A note on long-wave radiation terms in our external surface energy balance equation
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There was some confusion in class about some of the long-wave radiation terms in our external surface energy balance equation. I had drawn from disparate sources to develop the equation below for an example roof surface:

$$\alpha I_{solar} + \varepsilon_{surface}\sigma F_{sky}(T_{sky}^4 - T_{surf}^4) + \varepsilon_{surface}\sigma F_{air}(T_{air}^4 - T_{surface}^4) + h_{conv}(T_{air} - T_{surface}) - U(T_{surface} - T_{surface,interior}) = 0$$

This energy balance accounted for, from left to right, incoming short-wave solar radiation, net long-wave radiation exchange between the roof surface and the sky, net long-wave radiation exchange between the roof surface and the ***air***, convection between the surface and the surrounding outdoor air, and conduction between the roof exterior surface and the interior surface of the roofing material. Terms from this equation were taken directly from the EnergyPlus Engineering Reference Manual.

The question raised in class was why do we include a surface-to-air long-wave radiation term? We previously only described long-wave radiation exchange in terms of surface-to-surface interactions, so why this surface-to-air interaction? And why is it treated separately from surface-to-sky interactions? Here I will attempt to find an answer for us.

The theory behind surface-to-air long-wave interactions stems from the idea that water vapor and other compounds such as CO_2 in the atmosphere have the ability to absorb long-wave radiation. And we understand surface-to-sky long-wave interactions as a surface on earth exchanging thermal energy with a big black body: outer space. But sometimes (often) clouds get in the way, which I think distorts our nomenclature of "air" vs. "sky."

Let's start by discussing further how EnergyPlus treats these things. As we described in class, the EnergyPlus Engineering Reference Manual first discusses how long-wave radiation heat flux on an external surface is a function of the exchange between the surface, the sky, and the ground. Then out of nowhere they describe a separate surface-to-air exchange that should be accounted for in addition to any ground and/or sky interactions:

$$q''_{LWR} = \varepsilon\sigma F_{gnd}(T_{gnd}^4 - T_{surf}^4) + \varepsilon\sigma F_{sky}(T_{sky}^4 - T_{surf}^4) + \varepsilon\sigma F_{air}(T_{air}^4 - T_{surf}^4)$$

They describe view factors for surface-ground radiation and for surface-sky radiation, as follows:

$$F_{ground} = 0.5(1 - \cos\phi)$$

$$F_{sky} = 0.5(1 + \cos\phi)$$

where ϕ is the tilt angle of the surface. I showed these in class. However, I failed to mention that they also describe the view factor between a surface and the ***air***, stating, "**The view factor to the sky is further split between sky and air radiation by:**"

$$\beta = \sqrt{0.5(1 + \cos\phi)}$$

Although it is unclear exactly what is meant by "split by", we can probably assume the following:
For a vertical surface, $\phi = 90°$, $\cos(90°) = 1$, $F_{sky} = 0.5(1 + 0) = 0.5$, as we showed in class. Thus, $\beta = (0.5)^{1/2} = 0.707$.

But how do we account for this? Do we multiply the sky view factor by 0.707 to get 0.5(0.707) = 0.35? What then would be the air view factor?

Digging deeper into the references that the EnergyPlus Engineering Reference Manual uses, we find some more helpful relationships. For example, in McClellan and Pedersen (1997), in a large review of every type of interaction on the energy balance of an exterior building surface, provide the following:

The longwave portion of the balance may be written (Walton 1983):

$$q''_{LWR} = \varepsilon\sigma[F_a(T_o^4 - T_{so}^4) + F_{sky}(T_{sky}^4 - T_{so}^4) + F_g(T_g^4 - T_{so}^4)] \quad (4)$$

where

ε	=	longwave emittance of the surface,
σ	=	Stefan-Boltzmann constant,
F_a	=	view factor of wall surface to air temperature,
F_{sky}	=	view factor of wall surface to sky temperature,
T_{sky}	=	sky temperature,
F_g	=	view factor of wall surface to ground surface temperature,
T_g	=	ground surface temperature.

Does that look familiar? It's the same equation for long-wave radiation exchange as EnergyPlus uses. (Note that T_o is the outdoor air temperature in this nomenclature). And it's derived from yet another reference, Walton (1983), which I haven't been able to acquire. It is a reference from NIST that apparently outlines the development of one of the first building energy models.

Moving along within this reference, we see the following:

Longwave Radiant View Factors, F_{sky} and F_g

Longwave radiant view factors are also somewhat case-dependent. Perhaps the simplest estimation of these parameters is to base them on the surface tilt, such that

$$F_{sky} = \left[\frac{(1 + \cos\Sigma)}{2}\right] \cdot \cos\left(\frac{\Sigma}{2}\right) \quad (29)$$

$$F_g = (1 - \cos\Sigma)/2$$
$$(F_a = 1 - F_{sky} - F_g). \quad (30)$$

This is the formulation used in the BLAST program (Walton 1983). However, if the building is part of an urban environment, other buildings may dominate the surface's radiant view. Therefore, view factors should also be approximated based on the building's surroundings.

In this case, Σ refers to a surface's tilt angle. For a vertical surface with $\Sigma = 90°$,
F_{sky} = [(1 + cos(90°))/2]×cos(90°/2) = [(1)/2]×cos(45°) = [0.5]×0.707 = 0.35. Same as above.
F_{ground} would still equal 0.5, and importantly, Fair would be found by subtracting F_{sky} and F_{ground} from 1 (Equation 30 in their work). Therefore:
$F_{air} = 1 - F_{ground} - F_{sky} = 1 – 0.5 – 0.35 = 0.15$
For a horizontal surface, $\Sigma = 0°$:
$F_{ground} = 0$
F_{sky} = [(1 + cos(0°))/2]×cos(0°) = [(1+1)/2]×cos(0°) = [1]×1 = 1
$F_{air} = 1 - F_{ground} - F_{sky} = 1 – 0 – 1 = 0$

Now, we have at least resolved our problem. The sum of ground, air, and sky view factors equals 1. But does this even make sense? When a surface is facing upwards it can only "see" the sky and not the air? Only when you tilt it can it "see" the air? In my opinion: not really.

But, for our purposes, this may work out fine. The view factor will never be greater than 0.15 for the air, as most surfaces lie somewhere between horizontal and vertical. So air-surface interactions are rarely very meaningful.

Most other building energy simulation programs simply ignore this. The TRNSYS 16 "Multizone Building modeling with Type56 and TRNBuild" manual deals only with long-wave radiation exchange between the surface and the ***sky***. DOE-2 only includes surface and ***sky*** interactions as well. Even I have traditionally ignored any differences between sky and air long-wave radiation, resorting only to using sky interactions (e.g., Susorova et al., 2013). And it's important to note that because air and sky temperatures track each other (depending on cloudiness as we've discussed in class), if we assumed all view factors for surface-air interactions were zero, the increased view factor for surface-sky interactions would still account for some level of surface-air exchange anyway. So it's kind of a moot point.

Therefore, for our purposes, you can either lump all air/sky radiation into one term with the sky temperature alone, or you can follow the procedure above and separate sky/air radiation into two terms using the two view factors. I have chosen to make changes in my spreadsheet that I demonstrated in class to account for both view factors.

I should also note that the real story is that there are also many other methods for accounting for long-wave radiation exchange between surfaces on the earth and the sky and/or air temperature. Methods of doing so depend largely on where you look. Perhaps a clearer way of dealing with differences in sky and air long-wave exchange should be to use a different nomenclature. For example, Choi et al. (2008) reviewed several models for estimating "downwelling longwave radiation" for both clear and cloudy sky days, and compared the models to experiments performed outdoors in several locations in Florida. Downwelling simply means they were focusing on incoming long-wave radiation, regardless of what surface it was exchanging with.

In their work, which is more grounded in atmospheric science than some of the equations we use, they start by describing incoming clear-sky long-wave radiation with an equation we're familiar with:

$$q_{LWR,incoming,clear} = \varepsilon_{air} \sigma T_{air}^4$$

where the atmospheric emissivity is estimated as a function of air temperature and vapor pressure (similar to some of the methods we described in class for "sky" emissivities). For us to describe exchange with a surface, we would also include a term for our surface exchanging back with the same air temperature.

To account for cloudy days, they simply adjust emissivity according to cloud cover and still deal with the same air temperature as before. Cloud cover can be estimated from visual observations, including taking images of the sky. Although there are several ways to account for cloud cover, most of the models take into account either cloud cover alone or cloud cover and air temperature. The simplest form takes cloud cover alone:

$$q_{LWR,incoming,cloudy} = q_{LWR,incoming,clear}\left(1+0.2c\right)$$

where c = fractional cloud cover.

So the incoming long-wave radiation during a cloudy day ($c > 0$) will be higher than on a clear day. Think about it this way: surfaces cool off in part by radiating their energy back to a very cold, dark outer space. Clouds block this ability and re-entrain some of that energy, or more accurately, reduce the surface temperature differential driving force.

Many other applications of long-wave radiation exchange equations also rely on adjustments of air temperatures without treating "sky" temperatures explicitly. That makes sense to me, as "sky" temperatures are really just empirical functions of air temperatures adjusted for cloud cover and water vapor presence. So at the end of the day, what's the difference?

References

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