ENVE 576
Indoor Air Pollution
Spring 2013

Lecture 2: January 22, 2013
Human exposure patterns
Reactor models
Ventilation and air exchange

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Review from last time

- Course overview
- Introduction to indoor air
  - Topics
  - Research
  - Literature
- Some basic air fundamentals

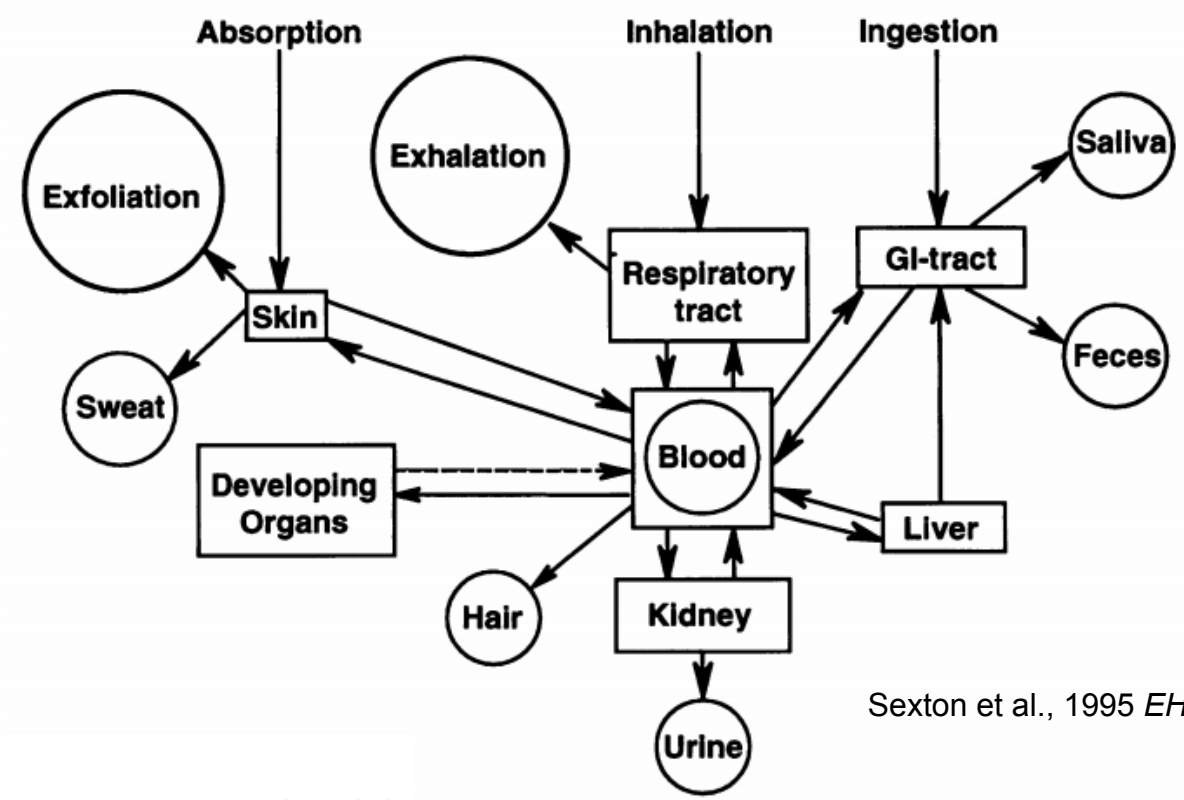
- *Quick!*
  - What is 50 μg/m^3 of NO_2 in ppb at standard temperature and pressure?

Today's objectives

- Human exposure patterns
  - Inhalation and intake fractions
- Reactor models
- Ventilation and air exchange rates

Human exposure pathways

- How do we come in contact with environmental contaminants?
  - Ingestion (water, food, pharmaceuticals, hand-to-mouth)
  - Inhalation (our focus)
  - Dermal uptake
  - Ocular (eyes)
  - Hand to body



Sexton et al., 1995 *EHP*

- We focus on inhalation exposure in this course

Inhalation exposure

- "Exposure" accounts for both the concentration of a substance that an occupant is subjected to *and* the amount of time the occupant is present with the substance

$$E = \int_0^t C(t)\,dt$$

Units are in [concentration × time]

E = exposure (concentration × time)
$C(t)$ = concentration (ppb, μg/m^3, #/cm^3)
t = time (hr, min, sec)

- If the formaldehyde concentration is 20 ppb in my bathroom and I am in there for 10 minutes, my exposure to formaldehyde is:

$$E = 20\ \text{ppb} * 10\ \text{minutes} = 200\ \text{ppb} \cdot \text{mins} = 3.3\ \text{ppb} \cdot \text{hrs}$$

Inhalation exposure

- Total exposure during a period of time is the sum of all exposures in individual microenvironments:

$$E_{total} = \sum_{i=1}^{n} \int_0^{t_i} C_i(t)\,dt$$

E_{total} = total exposure during period of time (concentration × time)
$C_i(t)$ = concentration in a particular microenvironment i (ppb, μg/m^3, #/cm^3)
t_i = time spent in microenvironment i (hr, min, sec)

- Microenvironments include bedrooms, offices, outdoors, transportation…

Inhalation exposure

- If we measure exposures to particular pollutants, we will often end up with time-averaged data (depending on the pollutant and monitoring device)
  - In this case, the integral is simplified:

$$E_{total} = \sum_{i=1}^{n} \overline{C}_i \cdot \Delta t_i$$

  - So what influences exposure?
    - Which microenvironment i
    - The average concentration C_i
    - Time spent in microenvironment Δt_i
  - So we need to know where people spend their time, how much time they spend there, and what the concentration they are exposed to in that environment

Inhalation dose

- "Dose" accounts for the actual amount that crosses a contact boundary
  - Inhalation dose is a function of the exposure (concentration), breathing rate, and the duration of exposure (and presumably breathing)
  - Dose is therefore a mass (or number) of substance ingested

$$D = \int_0^t C(t) Q_b(t)\, dt$$

D = inhalation dose (mg, μg, # of particles, # of cells, etc.)
$Q_b(t)$ = breathing rate at time t (m^3/day, m^3/hr, L/min, L/s, etc.)

$$D_{total} = \sum_{i=1}^{n} \int_0^{t_i} C_i(t) \cdot Q_b(t)\, dt$$

$$D_{total} = \sum_{i=1}^{n} \bar{C}_i \cdot \bar{Q}_b \cdot \Delta t_i$$

- Exposure can be measured, but dose is usually estimated
  - Unless you're doing toxicology work where you control the dose

Breathing rates

- So we need to know breathing rates
- One good source is the EPA Exposure Factors Handbook

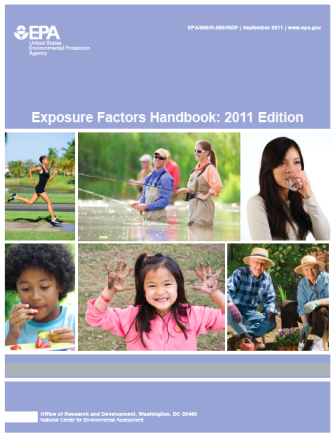


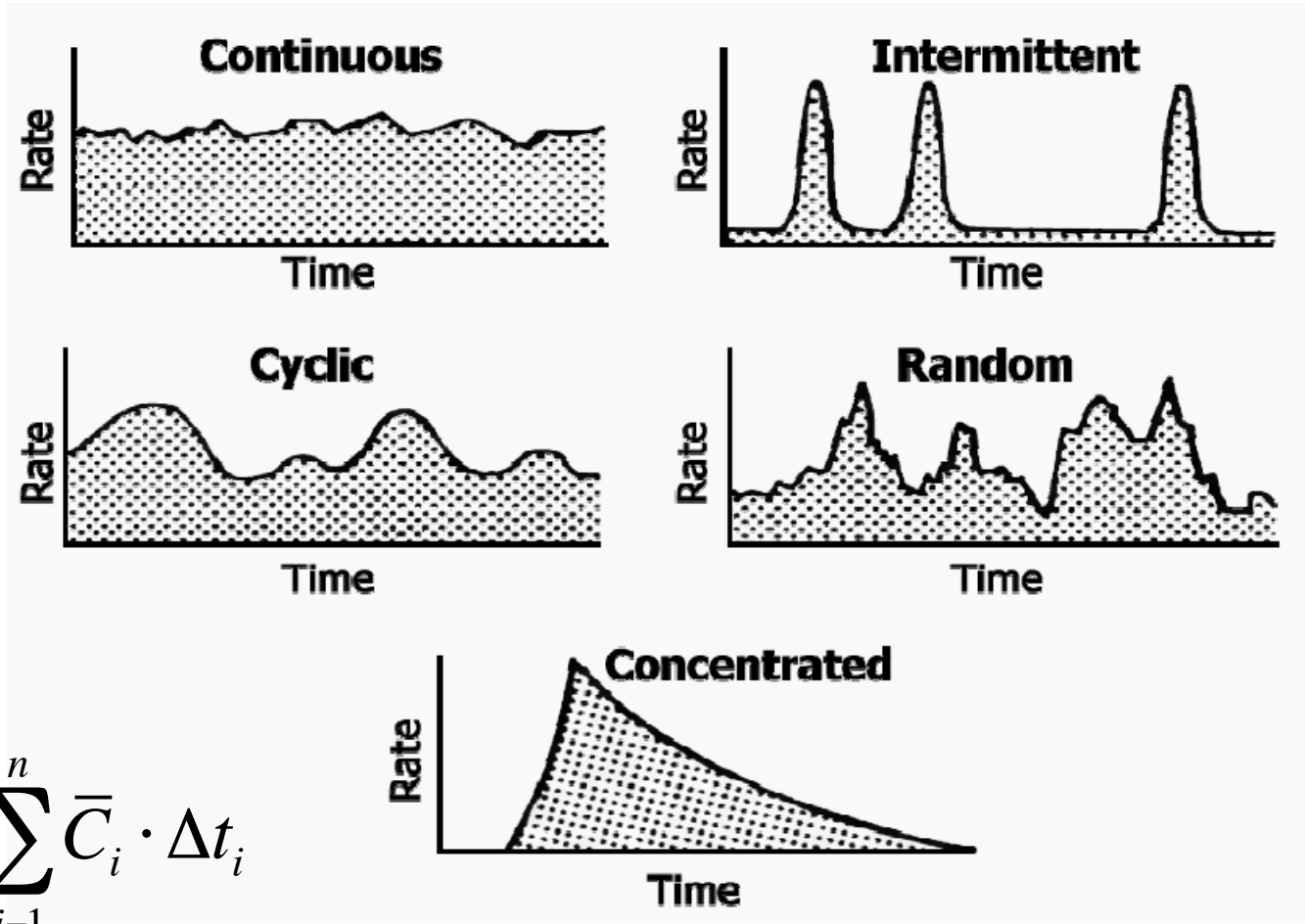
Table 6-4. Distribution Percentiles of Physiological Daily Inhalation Rates (PDIRs) (m^3/day) for Free-Living Normal-Weight Males and Females Aged 2.6 Months to 96 Years

Age Group (years)	N	Body Weight[a] (kg) Mean ± SD	Physiological Daily Inhalation Rates[b] (m^3/day) Mean ± SD	Percentile[c] 5th	10th	25th	50th	75th	90th	95th	99th
Males											
0.22 to <0.5	32	6.7 ± 1.0	3.38 ± 0.72	2.19	2.46	2.89	3.38	3.87	4.30	4.57	5.06
0.5 to <1	40	8.8 ± 1.1	4.22 ± 0.79	2.92	3.21	3.69	4.22	4.75	5.23	5.51	6.05
1 to <2	35	10.6 ± 1.1	5.12 ± 0.88	3.68	3.99	4.53	5.12	5.71	6.25	6.56	7.16
2 to <5	25	15.3 ± 3.4	7.60 ± 1.28	5.49	5.95	6.73	7.60	8.47	9.25	9.71	10.59
5 to <7	96	19.8 ± 2.1	8.64 ± 1.23	6.61	7.06	7.81	8.64	9.47	10.21	10.66	11.50
7 to <11	38	28.9 ± 5.6	10.59 ± 1.99	7.32	8.04	9.25	10.59	11.94	13.14	13.87	15.22
11 to <23	30	58.6 ± 13.9	17.23 ± 3.67	11.19	12.53	14.75	17.23	19.70	21.93	23.26	25.76
23 to <30	34	70.9 ± 6.5	17.48 ± 2.81	12.86	13.88	15.59	17.48	19.38	21.08	22.11	24.02
30 to <40	41	71.5 ± 6.8	16.88 ± 2.50	12.77	13.68	15.20	16.88	18.57	20.09	21.00	22.70
40 to <65	33	71.1 ± 7.2	16.24 ± 2.67	11.84	12.81	14.44	16.24	18.04	19.67	20.64	22.46
65 to ≤96	50	68.9 ± 6.7	12.96 ± 2.48	8.89	9.79	11.29	12.96	14.63	16.13	17.03	18.72
Females											
0.22 to <0.5	53	6.5 ± 0.9	3.26 ± 0.66	2.17	2.41	2.81	3.26	3.71	4.11	4.36	4.81
0.5 to <1	63	8.5 ± 1.0	3.96 ± 0.72	2.78	3.05	3.48	3.96	4.45	4.88	5.14	5.63
1 to <2	66	10.6 ± 1.3	4.78 ± 0.96	3.20	3.55	4.13	4.78	5.43	6.01	6.36	7.02
2 to <5	36	14.4 ± 3.0	7.06 ± 1.16	5.15	5.57	6.28	7.06	7.84	8.54	8.97	9.76
5 to <7	102	19.7 ± 2.3	8.22 ± 1.31	6.06	6.54	7.34	8.22	9.11	9.90	10.38	11.27
7 to <11	161	28.3 ± 4.4	9.84 ± 1.69	7.07	7.68	8.70	9.84	10.98	12.00	12.61	13.76
11 to <23	87	50.0 ± 8.9	13.28 ± 2.60	9.00	9.94	11.52	13.28	15.03	16.61	17.56	19.33
23 to <30	68	59.2 ± 6.6	13.67 ± 2.28	9.91	10.74	12.13	13.67	15.21	16.59	17.42	18.98
30 to <40	59	58.7 ± 5.9	13.68 ± 1.76	10.78	11.42	12.49	13.68	14.87	15.94	16.58	17.78
40 to <65	58	58.8 ± 5.1	12.31 ± 2.07	8.91	9.66	10.92	12.31	13.70	14.96	15.71	17.12
65 to ≤96	45	57.2 ± 7.3	9.80 ± 2.17	6.24	7.02	8.34	9.80	11.27	12.58	13.37	14.85

EPA Exposure Factors Handbook 2011

Exposure patterns

- We also need to know exposure concentrations and times
- What do exposure patterns look like?



$$E_{total} = \sum_{i=1}^{n} \bar{C}_i \cdot \Delta t_i$$

Relative inhalation exposures

- How do we compare two different microenvironments?
  - To help focus attention on the most important/relevant environments

$$E_{relative,i-j} = \frac{\bar{C}_i \cdot \Delta t_i}{\bar{C}_j \cdot \Delta t_j} = \frac{\bar{C}_i}{\bar{C}_j} \cdot \frac{\Delta t_i}{\Delta t_j}$$

- Example: indoor vs. outdoor ozone
  - Typical values:

$$\frac{\bar{C}_{in}}{\bar{C}_{out}} \approx 0.05 - 0.5 \qquad \frac{\Delta t_{in}}{\Delta t_{out}} \approx 10 - 20$$

$$\left.\begin{aligned} E_{relative,indoor-outdoor} &= \frac{\bar{C}_{in}}{\bar{C}_{out}} \cdot \frac{\Delta t_{in}}{\Delta t_{out}} = 0.05 \cdot (10) = 0.5 \text{ (min.)} \\ E_{relative,indoor-outdoor} &= \frac{\bar{C}_{in}}{\bar{C}_{out}} \cdot \frac{\Delta t_{in}}{\Delta t_{out}} = 0.5 \cdot (20) = 10 \text{ (max.)} \end{aligned}\right\} \text{1-2 is typical}$$

This means that at least ~33% of outdoor ozone exposure probably occurs indoors

- 0.5 as much indoor vs. outdoor exposure → 0.67 outdoors + 0.33 indoors
- And probably much more (as much as 90% in some cases)

Human activity patterns

- So we need to understand where we spend our time in order to understand what exposures are important
  - Δt_i
- Where do we spend our time?

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - A 2-year phone survey of ~9400 people in the U.S. (1992-1994)
  - Out of date, but remains highly cited as one of the first nationally representative activity surveys

Microenvironment number	Starting time	Ending time	Summary	Detailed activity	Simplified activity	Detailed location	Simplified location	Smoker? (1=Yes)	Time spent (min)
1	00:00	01:45	At night club	77	0	405	90	1	105
2	01:45	02:00	Traveled home after night club	79	0	301	30	0	15
3	02:00	11:00	Sleeping or napping	45	0	105	10	0	540
4	11:00	11:05	Brushed teeth	44	40	104	10	0	5
5	11:05	11:15	Preparing meals or snacks	10	10	101	10	0	10
6	11:15	11:25	Eating meals or snacks	43	70	102	10	0	10
7	11:25	11:30	Dressing or personal grooming	47	0	102	10	0	5
8	11:30	11:37	Traveling to play football	89	0	306	40	0	7
9	11:37	13:37	Playing flag football	80	60	507	50	0	120
10	13:37	13:44	Traveling to home	79	0	306	40	0	7
11	13:44	13:54	Preparing meals or snacks	10	10	201	10	0	10
12	13:54	13:57	Traveling to bar	79	0	301	30	0	3
13	13:57	15:30	At bar	77	0	405	90	1	93
14	15:30	15:33	Traveling from bar	79	0	301	30	0	3
15	15:33	16:30	Watching TV	91	0	102	10	0	57
16	16:30	17:00	Bathing or showering	40	40	104	10	0	30
17	17:00	19:00	Watching TV	91	0	102	10	0	120
18	19:00	19:10	Traveling to shopping	39	0	301	30	0	10
19	19:10	19:25	Shopping for food	30	0	414	90	0	15
20	19:25	19:35	Travel related to shopping for food	39	0	301	30	0	10
21	19:35	21:00	Watching TV	91	0	102	10	0	85
22	21:00	24:00	Studying	54	0	102	10	0	180

The respondent, whose diary is shown in this table, was a Hispanic male from Connecticut between the ages of 18 and 24 who was interviewed on a weekend in the fall. See the Sample and Data characteristics section for a description of the simplified (i.e., recoded) locations and activities.

Example time activity diary from NHAPS

Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - How much time do people spend in what environments?
  - And what fraction of people do that?
    - A nation of *doers*?

Location	*n*	Overall mean (min)	Doer %	Doer *n*	Doer mean (min)
NHAPS-nation					
In a residence	9196	990	99.4	9153	996
Office–factory	9196	78	20.0	1925	388
Bar–restaurant	9196	27	23.7	2263	112
Other indoor	9196	158	59.1	5372	267
In an enclosed vehicle	9196	79	83.2	7596	95
Outdoors	9196	109	59.3	5339	184

Variability between populations:

99.4% of people spent time at home

- 990 minutes on average, 990/1440 = 69% of the day

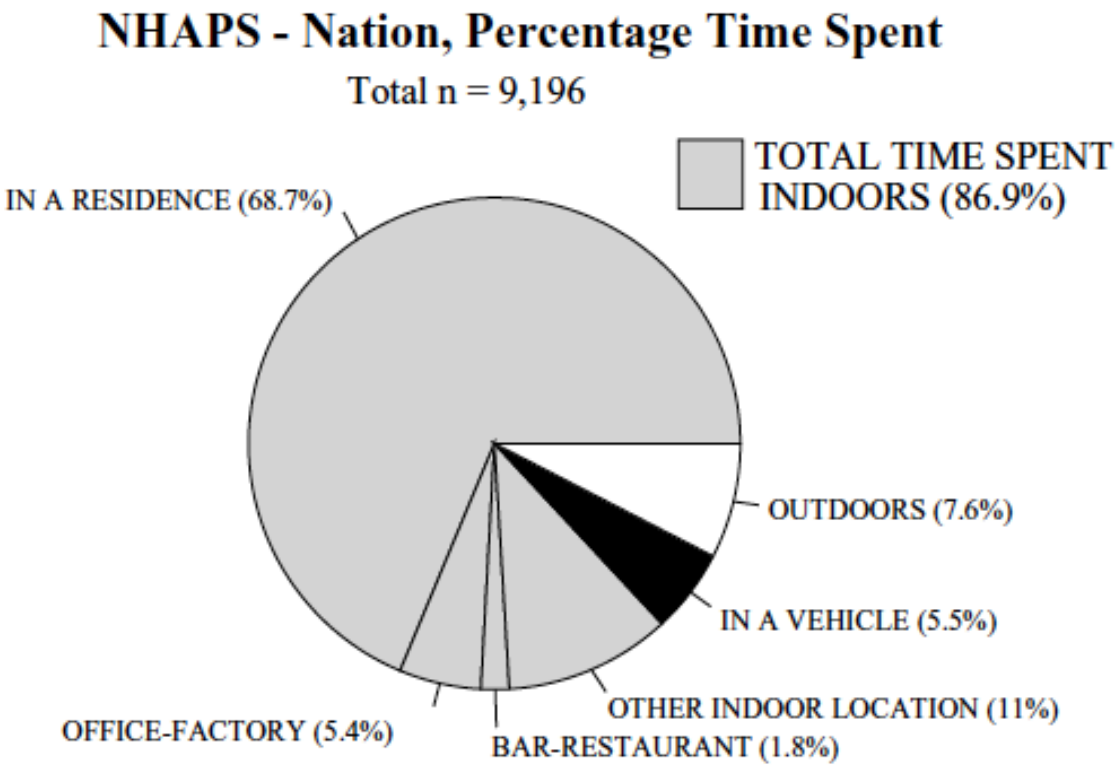
Only 59.3% of people went outdoors

- Of those that did, they spent 109 minutes outdoors, on average
- 109/1440 = 7.5% of the day

Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

Human activity patterns

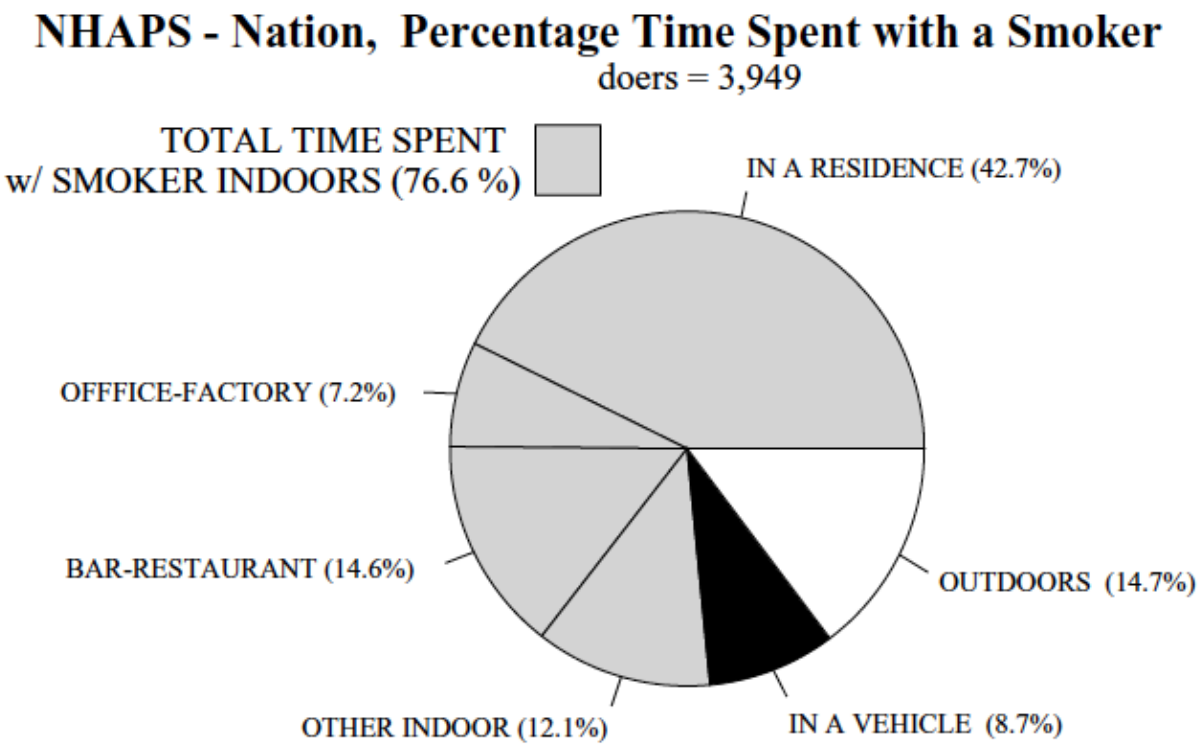
- The National Human Activity Pattern Survey (NHAPS)
  - Overall nationwide averages



Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

Human activity patterns

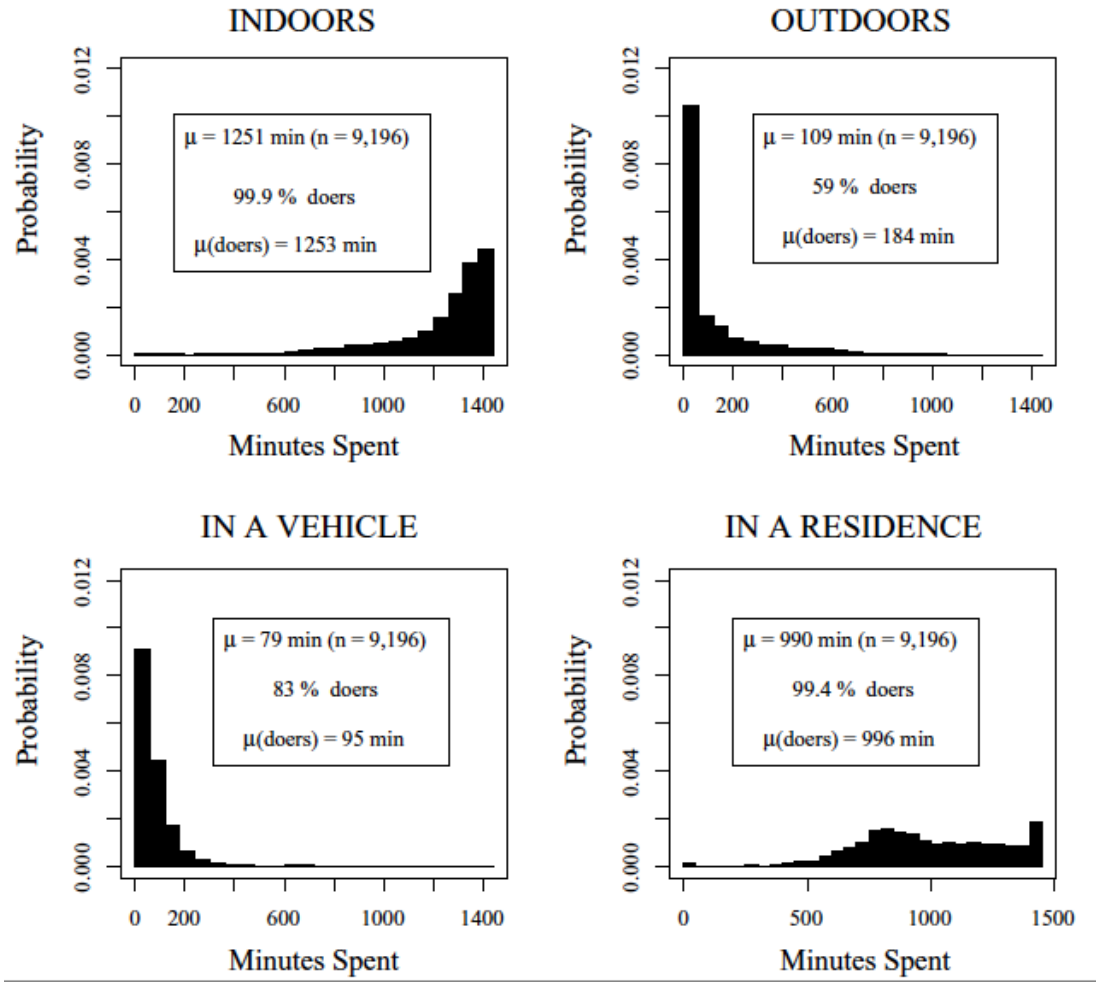
- The National Human Activity Pattern Survey (NHAPS)
  - Time spent with a smoker



Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

Human activity patterns

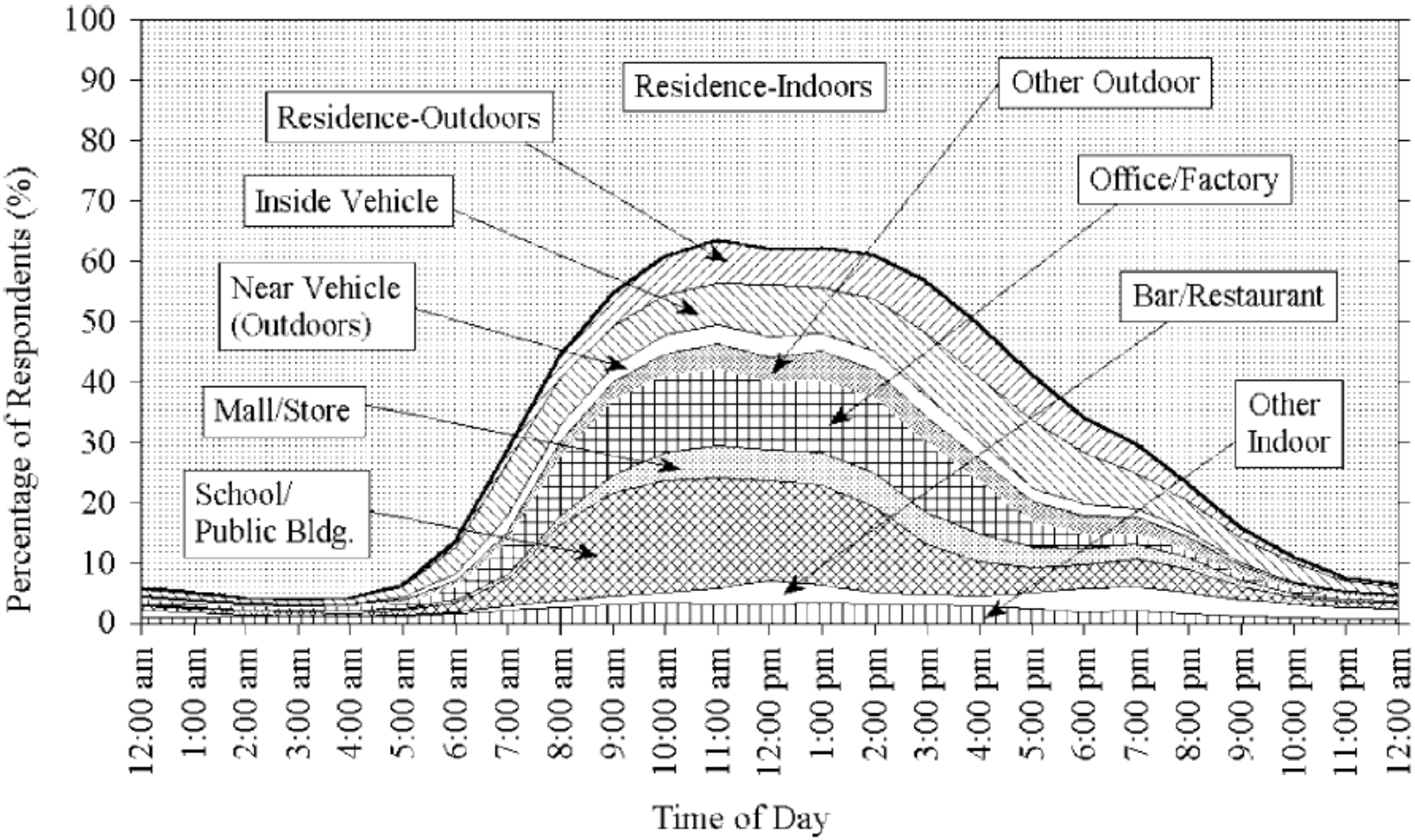
- The National Human Activity Pattern Survey (NHAPS)
  - Probability distributions in each environment



Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

Human activity patterns

- The National Human Activity Pattern Survey (NHAPS)
  - Time-varying activity patterns



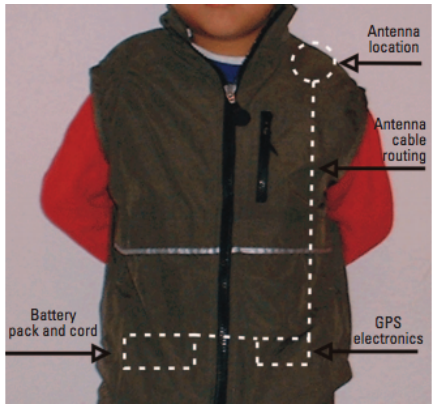
Klepeis et al., *J Exp. Anal. Environ. Epidem.* **2001**, 11, 231-252

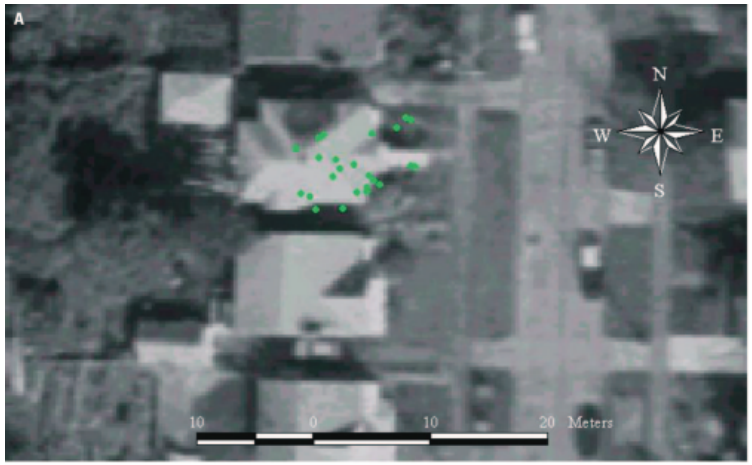
Human activity patterns

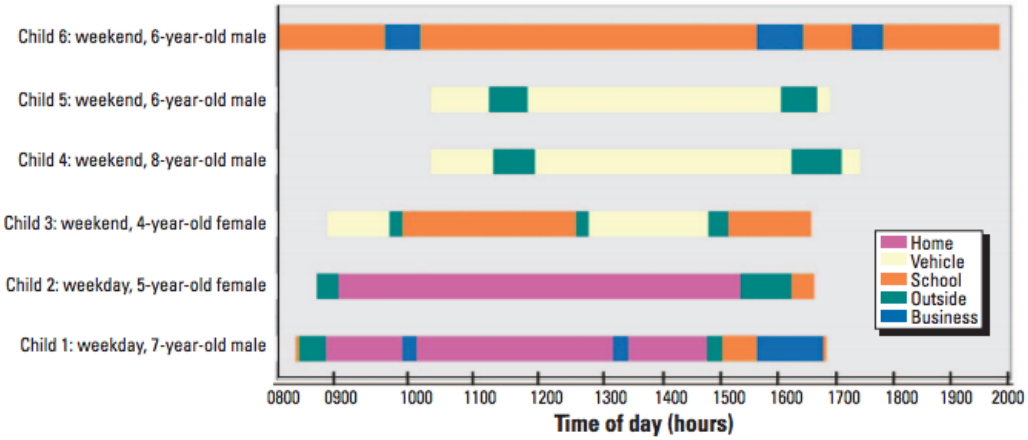
- What are some other ways to collect human activity data?

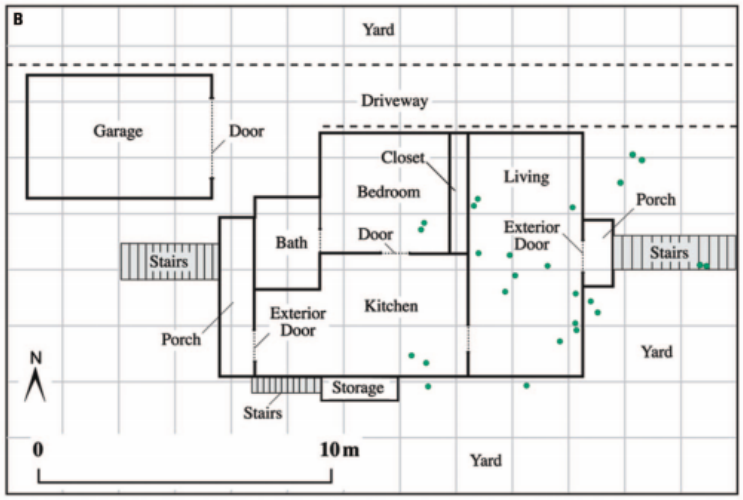
Human activity patterns

- What are some other ways to collect human activity data?
  - GPS instruments









Elgethun et al., *EHP* **2003**, 111, 115-122

Indoor exposures

- So we spend a lot of time indoors (Δt_{indoor} is large)
  - Do we also encounter large concentrations? (C_{indoor})

- Depends on what emissions we're talking about

- Let's first discuss "intake fractions"

Intake fractions

- Emissions to intake relationship
  - A 1 kg mass of pollutant is emitted into the environment from a point source 50 miles away from you
  - A 0.01 kg (10 g) mass of pollutant is emitted into your home air
  - Which is more important to you from an exposure standpoint?

- An intake fraction helps describe importance of emissions
  - Integrated intake of a pollutant released from a source and summed over all exposed individuals during a given exposure time, per unit of emitted pollutant:

$$iF = \frac{\sum_{people,time} \text{mass intake of pollutant by an individual}}{\text{mass of pollutant released into the environment}}$$

Bennett et al., 2002 *Environ Sci Technol*

Intake fractions

- Values of iF depend on several factors:
  - Chemical properties of the contaminant
  - Emission locations
  - Environmental conditions
  - Exposure pathways
  - Receptor (i.e., human) locations and activities
  - Population characteristics

Bennett et al., 2002 *Environ Sci Technol*

Intake fractions

- Lai et al. (2000) estimated IFs for several scenarios
  - Outdoor air basins, point releases, and line sources; vehicles; and indoors

Individual IFs

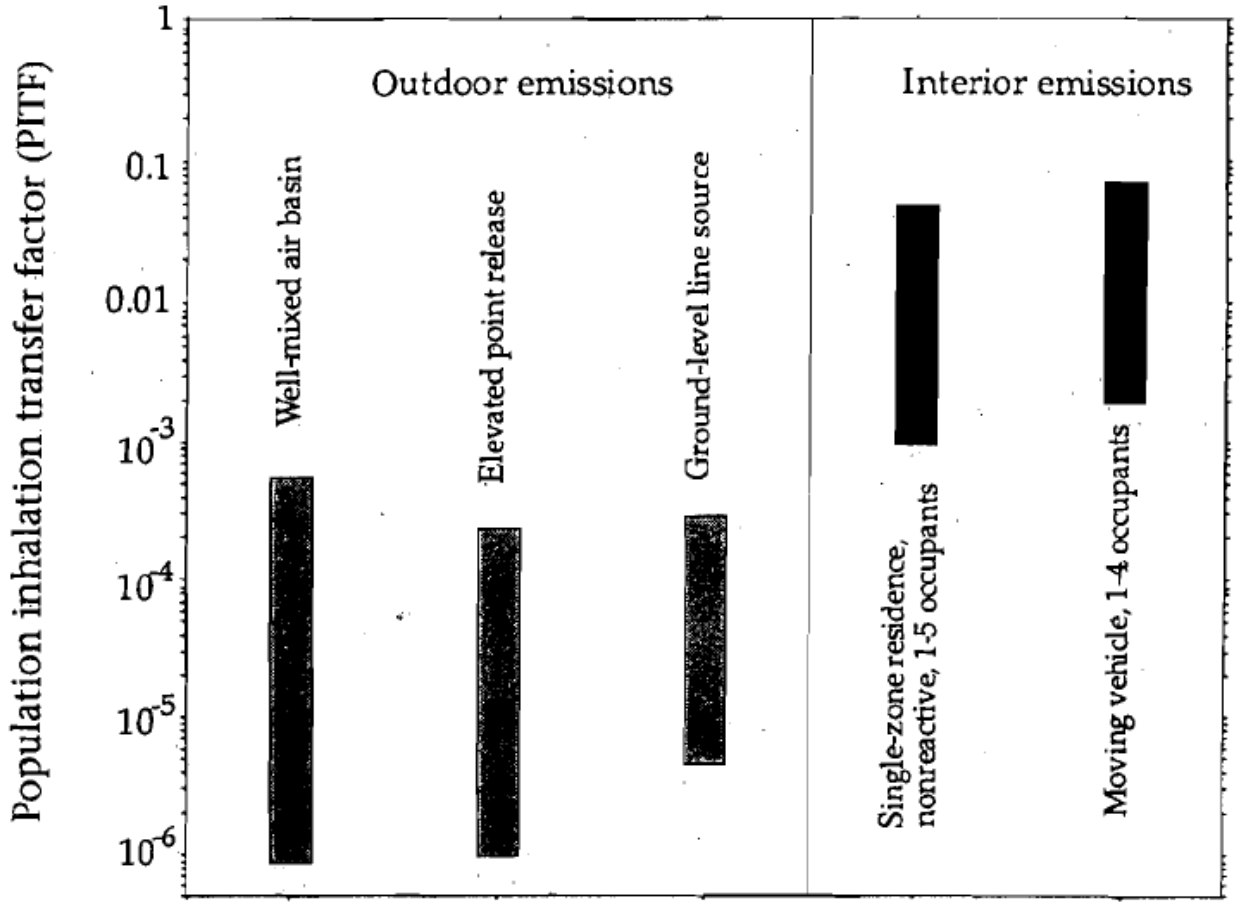
$$\mathrm{ITF} = \frac{\text{mass inhalation rate}}{\text{mass emission rate}} = \frac{CQ_B}{E}$$

Population wide IFs

$$\mathrm{PITF} = \sum_{i=1}^{N} \mathrm{ITF}_i$$

$$\mathrm{PITF} = \frac{\sum_{i=1}^{N} \int C_i(t) Q_{Bi}(t)\,dt}{\int E(t)\,dt}$$

$$\mathrm{PITF} = \frac{\sum_{i=1}^{N} C_i Q_{Bi}}{E}$$



Lai et al., 2000 *J Air Waste Manag Assoc*

Intake fraction example

- Benzene example
  - Benzene is emitted to outdoor air from motor vehicles
  - Benzene is also present in environmental tobacco smoke (ETS)

- Outdoor benzene in California's South Cost air basin (SoCAB)
  - 16,000 km^2 area
  - Home to 14 million people who drive vehicles ~0.5 billion km daily
    - They use ~59 million L of gasoline daily
    - ~280 mg of benzene is emitted per L of gasoline
    - Total emissions of ~17 metric tons (17000 kg) of benzene per day
    - Outdoor iFs range approximately 1×10^{-6} to 5×10^{-4}
      - Depending on meteorology and other factors

Bennett et al., 2002 *Environ Sci Technol*

Intake fraction example

- Benzene from ETS indoors
  - SoCAB is also home to ~1.9 million smokers
    - Consuming 42 million cigarettes daily
  - Assume that 50% of cigarettes consumed in the area are smoked in homes
  - Benzene emission factors for ETS are 280-610 μg per cigarette
    - Assume ~450 μg per cigarette
  - Total estimated residential emissions of benzene from ETS are ~9 kg/day
    - That is only ~0.05% of the total emitted by motor vehicles

  - But, the iF for a nonreactive pollutant in a residence is $\sim 7\times10^{-3}$
    - That is 10-100+ times as high as for outdoor emissions (1×10^{-6}-5×10^{-4})

  - Overall, vehicles account for inhalation of ~1 kg/day of benzene inhalation
    - Across the basin population
    - ETS accounts for ~60 g/day
    - So while ETS accounts for only 0.05% of the emissions, it accounts for ~6% of benzene intake in the area
      - Non-negligible amount (and *IF* is 120 times higher than *E*)

Bennett et al., 2002 *Environ Sci Technol*

Intake fractions

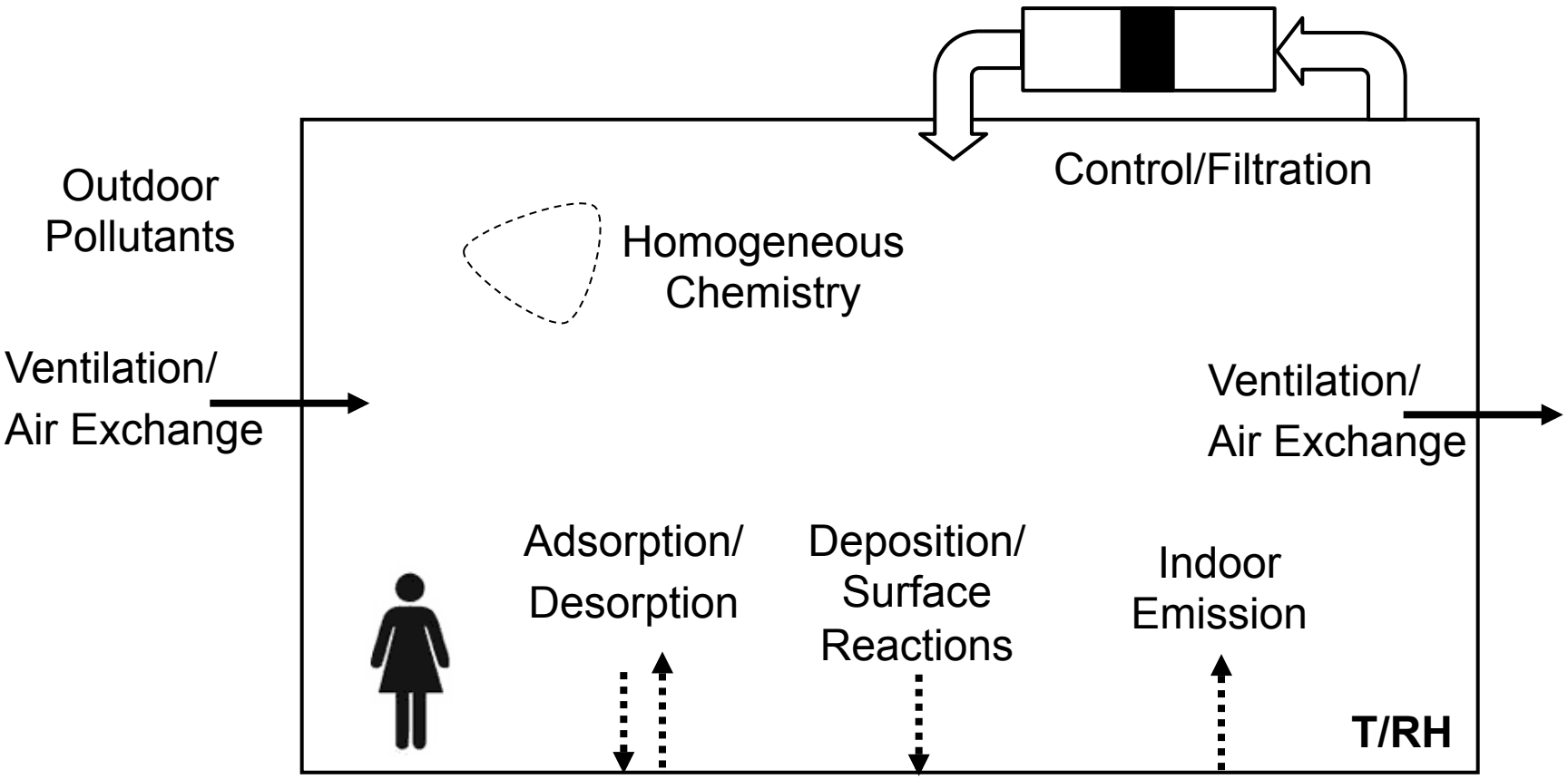
- So what was the answer to our original question?
  - A 1 kg mass of pollutant is emitted into the environment from a point source 50 miles away from you
  - A 0.01 kg (10 g) mass of pollutant is emitted into your home air
  - Which is more important to you from an exposure standpoint?

  - Indoor emission is ~1/100th of the outdoor mass emission
  - But indoor IF is ~10 to ~1000 higher
  - So the overall effect on intake is generally higher for the indoor source

Mass balances

- We've talked about time we spend indoors (Δt_{indoor})
- And we've talked about emission to intake ratios
- But do we also encounter large concentrations? (C_{indoor})
  - Need to be able to measure and predict C_{indoor}
  - And model what affects C_{indoor}
- We can do that with a mass balance

Indoor environment: Mass balance



Indoor environment: Mass balance

- Simplest case
  - Neglecting indoor physics/chemistry
    - No deposition, no reaction

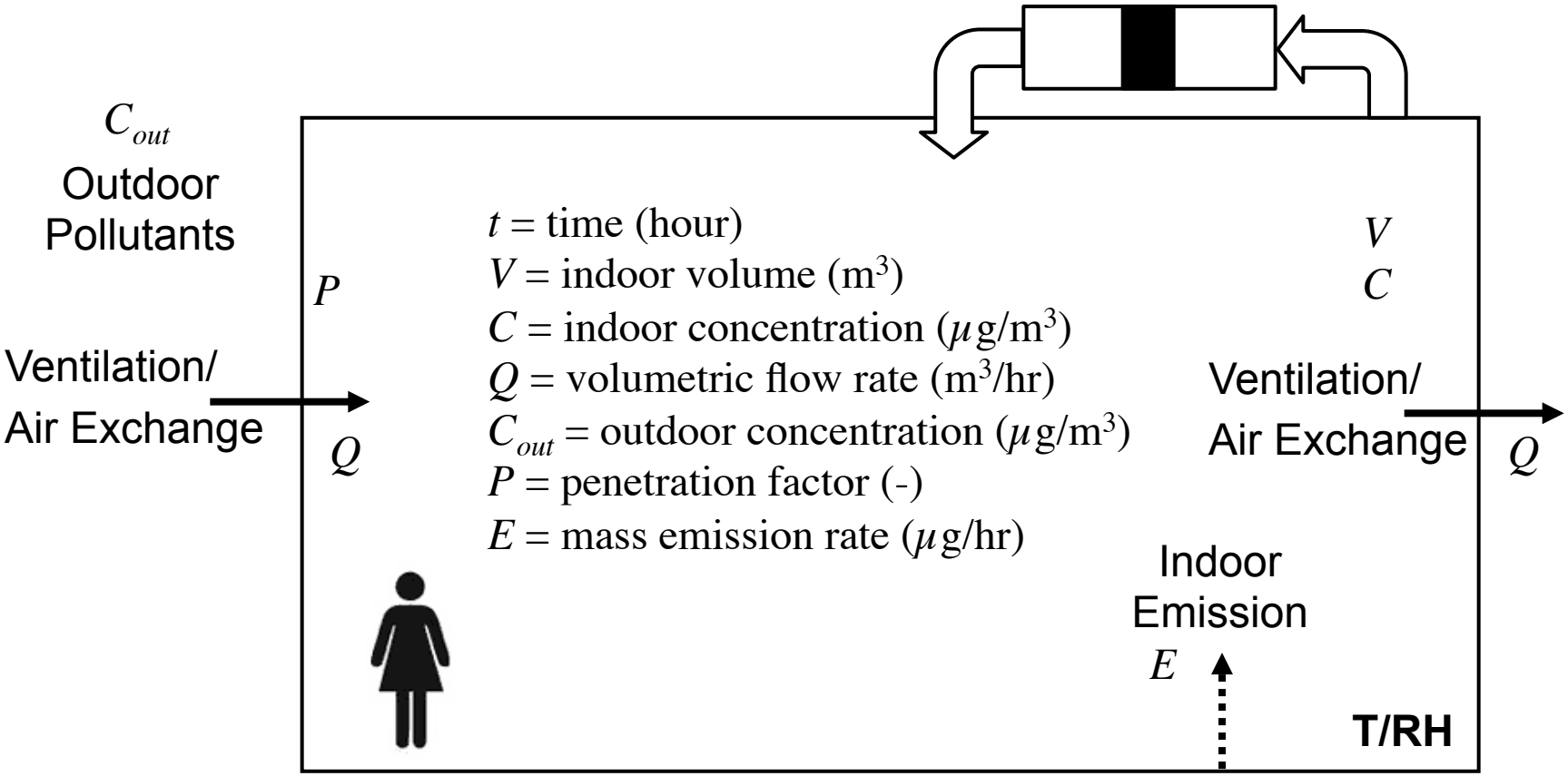
Mass accumulation rate [mass / time]	=	Mass flow in [mass / time]	+	Mass flow out [mass / time]	+	Mass emitted [mass / time]

$$\frac{dm}{dt} = \frac{dCV}{dt} = V\frac{dC}{dt} + \underbrace{C\frac{dV}{dt}}_{0}$$

Assumptions:

- Building/room can be treated as well-mixed
- Ventilation/air exchange rate is constant
- Outdoor pollutant concentration is constant
- Indoor sources emit at a constant rate

Indoor environment: Mass balance



$$V\frac{dC}{dt} = PQC_{out} - QC + E$$

Indoor environment: Mass balance

$$V\frac{dC}{dt} = PQC_{out} - QC + E$$

- Divide by volume

$$\frac{dC}{dt} = P\frac{Q}{V}C_{out} - \frac{Q}{V}C + \frac{E}{V}$$

$$\boxed{\frac{dC}{dt} = P\lambda C_{out} - \lambda C + \frac{E}{V}}$$

$$\lambda = \frac{Q}{V} = \text{air exchange rate } (\frac{1}{\text{hr}})$$

Indoor environment: Mass balance

- Assume steady-state conditions:

$$\underbrace{\cancel{\frac{dC}{dt}}}_{0} = P\lambda C_{out} - \lambda C + \frac{E}{V}$$

$$\boxed{C_{ss} = PC_{out} + \frac{E}{\lambda V}}$$

- If λ is large (and/or E is small): $PC_{out} >> E/\lambda V$
  - C approaches C_{out} (depending on P)
  - This means outdoor sources are relatively more important

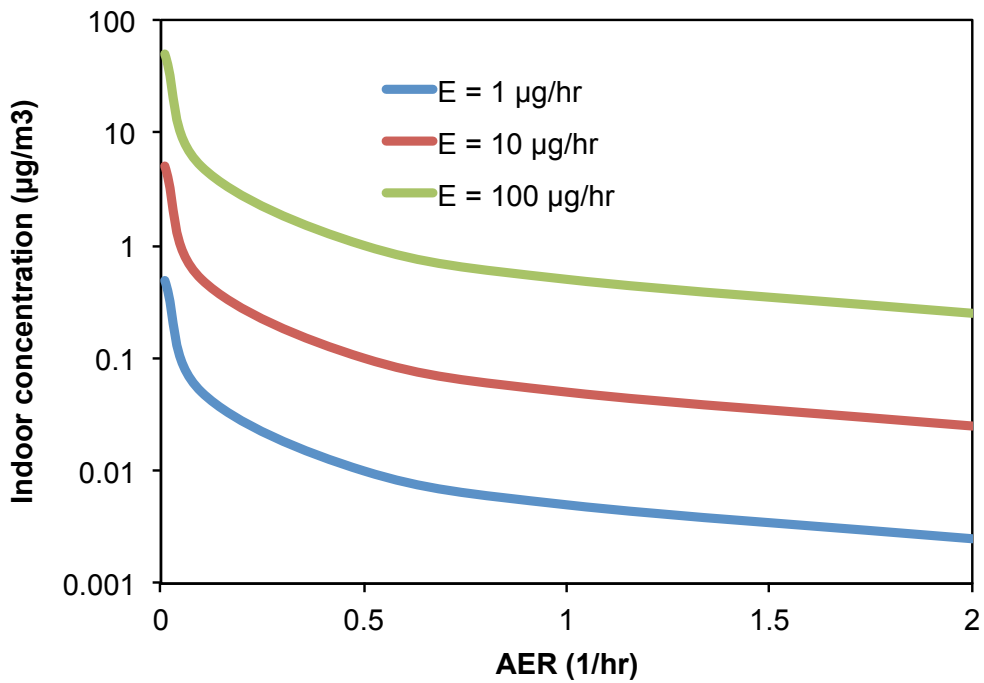
- If λ is small (and/or E is large): $PC_{out} << E/\lambda V$
  - C approaches $E/\lambda V$
  - This means indoor sources are relatively more important

Steady state mass balance

- Example steady state calculations: $C_{ss} = PC_{out} + \frac{E}{\lambda V}$

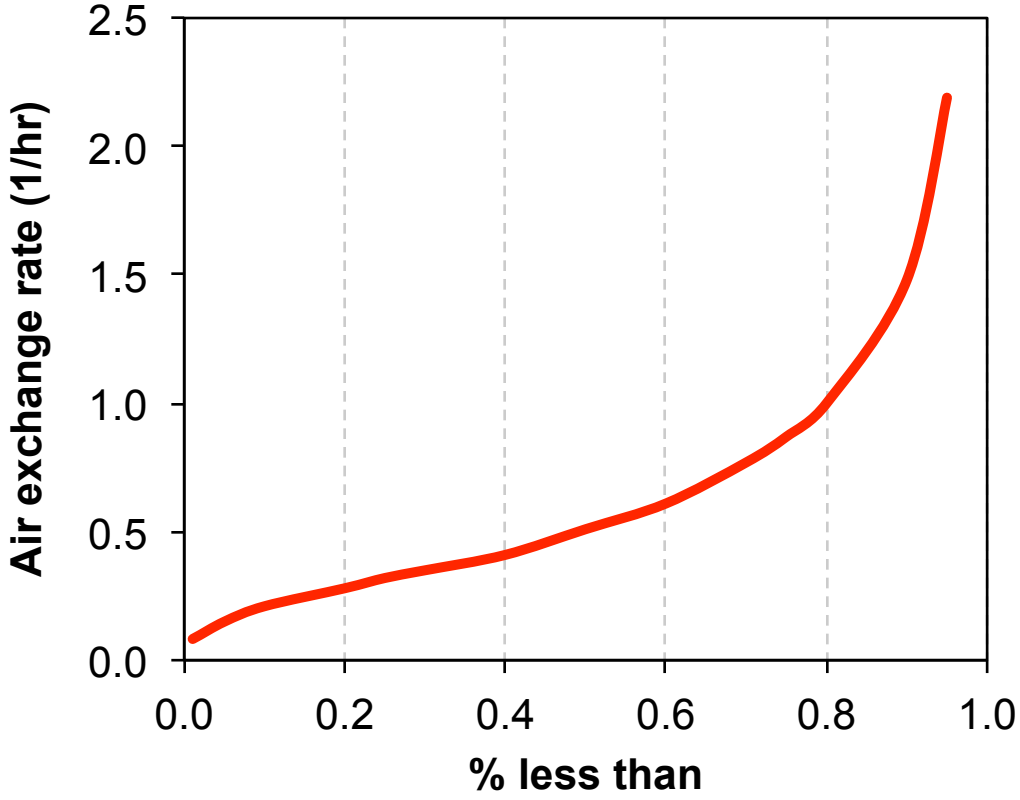
- Assume P = 1 and C_{out} = 0: $C_{ss} = \frac{E}{\lambda V}$

- Assume V = 200 m^3, how are C, E, and λ related?



What are typical values of λ (AER)?

- Distribution of AERs in ~2800 homes in the U.S.
  - Measured using PFT (perfluorocarbon tracer) in the early 1990s

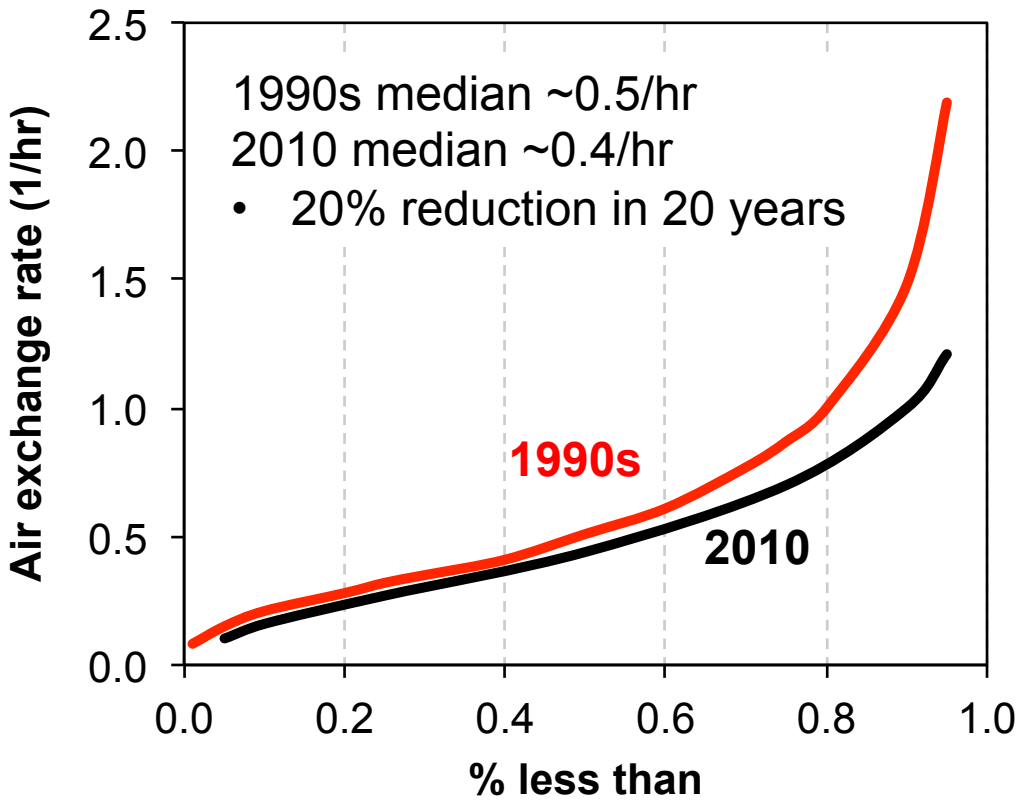


- What do you think this curve looks like now?

Murray and Burmaster, 1995 *Risk Analysis*

What are typical values of λ (AER)?

- Distribution of AERs U.S. homes
  - Early 1990s and revisited in 2010 (Persily et al. 2010)

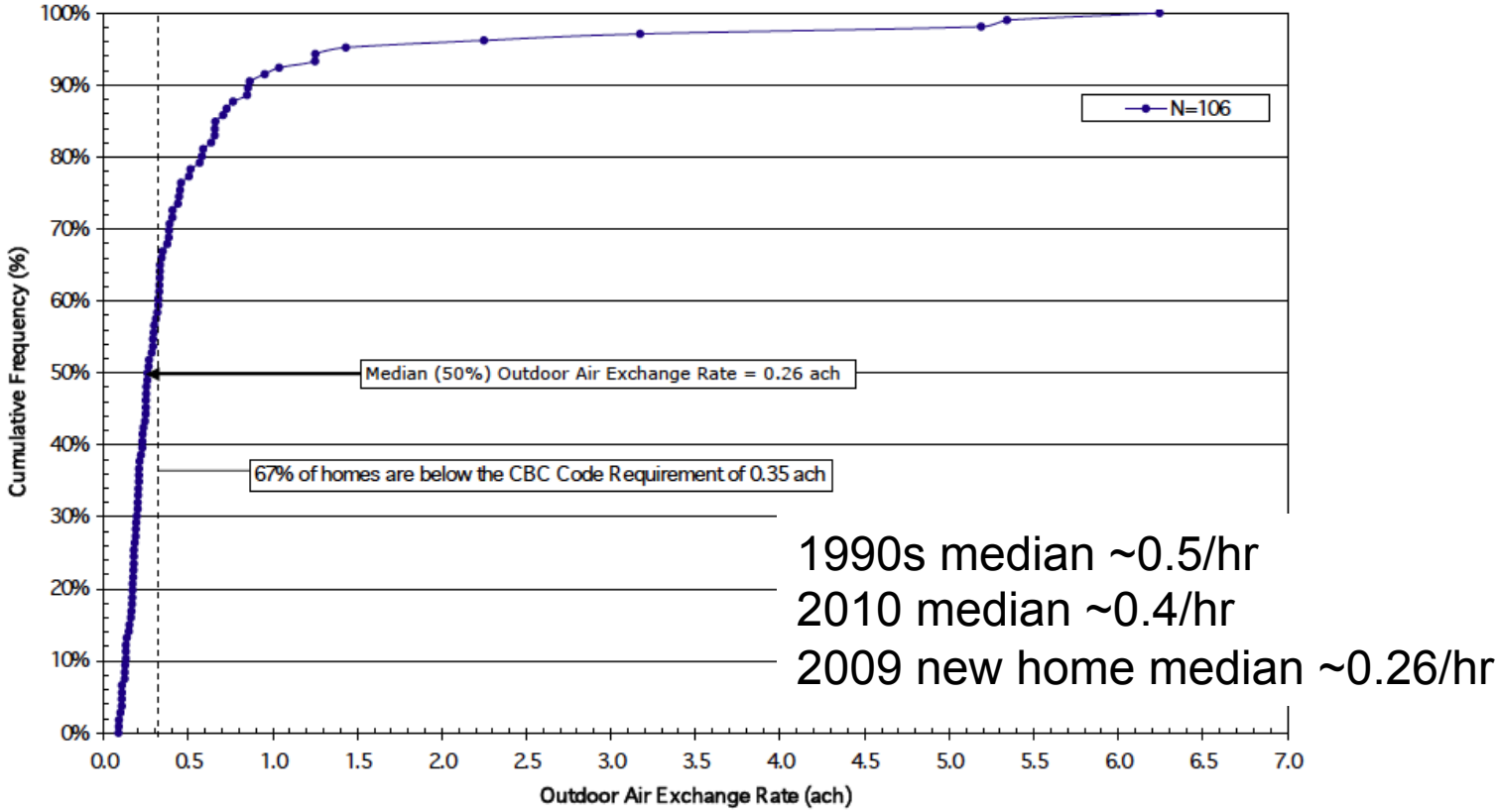


- What about brand new homes?

Murray and Burmaster, 1995 *Risk Analysis*; Persily et al. 2010 *Indoor Air*

What are typical values of λ (AER)?

- Distribution of AERs U.S. homes
  - Addition of 106 new homes (Offermann et al., 2009)

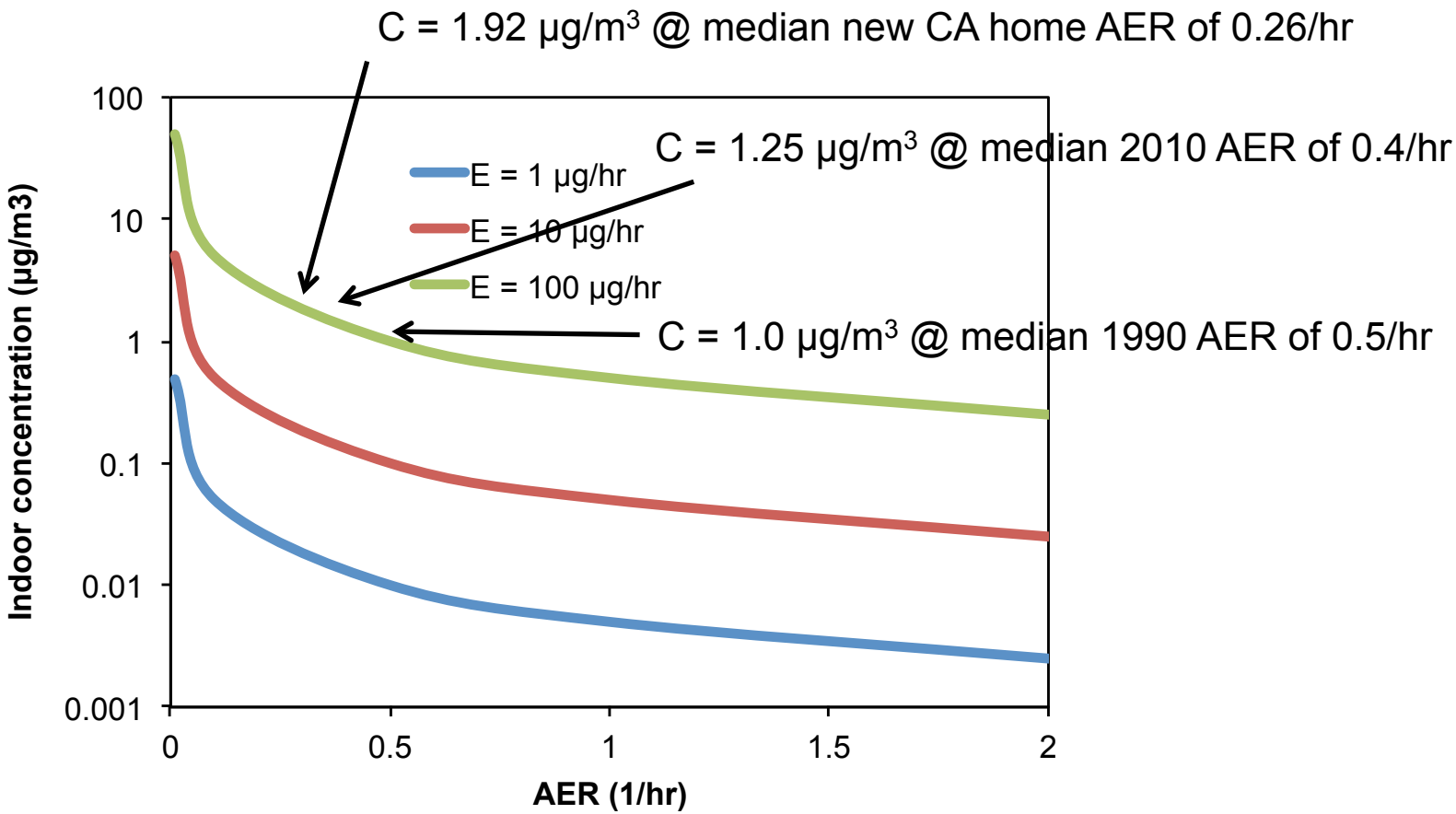


- Not uncommon for new homes to have AER = 0.05-0.20 per hour

Offermann et al. 2009 CEC PIER Report

Steady state mass balance with AER

- What do trends in AERs mean for indoor concentrations?
  - Nonreactive pollutants at steady date (without an outdoor source)



Limitations to previous mass balance

- Well-mixed assumption
  - Occupant exposure can be much higher than estimated near source
  - Cooking, cleaning, vicinity of smoker
  - Personal cloud or “pig pen” effect is where:

$$C_{personal} > C_{indoor}$$

- Assumption of no sinks or transformations
  - Adsorption, desorption, deposition, and reactions all ignored (for now)

- Assumption of no control of pollutants
  - No whole building filtration or portable air cleaner (for now)

- Also assumed steady-state
  - What about dynamic solution?

Dynamic solution to mass balance

- Start with basic mass balance:

$$\frac{dC}{dt} = P\lambda C_{out} - \lambda C + \frac{E}{V}$$

- Rearrange:

$$\frac{1}{P\lambda C_{out} - \lambda C + \frac{E}{V}} dC = dt$$

- Factor out (-1):

$$\frac{1}{\lambda C - P\lambda C_{out} - \frac{E}{V}} dC = -dt$$

- Substitute: $\text{Let } x = \text{denominator} = \lambda C - P\lambda C_{out} - \frac{E}{V}$
  - So that: $\frac{dx}{dC} = \lambda \longrightarrow dC = \frac{1}{\lambda} dx$

Dynamic solution to mass balance

Letting $x = \lambda C - P\lambda C_{out} - \frac{E}{V}$ and thus $\frac{dx}{dC} = \lambda$ transforms:

$$\frac{1}{\lambda C - P\lambda C_{out} - \frac{E}{V}} dC = -dt \xrightarrow{\text{into}} \frac{1}{\lambda}\left(\frac{1}{x}\right) dx = -dt$$

- We can now solve this simpler equation

Rearrange:

$$\left(\frac{1}{x}\right) dx = -\lambda dt$$

Integrate both sides:

$$\int_{x_0}^{x} \frac{1}{x} dx = -\lambda \int_0^t dt$$

Solution with x:

$$\ln(x)\Big|_{x_0}^{x} = -\lambda t$$

Substitute back in for x:

$$\ln\left\{\frac{\lambda C - P\lambda C_{out} - \frac{E}{V}}{\lambda C(t=0) - P\lambda C_{out} - \frac{E}{V}}\right\} = -\lambda t$$

Dynamic solution to mass balance

$$\ln\left\{\frac{\lambda C - P\lambda C_{out} - \dfrac{E}{V}}{\lambda C(t=0) - P\lambda C_{out} - \dfrac{E}{V}}\right\} = -\lambda t$$

- Raise *e* to both sides:

$$\frac{\lambda C - P\lambda C_{out} - \dfrac{E}{V}}{\lambda C(t=0) - P\lambda C_{out} - \dfrac{E}{V}} = e^{-\lambda t}$$

- Rearrange:

$$\lambda C - P\lambda C_{out} - \frac{E}{V} = \left\{\lambda C(t=0) - P\lambda C_{out} - \frac{E}{V}\right\} e^{-\lambda t}$$

Dynamic solution to mass balance

- Solve for C:
  - Which is *C* at time *t*, or *C(t)*

$$\lambda C - P\lambda C_{out} - \frac{E}{V} = \left\{ \lambda C(t=0) - P\lambda C_{out} - \frac{E}{V} \right\} e^{-\lambda t}$$

$$\boxed{C(t) = C(t=0)\underbrace{\cancel{e^{-\lambda t}}}_{0} + \left(PC_{out} + \frac{E}{\lambda V} \right)\left(1 - \underbrace{\cancel{e^{-\lambda t}}}_{0}\right)}$$

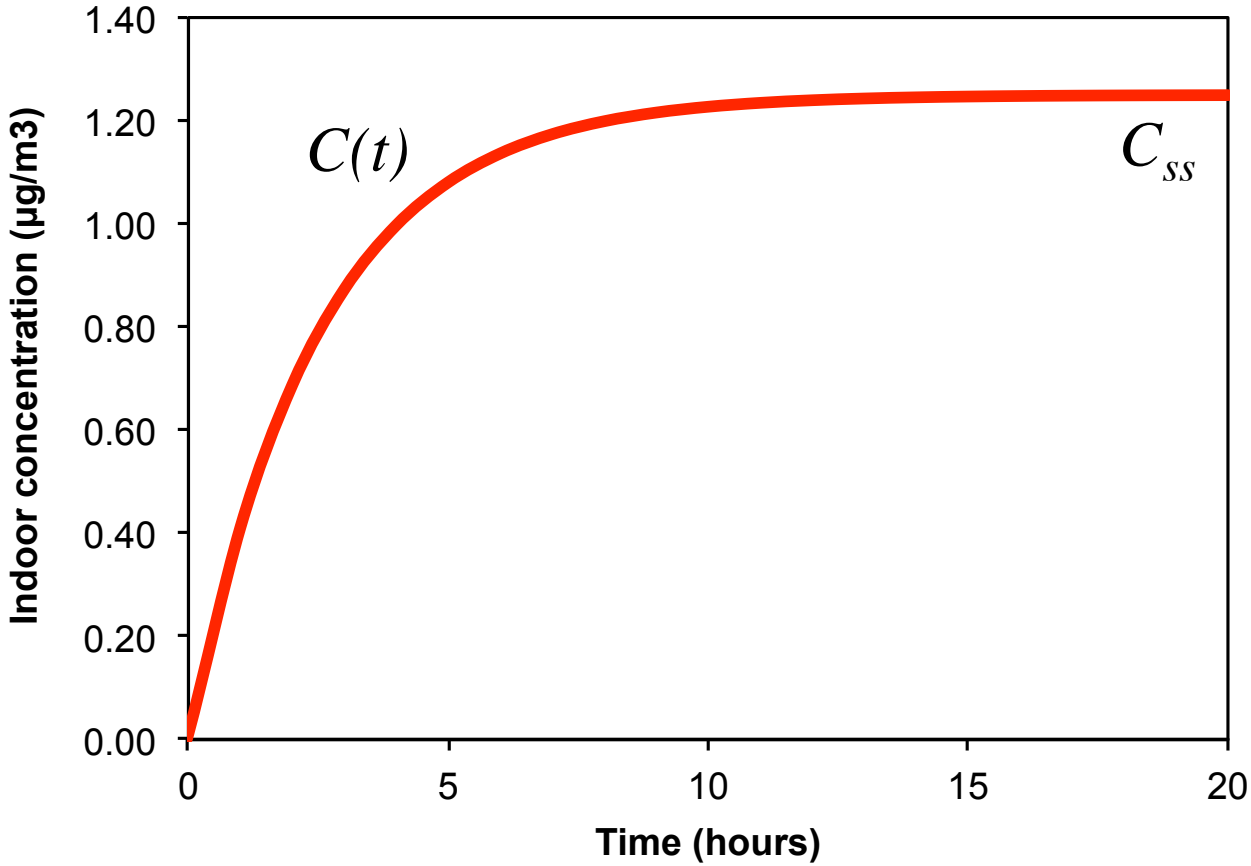
- What do these two terms represent?

- What happens as t → ∞ ?

$$\boxed{C(t \to \infty) = PC_{out} + \frac{E}{\lambda V} = \text{our steady state solution}}$$

Dynamic solution to mass balance

- Example concentration profile
  - V = 200 m^3, E = 100 μg/hr, λ = 0.4/hr, C_{out} = 0



Time to reach steady state

$$C(t) = C(t=0)e^{-\lambda t} + \left(PC_{out} + \frac{E}{\lambda V}\right)\left(1 - e^{-\lambda t}\right)$$

- If we assume an inert pollutant emitted indoors with an initial concentration of zero, how long would it take to achieve 95% of steady state?
- 95% of steady-state is reached when:

$$\left(1 - e^{-\lambda t}\right) = 0.95 \longrightarrow e^{-\lambda t} = 1 - 0.95 = 0.05$$

$$-\lambda t = \ln(0.05) \longrightarrow \lambda t = -\ln(0.05) = 3$$

$$\boxed{t = \frac{3}{\lambda}}$$

Consider λ = 0.1 hr^{-1}

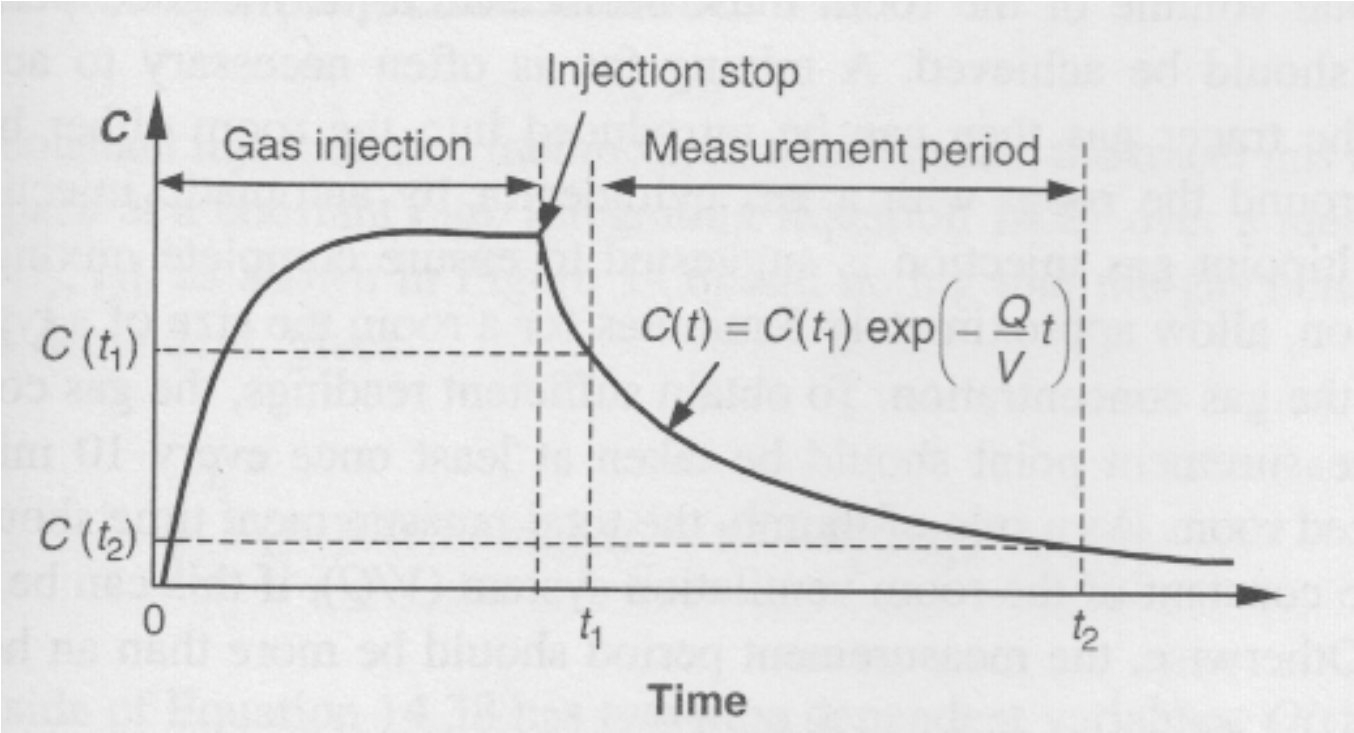
t to 95% steady state = 30 hours

Consider λ = 1 hr^{-1}

t to 95% steady state = 3 hours

How do we measure λ?

- One method is to inject an inert tracer gas, and measure the decay from C(t=0) after time t=0



How do we measure λ?

- One method is to inject an inert tracer gas, and measure the decay from C(t=0) after time t=0
  - In this case, E = 0
  - Assume P = 0 (reasonable for inert gas)

$$C(t) = C(t=0)e^{-\lambda t} + \left(PC_{out} + \underbrace{\cancel{\frac{E}{\lambda V}}}_{0}\right)\left(1 - e^{-\lambda t}\right)$$

$$C(t) = C(t=0)e^{-\lambda t} + C_{out}\left(1 - e^{-\lambda t}\right)$$

$$C(t) = C(t=0)e^{-\lambda t} + C_{out} - C_{out}e^{-\lambda t}$$

$$C(t) - C_{out} = \left\{C(t=0) - C_{out}\right\}e^{-\lambda t}$$

How do we measure λ?

$$C(t) - C_{out} = \left\{ C(t=0) - C_{out} \right\} e^{-\lambda t}$$

$$\frac{C(t) - C_{out}}{C(t=0) - C_{out}} = e^{-\lambda t}$$

- Take the natural log of both sides:

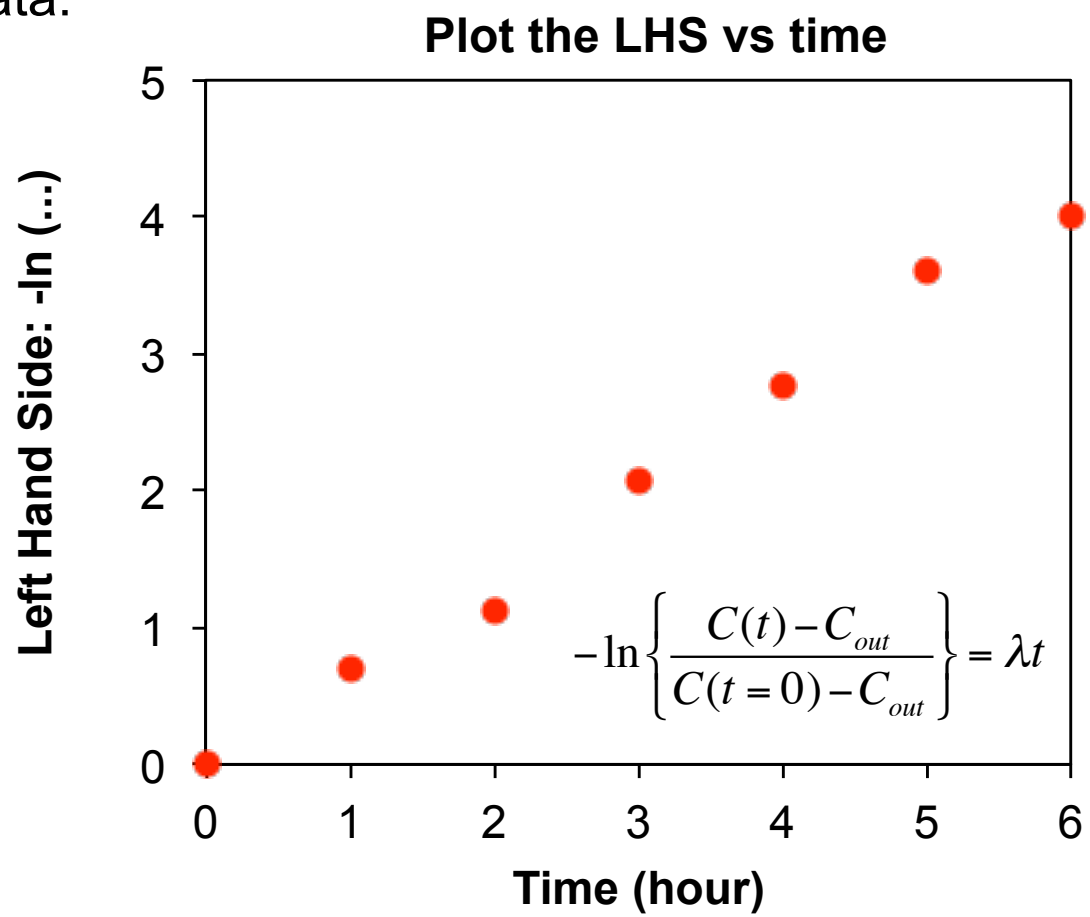
$$-\ln\left\{ \frac{C(t) - C_{out}}{C(t=0) - C_{out}} \right\} = \lambda t$$

- To find λ, plot left hand side versus right hand side
  - Slope of that line is λ

How do we measure λ?

- **Example:** You perform a tracer test with CO_2
  - You measure a constant outdoor concentration of 400 ppm
  - You elevate indoors to 2000 ppm, then leave for 6 hours
  - You record these data:

Time (hr)	C(t) (ppm)
0	2500
1	1450
2	900
3	660
4	530
5	460
6	430



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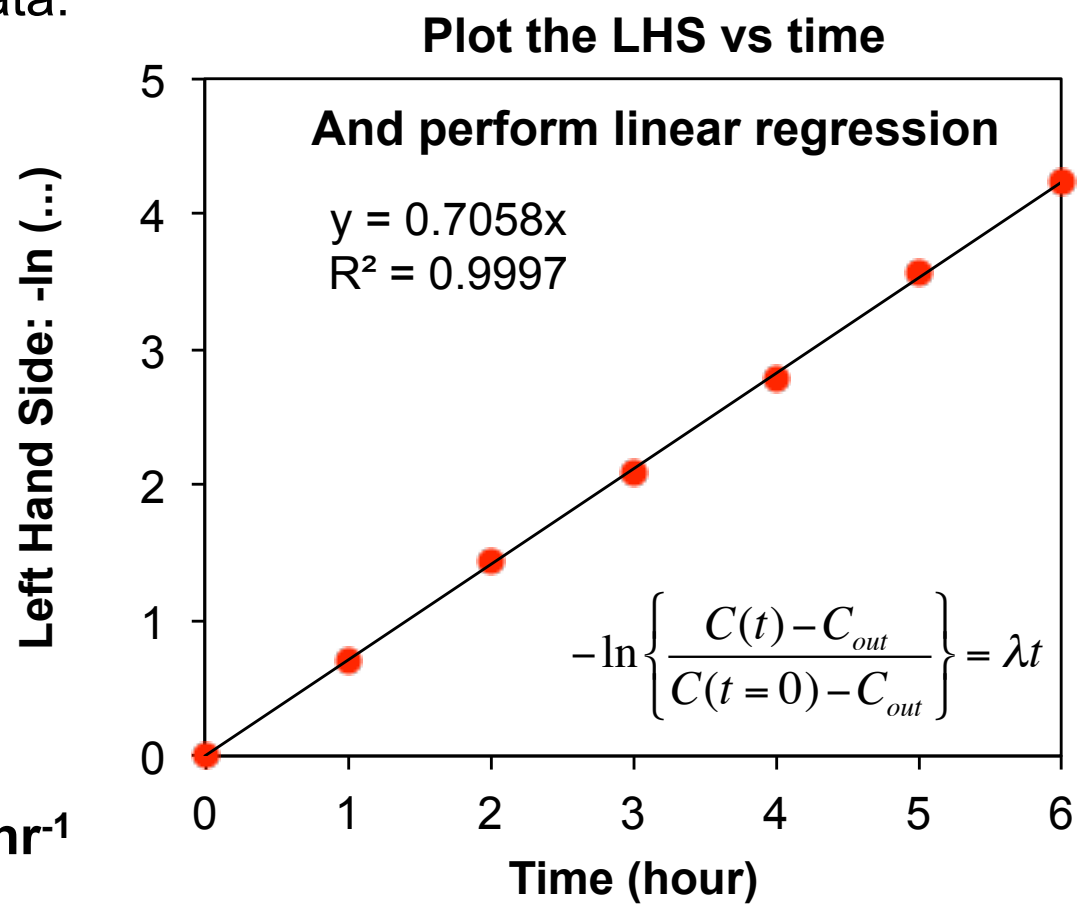


AER = λ = slope = 0.71 hr^{-1}

What makes a good tracer gas?

Characteristics

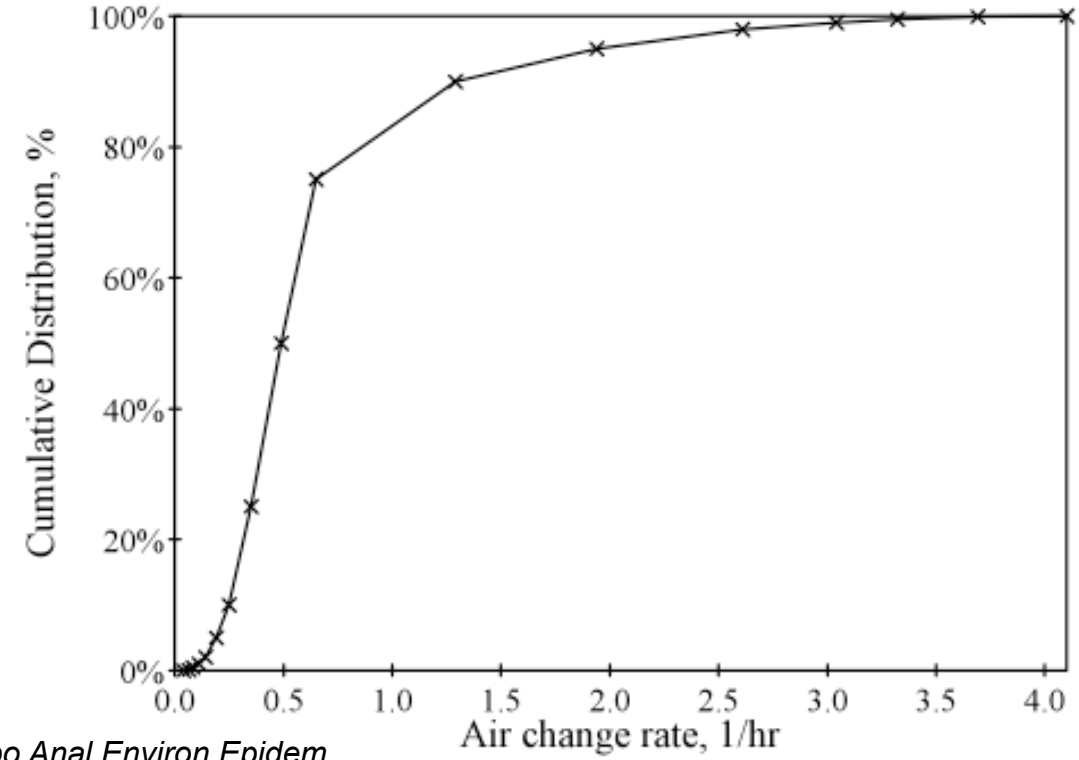
- Non-reactive (inert)
- Non-toxic
- Colorless
- Odorless
- Cheap
  - Gas
  - Sensor
- Low detection limits
- Portable

Commonly used gases

- Carbon dioxide (CO_2)
  - People are a source
  - Need to account for E/V
- Nitrous oxide (N_2O)
  - Laughing gas
  - Toxic at high levels
- Freon (CFC)
  - Global warming potential
- Helium (He)
  - Costs
- Sulfur hexafluoride (SF_6)
  - Global warming potential

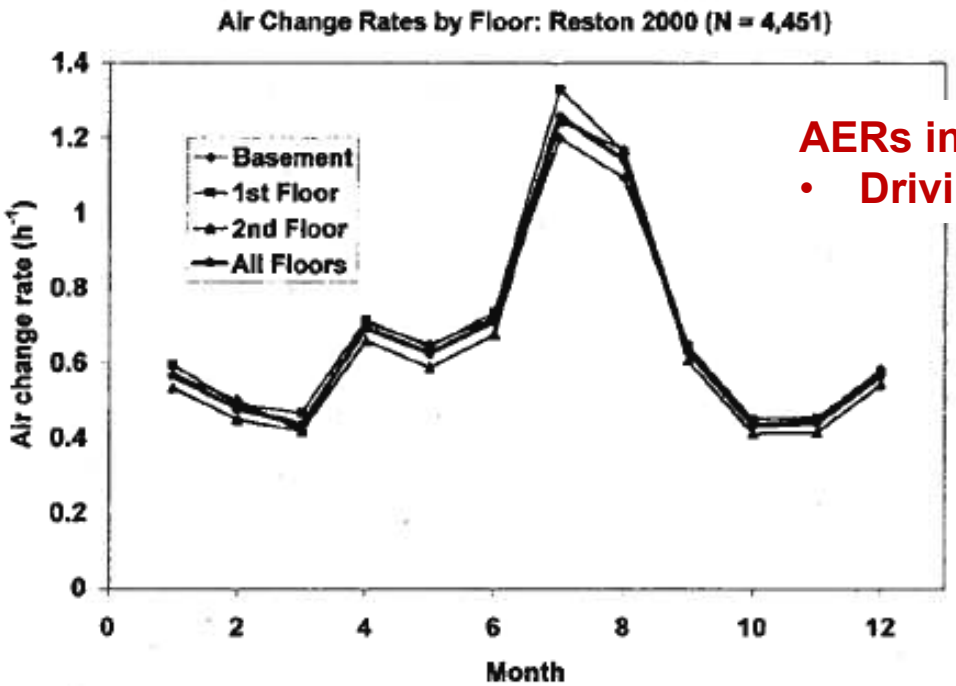
Variation in AER

- Air exchange rates differ from building to building
  - Differences vary by driving forces and building characteristics
- Example research: "Continuous measurements of air change rates in an occupied house for 1 year: the effect of temperature, wind, fans, and windows"
  - 4600 AERs measured by automated SF_6 system in one house for 2 years!



Wallace et al. 2002 *J Expo Anal Environ Epidem*

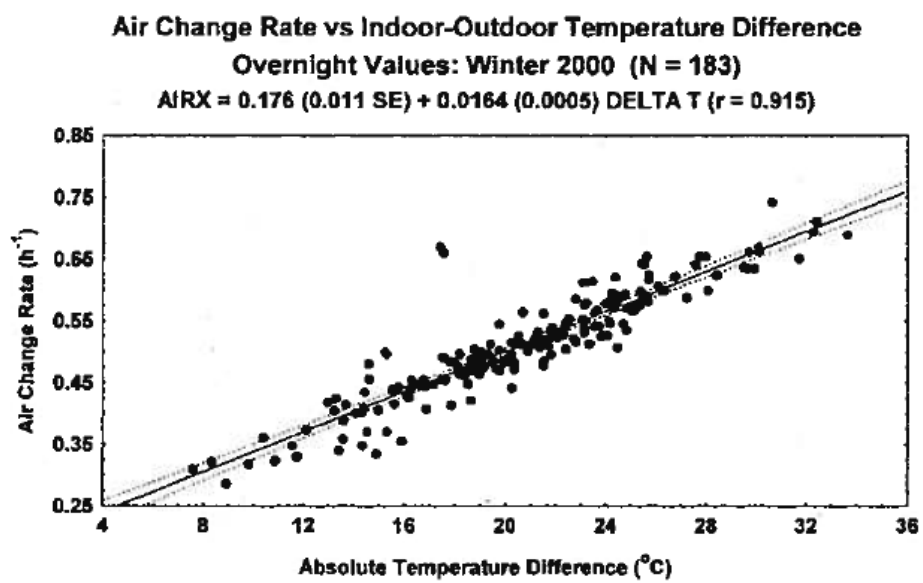
Variation in AER



AERs in individual buildings can vary by season

- **Driving forces: temperature, wind speed**

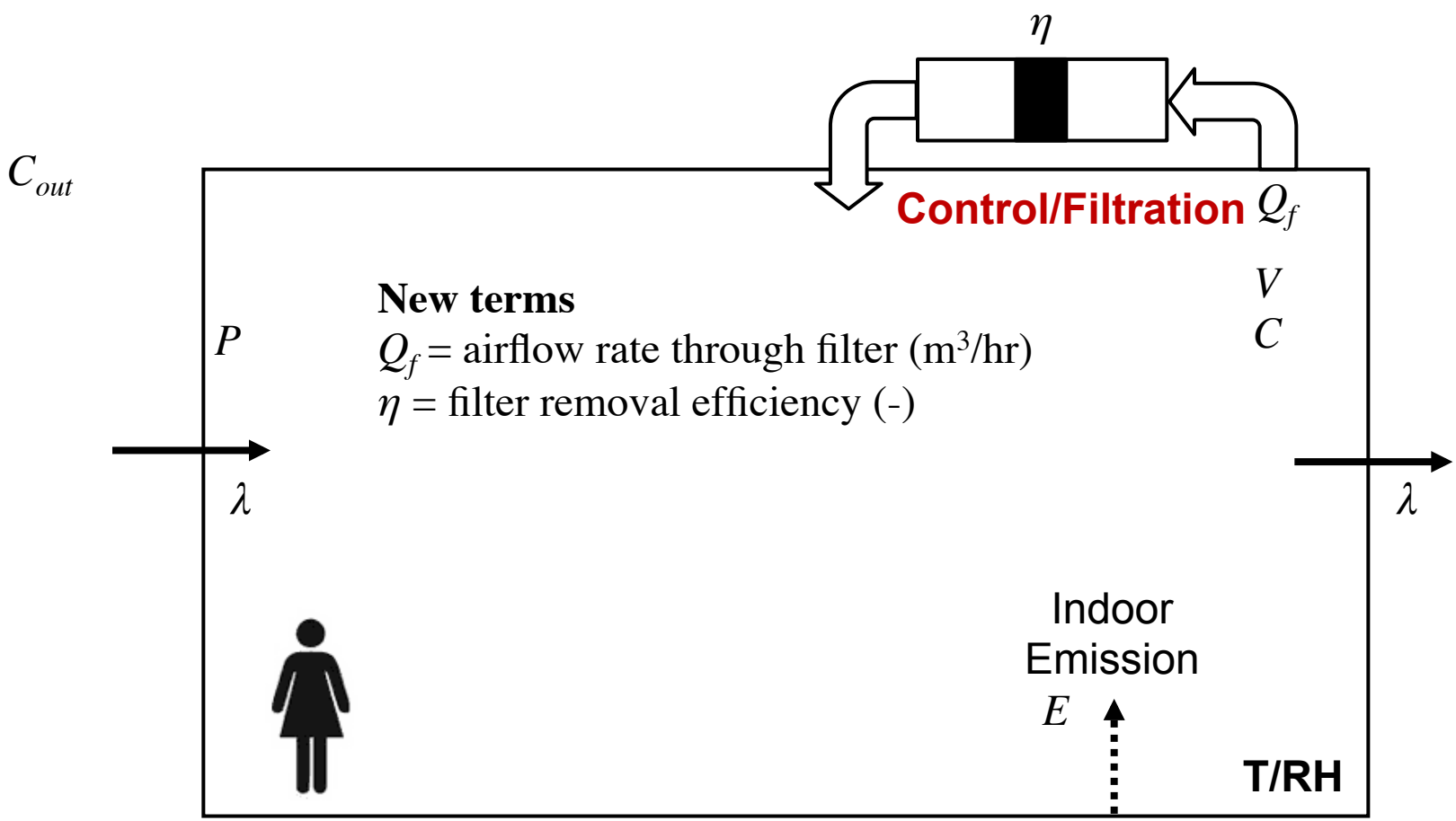
AERs can vary by I/O temperature within seasons



Wallace et al. 2002 *J Expo Anal Environ Epidem*

What if our pollutant has another loss term?

- Rarely are we working with inert pollutants
  - Other loss mechanisms are important
  - Deposition to surfaces, control by HVAC filter, reaction, evaporation…



Mass balance with filtration

- New term to mass balance:

$$V\frac{dC}{dt} = PQC_{out} - QC + E - \eta Q_f C$$

$$\underbrace{\cancel{\frac{dC}{dt}}}_{0} = P\lambda C_{out} - \lambda C + \frac{E}{V} - \frac{\eta Q_f}{V}C$$

- Assume steady state for now, divide by λ, and solve for C:

$$C = \frac{PC_{out} + \frac{E}{\lambda V}}{1 + \frac{\eta Q_f}{\lambda V}} \longrightarrow C = \frac{PC_{out} + \frac{E}{\lambda V}}{1 + \frac{CADR}{\lambda V}}$$

- CADR = Clear Air Delivery Rate $\longrightarrow$ $CADR = \eta Q_f$

Dynamic solution with filtration

$$V\frac{dC}{dt} = PQC_{out} - QC + E - \eta Q_f C$$

- Going through the same process as before but with a new loss term:

$$C(t) = \underbrace{C(t=0)e^{-\left(\lambda+\frac{CADR}{V}\right)t}}_{\text{how far from your initial concentration have you fallen?}} + \underbrace{\frac{PC_{out} + \frac{E}{\lambda V}}{1+\frac{CADR}{\lambda V}}}_{\text{steady-state solution}} \underbrace{\left(1 - e^{-\left(\lambda+\frac{CADR}{V}\right)t}\right)}_{\text{how close to steady-state have you risen?}}$$

Generalized steady and dynamic mass balance solutions

$$\frac{dC}{dt} = S - LC$$

$$S = sources = \lambda PC_{out} + \frac{E}{V}$$

$$L = losses = \lambda + \frac{CADR}{V} + k_{deposition} + k_{rxn} + \ldots$$

General steady-state solution: $C_{ss} = \frac{S}{L}$

General dynamic solution: $C(t) = C_0 e^{-Lt} + \frac{S}{L}\left(1 - e^{-Lt}\right)$

Assignment: HW1

- HW1 has been posted to BB
  - Covers AER estimation and basic steady state calcs
- Due 1 week from today in class
  - Upload a PDF, email me a PDF, or turn in hardcopy in class
- I will be at ASHRAE in Dallas Thursday through Tuesday morning
  - Ask any questions via email
- Tuesday class as scheduled, unless I get delayed in Dallas
  - In that case, I would email you all Tuesday about postponing

Next time

- Overview of indoor pollutants
  - Particles
  - Gas-phase compounds
  - Biological
- Typical concentrations measured in field studies
  - Will go into individual dynamics later in the course
- Read Weschler paper if interested
  - How have indoor pollutants changed since the 1950s?