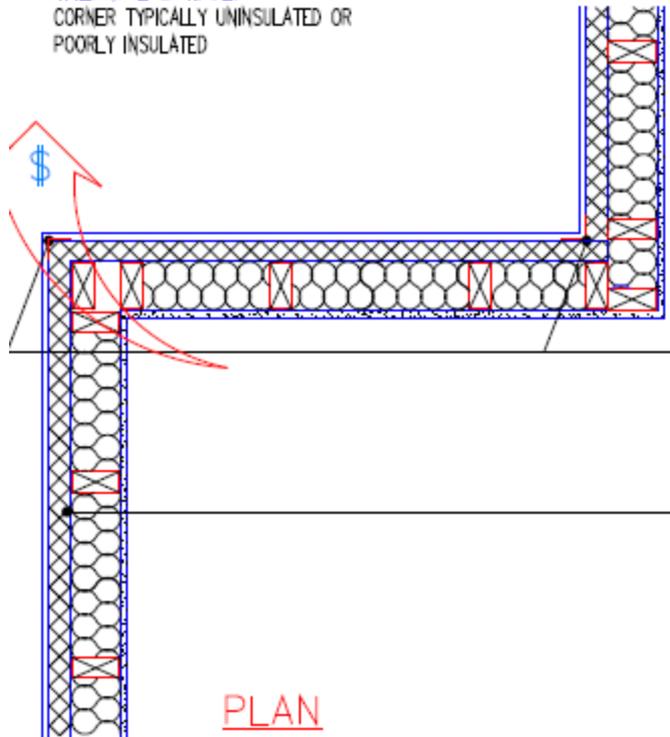
AT CORNER  
THERMAL MASS OUTSIDE OF  
INSULATION.



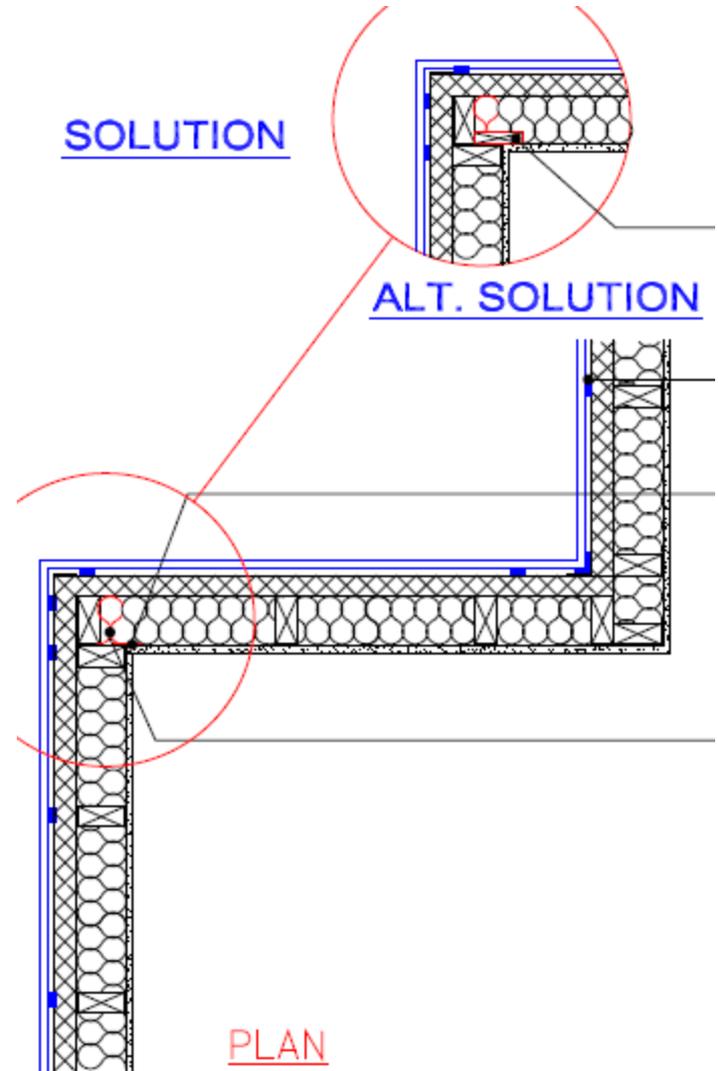
# Wood stud corners

## PROBLEM

THERMAL BRIDGE:  
CORNER TYPICALLY UNINSULATED OR  
POORLY INSULATED



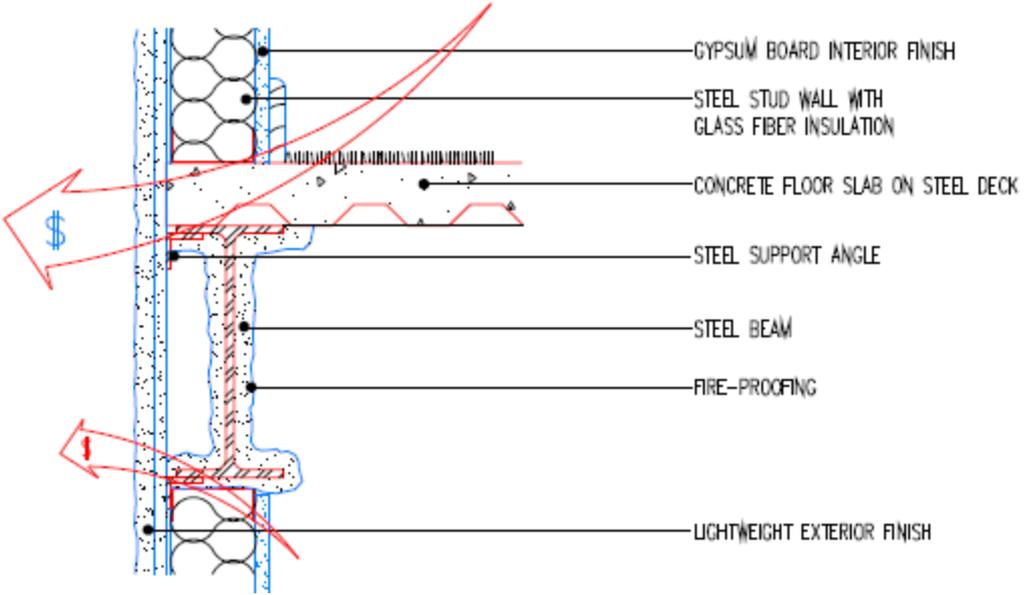
## SOLUTION



# Slab edge stud wall

## PROBLEM

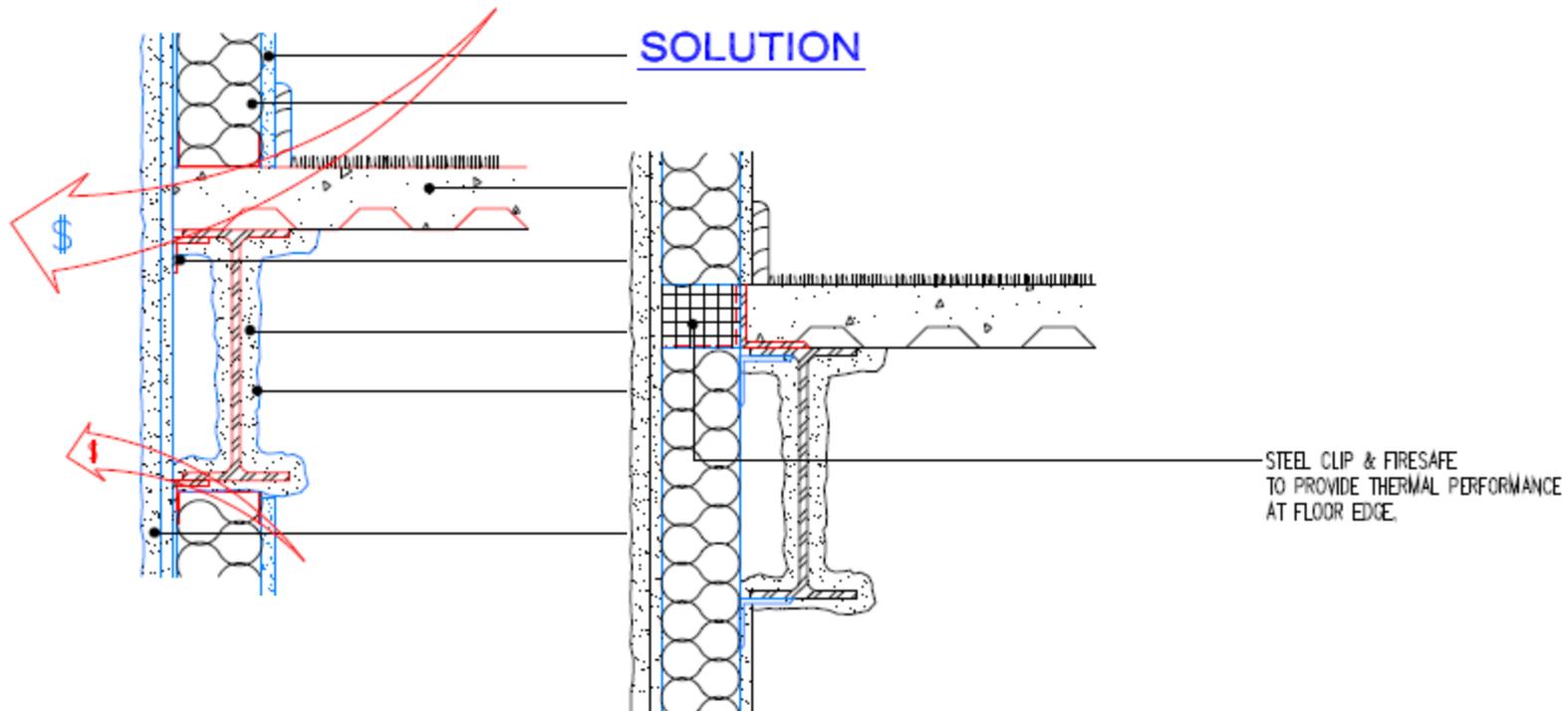
**THERMAL BRIDGE:**  
AT EDGE OF CONCRETE FLOOR  
SLAB.



# Slab edge stud wall

## PROBLEM

**THERMAL BRIDGE:**  
AT EDGE OF CONCRETE FLOOR  
SLAB.

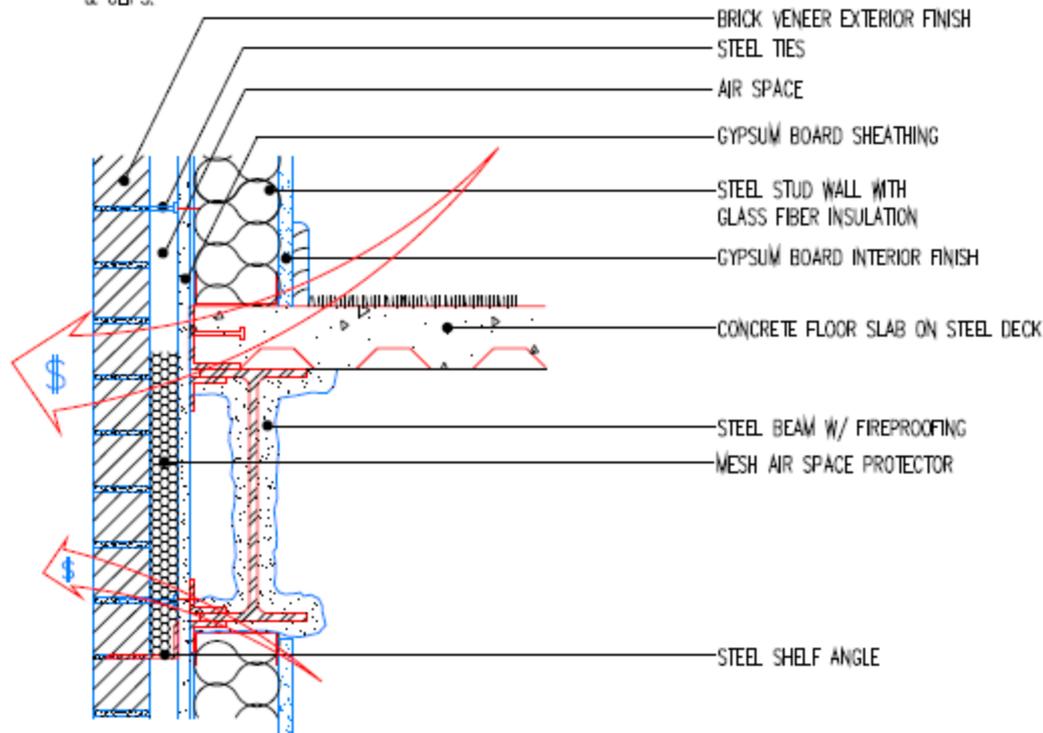


# Slab edge brick/stud wall

## PROBLEM

### THERMAL BRIDGE:

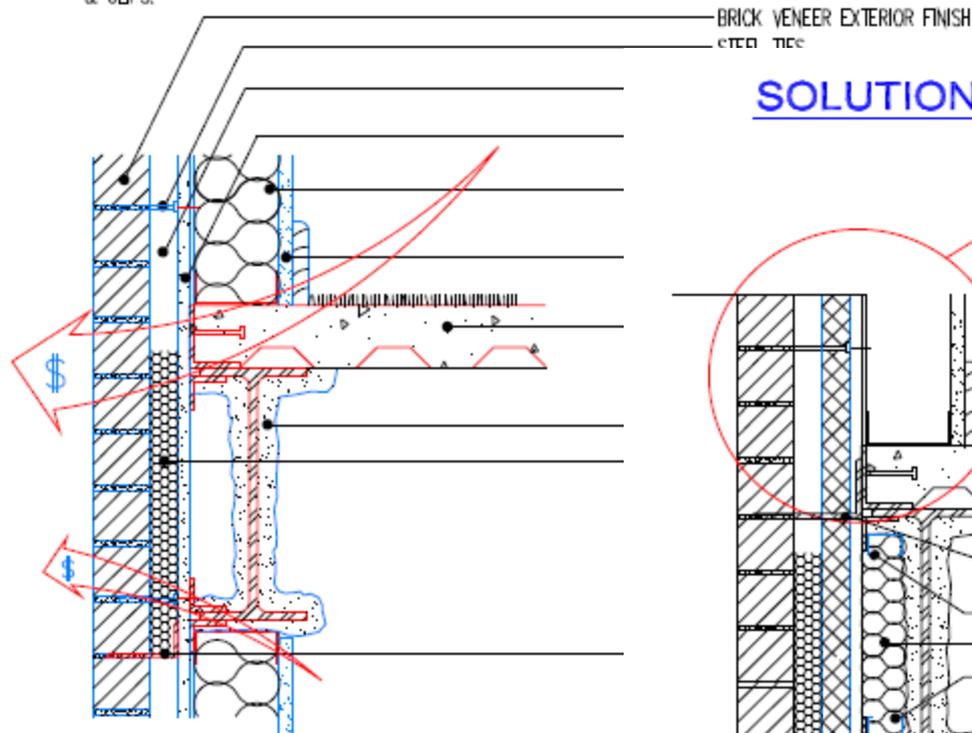
AT EDGE OF CONCRETE FLOOR  
& CLIPS.



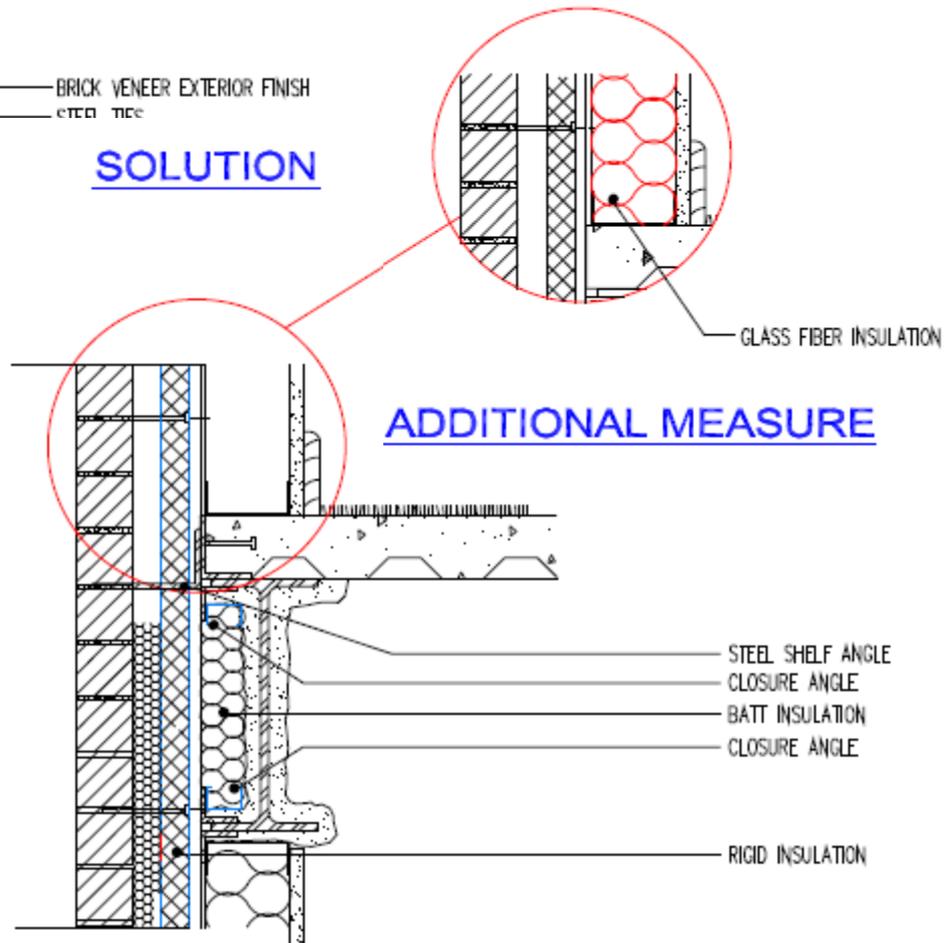
# Slab edge brick/stud wall

## PROBLEM

**THERMAL BRIDGE:**  
AT EDGE OF CONCRETE FLOOR  
& CLIPS.



## SOLUTION



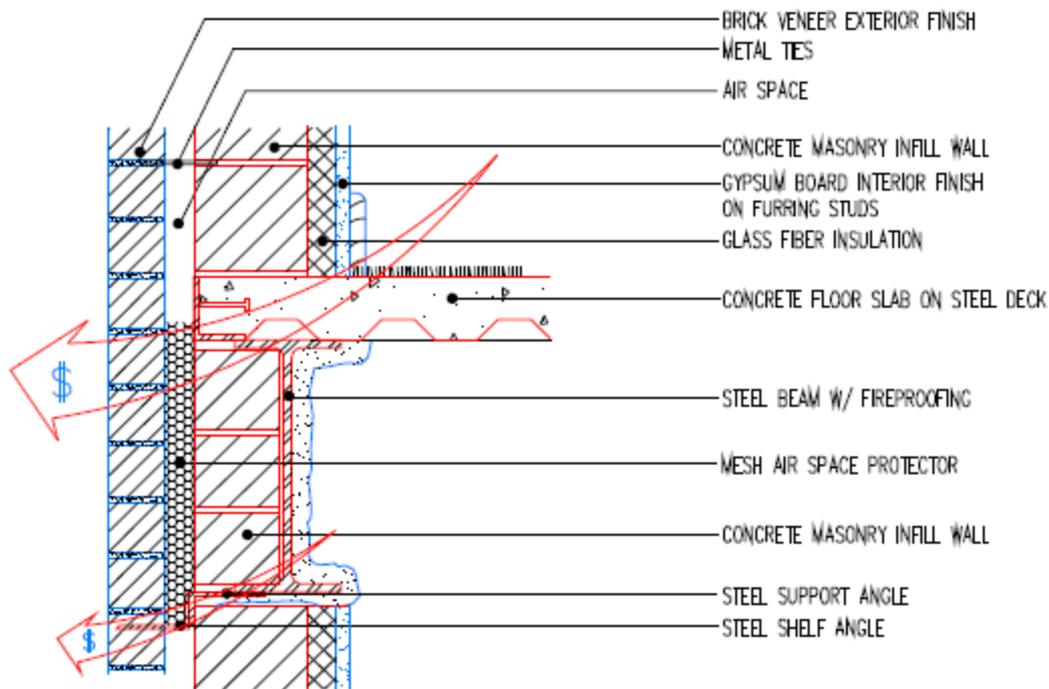
## ADDITIONAL MEASURE

STEEL SHELF ANGLE  
CLOSURE ANGLE  
BATT INSULATION  
CLOSURE ANGLE  
RIGID INSULATION

# Slab edge masonry wall

## PROBLEM

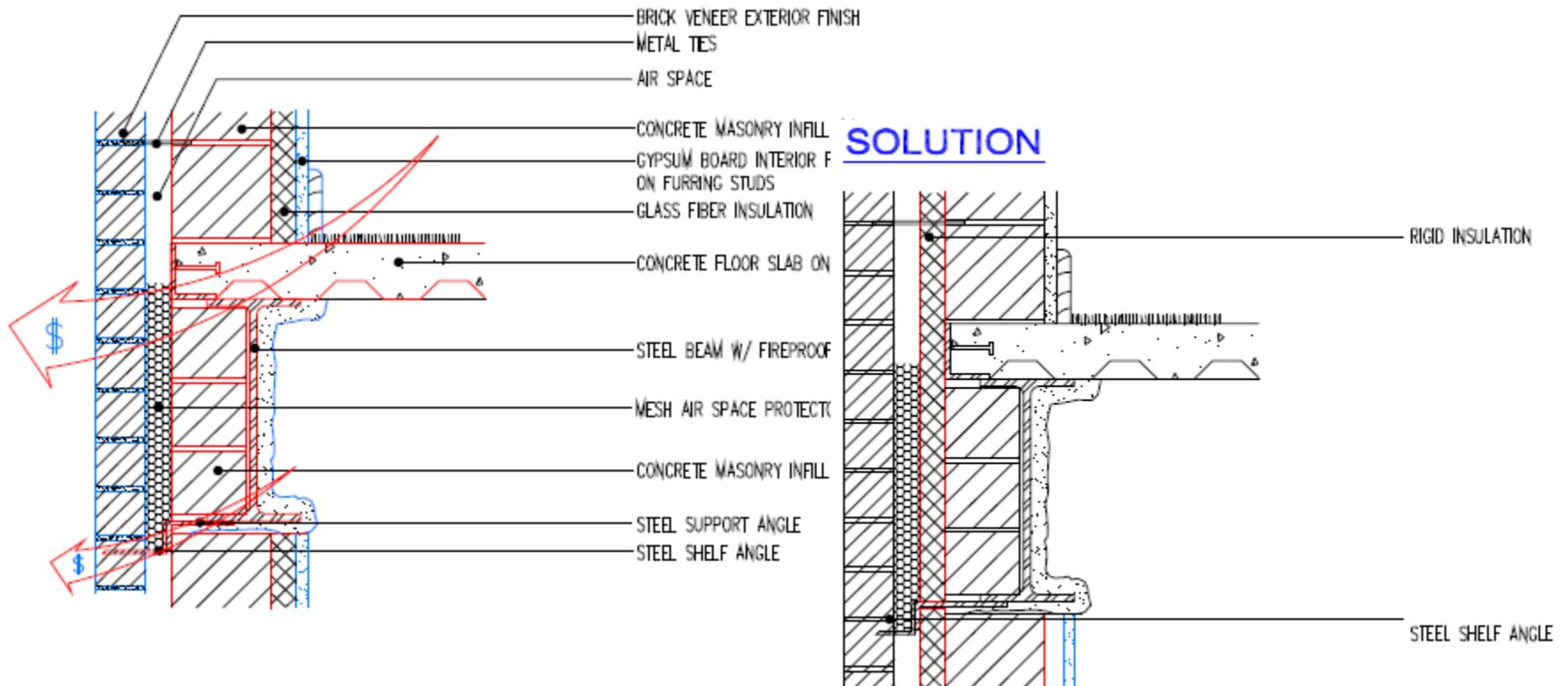
**THERMAL BRIDGE:**  
AT FLOOR SLAB EDGE &  
AND MASONRY INFILL



# Slab edge masonry wall

## PROBLEM

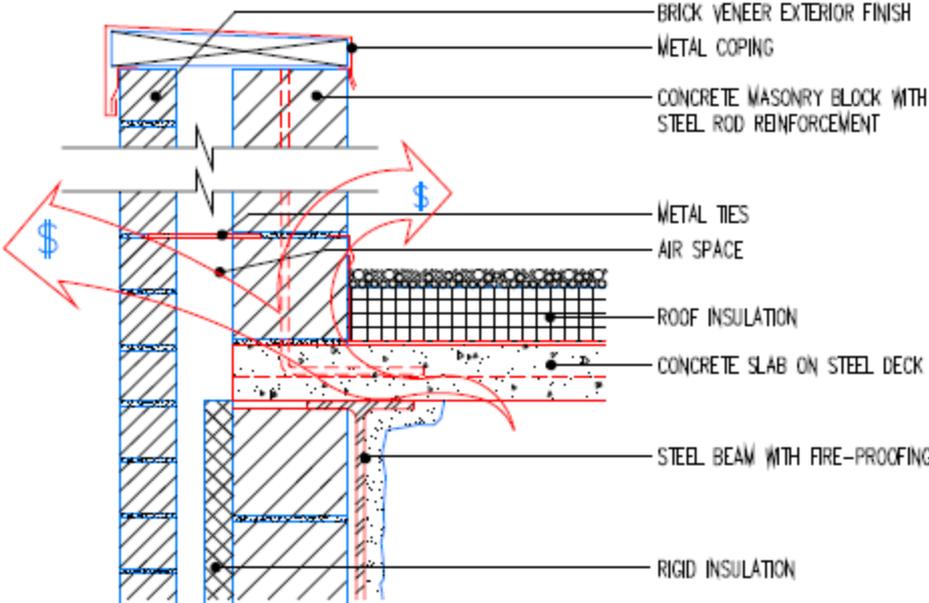
**THERMAL BRIDGE:**  
AT FLOOR SLAB EDGE &  
AND MASONRY INFILL



# Roof parapet

## PROBLEM

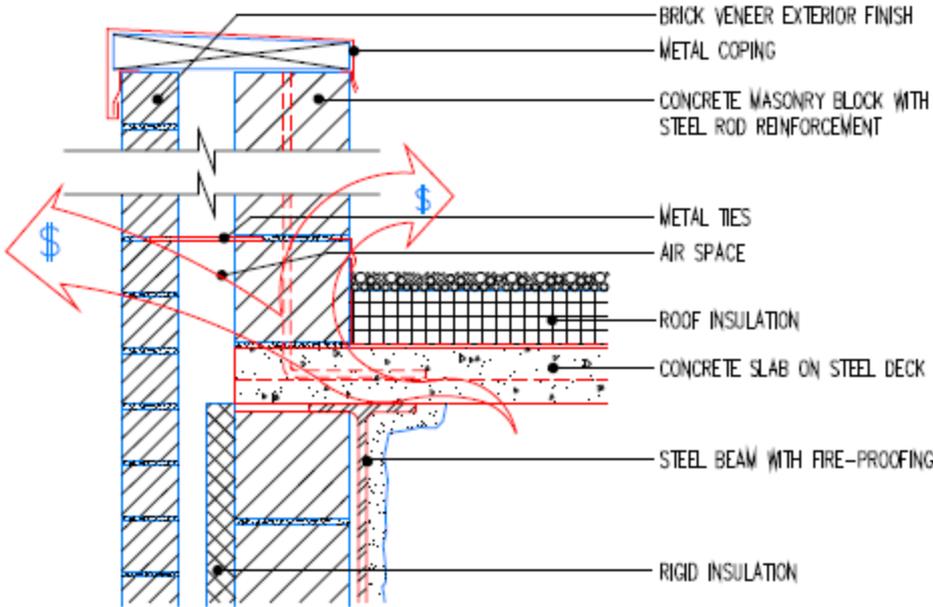
**THERMAL BRIDGE:**  
AT EDGE OF CONCRETE ROOF DECK



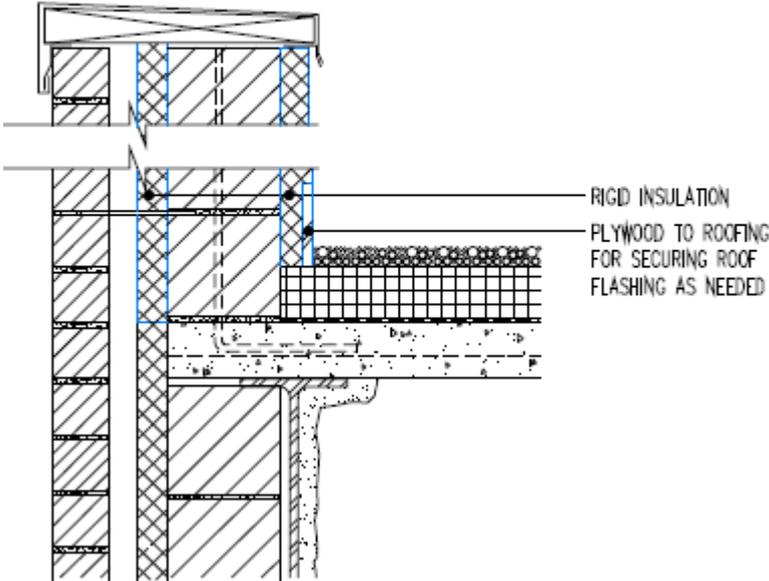
# Roof parapet

## PROBLEM

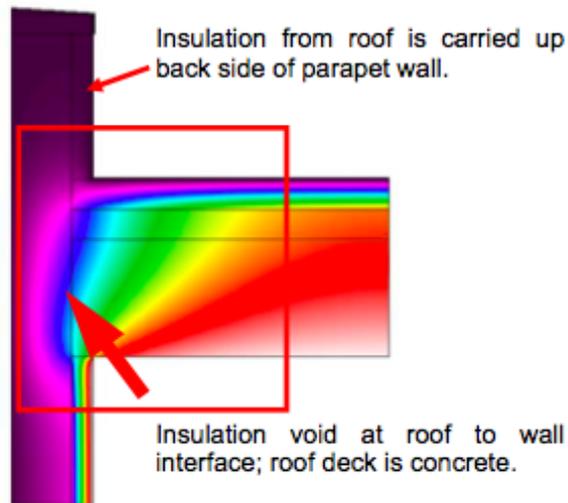
**THERMAL BRIDGE:**  
AT EDGE OF CONCRETE ROOF DECK



## SOLUTION



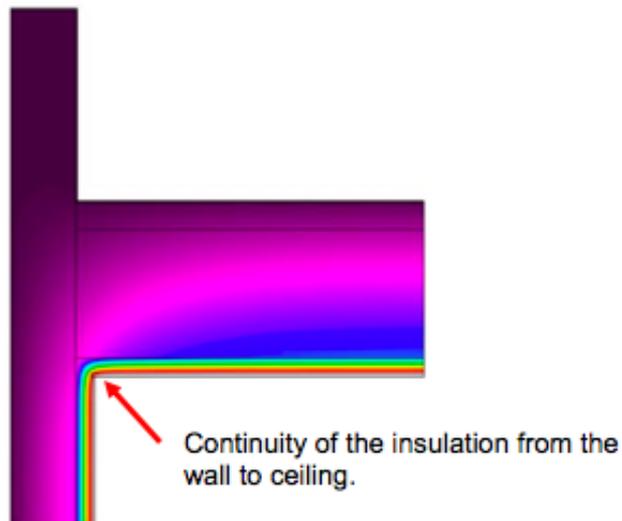
# Roof parapet



**Figure 2**

Results of a THERM model showing a thermal bridge at the roof to wall interface of a precast wall where roof insulation is not tied to wall insulation; although insulation now carries up the backside of the parapet wall, there is still no major effect on the bridge.

$$T_{\text{interior}} = 72^{\circ}\text{F}$$
$$T_{\text{exterior}} = 7^{\circ}\text{F}$$



**Figure 3**

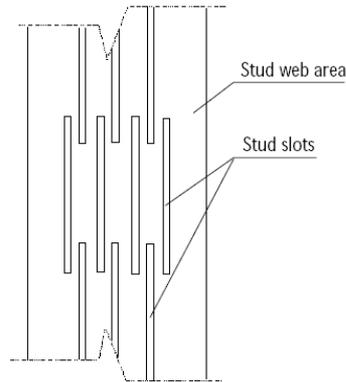
Results of a THERM model shows continuity of insulation below the roof deck; another option not shown here is to carry the wall insulation up by the roof deck and tie it into the roof insulation; structural considerations must be evaluated for this option. Window head below either option also requires careful evaluation.

$$T_{\text{interior}} = 72^{\circ}\text{F}$$
$$T_{\text{exterior}} = 7^{\circ}\text{F}$$

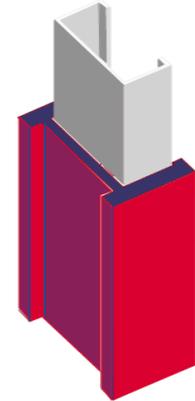
# Reducing framing effects: Modified steel studs

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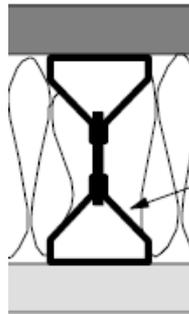
Slotted Studs



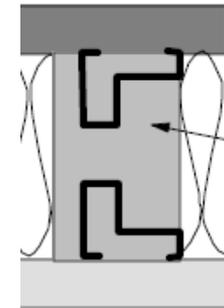
Insulated Studs



Triangular Studs



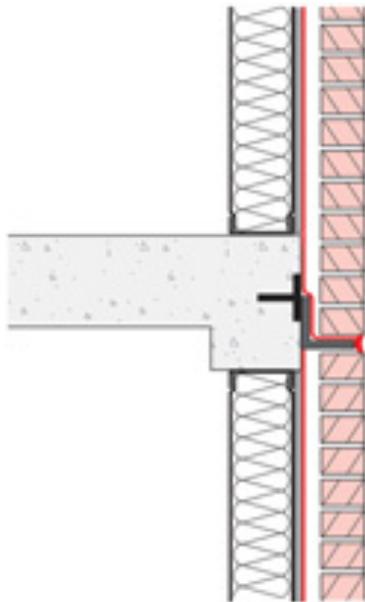
Small Studs + Foam Binder



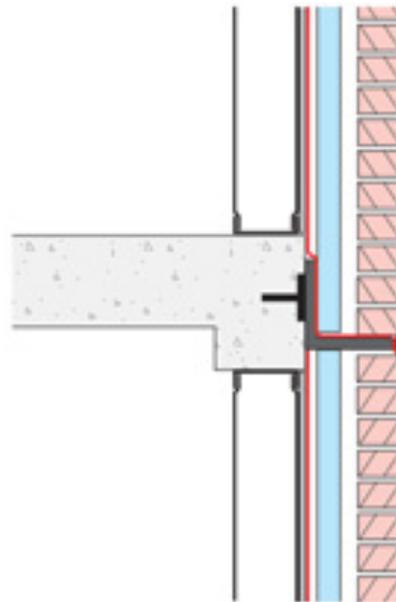
AISI/DOE report from 2003 on alternative steel options with better insulation:  
<http://steeltrp.com/finalreports/finalreports/9703NonPropFinalReport.pdf>

# Exterior wall connections

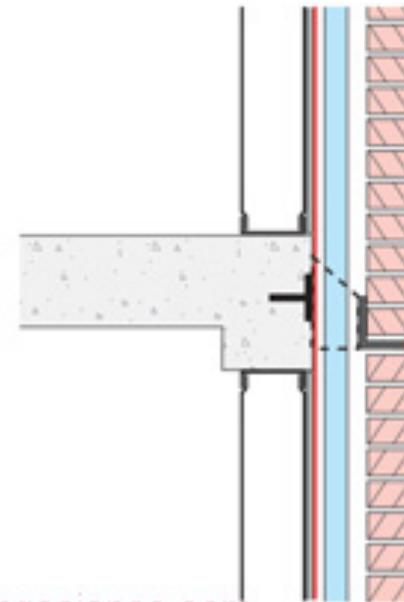
- Exterior veneer connections can also be significant thermal bridges
  - Here are three different design examples:



“The Ugly”



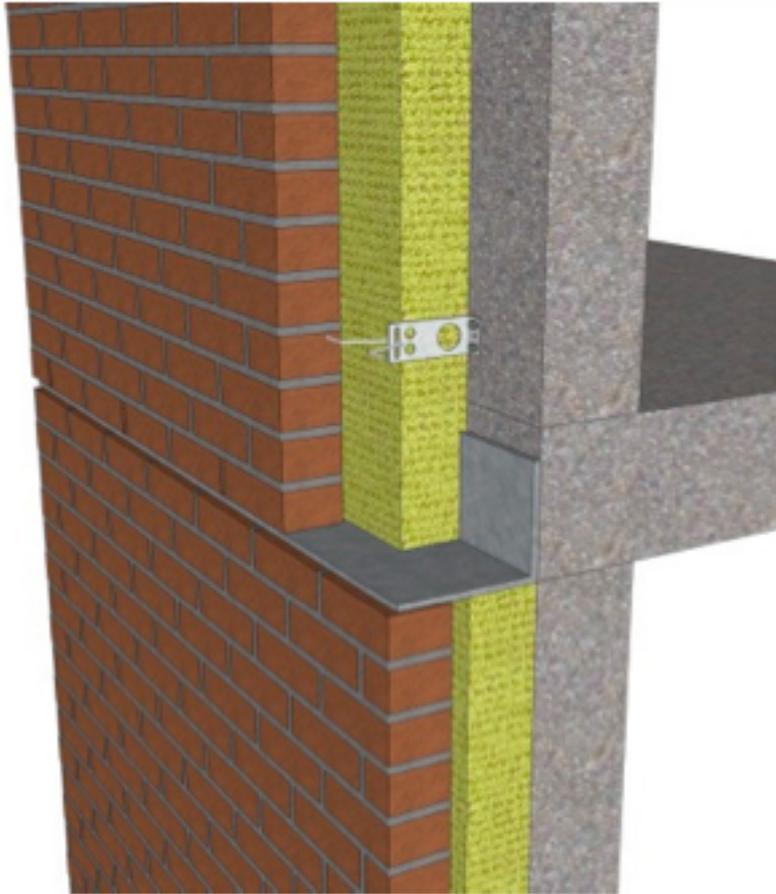
“The Bad”



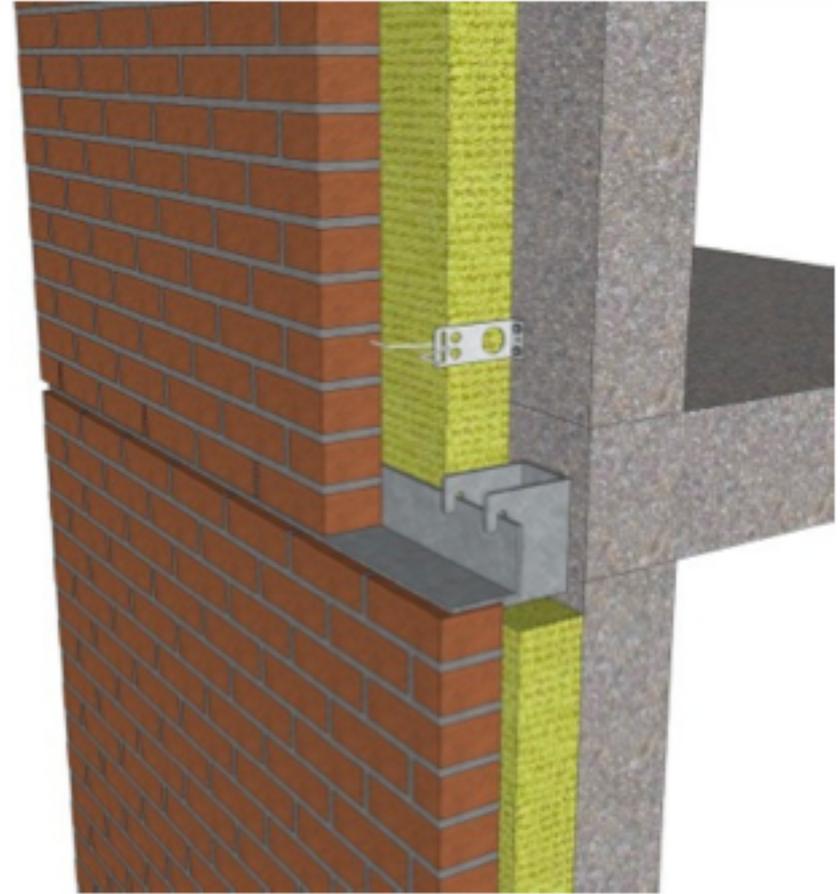
“The Good”

© buildingscience.com

# Thermally improved exterior wall connections



*Standard slab attached shelf angle*



*Intermittent shelf angle support*

# Ultimate thermal bridge?

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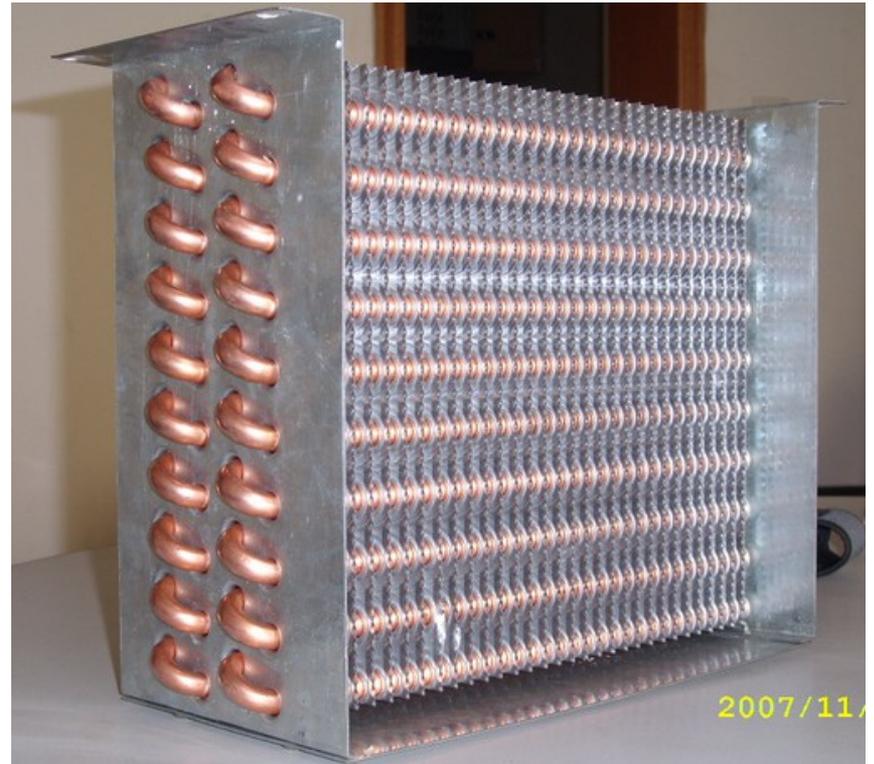


- This cantilevered balcony design provides a thermal connection from the interior to an exterior heating/cooling fin
- The balcony makes a fantastic heat exchanger!

# Ultimate thermal bridge?



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Balconies can act as a heat exchanger – heat exchangers are specifically designed to use fins to increase surface area for heat transfer (increasing UA)

# What about this building?



# Aqua Tower

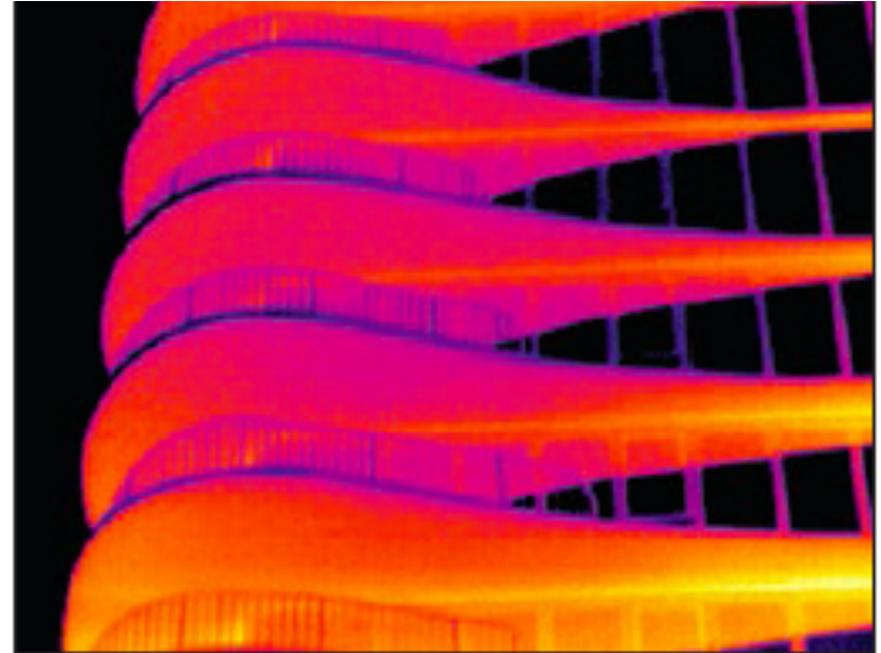
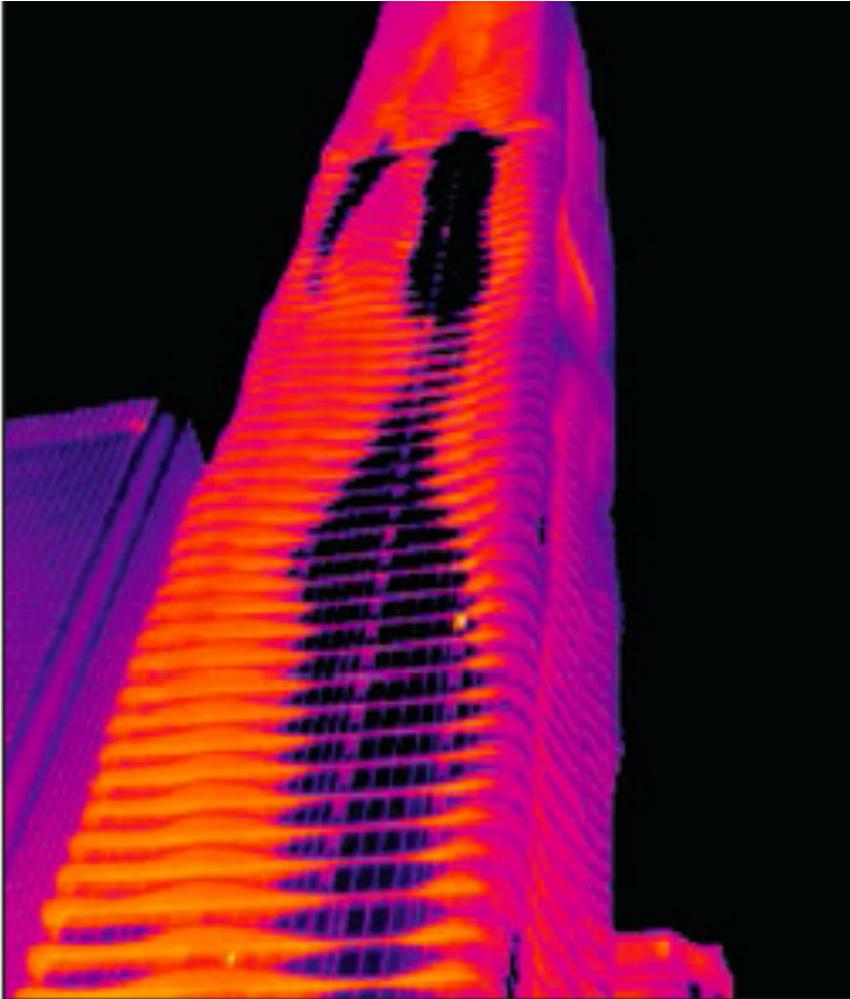
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- Designed by Jeanne Gang, of Studio Gang
- This design has won many awards but it consists of many cantilevered concrete slabs
- They did NOT use insulated cantilevers
  - Used “insulating paint” which isn’t insulating – it just changes the reflectivity/emissivity/absorptivity
  - May help solar gains in summer but does nothing for conduction, particularly in the winter



# “Thermal Bridge Redux” by J. Lstiburek, ASHRAE Journal July 2012

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**Photo 1f (left):** Infrared of Aqua Tower.  
**Photo 1g (right):** Infrared of Aqua Tower Balcony.

Thermographic images courtesy of Dave Robley, Thermographer, Fluke

# Solution 1: Hanging balconies

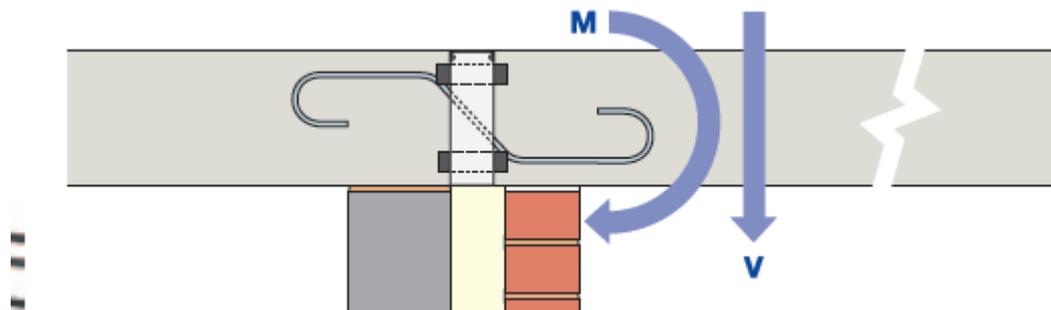
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- These balconies are precast concrete that is connected with offset point supports and tie rods
- A foam thermal break is easily incorporated into this design
- Of course, this wouldn't work for the Aqua aesthetic

## Solution 2: Insulated cantilever

- Insulated connectors for balcony cantilevers are now available on the market and used throughout Europe
- Search for: Ancon, Egccobox, Halfen



## Solution 2: Insulated cantilever

k (carbon steel) ~36-54 W/mK

k (stainless steel) ~16-24 W/mK



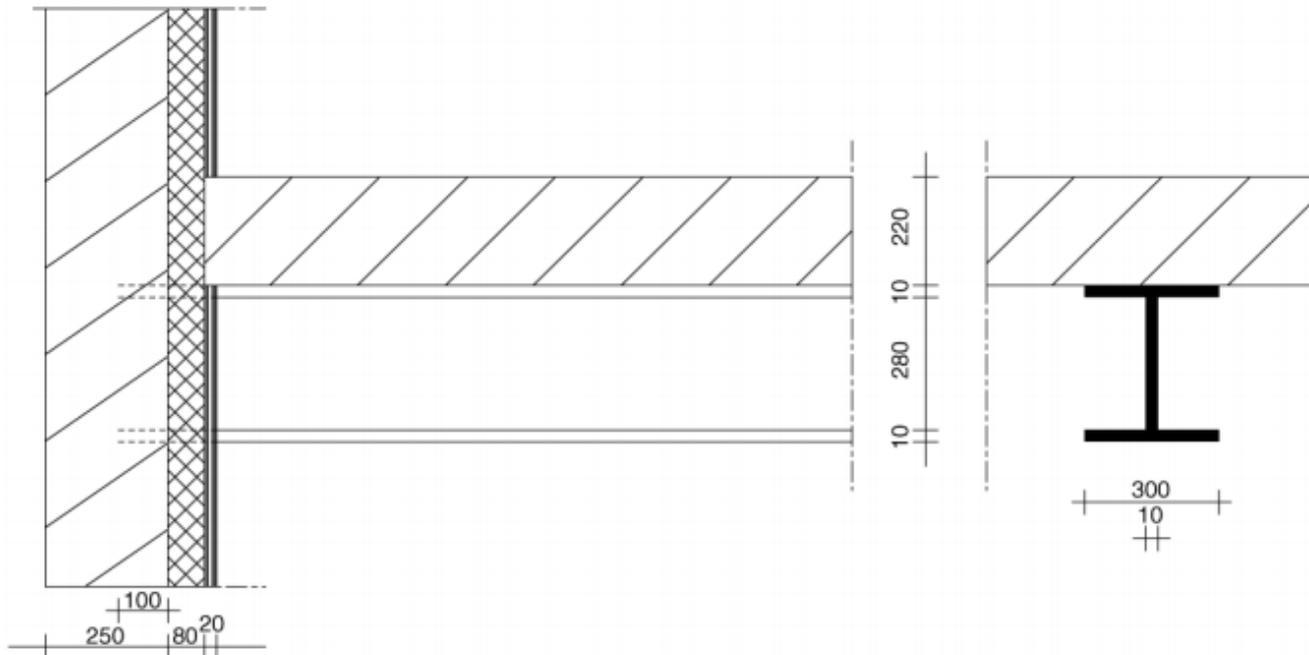
**Photo 2:** Premanufactured Thermal Break. High density graphite enhanced expanded polystyrene. Note the reinforcing rods penetrating the foam are stainless steel not carbon steel. Stainless steel has less than half the thermal conductivity of carbon steel. Neat, eh? Image courtesy of Schoeck Canada, Inc.



**Photo 3:** Belgrade Balcony. Nice view of the Danube River from a thermally broken balcony. OK, the river is out there somewhere. I have not been to Serbia recently, but apparently they can afford more efficiency than Chicago. Image courtesy of Beodom, Inc., Belgrade, Serbia.

# Thermal analysis of these even more complex geometries

- How do we estimate U and R values for complex geometries and combinations of materials like this one?



- Simple 1-D calculations can have significant errors
  - Hard (or impossible) to capture all phenomena
- Need to model 2-D or 3-D heat transfer using computer simulations

# **2-D AND 3-D HEAT TRANSFER USING NUMERICAL SOLVERS**

# Numerical solutions

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- In addition to our analytical solution methods, there is another way to solve these problems:
  - Numerical analysis using computers
- For assemblies with thermal bridges, it is probably a better solution to utilize finite-element or similar heat transfer software to estimate the U-value, R-value, and temperatures in and around the assembly

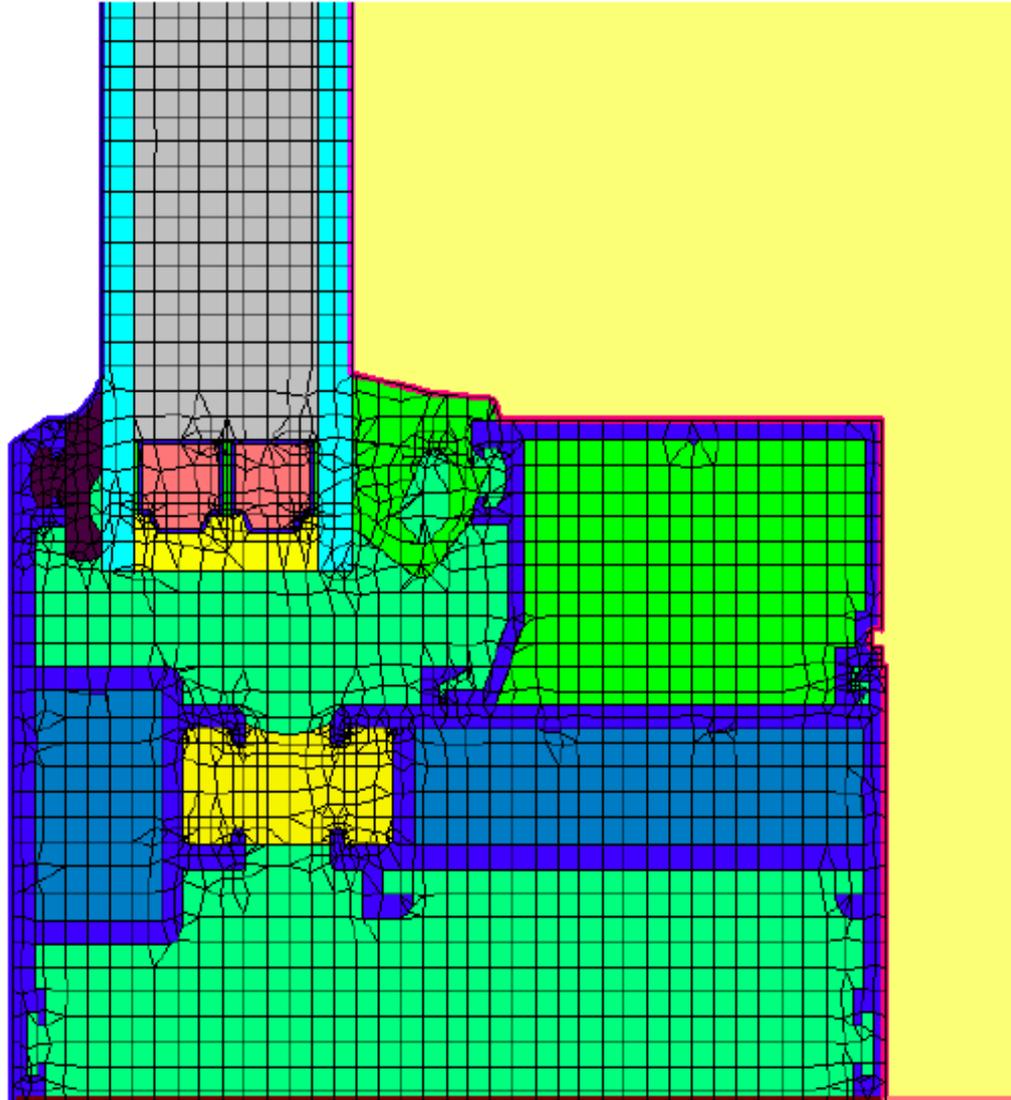
# Basic idea

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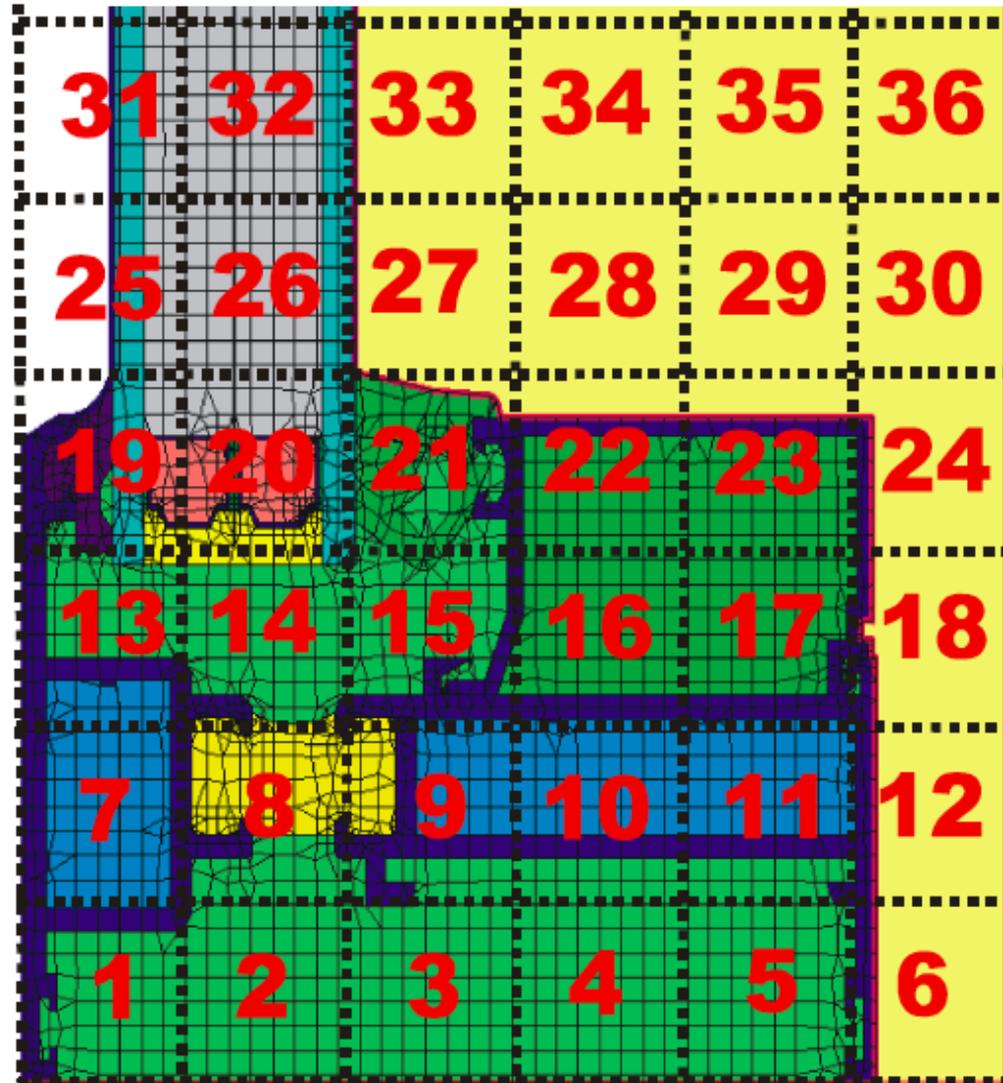
- Break assembly into a hundreds or thousands of homogenous elements
- Use the basic equations of heat transfer and write heat balances on each element to create a huge set of simultaneous equations
- Solve the simultaneous equations numerically to find heat flow and temperatures throughout the system

# Grid for numerical solver

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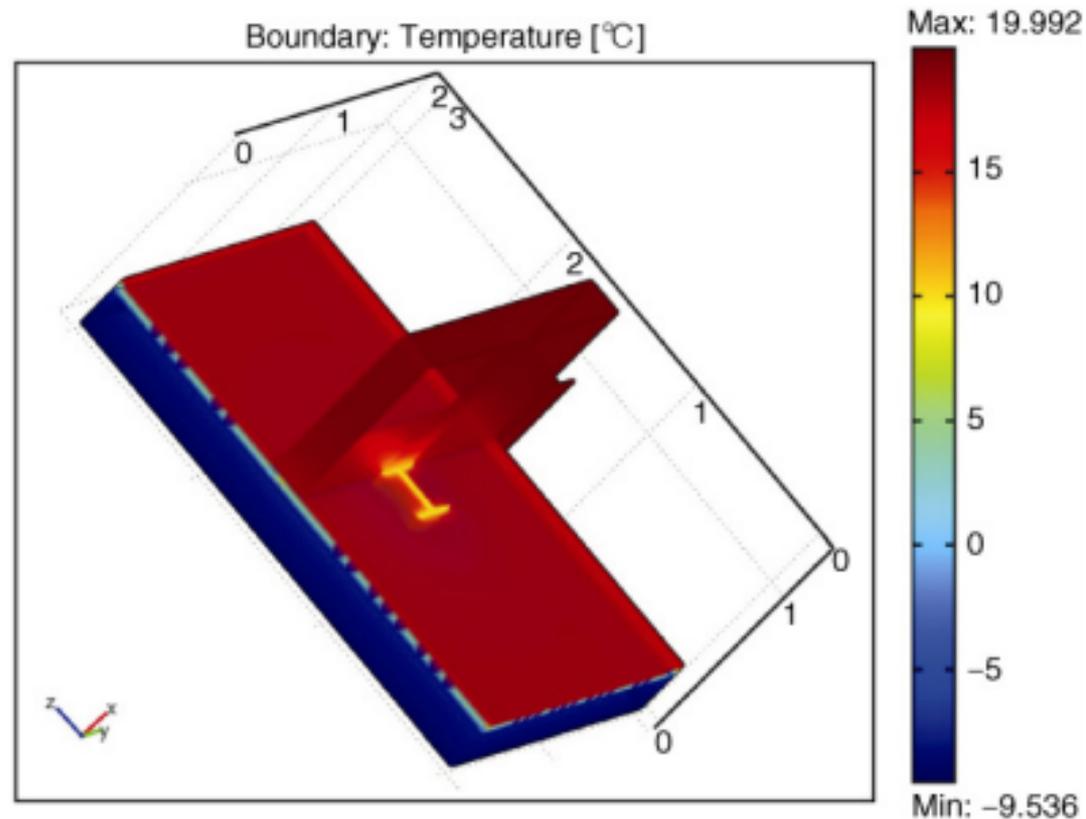
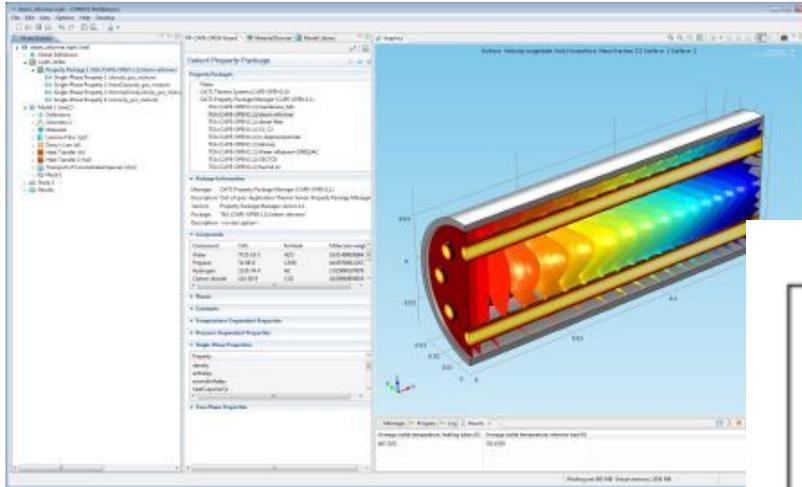


# Grid for numerical solver



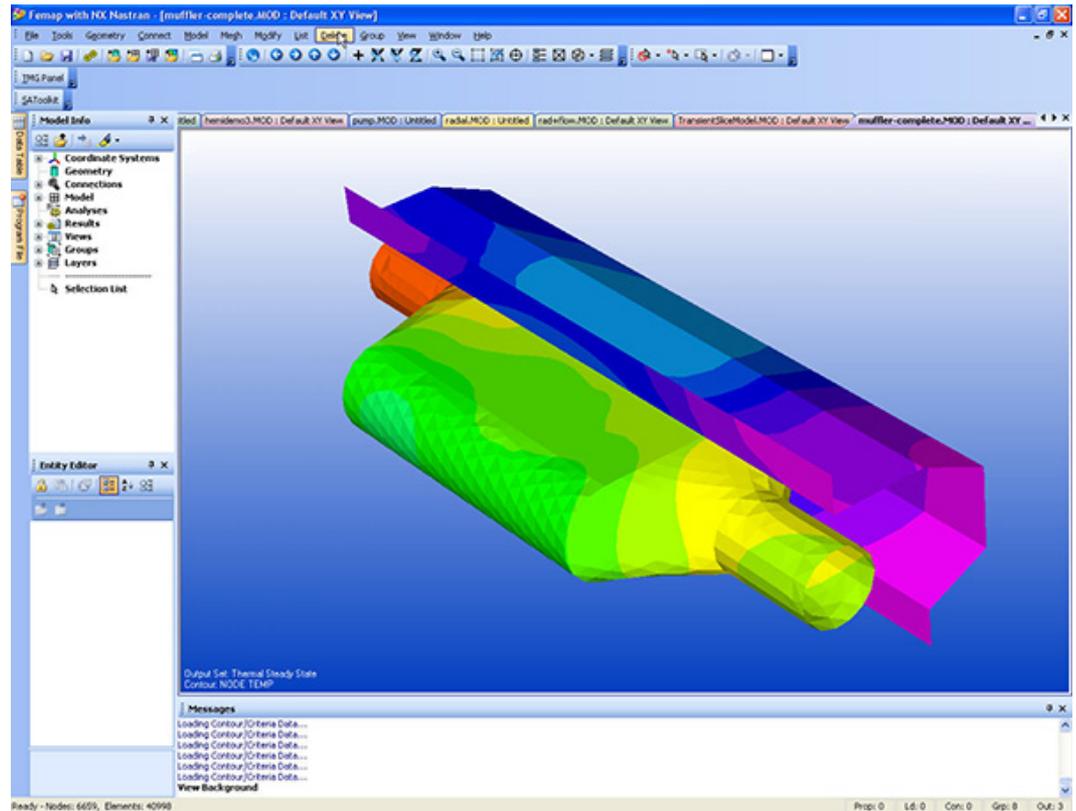
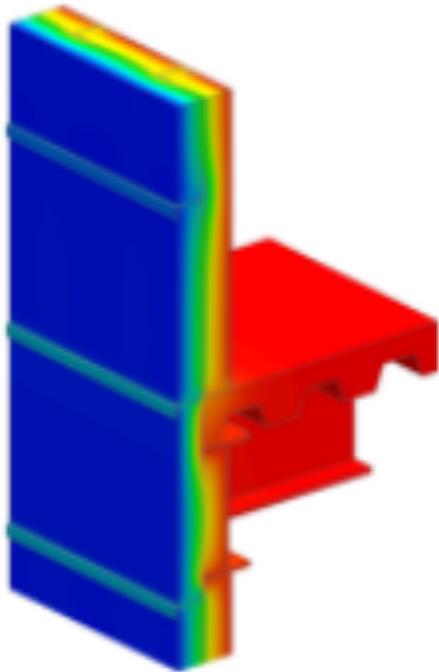
# 3-D solvers

- COMSOL finite element solver



# 3-D solvers

- Femap finite element analysis



# THERM – popular 2-D solver

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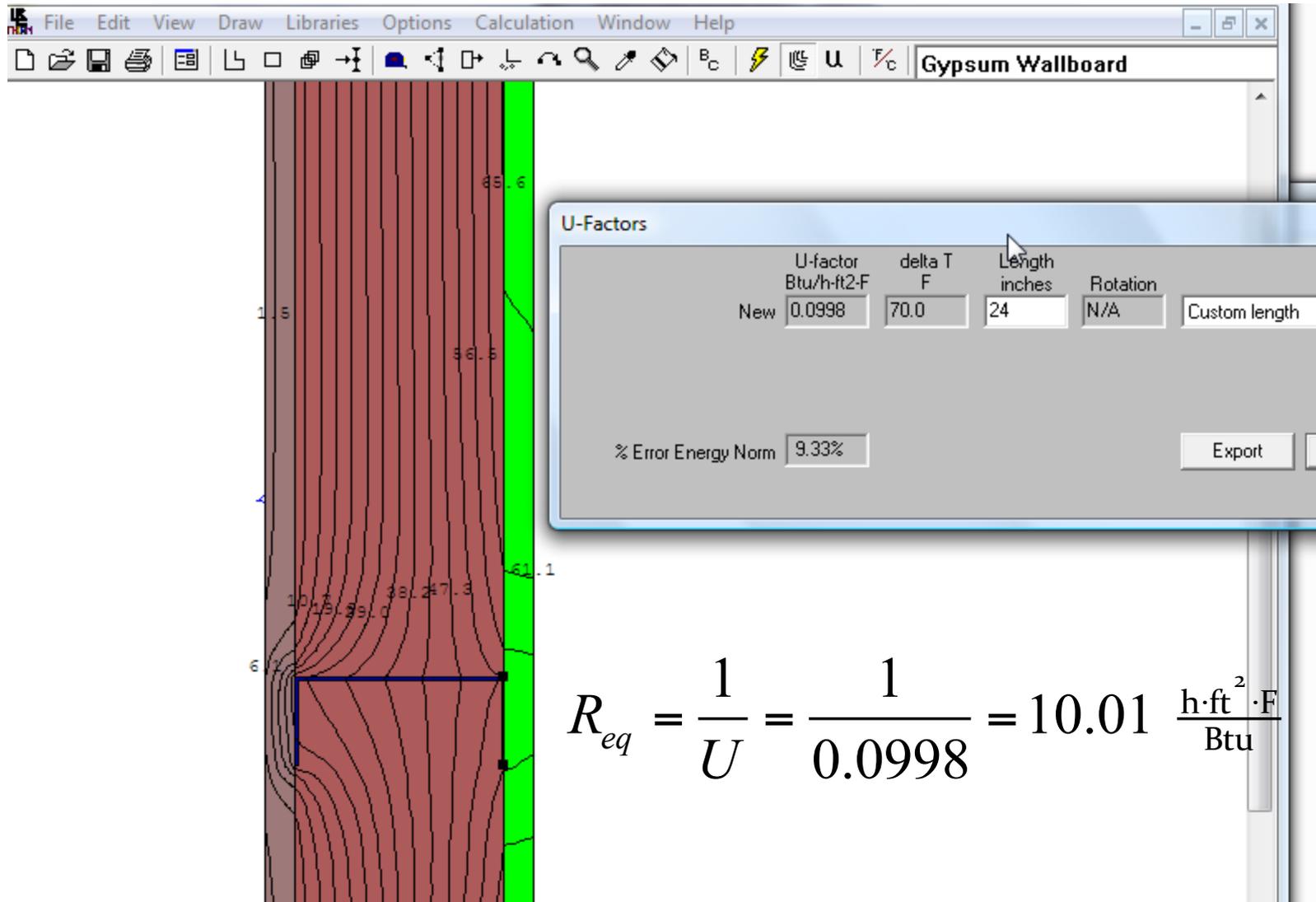
- A very popular 2-D heat transfer program used in the US – especially for window analysis - is called THERM
  - URL to download: <http://windows.lbl.gov/software/therm/therm.html>
  - Requires registration but is FREE
- THERM is used with WINDOW for designing windows
  - But can also be used as a stand alone program for any assembly
  - Very similar 2-D and 3-D programs often used in Europe are called heat2 and heat3
- You will use THERM on a HW
  - You can download from my website:  
<http://built-envi.com/wp-content/uploads/2012/09/THERM63Setup.exe>
  - Only runs on Windows ☹
  - It is also installed in the AM 218 computer lab

# Using THERM

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- In THERM you “draw” out an assembly
  - Assign materials
  - Assign boundary conditions
  - And let the program solve for the temperatures and heat flow throughout the assembly
  
- The program then can analyze that output to calculate a U value for the entire 2-D assembly
  - You can then calculate  $R = 1/U$  for the assembly

# Example screenshot: THERM analysis of steel stud



$$R_{eq} = \frac{1}{U} = \frac{1}{0.0998} = 10.01 \frac{\text{h}\cdot\text{ft}^2\cdot\text{F}}{\text{Btu}}$$

# Governing equations for THERM

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## 1-D Conduction

$$q_{cond} = -k \frac{dT}{dx} = -\frac{k}{L} dT$$

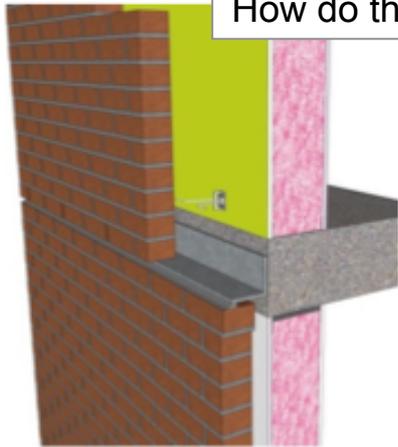
## 2-D Conduction

$$q_{cond} = -k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

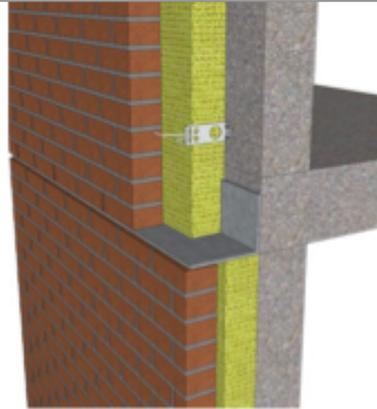
- Energy balance at a surface of each discretized element is the same as previously, except the conduction term contains X and Y components
- Set boundary conditions on interior and exterior and the solver will compute temperature throughout assembly
  - Then it can calculate a U-value for the whole assembly

# What you can do with THERM

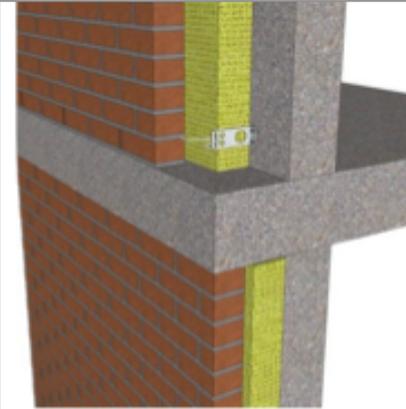
How do thermal bridges affect R-values of typical exterior wall connections?



Steel Stud Backup Wall

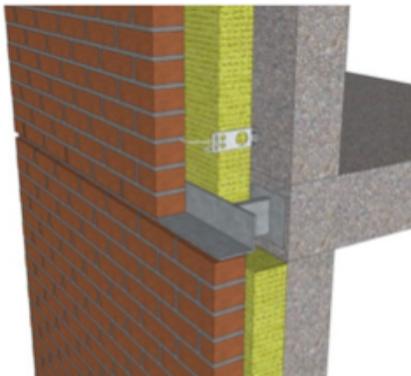


Concrete Backup Wall

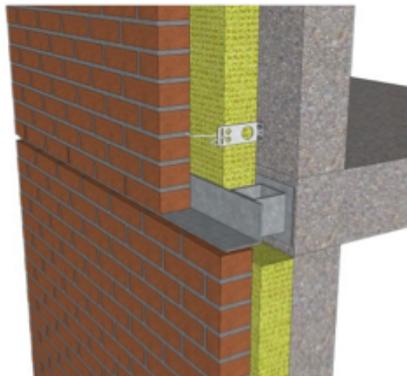


Exposed Concrete Slab

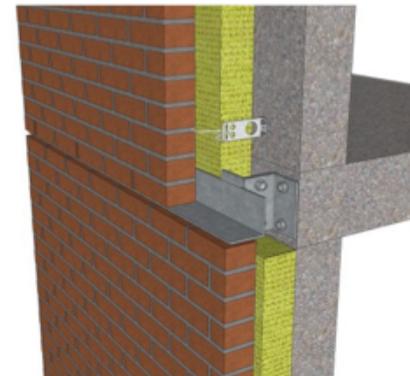
Knife Plate



HSS Structural Section



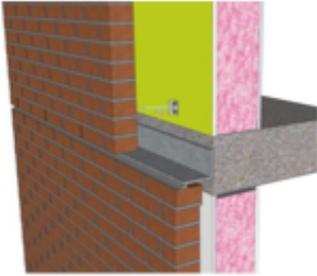
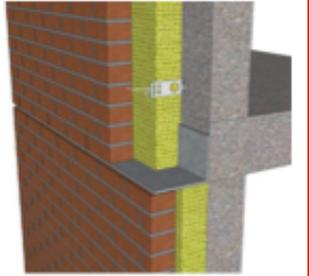
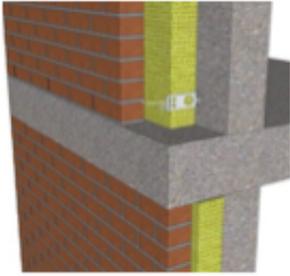
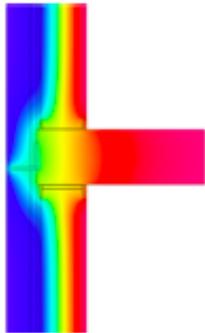
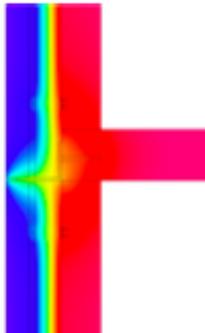
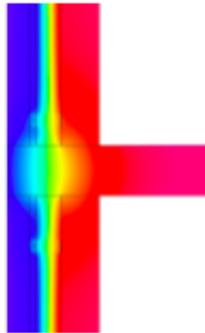
Overlapping Angles



And how do improved connections improve thermal performance?

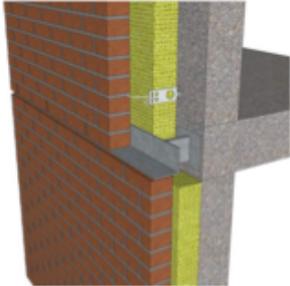
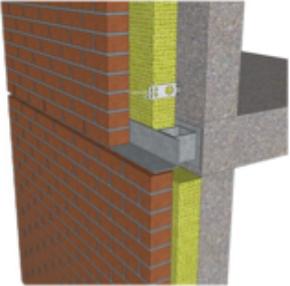
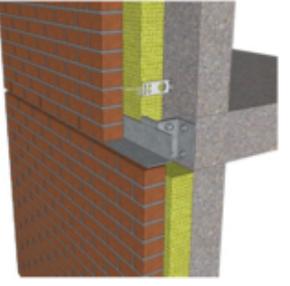
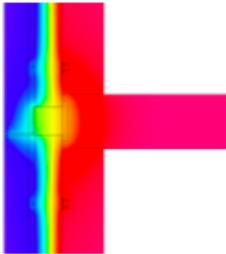
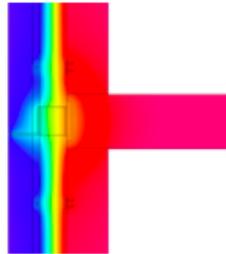
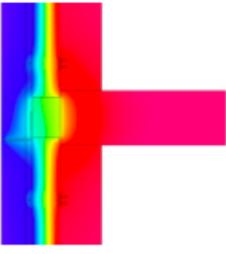
# What you can do with THERM

**Table 1: Summary of Nominal and Effective R-Values and U-Values for Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging**

	Steel Stud Backup	Poured Concrete Backup	Exposed Slab Edge
			
			
Nominal Insulation R-Value/U-Value	R-20 (RSI 3.52) U-0.05 (USI 0.284)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
Effective Assembly R-Value/U-Value	R-7.3 (RSI 1.29) U-0.137 (USI 0.777)	R-10.5 (RSI 1.84) U-0.096 (USI 0.543)	R-9 (RSI 1.58) U-0.112 (USI 0.634)
Effective Reduction	63.5%	37.5%	46.4%
Linear Transmission	-	$\psi = 0.339$ IP (0.586 SI)	$\psi = 0.478$ IP (0.827 SI)

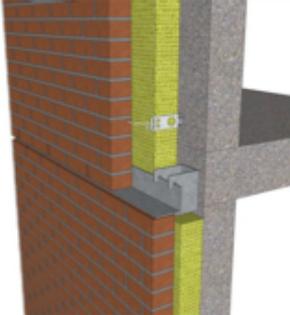
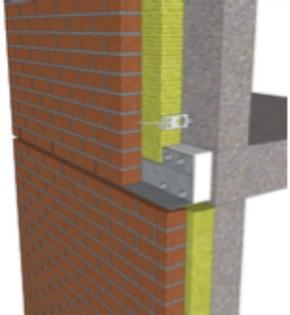
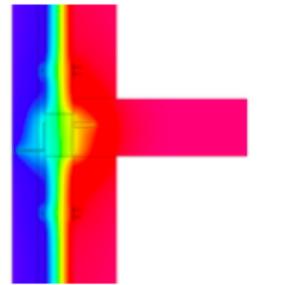
# What you can do with THERM

**Table 2: Summary of Nominal and Effective R-Values and U-Values for Typical Stand-Off Modifications to the Baseline Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging**

	Knife Plate	HSS Structural Section	Overlapping Angles
			
			
	shelf angle: 4"x4"x1/4" outside of insulation. 4"x4"x3/4" stand-off knife plates welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 4"x4"x1/4" HSS tube welded to embed plates at 48" o.c.	shelf angle 4"x4"x1/4" outside insulation. 2-6"x4"x5/16" angles bolted to slab edge at 48" o.c.
<b>Nominal Insulation R-Value/U-Value</b>	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
<b>Effective Assembly R-Value/U-Value</b>	R-14.8 (RSI 2.6) U-0.068 (USI 0.384)	R-14.8 (RSI 2.6) U-0.068 (USI 0.385)	R-15.0 (RSI 2.64) U-0.067 (USI 0.379)
<b>Effective Reduction</b>	16.4%	16.5%	15.3%
<b>Linear Transmission</b>	$\psi = 0.096$ IP (0.166 SI)	$\psi = 0.097$ IP (0.168 SI)	$\psi = 0.089$ IP (0.153 SI)

# What you can do with THERM

**Table 3: Summary of Nominal and Effective R-Values and U-values for Proprietary Masonry Shelf Angle Support Options Showing Impact of Thermal Bridging**

	Standoff Bracket	4-Bolt Cast-In
		
		
	shelf angle 4"x4"x1/4" outside insulation. Proprietary clip is 1/4" thick steel, 4"x4"x1/4" 6 "lg C-section. Non-welded connection. .	Shelf angle 4"x4"x1/4" outside insulation. Pre-manufactured cast-in place thermal break connection with 4 stainless steel bolts attached to 7"x7"x 3/8" plate.
<b>Nominal Insulation R-Value/U-Value</b>	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)	R-16.8 (RSI 2.95) U-0.060 (USI 0.339)
<b>Effective Assembly R-Value/U-Value</b>	R-14.9 (RSI 2.62) U-0.067 (USI 0.381)	R-16.4 (RSI 2.9) U-0.061 (USI 0.345)
<b>Effective Reduction</b>	16.4%	7%
<b>Linear Transmission</b>	$\psi = 0.091$ IP (0.158 SI)	$\psi = 0.037$ IP (0.064 SI)

# THERM demonstration

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