Lecture 2: January 22, 2013
Human exposure patterns
Reactor models
Ventilation and air exchange
Review from last time

• Course overview
• Introduction to indoor air
  – Topics
  – Research
  – Literature
• Some basic air fundamentals

• Quick!
  – What is 50 µg/m$^3$ of NO$_2$ in ppb at standard temperature and pressure?
Today’s objectives

• Human exposure patterns
  – Inhalation and intake fractions

• Reactor models

• Ventilation and air exchange rates
Human exposure pathways

• How do we come in contact with environmental contaminants?
  – Ingestion (water, food, pharmaceuticals, hand-to-mouth)
  – Inhalation (our focus)
  – Dermal uptake
  – Ocular (eyes)
  – Hand to body

• We focus on **inhalation** exposure in this course

Sexton et al., 1995 *EHP*
Inhalation exposure

• “Exposure” accounts for both the concentration of a substance that an occupant is subjected to and the amount of time the occupant is present with the substance

\[ E = \int_0^t C(t) \, dt \]

Units are in [concentration × time]

\[ E = \text{exposure (concentration} \times \text{time)} \]

\[ C(t) = \text{concentration (ppb, } \mu\text{g/m}^3, \text{#/cm}^3) \]

\[ t = \text{time (hr, min, sec)} \]

• If the formaldehyde concentration is 20 ppb in my bathroom and I am in there for 10 minutes, my exposure to formaldehyde is:

\[ E = 20 \text{ ppb} \times 10 \text{ minutes} = 200 \text{ ppb} \cdot \text{mins} = 3.3 \text{ ppb} \cdot \text{hrs} \]
Inhalation exposure

• Total exposure during a period of time is the sum of all exposures in individual microenvironments:

\[ E_{total} = \sum_{i=1}^{n} \int_{0}^{t_i} C_i(t) \ dt \]

- \( E_{total} \) = total exposure during period of time (concentration × time)
- \( C_i(t) \) = concentration in a particular microenvironment \( i \) (ppb, \( \mu \)g/m\(^3\), #/cm\(^3\))
- \( t_i \) = time spent in microenvironment \( i \) (hr, min, sec)

• Microenvironments include bedrooms, offices, outdoors, transportation…
Inhalation exposure

- If we measure exposures to particular pollutants, we will often end up with time-averaged data (depending on the pollutant and monitoring device)
  - In this case, the integral is simplified:

\[
E_{total} = \sum_{i=1}^{n} \bar{C}_i \cdot \Delta t_i
\]

- So what influences exposure?
  - Which microenvironment \( i \)
  - The average concentration \( C_i \)
  - Time spent in microenvironment \( \Delta t_i \)

- So we need to know where people spend their time, how much time they spend there, and what the concentration they are exposed to in that environment
Inhalation dose

- “Dose” accounts for the actual amount that crosses a contact boundary
  - Inhalation dose is a function of the exposure (concentration), breathing rate, and the duration of exposure (and presumably breathing)
  - Dose is therefore a mass (or number) of substance ingested

\[
D = \int_{0}^{t} C(t)Q_b(t)\,dt
\]

\(D\) = inhalation dose (mg, \(\mu\)g, # of particles, # of cells, etc.)
\(Q_b(t)\) = breathing rate at time \(t\) (m³/day, m³/hr, L/min, L/s, etc.)

\[
D_{\text{total}} = \sum_{i=1}^{n} \int_{0}^{t_i} C_i(t) \cdot Q_b(t)\,dt
\]

\[
D_{\text{total}} = \sum_{i=1}^{n} \overline{C_i} \cdot \overline{Q_b} \cdot \Delta t_i
\]

- Exposure can be measured, but dose is usually estimated
  - Unless you’re doing toxicology work where you control the dose
Breathing rates

- So we need to know breathing rates
- One good source is the EPA Exposure Factors Handbook

Table 6-4. Distribution Percentiles of Physiological Daily Inhalation Rates (PDIRs) (m³/day) for Free-Living Normal-Weight Males and Females Aged 2.6 Months to 96 Years

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Body Weight (kg)</th>
<th>Physiological Daily Inhalation Rates (m³/day)</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean ± SD</td>
<td>5th</td>
</tr>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.22 to &lt;0.5</td>
<td>32</td>
<td>6.7 ± 1.0</td>
<td>3.38 ± 0.72</td>
</tr>
<tr>
<td>0.5 to &lt;1</td>
<td>40</td>
<td>8.8 ± 1.1</td>
<td>4.22 ± 0.79</td>
</tr>
<tr>
<td>1 to &lt;2</td>
<td>35</td>
<td>10.6 ± 1.1</td>
<td>5.12 ± 0.88</td>
</tr>
<tr>
<td>2 to &lt;5</td>
<td>25</td>
<td>15.3 ± 3.4</td>
<td>7.84 ± 1.32</td>
</tr>
<tr>
<td>5 to &lt;7</td>
<td>96</td>
<td>19.8 ± 6.1</td>
<td>8.66 ± 1.23</td>
</tr>
<tr>
<td>11 to &lt;23</td>
<td>30</td>
<td>58.6 ± 13.9</td>
<td>17.23 ± 3.67</td>
</tr>
<tr>
<td>23 to &lt;30</td>
<td>34</td>
<td>70.9 ± 6.5</td>
<td>16.78 ± 2.81</td>
</tr>
<tr>
<td>30 to &lt;40</td>
<td>41</td>
<td>71.5 ± 6.8</td>
<td>16.88 ± 2.52</td>
</tr>
<tr>
<td>40 to &lt;65</td>
<td>53</td>
<td>71.6 ± 7.2</td>
<td>16.24 ± 2.67</td>
</tr>
<tr>
<td>65 to &lt;96</td>
<td>50</td>
<td>68.9 ± 6.7</td>
<td>12.96 ± 2.48</td>
</tr>
</tbody>
</table>

Females

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Body Weight (kg)</th>
<th>Physiological Daily Inhalation Rates (m³/day)</th>
<th>Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>Mean ± SD</td>
<td>5th</td>
</tr>
<tr>
<td>Males</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.22 to &lt;0.5</td>
<td>53</td>
<td>6.5 ± 0.9</td>
<td>3.26 ± 0.66</td>
</tr>
<tr>
<td>0.5 to &lt;1</td>
<td>63</td>
<td>8.5 ± 1.0</td>
<td>3.96 ± 0.72</td>
</tr>
<tr>
<td>1 to &lt;2</td>
<td>66</td>
<td>10.6 ± 1.3</td>
<td>4.78 ± 0.96</td>
</tr>
<tr>
<td>2 to &lt;5</td>
<td>36</td>
<td>14.4 ± 3.0</td>
<td>7.06 ± 1.16</td>
</tr>
<tr>
<td>5 to &lt;7</td>
<td>102</td>
<td>19.7 ± 2.3</td>
<td>8.22 ± 1.31</td>
</tr>
<tr>
<td>7 to &lt;11</td>
<td>161</td>
<td>23.8 ± 4.4</td>
<td>9.84 ± 1.69</td>
</tr>
<tr>
<td>11 to &lt;23</td>
<td>87</td>
<td>50.0 ± 8.9</td>
<td>13.28 ± 2.60</td>
</tr>
<tr>
<td>23 to &lt;30</td>
<td>68</td>
<td>59.2 ± 6.6</td>
<td>13.67 ± 2.28</td>
</tr>
<tr>
<td>30 to &lt;40</td>
<td>59</td>
<td>58.7 ± 5.9</td>
<td>13.68 ± 1.76</td>
</tr>
<tr>
<td>40 to &lt;65</td>
<td>58</td>
<td>58.8 ± 5.1</td>
<td>12.31 ± 2.07</td>
</tr>
<tr>
<td>65 to &lt;96</td>
<td>45</td>
<td>57.2 ± 7.3</td>
<td>9.80 ± 2.17</td>
</tr>
</tbody>
</table>
Exposure patterns

• We also need to know exposure concentrations and times
• What do exposure patterns look like?

\[ E_{total} = \sum_{i=1}^{n} \overline{C_i} \cdot \Delta t_i \]
Relative inhalation exposures

- How do we compare two different microenvironments?
  - To help focus attention on the most important/relevant environments

\[
E_{\text{relative, }i-j} = \frac{C_i \cdot \Delta t_i}{C_j \cdot \Delta t_j} = \frac{C_i}{C_j} \cdot \frac{\Delta t_i}{\Delta t_j}
\]

- Example: indoor vs. outdoor ozone
  - Typical values:

\[
\frac{C_{\text{in}}}{C_{\text{out}}} \approx 0.05 - 0.5 \quad \frac{\Delta t_{\text{in}}}{\Delta t_{\text{out}}} \approx 10 - 20
\]

\[
E_{\text{relative, indoor-outdoor}} = \frac{C_{\text{in}}}{C_{\text{out}}} \cdot \frac{\Delta t_{\text{in}}}{\Delta t_{\text{out}}} = 0.05 \cdot (10) = 0.5 \text{ (min.)}
\]

\[
E_{\text{relative, indoor-outdoor}} = \frac{C_{\text{in}}}{C_{\text{out}}} \cdot \frac{\Delta t_{\text{in}}}{\Delta t_{\text{out}}} = 0.5 \cdot (20) = 10 \text{ (max.)}
\]

This means that at least ~33% of outdoor ozone exposure probably occurs indoors
- 0.5 as much indoor vs. outdoor exposure → 0.67 outdoors + 0.33 indoors
- And probably much more (as much as 90% in some cases)
Human activity patterns

• So we need to understand where we spend our time in order to understand what exposures are important
  – $\Delta t_i$

• Where do we spend our time?
Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - Out of date, but remains highly cited as one of the first nationally representative activity surveys

<table>
<thead>
<tr>
<th>Microenvironment number</th>
<th>Starting time</th>
<th>Ending time</th>
<th>Summary</th>
<th>Detailed activity</th>
<th>Simplified activity</th>
<th>Detailed location</th>
<th>Simplified location</th>
<th>Smoker? (1 = Yes)</th>
<th>Time spent (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>00:00</td>
<td>01:45</td>
<td>At night club</td>
<td>77</td>
<td>0</td>
<td>405</td>
<td>90</td>
<td>1</td>
<td>105</td>
</tr>
<tr>
<td>2</td>
<td>01:45</td>
<td>02:00</td>
<td>Traveling home after night club</td>
<td>79</td>
<td>0</td>
<td>301</td>
<td>30</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>02:00</td>
<td>11:00</td>
<td>Sleeping or napping</td>
<td>45</td>
<td>0</td>
<td>105</td>
<td>10</td>
<td>0</td>
<td>540</td>
</tr>
<tr>
<td>4</td>
<td>11:00</td>
<td>11:05</td>
<td>Brushed teeth</td>
<td>44</td>
<td>40</td>
<td>104</td>
<td>10</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>11:05</td>
<td>11:15</td>
<td>Preparing meals or snacks</td>
<td>10</td>
<td>10</td>
<td>101</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>11:15</td>
<td>11:25</td>
<td>Eating meals or snacks</td>
<td>43</td>
<td>70</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>11:25</td>
<td>11:30</td>
<td>Dressing or personal grooming</td>
<td>47</td>
<td>0</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>11:30</td>
<td>11:37</td>
<td>Traveling to play football</td>
<td>89</td>
<td>0</td>
<td>306</td>
<td>40</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>11:37</td>
<td>13:37</td>
<td>Playing flag football</td>
<td>80</td>
<td>60</td>
<td>507</td>
<td>50</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>10</td>
<td>13:37</td>
<td>13:44</td>
<td>Traveling to home</td>
<td>79</td>
<td>0</td>
<td>306</td>
<td>40</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>13:44</td>
<td>13:54</td>
<td>Preparing meals or snacks</td>
<td>10</td>
<td>10</td>
<td>201</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>13:54</td>
<td>13:57</td>
<td>Traveling to bar</td>
<td>79</td>
<td>0</td>
<td>301</td>
<td>30</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>13:57</td>
<td>15:30</td>
<td>At bar</td>
<td>77</td>
<td>0</td>
<td>405</td>
<td>90</td>
<td>1</td>
<td>93</td>
</tr>
<tr>
<td>14</td>
<td>15:30</td>
<td>15:33</td>
<td>Traveling from bar</td>
<td>79</td>
<td>0</td>
<td>301</td>
<td>30</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>15:33</td>
<td>16:30</td>
<td>Watching TV</td>
<td>91</td>
<td>0</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>57</td>
</tr>
<tr>
<td>16</td>
<td>16:30</td>
<td>17:00</td>
<td>Bathing or showering</td>
<td>40</td>
<td>40</td>
<td>104</td>
<td>10</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>17</td>
<td>17:00</td>
<td>19:00</td>
<td>Watching TV</td>
<td>91</td>
<td>0</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>120</td>
</tr>
<tr>
<td>18</td>
<td>19:00</td>
<td>19:10</td>
<td>Traveling to shopping</td>
<td>39</td>
<td>0</td>
<td>301</td>
<td>30</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>19</td>
<td>19:10</td>
<td>19:25</td>
<td>Shopping for food</td>
<td>30</td>
<td>0</td>
<td>414</td>
<td>90</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>20</td>
<td>19:25</td>
<td>19:35</td>
<td>Travel related to shopping</td>
<td>39</td>
<td>0</td>
<td>301</td>
<td>30</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>21</td>
<td>19:35</td>
<td>21:00</td>
<td>Watching TV</td>
<td>91</td>
<td>0</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>85</td>
</tr>
<tr>
<td>22</td>
<td>21:00</td>
<td>24:00</td>
<td>Studying</td>
<td>54</td>
<td>0</td>
<td>102</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

Example time activity diary from NHAPS

Human activity patterns

• The National Human Activity Pattern Survey (NHAPS)
  – How much time do people spend in what environments?
  – And what fraction of people do that?
    • A nation of doers?

<table>
<thead>
<tr>
<th>Location</th>
<th>n</th>
<th>Overall mean (min)</th>
<th>Doer %</th>
<th>Doer n</th>
<th>Doer mean (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NHAPS - nation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In a residence</td>
<td>9196</td>
<td>990</td>
<td>99.4</td>
<td>9153</td>
<td>996</td>
</tr>
<tr>
<td>Office - factory</td>
<td>9196</td>
<td>78</td>
<td>20.0</td>
<td>1925</td>
<td>388</td>
</tr>
<tr>
<td>Bar - restaurant</td>
<td>9196</td>
<td>27</td>
<td>23.7</td>
<td>2263</td>
<td>112</td>
</tr>
<tr>
<td>Other indoor</td>
<td>9196</td>
<td>158</td>
<td>59.1</td>
<td>5372</td>
<td>267</td>
</tr>
<tr>
<td>In an enclosed vehicle</td>
<td>9196</td>
<td>79</td>
<td>83.2</td>
<td>7596</td>
<td>95</td>
</tr>
<tr>
<td>Outdoors</td>
<td>9196</td>
<td>109</td>
<td>59.3</td>
<td>5339</td>
<td>184</td>
</tr>
</tbody>
</table>

Variability between populations:
99.4% of people spent time at home
• 990 minutes on average, 990/1440 = 69% of the day
Only 59.3% of people went outdoors
• Of those that did, they spent 109 minutes outdoors, on average
• 109/1440 = 7.5% of the day

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - Overall nationwide averages

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - Time spent with a smoker

Human activity patterns

• The National Human Activity Pattern Survey (NHAPS)
  − Probability distributions in each environment

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - Time-varying activity patterns
Human activity patterns

• What are some other ways to collect human activity data?
Human activity patterns

• What are some other ways to collect human activity data?
  – GPS instruments

Elgethun et al., *EHP* 2003, 111, 115-122
Indoor exposures

- So we spend a lot of time indoors ($\Delta t_{\text{indoor}}$ is large)
  - Do we also encounter large concentrations? ($C_{\text{indoor}}$)

- Depends on what emissions we’re talking about

- Let’s first discuss “intake fractions”
Intake fractions

• Emissions to intake relationship
  – A 1 kg mass of pollutant is emitted into the environment from a point source 50 miles away from you
  – A 0.01 kg (10 g) mass of pollutant is emitted into your home air
  – Which is more important to you from an exposure standpoint?

• An intake fraction helps describe importance of emissions
  – Integrated intake of a pollutant released from a source and summed over all exposed individuals during a given exposure time, per unit of emitted pollutant:

\[
iF = \sum_{people,time} \frac{\text{mass intake of pollutant by an individual}}{\text{mass of pollutant released into the environment}}
\]

Bennett et al., 2002 *Environ Sci Technol*
Intake fractions

• Values of iF depend on several factors:
  – Chemical properties of the contaminant
  – Emission locations
  – Environmental conditions
  – Exposure pathways
  – Receptor (i.e., human) locations and activities
  – Population characteristics
Intake fractions

- Lai et al. (2000) estimated IFs for several scenarios
  - Outdoor air basins, point releases, and line sources; vehicles; and indoors

Individual IFs

\[
\text{ITF} = \frac{\text{mass inhalation rate}}{\text{mass emission rate}} = \frac{CQ_b}{E}
\]

Population wide IFs

\[
\text{PITF} = \sum_{i=1}^{N} \text{ITF}_i
\]

\[
\text{PITF} = \frac{\sum i \int C_i(t)Q_{bi}(t)dt}{\int E(t)dt}
\]

\[
\text{PITF} = \frac{\sum_{i=1}^{N} C_iQ_{bi}}{E}
\]
Intake fraction example

• Benzene example
  – Benzene is emitted to outdoor air from motor vehicles
  – Benzene is also present in environmental tobacco smoke (ETS)

• Outdoor benzene in California’s South Cost air basin (SoCAB)
  – 16,000 km² area
  – Home to 14 million people who drive vehicles ~0.5 billion km daily
    • They use ~59 million L of gasoline daily
    • ~280 mg of benzene is emitted per L of gasoline
    • Total emissions of ~17 metric tons (17000 kg) of benzene per day
    • Outdoor iFs range approximately $1 \times 10^{-6}$ to $5 \times 10^{-4}$
      – Depending on meteorology and other factors

Bennett et al., 2002 Environ Sci Technol
### Intake fraction example

- **Benzene from ETS indoors**
  - SoCAB is also home to ~1.9 million smokers
    - Consuming 42 million cigarettes daily
  - Assume that 50% of cigarettes consumed in the area are smoked in homes
  - Benzene emission factors for ETS are 280-610 µg per cigarette
    - Assume ~450 µg per cigarette
  - Total estimated residential emissions of benzene from ETS are ~9 kg/day
    - That is only ~0.05% of the total emitted by motor vehicles
  - But, the iF for a nonreactive pollutant in a residence is ~7×10^{-3}
    - That is 10-100+ times as high as for outdoor emissions (1×10^{-6}-5×10^{-4})
  - Overall, vehicles account for inhalation of ~1 kg/day of benzene inhalation
    - Across the basin population
    - ETS accounts for ~60 g/day
    - So while ETS accounts for only 0.05% of the emissions, it accounts for ~6% of benzene intake in the area
      - Non-negligible amount (and IF is 120 times higher than E)

Bennett et al., 2002 *Environ Sci Technol*
Intake fractions

• So what was the answer to our original question?
  – A 1 kg mass of pollutant is emitted into the environment from a point source 50 miles away from you
  – A 0.01 kg (10 g) mass of pollutant is emitted into your home air
  – Which is more important to you from an exposure standpoint?

  – Indoor emission is \(~1/100^{th}\) of the outdoor mass emission
  – But indoor IF is \(~10\) to \(~1000\) higher
  – So the overall effect on intake is generally higher for the indoor source
Mass balances

- We’ve talked about time we spend indoors ($\Delta t_{\text{indoor}}$)
- And we’ve talked about emission to intake ratios
- But do we also encounter large concentrations? ($C_{\text{indoor}}$)
  - Need to be able to measure and predict $C_{\text{indoor}}$
  - And model what affects $C_{\text{indoor}}$
- We can do that with a mass balance
Indoor environment: Mass balance

Outdoor Pollutants
Ventilation/Air Exchange
- Homogeneous Chemistry
- Adsorption/Desorption
- Deposition/Surface Reactions
- Indoor Emission
- Control/Filtration

T/RH
Indoor environment: Mass balance

- Simplest case
  - Neglecting indoor physics/chemistry
    - No deposition, no reaction

\[
\text{Mass accumulation rate [mass / time]} = \text{Mass flow in [mass / time]} + \text{Mass flow out [mass / time]} + \text{Mass emitted [mass / time]}
\]

\[
\frac{dm}{dt} = \frac{dCV}{dt} = V \frac{dC}{dt} + C \frac{dV}{dt}
\]

Assumptions:
- Building/room can be treated as well-mixed
- Ventilation/air exchange rate is constant
- Outdoor pollutant concentration is constant
- Indoor sources emit at a constant rate
Indoor environment: Mass balance

\[ V \frac{dC}{dt} = PQC_{\text{out}} - QC + E \]
Indoor environment: Mass balance

\[ V \frac{dC}{dt} = PQC_{out} - QC + E \]

- Divide by volume

\[ \frac{dC}{dt} = P \frac{Q}{V} C_{out} - \frac{Q}{V} C + \frac{E}{V} \]

\[ \frac{dC}{dt} = P\lambda C_{out} - \lambda C + \frac{E}{V} \]

\[ \lambda = \frac{Q}{V} = \text{air exchange rate} \left( \frac{1}{\text{hr}} \right) \]
Indoor environment: Mass balance

• Assume steady-state conditions:

\[
\frac{dC}{dt} = P\lambda C_{out} - \lambda C + \frac{E}{V}
\]

\[
C_{ss} = PC_{out} + \frac{E}{\lambda V}
\]

• If \( \lambda \) is large (and/or \( E \) is small): \( PC_{out} >> \frac{E}{\lambda V} \)
  – \( C \) approaches \( C_{out} \) (depending on \( P \))
  – This means outdoor sources are relatively more important

• If \( \lambda \) is small (and/or \( E \) is large): \( PC_{out} << \frac{E}{\lambda V} \)
  – \( C \) approaches \( \frac{E}{\lambda V} \)
  – This means indoor sources are relatively more important
Steady state mass balance

- Example steady state calculations: \[ C_{ss} = PC_{out} + \frac{E}{\lambda V} \]

- Assume \( P = 1 \) and \( C_{out} = 0 \): \[ C_{ss} = \frac{E}{\lambda V} \]

- Assume \( V = 200 \text{ m}^3 \), how are \( C \), \( E \), and \( \lambda \) related?
What are typical values of $\lambda$ (AER)?

- Distribution of AERs in ~2800 homes in the U.S.
  - Measured using PFT (perfluorocarbon tracer) in the early 1990s

What do you think this curve looks like now?

Murray and Burmaster, 1995 *Risk Analysis*
What are typical values of \( \lambda \) (AER)?

- Distribution of AERs U.S. homes
  - Early 1990s and revisited in 2010 (Persily et al. 2010)

\[
\begin{align*}
\text{Air exchange rate (1/hr)} \\
\% \text{ less than} \\
1990s \text{ median } &\sim 0.5/\text{hr} \\
2010 \text{ median } &\sim 0.4/\text{hr} \\
\cdot &\text{ 20\% reduction in 20 years}
\end{align*}
\]

- What about brand new homes?

Murray and Burmaster, 1995 *Risk Analysis*; Persily et al. 2010 *Indoor Air*
What are typical values of $\lambda$ (AER)?

- Distribution of AERs U.S. homes
  - Addition of 106 new homes (Offermann et al., 2009)

  - 1990s median ~0.5/hr
  - 2010 median ~0.4/hr
  - 2009 new home median ~0.26/hr

- Not uncommon for new homes to have AER = 0.05-0.20 per hour

Offermann et al. 2009 CEC PIER Report
Steady state mass balance with AER

- What do trends in AERs mean for indoor concentrations?
  - Nonreactive pollutants at steady date (without an outdoor source)

\[
C = 1.92 \, \mu g/m^3 \text{ @ median new CA home AER of } 0.26/\text{hr} \\
C = 1.25 \, \mu g/m^3 \text{ @ median 2010 AER of } 0.4/\text{hr} \\
C = 1.0 \, \mu g/m^3 \text{ @ median 1990 AER of } 0.5/\text{hr}
\]
Limitations to previous mass balance

- **Well-mixed assumption**
  - Occupant exposure can be much higher than estimated near source
  - Cooking, cleaning, vicinity of smoker
  - Personal cloud or “pig pen” effect is where:
    \[ C_{\text{personal}} > C_{\text{indoor}} \]

- **Assumption of no sinks or transformations**
  - Adsorption, desorption, deposition, and reactions all ignored (for now)

- **Assumption of no control of pollutants**
  - No whole building filtration or portable air cleaner (for now)

- **Also assumed steady-state**
  - What about dynamic solution?
Dynamic solution to mass balance

• Start with basic mass balance:

\[
\frac{dC}{dt} = P\lambda C_{out} - \lambda C + \frac{E}{V}
\]

• Rearrange:

\[
\frac{1}{P\lambda C_{out} - \lambda C + \frac{E}{V}}dC = dt
\]

• Factor out (-1):

\[
\frac{1}{\lambda C - P\lambda C_{out} - \frac{E}{V}}dC = -dt
\]

• Substitute: Let \(x = \text{denominator} = \lambda C - P\lambda C_{out} - \frac{E}{V}\)

  – So that: \(\frac{dx}{dC} = \lambda \quad \quad \quad \quad \quad dC = \frac{1}{\lambda}dx\)
Dynamic solution to mass balance

Letting \( x = \lambda C - P\lambda C_{out} - \frac{E}{V} \) and thus \( \frac{dx}{dC} = \lambda \) transforms:

\[
\frac{1}{\lambda C - P\lambda C_{out} - \frac{E}{V}} dC = -dt
\]

\[
\frac{1}{\lambda} \left( \frac{1}{x} \right) dx = -dt
\]

• We can now solve this simpler equation

Rearrange: \( \left( \frac{1}{x} \right) dx = -\lambda dt \)

Integrate both sides: \( \int_{x_0}^{x} \frac{1}{x} dx = -\lambda \int_{0}^{t} dt \)

Solution with \( x \):

\( \ln(x)|_{x_0}^{x} = -\lambda t \)

Substitute back in for \( x \):

\[
\ln \left\{ C(t) - P\lambda C_{out} - \frac{E}{V} \right\} = -\lambda t
\]

\[
\ln \left\{ \frac{\lambda C(t = 0) - P\lambda C_{out} - E}{V} \right\} = -\lambda t
\]
Dynamic solution to mass balance

\[
\ln \left\{ \frac{\lambda C - P\lambda C_{out} - \frac{E}{V}}{\lambda C(t = 0) - P\lambda C_{out} - \frac{E}{V}} \right\} = -\lambda t
\]

- Raise \( e \) to both sides:

\[
\frac{\lambda C - P\lambda C_{out} - \frac{E}{V}}{\lambda C(t = 0) - P\lambda C_{out} - \frac{E}{V}} = e^{-\lambda t}
\]

- Rearrange:

\[
\lambda C - P\lambda C_{out} - \frac{E}{V} = \left\{ \lambda C(t = 0) - P\lambda C_{out} - \frac{E}{V} \right\} e^{-\lambda t}
\]
Dynamic solution to mass balance

• Solve for C:
  – Which is C at time t, or C(t)

\[ \lambda C - P \lambda C_{out} - \frac{E}{V} = \left\{ \lambda C(t = 0) - P \lambda C_{out} - \frac{E}{V} \right\} e^{-\lambda t} \]

\[ C(t) = C(t = 0) e^{-\lambda t} + \left( P C_{out} + \frac{E}{\lambda V} \right) \left( 1 - e^{-\lambda t} \right) \]

• What do these two terms represent?

• What happens as t \to \infty ?

\[ C(t \to \infty) = P C_{out} + \frac{E}{\lambda V} = \text{our steady state solution} \]
Dynamic solution to mass balance

- Example concentration profile
  - $V = 200 \, m^3$, $E = 100 \, \mu g/hr$, $\lambda = 0.4/\text{hr}$, $C_{out} = 0$
Time to reach steady state

\[ C(t) = C(t = 0)e^{-\lambda t} + \left( PC_{out} + \frac{E}{\lambda V} \right)(1 - e^{-\lambda t}) \]

- If we assume an inert pollutant emitted indoors with an initial concentration of zero, how long would it take to achieve 95% of steady state?

- 95% of steady-state is reached when:

\[
\left(1 - e^{-\lambda t}\right) = 0.95 \quad \rightarrow \quad e^{-\lambda t} = 1 - 0.95 = 0.05
\]

\[-\lambda t = \ln(0.05) \quad \rightarrow \quad \lambda t = -\ln(0.05) = 3\]

\[ t = \frac{3}{\lambda} \]

Consider \( \lambda = 0.1 \) hr\(^{-1} \)

t to 95% steady state = 30 hours

Consider \( \lambda = 1 \) hr\(^{-1} \)

t to 95% steady state = 3 hours
How do we measure $\lambda$?

• One method is to inject an inert tracer gas, and measure the decay from $C(t=0)$ after time $t=0$
How do we measure $\lambda$?

- One method is to inject an inert tracer gas, and measure the decay from $C(t=0)$ after time $t=0$
  - In this case, $E = 0$
  - Assume $P = 0$ (reasonable for inert gas)

\[
C(t) = C(t = 0)e^{-\lambda t} + \left( P C_{out} + \frac{E}{\lambda V} \right) \left( 1 - e^{-\lambda t} \right)
\]

\[
C(t) = C(t = 0)e^{-\lambda t} + C_{out} \left( 1 - e^{-\lambda t} \right)
\]

\[
C(t) = C(t = 0)e^{-\lambda t} + C_{out} - C_{out}e^{-\lambda t}
\]

\[
C(t) - C_{out} = \left\{ C(t = 0) - C_{out} \right\} e^{-\lambda t}
\]
How do we measure $\lambda$?

$$C(t) - C_{out} = \left\{C(t = 0) - C_{out}\right\} e^{-\lambda t}$$

$$\frac{C(t) - C_{out}}{C(t = 0) - C_{out}} = e^{-\lambda t}$$

- Take the natural log of both sides:

$$-\ln \left\{ \frac{C(t) - C_{out}}{C(t = 0) - C_{out}} \right\} = \lambda t$$

- To find $\lambda$, plot left hand side versus right hand side
  - Slope of that line is $\lambda$
How do we measure $\lambda$?

**Example:** You perform a tracer test with $\text{CO}_2$
- You measure a constant outdoor concentration of 400 ppm
- You elevate indoors to 2000 ppm, then leave for 6 hours
- You record these data:

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>$C(t)$ (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2500</td>
</tr>
<tr>
<td>1</td>
<td>1450</td>
</tr>
<tr>
<td>2</td>
<td>900</td>
</tr>
<tr>
<td>3</td>
<td>660</td>
</tr>
<tr>
<td>4</td>
<td>530</td>
</tr>
<tr>
<td>5</td>
<td>460</td>
</tr>
<tr>
<td>6</td>
<td>430</td>
</tr>
</tbody>
</table>

Plot the LHS vs time

$$-\ln \left( \frac{C(t) - C_{out}}{C(t = 0) - C_{out}} \right) = \lambda t$$
How do we measure $\lambda$?

**Example:** You perform a tracer test with $\text{CO}_2$
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</table>

Plot the LHS vs time

And perform linear regression

$$y = 0.7058x$$

$$R^2 = 0.9997$$

Left Hand Side: $-\ln \left( \frac{C(t) - C_{out}}{C(t = 0) - C_{out}} \right) = \lambda t$

AER = $\lambda$ = slope = 0.71 hr$^{-1}$
What makes a good tracer gas?

Characteristics
- Non-reactive (inert)
- Non-toxic
- Colorless
- Odorless
- Cheap
  - Gas
  - Sensor
- Low detection limits
- Portable

Commonly used gases
- Carbon dioxide (CO$_2$)
  - People are a source
  - Need to account for E/V
- Nitrous oxide (N$_2$O)
  - Laughing gas
  - Toxic at high levels
- Freon (CFC)
  - Global warming potential
- Helium (He)
  - Costs
- Sulfur hexafluoride (SF$_6$)
  - Global warming potential
Variation in AER

• Air exchange rates differ from building to building
  – Differences vary by driving forces and building characteristics
• Example research: “Continuous measurements of air change rates in an occupied house for 1 year: the effect of temperature, wind, fans, and windows”
  – 4600 AERs measured by automated SF$_6$ system in one house for 2 years!

Wallace et al. 2002 J Expo Anal Environ Epidem
Variation in AER

AERs in individual buildings can vary by season
- Driving forces: temperature, wind speed

AERs can vary by I/O temperature within seasons

Wallace et al. 2002 J Expo Anal Environ Epidem
What if our pollutant has another loss term?

- Rarely are we working with inert pollutants
  - Other loss mechanisms are important
  - Deposition to surfaces, control by HVAC filter, reaction, evaporation...

\[ C_{\text{out}} \]

\[ P \]

\[ \lambda \]

New terms
\[ Q_f = \text{airflow rate through filter (m}^3/\text{hr)} \]
\[ \eta = \text{filter removal efficiency (-)} \]

Indoor Emission
\[ E \]

Control/Filtration
\[ Q_f \]

T/RH
Mass balance with filtration

• New term to mass balance:

\[
V \frac{dC}{dt} = P Q C_{out} - Q C + E - \eta Q_f C
\]

• Assume steady state for now, divide by \( \lambda \), and solve for \( C \):

\[
C = \frac{P C_{out} + \frac{E}{\lambda V}}{1 + \frac{\eta Q_f}{\lambda V}}
\]

\[
C = \frac{P C_{out} + \frac{E}{\lambda V}}{1 + \frac{1}{\lambda V} \frac{CADR}{\lambda V}}
\]

• \( CADR = \) Clear Air Delivery Rate

\[
CADR = \eta Q_f
\]
Dynamic solution with filtration

\[
V \frac{dC}{dt} = P Q C_{\text{out}} - Q C + E - \eta Q_f C
\]

- Going through the same process as before but with a new loss term:

\[
C(t) = C(t = 0) e^{-\left(\frac{\lambda + \text{CADR}}{V}\right) t} + \frac{P C_{\text{out}} + \frac{E}{\lambda V}}{1 + \frac{\text{CADR}}{\lambda V}} \left(1 - e^{-\left(\frac{\lambda + \text{CADR}}{V}\right) t}\right)
\]

- how far from your initial concentration have you fallen?
- steady-state solution
- how close to steady-state have you risen?
Generalized steady and dynamic mass balance solutions

\[ \frac{dC}{dt} = S - LC \]

\[ S = \text{sources} = \lambda PC_{\text{out}} + \frac{E}{V} \]

\[ L = \text{losses} = \lambda + \frac{CADR}{V} + k_{\text{deposition}} + k_{\text{rxn}} + \ldots \]

General steady-state solution: \[ C_{ss} = \frac{S}{L} \]

General dynamic solution: \[ C(t) = C_0 e^{-Lt} + \frac{S}{L} \left(1 - e^{-Lt}\right) \]
Assignment: HW1

- HW1 has been posted to BB
  - Covers AER estimation and basic steady state calcs

- Due 1 week from today in class
  - Upload a PDF, email me a PDF, or turn in hardcopy in class

- I will be at ASHRAE in Dallas Thursday through Tuesday morning
  - Ask any questions via email

- Tuesday class as scheduled, unless I get delayed in Dallas
  - In that case, I would email you all Tuesday about postponing
Next time

- Overview of indoor pollutants
  - Particles
  - Gas-phase compounds
  - Biological

- Typical concentrations measured in field studies
  - Will go into individual dynamics later in the course

- Read Weschler paper if interested
  - How have indoor pollutants changed since the 1950s?