# CAE 331/513
# Building Science
Fall 2013

## Lecture 2: August 26, 2013
Elements of heat transfer in buildings

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# Last time

- Introduced the topic of building science
  - Buildings use a lot of energy
  - Buildings cost a lot of money
  - Buildings have major impacts on both indoor and outdoor environments

- Reviewed basic units
- Assigned HW 1 (due today)
- Graduate students: $1^{st}$ blog post was due today

# Blog posts

- Your first blog posts were due today:
- www.iitbuildingscience.wordpress.com

# HW 1

- HW1 is due today
- Will be graded and returned by next class period
  - 2 weeks from today
- Will go over solutions next time
  - And will post PDF of solutions to Blackboard before then

# Objectives for today's lecture

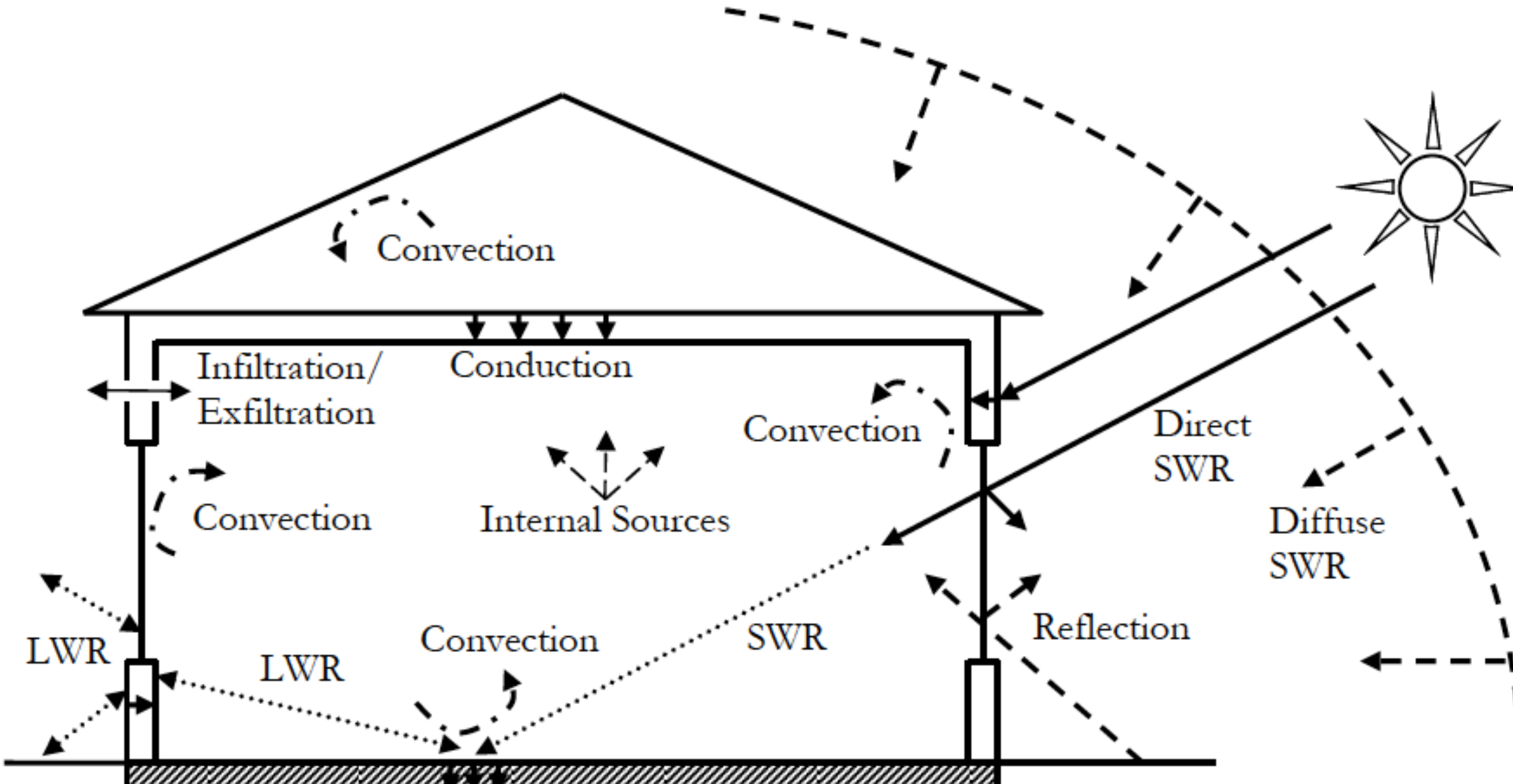
- Introduce the elements of heat transfer in buildings
  - Generally follows Kreider Chapter 2
- Review of heat transfer fundamentals
  - Conduction
  - Convection
  - Radiation
- In context: heat transfer in buildings
  - Walls, roofs, windows, floors
  - Description of building systems

# Heat transfer

- Heat transfer is the transfer of thermal energy between objects of different temperatures
- In building science, we work with temperature differences between the interior and exterior of the building
- The element that separates indoors from outdoors is the building enclosure (or building envelope)
  - Walls, roofs, floors, and fenestration (i.e., windows, doors, skylights)
- We also have internal heat gains that contribute to higher temperatures in indoor environments

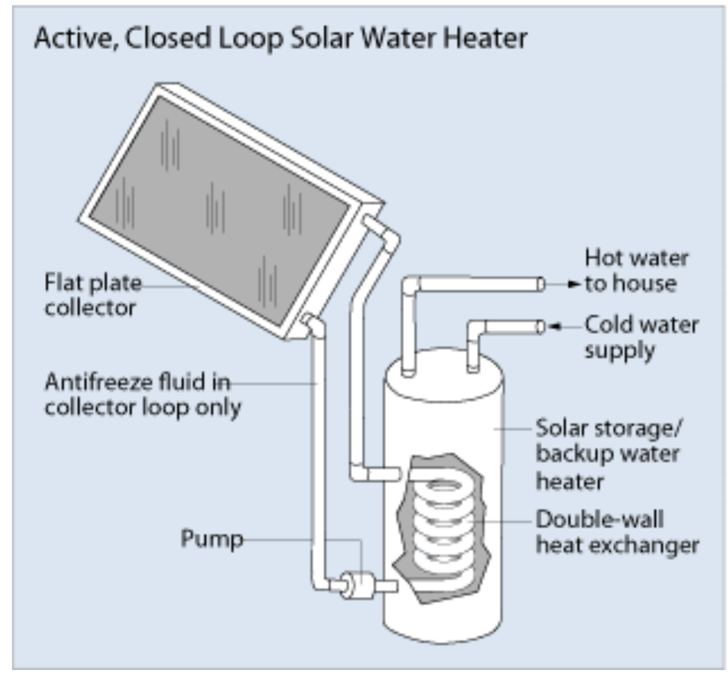
# Examples of heat transfer in a building

- Conduction of heat through a building's skin
- Transmission of solar radiation through windows
- Cooling of occupants by ventilation

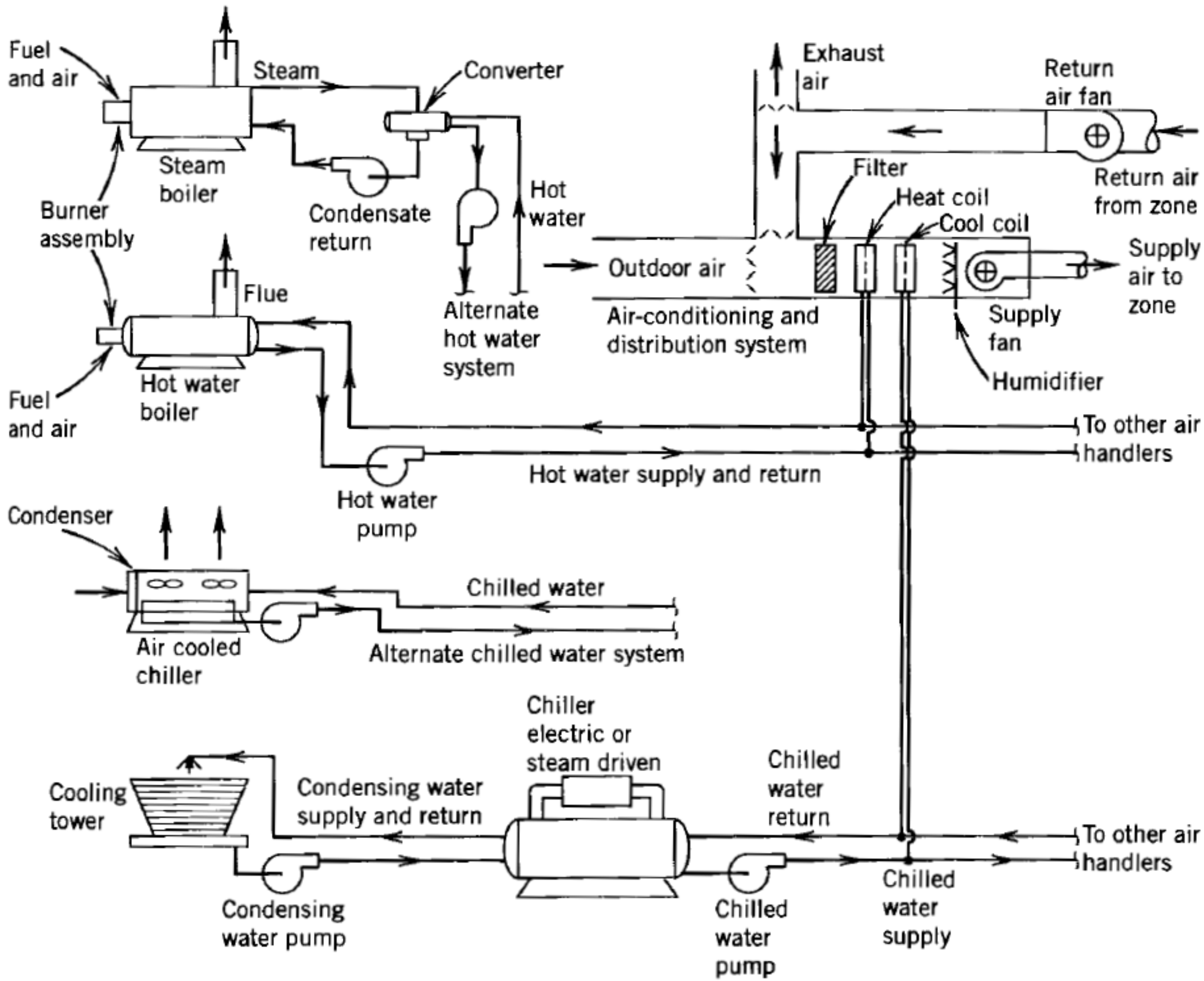


# More heat transfer examples

- We also deal with heat transfer within HVAC systems, water heating, and many other applications



# Heat transfer in HVAC systems



# Building heat transfer

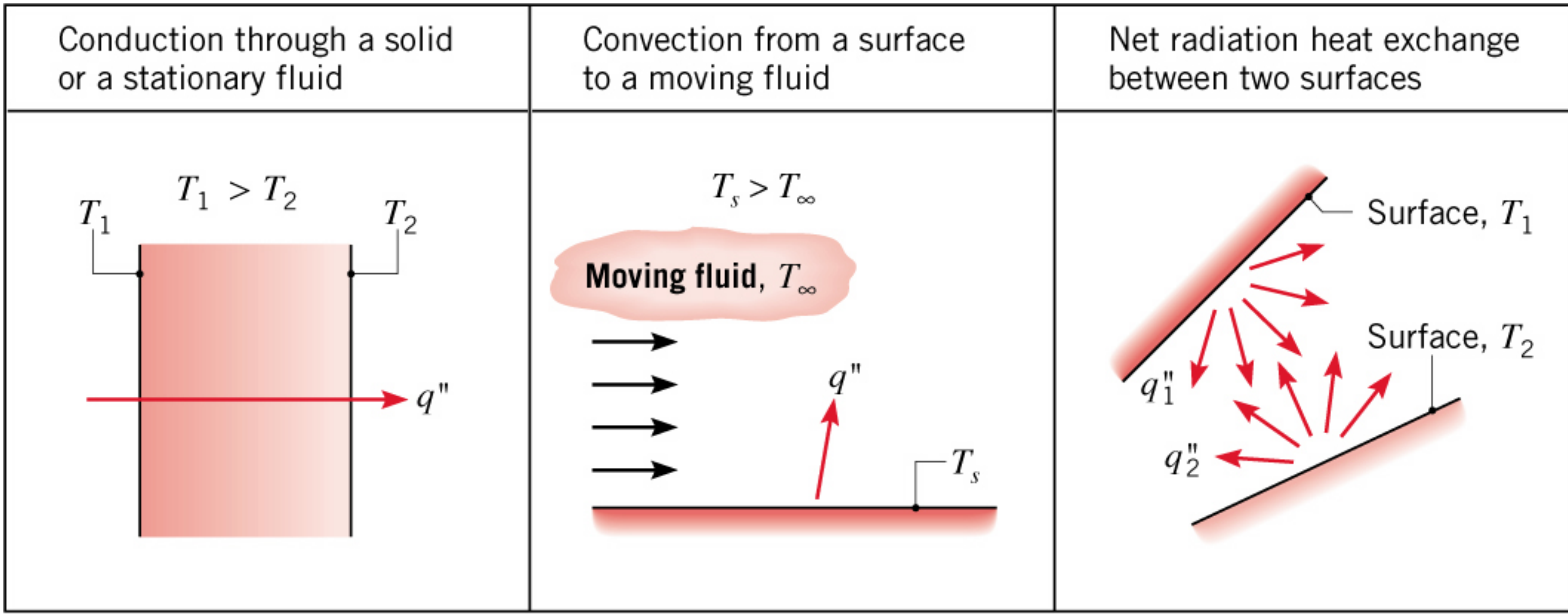
- Heat transfer is the science and art of predicting the rate at which heat flows through substances under various external conditions

- The laws of heat transfer govern the rate at which heat energy must be supplied to or removed from a building to maintain the comfort of occupants or to meet other thermal requirements of buildings

- We will review heat transfer fundamentals here, and use these concepts later in the course to estimate heating and cooling needs for whole buildings

# Heat transfer

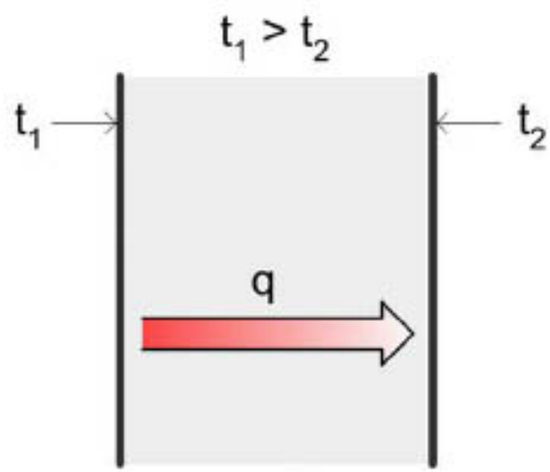
- Three modes of heat transfer




**Conduction** **Convection** **Radiation**

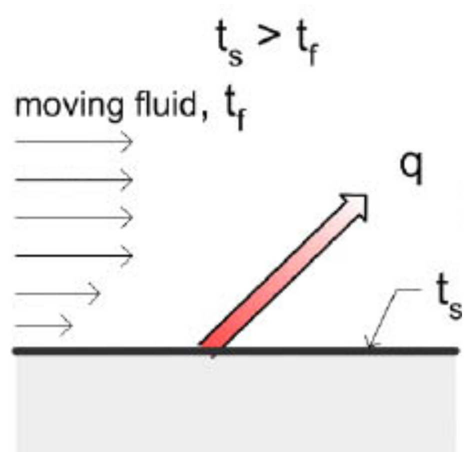
# Conduction

- **Conduction** heat transfer is a result of molecular-level kinetic energy transfers in solids, liquids, and gases
  - Think: analogous electrical conduction in solids
- Conduction heat flow occurs in the direction of decreasing temperature
  - From high T to low T
- Example: heat loss through opaque walls in winter



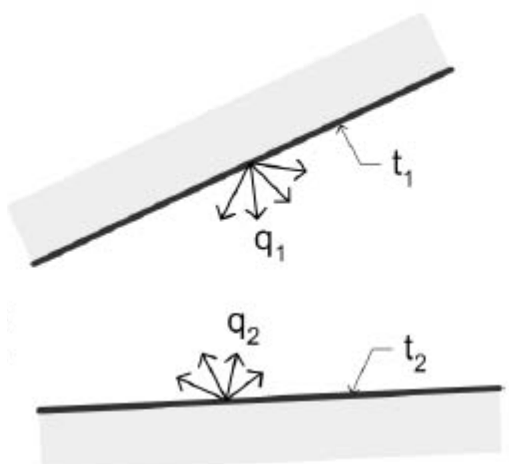
# Convection

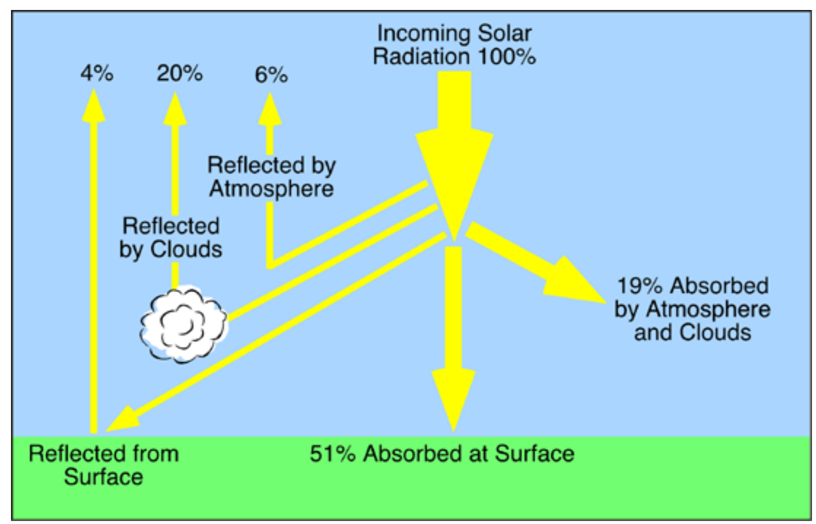
- **Convection** heat transfer is a result of larger-scale motions of a fluid, either liquid or gas
- The higher the velocity of fluid flow, the higher the rate of convection heat transfer
  - Also the greater the temperature difference the greater the heat flow
- Example: when a cold wind blows over a person's skin and removes heat from it



# Radiation

- **Radiation** heat transfer is the transport of energy by electromagnetic waves
  - Exchange between two surfaces at different temperatures
- Radiation must be absorbed by matter to produce internal energy
- Example: energy transported from the sun to the earth



# Conduction

- **Conduction** follows Fourier's Law: $q = -k\nabla T$

$$q = -k\nabla T = -k\left(\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} + \frac{\partial T}{\partial z}\right)$$

where:

$q =$ heat flux per unit area [Btu/(h·ft$^2$) or W/m$^2$]

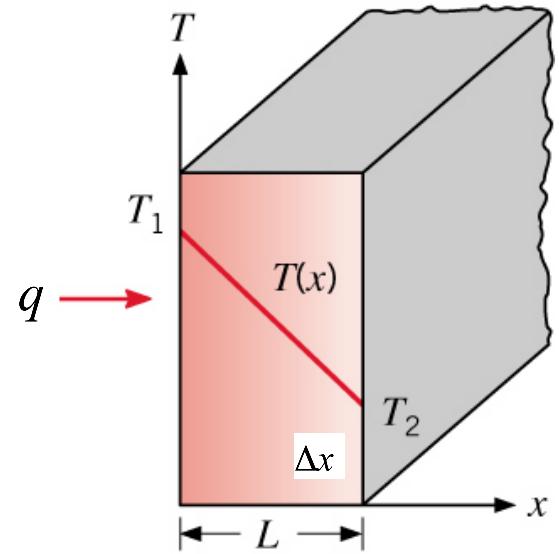
$k =$ thermal conducitivity [Btu/(h·ft·°F) or W/(m·K)]

$T =$ temperature [°F or K]

- In 1-D, this becomes: $q = -k\dfrac{dT}{dx}$

# Simplified conduction: 1-dimension

If a material has uniform thermal conductivity throughout & consists of parallel surfaces with uniform temperatures, then:




Δx = L

$$q = k\frac{\Delta T}{\Delta x} = k\frac{T_1 - T_2}{x_2 - x_1} = \frac{k}{L}\left(T_1 - T_2\right)$$

Here $T_1$ and $T_2$ are the surface temperatures at $x_1$ and $x_2$

Notice that this equation differs from the last by a minus sign

I suggest you use the $\Delta T / \Delta x$ formulation and note that heat will always flow from high to low temperature

# Conduction

Remember:

- To get $Q$ in [W], simply multiply $q$ [W/m$^2$] by $A$ [m$^2$]

$$Q = qA = A\frac{k}{L}\left(T_1 - T_2\right)$$

where:

$Q =$ heat flux [Btu/h or W]

$A =$ area normal to heat flow [m$^2$]

# Thermal conductance and resistance

- Conductivity and length can also be described in other terms

$$Q = A\boxed{\frac{k}{L}}\left(T_1 - T_2\right)$$

$$\frac{k}{L} = U \qquad \text{and} \qquad R = \frac{1}{U}$$

where:

$U$ = unit thermal conductance [$\frac{\mathrm{Btu}}{\mathrm{h \cdot ft^2 \cdot {}^\circ F}}$] or [$\frac{\mathrm{W}}{\mathrm{m^2 K}}$]

$R$ = unit thermal resistance [$\frac{\mathrm{h \cdot ft^2 \cdot {}^\circ F}}{\mathrm{Btu}}$] or [$\frac{\mathrm{m^2 K}}{\mathrm{W}}$]

# Thermal resistance of common materials (SI units)

- We will often be concerned more with the ability of a material to **resist** heat flow rather than conduct it

$$q = \frac{k}{L}\left(T_1 - T_2\right) = U\left(T_1 - T_2\right) = \frac{1}{R}\left(T_1 - T_2\right)$$

Here the thermal conductivity (k) divided by thickness (L) yields "Conductance" of a material, with units of $[\mathrm{W/(m^2 \cdot K)}]$. Conductance is also called the U-value.

The inverse of conductance (C) is the resistance (R), or R-value. Where 1/C = R, with units of $[\mathrm{(m^2 \cdot K)/W}]$.

Therefore:

$C = U = \dfrac{k}{L} =$ unit thermal conductance = U-value $[\mathrm{W/(m^2 \cdot K)}]$

$R = \dfrac{1}{U} = \dfrac{L}{k} =$ unit thermal resistance = R-value $[\mathrm{(m^2 \cdot K)/W}]$

# Units of R and U-Value

- R values are typically used for insulating materials
  - For example: wall insulation materials
- U values are typically used for conductive materials
  - For example: windows
- SI units are easier for most to work with, but most products in the US are sold in IP units
  - So, REMEMBER THIS CONVERSION!
  - R(IP) = R(SI) × 5.678

**R-SI**

$$1\,\frac{\mathrm{m^2K}}{\mathrm{W}} = 5.678\,\frac{\mathrm{h{\cdot}ft^2{\cdot}{}^\circ F}}{\mathrm{Btu}}$$

**R-IP**

# R-values and R-values

- Our textbook defines two different R-values
  - $R$ (resistance to heat transfer):

$$R = \frac{1}{\left(A\frac{k}{L}\right)} = \frac{L}{kA} \qquad [\frac{\mathrm{K}}{\mathrm{W}}]$$

  - $R_{th}$ (unit thermal resistance, or "R-value"):

$$R_{th} = \frac{1}{\left(\frac{k}{L}\right)} = \frac{L}{k} \qquad [\frac{\mathrm{m^2K}}{\mathrm{W}}]$$

This unit R-value is most useful for our purposes

- It is independent of area

# Thermal conductivity of building materials

- Thermal transmission data for some typical materials:

**TABLE 2.2**

Representative Magnitudes of Thermal Conductivity

| Material | Conductivity, Btu/(h · ft · °F) | Conductivity, W/(m · K) |
|---|---|---|
| Atmospheric-pressure gases | 0.004–0.10 | 0.007–0.17 |
| Insulating materials | 0.02–0.12 | 0.034–0.21 |
| Nonmetallic liquids | 0.05–0.40 | 0.086–0.69 |
| Nonmetallic solids (brick, stone, concrete) | 0.02–1.50 | 0.034–2.6 |
| Metal alloys | 8–70 | 14–120 |
| Pure metals | 30–240 | 52–410 |

- These can also be found in the ASHRAE Handbook, Ch. 26

# Thermal conductivity of building materials

**TABLE 2.3**

Values of Thermal Conductivity for Building Materials

| Material | $k$, Btu/(h · ft · °F) | $T$, °F | $k$, W/(m · K) | $T$, °C |
|---|---|---|---|---|
| Construction materials | | | | |
| Asphalt | 0.43–0.44 | 68–132 | 0.74–0.76 | 20–55 |
| Cement, cinder | 0.44 | 75 | 0.76 | 24 |
| Glass, window | 0.45 | 68 | 0.78 | 20 |
| Concrete | 1.0 | 68 | 1.73 | 20 |
| Marble | 1.2–1.7 | — | 2.08–2.94 | — |
| Balsa | 0.032 | 86 | 0.055 | 30 |
| White pine | 0.065 | 86 | 0.112 | 30 |
| Oak | 0.096 | 86 | 0.166 | 30 |
| Insulating materials | | | | |
| Glass fiber | 0.021 | 75 | 0.036 | 24 |
| Expanded polystyrene | 0.017 | 75 | 0.029 | 24 |
| Polyisocyanurate | 0.012 | 75 | 0.020 | 24 |
| Gases at atmospheric pressure | | | | |
| Air | 0.0157 | 100 | 0.027 | 38 |
| Helium | 0.0977 | 200 | 0.169 | 93 |
| Refrigerant 12 | 0.0048 | 32 | 0.0083 | 0 |
| | 0.0080 | 212 | 0.0038 | 100 |
| Oxygen | 0.00790 | −190 | 0.0137 | −123 |
| | 0.02212 | 350 | 0.0383 | 175 |

*Source:* Courtesy of Karlekar, B. and Desmond, R.M., *Engineering Heat Transfer*, West Publishing, St. Paul, MN, 1982. With permission.

# Thermal properties of building materials

**Table 1 Building and Insulating Materials: Design Values[a]**

| Description | Density, kg/m³ | Conductivity[b] $k$, W/(m·K) | Resistance $R$, (m²·K)/W | Specific Heat, kJ/(kg·K) | Reference[l] |
|---|---|---|---|---|---|
| **Insulating Materials** | | | | | |
| *Blanket and batt[c,d]* | | | | | |
| Glass-fiber batts | | | | 0.8 | Kumaran (2002) |
| | 7.5 to 8.2 | 0.046 to 0.048 | — | — | Four manufacturers (2011) |
| | 9.8 to 12 | 0.040 to 0.043 | — | — | Four manufacturers (2011) |
| | 13 to 14 | 0.037 to 0.039 | — | — | Four manufacturers (2011) |
| | 22 | 0.033 | — | — | Four manufacturers (2011) |
| Rock and slag wool batts | — | — | — | 0.8 | Kumaran (1996) |
| | 32 to 37 | 0.036 to 0.037 | — | — | One manufacturer (2011) |
| | 45 | 0.033 to 0.035 | — | — | One manufacturer (2011) |
| Mineral wool, felted | 16 to 48 | 0.040 | — | — | CIBSE (2006), NIST (2000) |
| | 16 to 130 | 0.035 | — | — | NIST (2000) |
| *Board and slabs* | | | | | |
| Cellular glass | 120 | 0.042 | — | 0.8 | One manufacturer (2011) |
| Cement fiber slabs, shredded wood with Portland cement binder | 400 to 430 | 0.072 to 0.076 | — | — | |
| with magnesia oxysulfide binder | 350 | 0.082 | — | 1.3 | |
| Glass fiber board | — | — | — | 0.8 | Kumaran (1996) |
| | 24 to 96 | 0.033 to 0.035 | — | — | One manufacturer (2011) |
| Expanded rubber (rigid) | 64 | 0.029 | — | 1.7 | Nottage (1947) |
| Extruded polystyrene, smooth skin | — | — | — | 1.5 | Kumaran (1996) |
| aged per Can/ULC *Standard* S770-2003 | 22 to 58 | 0.026 to 0.029 | — | — | Four manufacturers (2011) |
| aged 180 days | 22 to 58 | 0.029 | | | One manufacturer (2011) |
| European product | 30 | 0.030 | | | One manufacturer (2011) |
| aged 5 years at 24°C | 32 to 35 | 0030 | — | — | One manufacturer (2011) |
| blown with low global warming potential (GWP) (<5) blowing agent | | 0.035 to 0.036 | — | — | One manufacturer (2011) |
| Expanded polystyrene, molded beads | — | — | — | 1.5 | Kumaran (1996) |
| | 16 to 24 | 0.035 to 0.037 | — | — | Independent test reports (2008) |
| | 29 | 0.033 | — | — | Independent test reports (2008) |

# Thermal properties of building materials

**Table 1 Building and Insulating Materials: Design Values[a] (*Continued*)**

| Description | Density, kg/m³ | Conductivity[b] $k$, W/(m·K) | Resistance $R$, (m²·K)/W | Specific Heat, kJ/(kg·K) | Reference[l] |
|---|---|---|---|---|---|
| | 1760 | 0.71 to 0.85 | — | — | Valore (1988) |
| | 1600 | 0.61 to 0.74 | — | — | Valore (1988) |
| | 1440 | 0.52 to 0.62 | — | — | Valore (1988) |
| | 1280 | 0.43 to 0.53 | — | — | Valore (1988) |
| | 1120 | 0.36 to 0.45 | — | — | Valore (1988) |
| Clay tile, hollow | | | | | |
| 1 cell deep .......... 75 mm | — | — | 0.14 | 0.88 | Rowley and Algren (1937) |
| .......... 100 mm | — | — | 0.20 | — | Rowley and Algren (1937) |
| 2 cells deep .......... 150 mm | — | — | 0.27 | — | Rowley and Algren (1937) |
| .......... 200 mm | — | — | 0.33 | — | Rowley and Algren (1937) |
| .......... 250 mm | — | — | 0.39 | — | Rowley and Algren (1937) |
| 3 cells deep .......... 300 mm | — | — | 0.44 | — | Rowley and Algren (1937) |
| Lightweight brick .......... | 800 | 0.20 | — | — | Kumaran (1996) |
| | 770 | 0.22 | — | — | Kumaran (1996) |
| *Concrete blocks*[f, g] | | | | | |
| Limestone aggregate | | | | | |
| ~200 mm, 16.3 kg, 2200 kg/m³ concrete, 2 cores .......... | — | — | — | — | |
| with perlite-filled cores.......... | — | — | 0.37 | — | Valore (1988) |
| ~300 mm, 25 kg, 2200 kg/m³ concrete, 2 cores .......... | — | | — | — | |
| with perlite-filled cores.......... | — | — | 0.65 | — | Valore (1988) |
| Normal-weight aggregate (sand and gravel) | | | | | |
| ~200 mm, 16 kg, 2100 kg/m³ concrete, 2 or 3 cores… | — | — | 0.20 to 0.17 | 0.92 | Van Geem (1985) |
| with perlite-filled cores.......... | — | — | 0.35 | — | Van Geem (1985) |
| with vermiculite-filled cores.......... | — | — | 0.34 to 0.24 | — | Valore (1988) |
| ~300 mm, 22.7 kg, 2000 kg/m³ concrete, 2 cores .......... | — | — | 0.217 | 0.92 | Valore (1988) |
| Medium-weight aggregate (combinations of normal and lightweight aggregate) | | | | | |
| ~200 mm, 13 kg, 1550 to 1800 kg/m³ concrete, 2 or 3 cores | — | — | 0.30 to 0.22 | — | Van Geem (1985) |
| with perlite-filled cores.......... | — | — | 0.65 to 0.41 | — | Van Geem (1985) |
| with vermiculite-filled cores.......... | — | — | 0.58 | — | Van Geem (1985) |
| with molded-EPS-filled (beads) cores.......... | — | — | 0.56 | — | Van Geem (1985) |
| with molded EPS inserts in cores .......... | — | — | 0.47 | — | Van Geem (1985) |

# Actual building materials

- Insulation manufacturers often sell their products in terms of "R-value per inch"


**PRODUCT OVERVIEW**

FOAMULAR 150 extruded polystyrene (XPS) rigid foam insulation contains hundreds of millions of densely packed closed cells to provide exceptional thermal performance. It's also virtually impervious to moisture, unlike other plastic foam insulation products, preventing loss of R-value due to moisture penetration. FOAMULAR weighs considerably less than plywood, OSB or other non-insulation materials so it's easier, faster and safer to install. Plus, the product's built-in rigidity means it can be scored and snapped, cut, or sawed with common tools. Sagging and settling are never a problem. Retains it's long-term R-value year after year, even following prolonged exposure to water leakage, humidity, condensation, ground water and freeze/thaw cycling. Contains a minimum of 20% certified recycled content, certified GreenGuard Indoor Air Quality for Children and Schools, Energy Star Seal and Insulate Program, and NAHB Green approved. Owens Corning Foam Insulation, LLC now warrants a Lifetime Limited Warranty on FOAMULAR Extruded Polystyrene (XPS) Foam Insulation products. This new, enhanced warranty indicates that for the lifetime of the product, FOAMULAR XPS Insulation products are free from defects in material and/or workmanship that materially affect the performance of the product in a building installation.

- Exceptional thermal performance at r-5 per in.
- Virtually impervious to moisture penetration
- For exterior wall sheathing, wall furring, perimeter/foundation, cavity wall, crawlspace, pre-cast concrete, under slab and other applications
- Fast, easy installation
- Available in a wide range of sizes, thicknesses and edge trims
- Compressive strength of 15 psi; astm c578 type x
- Will retain at least 90 percent of their advertised r-value
- MFG Model # : 45W
- MFG Part # : 270895

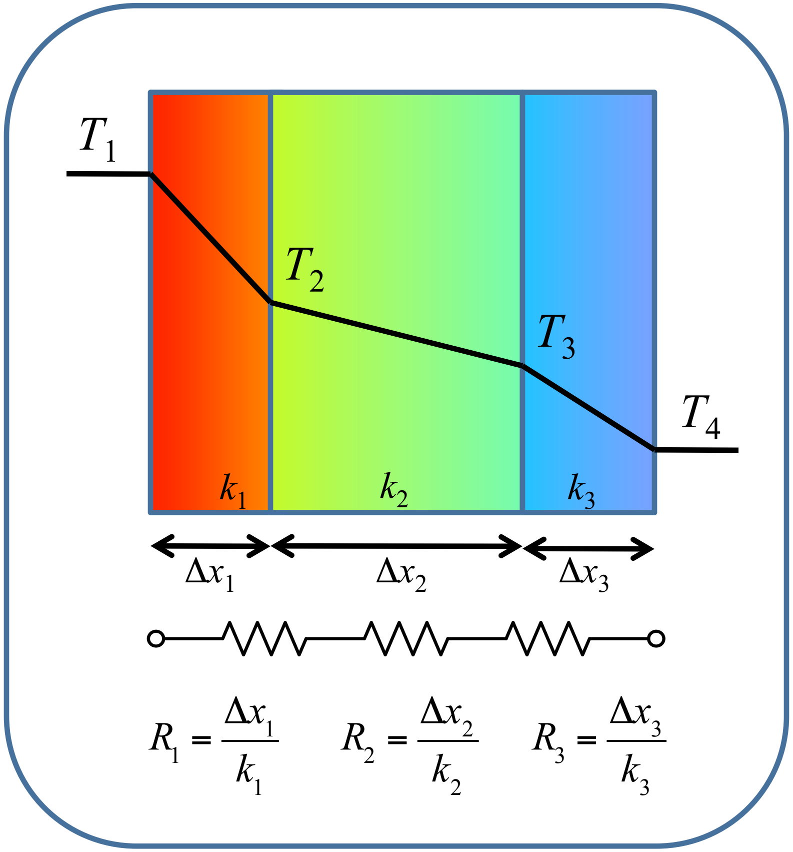
**Owens Corning FOAMULAR 2 inch x 48 inch x 8 feet foamboard**
Extruded polystyrene rigid foam insulation – closed cell

# Resistances of series

- Just as in electrical circuits, the overall thermal resistance of a series of elements can be expressed as the sum of the resistances of each layer:

- $R_{total} = R_1 + R_2 + R_3 + \ldots$

$$q = \frac{1}{R_{total}}\left(T_1 - T_4\right)$$




$$q = U_{total}\left(T_1 - T_4\right)$$

$$R_{total} = \frac{1}{U_{total}}$$

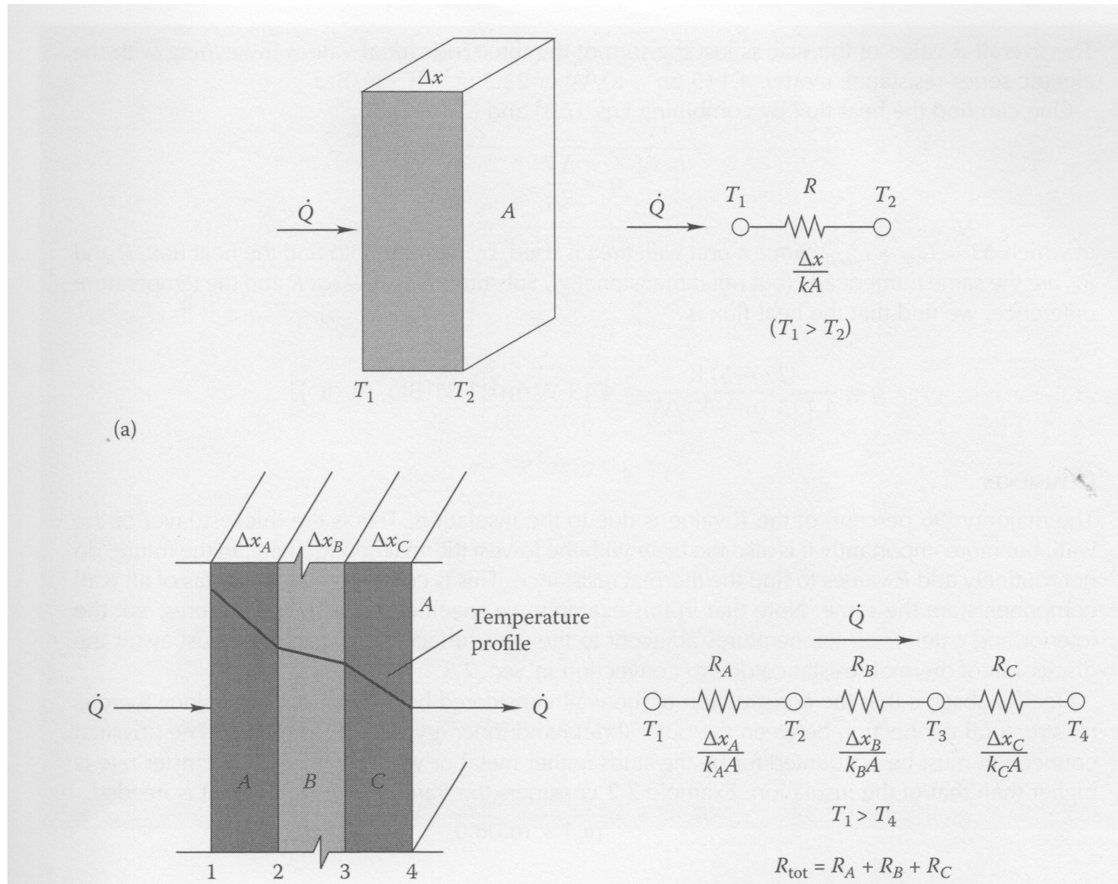
# Example 2.1

- R-value calculation for a building wall:

The outside wall of a home consist of a 10-cm layer of brick (k = 0.68 W/mK), a 15-cm layer of fiberglass insulation (k = 0.038 W/mK), and a 1-cm layer of gypsum board (k = 0.48 W/mK).

What is the overall R-value?

What is the heat flux through the wall if the interior surface temperature is 22°C and the exterior surface is 5°C?

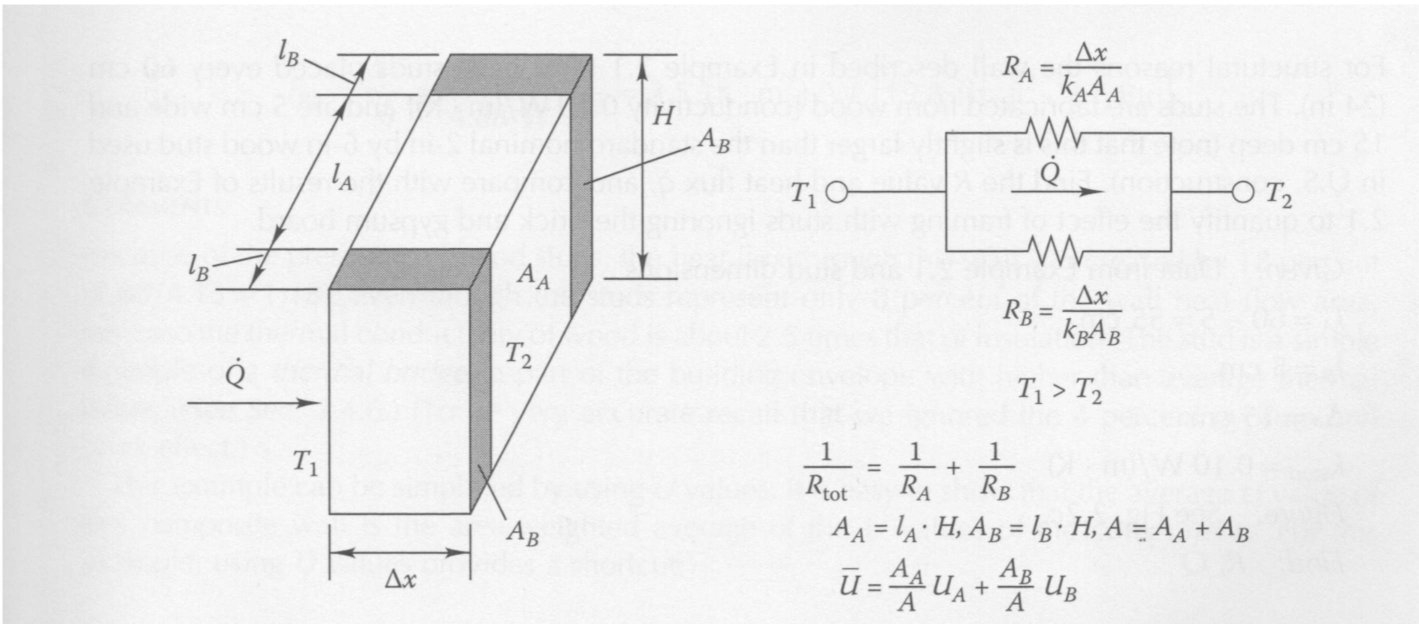


# What about more realistic constructions?

- Building walls rarely exist in complete, homogenous layers
- Structural elements – studs – are usually located within the envelope matrix at regular intervals

# Accounting for structural elements (studs)

- Parallel-resistance heat flow




- Note: this can also be done with weighted average U values

$$U_{total} = \frac{A_1}{A_{total}} U_1 + \frac{A_2}{A_{total}} U_2 + \ldots$$

# Accounting for structural elements (studs)

- Example 2.2: For structural reasons the wall described in Example 2.1 must have studs placed every 60 cm (24 inches)
- The studs are wood (k = 0.10 W/mK) and are 5 cm wide and 15 cm deep
- Find the R value of this assembly and heat flux, and compare to Example 2.1

- Structural elements form "thermal bridges"

# Thermal resistances of other shapes: cylinders

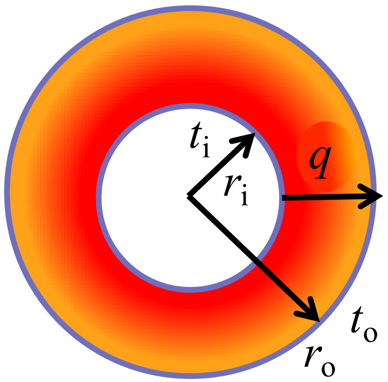
- Thermal resistances can be defined for non-planar elements
  - For example, to determine heat transfer through the walls of pipes and pipe insulation, we should understand heat transfer through cylinders

- Example: a hollow cylinder with length $L$, inner radius $r_i$ and outer radius $r_o$:

$$q = \frac{2\pi kL}{\ln\left(r_o/r_i\right)}\left(T_i - T_o\right) = \frac{\Delta T}{R_{cyl}}$$

so

$$R_{cyl} = \frac{\ln\left(r_o/r_i\right)}{2\pi kL}$$




$$R'_{cyl} = \frac{\ln\left(r_o/r_i\right)}{2\pi kL}$$

# Conduction in other geometries

- Often heat transfer is NOT 1-dimensional
  - Example: a pipe is carrying a heated or cooled fluid from a central plant, underground to a building for heating or cooling
    - Interactions with surroundings in all directions

- One way to account for this is the use of a "shape factor"
  - Shape factors account for 2-dimensional effects

$$Q = kS\Delta T$$

Where $S$ = shape factor [m]

- Another approach is a full two-dimensional analysis, although we don't cover in this class
  - Some coverage in CAE 463/524 Building Enclosure Design

# More shape factors

- Example 2.3: Heat loss from a buried pipe
  - A pipe with an outer surface temperature of 100°C and a radius of 15 cm is buried 30 cm deep in earth with a thermal conductivity of 1.7 W/mK. If the surface temperature of the pipe is 20°C and the pipe is uninsulated and 10 m long, what is the heat loss for the pipe?

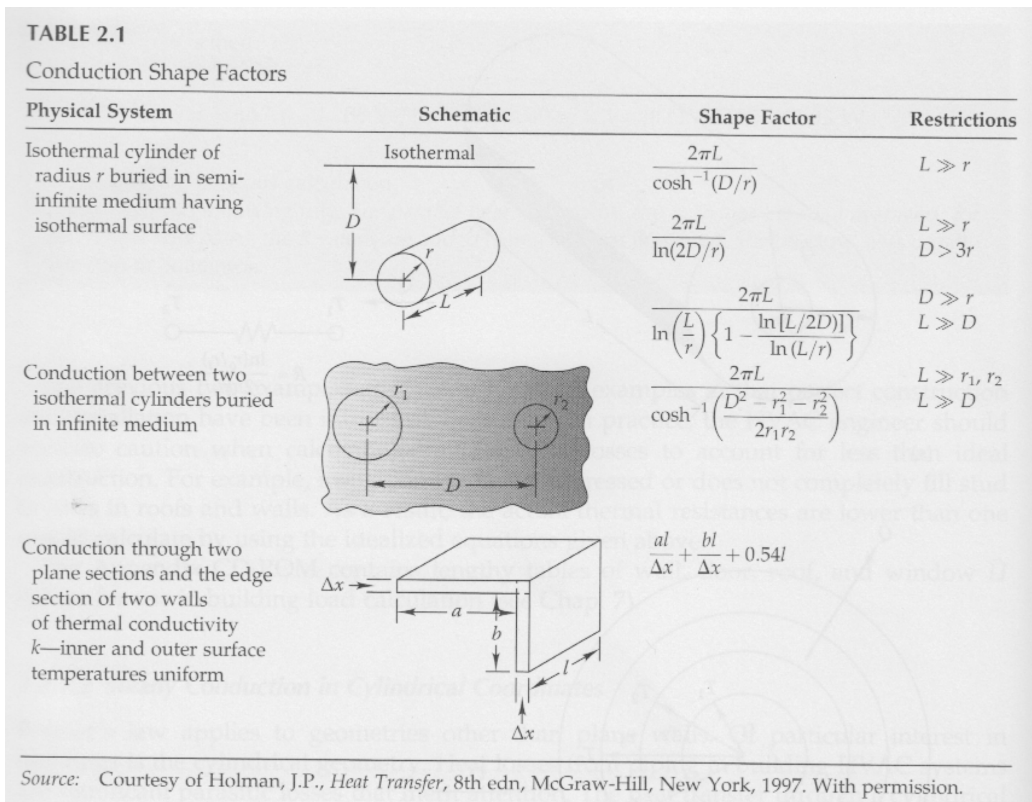


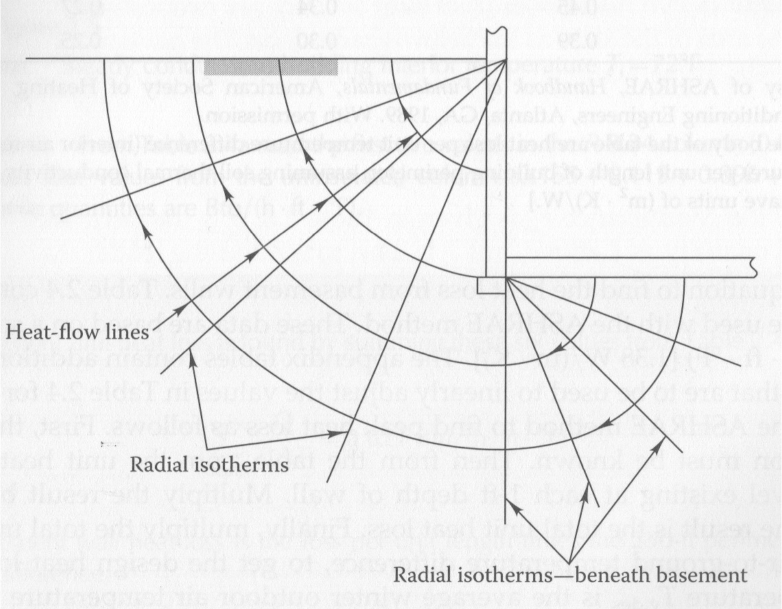
**TABLE 2.1**
Conduction Shape Factors

| Physical System | Schematic | Shape Factor | Restrictions |
|---|---|---|---|
| Isothermal cylinder of radius $r$ buried in semi-infinite medium having isothermal surface | Isothermal | $\dfrac{2\pi L}{\cosh^{-1}(D/r)}$ | $L \gg r$ |
| | | $\dfrac{2\pi L}{\ln(2D/r)}$ | $L \gg r$<br>$D > 3r$ |
| | | $\dfrac{2\pi L}{\ln\left(\dfrac{L}{r}\right)\left\{1 - \dfrac{\ln[L/2D)]}{\ln(L/r)}\right\}}$ | $D \gg r$<br>$L \gg D$ |
| Conduction between two isothermal cylinders buried in infinite medium | | $\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 - r_1^2 - r_2^2}{2r_1 r_2}\right)}$ | $L \gg r_1, r_2$<br>$L \gg D$ |
| Conduction through two plane sections and the edge section of two walls of thermal conductivity $k$—inner and outer surface temperatures uniform | | $\dfrac{al}{\Delta x} + \dfrac{bl}{\Delta x} + 0.54l$ | |

*Source:* Courtesy of Holman, J.P., *Heat Transfer*, 8th edn, McGraw-Hill, New York, 1997. With permission.

# Ground coupled heat transfer

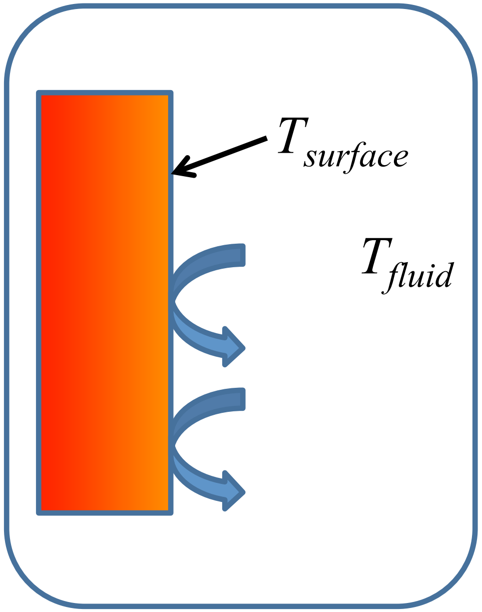
- What about heat transfer through a basement or floor slab of a building?




- This is fairly complicated
  - Depends on estimation method, soil type, etc.
  - Because this is one of the least accurate procedures of any building thermal analysis, and the level of complexity is rather high, we will save this for CAE 463/524

# Convection

- Convective heat transfer occurs between a solid and a moving fluid
  - Since heat transfer to a still fluid causes buoyancy which moves the fluid, **all** solid-fluid heat transfer is convective
- The heat transfer coefficient, $h_{conv}$, relates the heat transfer to the difference between the solid wall temp, $T_{surface}$, and the effective temperature of the fluid far from the surface, $T_{fluid}$




$$q_{conv} = h_{conv}\left(T_{fluid} - T_{surface}\right) = \frac{T_{fluid} - T_{surface}}{R_{conv}} = \frac{\Delta T}{R_{conv}}$$

where $T_{fluid}$ = fluid temp far enough not to be affected by $T_{surface}$

$h_{conv}$ = conductive heat transfer coefficient [W/(m$^2\cdot$K)]

and $R_{conv} = \dfrac{1}{h_{conv}}$ = convective thermal resistance [(m$^2\cdot$K)/W]

# Types of convective heat transfer

- In general, the higher the velocity of fluid flow, the higher the rate of convection heat transfer

- Two kinds of convection exist:
  - Natural (or free) convection: Results from density differences in the fluid caused by contact with the surface to or from which the heat transfer occurs
    - Buoyancy is the main driver
    - Example: The gentle circulation of air in a room caused by the presence of a solar-warmed window or wall (no mechanical system) is a manifestation of natural/free convection
  - Forced convection: Results from a force external to the problem (other than gravity or other body forces) moves a fluid past a warmer or cooler surface
    - Usually much higher velocities, driven by mechanical forces (e.g. fans)
    - Example: Heat transfer between cooling coils and an air stream

# Q versus q for convection

- Same story as conduction…

$$q_{conv} = h_{conv}\left(T_{fluid} - T_{surface}\right) \qquad \left[\frac{\text{W}}{\text{m}^2}\right]$$

- To get $Q$, just multiply by surface area, $A$

$$Q_{conv} = h_{conv}A\left(T_{fluid} - T_{surface}\right) \qquad \left[\text{W}\right]$$

Also known as Newton's law of cooling

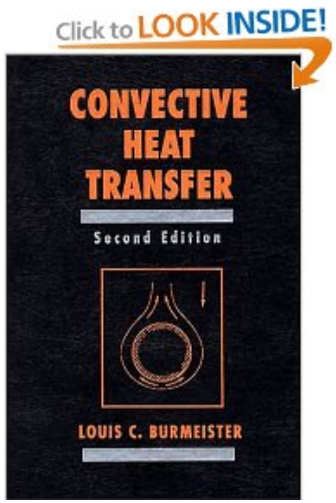
**TABLE 2.9**

Magnitude of Convection Coefficients

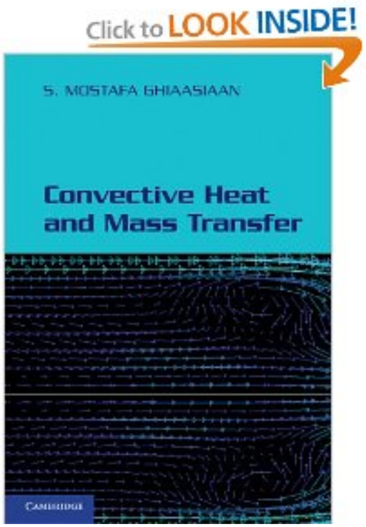
| Arrangement | $W/(m^2 \cdot K)$ | $Btu/(h \cdot ft^2 \cdot F)$ |
|---|---|---|
| Air, free convection | 6–30 | 1–5 |
| Superheated steam or air, forced convection | 30–300 | 5–50 |
| Oil, forced convection | 60–1800 | 10–300 |
| Water, forced convection | 300–6000 | 50–1000 |
| Water, boiling | 3000–60,000 | 500–10,000 |
| Steam, condensing | 6000–120,000 | 1000–20,000 |

The conversion between SI and USCS units is 5.678 $W/(m^2 \cdot K) = 1$ $Btu/(h \cdot ft^2 \cdot °F)$.

# Convection is really a field of its own

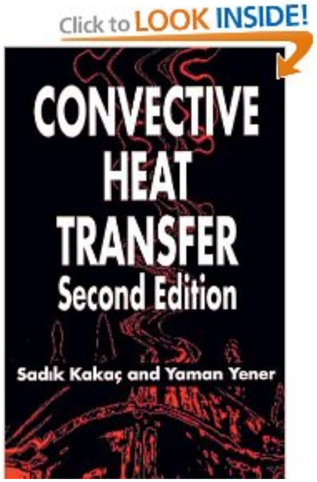













# Important notes on convection in building science

- Convective heat transfer coefficients can depend upon details of the surface-fluid interface
  - Rough surfaces have higher rates of convection
  - Orientation matters for *natural* convection
  - Natural heat transfer coefficients can depend upon the actual fluid temperature and not just the temperature difference

# Convective heat transfer coefficient, $h_{conv}$

- The convective heat transfer coefficient, $h_{conv}$, will take on many forms depending upon whether the convection is forced or natural
  - Natural convection occurs when buoyancy effects induce air motion
    - *Temperature-dependent density differences*

$$\rho = \frac{n}{V} = \frac{P}{RT}$$

**Hold P and R constant…**

$T \downarrow \rho \uparrow \qquad T \uparrow \rho \downarrow$

  - Forced convection occurs when an external force (e.g. fan or wind) imposes air motion (more random and chaotic)
  - $h_c$ is also known as the film coefficient or the surface conductance
- The next few slides show some of the important convective equations that arise in computing heat transfer to/from walls, floors

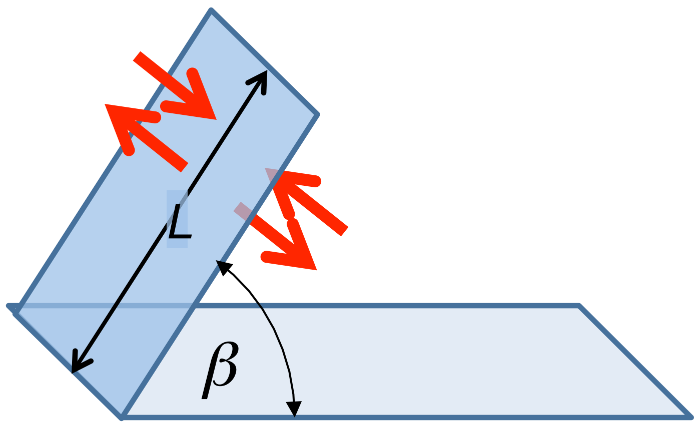
# External flows for buildings

- Flows over unconfined geometries, such as airflow over the wall of a building or across a bank of tubes in a heat exchanger

- Laminar versus turbulent
  - When the temperature differences are high enough the natural motion is turbulent, the result is more mixing and higher heat transfer
    - So, for high temperature differences, the heat transfer coefficient is larger and has a different equation than for lower $\Delta T$
  - Nearly all forced convection is turbulent
  - Free convection can be either

- Laminar flow occurs for cases when: $L^3 \Delta T < 1.0$ in SI units

# Free convection in air from a tilted surface




$h_{\mathrm{conv}}$ in [W/(m$^2$ K)]

For natural convection to or from either side of a vertical surface or a sloped surface with $\beta > 30°$

For laminar: $$h_{conv} = 1.42\left(\frac{\Delta T}{L}\sin\beta\right)^{\frac{1}{4}}$$ [Kreider 2.18SI]

For turbulent: $$h_{conv} = 1.31\left(\Delta T\sin\beta\right)^{\frac{1}{3}}$$ [Kreider 2.19SI]

Note that these equations are dimensional, so they are different for IP and SI

# Free convection from horizontal pipes in air

- For cylindrical pipes of outer diameter, $D$, in [m]

For laminar: $$h_{conv} = 1.32\left(\frac{\Delta T}{D}\right)^{\frac{1}{4}}$$ [Kreider 2.20SI]

For turbulent: $$h_{conv} = 1.24\left(\Delta T\right)^{\frac{1}{3}}$$ [Kreider 2.21SI]

# Free convection for surfaces

- Warm horizontal surfaces facing up
  - e.g. up from a warm floor or up to a cold ceiling


laminar: $h_{conv} \approx 1.32\left(\frac{\Delta T}{L}\right)^{1/4}$ [Kreider 2.22SI]

turbulent: $h_{conv} \approx 1.52\left(\Delta T\right)^{1/3}$ [Kreider 2.23SI]

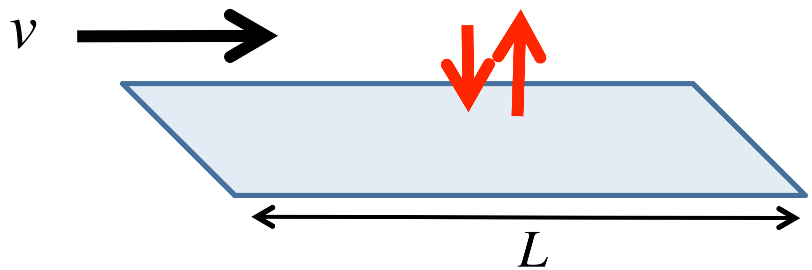
# Free convection for surfaces

- Warm horizontal surface facing down
  - Convection is reduced because of stratification
    - e.g. a warm ceiling facing down (works against buoyancy)
    - Also applies for cooled flat surfaces facing up (like a cold floor)


$$h_{conv} \approx 0.59\left(\frac{\Delta T}{L}\right)^{1/4}$$ both laminar and turbulent

# Forced convection over planes

- Does not depend on their orientation




laminar: $h_{conv} \approx 2.0\left(\dfrac{v}{L}\right)^{1/2}$ [Kreider 2.24SI]

turbulent: $h_{conv} \approx 6.2\left(\dfrac{v^4}{L}\right)^{1/5}$ [Kreider 2.25SI]

*Velocity is in m/s

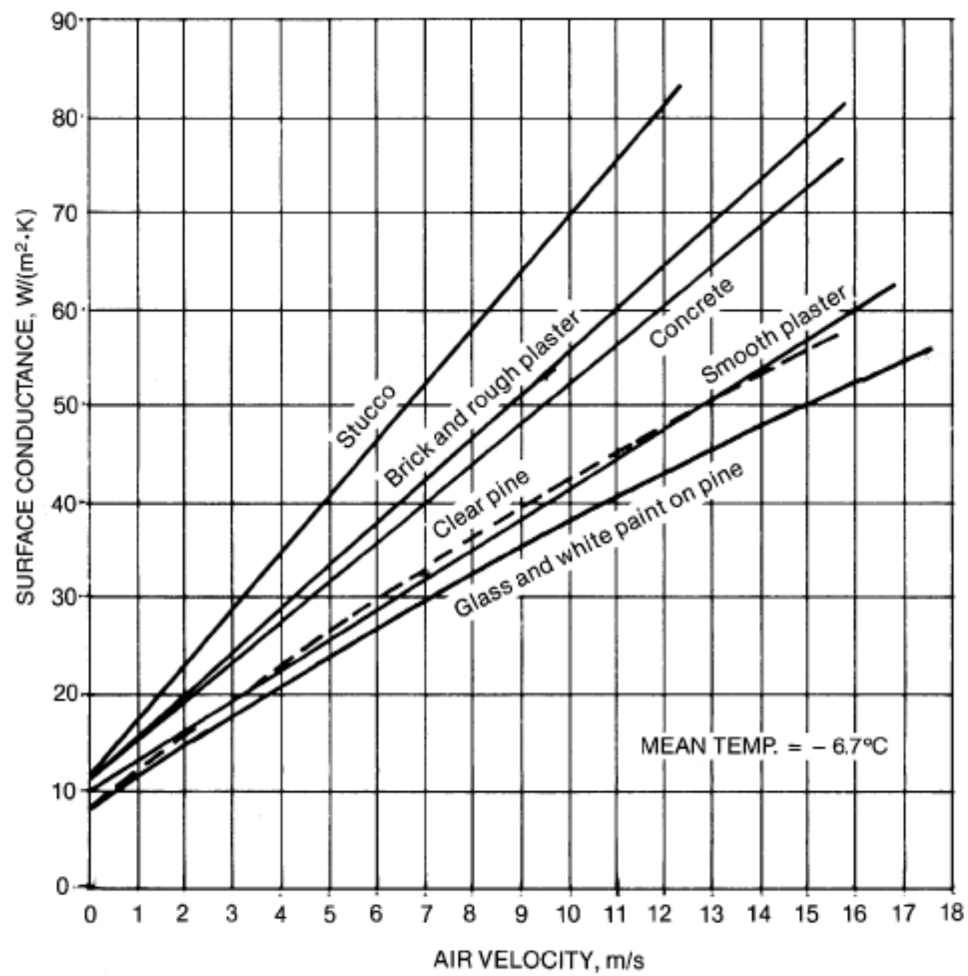
# Convection example

- Estimate the convective heat transfer coefficient along a wall in the classroom, assuming either forced or natural convection

# $h_{conv}$ for forced convection on exterior walls

- For forced convection $h_{conv}$ depends upon surface roughness and air velocity but not orientation

- The figure at right is taken from ASHRAE HOF



# Most popular general $h_{conv}$ for forced convection

There are two relationships for $h_{conv}$ (forced convection) which are commonly used, depending on wind speed:

- For $1 < v_{wind} < 5$ m/s

  $h_{conv} = 5.6 + 3.9 v_{wind}$  [W/(m²·K)]  [Straube 5.15]

- For $5 < v_{wind} < 30$ m/s

  $h_{conv} = 7.2 v_{wind}^{0.78}$  [W/(m²·K)]  [Straube 5.16]

*Good for use with external surfaces

# Internal flows within building HVAC systems

- Flows of fluids confined by boundaries (such as the sides of a duct) are called internal flows

- Mechanisms of convection are different
  - And so are the equations

# Forced convection for fully developed turbulent flow

- Air through ducts and pipes

$$h_{conv} \approx 8.8\left(\frac{v^4}{D_h}\right)^{1/5}$$ [Kreider 2.26SI]

$D_h$ = the hydraulic diameter: 4 times the ratio of the flow conduit's cross-sectional area divided by the perimeter of the conduit

$$D_h = \frac{4\left(\frac{\pi D^2}{4}\right)}{\pi D}$$ [Kreider 2.27SI]

- Water flow through pipes:

$$h_{conv} \approx 3580(1+0.015T)\left(\frac{v^4}{D_h}\right)^{1/5}$$ [Kreider 2.28SI]

# Convective R-value

- Convective heat transfer can be considered an conductive layer in contact with air and so we can assign an R-value to it

- Looking back at our definitions we can see that the equivalent R value is simply the reciprocal of the heat transfer coefficient so

$$R_{conv} = \frac{1}{h_{conv}}$$

# Convection example

- What were the convective resistances from the previous example of the classroom wall?

- How does the convective thermal resistance compare to that of insulation in building walls and roofs?

# Typical convective surface resistances

- We often use the values given below for most conditions

| Surface Conditions | Horizontal Heat Flow | Upwards Heat Flow | Downwards Heat Flow |
|---|---|---|---|
| Indoors: $R_{in}$ | 0.12 $m^2K/W$ (SI)<br>0.68 $h\cdot ft^2\cdot {}^\circ F/Btu$ (IP) | 0.11 $m^2K/W$ (SI)<br>0.62 $h\cdot ft^2\cdot {}^\circ F/Btu$ (IP) | 0.16 $m^2K/W$ (SI)<br>0.91 $h\cdot ft^2\cdot {}^\circ F/Btu$ (IP) |
| $R_{out}$: 6.7 m/s wind (Winter) | | 0.030 $m^2K/W$ (SI)<br>0.17 $h\cdot ft^2\cdot {}^\circ F/Btu$ (IP) | |
| $R_{out}$: 3.4 m/s wind (Summer) | | 0.044 $m^2K/W$ (SI)<br>0.25 $h\cdot ft^2\cdot {}^\circ F/Btu$ (IP) | |

# Convection and heat exchangers

- Heat exchangers are used widely in buildings
- Heat exchanges are devices in which two fluid streams, usually separated from each other by a solid wall, exchange thermal energy by convection
  - One fluid is typically heated, one is cooled
  - Fluids may be gases, liquids, or vapors

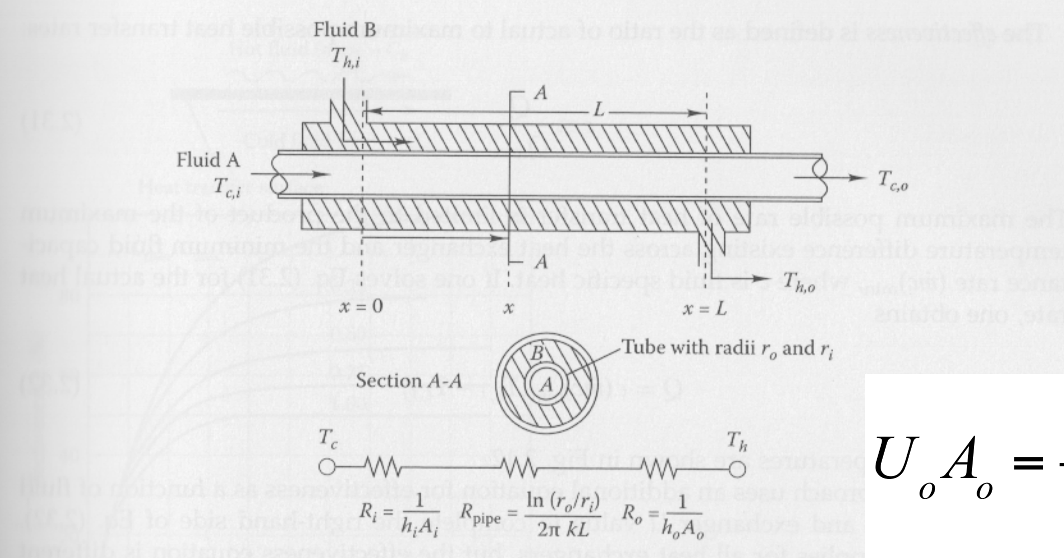
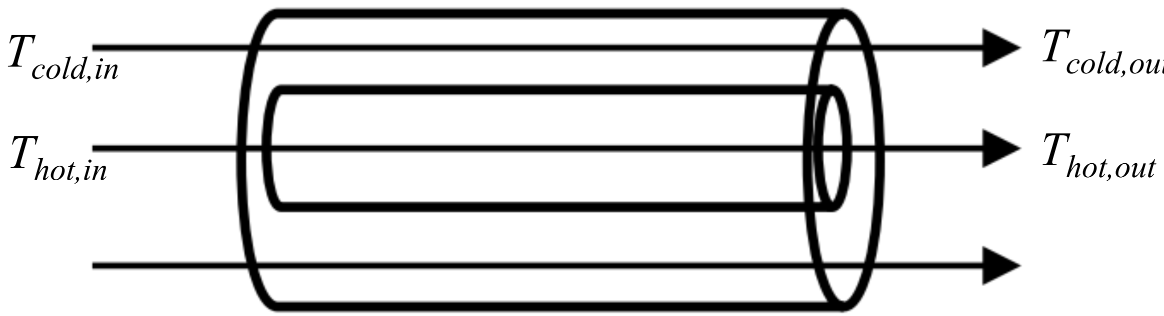


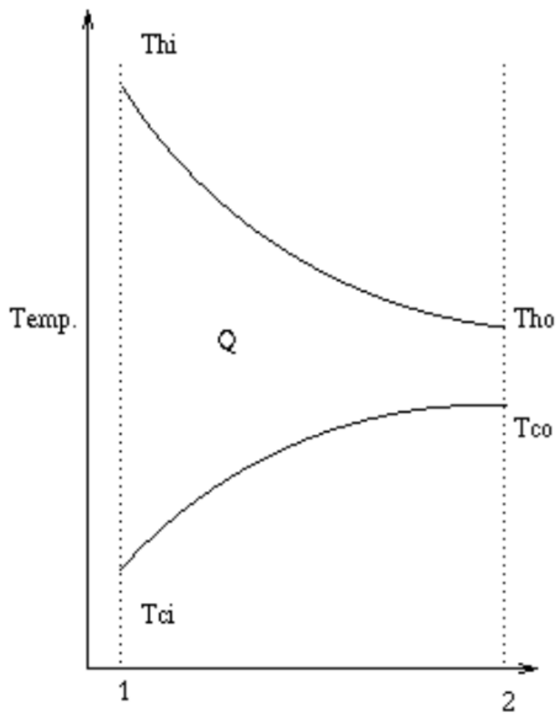

FIGURE 2.12
Schematic diagram of parallel-flow shell-and-tube heat exchanger showing fluid temperatures and equivalent thermal circuit.

$$U_o A_o = \frac{1}{R_{conv,i} + R_{pipe} + R_{conv,o}}$$

# Heat exchangers

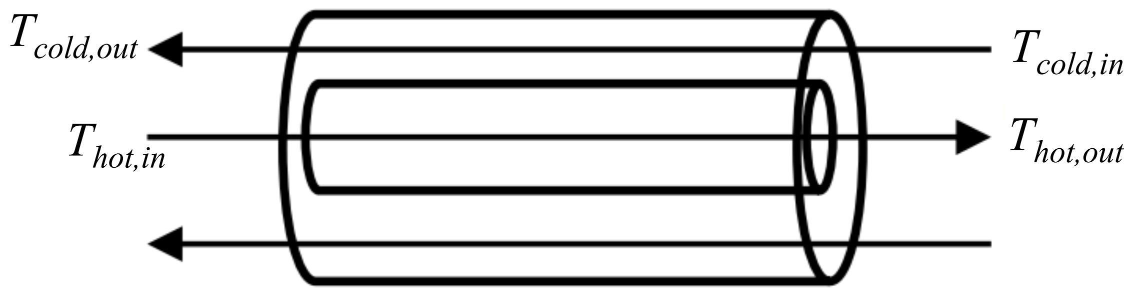
- Parallel flow: fluids flowing in the same direction

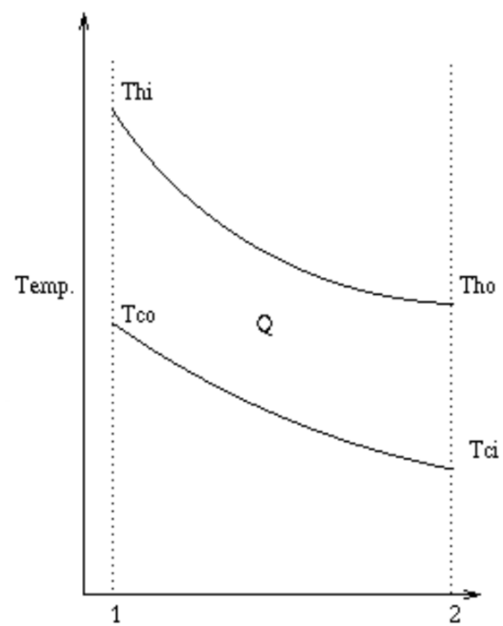


# Heat exchangers

- Counterflow: one fluid flows in the opposite direction
  - More efficient than parallel flow



# Heat exchangers

- Heat transfer in heat exchangers

$$Q = UA\Delta T_{mean}$$

$$\Delta T_{mean} = \frac{\Delta T_{hot} - \Delta T_{cold}}{\ln\left(\dfrac{\Delta T_{hot}}{\Delta T_{cold}}\right)}$$

- Method for predicting heat transfer rate in heat exchangers:
  - $\epsilon$-NTU method: Effectiveness number-of-transfer-units approach

# Heat exchangers: $\in$-NTU method

- Define effectiveness: ratio of actual to maximum possible heat transfer rates

$$\in = \frac{Q}{Q_{\max}}$$

- This maximum rate of heat transfer is limited to the product of the maximum temperature difference across the heat exchanger and the minimum fluid capacitance rate:

$$Q = \in (\dot{m}C_p)_{\min}\left(T_{hot,in} - T_{cold,in}\right)$$

- The idea is that heat transfer will almost never be its maximum because the hot and cold T's are constantly changing (and changing the driving force)

# Heat exchangers: ∊-NTU method

- The effectiveness of different types of heat exchangers can be described with various equations, all using the term number of transfer units, or “NTU”

$$NTU = \frac{U_o A_o}{(\dot{m}C_p)_{\min}}$$

Where the denominator is the smaller of the two fluid capacitance rates: $\dot{C}_{\min} = (\dot{m}C_p)_{\min}$

**TABLE 2.10**

Heat Exchanger Effectiveness Relations $N = \mathrm{NTU} = \frac{U_o A_o}{\dot{C}_{\min}}$  $C = \frac{\dot{C}_{\min}}{\dot{C}_{\max}}$

| Flow Geometry | Relation |
|---|---|
| Double pipe | |
| Parallel flow | $\varepsilon = \frac{1 - \exp[-N(1+C)]}{1+C}$ |
| Counterflow | $\varepsilon = \frac{1 - \exp[-N(1-C)]}{1 - C\exp[-N(1-C)]}$ |
| Crossflow | |
| Both fluids unmixed | $\varepsilon = 1 - \exp\left\{\frac{1}{Cn}[\exp(-NCn) - 1]\right\}$ where $n = N^{-0.22}$ |
| Both fluids unmixed | $\varepsilon = N\left[\frac{N}{1-\exp(-N)} + \frac{NC}{1-\exp(-NC)} - 1\right]^{-1}$ |
| $\dot{C}_{\max}$ mixed, $\dot{C}_{\min}$ unmixed | $\varepsilon = \frac{1}{C}\{1 - \exp[-C + C\ \exp(-N)]\}$ |
| $\dot{C}_{\max}$ unmixed, $\dot{C}_{\min}$ mixed | $\varepsilon = 1 - \exp\left\{-\frac{1}{C}[1 - \exp(-NC)]\right\}$ |
| Shell and tube | |
| One shell pass; two, four, six tube passes | $\varepsilon = 2\left[1 + C + \sqrt{1+C^2}\frac{1+\exp(-N\sqrt{1+C^2})}{1-\exp(-N\sqrt{1+C^2})}\right]^{-1}$ |

# Heat exchangers: ∈-NTU method

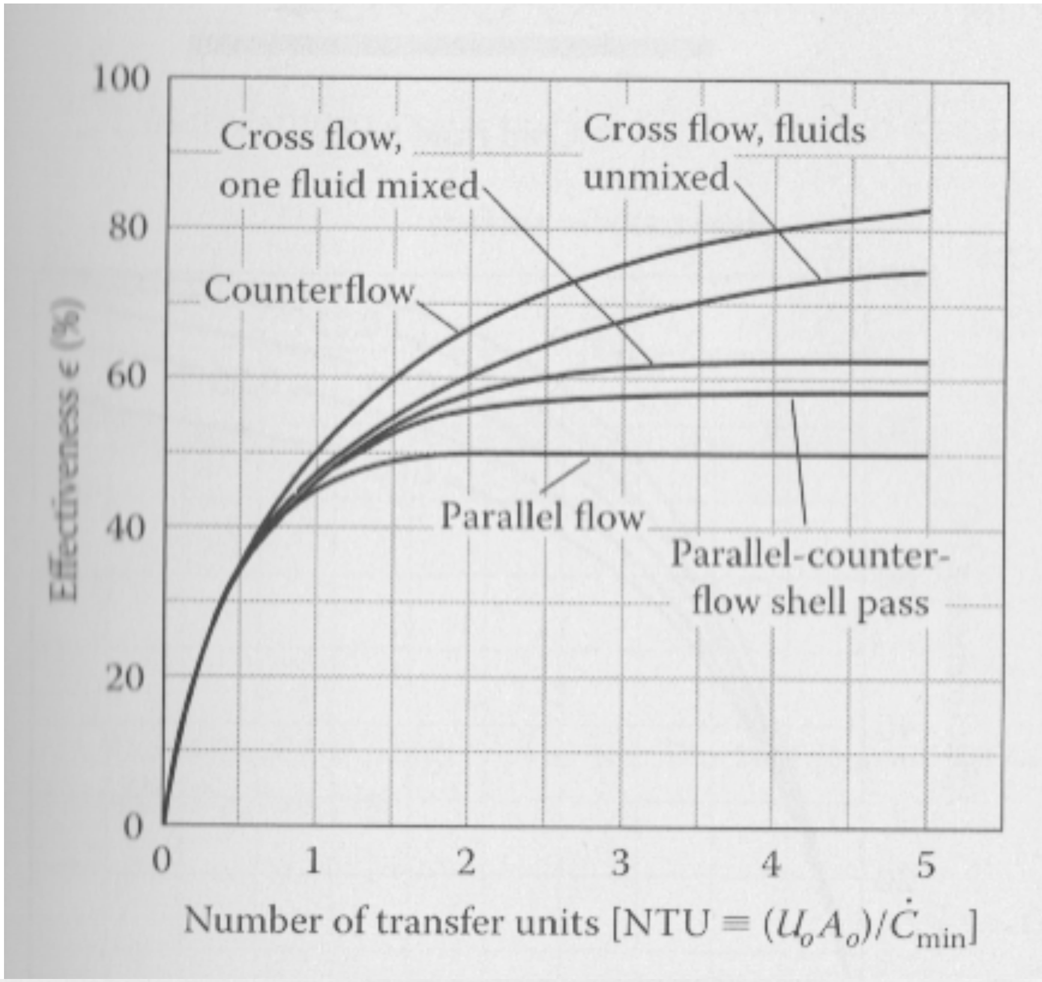
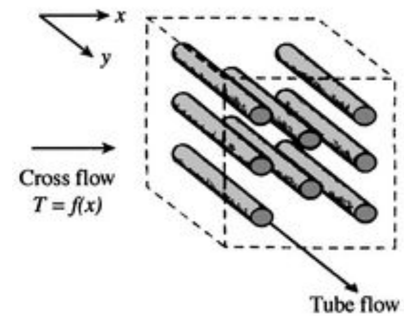



**FIGURE 2.14**
Comparison of effectiveness of several heat exchanger designs for equal hot- and cold-side capacitance rates, $\dot{C}_{min} = \dot{C}_{max}$.

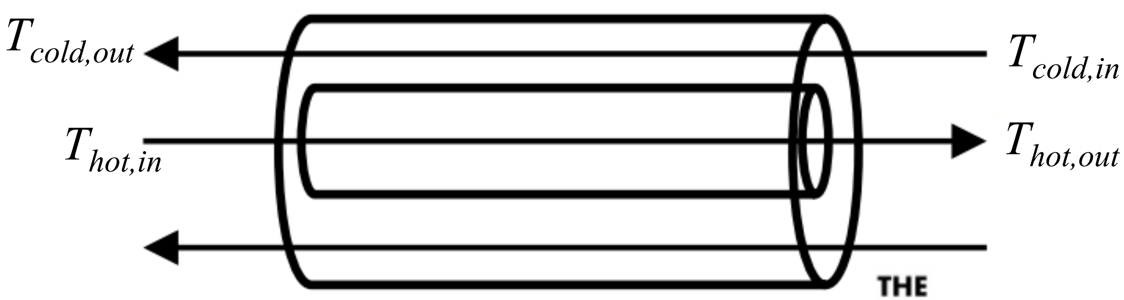


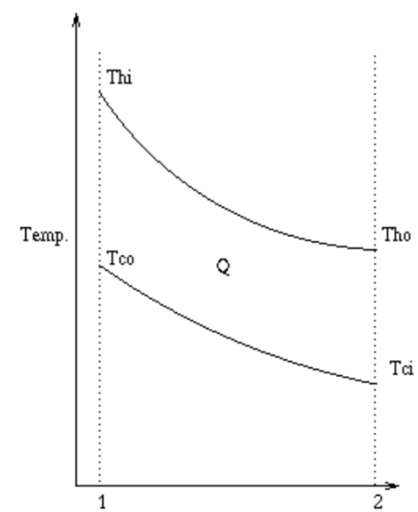

**This subject is covered in detail in CAE 464 HVAC Design**

# Heat exchanger example

- Example 2.7: Potable service water is heated in a building from 20°C at a rate of 70 kg/min by using nonpotable pressurized water from a boiler at 110°C in a single-pass counterflow heat exchanger

- Find the heat transfer rate if the hot water flow is 90 kg/min
- Also find exit temperatures of both streams
  - Note: The overall U value is 320 W/(m$^2$K) and the transfer area is 20 m$^2$



# Bulk convective heat transfer: “Advection”

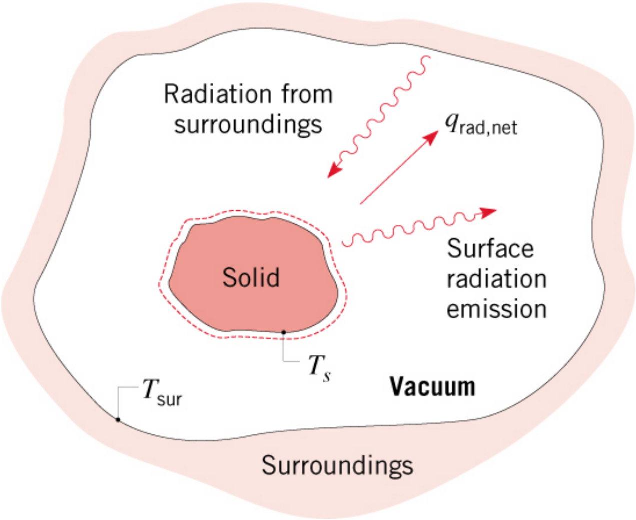
- Bulk convective heat transfer, or advection, is more direct than convection between surfaces and fluids
- Bulk convective heat transfer is the transport of heat by airflow
  - Air has a capacity to store heat, so air flowing into or out of a building carries heat with it

$$Q_{bulk} = \dot{m} C_p \Delta T \qquad [\mathrm{W}]=[\frac{\mathrm{kg}}{\mathrm{s}} \cdot \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}} \cdot \mathrm{K}]$$

*m “dot”* = mass flow rate of air (kg/s)
$C_p$ = specific heat capacity of air [J/(kgK)]

# Radiation

- All objects above absolute zero radiate electromagnetic energy due to the emission of photons from electrons changing energy levels




- Net radiation heat transfer occurs when an object radiates a different amount of electromagnetic energy than it absorbs

- If all the surrounding objects are at the same temperature, the net heat transfer will be zero

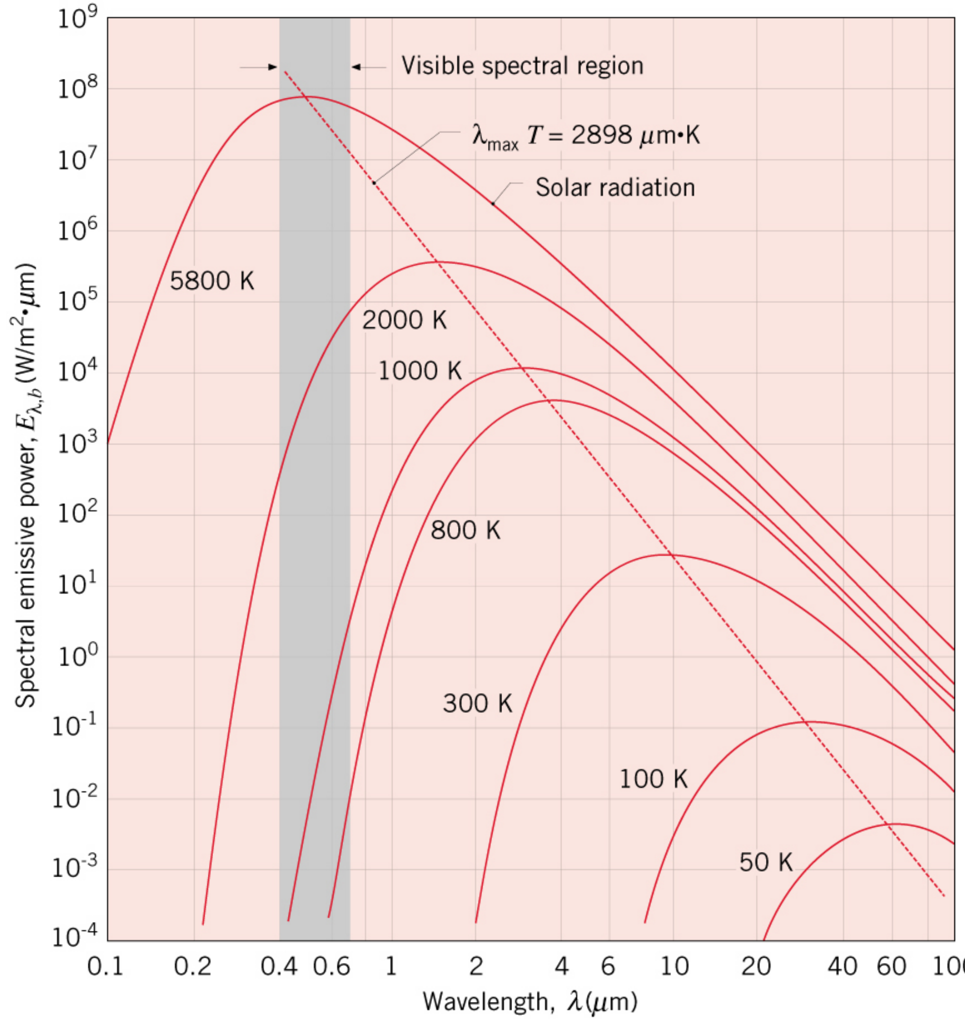
# Black body radiation

- Radiation from a perfect radiator follows the black body curve

- The peak of the black body curve depends on the object's temperature

- Peak radiation from the sun is in the visible region
  - About 0.4 to 0.7 µm

- Radiation involved in building surfaces is in the infrared region
  - Greater than 0.7 µm



# Black body radiation

- For any object with an absolute surface temperature, T, the radiation from the object will be:

$$q_{rad} = \varepsilon \sigma T^4$$

where

$\varepsilon$=emissivity

$\sigma$ = Stefan-Boltzmann constant = $5.670 \times 10^{-8}\ \frac{\mathrm{W}}{\mathrm{m^2 \cdot K^4}}$

$T$ = Absolute temperature in Kelvin

# Emissivity and absorptivity

- Real surfaces emit less radiation than ideal “black” ones
  - The ratio of energy radiated by a given body to a perfect black body at the same temperature is called the emissivity: $\varepsilon$

- $\varepsilon$ is dependent on wavelength, but for most common building materials (e.g. brick, concrete, wood…), $\varepsilon$ = 0.9 at most wavelengths

# Emissivity of common building materials

**TABLE 2.11**

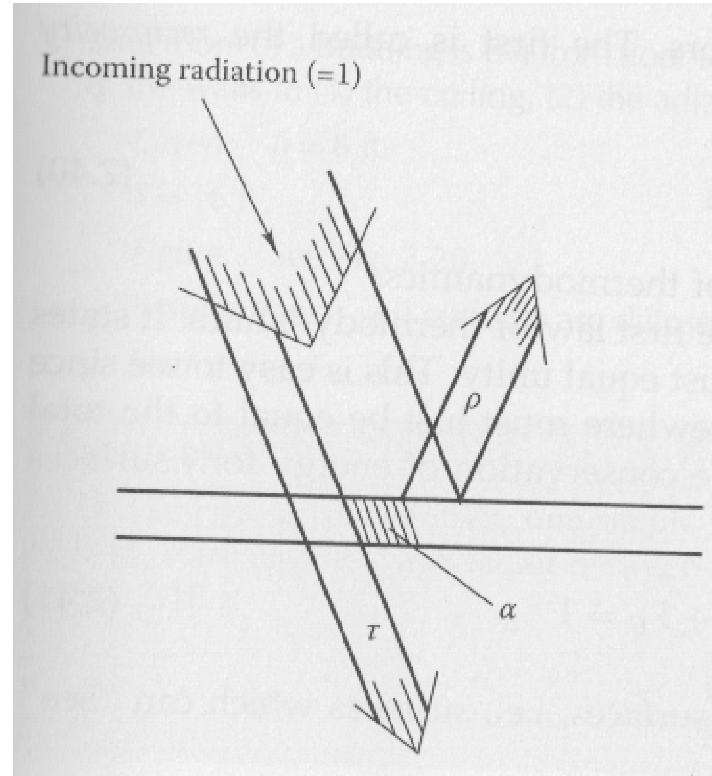
Emissivities of Some Common Building Materials at Specified Temperatures

| Surface | Temperature, °C | Temperature, °F | $\epsilon$ |
|---|---|---|---|
| Brick | | | |
| Red, rough | 40 | 100 | 0.93 |
| Concrete | | | |
| Rough | 40 | 100 | 0.94 |
| Glass | | | |
| Smooth | 40 | 100 | 0.94 |
| Ice | | | |
| Smooth | 0 | 32 | 0.97 |
| Marble | | | |
| White | 40 | 100 | 0.95 |
| Paints | | | |
| Black gloss | 40 | 100 | 0.90 |
| White | 40 | 100 | 0.89–0.97 |
| Various oil paints | 40 | 100 | 0.92–0.96 |
| Paper | | | |
| White | 40 | 100 | 0.95 |
| Sandstone | 40–250 | 100–500 | 0.83–0.90 |
| Snow | −12–−6 | 10–20 | 0.82 |
| Water | | | |
| 0.1 mm or more thick | 40 | 100 | 0.96 |
| Wood | | | |
| Oak, planed | 40 | 100 | 0.90 |
| Walnut, sanded | 40 | 100 | 0.83 |
| Spruce, sanded | 40 | 100 | 0.82 |
| Beech | 40 | 100 | 0.94 |

*Source:* Courtesy of Sparrow, E.M. and Cess, R.D., *Radiation Heat Transfer*, augmented edn, Hemisphere, New York, 1978. With permission.

# Absorptivity, transmissivity, and reflectivity

- The absorptivity, $\alpha$, is the fraction of energy hitting an object that is actually absorbed
- Transmissivity, $\tau$, is a measure of how much radiation passes through an object
- Reflectivity, $\rho$, is a measure of how much radiation is reflected off an object

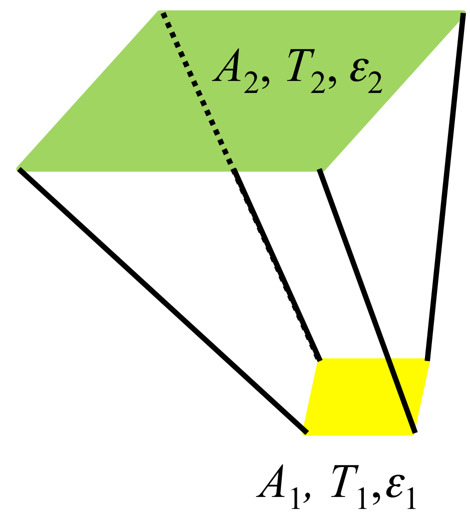



$$\alpha + \tau + \rho = 1$$

# Radiation heat transfer between surfaces

- If a material follows Kirchoff's law, (absorptivity = emissivity) we can write the net heat transfer between surfaces 1 and 2 as:

$$Q_{1\to 2} = \frac{A_1 \sigma \left(T_1^4 - T_2^4\right)}{\frac{\rho_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{\rho_2 A_1}{\varepsilon_2 A_2}}$$




where $\varepsilon_1$ and $\varepsilon_2$ are the surface emittances,

$A_1$ and $A_2$ are the surface areas

and $F_{1\to 2}$ is the view factor from surface 1 to 2

$F_{1\to 2}$ is a function of geometry only

# View factors

- Radiation travels only in a straight line
  - Areas and angle of incidence between two exchanging surfaces influences radiative heat transfer

**Some common view factors**

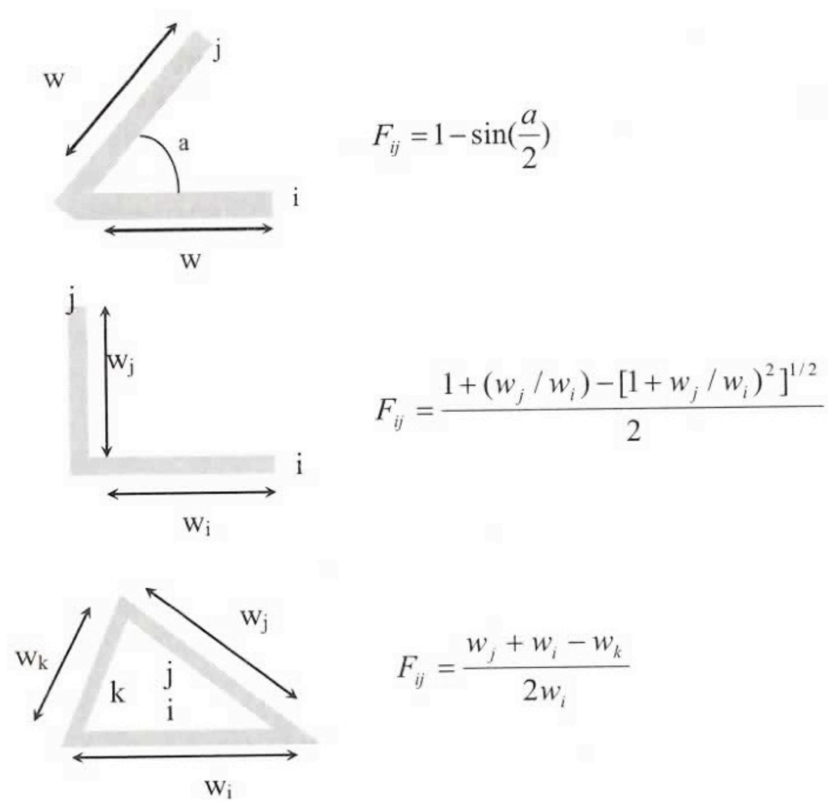
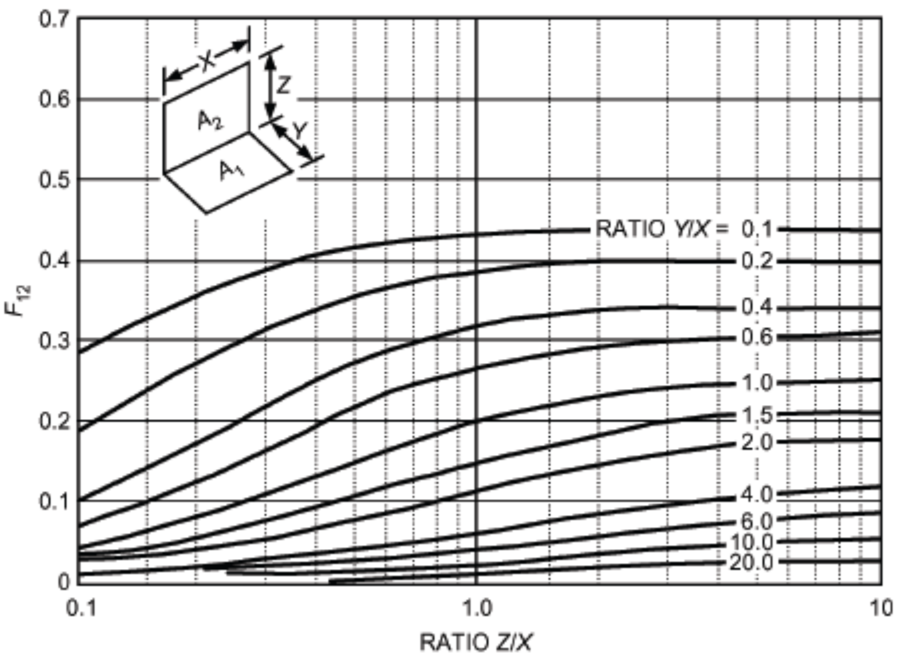


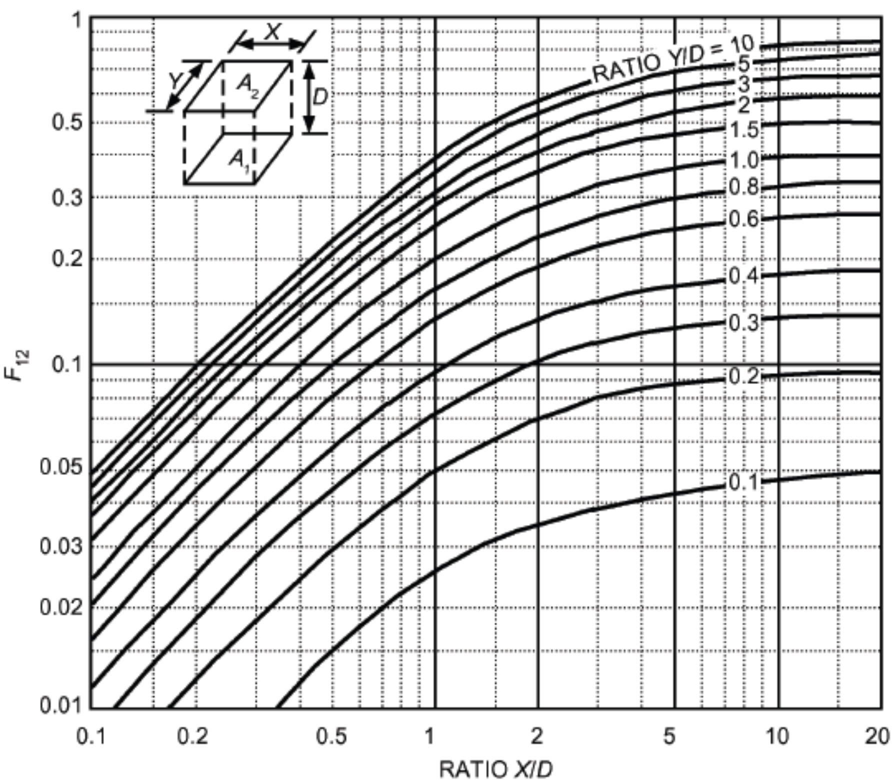

Figure 5.6: View factors for common situations in building enclosures [Hagentoft 2000]

# Common view factors

- Other common view factors from ASHRAE HOF




A. PERPENDICULAR RECTANGLES WITH COMMON EDGE




B. ALIGNED PARALLEL RECTANGLES

# Simplifying radiation

- We will sometimes simplify the equation for radiation heat transfer
  - For example, when two parallel planes are close together
  - $F_{12} = 1; A_1 = A_2 = A$
- From: $$Q_{1\rightarrow 2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{\rho_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{\rho_2 A_1}{\varepsilon_2 A_2}}$$ To: $$Q_{1\rightarrow 2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{\rho_1}{\varepsilon_1} + \frac{\rho_2}{\varepsilon_2}}$$
- Remember that $\alpha + \tau + \rho = 1$ … if we assume $\tau$ = 0 (opaque): $\rho = 1 - \varepsilon$

$$Q_{1\rightarrow 2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

# Simplifying radiation

- We can also define a radiation heat transfer coefficient that is analogous to other heat transfer coefficients

$$h_{rad} = \frac{q_{1\rightarrow 2}}{T_1 - T_2}$$

- When $A_1 = A_2$, and $T_1$ and $T_2$ are within ~50°F of each other, we can approximate $h_{rad}$ with a simpler equation:

$$h_{rad} = \frac{4\sigma T_{avg}^3}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

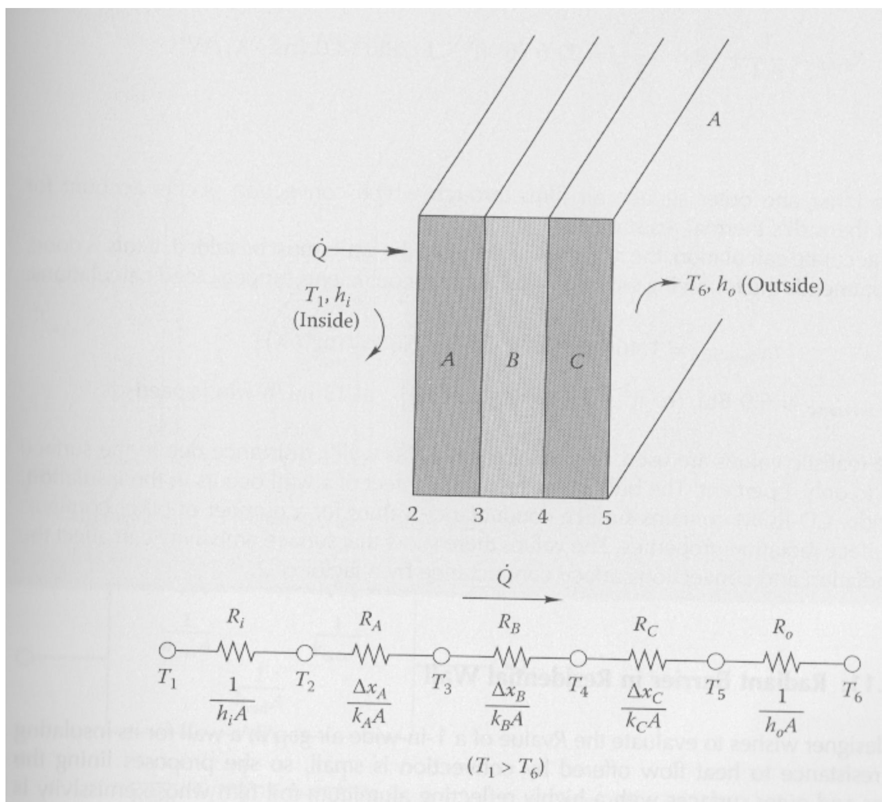
where

$$T_{avg} = \frac{T_1 + T_2}{2}$$

# COMBINED-MODE HEAT TRANSFER

# Combined mode heat transfer

- Nearly all heat transfer situations in buildings include more than one mode of heat transfer
- When more than one heat transfer mode is present, we can compute heat loss using resistances (of all kinds) in series



# Combined modes of heat transfer

- Example 2.10: Convection and wall R-values

- Repeat example 2.2 for a stud wall to include the effect of inner and outer surface convection coefficients

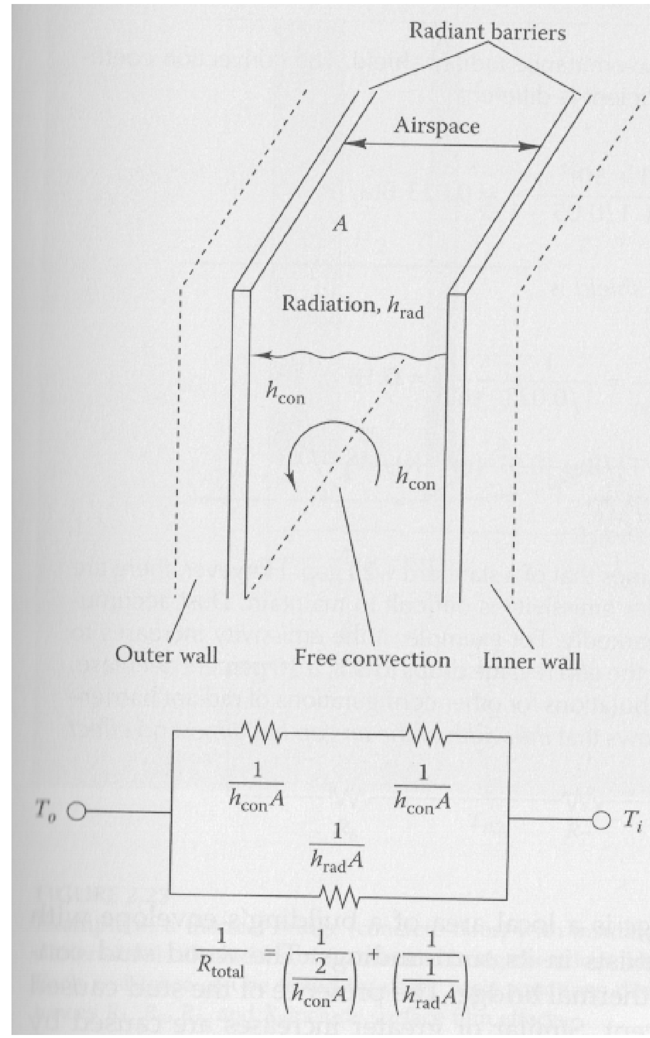
- Assume the same interior surface resistance from our previous classroom problem
  - Assume the outer surface coefficient comes from slide 49 with wind speed of ~2.5 m/s and winter conditions

# Combined modes of heat transfer

- Example 2.11: Radiant barrier in a residential wall

A building designer wishes to evaluate the R-value of a 1 inch wide air gap in a wall for its insulation effect. The resistance to heat flow offered by convection is small, so she proposes lining the cavity's inner and outer surfaces with a highly reflecting aluminum foil film whose emissivity is 0.05.

Find the R-value of this cavity, including both radiation and convection effects, if the surface temperatures facing the gap are 7.2°C and 12.8°C.



# Next time

- No class next Monday September 2nd (Labor Day)
- The following class, on Monday September 9th:
  - HW 2 due
  - Blog post #2 due (graduate students)
  - Continuing elements of heat transfer in buildings
    - Solar radiation
    - Windows
    - Building energy balances