

CAE 463/524

Building Enclosure Design

Fall 2012

Lecture 10: November 5, 2012

Heat transfer – floors, roofs, and thermal mass

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Housekeeping

- Ray Clark's lecture last week
- Exam
 - Grades and solutions: both on Blackboard
- Campus project due next week in class
 - Firm deadline
 - Any questions?
- Final project
 - Need to schedule final presentations
 - Approximately half on 11/26 in class, half on 12/3 7-9 PM

Housekeeping: final presentation scheduling

- Final project scheduling
 - Need to schedule final presentations
 - Approximately half on 11/26 in class, half on 12/3 7-9 PM
 - **Please tell me what day you prefer to present ASAP**

Team	Member	Member	Topic	11/26 or 12/3?
1	Russo	Foley	Green roofs	
2	Vagner	Diaz	Electrochromic windows	
3	Espinoza	Wright	Cool roofs	
4	Angulo	Huo	Double skin facades	
5	Sebastian	Morris	Phase change materials	
6	Kayo	Gonzalez, Alv	Building integrated photovoltaics	
7	Zwang	Gomez Soriano	Exterior insulated finish systems	
8	El Orch	Gonzalez, Ar	Strawbale construction	
9	Mcgreal	Diaz	Conventional construction	
10	Daras Ballester	Zylstra	High performance glass	
11	Mejia	n/a	Sun shading and thermal mass	

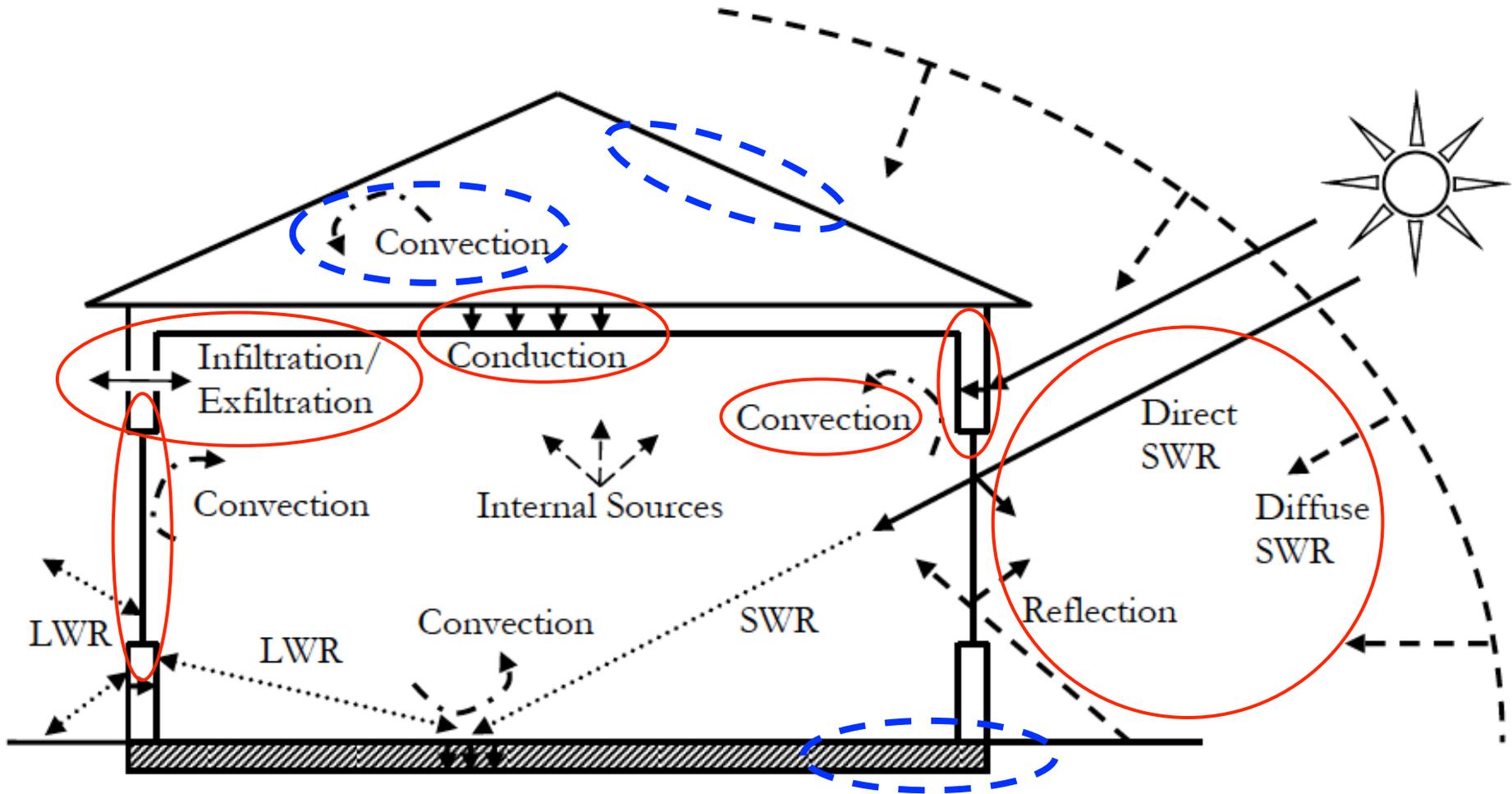
Last time we met

- Two weeks ago: Glazing
 - Light transmittance and heat transfer through windows
 - Solar heat gain coefficients, SHGC
 - Visible light transmission, VT
- Last week: Ray Clark, SOM
 - Daylighting and energy use

Today's lecture

- Heat transfer through floors and roofs
- Thermal mass
 - Unsteady conduction
- Will lead into energy modeling for enclosure design
 - Next lecture (Lecture 11)

Building heat transfer: what have we learned?



FLOORS AND BELOW-GRADE WALLS

(1) Below-grade and (2) on-grade heat transfer

Below-grade heat flow

- Where does heat flow?
 - Depends on surface and **ground temperature** distributions

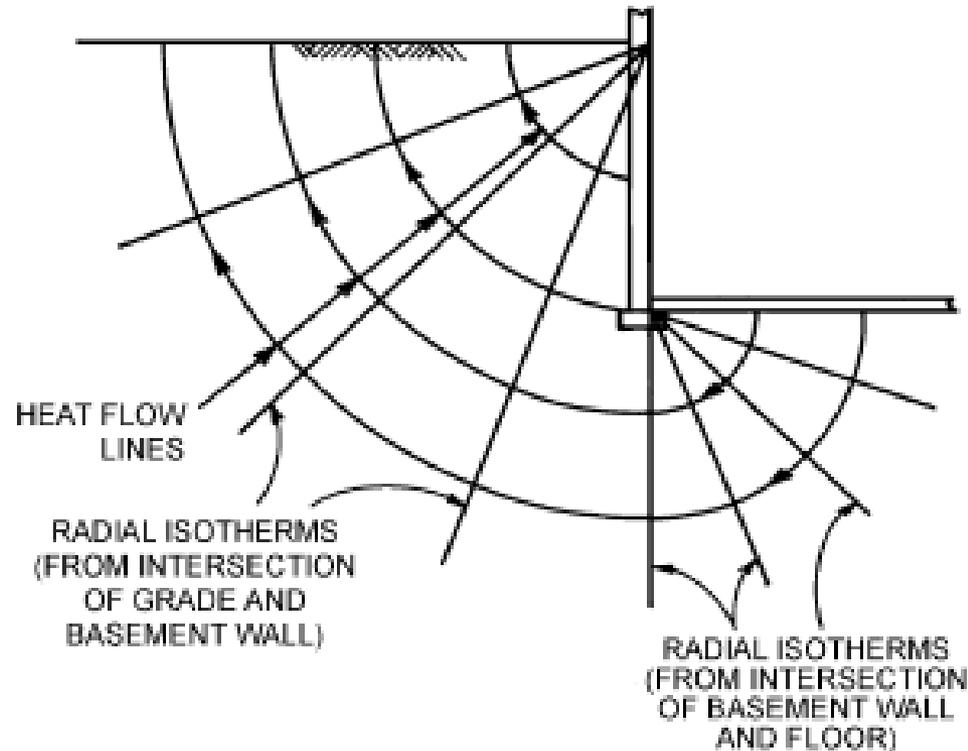


Fig. 4 Heat Flow from Basement

Mean ground temperatures

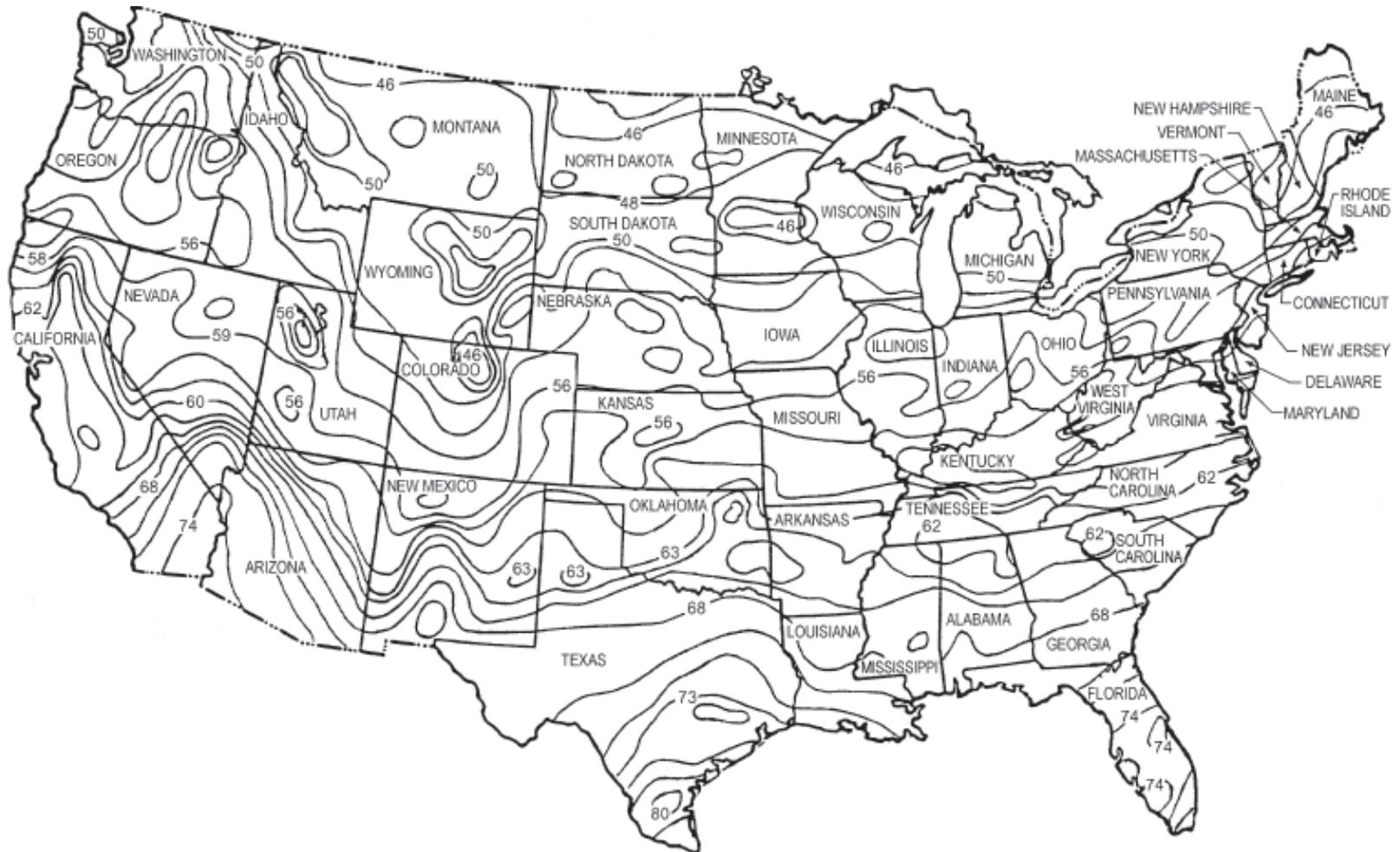
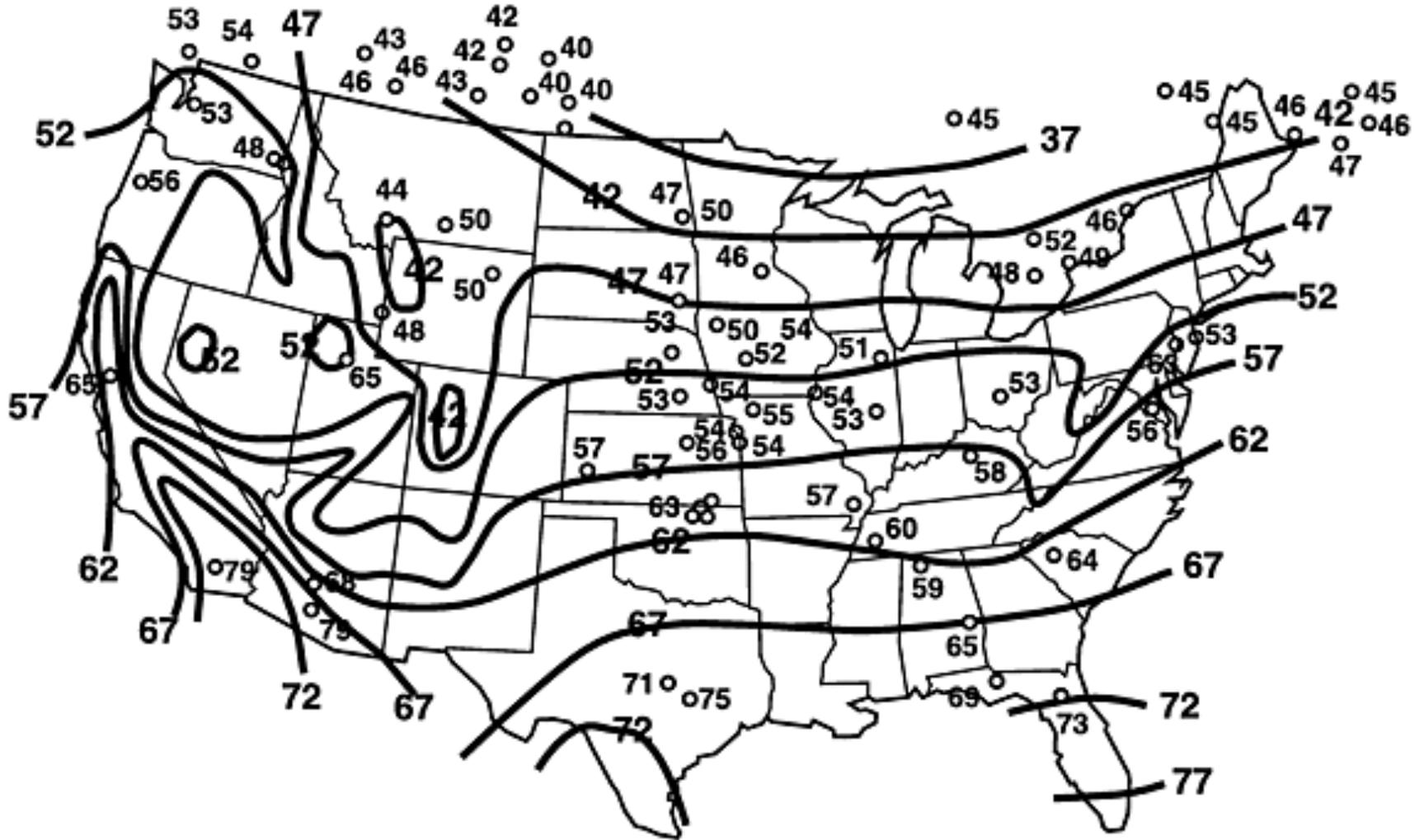
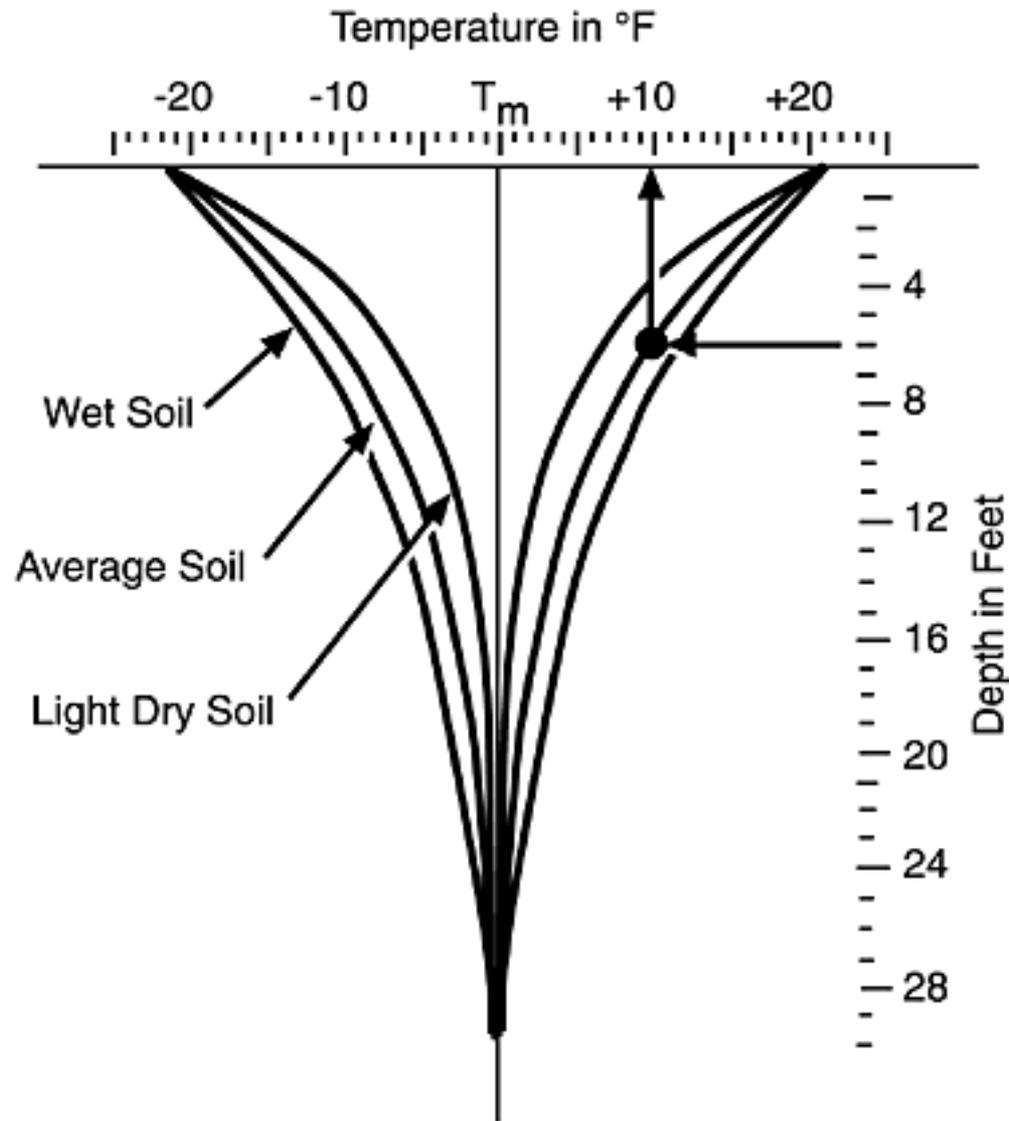


Fig. 17 Approximate Groundwater Temperatures (°F) in the Continental United States

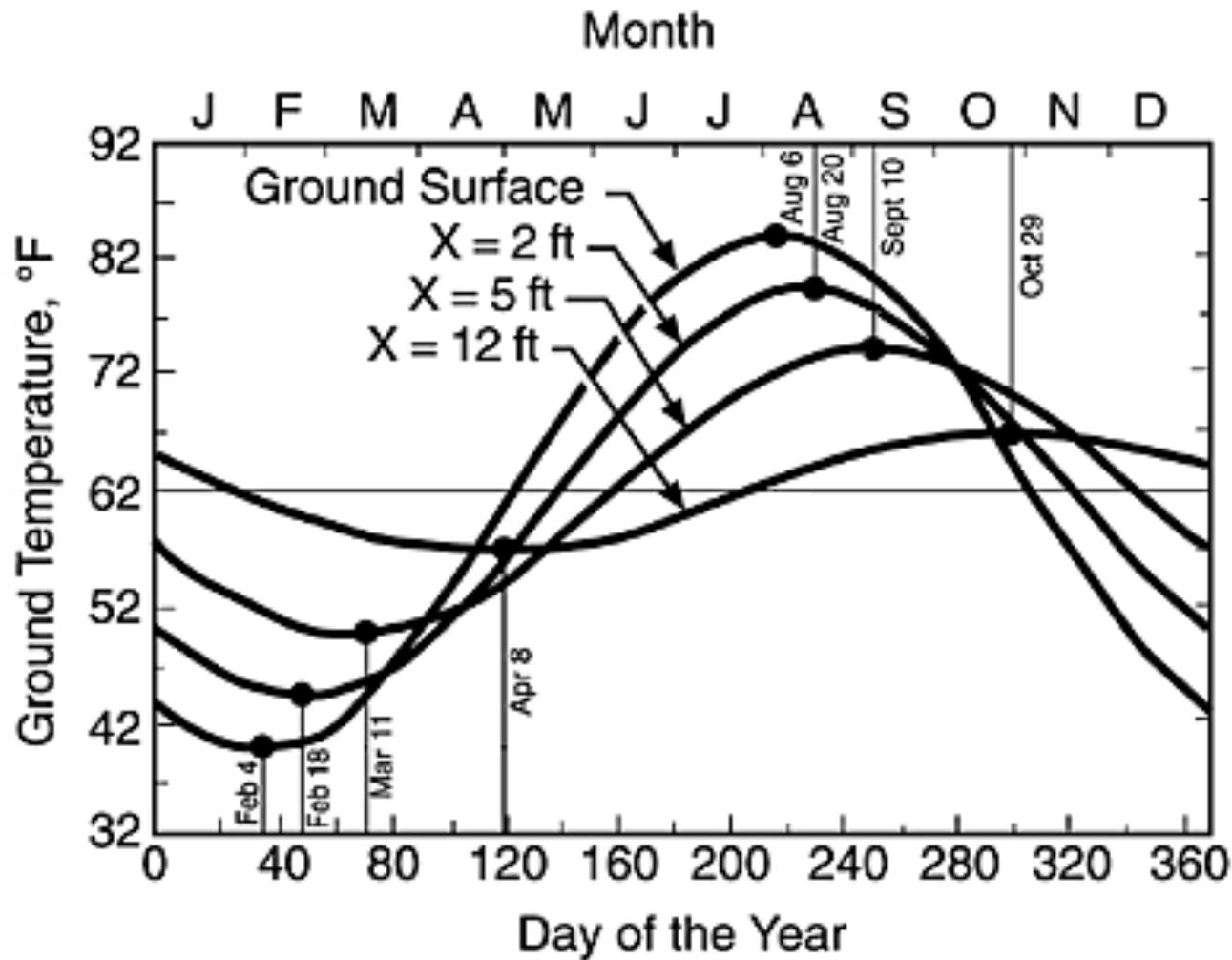
Mean ground temperatures



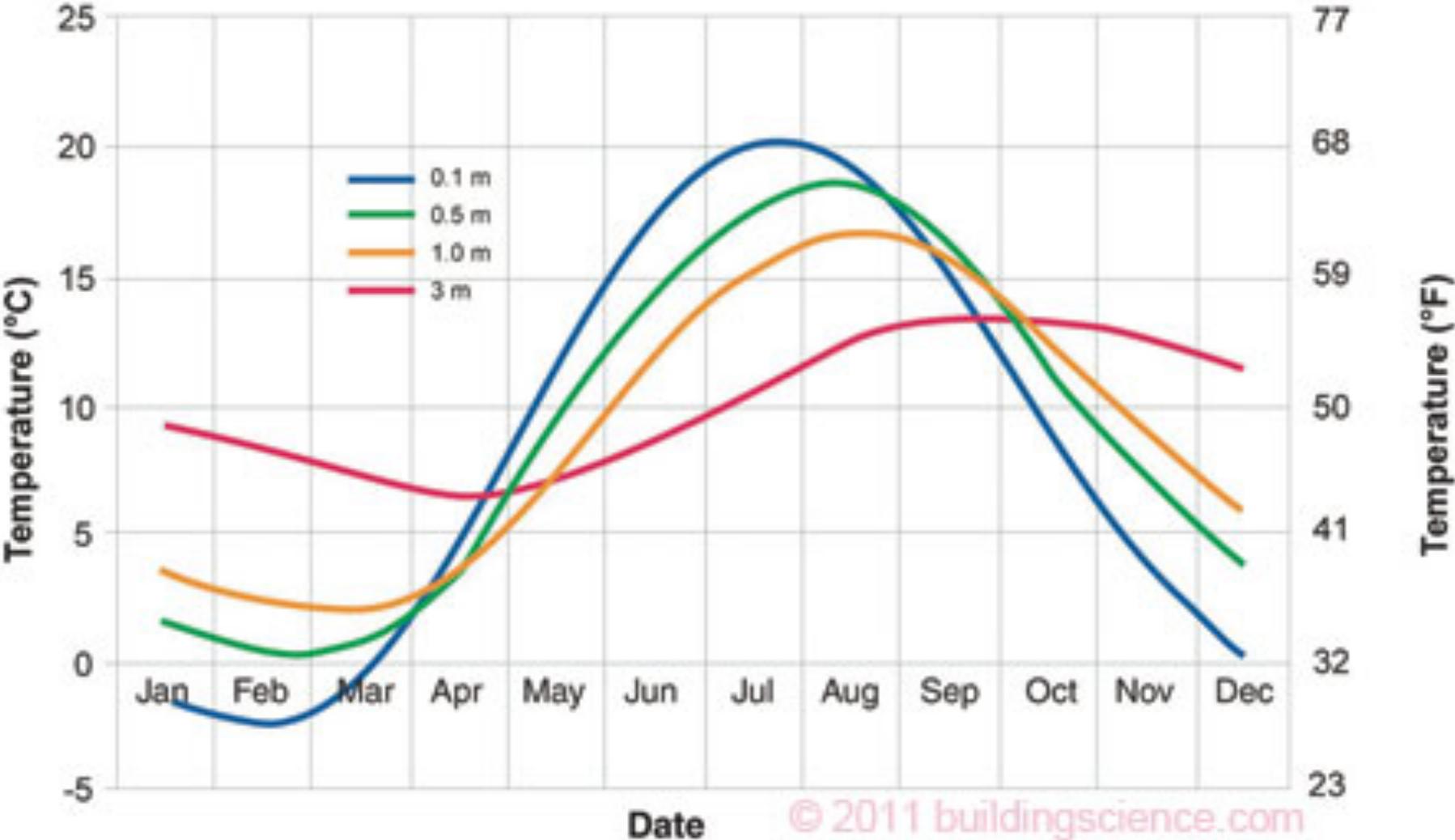
Ground temperatures and depth



Ground temperatures and season



Ground temperatures and season: cold climate



Design ground temperatures

- Design (worst-case) ground temperatures
 - Adjust mean ground temperature by a peak seasonal amplitude:

$$T_{gr} = T_{gm} - T_A$$

where

T_A = the ground temperature variation amplitude (right)

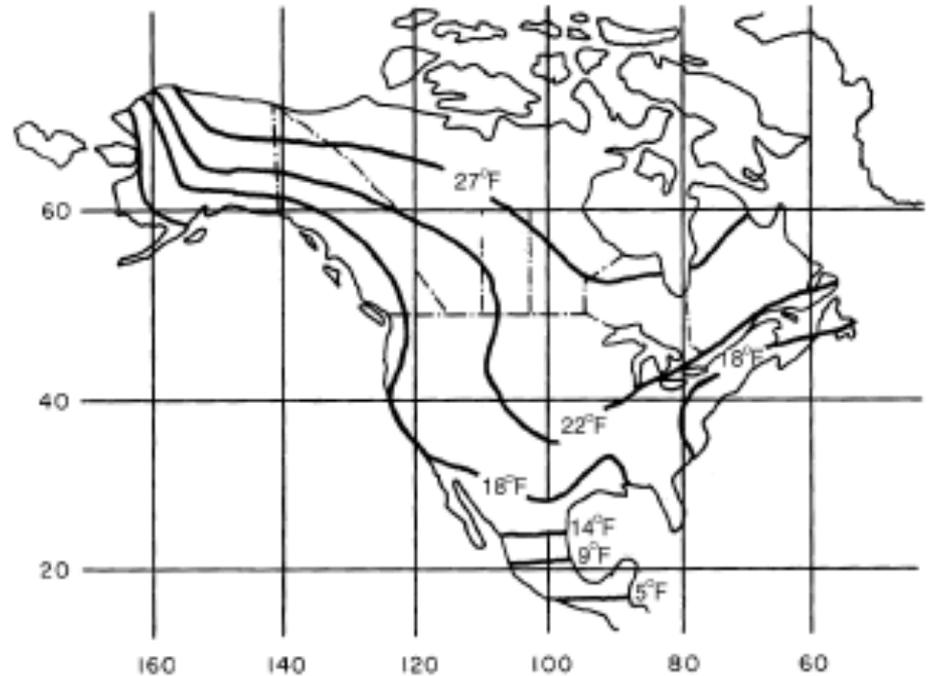
T_{gm} = mean ground temperature

*Note for Chicago:

$$T_A \approx 22^\circ\text{F} \quad T_{gm} \approx 54^\circ\text{F}$$

$$T_{gr} \approx 54 - 22 = 32^\circ\text{F}$$

Alternatively, T_{gr} can be estimated as the mean air temperature in the coldest month



Ground Temp **Amplitude**, T_A

Below-grade heat transfer

- Heat transfer through below-grade walls and floors
 - Conduction is truly 2-D
 - 1-D modeling is not appropriate
- Heat transfer through walls
 - Between inside and surrounding soil (not exterior air)
 - Depends on the wall area
 - Wall area depends upon perimeter but also height
- Heat transfer through the floor
 - Between inside and the soil below
 - Depends on the floor area
- ASHRAE HOF has some guidelines for transforming 2-D into 1-D

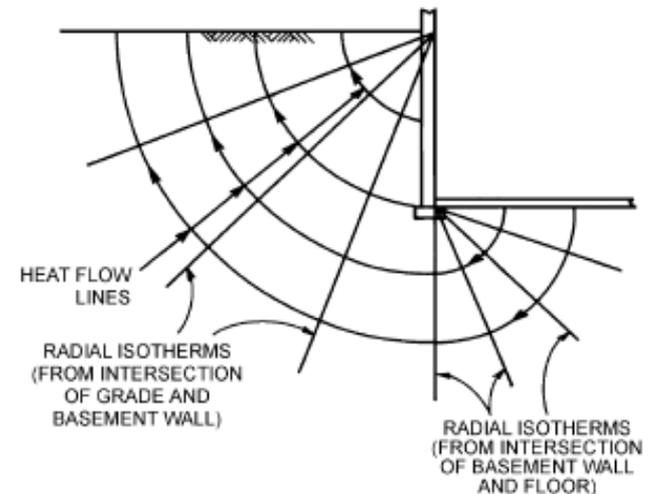


Fig. 4 Heat Flow from Basement

Below-grade heat transfer

$$Q = AU_{avg} (T_i - T_{gr}) \text{ [W]}$$

$$q = U_{avg} (T_i - T_{gr}) \text{ [W/m}^2\text{]}$$

where

A is the wall or floor area below grade [m^2] (analyze any wall portion above- grade in normal way)

T_i is the below grade inside temp [K]

T_{gr} is the **design** ground surface temp [K]

U_{avg} is the average U factor for the below grade surface [$\text{W}/(\text{m}^2\text{K})$] (see following slides)

Below grade depth parameters

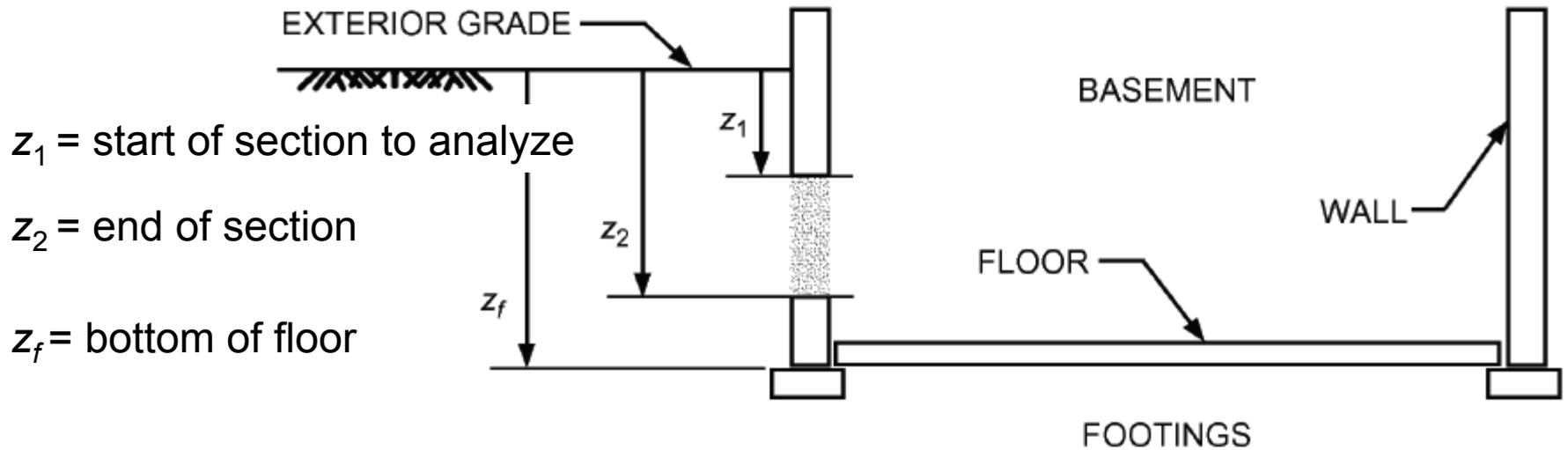


Fig. 14 Below-Grade Parameters

$U_{avg,bf}$ for below-grade floors

- For **average below-grade floor** value with a floor depth of height z_f from ground (“grade”)

$$U_{avg,bf} = \frac{2k_{soil}}{\pi w_b} \times \left[\ln\left(\frac{w_b}{2} + \frac{z_f}{2} + \frac{k_{soil}R_{other}}{\pi}\right) - \ln\left(\frac{k_{soil}R_{other}}{\pi}\right) \right]$$

k_{soil} = soil thermal conductivity ≈ 1.4 W/mK

R_{other} = R value of floor + insulation + convection [m²K/W]

w_b = shortest dimension of basement width [m]

z_f = floor depth below grade [m]

Pre-computed tables for $U_{avg,bf}$

Table 17 Average U-Factor for Basement Floors

z_f (depth of floor below grade), m	$U_{avg,bf}$, W/(m ² ·K)			
	w_b (shortest width of basement), m			
	6	7	8	9
0.3	0.370	0.335	0.307	0.283
0.6	0.310	0.283	0.261	0.242
0.9	0.271	0.249	0.230	0.215
1.2	0.242	0.224	0.208	0.195
1.5	0.220	0.204	0.190	0.179
1.8	0.202	0.188	0.176	0.166
2.1	0.187	0.175	0.164	0.155

Soil conductivity is 1.4 W/(m·K); floor is uninsulated. For other soil conductivities and insulation, use Equation (38).

- Assuming **un-insulated concrete** floor

$U_{avg,bw}$ for below-grade walls

$$U_{avg,bw} = \frac{2k_{soil}}{\pi(z_1 - z_2)} \times \left[\ln\left(z_2 + \frac{2k_{soil}R_{other}}{\pi}\right) - \ln\left(z_1 + \frac{2k_{soil}R_{other}}{\pi}\right) \right]$$

- k_{soil} = soil thermal conductivity ≈ 1.4 W/mK
- R_{other} = R value of wall, insulation and inside surface resistance [m²K/W]
- z_1, z_2 = depths of top and bottom of wall segment under consideration [m]

Pre-computed tables for $U_{avg,bw}$

Table 16 Average U-Factor for Basement Walls with Uniform Insulation

Depth, m	$U_{avg,bw}$ from grade to depth, W/(m ² ·K)			
	Uninsulated	R-0.88	R-1.76	R-2.64
0.3	2.468	0.769	0.458	0.326
0.6	1.898	0.689	0.427	0.310
0.9	1.571	0.628	0.401	0.296
1.2	1.353	0.579	0.379	0.283
1.5	1.195	0.539	0.360	0.272
1.8	1.075	0.505	0.343	0.262
2.1	0.980	0.476	0.328	0.252
2.4	0.902	0.450	0.315	0.244

Soil conductivity = 1.4 W/(m·K); insulation is over entire depth. For other soil conductivities and partial insulation, use Equation (37).

- Assuming **concrete** walls

Below-grade example problem

Determine the heat flow, Q , through a basement enclosure

- Basement is 60 ft x 25 ft
- Walls are 5 ft below grade
 - Covered with R-4 (IP) insulation
 - Walls and floor are 6" concrete

Other information:

- $k_{\text{soil}} = 0.8 \text{ Btu}/(\text{h ft}^2 \text{ } ^\circ\text{F})$
- $R_{\text{wall}} = 4.0 \text{ (insul.)} + 0.68 \text{ (convection)} + 0.60 \text{ (6" concrete)} = 5.28 \text{ (IP)}$
- $R_{\text{floor}} = 0.61 \text{ (convection)} + 0.6 \text{ (6" concrete)} = 1.21 \text{ (IP)}$
- $z_1 = 0, z_2 = 5 \text{ ft}, z_f = 5 \text{ ft}, w_b = 25 \text{ ft}$
- $A_{\text{wall}} = 2*(5*25) + 2*(5*60) = 850 \text{ ft}^2$
- $A_{\text{floor}} = 25*60 = 1500 \text{ ft}^2$
- Assume $T_{\text{in}} = 65^\circ\text{F}$ and $T_{\text{gr}} = 40^\circ\text{F}$

Below-grade example problem

$$U_{avg,bw} = \frac{2(0.8)}{\pi(5-0)} \left[\ln \left(5 + \frac{2(0.8)5.28}{\pi} \right) - \ln \left(0 + \frac{2(0.8)5.28}{\pi} \right) \right] = 0.11$$

$$U_{avg,bf} = \frac{2(0.8)}{\pi 25} \left[\ln \left(\frac{25}{2} + \frac{5}{2} + \frac{(0.8)1.21}{\pi} \right) - \ln \left(\frac{5}{2} + \frac{(0.8)1.21}{\pi} \right) \right] = 0.034$$

$$Q_{bw} = 850(0.11)(65 - 40) = 2.27 \text{ kBtu/hr} = 665 \text{ W}$$

$$Q_{bf} = 1500(0.034)(65 - 40) = 1.30 \text{ kBtu/hr} = 380 \text{ W}$$

Notice that the heat transfer through the wall is almost twice that through the floor

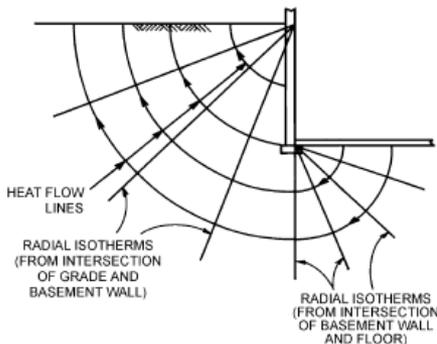


Fig. 4 Heat Flow from Basement

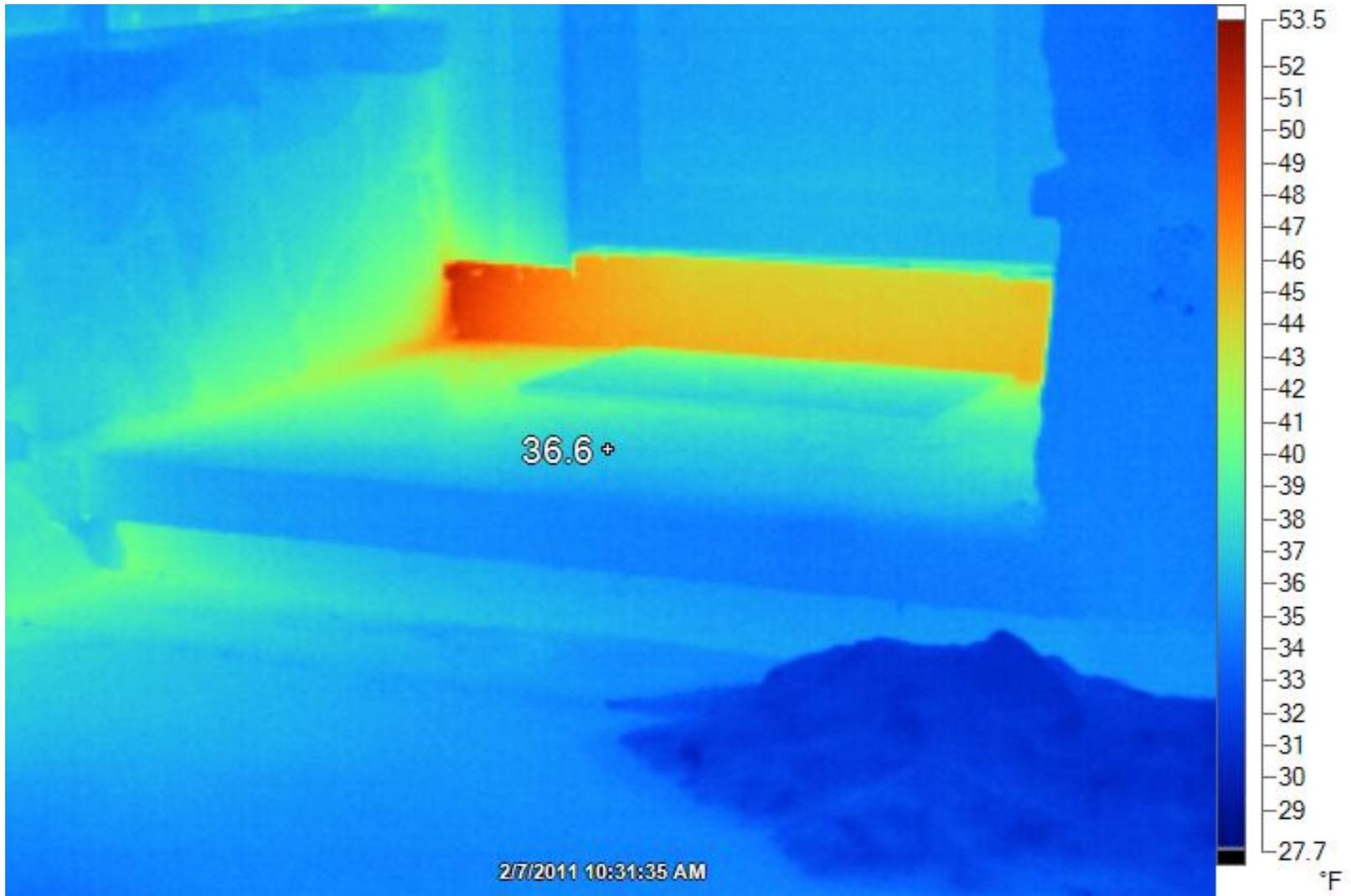
On-grade heat transfer



On-grade heat transfer

- Heat transfer for slab-on-grade floors
 - Concrete slabs can be heated or unheated
 - In either case:
 - Ground is often at a lower temperature than indoor air
 - Soil and concrete are fairly conductive
 - Perimeter can be exposed directly to outdoor air
 - It turns out that the **perimeter** is important for both energy and comfort
 - Need to insulate the perimeter in either case

Slab-on-grade floors



Slab-on-grade floors

- Heat transfer through slab-on-grade floors
 - Function of perimeter of slab (not area)

$$Q = pF_p (T_i - T_o)$$



where T_i and T_o are the inside and outside temps [K]

p is the perimeter of the exposed floor surface [m]

F_p is the heat loss coefficient per unit length of perimeter [W/mK]

Design considerations

- To reduce heat transfer through slab on grade floors, we obviously need to:
 - Reduce the perimeter length, and/or
 - Decrease the heat loss coefficient, F_p
- Decreasing F_p is as simple as adding insulation to the foundation exterior
 - No need to exceed $R = 8$ (IP)

Figure 3. Insulated Form Board Field Installation



Heat loss coefficient: F_p

Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

Construction	Insulation	F_p , W/(m·K)
200 mm block wall, brick facing	Uninsulated	1.17
	R-0.95 (m ² ·K)/W from edge to footer	0.86
4 in. block wall, brick facing	Uninsulated	1.45
	R-0.95 (m ² ·K)/W from edge to footer	0.85
Metal stud wall, stucco	Uninsulated	2.07
	R-0.95 (m ² ·K)/W from edge to footer	0.92
Poured concrete wall with duct near perimeter*	Uninsulated	3.67
	R-0.95 (m ² ·K)/W from edge to footer	1.24

*Weighted average temperature of the heating duct was assumed at 43°C during heating season (outdoor air temperature less than 18°C).

Heat loss coefficient: F_p

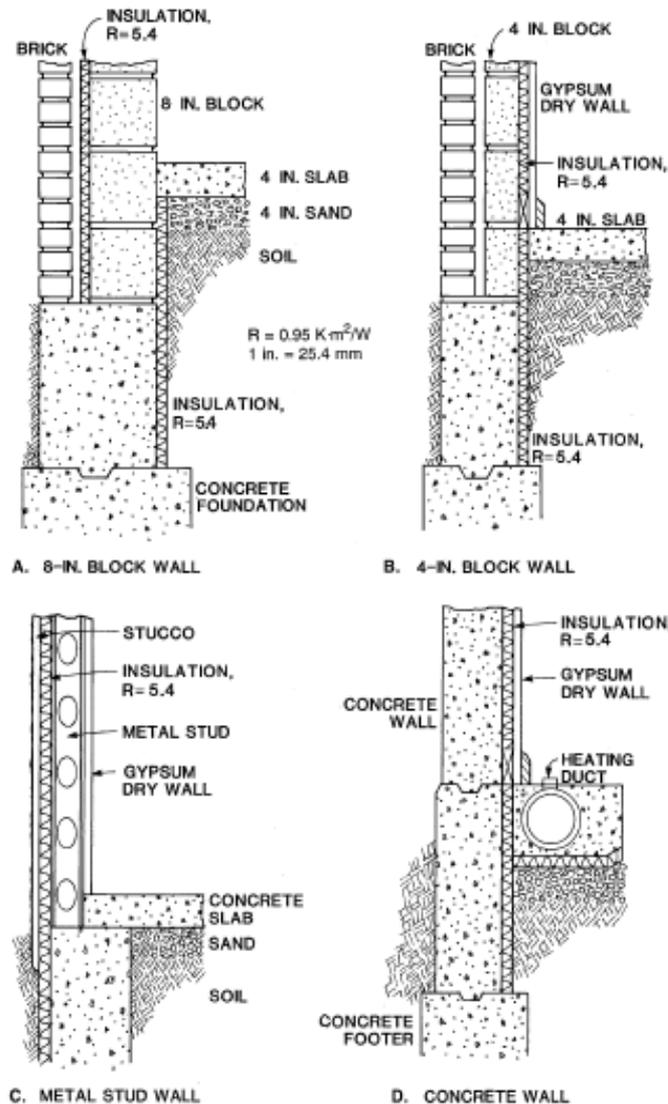


Table 18 Heat Loss Coefficient F_p of Slab Floor Construction

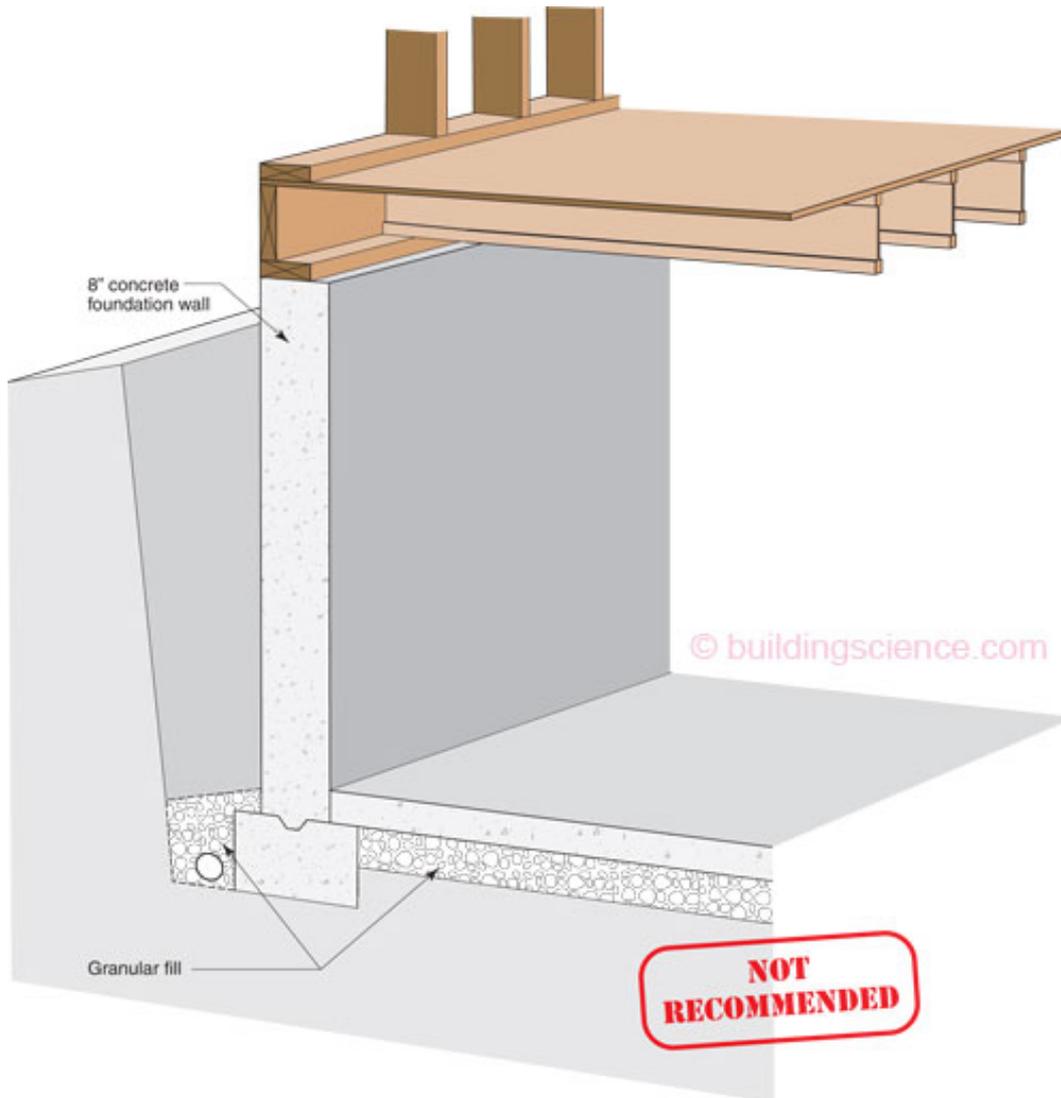
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Fig. 8 Slab-on-Grade Foundation Insulation

Slab-on-grade and below-grade enclosures

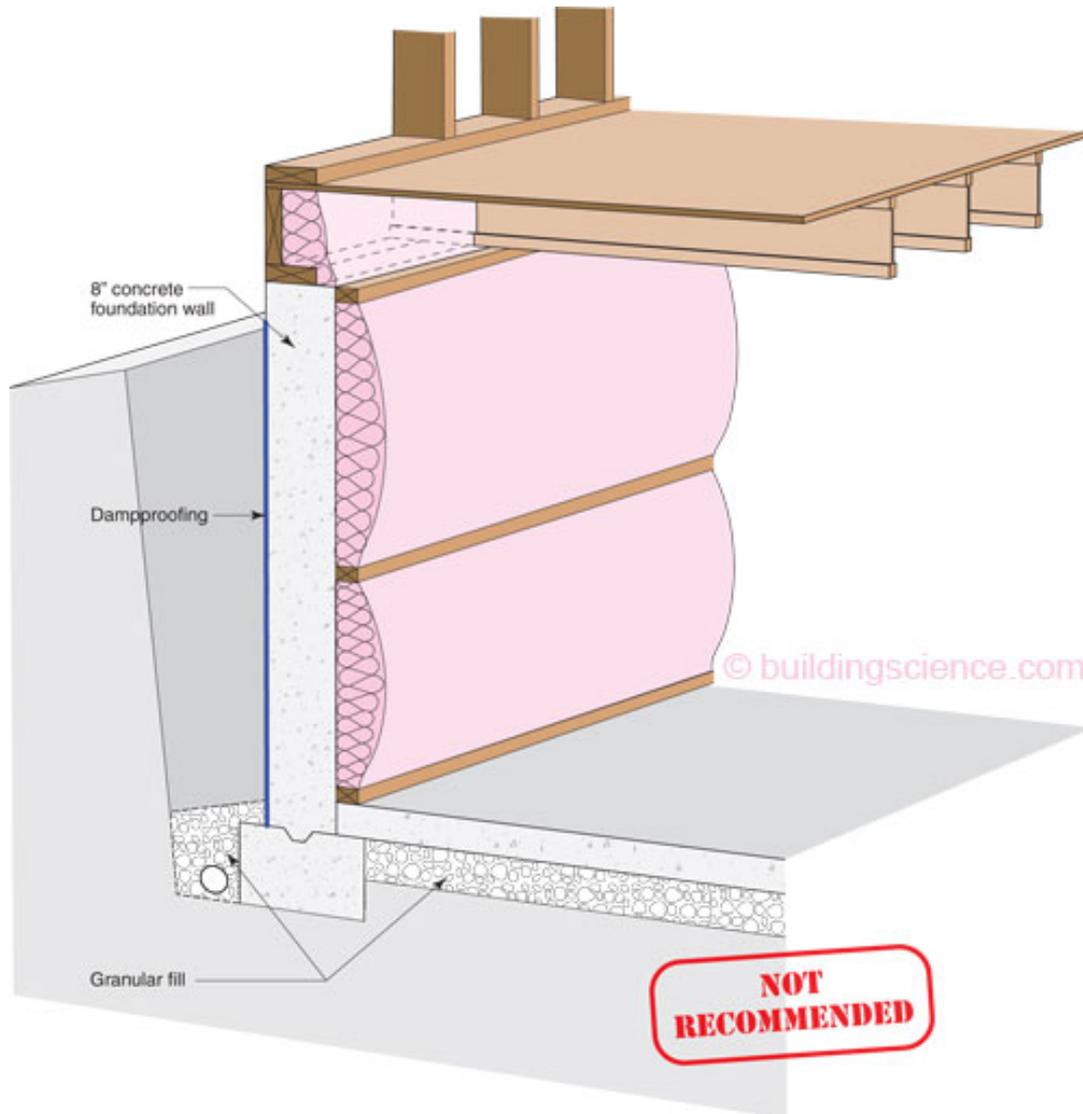
Un-insulated concrete foundation wall and slab



- No thermal control
- Not even allowed by code if basement is conditioned
- No moisture control
- Water vapor diffusion and capillary action are near-constant moisture sources

Slab-on-grade and below-grade enclosures

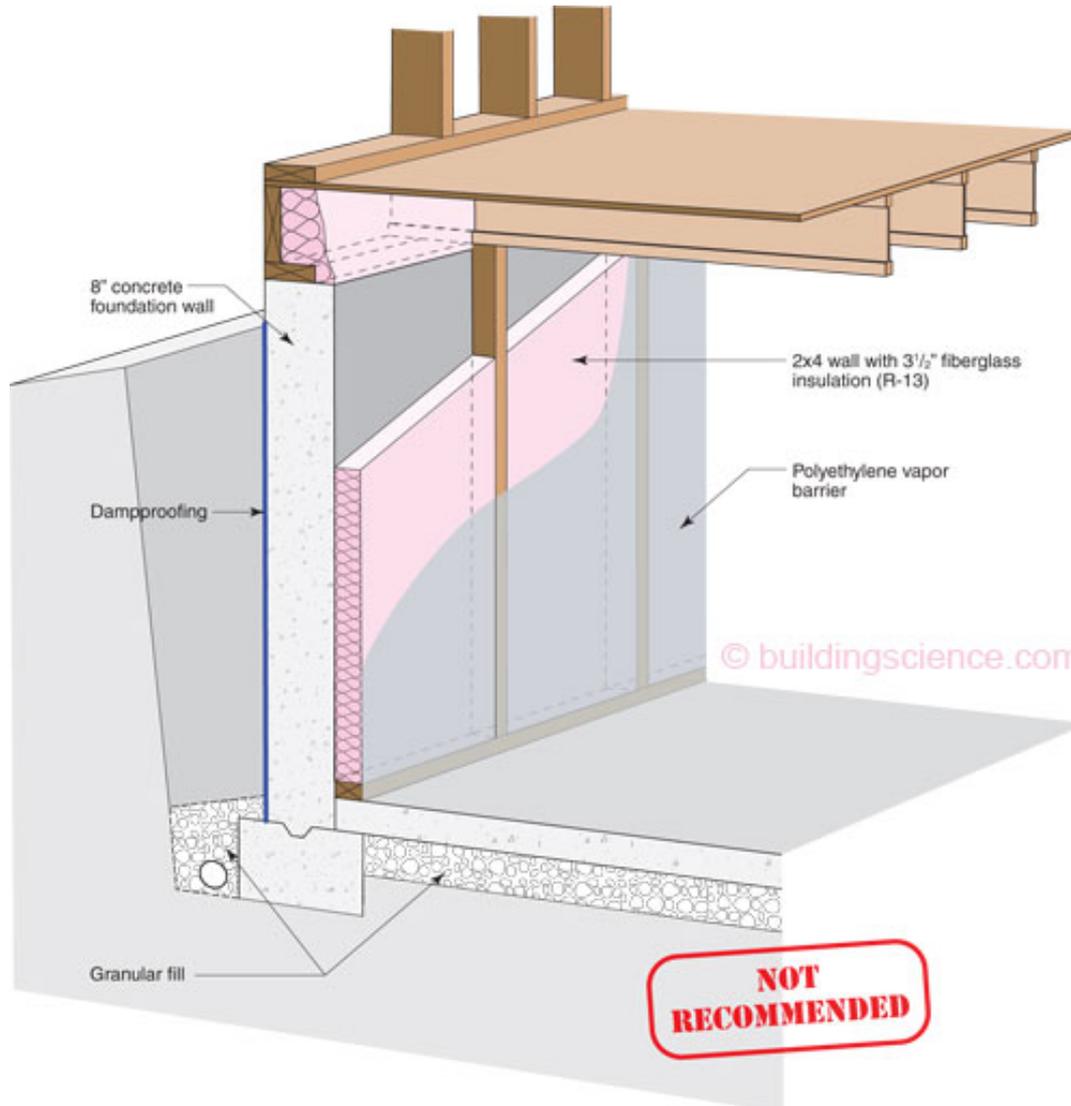
Code minimum R-10 (IP) continuous insulation in a framed wall



- Slab not insulated
- Sometimes wall insulation batt is covered with vapor barrier
- Better thermal control
- Inexpensive
- Moisture issues (batt is air and vapor permeable)
- High RH at concrete wall most of the year

Slab-on-grade and below-grade enclosures

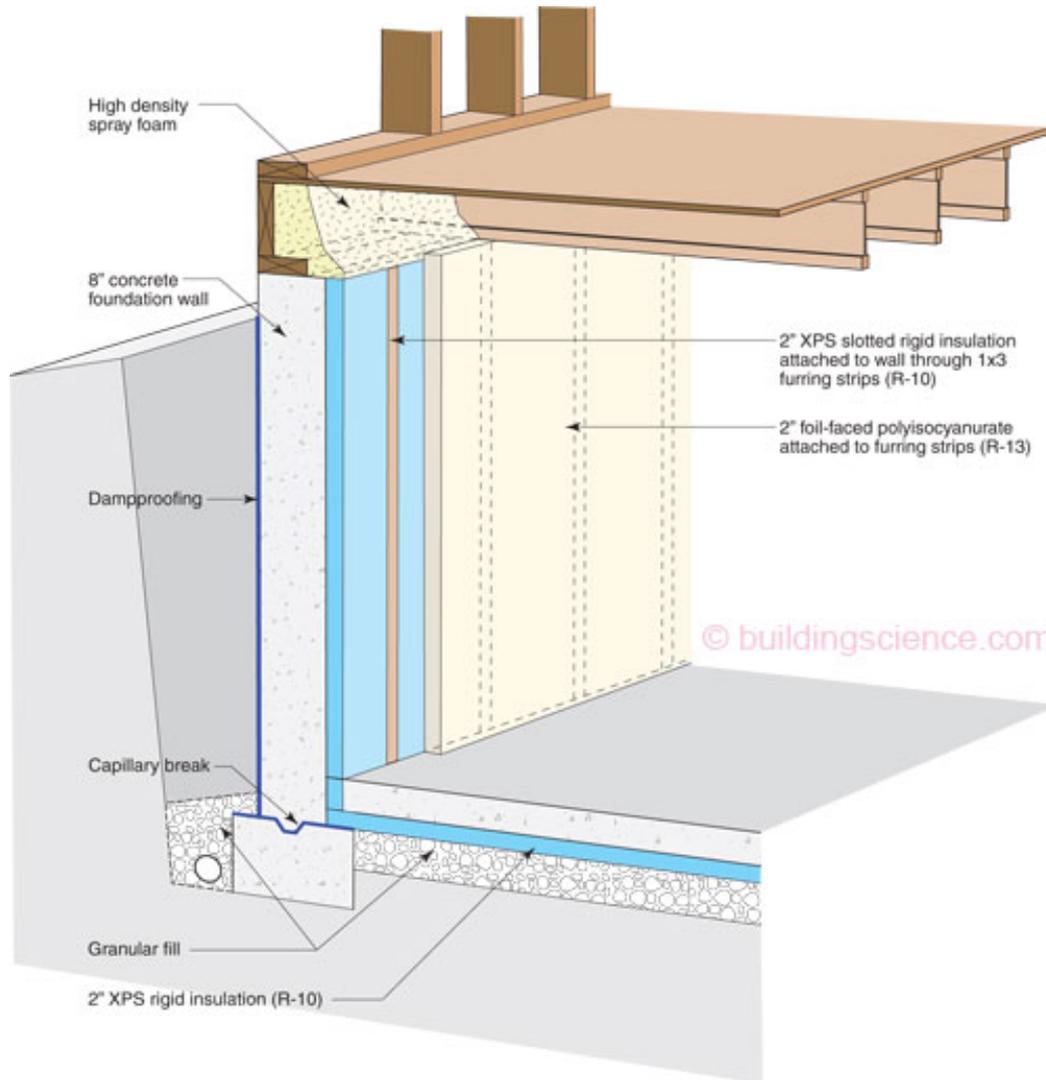
R-13 (IP) insulation in a 2x4 framed wall



- Similar to last construction
- Moisture issues
- High RH at concrete wall most of the year
- Particularly a problem if there is any air leakage

Slab-on-grade and below-grade enclosures

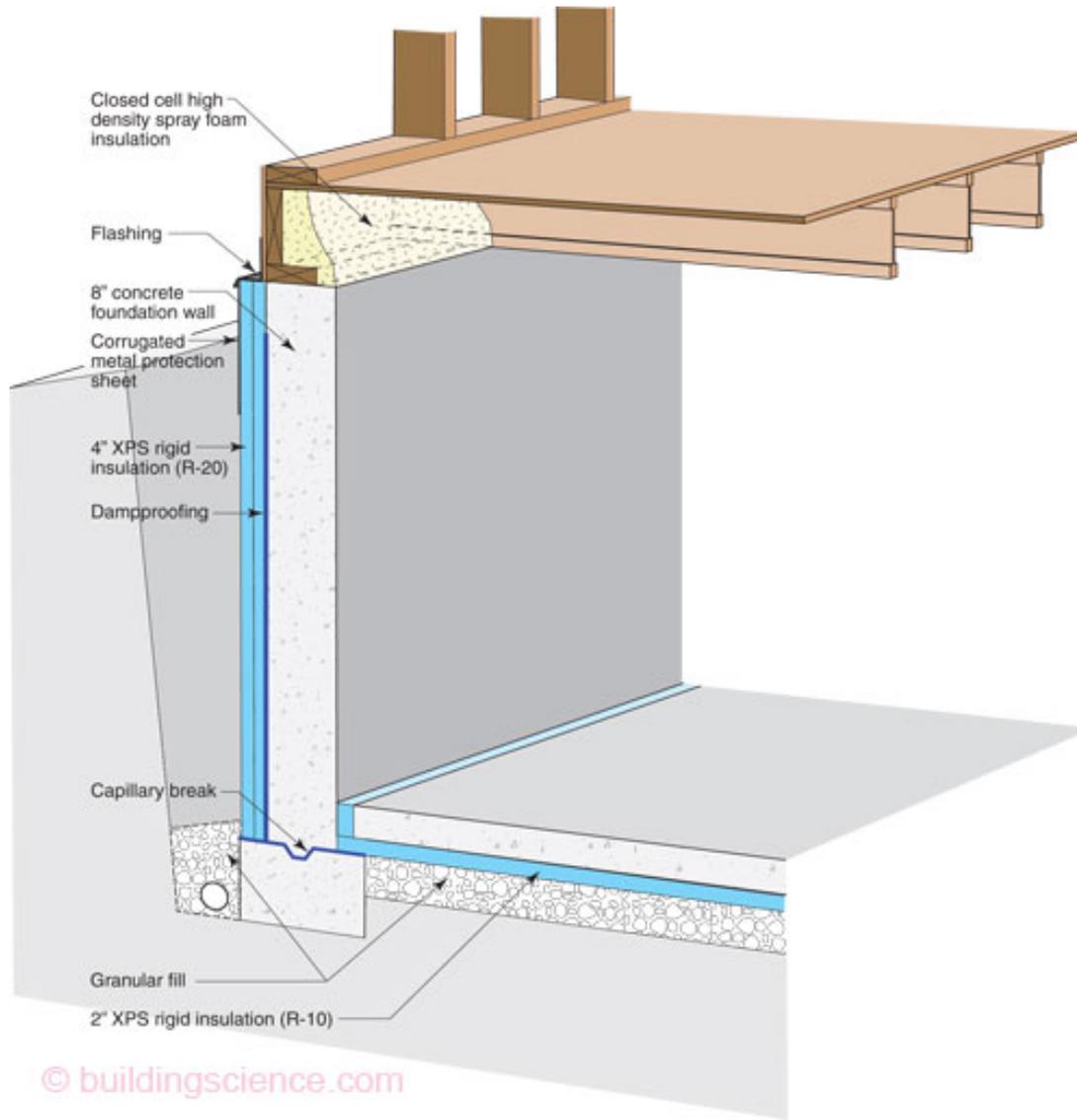
2" XPS rigid insulation + 2" foil-face polyisocyanurate foam board



- Very good thermal control (R-23 walls)
- Water vapor diffusion is prevented
- Capillary action is prevented by the thermal/capillary break at the edge of the slab and top of footing

Slab-on-grade and below-grade enclosures

Rigid XPS exterior insulation



- Very good thermal control (R-20 walls)
- Exterior insulation can be joined with first floor insulation
- Excellent resistance to vapor diffusion
- Capillary action is a potential problem (through the footing)
 - Need a break
- Exposed concrete provides moisture buffer after it dries
- May be hard to construct

Slab-on-grade and below-grade enclosures

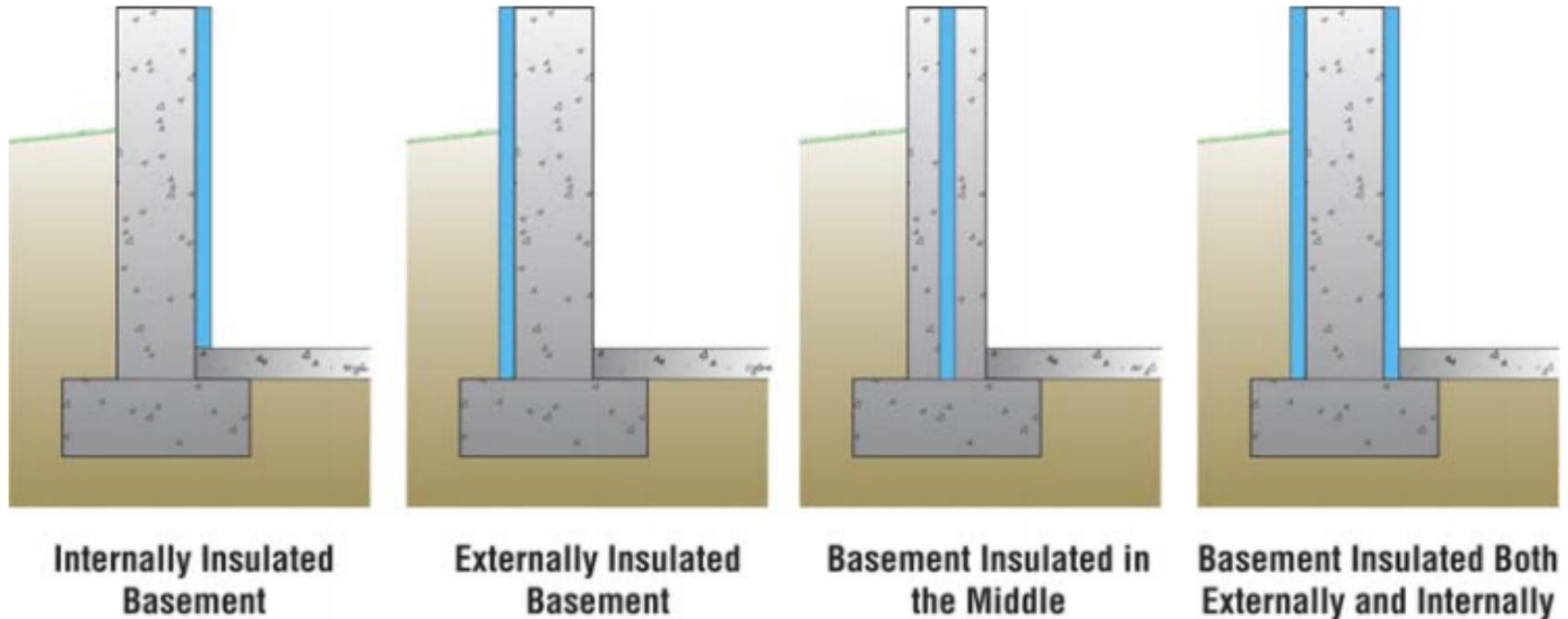


Figure 4: Generic Insulation Approaches

- Interior insulation most common, least expensive, has most moisture problems
- Exterior insulation best location from physics perspective, has practical problems with protection, thermal bridging and insects
- Insulation in middle is most expensive approach, has fewest moisture and insect problems, but is the most difficult to construct
- Insulation on both sides has similar problems to the exterior insulation approach with the additional cost of the interior layer

Slab-on-grade and below-grade enclosures



Photograph 1: Exterior Basement Insulation

- Ideal location from the physics perspective
- Practical problems with protection, insect control and thermal bridging of brick veneers

Slab-on-grade and below-grade enclosures



Photograph 2: Interior Frame Wall With Plastic Vapor Barrier

- Plastic vapor barrier prevents inward drying
- Common outcome are odor, mold, decay and corrosion problems

Slab-on-grade and below-grade enclosures



Photograph 3: "Blanket Insulation"

- aka "the diaper"
- Plastic film on interior of blanket insulation prevents inward drying

Slab-on-grade and below-grade enclosures

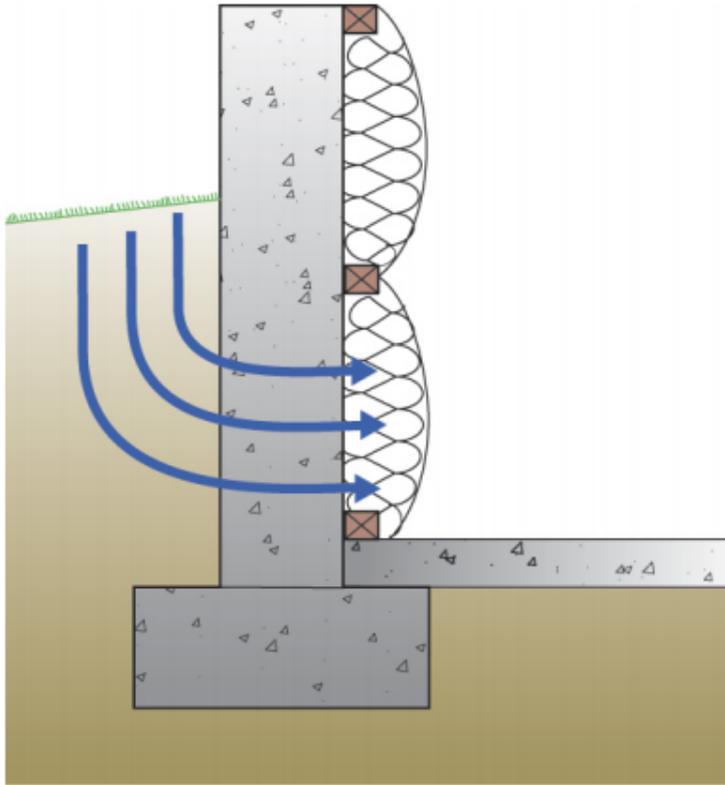


Figure 6: Groundwater Entry

- Interior insulation layer is typically water sensitive and prevents inward drying

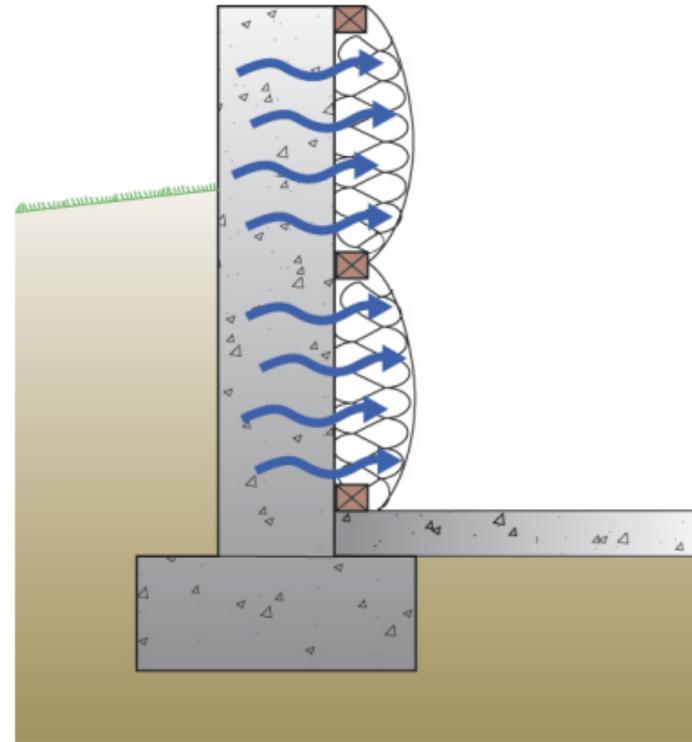


Figure 7: Moisture of Construction

- Interior insulation layer is typically water sensitive and prevents inward drying
- Several thousand pounds of water in freshly placed concrete attempts to dry inward

Slab-on-grade and below-grade enclosures

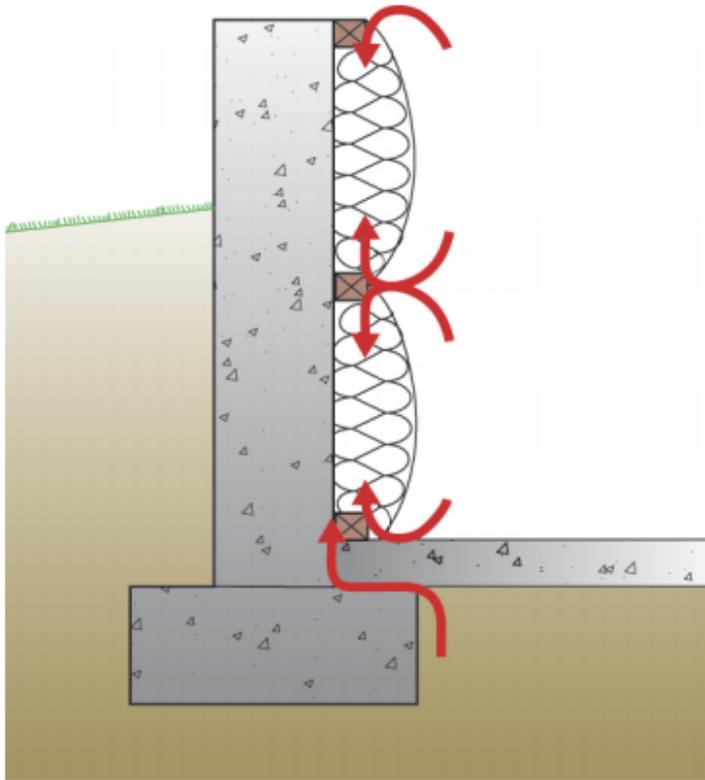


Figure 9: Condensation From Interior Air Leakage

- Interior insulation layer is typically not airtight and does not prevent interior air from condensing on concrete foundation wall

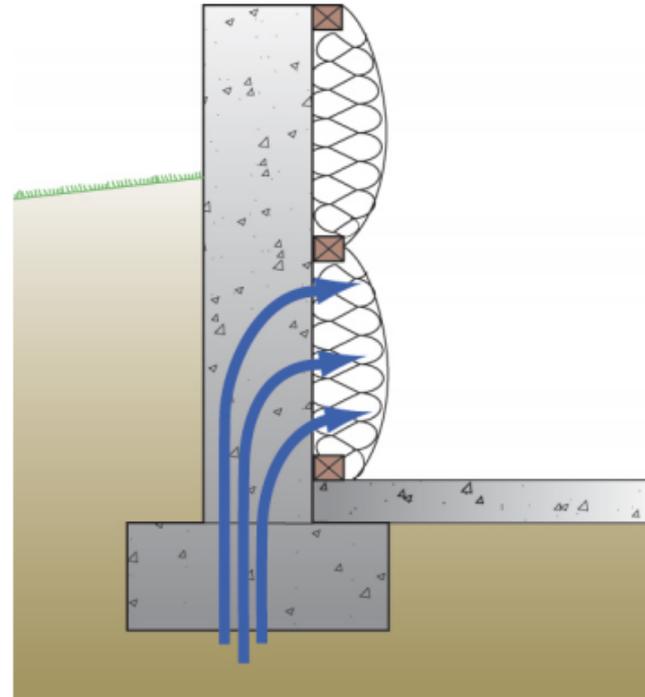


Figure 8: Capillary Rise Through Footing

- Interior insulation layer is typically water sensitive and prevents inward drying

Using THERM for finding U_{avg}

- You can use THERM to model 2-D heat transfer but be careful ...
 - Must create new convection boundary conditions for interior convection to/from floor
 - Must model a large area of soil around the foundation as a solid with adiabatic boundary conditions
 - Must model outside soil/air interface with new exterior convection
- Basement floor is a 3-D problem and so it cannot be easily modeled

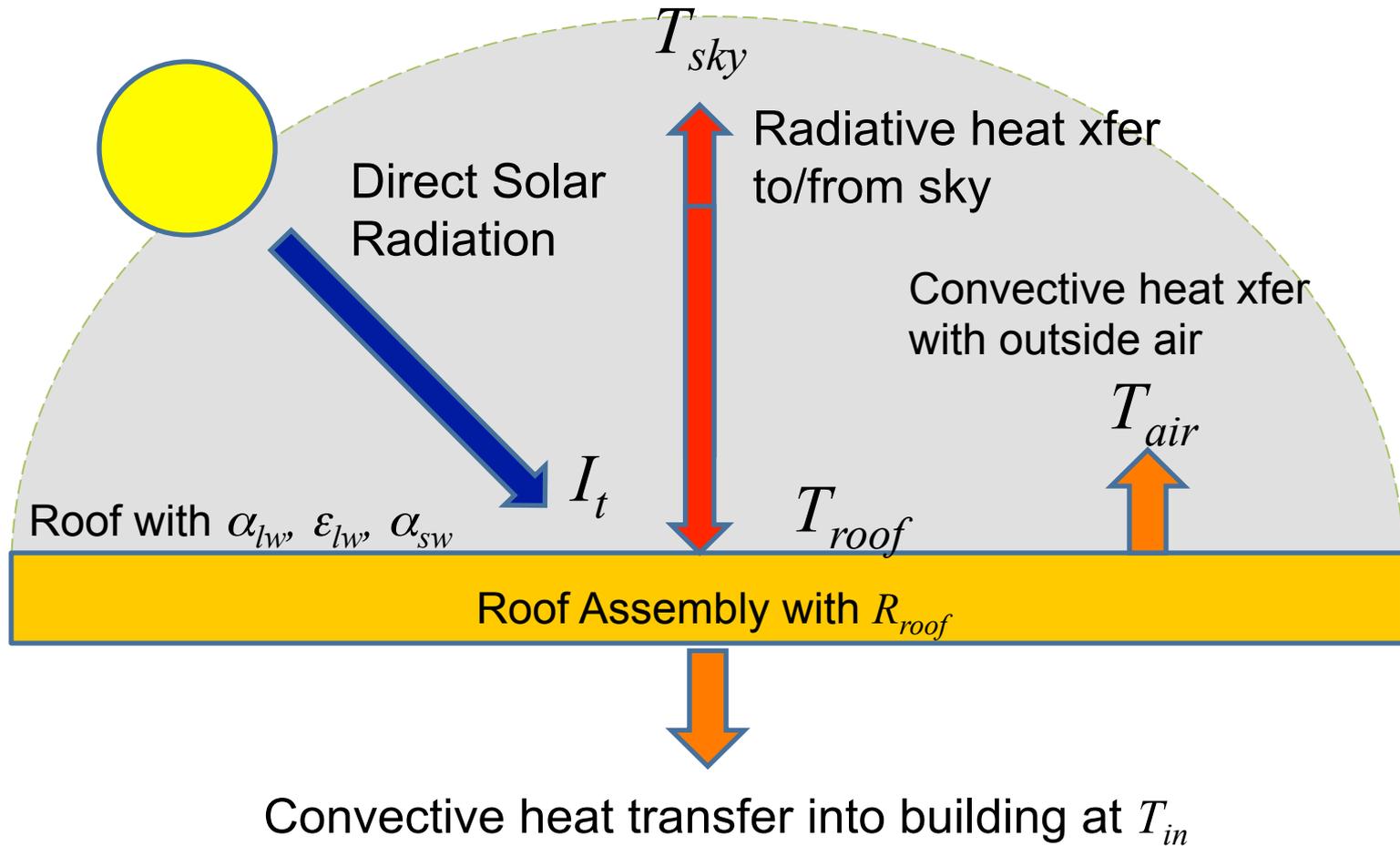
ROOFS

The roof is on fire!?

Radiative heating of roofs

- For some building elements, especially roofs, radiative transfer can dominate
 - You've seen example problems like this in your HW and your exam problem on roof surface temperatures
- Roofs can undergo huge temperature changes throughout the day
 - Can create huge thermal stresses
- We need to know how to estimate surface temperatures on roofs and their impacts on indoor loads
 - Includes radiation, convection, and conduction

Heat Balance on Roof with $T=T_{roof}$



Important terms

- I_t = total wavelength solar irradiation [W/m²]
 - We ignore ground reflected solar radiation for roofs
- α_{sw} = short wavelength absorptivity
- α_{lw} = long wavelength absorptivity
- ε_{lw} = long wavelength emissivity
 - Note: for most materials $\alpha_{lw} = \varepsilon_{lw}$
- R_{roof} = R value of the roof assembly [m²K/W]

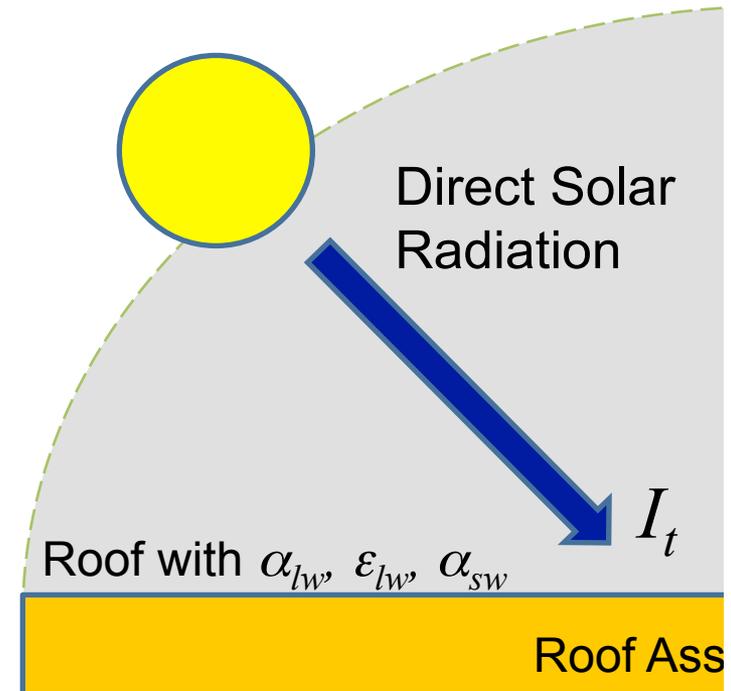
Basic premise

- As a starting point, assume the roof temperature has reached steady state
- If we have reached steady state, the amount of heat hitting the roof must equal the heat leaving the roof
 - Heat from sun is short wavelength
 - Heat exchange with sky is long wavelength
 - Heat exchange with air and room are convective
 - Also have conduction through roofing material itself

Solar irradiation

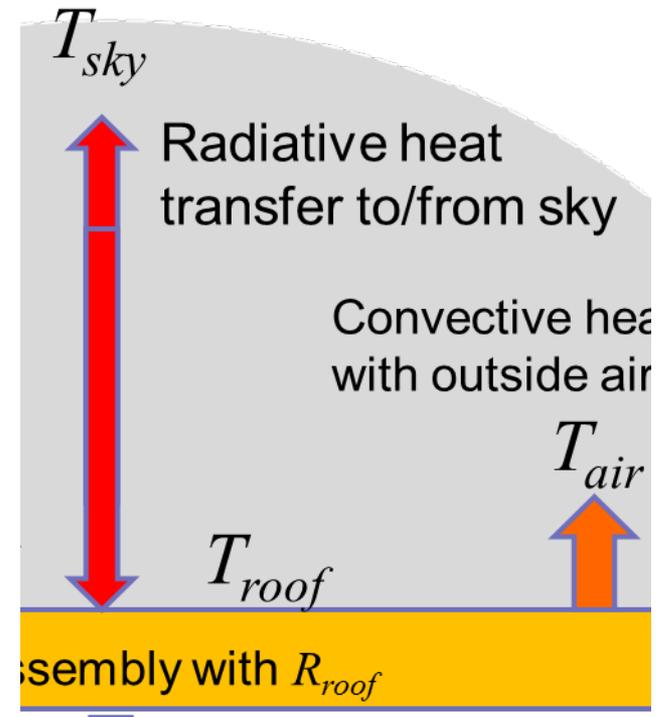
- Total irradiance is I_t
$$I_t = I_{dir} \cos \theta + I_{diffuse}$$
 - We ignore ground reflected solar radiation for roofs
 - We know how to get these values from Lecture 2
- Radiation is short wavelength
 - Fraction absorbed is α_{sw}
- Heat transfer to the roof from direct solar irradiance is:

$$q_{solar} = \alpha_{sw} I_t$$



Net radiation to sky

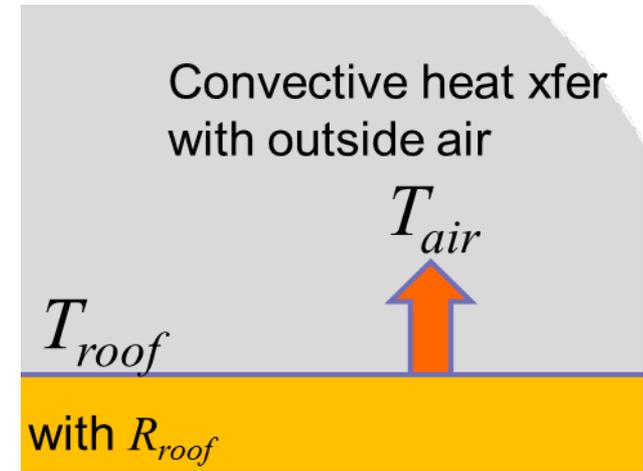
- There is a radiant heat exchange between the roof at T_{roof} and the sky at T_{sky}
- This radiation is long wavelength and if we assume that the view factor from roof-to-sky $F_{12} \approx 1$ and $\alpha_{lw} = \epsilon_{lw}$



$$q_{\text{net to sky}} = \epsilon_{lw} \sigma \left(T_{roof}^4 - T_{sky}^4 \right)$$

Convective transfer to outside air

- The roof at T_{roof} will have convective heat transfer with the surrounding air at T_{air}

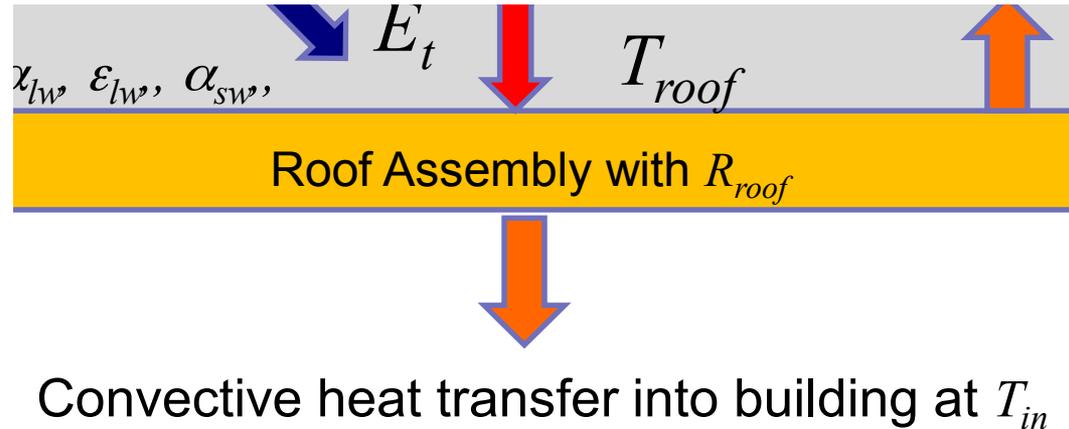


$$q_{to\ air} = h_c (T_{roof} - T_{air})$$

- Note: h_c will depend upon the wind speed, temperature difference, and/or orientation of transfer

Conductive transfer to inside

- The roof at T_{roof} also conducts through the assembly with R_{roof} and convectively into the building at T_{in}



$$q_{\text{to inside}} = \frac{(T_{roof} - T_{in})}{R_{roof}}$$

Thermal equilibrium

- If the roof is hot:

$$T_{roof} > T_{sky}, T_{air}, T_{in}$$

- If the roof temperature is steady, the heat hitting the roof from the sun **MUST** be equal to the heat leaving the roof through radiation to sky, outside air and inside to the building
- With this info we can write a simple heat balance equation...

Complete heat balance equation

$$q_{\text{solar}} = q_{\text{net to sky}} + q_{\text{to air}} + q_{\text{to inside}}$$

$$\alpha_{sw} I_t = \sigma \epsilon_{lw} \left(T_{\text{roof}}^4 - T_{\text{sky}}^4 \right) + h_c \left(T_{\text{roof}} - T_{\text{air}} \right) + \frac{\left(T_{\text{roof}} - T_{\text{in}} \right)}{R_{\text{roof}}}$$

- In most cases, the only unknown in the above is T_{roof}
- Solving for T_{roof} is difficult because this is a 4th order equation
- Additionally, h_c depends upon T_{roof} which makes the equation transcendental
- So, we must solve for T_{roof} numerically
- **We have had a HW problem like this before**

Solving for T_{roof}

$$q_{net} = \alpha_{sw} I_t - \sigma \epsilon_{lw} (T_{roof}^4 - T_{sky}^4) - h_c (T_{roof} - T_{out}) - \frac{(T_{roof} - T_{in})}{R_{roof}}$$

Rewrite the previous equation to find the difference between incident solar energy and the radiated, convected, and transmitted to interior

At steady state $q_{net}=0$, T_{roof} is the temp that sets $q_{net}=0$

So, in a spreadsheet you would set it up to calculate q_{net} and adjust T_{roof} until $q_{net}=0$

- Do this either by hand or use “goal seek” to adjust it automatically for you

How can we use this for design?

- We can use this equation (or a spreadsheet that implements it) to find out how adjusting material parameters such as α_{sw} , ε_{lw} , and R_{roof} change T_{roof} for a given I_t , T_{sky} , and T_{air}
 - What can you accomplish by changing T_{roof} ?

Lower roof temperatures means:

- Less stress on roofing materials
- Lower near-roof air temperatures for intakes for HVAC equipment
- Less conduction to the inside (reduced loads)

Understanding the terms

- We usually want to reduce roof temps in the hot summer when the heat gain is all solar
 - We need to absorb less solar energy and emit more thermal energy

$\alpha_{sw} I_t$ = short wavelength (solar) heat gain

- Reduce this by reducing α_{sw}

$\sigma \varepsilon_{lw} (T_{roof}^4 - T_{sky}^4)$ = long wavelength (IR) heat loss

- Increase this by increasing ε_{lw}

Understanding the terms

$h_c (T_{roof} - T_{out})$ = convective heat transfer from roof.

We cannot really change this as it depends on wind speed and the outside air temperature.

$\frac{(T_{roof} - T_{in})}{R_{roof}}$ = conductive heat transfer through roof.

Because modern designs often have very high R values for roofs, this term is often quite small. But we still do not want to reduce R to increase this term as a way of reducing reduce the roof temp.

Computing T_{sky}

- The effective sky temperature, T_{sky} , is related to the outside air temp, T_{air} , the humidity ratio, and cloud cover
 - One common equation developed by Walton^a is:

$$T_{sky} = \left(0.787 + 0.764F_{cloud} \ln \left(\frac{T_{dew}}{T_{ice}} \right) \right)^{1/4} T_{air} \quad \text{all } T \text{ in K or R}$$

T_{dew} = Dew Point Temp of Air, T_{ice} = Freezing temp of H₂O

$F_{cloud} = 1 + 0.024N - 0.035N^2 + 0.00028N^3$ where

N = tenths cloud cover $0 < N < 1$

^{a)} Walton, G.N., *Thermal Analysis Research Program- Reference Manual*, NBSIR 83-2655, U.S. Department of Commerce, March 1983, Update 1985.

Computing T_{roof}

- Now that we have I_t , and T_{sky} we can compute T_{roof} if we have the roofing material properties and air temperature
- This is easily solved using goal seek or solver in excel, with EES, with Mathcad, or other similar software
 - Or write your own program

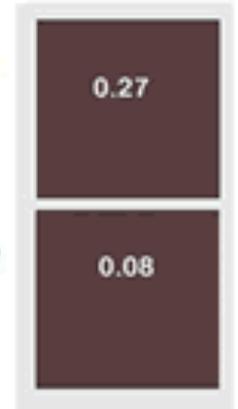
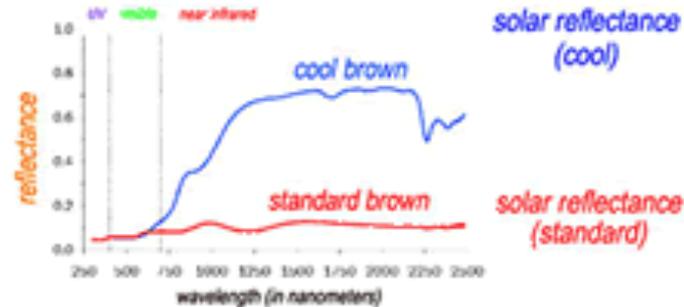
Design considerations

- Having a high R value for the roof construction will limit heat transmission to the inside of the building
 - This is good for reducing the building cooling load
 - This is actually somewhat bad for the surface temperature as heat transfer will mostly be convection and radiation to sky

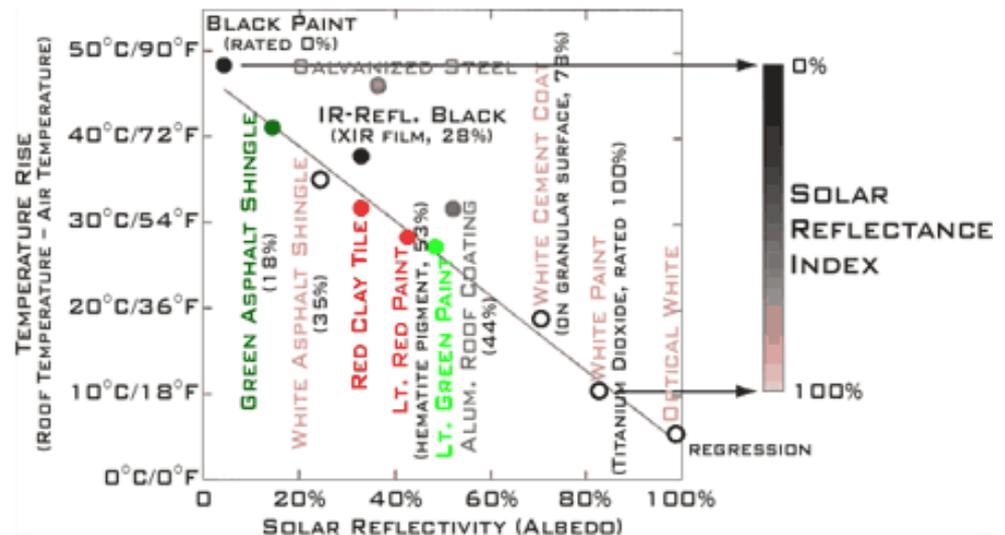
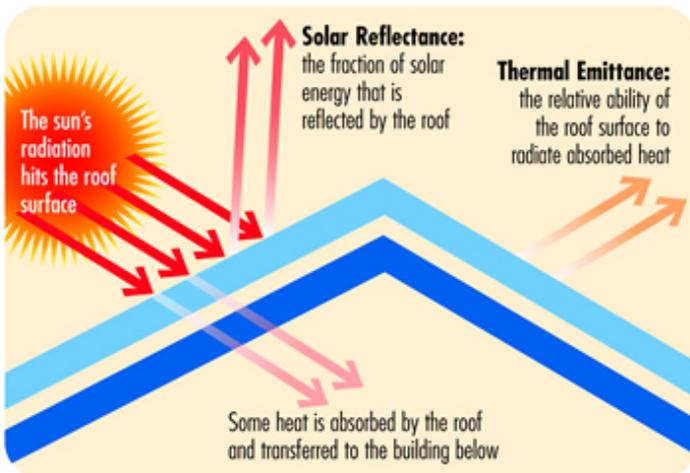
Cool roofs

Special roofing materials allow for “cool roofs”

- Low α_{sw}
- High ε_{lw}
- Designed to reduce T_{roof}



courtesy BASF Coatings



Roof examples

- Let $I_t = 380 \text{ Btu/hr}\cdot\text{ft}^2 = 1200 \text{ W/m}^2$
- $T_{\text{air}} = 90^\circ\text{F}$, RH=70%
- $T_{\text{in}} = 70^\circ\text{F}$, $R_{\text{roof}} = 20 \text{ hr}\cdot\text{ft}^2\cdot\text{F/Btu}$
- $V_{\text{air}} = 0 \text{ mph}$ (no wind)
- $\alpha_{\text{sw}} = 0.9$
- $\alpha_{\text{lw}} = 0.9$

- Any guesses as to T_{roof} ?

Roof examples: spreadsheet program

Environmental Information

E_t	380	[Btu/(hr ft ² °F)]	=	1198.87	[W/m ²]
V_{roof}	0	[mph]	=	0	[m/s]
Outdoor RH	70	90	=	70	[%]
T_{air}	90	[°F]	=	32.2222	[°C]
T_{in}	70	[°F]	=	21.1111	[°C]

Roof Information

α_{sw}	0.9				
α_{lw}	0.9				
R_{roof}	20	[hr ft ² °F/Btu]	=	3.5222	[m ² K/W]
F_{sky}	1.0	View factor to sky from roof. This is usually close to or equal to 1.0			

Click Here to Find

The click here runs a macro adjusting Troof until qnet=0

T_{roof}	210.6	[°F]	=	99.2	[°C]
q_{net}	0.00	[Btu/(hr ft ²)]	=	0.0	[W/m ²] = net gain into membrane

Roof examples: spreadsheet program

- This spreadsheet estimates a peak roof temp of:

$$T_{roof} = 210.6^{\circ}\text{F} = 99^{\circ}\text{C}$$

- You can almost boil water, fry an egg, cook a pot roast
- Imagine the thermal expansion of the roof material at that temperature!
- Imagine how hot the air right next to the roof is
 - Note that this is where the HVAC OA inlets are often located
 - Often evaporator coils and/or condenser coils as well

Roof examples: spreadsheet program

- What is the temp at night when $T_{air} = 70^\circ\text{F}$ and $I_t = 0$?

Environmental Information

E_t	0	[Btu/(hr ft ² °F)]	=	0	[W/m ²]
V_{roof}	0	[mph]	=	0	[m/s]
Outdoor RH	70	90	=	70	[%]
T_{air}	70	[°F]	=	294.261	[K] = 21.1 [°C]
T_{in}	70	[°F]	=	294.261	[K] = 21.1 [°C]

Roof Information

α_{sw}	0.9	=	0.9		
α_{lw}	0.9	=	0.9		
R_{roof}	20	[hr ft ² °F/Btu]	=	3.52237	[m ² K/W]
F_{sky}	1.0	View factor to sky from roof. This is usually close to or equal to 1.0			

Click Here to Find T_{roof}

The click here runs a macro adjusting T_{roof} until $q_{net}=0$

T_{roof} 50.0 [°F] = 283.13 [K]

Only 50°F = 10°C → Huge swings from day to night

Roof examples: spreadsheet program

- What if we drop solar absorption by installing a cool roof with $\alpha_{sw}=0.2$?

Roof Information

α_{sw}	0.2	=	0.2
α_{lw}	0.9	=	0.9
R_{roof}	20 [hr ft ² °F/Btu]	=	3.52237 [m ² K/W]
F_{sky}	1.0	View factor to sky from roof. This is usually close to or equal to 1.0	

Click Here to Find T_{roof}

The click here runs a macro adjusting Troof until qnet=0

T_{roof}	117.3 [°F]	=	320.518 [K]
$q_{net} =$	0.0 [W/m ²]	=	0.00 [Btu/(hr ft ² °F)] net gain into membrane

Daytime peak is reduced to **117°F (47°C)**

Roof examples: spreadsheet program

- What if we had a 10 mph wind and normal roof?

V_{roof}	10	[mph]	=	4.4	[m/s]	
Outdoor RH	70		90	=	70	[%]
T_{air}	90	[°F]	=	305.372	[K]	= 32.2 [°C]
T_{in}	70	[°F]	=	294.261	[K]	= 21.1 [°C]
Roof Information						
α_{sw}	0.9		=	0.9		
α_{lw}	0.9		=	0.9		
R_{roof}	20	[hr ft ² °F/Btu]	=	3.52237	[m ² K/W]	
F_{sky}	1.0	View factor to sky from roof. This is usually close to or equal to 1.0				
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; background-color: #e0e0e0;"> <p>Click Here to Find T_{roof}</p> </div> <div style="border: 1px solid black; padding: 5px; background-color: #e0e0e0;"> <p>The click here runs a macro adjusting Troof until qnet=0</p> </div> </div>						
T_{roof}	149.9	[°F]	=	338.67	[K]	

Daytime peak is **150°F (66°C)**

→ Lower than zero wind but still very high

Roof examples: spreadsheet program

- What if we had a 10 mph wind and a cool roof?

Environmental Information

E_t	380	[Btu/(hr ft ² °F)]	=	1199.06	[W/m ²]
V_{roof}	10	[mph]	=	4.4	[m/s]
Outdoor RH	70	90	=	70	[%]
T_{air}	90	[°F]	=	305.372	[K] = 32.2 [°C]
T_{in}	70	[°F]	=	294.261	[K] = 21.1 [°C]

Roof Information

α_{sw}	0.2	=	0.2		
α_{lw}	0.9	=	0.9		
R_{roof}	20	[hr ft ² °F/Btu]	=	3.52237	[m ² K/W]
F_{sky}	1.0	View factor to sky from roof. This is usually close to or equal to 1.0			

[Click Here to Find \$T_{\text{roof}}\$](#)

The click here runs a macro adjusting T_{roof} until $q_{\text{net}}=0$

T_{roof}	99.7	[°F]	=	310.734	[K]
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Daytime peak is reduced to **100°F (38°C)**

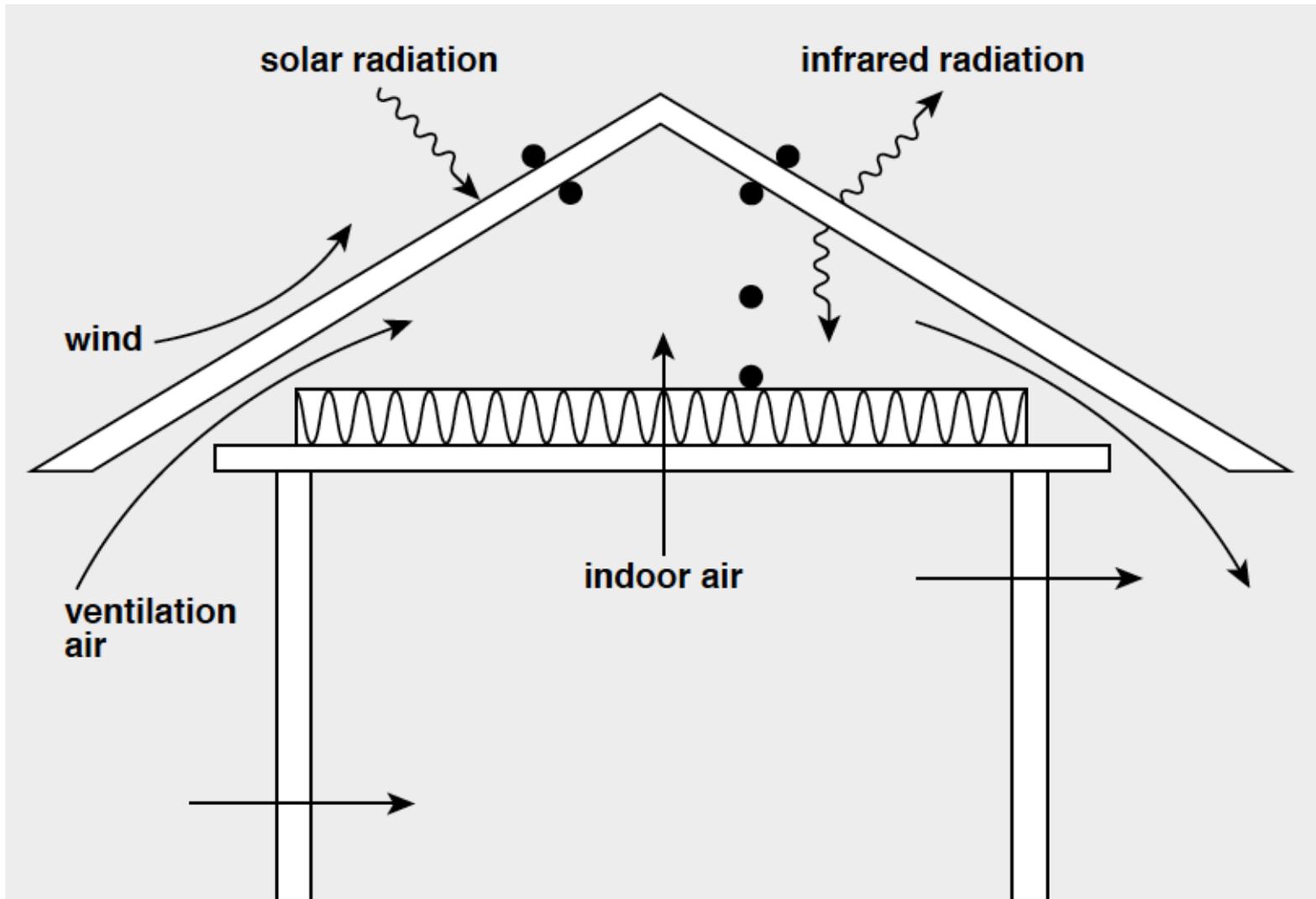
Roof example summary

- Stale hot day with high solar radiation

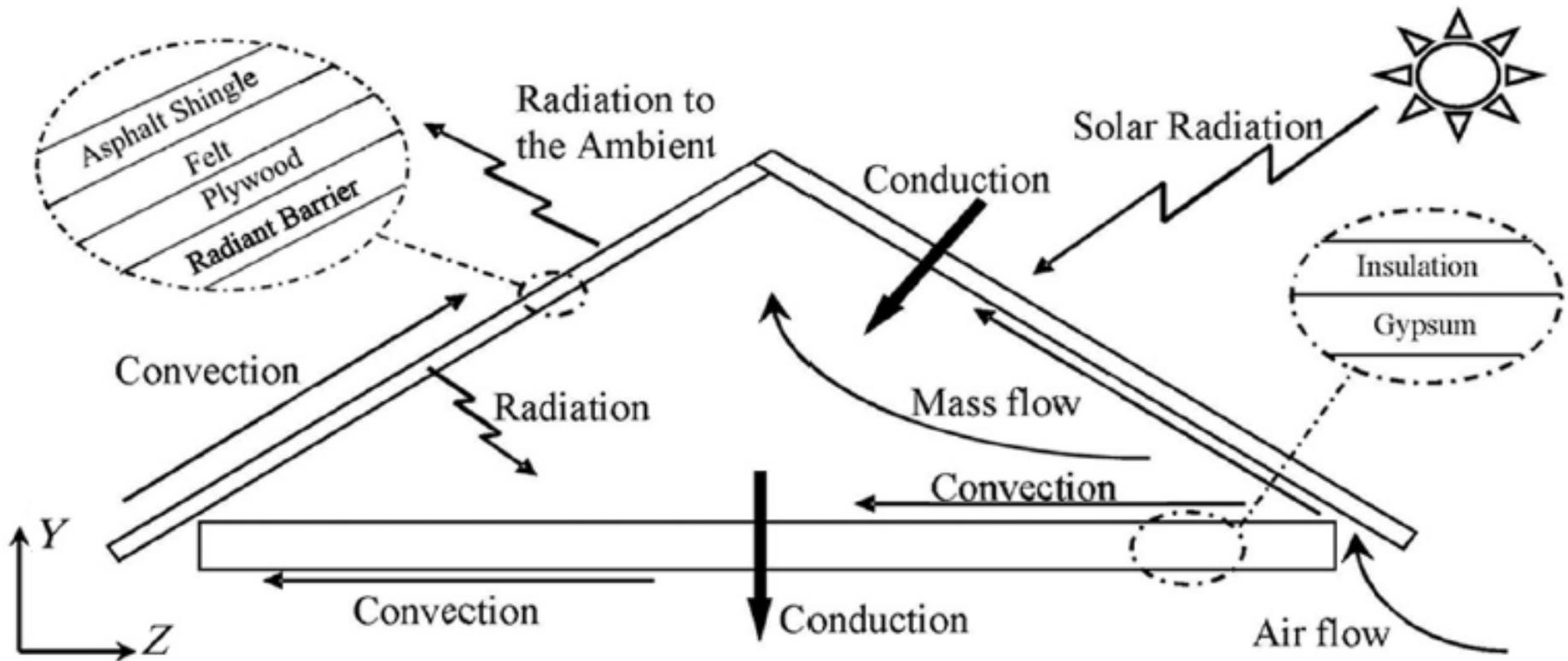
Roof type/condition	T_{roof} , °F (°C)
Normal black roof, no wind	210°F (99°C)
Normal black roof, 10 mph wind	150°F (66°C)
Cool roof, no wind	117°F (47°C)
Cool roof, 10 mph wind	100°F (38°C)

- Roofing material can reduce peak roof temperature by ~100°F (50°C)
 - From boiling to not-so-hot

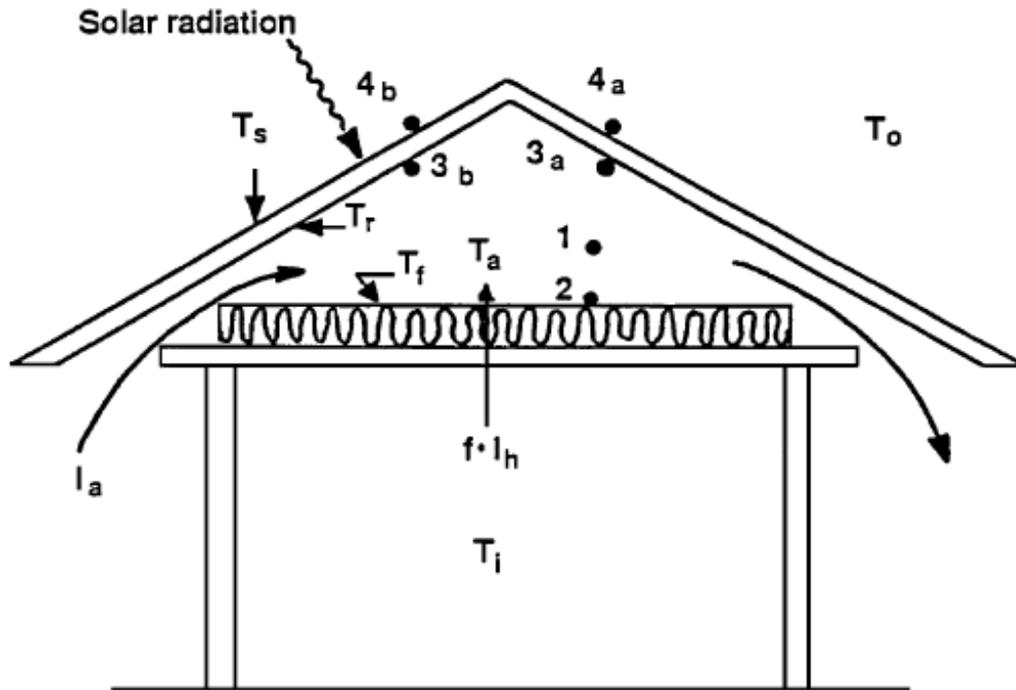
What about attics?



Attic heat transfer



Attic heat transfer



● **Nodes for heat balance**

- $T_a, T_i, T_f,$ **Temperatures of attic, indoor air, attic floor**
- T_o, T_r, T_s **outside air, underside of roof, surface of roof**
- f **Fraction of air entering attic**
- I_a, I_h **Ventilation rates for attic and house, vol. changes/hour**

Attic Air—The heat balance for the attic air (node 1) is given by

$$A_c h_f (T_f - T_a) + f I_h V_h \rho C_p (T_i - T_a) = \frac{A_r}{2} h_{r,1} (T_a - T_{r,1}) + \frac{A_r}{2} h_{r,2} (T_a - T_{r,2}) + \frac{A_{es}}{R_{es}} (T_a - T_o) + I_a \rho C_p V_a (T_a - T_o) \quad (2)$$

where

- $A_c, A_r,$
 A_{es} = surface areas of ceiling, total roof area, and combined area of soffit and end walls (ft^2)
- C_p = specific heat of air ($\text{Btu/lb} \cdot ^\circ\text{F}$)
- f = fraction of house exfiltration that transfers into attic
- $h_f, h_{r,n}$ = convective heat transfer coefficients at attic floor and underside of roof (surface n), respectively ($\text{Btu/h} \cdot \text{ft}^2 \cdot ^\circ\text{F}$)
- I_h = house exfiltration rate (h^{-1})
- I_a = attic ventilation rate, i.e., outdoor air entering attic (h^{-1})
- $R_{e,s}$ = average thermal resistance of end walls and eaves ($\text{h} \cdot \text{ft}^2 \cdot ^\circ\text{F}/\text{Btu}$)
- T_a = attic air temperature ($^\circ\text{R}$)
- T_f = temperature of attic floor surface ($^\circ\text{R}$)
- T_i = indoor air temperature ($^\circ\text{R}$)
- T_o = outdoor air temperature ($^\circ\text{R}$)
- $T_{r,1}$ = temperature of roof sheathing underside a ($^\circ\text{R}$)
- $T_{r,2}$ = temperature of roof sheathing underside b ($^\circ\text{R}$)
- V_a = volume of attic space (ft^3)
- V_h = volume of house (ft^3)
- ρ = density of air (lb/ft^3)

Attic heat transfer

Attic Floor—The heat balance for the attic floor (node 2) is

$$\frac{T_i - T_f}{R_c} = h_f(T_f - T_a) + \frac{F_1}{2}(T_f - T_{r,1}) + \frac{F_2}{2}(T_f - T_{r,2}) \quad (3)$$

where

R_c = thermal resistance of attic floor (ceiling)
(h·ft²·°F/Btu)

F_1, F_2 = radiative heat transfer coefficients between
attic floor and undersides of roof
(Btu/h·ft²·°F), with the following
definition:

$$F_n = \frac{\sigma}{(1/\epsilon_f) + (1/\epsilon_r) - 1} (T_f^2 + T_{r,n}^2)(T_f + T_{r,n}) \quad n = 1, 2$$

where

ϵ_f = emissivity of attic floor surface
(assumed to be 0.9)

ϵ_r = emissivity of roof sheathing surface
(underside, assumed to be 0.9)

σ = Stefan–Boltzman constant (Btu/h·ft²·°R⁴)

Attic heat transfer

Sheathing—The heat balance at the underside of sheathing (nodes 3a and 3b) is

$$\frac{T_{r,n} - T_{s,n}}{R_r} = h_r(T_a - T_{r,n}) + \frac{A_c}{A_r} F_n (T_f - T_{r,n}) + \frac{2}{A_r} L_h W_{r,n} \quad n = 1, 2 \quad (5)$$

where

- L_h = latent heat of vaporization (1,050 Btu/lb)
- R_r = thermal resistance of roof (h·ft²·°F/Btu)
- $T_{s,n}$ = temperature of outside roof surface (°R)
- $W_{r,n}$ = rate of moisture adsorption into sheathing (lb/h)

Attic heat transfer

The heat balance at the top surface of sheathing (nodes 4a and 4b) is

$$\frac{T_{s,n} - T_{r,n}}{R_r} = \alpha I_n + (h_{o,n} + h_{IR})(T_o - T_{s,n}) - L_{IR} \quad n = 1, 2 \quad (6)$$

where

- h_{IR}, L_{IR} = adjustments for infrared radiation exchange with sky (see Eqs. (7), (8), and (9))
- $h_{o,n}$ = convective heat transfer coefficient at exterior roof surface n (Btu/h·ft²·°F)
- I_n = total solar radiation incident on roof surface (Btu/h·ft²)
- α = solar absorptance

Attic heat transfer

$$h_{\text{IR}} = 4\varepsilon_s \sigma T_o^3 \quad (7)$$

$$L_{\text{IR}} = \varepsilon_s \sigma T_o^4 (1 - \varepsilon_{\text{IR}}) \quad (8)$$

where

ε_s = emissivity of roof shingles (assumed to be 0.9)

ε_{IR} = sky emissivity with clouds

The sky emissivity is calculated with equations from Martin and Berdahl (1984). The emissivity for a clear sky is

$$\varepsilon_0 = 0.711 + 0.56 \frac{T_d}{100} + 0.73 \left(\frac{T_d}{100} \right)^2 + 0.013 \cos \left(\frac{2\pi t}{24} \right) \quad (9)$$

where

ε_0 = emissivity of clear sky

T_d = outdoor dew point temperature (°C)

t = time (h)

and the emissivity of the sky with clouds is

$$\varepsilon_{\text{IR}} = \varepsilon_0 + 0.784C(1 - \varepsilon_0) \quad (10)$$

where C = total cloud cover as recorded by National Climatic Center (Asheville, NC). Values of C range from 0 to 10.

Attic heat transfer

Sheathing surface temperature

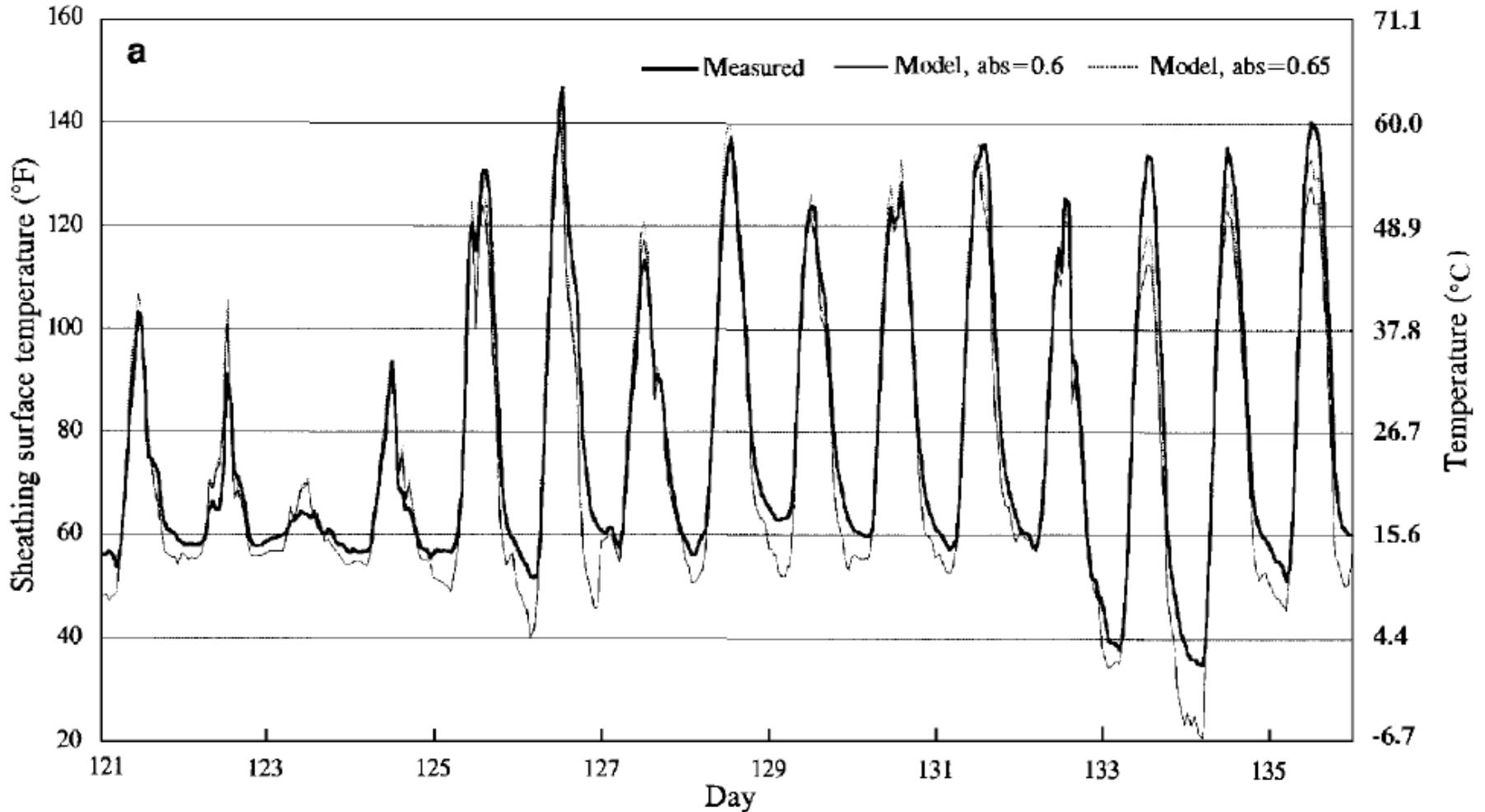


Figure 4—Exterior sheathing surface temperature, bay 2, May 1–15, 1993.

Attic heat transfer

Attic air temperature

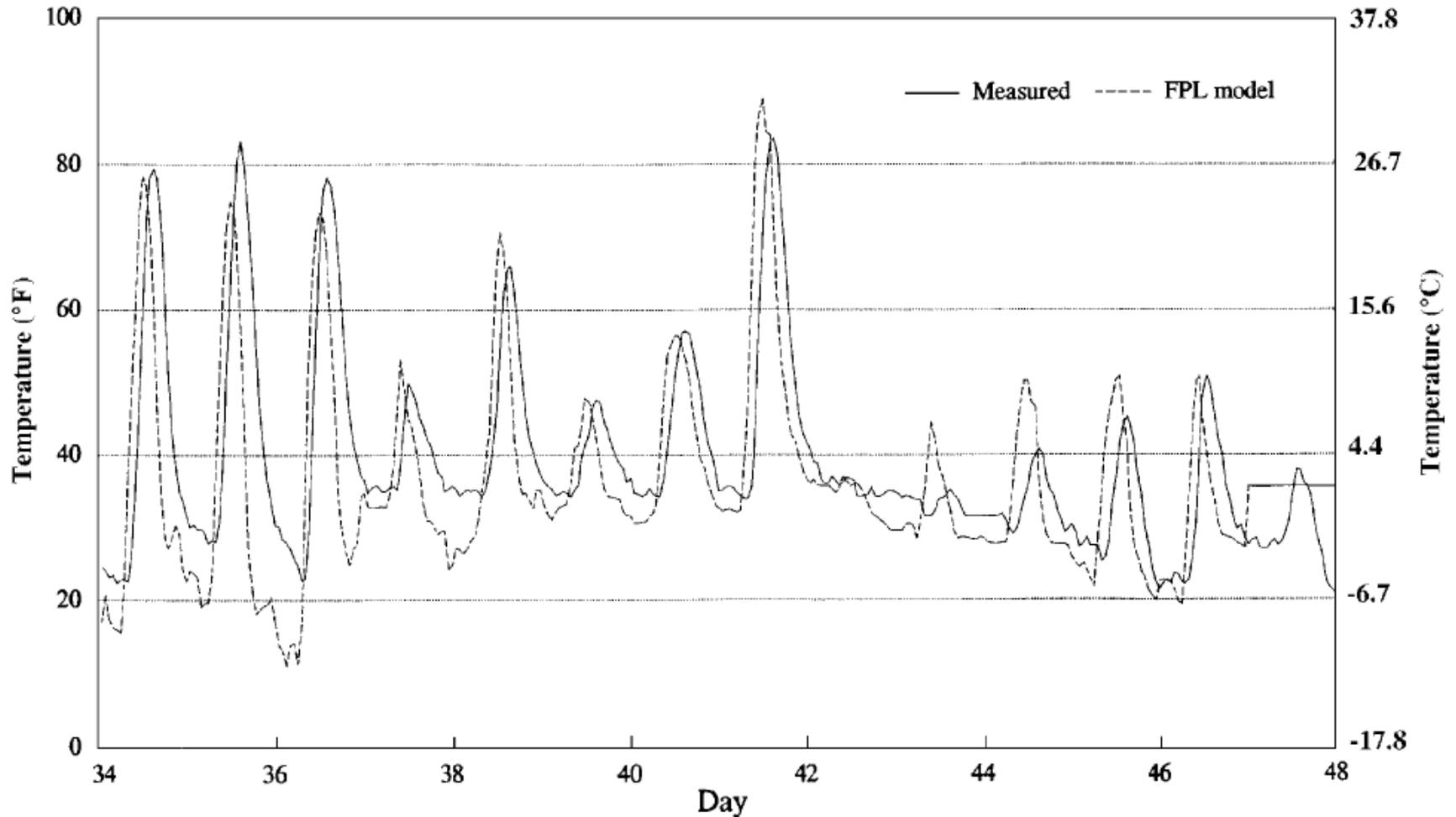


Figure 5—Attic air temperature, bay 2, February 3–16, 1993; absorptance = 0.65.

Attic heat transfer summary

- Attic temperature is important
 - HVAC equipment is often located in attics
 - Impacts conduction across ceiling into interior space
- Ways to reduce attic temperatures?
 - Increase convective heat transfer
 - Attic ventilation
 - Reduce exterior roof surface temperature
 - ‘Cool roof’ low absorptivity materials on the exterior
 - Reduce attic floor surface and underside sheathing temperatures
 - Low-emissivity materials inside attic
 - Attic floor or underside of sheathing
 - Radiant barriers

Radiant barriers

- **Radiant barriers** typically have LW emissivity less than ~ 0.1
 - Approximately 90% of materials have emissivity of 0.9
- Inhibits heat flow through radiation only
 - Doesn't directly impact convection or conduction



Impacts of radiant barriers

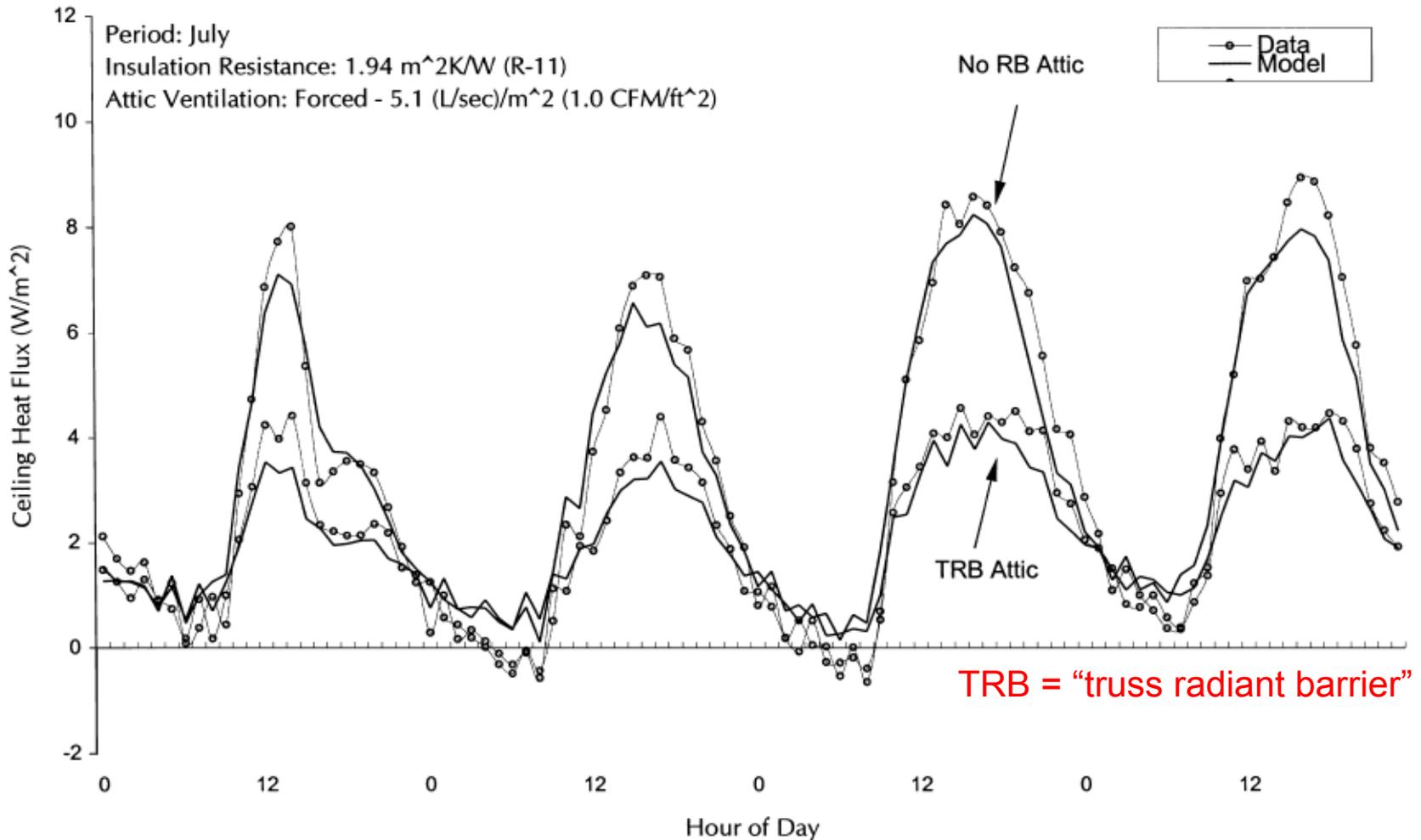


Fig. 6. Ceiling heat fluxes (TRB case, insulation resistance: 1.94 m² K/W, R-11; with attic airflow rate: 5.1 l/s/m², 1.0 CFM/ft²).

Impacts of radiant barriers

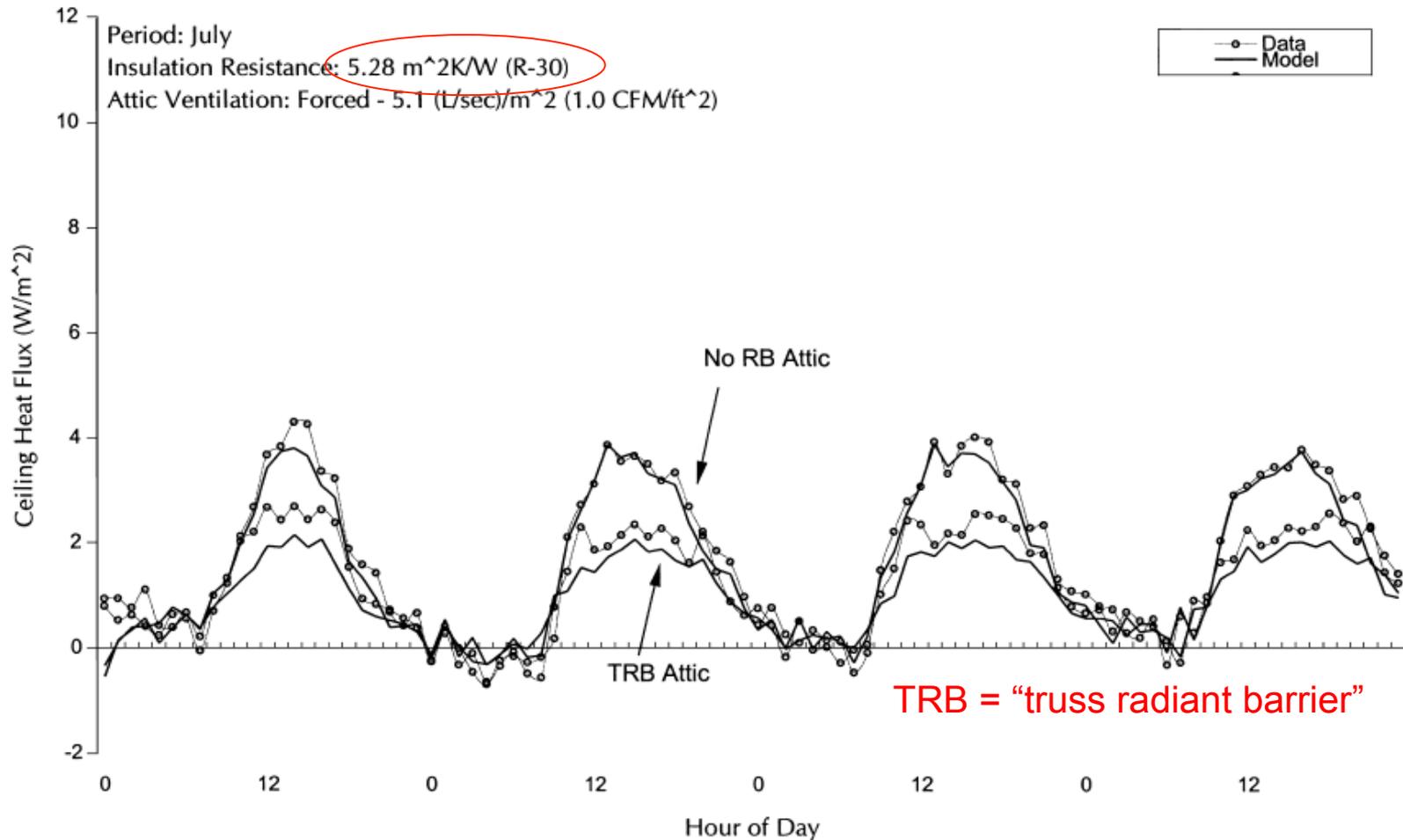
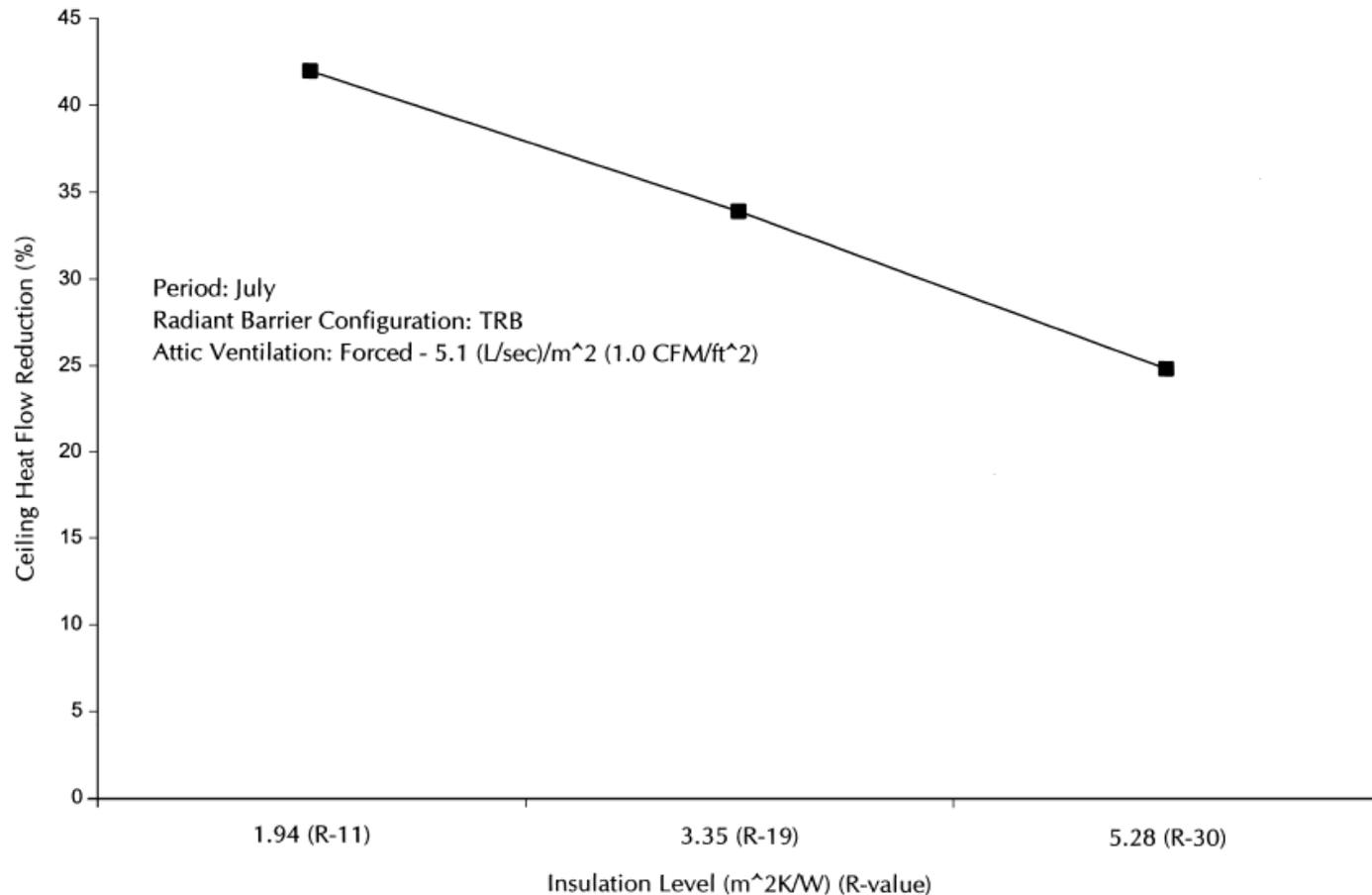


Fig. 7. Ceiling heat fluxes (TRB case, insulation resistance: 5.28 m² K/W, R-30; with attic airflow rate: 5.1 l/s/m², 1.0 CFM/ft²).

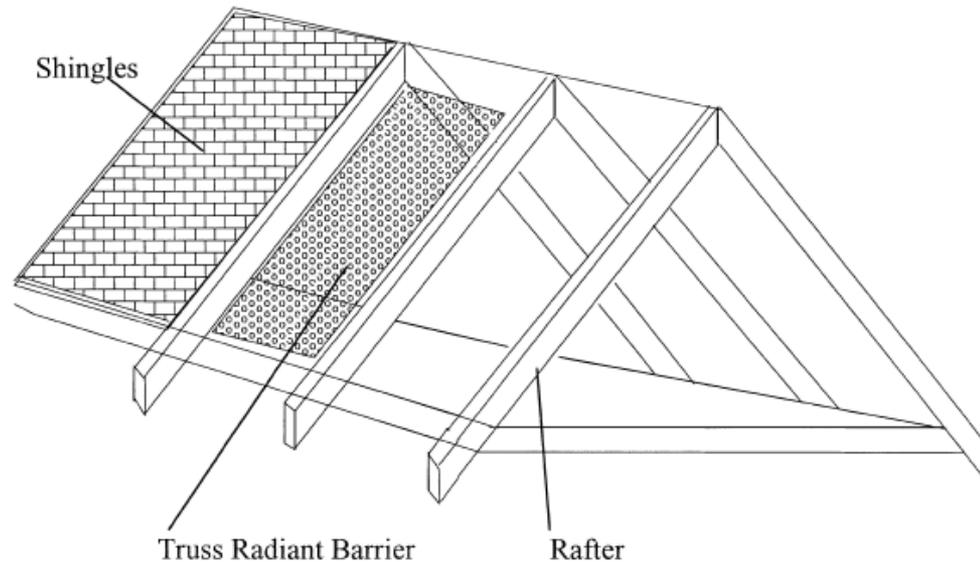
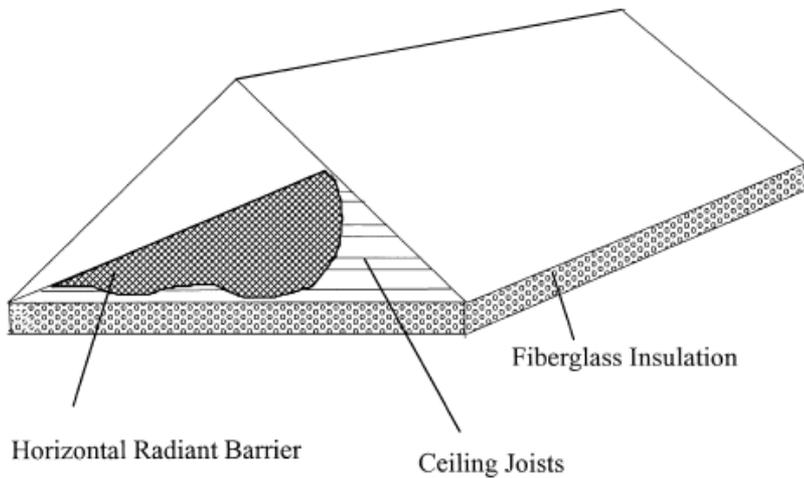
Impacts of radiant barriers

- Radiant barriers have less of an impact on well-insulated roofs



Impacts of radiant barriers

- Should we install radiant barriers on attic floors or underside of roof sheathing?



Impacts of radiant barriers

- Should we install radiant barriers on attic floors or underside of roof sheathing?

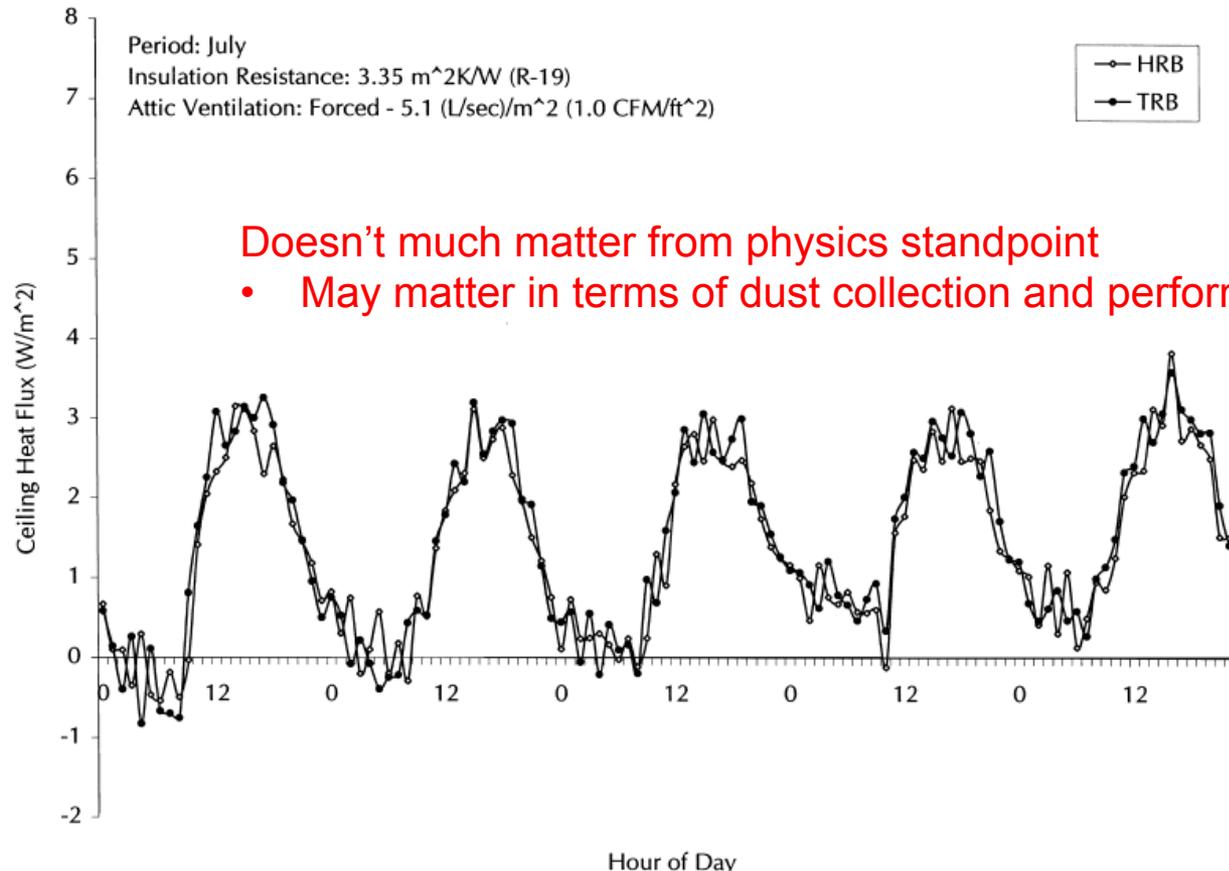


Fig. 9. Comparison between HRB and TRB configurations — experimental (insulation resistance: 3.35 m² K/W, R-19; with airflow rate: 5.1 l/s/m², 1.0 CFM/ft²).

Roof and attic heat transfer summary

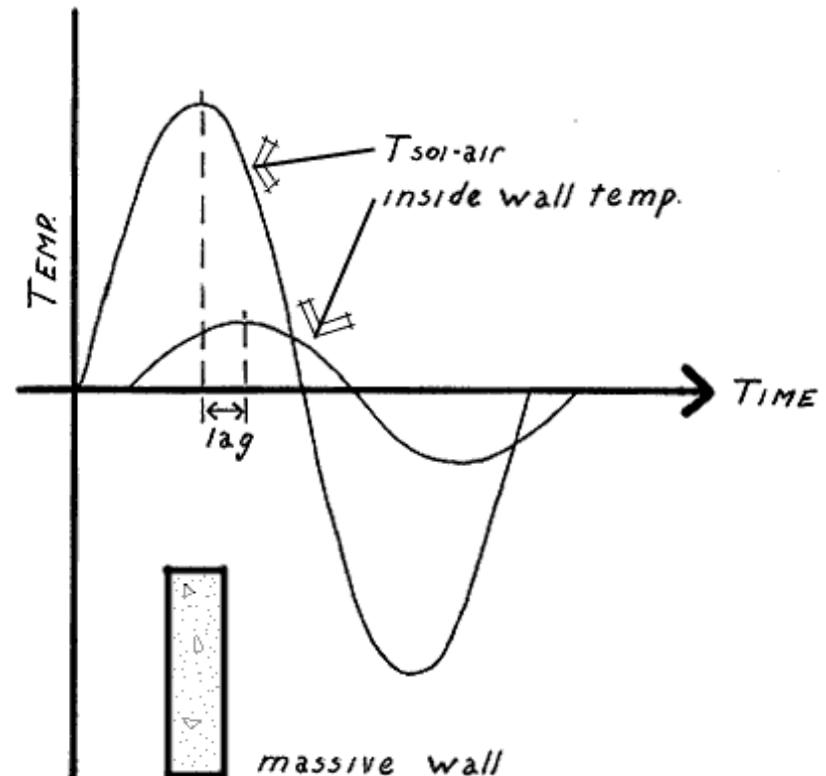
- Roof surfaces can be extremely hot
 - Several energy and material longevity impacts
 - We can alter materials in design to reduce roof temperatures
- Attic air can also be very warm
 - Also has energy and comfort impacts
 - We now know what governs attic air temperatures
 - We know radiant barriers can help
 - Particularly if you have a poorly insulated roof
 - Better off just insulating roof if you can
- For both of these, we can use our modeling approaches in the design phase

THERMAL MASS

Heat storage and release

Thermal mass

- Thermal mass refers to materials that have the capacity to store thermal energy for extended periods of time
- Thermal mass can be used effectively to absorb daytime heat gains
 - Reduces peak cooling load
 - Releases heat during the night (can reduce heat load)



Historical use of thermal mass



- Thermal mass is **NOT** a new idea
- The use of thermal mass in construction dates to the beginning of history
 - Stone caves are great examples of ancient thermal mass buildings
 - Mud-brick houses have been used for thousands of years by numerous civilizations in hot climates
 - Helps buffer harsh exterior conditions

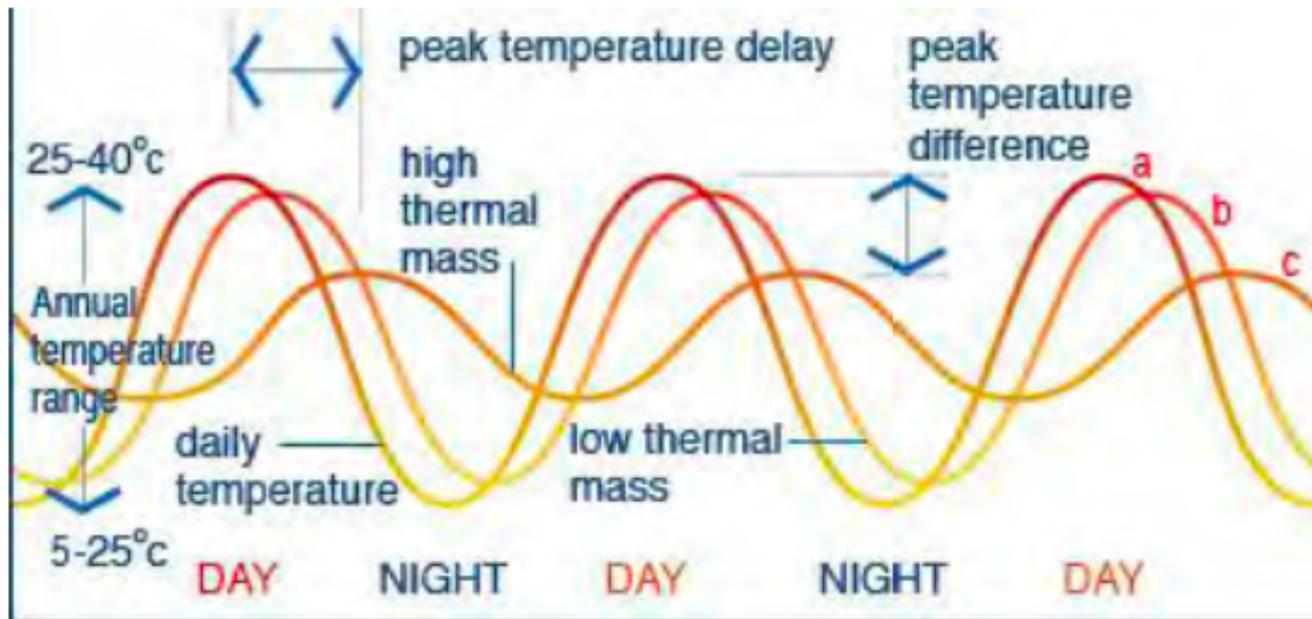


Types of thermal mass

- Traditionally, materials with high **thermal** mass have also had a high mass themselves:
 - Water, earth, stone, brick, cement, concrete, thick tiles
 - All these materials have high densities and high heat capacities
- More recently, phase change materials are being used:
 - Solid-liquid salts, paraffin wax, crystalline hydrocarbons
 - These are materials that melt at room temperature and can store/release large amounts of latent heat with much lower mass than traditional materials
 - Will not cover → we have a final project on PCM

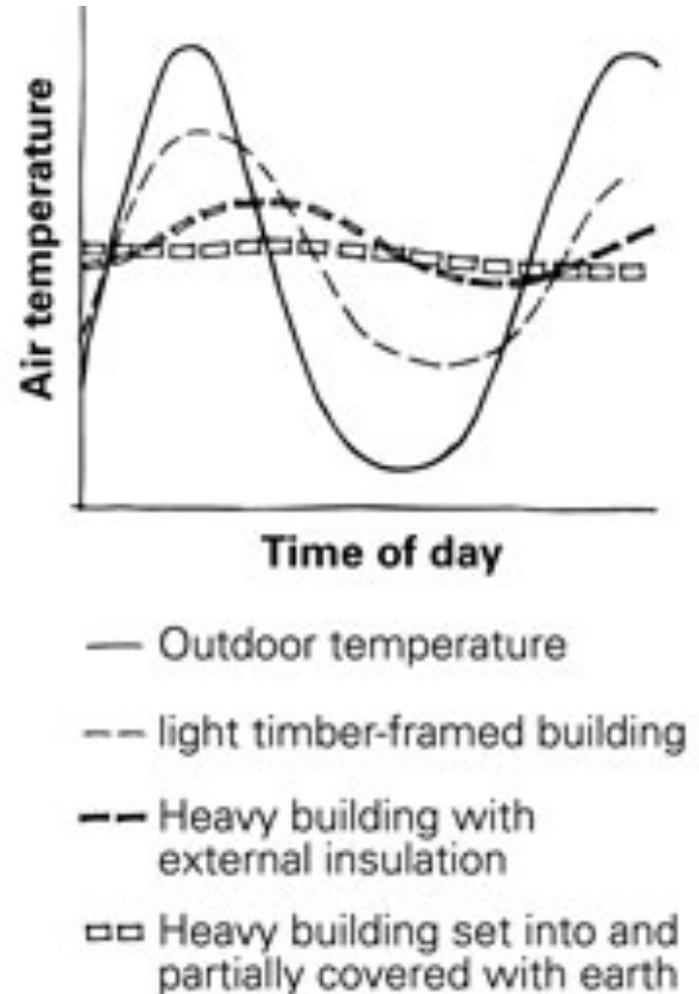
Thermal mass

- All materials/constructions have some thermal mass
 - But constructions with a high thermal mass have large effects on the dynamic energy transfer in a building
 - Thermal mass can be both useful and detrimental to maintaining thermal comfort with changing heating and cooling loads



Thermal mass: Why do we care?

- A high thermal mass will be slow to heat up
 - But also slow to cool down
 - Can store large amounts of heat
- The result is that exterior temperatures can fluctuate greatly
 - But the interior temperature will fluctuate less



What thermal mass is not

- Thermal mass is not the same as thermal insulation/thermal resistance
 - In fact, most materials with high thermal resistance have low thermal mass
- Some material in a construction needs to have a high **heat capacity** for the construction to have a high thermal mass

Heat Capacity, HC

- The heat capacity (HC) of a material is the product of the surface mass density of the material and the specific heat
 - $HC = \rho L C_p$ [J/m²K]
 - HC is a measure of the ability of a material to store energy per unit area
 - L = length [m]
 - ρ = density [kg/m³]
 - C_p = specific heat capacity [J/kgK]
 - You sometimes also see $HC^*A = \rho L A C_p$ [J/K]
- Heat capacity is important to thermal mass, but needs to be compared with thermal conductivity to get the whole story

Thermal Diffusivity, α

- Thermal diffusivity, α , is the measure of how fast heat can travel through an object
- α is proportional to conductivity but inversely proportional to density and specific heat:

$$\alpha = \frac{k}{\rho C_p} \quad [\text{m}^2/\text{s}]$$

- The lower the α , the better the material is as a thermal mass (low conductivity relative to storage ability)
 - The time lag between peak internal and external temperature is related to the diffusivity of the walls
 - Steel has a high ρC_p but also a high k so it is not as good a thermal mass as concrete or masonry

Thermal properties

- All three material properties can be found in ASHRAE HOF chapter on thermal transmission data (Ch. 25 in 2005)
 - Thermal conductivity, density, and specific heat

Description	Density, kg/m ³	Conductivity ^b (<i>k</i>), W/(m·K)	Conductance (<i>C</i>), W/(m ² ·K)	Resistance ^c (<i>R</i>)		Specific Heat, kJ/(kg·K)
				1/ <i>k</i> , (m·K)/W	For Thickness Listed (1/ <i>C</i>), (m ² ·K)/W	
<i>Gypsum partition tile</i>						
75 by 300 by 760 mm, solid	—	—	4.50	—	0.222	0.79
75 by 300 by 760 mm, 4 cells	—	—	4.20	—	0.238	—
100 by 300 by 760 mm, 3 cells	—	—	3.40	—	0.294	—
<i>Concretes^o</i>						
Sand and gravel or stone aggregate concretes (concretes	2400	1.4-2.9	—	0.69-0.35	—	—
with more than 50% quartz or quartzite sand have	2240	1.3-2.6	—	0.77-0.39	—	0.8-1.0
conductivities in the higher end of the range)	2080	1.0-1.9	—	0.99-0.53	—	—
Limestone concretes	2240	1.60	—	0.62	—	—
	1920	1.14	—	0.88	—	—
	1600	0.79	—	1.26	—	—

Modeling thermal mass

- There are several ways to model conduction and thermal mass together
- Lumped capacitance:

$$C \frac{dT_1}{dt} = hA(T_o - T_1) + \frac{T_2 - T_1}{R}$$

$$C \frac{dT_2}{dt} = hA(T_i - T_2) + \frac{T_1 - T_2}{R}$$

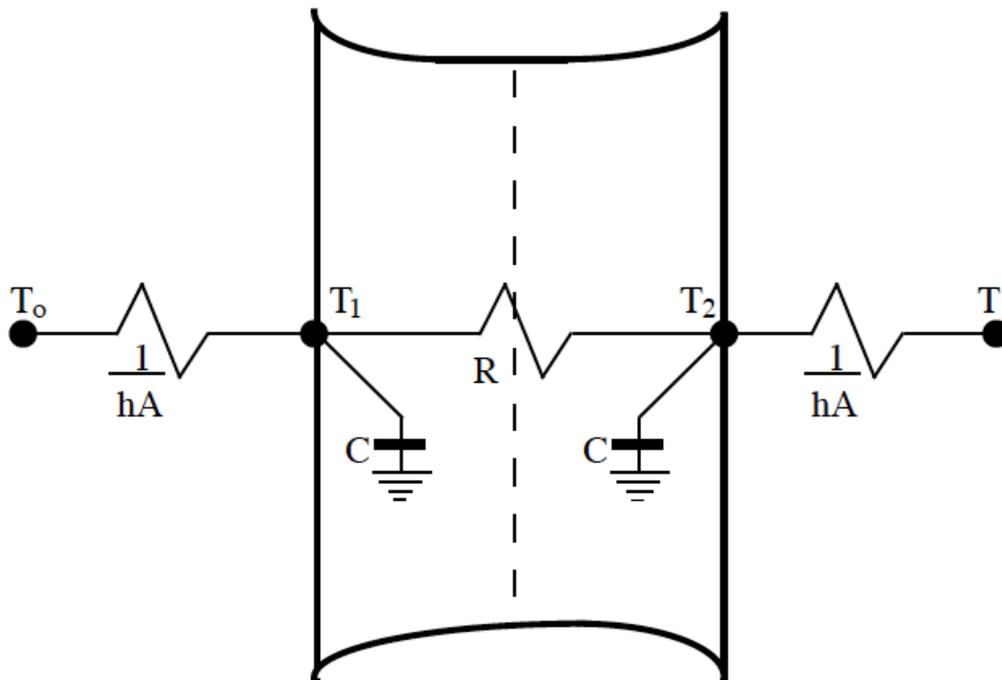


Figure 9. Two Node State Space Example.

where:

$$R = \frac{\ell}{kA},$$

$$C = \frac{\rho c_p \ell A}{2}$$

Modeling thermal mass

- Lumped capacitance: wall example

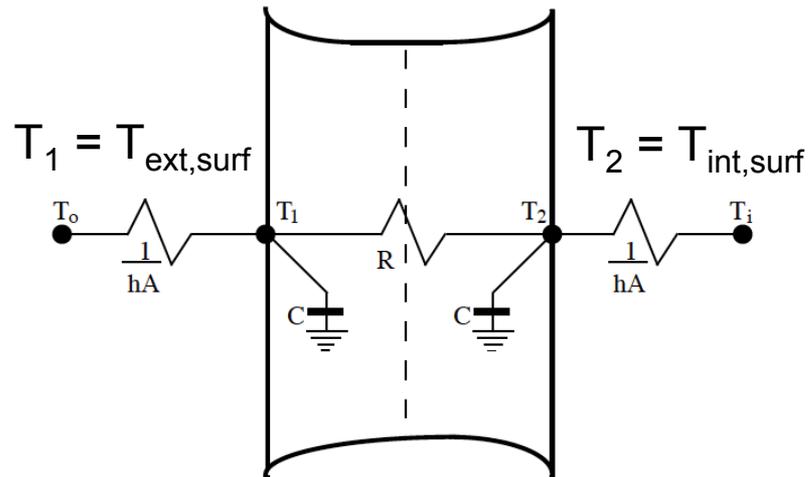


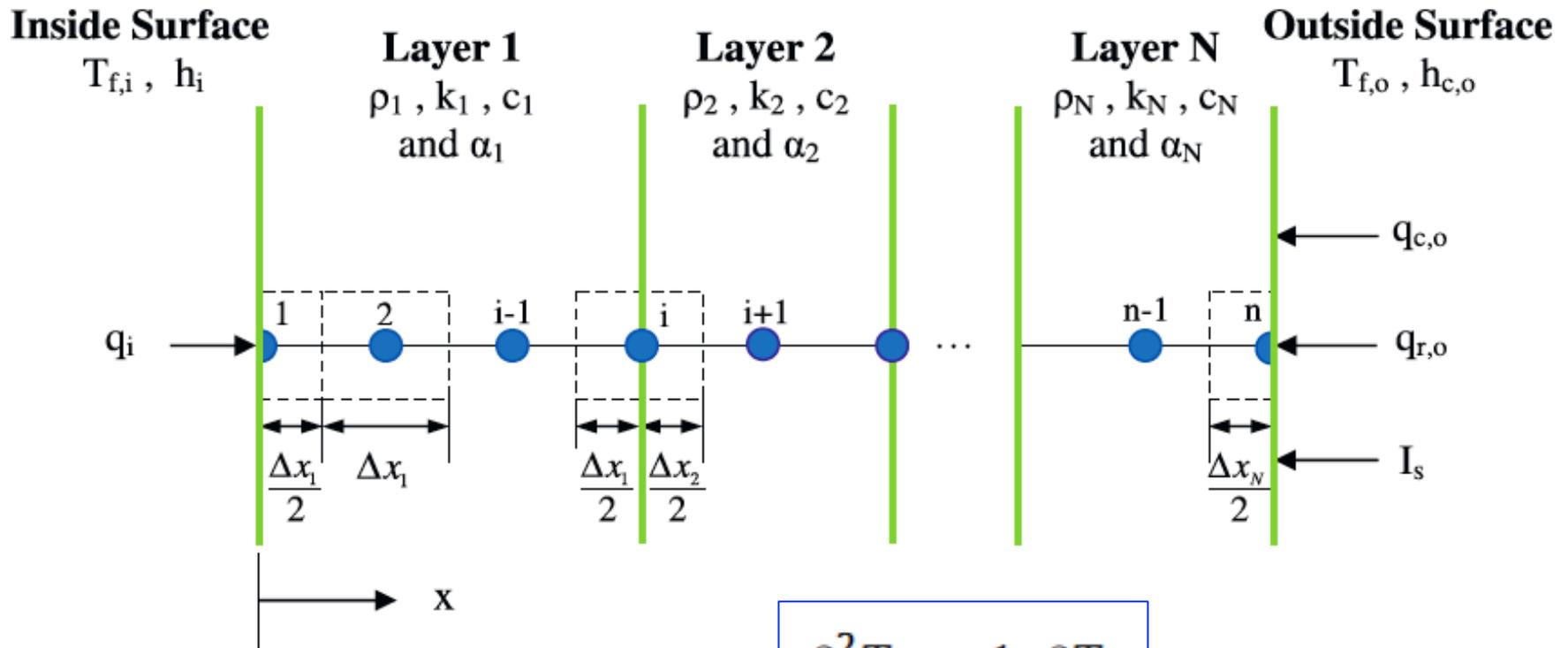
Figure 9. Two Node State Space Example.

Time-varying surface energy balance:

$$\frac{\rho C_p L}{2} \left(\frac{T_{ext} - T_{ext}^{n-1}}{\Delta t} \right) = I_{tot} \alpha_{sw} + h_{conv} (T_{out} - T_{ext}) + \varepsilon \varepsilon F \sigma (T_{sky}^4 - T_{ext}^4) - \frac{k}{L} (T_{ext} - T_{int})$$

Modeling thermal mass

- Conduction and thermal mass together can also be modeled using discrete nodes:



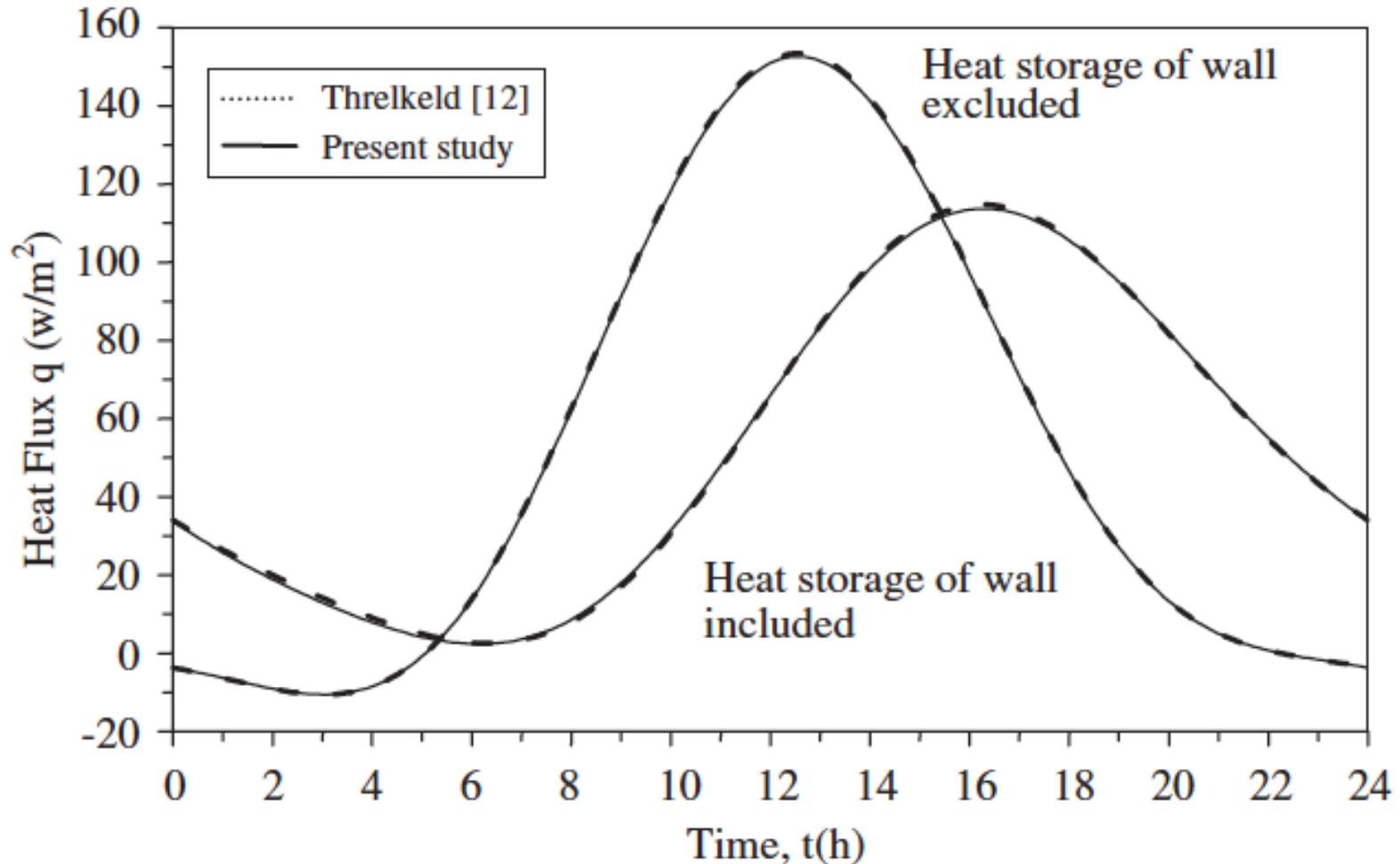
$$\frac{\partial^2 T_j}{\partial x^2} = \frac{1}{\alpha_j} \frac{\partial T_j}{\partial t}$$

Time lags and decrement factors

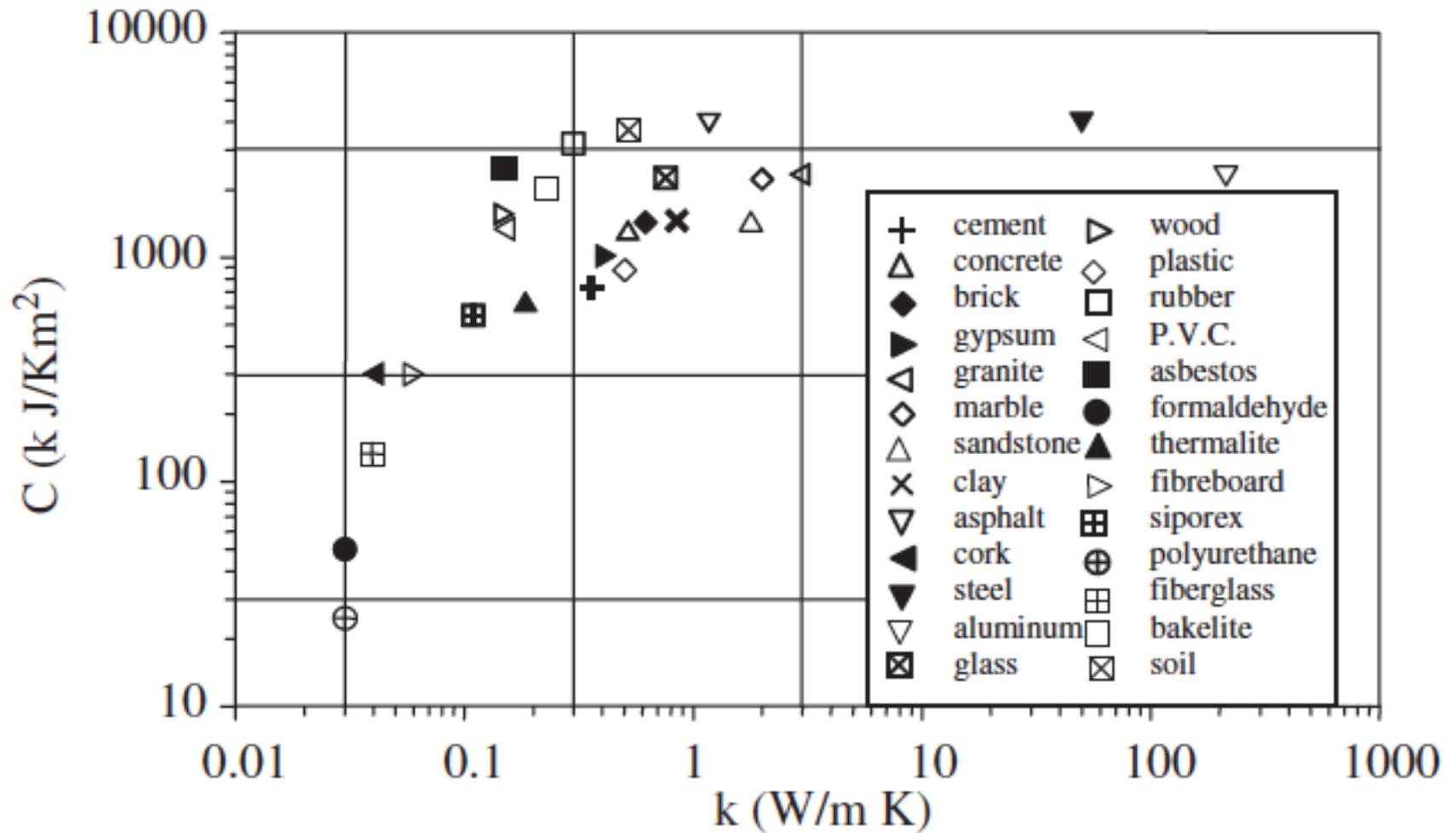
- These models (and measurements) can be used to describe ‘time lags’ and ‘decrement factors’
- Time lag tells you:
 - For a given peak exterior surface temperature at a certain time
 - How much later does the peak interior surface temperature actually occur?
 - Shift due to thermal lag effects
- Decrement factor tells you:
 - How much lower is the peak temperature swing (amplitude) with an enclosure with high thermal mass relative to no thermal mass
 - e.g., How squished is the peak temperature profile?

Modeling thermal mass impacts

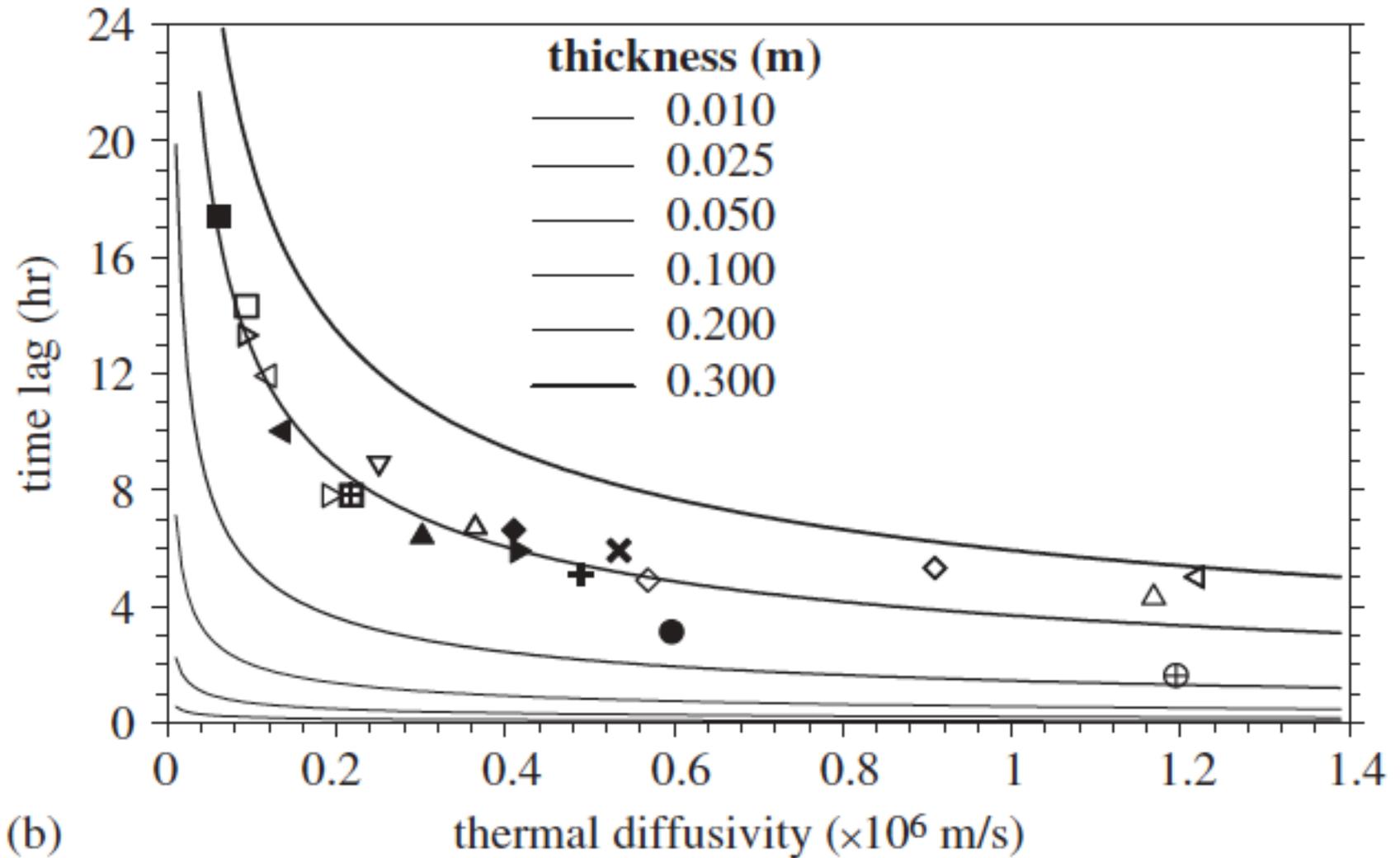
Decrement factors and time lags



Material properties



Modeled time lag of several materials with varying thickness



(b)

Why are these important?

- Time lag
 - Doesn't impact energy use directly
 - But impacts **time** of energy use
 - Can shift peak loads
 - Meaningful for peak loads on aggregate basis
 - Also for energy markets with dynamic pricing
- Decrement factor
 - **Does** impact energy use
 - Dampens rate of conduction through enclosure
 - Can allow for smaller HVAC equipment
 - Lower upfront costs
 - Important in design phase

Thermal time constant (TTC)

- The thermal time constant is defined as the sum of the product of the heat capacity of a layer i and the cumulative thermal resistance up to layer i

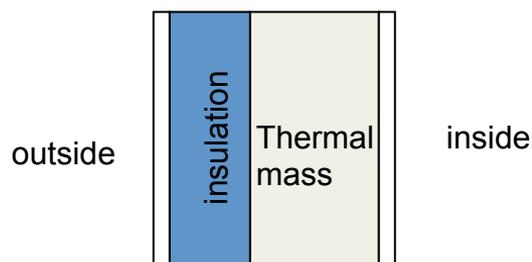
$$TTC = \sum_i \rho_i L_i C_{pi} R_{o \rightarrow i}$$

Units of time [sec]

- TTC is a measure of time it takes heat to propagate through the wall and is a kind of “effective” thermal insulating capability
 - The higher the TTC , the lower the overall heat transfer through the structure

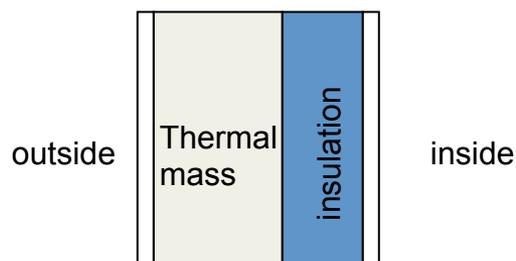
Example TTC calculation

Wall 1: exterior insulation



TTC = 43.8

Wall 2: interior insulation



TTC = 7.8

TABLE 3-8. CALCULATION OF THE THERMAL TIME CONSTANT OF 2 WALLS (METRIC)

Wall #1

LAYER	THICK l_i (m)	DENSITY ρ_i (Kg/m ³)	RESIST. r_i	CUMULAT. RESIST.	HC ρ^*c	QR _i Hr
Ext. surface						0.03
Ext. plaster	0.02	1800	0.025	0.0425	414	0.35
Polystyrene	0.025	30	0.71	0.41	12	0.12
Concrete	0.10	2200	0.06	0.795	506	40.2
Int. plaster	0.01	1600	0.014	0.832	368	3.1
Wall's TTC						43.8

Wall #2

LAYER	THICK l_i (m)	DENSITY ρ_i (Kg/m ³)	RESIST. r_i	CUMULAT. RESIST.	HC ρ^*c	QR _i Hr
Ext. surface						0.03
Ext. plaster	0.02	1800	0.025	0.0425	414	0.35
Concrete	0.10	2200	0.06	0.085	506	4.3
Polystyrene	0.025	30	0.71	0.47	12	0.14
Int. plaster	0.01	1600	0.014	0.832	368	3.1
Wall's TTC						7.8

TTC example

- The Thermal Time Constant (TTC) of the concrete wall with exterior insulation is nearly 5x larger than the concrete wall with interior insulation
 - This means the wall with exterior insulation will be a better thermal mass
- The assembly with the interior insulation has the large thermal mass directly exposed to the large temperature swings of the outdoors
 - By placing the insulation between the exterior air and the thermal mass, it takes longer to “charge” and “discharge” the thermal mass with heat

Next lecture

- Energy use and the enclosure
 - Energy modeling in design phase
 - Energy code requirements