# CAE 463/524 **Building Enclosure Design** Fall 2013

#### Lecture 3: September 4, 2013 - CORRECTED

**Energy balances** Solar orientation Conduction in building enclosures

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## Last time

- Review of building science
  - Psychrometrics
  - Modes of heat transfer

## **Today's objectives**

- Example calculations for single-mode heat transfer
- Bring all the heat transfer modes together
- Solar orientation and enclosures
- Assign HW #1

## Single-mode heat transfer examples

• Let's perform some example calculations, first treating conduction, convection, and radiation individually



# **BASIC HEAT TRANSFER THROUGH BUILDING ENCLOSURES**

## **Example 2.1: Single-layer conduction**

- A 2 m wide, 3 m high, and 50 mm thick piece of extruded polystyrene material has a surface temperature of 20°C on one side and 40°C on the other
  - a) Calculate steady state heat flow rate and heat flux
  - b) Calculate conductance (U-value)
  - c) Calculate resistance (R-value)

#### ASHRAE HOF (2005 Ch. 25):

Table 4 Typical Thermal Properties of Common Building and Insulating Materials-Design Values<sup>a</sup> (Continued)

|   |                               |  |  | Resista                  |   |                                |
|---|-------------------------------|--|--|--------------------------|---|--------------------------------|
| Description   | Density,<br>kg/m <sup>3</sup> | Conductivity <sup>b</sup><br>(k),<br>W/(m·K) | Conductance<br>(C),<br>W/(m <sup>2</sup> ·K) | 1/k,<br>( <b>m</b> ·K)/W | For Thickness<br>Listed (1/C),<br>(m <sup>2</sup> ·K)/W | Specific<br>Heat,<br>kJ/(kg·K) |
| Expanded polystyrene, extruded (smooth skin surface)<br>(HCFC-142b exp.) <sup>h</sup> | 29-56                         | 0.029  | _  | 34.7                     | _   | 1.21                           |

## A note on insulation materials

- All materials in an enclosure assembly will have some resistance to heat transfer
- Materials with thermal conductivities (k) less than about 0.05 W/mK are used specifically for insulation
  - 0.05 W/mK divided by 3-inches of typical thickness (0.076 m) yields U-value of ~0.66 W/m<sup>2</sup>K
  - $R = 1/U = 1/0.66 = -1.5 m^2 K/W RSI (or -R-9 in English units)$

| Example from       | Specification<br>Compliance | R-Value<br>(hr•ft²•°F/Btu) | RSI-Value<br>(m²•°C/Watts) | Thick<br>(in) | ness**<br>(mm) |
|--------------------|-----------------------------|----------------------------|----------------------------|---------------|----------------|
| product literature | ASTM C 665                  | 38c                        | 6.7                        | 10 1⁄4        | 260            |
|                    | Kraft-Faced                 | 38                         | 6.7                        | 13            | 330            |
|                    | Type II, Class C            | 30c                        | 5.3                        | 81⁄4          | 210            |
|                    | Category 1                  | 30                         | 5.3                        | 10 1⁄4        | 260            |
|                    |                             | 25                         | 4.4                        | 81⁄2          | 216            |
|                    |                             | 22                         | 3.9                        | 7 1⁄2         | 191            |
|                    |                             | 21                         | 3.7                        | 51⁄2          | 140            |
|                    |                             | 19                         | 3.3                        | 6½            | 165            |
| Johns Manville     |                             | 15                         | 2.6                        | 3½,3%         | 89, 92         |
|                    |                             | 13                         | 2.3                        | 3½,3%         | 89, 92         |
|                    |                             | 11                         | 1.9                        | 31⁄2,35⁄8     | 89, 92         |

#### AVAILABLE FORMS\*

## Another note on insulation materials

- Still air is also a low-cost insulator
  - Density ~1.2 kg/m<sup>3</sup>
  - Conductivity,  $k \sim 0.03$  W/mK
  - So many insulation materials rely on creating air voids
- Example: fiberglass insulation
  - Glass, with a density of 2500 kg/m<sup>3</sup> and k = 1 W/mK, is spun into fibers and made into a fiberglass insulation batt, which is ~99.4% air voids (~0.6% glass fibers) by volume
    - Yields a product with a density of 16 kg/m<sup>3</sup> and thermal conductivity of 0.043 W/mK
    - Both values are very close to that of still air

## **Example 2.2: Convection**

- The interior face of an insulated exterior enclosure wall 2.4 m wide and 2.4 m high is 3°C cooler than the indoor air (T<sub>indoor</sub> = 21°C)
  - a) Calculate convective heat transfer coefficient at the face
  - b) Calculate rate of convective heat transfer



Figure 5.4: Natural convection coefficients for laminar flow

## Example 2.3: Bulk convection

- An 800 m<sup>3</sup> building has an outdoor air exchange rate of 0.5 air changes per hour. The outdoor temperature is 35°C. The indoor air temperature is 20°C.
  - a) Calculate the rate at which heat is added to the indoor air from outdoors

## **Example 2.4: Radiation**

- Interior surfaces of two perpendicular walls (both are 2.4 m by 2.4 m) are 3°C different from each other. One is at 294 K, the other at 291 K. They both have an emissivity of 0.90.
  - a) Calculate the rate of radiative heat transfer between the two surfaces
  - b) What if the emissivity of one surface decreases to 0.1?



### **Combined heat transfer**

- In some cases, heat transfer from a surface is dominated by either convection or radiation
  - In many cases both are about the same magnitude
- In cavities (window spaces, wall cavities, crawl spaces) this is usually the case
  - So, heat transfer is fairly complicated
- We need to be able to describe all heat transfer mechanisms acting on each surface of an enclosure to understand how the enclosure affects heat, air, and moisture performance

## Bringing all the modes together

Once you have this

• Exterior surface example: roof



## Bringing all the modes together

• Exterior surface example: roof

 $\sum q = 0$ 

We can use this equation to estimate indoor and outdoor surface temperatures At steady state, net energy balance is zero

 Because of T<sup>4</sup> term, often requires iteration

Solar gain

Surface-sky radiation

Surface-air radiation

Convection on external wall Conduction through wall  $\alpha I_{solar}$   $+\varepsilon_{surface}\sigma F_{sky}(T_{sky}^{4} - T_{surf}^{4})$   $+\varepsilon_{surface}\sigma F_{air}(T_{air}^{4} - T_{surface}^{4})$   $+h_{conv}(T_{air} - T_{surface})$   $-U(T_{surface} - T_{surface,interior}) = 0$ 



$$(q_{sw,solar} + q_{lw,surface-sky} + q_{lw,surface-air} + q_{convection} = 0$$

## A note on sign conventions

- Move from left to right (or top to bottom)
- Assume that the temperature to the left (or upstream) is higher than the temperature to the right (or downstream)
  - The signs will work themselves out and let you know if that is not the case
  - Be consistent!

#### **Correction:** Accounting for *air* and *sky* LWR separately

- For a more detailed description of a discussion we had about whether or not to account for sky and air temperatures separately in long-wave radiation exchanges, please see the document I put on Blackboard and on my website
- I should note that some energy simulation programs deal only with surface-to-sky interactions, where sky temperatures are estimated from air temperatures and adjusted based on level of cloudiness and/or presence of water vapor (which absorb infrared radiation)
- Others, like EnergyPlus, which I've referenced in these slides, treat surface-air and surface-sky interactions separately
- Other disciplines don't even work with "sky" temperatures, but use air temperatures alone, although they are still adjusted for cloudiness and presence of water vapor
  - To-may-to vs. to-mah-to

#### **Correction:** Accounting for *air* and *sky* LWR separately

- For our purposes, I suggest we follow the EnergyPlus manual, with the following important changes that I failed to mention previously:
  - We should account for differences in surface-sky and surface-air long-wave radiation exchange simply by adjusting their respective view factors

#### Longwave Radiant View Factors, Fsky and Fg

Longwave radiant view factors are also somewhat casedependent. Perhaps the simplest estimation of these parameters is to base them on the surface tilt, such that

$$F_{sky} = \left[\frac{(1+\cos\Sigma)}{2}\right] \cdot \cos\left(\frac{\Sigma}{2}\right)$$
(29)

$$F_{g} = (1 - \cos \Sigma)/2$$

$$(F_{a} = 1 - F_{sky} - F_{g}).$$
(30)

This is the formulation used in the BLAST program (Walton 1983). However, if the building is part of an urban environment, other buildings may dominate the surface's radiant view. Therefore, view factors should also be approximated based on the building's surroundings.

**Source:** McClellan, T.M., Pedersen, C.O. (1997). Investigation of outside heat balance models for use in heat balance cooling load calculation procedure. *ASHRAE Transactions* 103:469-484.

# **Correction:** A note on typical view factors, $F_{1-2}$

• Some typical view factors from surfaces to ground or sky

-  $F_{surface-air}$  typically 1.0  $F_{surface-air}$  ranges from 0 to 0.15 depending on tilt



\*Note that other surrounding buildings complicate view factors, but their net temperature differences probably aren't that different so long-wave radiation can be negligible

### A note on sky temperatures

- Many ways to get sky temperature
  - Varying levels of detail and accuracy
- For a partly cloudy night sky:  $T_{sky} = T_{air} \left[ 0.8 + \frac{(T_{dewpoint} 273)}{250} \right]^{1/4}$

• For daytime: 
$$T_{sky} = \left(\varepsilon_{sky}T_{air}^4\right)^{0.25}$$

$$\varepsilon_{sky} = \left[0.787 + 0.764 \ln\left(\frac{T_{dewpoint}}{273}\right)\right] \left(1 + 0.0224N - 0.0035N^2 + 0.00028N^3\right)$$

- For a clear sky: N = 0

Where N = cloud cover (tenths)

- For 50% cloud cover, N = 0.5

## **Example 2.5: Roof surface temperature**

Estimate the surface temperature that might be reached by a bituminous roof (absorptance of 0.9) installed over a highly insulating substrate (R-20 IP) exposed to intense sun (q<sub>solar</sub> = 1000 W/m<sup>2</sup>) on a calm, cloudless day with an ambient temperature of 20°C and RH = 30%

Indoor surface temperature is 22°C

## Example 2.5: Solution (original, incorrect)

| Surface energy balance          |     | Add     | Subtract |
|---------------------------------|-----|---------|----------|
|                                 |     | $W/m^2$ | $W/m^2$  |
| Solar (short-wave)              |     | 900     |          |
| Surface-sky long-wave radiation |     | -364    |          |
| Surface-air long-wave radiation |     | -286    |          |
| Convection on roof              |     | -239    |          |
| Conduction through roof         |     |         | 12       |
| -                               | SUM | 0       |          |

| Given     | alpha                   | 0.9        | bituminous membrane     |                 |
|-----------|-------------------------|------------|-------------------------|-----------------|
| Given     | Itotal, W/m2            | 1000       |                         |                 |
| Assume    | Fsurface-sky            | 1          |                         |                 |
| Assume    | Fsurface-air            | 1          |                         |                 |
| Assume    | e,surface               | 0.9        |                         |                 |
| Given     | Tair,out, K             | 293.15     | 20 degC                 |                 |
| Assume    | Tair,out,dewpoint, K    | 275.06     | 1.91 degC               | psych chart     |
| Calculate | e,sky                   | 0.79       | N = 0                   |                 |
| Calculate | Tsky, K                 | 276.61     | Tsky equation for clean | r day           |
| Guess     | Tsurface, K             | 337.55     | 64.4 degC               | Adjust T until  |
| Given     | Tsurf,in, K             | 295.15     | 22.0 degC               | sum of all heat |
| Constant  | stef-boltz, W/(m2K4)    | 5.6704E-08 |                         | transfer modes  |
| Given     | R-value IP, h-ft2-F/Btu | 20         |                         |                 |
| Given     | R-value, SI             | 3.52       |                         | equals zero     |
| Given     | U-value, W/m2K          | 0.28       |                         | 21              |

## Example 2.5: Solution (corrected)

| Surface energy balance          |     | Add     | Subtract |
|---------------------------------|-----|---------|----------|
|                                 |     | $W/m^2$ | $W/m^2$  |
| Solar (short-wave)              |     | 900     |          |
| Surface-sky long-wave radiation |     | -512    |          |
| Surface-air long-wave radiation |     | 0       |          |
| Convection on roof              |     | -372    |          |
| Conduction through roof         |     |         | 17       |
|                                 | SUM | -1      |          |

| Given     | alpha                   | 0.9        | bituminous men    | nbrane       |                        |
|-----------|-------------------------|------------|-------------------|--------------|------------------------|
| Given     | Itotal, W/m2            | 1000       |                   |              |                        |
| Assume    | Fsurface-sky            | 1          |                   |              |                        |
| Assume    | Fsurface-air            | 0          |                   |              |                        |
| Assume    | e,surface               | 0.9        |                   |              |                        |
| Given     | Tair,out, K             | 293.15     | 20                | degC         |                        |
| Assume    | Tair,out,dewpoint, K    | 275.06     | 1.91              | degC         | psych chart            |
| Calculate | e,sky                   | 0.79       | N = 0             | U            | 1 2                    |
| Calculate | Tsky, K                 | 276.61     | Tsky equation for | or clear day |                        |
| Guess     | Tsurface, K             | 355.05     | 81.9              | degC         |                        |
| Given     | Tsurf,in, K             | 295.15     | 22.0              | degC         |                        |
| Constant  | stef-boltz, W/(m2K4)    | 5.6704E-08 |                   | U            |                        |
| Given     | R-value IP, h-ft2-F/Btu | 20         |                   |              | Aujust I surface until |
| Given     | R-value, SI             | 3.52       |                   |              | sum of all neat        |
| Given     | U-value, W/m2K          | 0.28       |                   |              | transfer modes         |

Note that there is no assumed surface-to-air radiation for this horizontal surface The roof surface is expected to be about 17 degrees C warmer than before!

## Bringing all the modes together

• Similarly, for a vertical surface:

$$q_{solar} + q_{lwr} + q_{conv} - q_{cond} = 0$$



$$\alpha I_{solar}$$

$$+ \varepsilon_{surface} \sigma F_{sky} (T_{sky}^{4} - T_{surf}^{4})$$

$$+ \varepsilon_{surface} \sigma F_{air} (T_{air}^{4} - T_{surface}^{4})$$

$$+ \varepsilon_{surface} \sigma F_{ground} (T_{air}^{4} - T_{ground}^{4})$$

$$+ h_{conv} (T_{air} - T_{surface})$$

$$- U (T_{surface} - T_{surface, interior}) = 0$$

Ground

## Bringing all modes (and nodes) together

- For an example room like this, you would setup a system of equations where the temperature at each node (either a surface or within a material) is unknown
  - 12 material nodes + 1 indoor air node

Heat Xfer @ external surfaces: Radiation and convection



# Bringing all modes (and nodes) together

- To get the impact on indoor air temperature (and close the system of equations)
  - Write an energy balance on the indoor air node
  - Air impacted directly only by convection (bulk and/or surface)

$$(V_{room}\rho_{air}c_{p,air})\frac{dT_{air,in}}{dt} = \sum_{i=1}^{n} h_i A_i \left(T_{i,surf} - T_{air,in}\right) + \dot{m}c_p \left(T_{out} - T_{air,in}\right) + Q_{HVAC}$$

#### In plain English:

The change in indoor air temperature is equal to the sum of convection from each interior surface plus outdoor air delivery (by infiltration or dedicated outdoor air supply), plus the bulk convective heat transfer delivered by the HVAC system



# SOLAR ORIENTATION

## **Solar radiation**

- The sun is the source of most energy on the earth
- Need to have a working knowledge of earth's relationship to the sun
- Should be able to estimate solar radiation intensity
  - Understand thermal effects of solar radiation and how to control or utilize them
  - Need to estimate solar gains on a building
  - Need to predict intensity of solar radiation and the direction at which it strikes building surfaces
    - Start with relationships between the sun and the earth

# Solar radiation: earth-sun relationship

- Earth rotates about its axis every 24 hours
- Earth revolves around sun every 365.2425 days
- Earth is titled at an angle of 23°27'



- Therefore, different locations on earth receive different levels of solar radiation during different times of the year (and different times of the day)
  - The greatest amount of solar radiation is delivered to northern hemisphere on **June 21**
  - Least amount of solar energy delivered on December 21
- There are methods of determining the amount of flux of solar radiation to surfaces on the earth

- The position of a point P on the earth's surface w/r/t the sun's rays can be calculated if we know:
  - Latitude of point on earth, / (degrees)
  - Hour angle of the point on earth, h (degrees)
  - Sun's declination, d (degrees)



Figure 13.3 Latitude, hour angle, and sun's declination.

• Sun's declination, *d*, can be estimated by:

$$d = 23.45 \sin\left(360 \,\frac{284 \,+\, n}{365}\right)$$

Where *n* is the day of the year, which you can determine by counting on your hands, looking up online, or using this table:

| Month  | n for the Day of the Month, D   | Month  | n for the Day of the Month, D   |  |
|--|---|--|---|--|
| January<br>February<br>March<br>April<br>May | $ \begin{array}{r} D\\ 31 + D\\ 59 + D\\ 90 + D\\ 120 + D\\ 151 + D \end{array} $ | July<br>August<br>September<br>October<br>November | $     \begin{array}{r}       181 + D \\       212 + D \\       243 + D \\       273 + D \\       304 + D \\       334 + D     \end{array} $ | Where <i>D</i> is<br>the day of the<br>month |

**TABLE 13.1** Variation in *n* throughout the Year for Eq. (13.1)



*d* is **positive** when sun's rays are **north** of the equator

- Now we have latitude (/) and sun's declination (d)
  - Need hour angle (h)
- It's all about time:
- Greenwich Civil Time = time at line of zero longitude
- Local Civil Time (CT) is governed by your longitude
  - 1/15<sup>th</sup> of an hour (4 mins) of time for each degree difference in longitude
    - Central Standard Time is 90 degrees from 0
    - 4 min per degree \* 90 degrees = 360 minutes = 6 hours
- Time also measured by apparent diurnal motion of the sun
  - Apparent Solar Time (AST), Local Solar Time (LST), or Solar Time (ST)
    - Interchangeable
  - Slightly different than a civil day because of irregularities of the earth's rotation and shape of earth's orbit
  - The difference between solar time (LST) and civil time (CT) is called the Equation of Time (E)



## Calculating solar time (LST)

• Local **solar** time:

$$LST = CT + \left(\frac{1}{15}\right)(L_{std} - L_{loc}) + E - DT$$

Where:

*LST* = local solar time (hour)

*CT* = clock time (hour)

 $L_{std}$  = standard meridian longitude for local time zone (degrees west)

 $L_{loc}$  = longitude of actual location (degrees west)

*E* = Equation of Time (hour)

*DT* = Daylight savings time correction (hour)

\*DT = 1 if on DST; otherwise 0

\*\*Note that all times should be converted to decimal format from 0 to 24. For example, 3:45 PM = 15.75 hours

• Equation of Time:  $E = 0.165 \sin 2B - 0.126 \cos B - 0.025 \sin B$ 

where 
$$B = \frac{360(n-81)}{364}$$
 and *n* is the day of the year. *B* is in degrees

## Calculating solar time (LST)

• Finally, the solar hour angle, h, can be calculated:

h = 15(LST - 12) degrees

*h* is **positive** *after* solar noon and **negative** *before* 

LST is in 24 hour format

 Again, you can either calculate these values, use a website\*, or look them up in a table like this:

|           | Day                     |                       |                         |                       |                         |                       |                         |                      |  |
|-----------|-------------------------|-----------------------|-------------------------|-----------------------|-------------------------|-----------------------|-------------------------|----------------------|--|
|           |                         | 7                     |                         | 14                    |                         | 21                    |                         | 28                   |  |
| Month     | Declination,<br>Degrees | Eq. of Time,<br>Hours | Declination,<br>Degrees | Eq. of Time,<br>Hours | Declination,<br>Degrees | Eq. of Time,<br>Hours | Declination,<br>Degrees | Eq. of Time<br>Hours |  |
| January   | -22.4                   | -0.10                 | -21.4                   | -0.15                 | -20.1                   | -0.19                 | - 18.5                  | -0.22                |  |
| February  | -15.8                   | -0.24                 | -13.6                   | -0.24                 | -11.2                   | -0.24                 | -8.7                    | -0.22                |  |
| March     | -6.0                    | -0.20                 | -3.2                    | -0.17                 | -0.4                    | -0.13                 | 2.4                     | -0.09                |  |
| April     | 6.4                     | -0.04                 | 9.0                     | -0.01                 | 11.6                    | 0.02                  | 13.9                    | 0.04                 |  |
| May       | 16.7                    | 0.06                  | 18.5                    | 0.06                  | 20.1                    | 0.06                  | 21.4                    | 0.05                 |  |
| June      | 22.7                    | 0.02                  | 23.3                    | 0.00                  | 23.45                   | -0.03                 | 23.3                    | -0.05                |  |
| July      | 22.6                    | -0.08                 | 21.7                    | -0.09                 | 20.4                    | -0.10                 | 18.9                    | -0.10                |  |
| August    | 16.3                    | -0.09                 | 14.1                    | -0.07                 | 11.8                    | -0.04                 | 9.2                     | -0.01                |  |
| September | 5.4                     | 0.05                  | 2.6                     | 0.09                  | -0.2                    | 0.13                  | -3.0                    | 0.17                 |  |
| October   | -6.6                    | 0.22                  | -9.2                    | 0.25                  | -11.8                   | 0.27                  | -14.1                   | 0.27                 |  |
| November  | -17.1                   | 0.27                  | -18.9                   | 0.25                  | -20.4                   | 0.22                  | -21.7                   | 0.18                 |  |
| December  | -22.8                   | 0.12                  | -23.3                   | 0.07                  | -23.45                  | 0.02                  | -23.3                   | -0.04                |  |

\*NOAA has website for this: <u>http://www.esrl.noaa.gov/gmd/grad/solcalc/</u>

# Calculating solar time (LST) and hour angle (h)

- Example problem 2.6
- Determine the local solar time and sun's hour angle in Minneapolis, MN (L<sub>loc</sub> = 93°W) at 2:25 PM Central Daylight Savings Time on July 21

Once we have our local latitude *I*, the sun's declination angle *d*, and the hour angle *h*, we can move on to other important relationships:



#### Three important angles (°)

#### $\theta_{H}$ = sun's zenith angle

angle between the sun's rays and the local vertical

#### $\beta$ = altitude angle

angle in a vertical plane between the sun's rays and the projection of the earth's horizontal plane

#### $\phi$ = solar azimuth angle

angle in the horizontal plane measured from south to the horizontal projection of the sun's rays

#### \*Note that $I_{DN}$ represents the sun's rays

• Relationships between I, h, d, and  $\theta_H$ ,  $\beta$ , and  $\phi$  can all be described in this figure:



#### \*Don't worry if this doesn't all make sense; there are formulas!

• After a lot of complex geometry/trigonometry...

 $\cos \theta_{H} = \cos I \cos h \cos d + \sin I \sin d$   $\sin \beta = \cos I \cos h \cos d + \sin I \sin d$  $\cos \phi = (\cos d \sin I \cos h - \sin d \cos I) / \cos \beta$ 

A note on sign conventions for all of these relationships: North latitudes (*I*) are positive, south latitudes are negative Declination (*d*) is positive when sun's rays are north of equator Hour angle (*h*) is negative before solar noon, positive after Azimuth angle ( $\phi$ ) is negative east of south and positive west of south

> Note that  $\beta$  for solar noon = 90 degrees - | *I* - *d* | Also note that  $\beta + \theta_H = 90$  degrees

\*\*Keep units consistent

- Last but not least...
- The previous relationships identify a point on the earth's surface in relation to the sun
  - All valid for horizontal surfaces
  - Buildings are not horizontal surfaces!
- Surface-sun relationships:



Figure 13.9 Definitions of surface azimuth, surface tilt, and surfacesolar azimuth angles and the relation of sun's rays to a tilted surface.

## Surface-sun relationships

#### More important angles (°)

 $\theta$  = incidence angle

angle between the solar rays and the surface normal

#### $\Sigma$ = surface tilt angle

angle between surface normal and the vertical

Vertical surface:  $\Sigma = 90^{\circ}$ Horizontal surface:  $\Sigma = 0^{\circ}$ 

#### $\Psi$ = surface azimuth angle

angle between south and the horizontal projection of the surface normal

#### $\gamma$ = surface-solar azimuth angle angle between horizontal projection of solar rays and the horizontal projection of the surface normal $\gamma = | \phi - \Psi |$



Figure 13.9 Definitions of surface azimuth, surface tilt, and surfacesolar azimuth angles and the relation of sun's rays to a tilted surface.

\*Sign convention:  $\Psi$  is negative for a surface that faces east of south and positive for a surface that faces west of south

#### **Tilted surface:**

 $\cos \theta = \cos \beta \cos \gamma \sin \Sigma + \sin \beta \cos \Sigma$ 

Vertical surface ( $\Sigma = 90^{\circ}$ ): cos  $\theta = \cos \beta \cos \gamma$ 

### **Surface-sun relationships**

- Example problem 2.7
- Calculate sun's altitude (β) and azimuth (φ) angles at 7:30 am solar time on August 7 for a location at 40 degrees north latitude

## Surface-sun relationships

- Example problem 2.8
- Calculate sun's incidence angle for a vertical surface that faces 25 degrees east of south and has a tilt angle of 60 degrees at 3:00 pm solar time on June 7 for a location at 36 degrees north latitude

#### Translation:

Find  $\theta$ 

Given: *Ψ, Σ, Ι, h, β, φ* 



Figure 13.9 Definitions of surface azimuth, surface tilt, and surfacesolar azimuth angles and the relation of sun's rays to a tilted surface.

## Solar flux

- Once we know earth-surface-sun relationships, we can eventually get to the effects of those relationships on actual solar radiation
- Solar radiation intensity is roughly constant at the outer layer of the atmosphere
  - 1367 W/m<sup>2</sup> varying a few percent depending on time of year
- The earth's atmosphere depletes some direct solar radiation
  - Intercepted by other air molecules, water molecules, dust particles
  - Remaining reaches earth's surface unchanged in wavelength
    - Direct radiation
  - The deflected radiation turns aside from the direct beam
    - Diffuse radiation

# Solar flux

- Estimating intensity of direct normal solar radiation
  - Many, *many* ways to estimate this
  - ASHRAE uses a relationship for "average clear days"

$$I_{DN} = A e^{-\frac{B}{\sin\beta}}$$

Where:

 $I_{DN}$  = direct normal solar radiation (W/m<sup>2</sup>)

A = apparent direct normal solar flux at outer edge of earth's atmosphere (W/m<sup>2</sup>)

*B* = empirically determined atmospheric extinction coefficient (dimensionless)

 $\beta$  = altitude angle

• Estimating intensity of diffuse horizontal radiation

$$I_{dH} = CI_{DN}$$

Where:

 $I_{dH}$  = diffuse horizontal solar radiation (W/m<sup>2</sup>)

C = empirically determined coefficient for typical "clear days" (dimensionless)

### Typical clear day values for solar radiation

**TABLE 13.3** Coefficients for Average Clear Day Solar Radiation Calculations

 for the Twenty-First Day of Each Month, Base Year 1964

|           |                             | 4               | В         | С            |                     |                         |
|-----------|-----------------------------|-----------------|-----------|--------------|---------------------|-------------------------|
|           | $\frac{Btu}{hr \cdot ft^2}$ | $\frac{W}{m^2}$ | Dimensior | nless Ratios | Declination,<br>deg | Equation of<br>Time, hr |
| January   | 390                         | 1230            | 0.142     | 0.058        | -20.0               | -0.19                   |
| February  | 385                         | 1215            | 0.144     | 0.060        | -10.8               | -0.23                   |
| March     | 376                         | 1186            | 0.156     | 0.071        | 0.0                 | -0.13                   |
| April     | 360                         | 1136            | 0.180     | 0.097        | 11.6                | 0.02                    |
| May       | 350                         | 1104            | 0.196     | 0.121        | 20.0                | 0.06                    |
| June      | 345                         | 1088            | 0.205     | 0.134        | 23.45               | -0.02                   |
| July      | 344                         | 1085            | 0.207     | 0.136        | 20.6                | -0.10                   |
| August    | 351                         | 1107            | 0.201     | 0.122        | 12.3                | -0.04                   |
| September | 365                         | 1151            | 0.177     | 0.092        | 0                   | 0.13                    |
| October   | 378                         | 1192            | 0.160     | 0.073        | -10.5               | 0.26                    |
| November  | 387                         | 1221            | 0.149     | 0.063        | -19.8               | 0.23                    |
| December  | 391                         | 1233            | 0.142     | 0.057        | -23.45              | 0.03                    |

Source: Adapted by permission from ASHRAE Handbook, Fundamentals Edition, 1993.

$$I_{DN} = A e^{-B/\sin\beta}$$

# Solar flux to building surfaces (finally!)

• Solar radiation striking a surface:  $I_{solar} = I_D + I_d + I_R$ 

• Direct  $(I_D)$ :  $I_D = I_{DN} \cos \theta$ 

Where:

 $\theta$  = incidence angle, or the angle between the solar rays and the surface normal  $I_{DN}$  = direct normal solar radiation (W/m<sup>2</sup>)

• Diffuse 
$$(I_d)$$
:  $I_d = I_{dH} \frac{1 + \cos \Sigma}{2}$ 

Where:

 $\Sigma$  = surface tilt angle, or the angle between surface normal and surface vertical  $I_{dH}$  = diffuse horizontal solar radiation (W/m<sup>2</sup>)

# Solar flux to building surfaces (finally!)

- Reflected  $(I_R)$ 
  - Radiation striking a surface after reflecting off surrounding surfaces
  - Similar to diffuse
  - Usually concerned with reflection from the ground

$$I_R = \frac{\rho_g I_H (1 - \cos \Sigma)}{2}$$

Where:

 $\rho_g$  = solar reflectance of the ground (depends on surface, usually 0.1-0.4)  $I_H$  = total solar flux striking the horizontal ground (W/m<sup>2</sup>)

$$I_{\rm H} = I_{DN} \cos\theta_{\rm H} + I_{dH}$$

## Solar flux to building surfaces

- Reflected  $(I_R)$ 
  - Values of reflectance  $(\rho_g)$ for common ground surfaces



Figure 13.21 Solar reflectance for various ground surfaces. [Reprinted by permission from ASHRAE Trans., 69 (1963), 31.]

## Solar flux to building surfaces

- Example problem 2.9
- Find the solar flux incident on the tilted surface used in the previous problem
  - Assume a ground reflectance of 0.15

## **Refined solar data**

 Now, you could make all of these calculations for every hour of the day...

#### OR

- You can build calculators or download data
- For hourly sun positions, you can build a calculator or use one from the internet
  - <u>http://www.susdesign.com/sunposition/index.php</u>
- For hourly solar data (direct + diffuse in W/m<sup>2</sup>)
  - <u>http://rredc.nrel.gov/solar/old\_data/nsrdb/</u>
  - You may be familiar with "typical meteorological years"
    - These data inform those databases
- For visualizing geometry, using something like IES-VE
  - Show videos (videos can be download on course website)

## Solar orientation videos/software

- <u>http://built-envi.com/courses/cae-463524-building-enclosure-design-fall-2013/</u>
- <u>http://built-envi.com/wp-content/uploads/2013/07/</u> solar position ies.zip
  - 56 mb zip file of several videos

## Bringing all the modes together

• Energy balance for a vertical surface:



Ground

We need to understand conduction through enclosures that are more complex than just single materials

# Wrapping up

• HW #1 will be due Wed, September 11 in class (1 week)

#### **Next series of topics:**

- Heat transfer through layers of enclosure elements
  - Thermal networks
- Heat transfer through more complex enclosure assemblies
   Various methods