CAE 463/524
Building Enclosure Design

Fall 2012

Lecture 3: Heat transfer (continued)

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Housekeeping

- HW 1 is graded
  - Will post solutions to Blackboard after class today
  - Will also post grades online to Blackboard
    - Most of you did very well

- Will assign another HW at the end of today’s lecture
  - HW 2
  - Due September 17… 1 week from today

- Project groups
  - Intermediate campus project: groups of 3
  - Final project: groups of 2

Housekeeping: Campus project

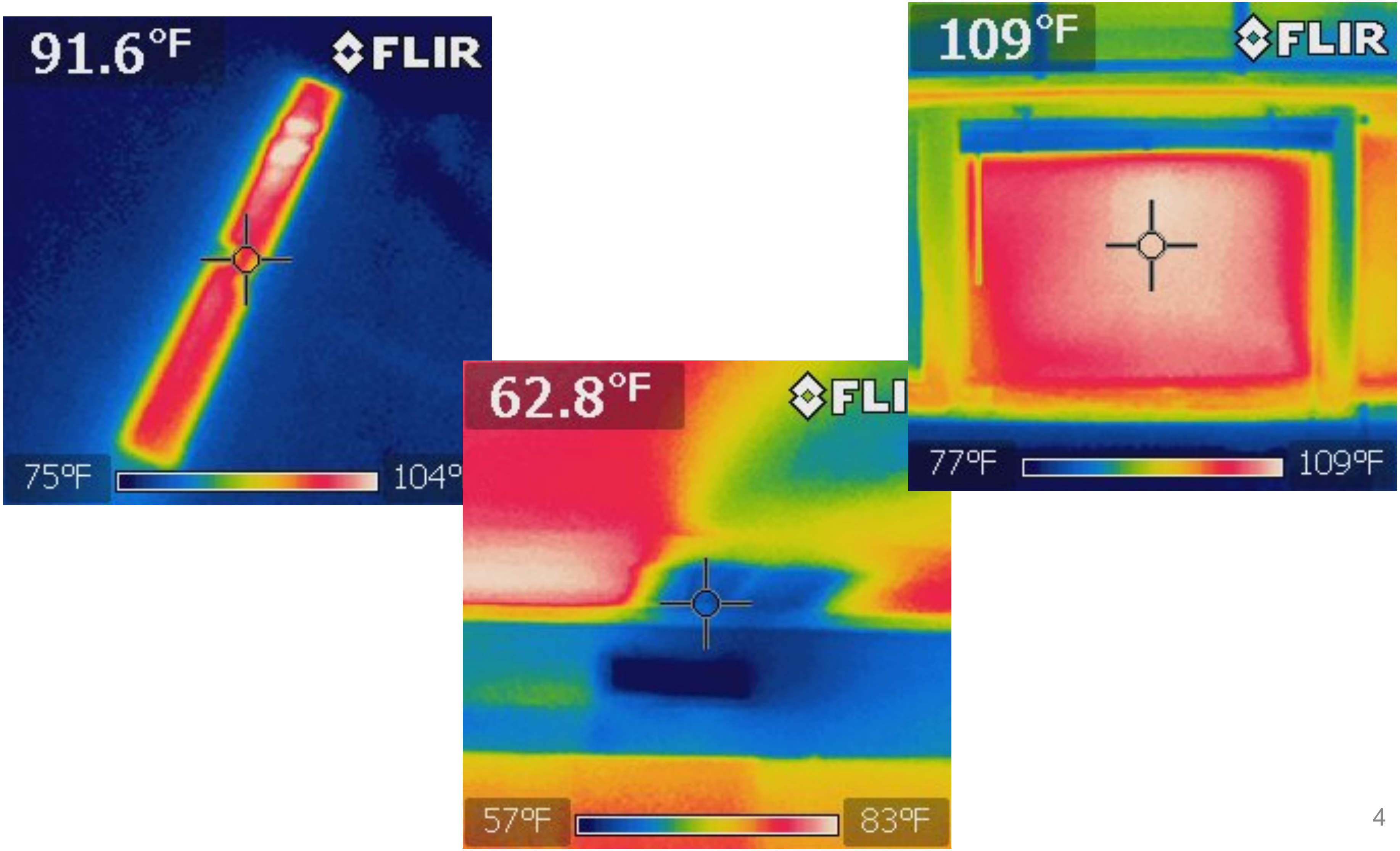
- Continue thinking about a campus building to assess
  - Email me your team (of 3) + building
  - Will spend some time next class (September 17, 2012) assigning teams randomly for those that don't self-organize
    - One student has claimed Alumni Memorial Hall (Lynda Russo)
    - One student has claimed Crown Hall (Dan Sebastian)
    - One student has claimed Siegel Hall (Melissa Angulo)

  - Will wait until ~October for thermal imaging
    - I have received our infrared camera (!)
    - Can do walkthroughs earlier
    - Talk to Facilities ahead of time

  - Will be due in early- to mid-November (tentatively Nov. 5)
    - I will provide example report(s) in the next few weeks
    - Mostly qualitative, lots of freedom

Campus project

- Some example infrared images (of my office)



Housekeeping: Final project

- Teams of 2
  - We have 21 students, so will have one team of 3 (or one team of 1)
- Topic justification
  - Due October 1
  - ½ to 1 page written justification to me of why you have chosen the high performance enclosure you have chosen
  - If you don't have a team gathered by September 24
    - Will randomly assign
- Group consultation
  - Week of October 15 (and again later if needed)
- Final report
  - November 26 (last day of class)
- Final presentation
  - Given in class November 26 (last day of class)

Final project possible topics

Email me as soon as you have a team and/or topic

List of example technologies/designs

- Green roofs **(Lynda + Stuart = Team 1, also Patrick)**
- Green walls
- Double skin facades **(Yechen, Daniel, Alejandro, Luciana, Juan)**
- Building integrated photovoltaics **(maybe Luciana, Giovanni)**
- Electrochromic windows **(Inna and Giovanni = Team 2?)**
- Phase change materials **(maybe Daniel)**
- Bio-based insulation materials (mushrooms, straw)
  - **Strawbale construction (Zeineb)**
- Structural insulated panels
- Cool roofs
- SmartWrap^{TM}
- Conventional construction **(Tommy)**
  - Also design for cooling in tropical climate **(Nestor)**

Review from last time

- Solar orientation
  - Learned about angles between surfaces on a point on the earth and the sun
  - Learned how that impacts direct, diffuse, and reflected radiation hitting a surface
    - Learned to predict at any time of the year at any location
    - Also learned where to download some of these data
    - One problem on these relationships on **HW 2**

- Heat transfer fundamentals
  - Learned about conduction, convection, and radiation
  - Just started to piece them together in terms of building enclosures
    - Will continue heat transfer today

1. HEAT TRANSFER IN BUILDING ENCLOSURES

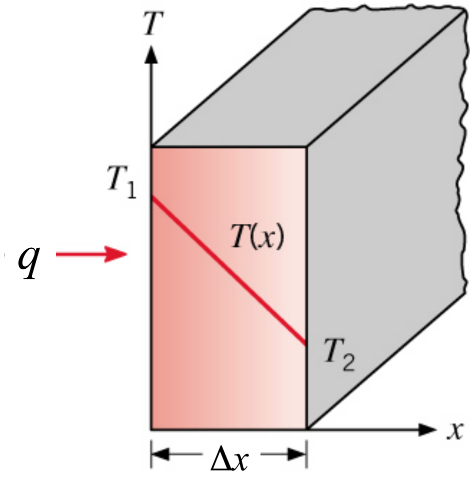
Conduction

- **In building enclosures:** Heat flow through solids

$$q_{cond} = -k\nabla T$$

which, in 1-D, becomes:

$$q_{cond} = -k\frac{dT}{dx}$$



Δx = L

And in 1-D through a material with uniform conductivity and thickness:

$$q_{cond} = k\frac{T_1 - T_2}{x_2 - x_1} = k\frac{\Delta T}{\Delta x} = \frac{k}{L}\left(T_1 - T_2\right) = U\left(T_1 - T_2\right) = \frac{1}{R}\left(T_1 - T_2\right)$$

Units:

q_{cond} = [W/m^2], k = conductivity [W/mK], L = thickness [m], U = conductance [W/(m^{2}K)]

T_1 and T_2 are surface temperatures = [K], R = resistance [(m^{2}K)/W]

Convection

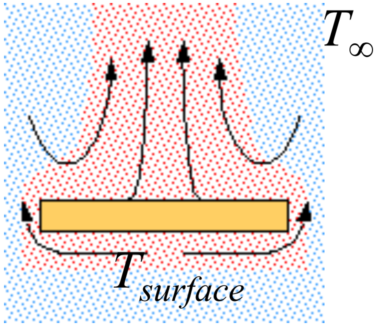
- **In building enclosures:**

Heat flow between surfaces and air (due to air movement)

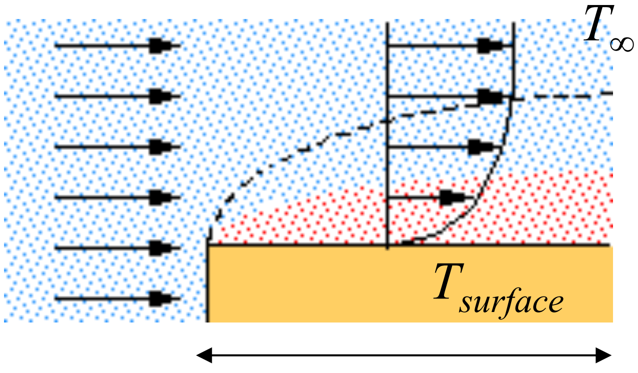
$$q_{conv} = h_{conv}\left(T_\infty - T_{surface}\right) = \frac{T_\infty - T_{surface}}{R_{conv}} = \frac{\Delta T}{R_{conv}}$$

Note that h_{conv} is a function of flow characteristics (laminar or turbulent), temperature differences, and air velocity/wind speed

Natural convection



Forced convection



L – characteristic length

Units:

q_{conv} = [W/m^2], h_{conv} = heat transfer coefficient [W/(m^{2}K)], $T_{surface}$ is the temperature at the building surface = [K], T_∞ is the temperature far away = [K],
R_{conv} = convective thermal resistance [(m^{2}K)/W]

Summary of h_c equations for natural convection (SI)

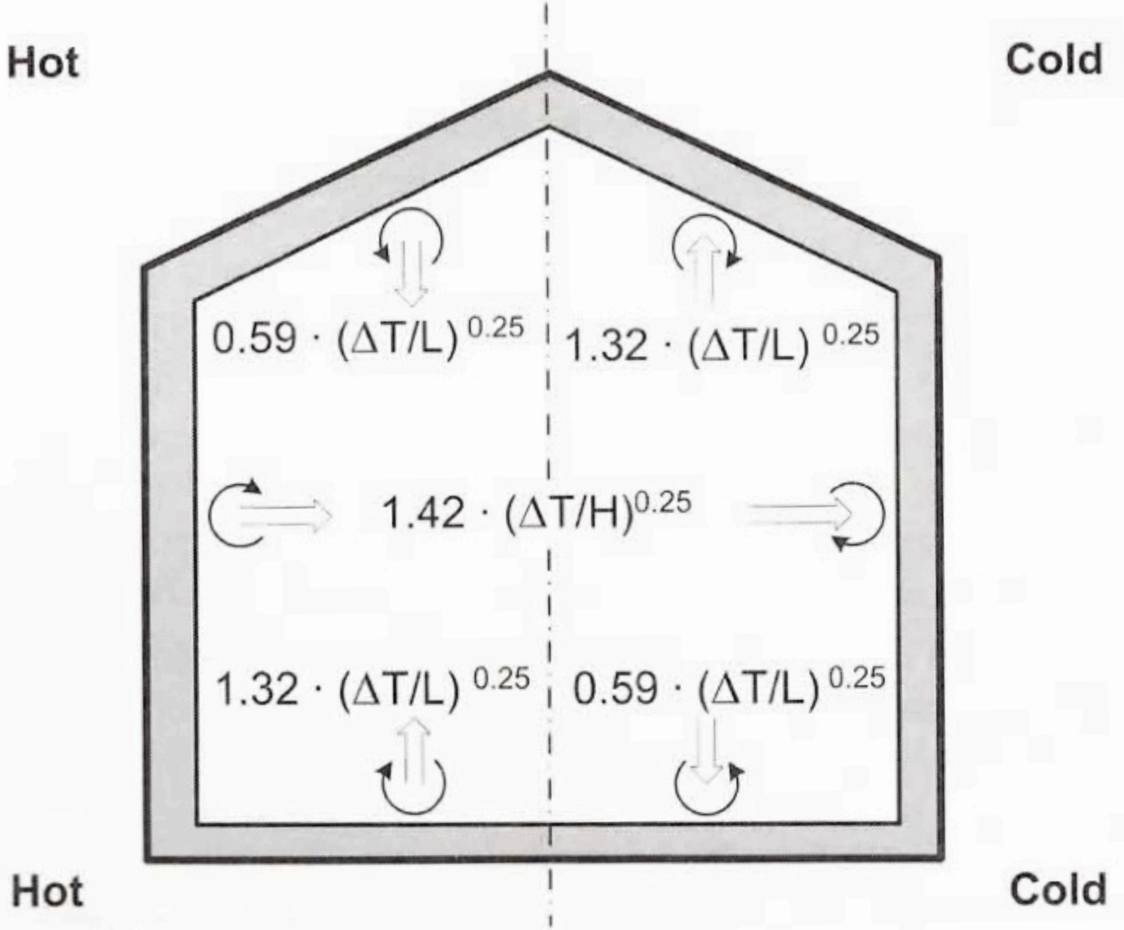


Figure 5.4: Natural convection coefficients for laminar flow

Most popular general h_c for forced convection

There are two relationships for h_c (forced convection) which are commonly used, depending on wind speed:

- For $1 < v_{wind} < 5$ m/s

  $h_c = 5.6 + 3.9v_{wind}$ [W/(m^2·K)] [Straube 5.15]

- For $5 < v_{wind} < 30$ m/s

  $h_c = 7.2v_{wind}^{0.78}$ [W/(m^2·K)] [Straube 5.16]

***Good for use with external surfaces**

Radiation

- **For building enclosures:** Radiation comes in two forms

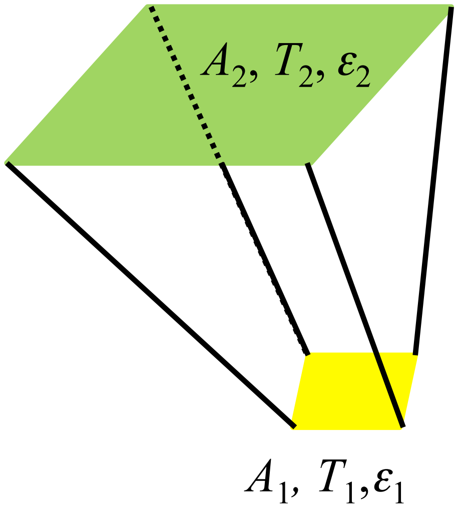
- **Short-wave radiation:** Energy from the sun's rays
  - Direct + diffuse + reflected
  - Denoted by "I_{total}" or "I_{SW}"
  - When the sun isn't shining: $I_{\text{SW}} = 0$
  - Multiply by a material's absorptivity for impact on surface
    - Learned to get I_{SW} last lecture
    - Fraction of short wave radiation absorbed = α_{sw}
    - Low α_{sw} materials absorb less solar radiation

$$\boxed{q_{SWR} = \alpha I_{SW}}$$

- **Long-wave radiation**
  - Heat flow by electromagnetic waves between surfaces and black bodies
  - Occurs even without sunshine

Long-wave radiation

- **In building enclosures:** Heat flow by electromagnetic waves between surfaces and black bodies



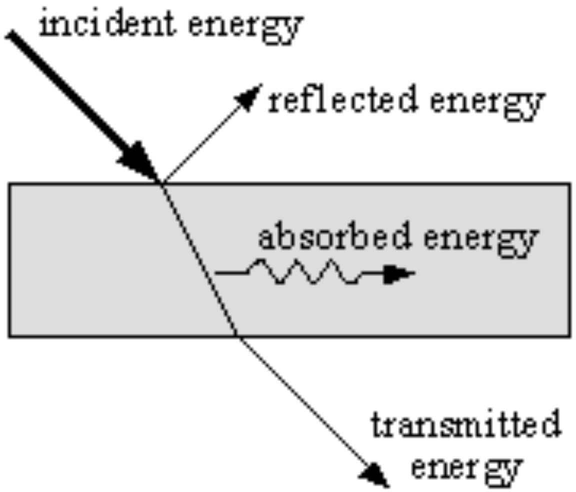
If a material has absorptivity = emissivity, the net radiation heat transfer between two surfaces is:

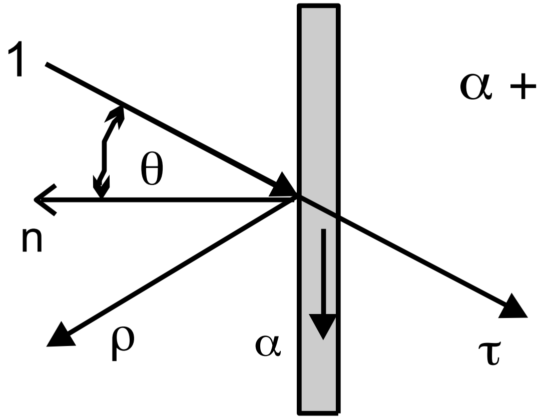
$$q_{r,1\to2} = \frac{Q_{1\to2}}{A_1} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2} + \frac{1}{F_{12}}}$$

where ε_1 and ε_2 are the surface emittances,
A_1 and A_2 are the surface areas [m^2]
and $F_{1\to2}$ is the view factor from surface 1 to 2
$F_{1\to2}$ is a function of geometry only

Units:
q_r = [W/m^2], T_1 and T_2 are surface temps [K], F_{12} is the view factor [-], ε is emissivity [value between 0 and 1], σ is the Stefan-Boltzmann constant = 5.6704×10^{-8} W/(m^{2}K^4)

Note on emissivity (ε) and absorptivity (α)





$$\alpha + \rho + \tau = 1$$

absorbed *(α)*, transmitted *(τ)*, and reflected *(ρ)* radiation

- **Absorptivity (α):** fraction of total energy striking on an object that is absorbed
- **Emissivity (ε):** ability of a material's surface to emit energy by radiation (0 to 1)

Kirchoff's Law:

- Emissivity (ε) is same as Absorptivity (α) for most "gray" surfaces
  - Therefore "low-e" materials are also typically low-α materials, which absorb less heat
- These properties can depend on wavelength
- Black surface: $\varepsilon = \alpha = 1$

Note on emissivity (ε) and absorptivity (α)

- ASHRAE HOF has emissivity (long-wave) and absorptivity (short-wave) values for many materials in their heat transfer chapter. Manufacturers also report these

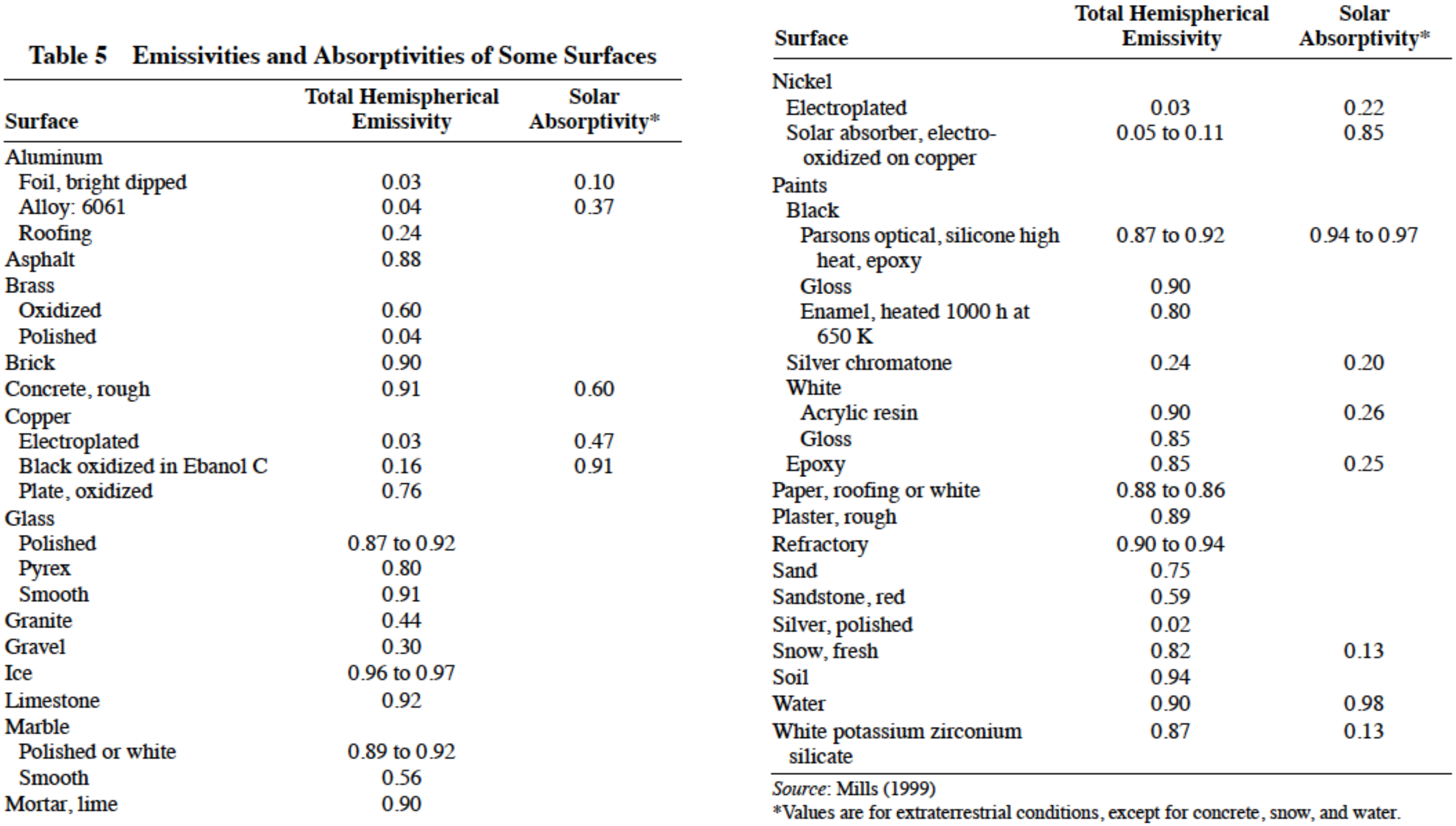


Table 5 Emissivities and Absorptivities of Some Surfaces

Surface	Total Hemispherical Emissivity	Solar Absorptivity*
Aluminum		
Foil, bright dipped	0.03	0.10
Alloy: 6061	0.04	0.37
Roofing	0.24	
Asphalt	0.88	
Brass		
Oxidized	0.60	
Polished	0.04	
Brick	0.90	
Concrete, rough	0.91	0.60
Copper		
Electroplated	0.03	0.47
Black oxidized in Ebanol C	0.16	0.91
Plate, oxidized	0.76	
Glass		
Polished	0.87 to 0.92	
Pyrex	0.80	
Smooth	0.91	
Granite	0.44	
Gravel	0.30	
Ice	0.96 to 0.97	
Limestone	0.92	
Marble		
Polished or white	0.89 to 0.92	
Smooth	0.56	
Mortar, lime	0.90	
Nickel		
Electroplated	0.03	0.22
Solar absorber, electro-oxidized on copper	0.05 to 0.11	0.85
Paints		
Black		
Parsons optical, silicone high heat, epoxy	0.87 to 0.92	0.94 to 0.97
Gloss	0.90	
Enamel, heated 1000 h at 650 K	0.80	
Silver chromatone	0.24	0.20
White		
Acrylic resin	0.90	0.26
Gloss	0.85	
Epoxy	0.85	0.25
Paper, roofing or white	0.88 to 0.86	
Plaster, rough	0.89	
Refractory	0.90 to 0.94	
Sand	0.75	
Sandstone, red	0.59	
Silver, polished	0.02	
Snow, fresh	0.82	0.13
Soil	0.94	
Water	0.90	0.98
White potassium zirconium silicate	0.87	0.13

Source: Mills (1999)

*Values are for extraterrestrial conditions, except for concrete, snow, and water.

Simplifying long-wave radiation

- We will sometimes simplify the equation for radiation heat transfer
  - Particularly when emissivities & areas are equal
- From:

$$q_{r,1\to 2}=\frac{Q_{1\to 2}}{A_1}=\frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1}+\frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}+\frac{1}{F_{12}}}$$

- To:

$$q_{r,1\to 2}=\frac{Q_{1\to 2}}{A_1}=\varepsilon_1\varepsilon_2\sigma F_{12}\left(T_1^4-T_2^4\right)$$

So you can see that “low-e” materials, with low values of ε, reduce heat transfer by radiation

Notes on radiation heat transfer

- Radiation heat transfer depends upon the view factor, which is a function of geometry
  - Also depends on both absolute T and differential T
- The view factor from one side of a wall cavity to the other does not change much at all as the cavity gets wider
  - This means that radiation heat transfer does not change much as air cavity spacing is increased
- Radiation heat transfer depends on T^4, which complicates things mathematically
- Obvious importance of low-emissivity and low-absorptivity materials

Multi-surface radiative transfer

- If we have more than two dominant surfaces we have a fairly complicated system
  - Best solved using computers rather than by hand

- To do these by hand we would need to develop full thermal networks for multi-surface radiation
  - We are not going to do that today

Two additional forms of heat transfer

- Bulk convective flow

$$Q_{bulk} = \dot{m} C_p \Delta T \qquad [\mathrm{W}] = [\frac{\mathrm{kg}}{\mathrm{s}} \cdot \frac{\mathrm{J}}{\mathrm{kg} \cdot \mathrm{K}} \cdot \mathrm{K}]$$

m “dot” = mass flow rate of air (kg/s)
C_p = specific heat capacity of air [J/(kgK)]

- State change
  - When materials change state, they release or absorb a material-specific amount of latent energy
    - Usually concerned with water evaporation/condensation
  - The amount of heat energy required to change a material from one temperature to another is:

$$E = C_p m \Delta T$$

where C_p is the specific heat capacity [kJ/(kgK)]

m is the mass (kg), ΔT is the temperature difference (K)

Single-mode heat transfer examples

- Let’s perform some example calculations, first treating conduction, convection, and radiation individually

Example 3.1: Single-layer conduction

- A 2 m wide, 3 m high, and 50 mm thick piece of extruded polystyrene material has a surface temperature of 20°C on one side and 40°C on the other
  - a) Calculate heat flow rate (Q) and heat flux (q)
  - b) Calculate conductance (U-value)
  - c) Calculate resistance (and R-value)

ASHRAE HOF (2005 Ch. 25):

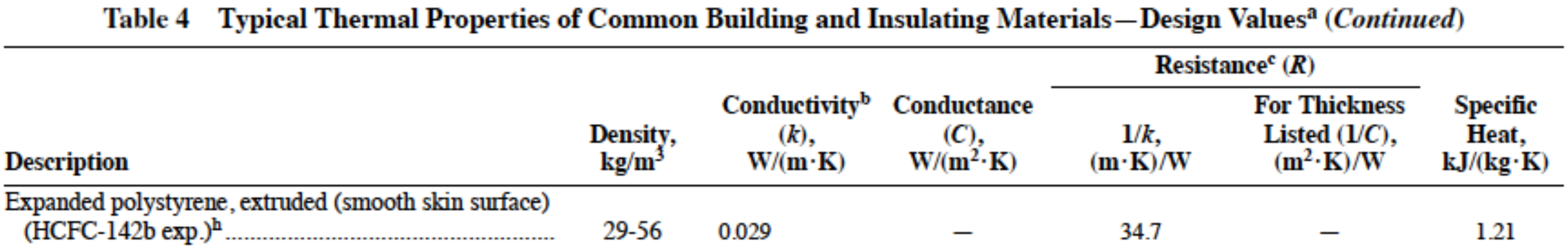


Table 4 Typical Thermal Properties of Common Building and Insulating Materials—Design Values[a] (*Continued*)

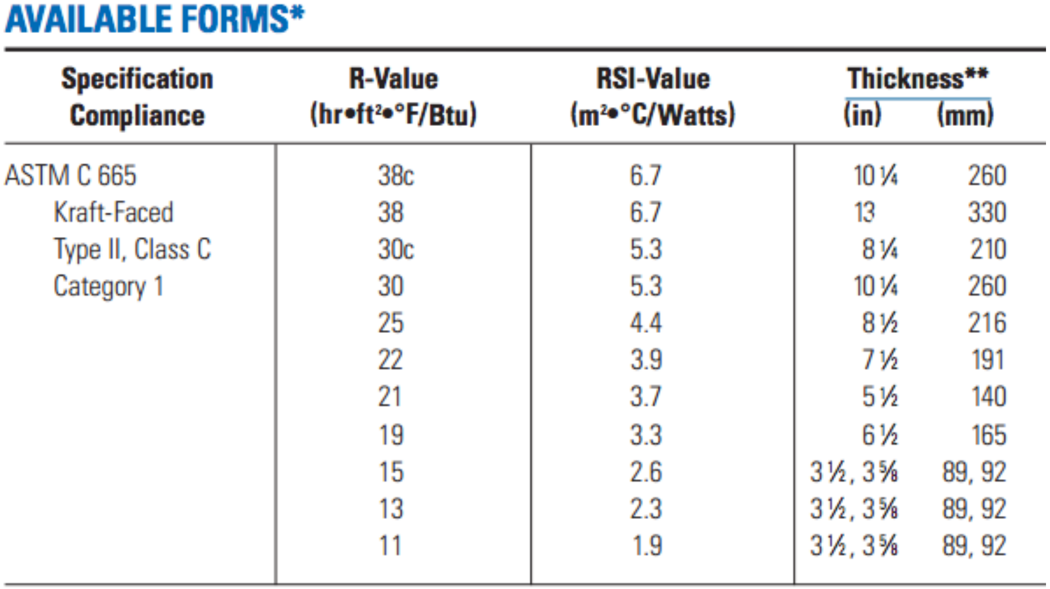
Description	Density, kg/m^3	Conductivity[b] (*k*), W/(m·K)	Conductance (*C*), $\text{W/(m}^2\cdot\text{K)}$	Resistance[c] (*R*) 1/*k*, (m·K)/W	Resistance[c] (*R*) For Thickness Listed (1/*C*), $(\text{m}^2\cdot\text{K})/\text{W}$	Specific Heat, kJ/(kg·K)
Expanded polystyrene, extruded (smooth skin surface) (HCFC-142b exp.)[h]	29-56	0.029	—	34.7	—	1.21

A note on insulation materials

- All materials in an enclosure assembly will have some resistance to heat transfer
- Materials with thermal conductivities (k) less than about 0.05 W/(mK) are used specifically for insulation
  - 0.05 W/(mK) divided by 3-inches of typical thickness (0.076 m) yields U-value of ~0.66 W/(m^2K)
  - R = 1/U = 1/0.66 = ~1.5 (m^2K)/W RSI (or ~R-9 in English units)

Example from product literature

Johns Manville



AVAILABLE FORMS*

Specification Compliance	R-Value (hr•ft²•°F/Btu)	RSI-Value (m²•°C/Watts)	Thickness**	
			(in)	(mm)
ASTM C 665 Kraft-Faced Type II, Class C Category 1	38c	6.7	10 ¼	260
	38	6.7	13	330
	30c	5.3	8 ¼	210
	30	5.3	10 ¼	260
	25	4.4	8 ½	216
	22	3.9	7 ½	191
	21	3.7	5 ½	140
	19	3.3	6 ½	165
	15	2.6	3 ½, 3 ⅝	89, 92
	13	2.3	3 ½, 3 ⅝	89, 92
	11	1.9	3 ½, 3 ⅝	89, 92

Another note on insulation materials

- **Still air** is also a low-cost insulator
  - – Density ~1.2 kg/m^3
  - – Conductivity, k ~ 0.03 W/(mK)
  - – So many insulation materials rely on creating air voids

- Example: fiberglass insulation
  - – Glass, with a density of 2500 kg/m^3 and k = 1 W/(mK), is spun into fibers and made into a fiberglass insulation batt, which is ~99.4% air voids (~0.6% glass fibers) by volume
    - Yields a product with a density of 16 kg/m^3 and thermal conductivity of 0.043 W/(mK)
    - Both values are very close to that of **still** air

Example 3.2: Convection

- The interior face of an insulated exterior enclosure wall 2.4 m wide and 2.4 m high is 3°C cooler than the interior air (T_{indoor} = 21°C)
  a) Calculate convective heat transfer coefficient at the face
  b) Calculate rate of convective heat transfer

Example 3.3: Bulk convection

- A building has an outdoor air delivery rate of 400 m^3 per hour. The outdoor temperature is 35°C. The indoor air temperature is 20°C.
  - a) Calculate the rate at which heat is added to the indoor air from outdoors

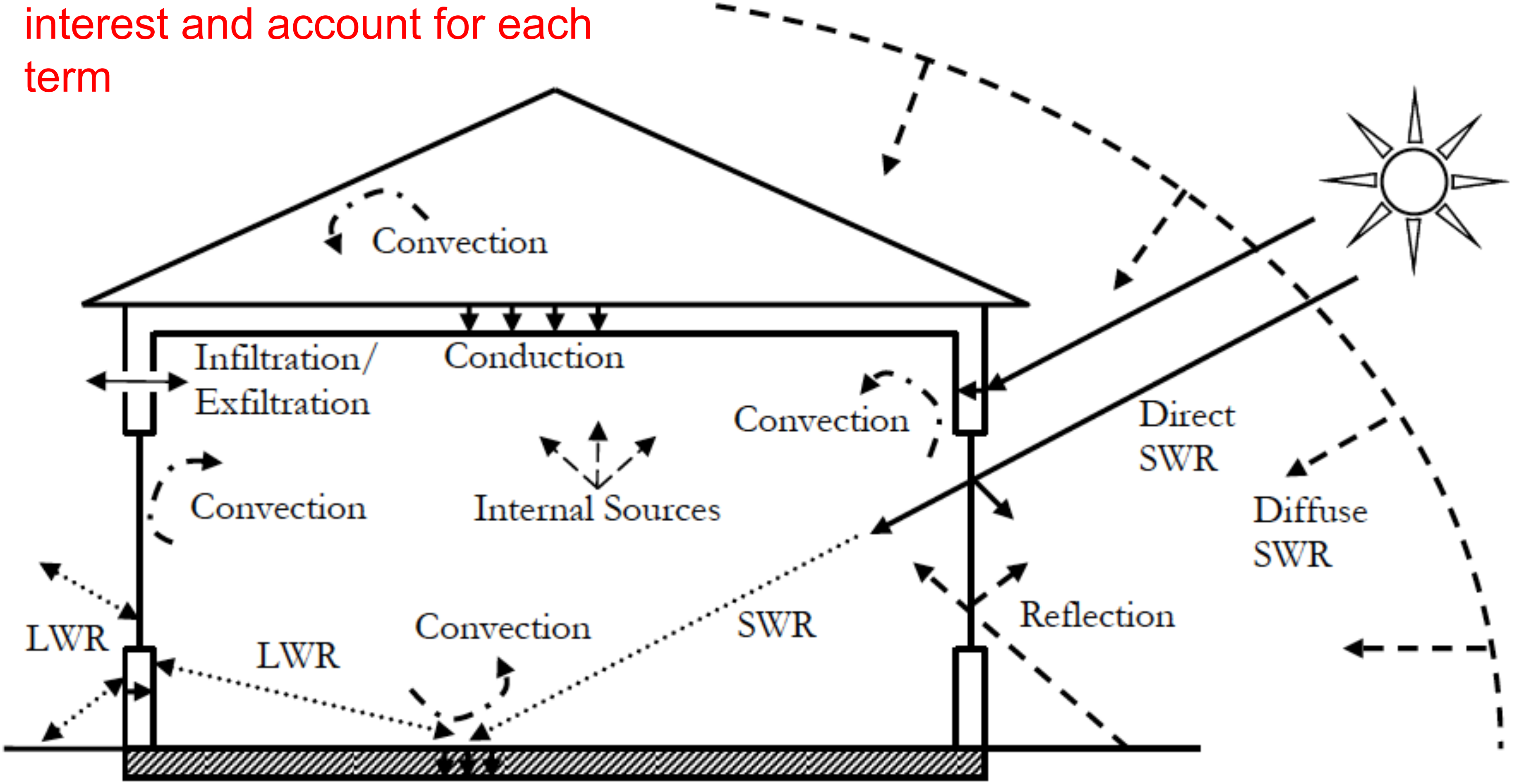
Example 3.4: Radiation

- Interior surfaces of two perpendicular walls (both are 2.4 m by 2.4 m) are 3°C different from each other. One is at 294 K, the other at 291 K. They both have an emissivity of 0.90.
  - a) Calculate the rate of long-wave radiative heat transfer between the two surfaces
  - b) What if the emissivity of one surface decreases to 0.1?

2. COMBINING HEAT TRANSFER MODES IN BUILDING ENCLOSURES

Bringing all the modes together

Write an energy balance at the building surface or element of interest and account for each term



Combined heat transfer

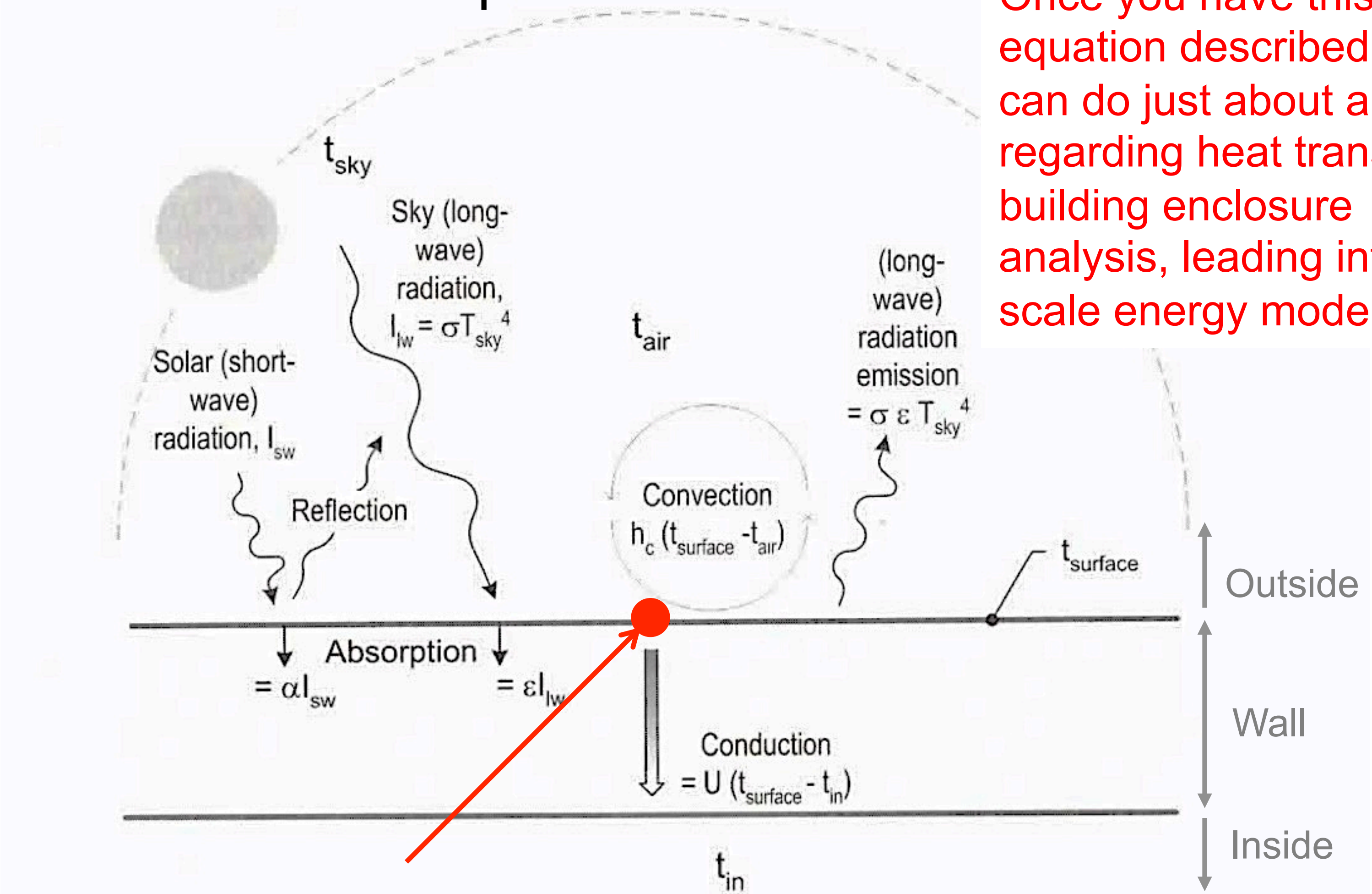
- In some cases, heat transfer from a surface is dominated by either convection or radiation
  - In many cases both are about the same magnitude

- In cavities (window spaces, wall cavities, crawl spaces) this is usually the case
  - So, heat transfer is fairly complicated

Bringing all the modes together

- Exterior surface example

Once you have this equation described, you can do just about anything regarding heat transfer in building enclosure analysis, leading into full-scale energy modeling



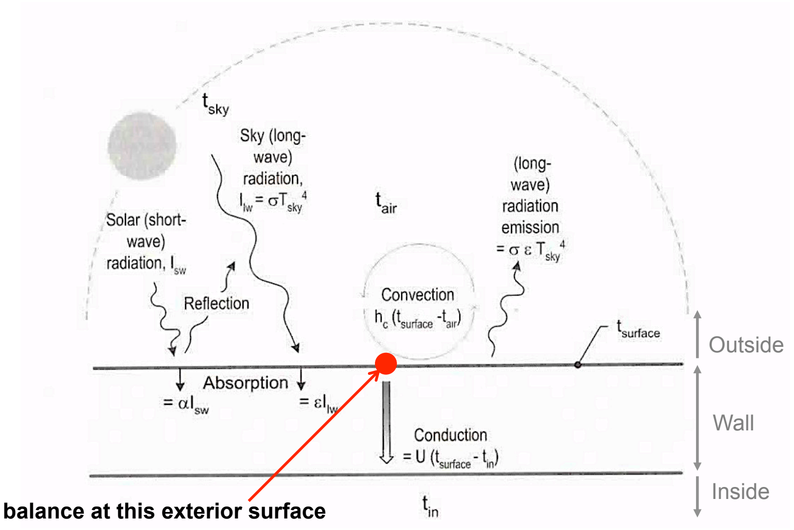
Steady-state energy balance at this exterior surface: What enters must also leave (no storage)

$$\boxed{\alpha I_{total} + F_{surface-sky}\varepsilon_{surface}\varepsilon_{sky}\sigma(T_{sky}^4 - T_{surface}^4) + F_{surface-ground}\varepsilon_{surface}\varepsilon_{ground}\sigma(T_{ground}^4 - T_{surface}^4) + h_{conv}(T_{air,out} - T_{surface}) = U(T_{surface} - T_{surface,in})}$$

Bringing all the modes together

- Exterior surface example

$$
\begin{aligned}
&q_{sw,\, incident\ solar} \\
&+q_{lw,\, net\ to\ sky} \\
&+q_{lw,\, net\ to\ ground} \\
&+q_{convective,\, net\ to\ outdoor\ air} \\
&= q_{conductive,\, to\ inside}
\end{aligned}
$$



Energy balance at this exterior surface

$$\boxed{\alpha I_{total} + F_{surface-sky}\varepsilon_{surface}\varepsilon_{sky}\sigma(T_{sky}^4 - T_{surface}^4) + F_{surface-ground}\varepsilon_{surface}\varepsilon_{ground}\sigma(T_{ground}^4 - T_{surface}^4) + h_{conv}(T_{air,out} - T_{surface}) = U(T_{surface} - T_{surface,in})}$$

- Energy balance on exterior surface

Solar gain	$+\alpha I_{total}$
Surface-sky radiation	$+F_{surface-sky}\varepsilon_{surface}\varepsilon_{sky}\sigma(T_{sky}^4 - T_{surface}^4)$
Surface-ground radiation	$+F_{surface-ground}\varepsilon_{surface}\varepsilon_{ground}\sigma(T_{ground}^4 - T_{surface}^4)$
Convection on external wall	$+h_{conv}(T_{air,out} - T_{surface})$
Conduction through wall	$= U(T_{surface} - T_{surface,in})$

$$\boxed{\sum q = 0}$$

A note on sign conventions

- Move from left to right (or top to bottom)
- Assume that the temperature to the left (or upstream) is higher than the temperature to the right (or downstream)
  - The signs will work themselves out and let you know if that is not the case
  - Be consistent!

(A note on solar absorptance of materials, α)

- Generally similar to values of emissivity, ε
  - But not always
  - Particular differences between long wave and short wave ε

*Note that ε_{lw} for sky typically taken as 1 or adjusted slightly for cloud cover

Source: McQuiston, Parker, and Spitler, *HVAC Analysis and Design*

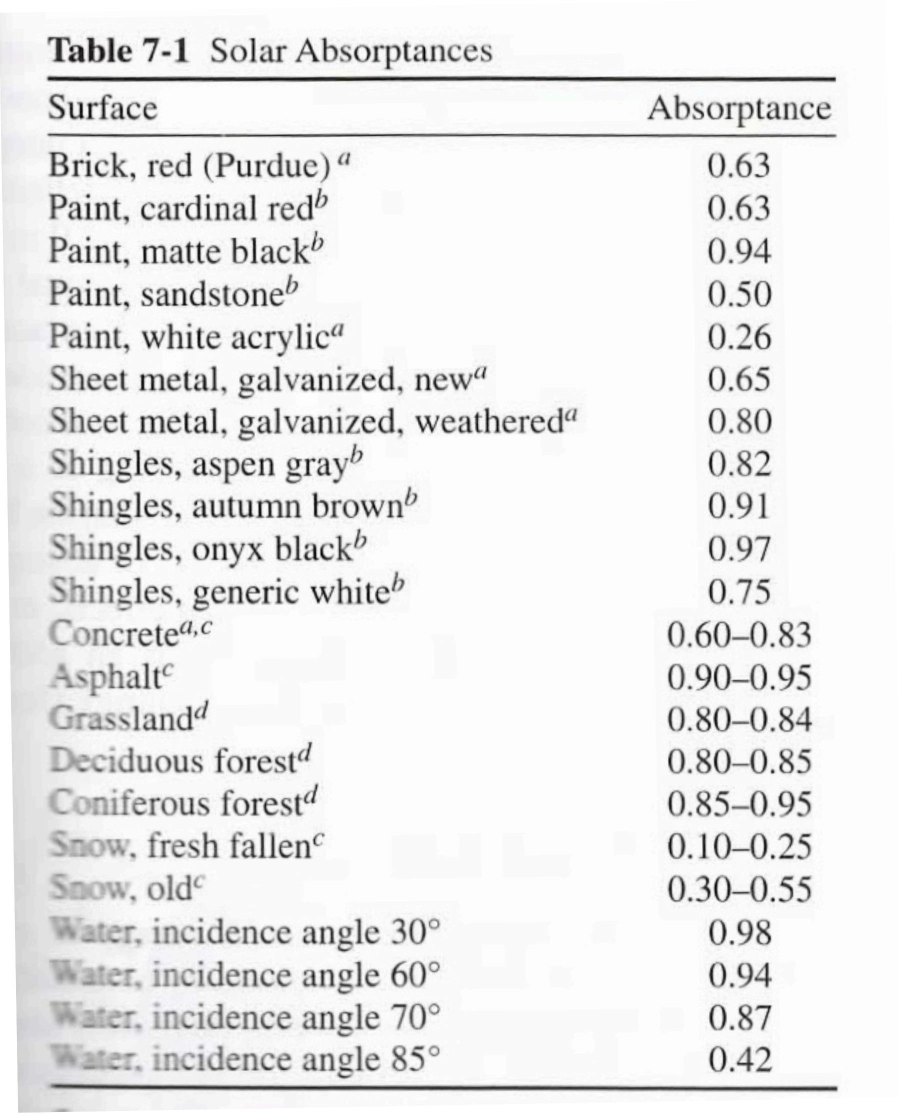


Table 7-1 Solar Absorptances

Surface	Absorptance
Brick, red (Purdue)[a]	0.63
Paint, cardinal red[b]	0.63
Paint, matte black[b]	0.94
Paint, sandstone[b]	0.50
Paint, white acrylic[a]	0.26
Sheet metal, galvanized, new[a]	0.65
Sheet metal, galvanized, weathered[a]	0.80
Shingles, aspen gray[b]	0.82
Shingles, autumn brown[b]	0.91
Shingles, onyx black[b]	0.97
Shingles, generic white[b]	0.75
Concrete[a,c]	0.60–0.83
Asphalt[c]	0.90–0.95
Grassland[d]	0.80–0.84
Deciduous forest[d]	0.80–0.85
Coniferous forest[d]	0.85–0.95
Snow, fresh fallen[c]	0.10–0.25
Snow, old[c]	0.30–0.55
Water, incidence angle 30°	0.98
Water, incidence angle 60°	0.94
Water, incidence angle 70°	0.87
Water, incidence angle 85°	0.42

(A note on sky and ground temperatures, T_{sky} and T_{ground})

- Many ways to get sky temperature
  - Many models with varying levels of detail and accuracy
- For a partly cloudy night sky

$$T_{sky} = T_{air}\left[0.8 + \frac{(T_{dewpoint} - 273)}{250}\right]^{1/4}$$

- We also usually assume ground temperature is equal to air temperature

$$T_{ground} = T_{air}$$

Another way to calculate T_{sky}

- During day time

$$T_{sky} = \left(0.787 + 0.764 F_{cloud} \ln\left(\frac{T_{dew}}{T_{ice}}\right)\right)^{1/4} T_{air} \text{ all } T \text{ in K or R}$$

T_{dew} = Dew Point Temp of Air, T_{ice} = Freezing temp of $H_2 0$

$F_{cloud} = 1 + 0.024N - 0.035N^2 + 0.00028N^3$ where

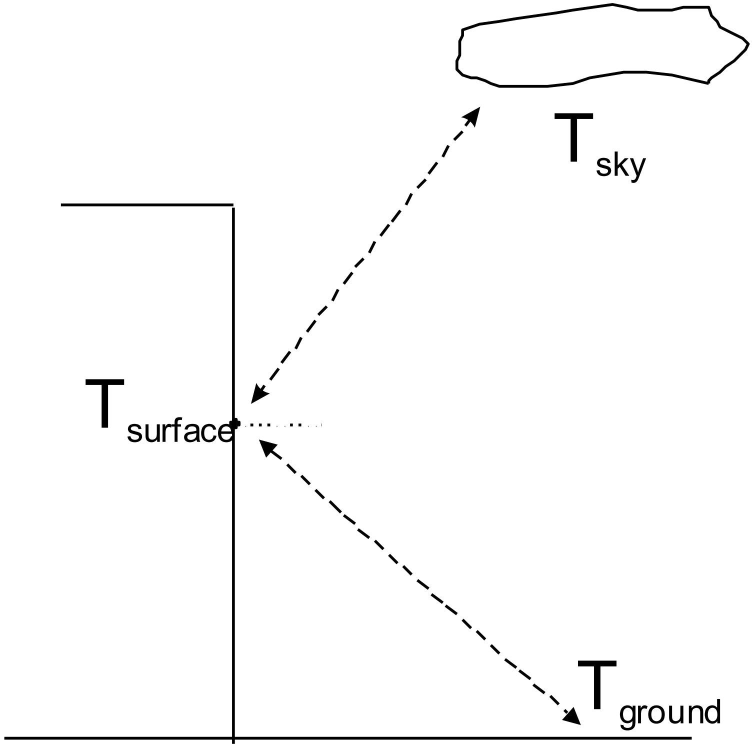
N = tenths cloud cover $0 < N < 1$

***Fractional cloud cover, N, is sometimes provided by weather data**

Source: Walton, G.N., *Thermal Analysis Research Program- Reference Manual*, NBSIR **83-2655**, U.S. Department of Commerce, March 1983, Update 1985.

(A note on typical view factors, F_{1-2})

- Some typical view factors from surfaces to ground or sky



View ("shape") factors for:

Vertical surfaces:

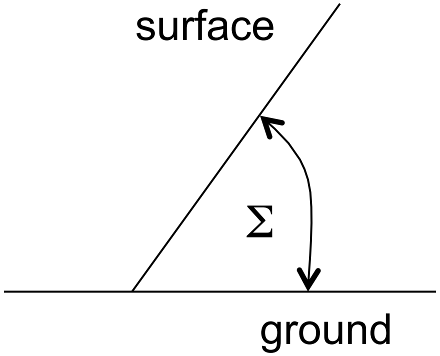
- To sky ($F_{surface\text{-}sky}$) 0.5
- To ground ($F_{surface\text{-}ground}$) 0.5

Horizontal surfaces:

- To sky ($F_{surface\text{-}sky}$) 1
- To ground ($F_{surface\text{-}ground}$) 0

3) Tilted surfaces

- To sky $(1+\cos\Sigma)/2$
- To ground $(1-\cos\Sigma)/2$



*Note that other surrounding buildings complicate view factors, but their net temperature differences probably aren't that different so long-wave radiation can be negligible

Bringing all the modes together

- Back to the exterior surface example

We can use this equation to estimate surface temperatures indoors and/or out

- At steady state, net energy balance is zero
- Because of T^4 term, often requires iteration

- Energy balance on exterior surface

Solar gain	$+\alpha I_{total}$
Surface-sky radiation	$+F_{surface-sky}\varepsilon_{surface}\varepsilon_{sky}\sigma(T_{sky}^4 - T_{surface}^4)$
Surface-ground radiation	$+F_{surface-ground}\varepsilon_{surface}\varepsilon_{ground}\sigma(T_{ground}^4 - T_{surface}^4)$
Convection on external wall	$+h_{conv}(T_{air,out} - T_{surface})$
Conduction through wall	$= U(T_{surface} - T_{surface,in})$

Bringing all modes (and **nodes**) together

- So far we’ve only looked at the energy balance at a single external surface
  - Steady state
  - Ignoring indoor impacts and impacts of heat storage
- In reality, we are usually dealing with unsteady conduction through envelope materials
  - And our goal is also typically to find impacts on indoor air temperatures so we can determine the actual impact of enclosures on heating or cooling loads
- To move forward, we can use a system of equations based on nodes within building elements
  - I will introduce this today but we will not focus on it
  - Reserved for a future course on building energy modeling

Bringing all modes (and **nodes**) together

- For an example room like this, you would setup a system of equations where the temperature at each node (either a surface or within a material) is unknown
  - 12 material nodes + 1 indoor air node

At surface nodes:

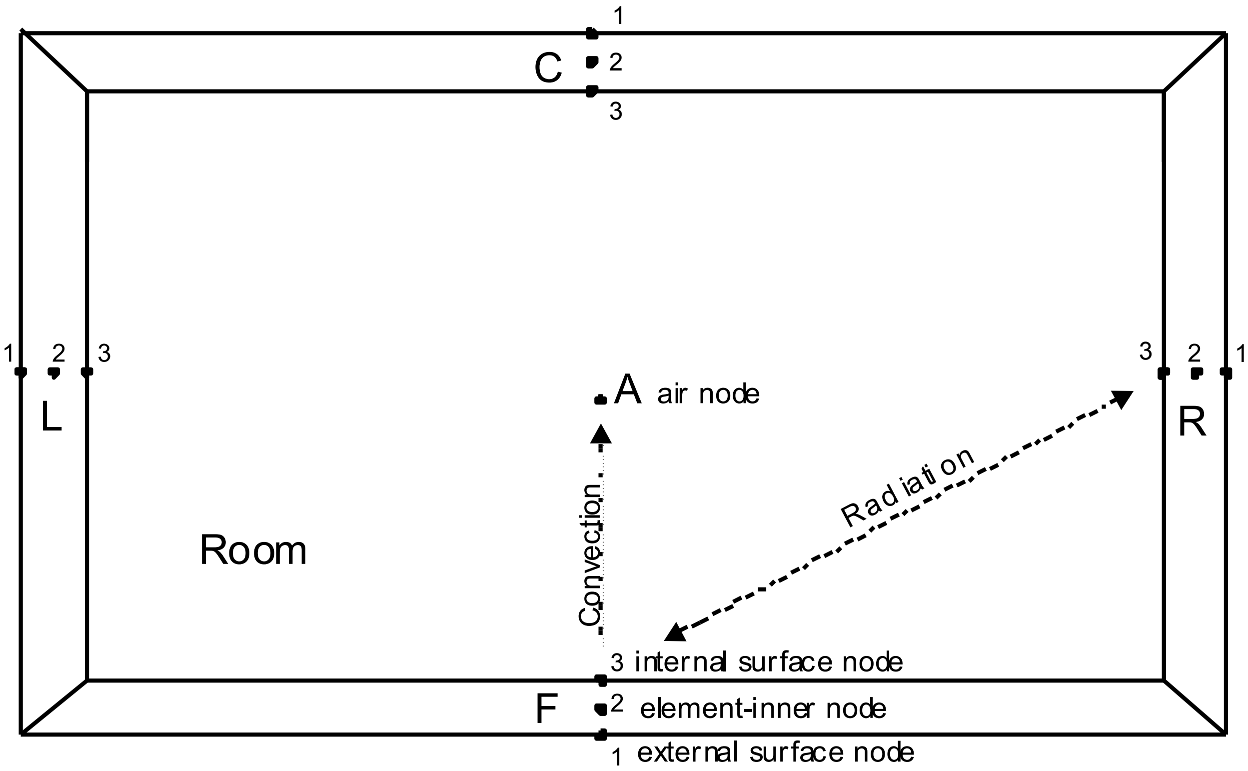
$$\sum q = 0$$

At nodes inside materials:

$$mc_p \frac{\partial T}{\partial \tau} = \sum q_{at\ boundaries}$$

Based on density and heat capacity of material… "Thermal mass"

Heat Xfer @ external surfaces:
Radiation and convection

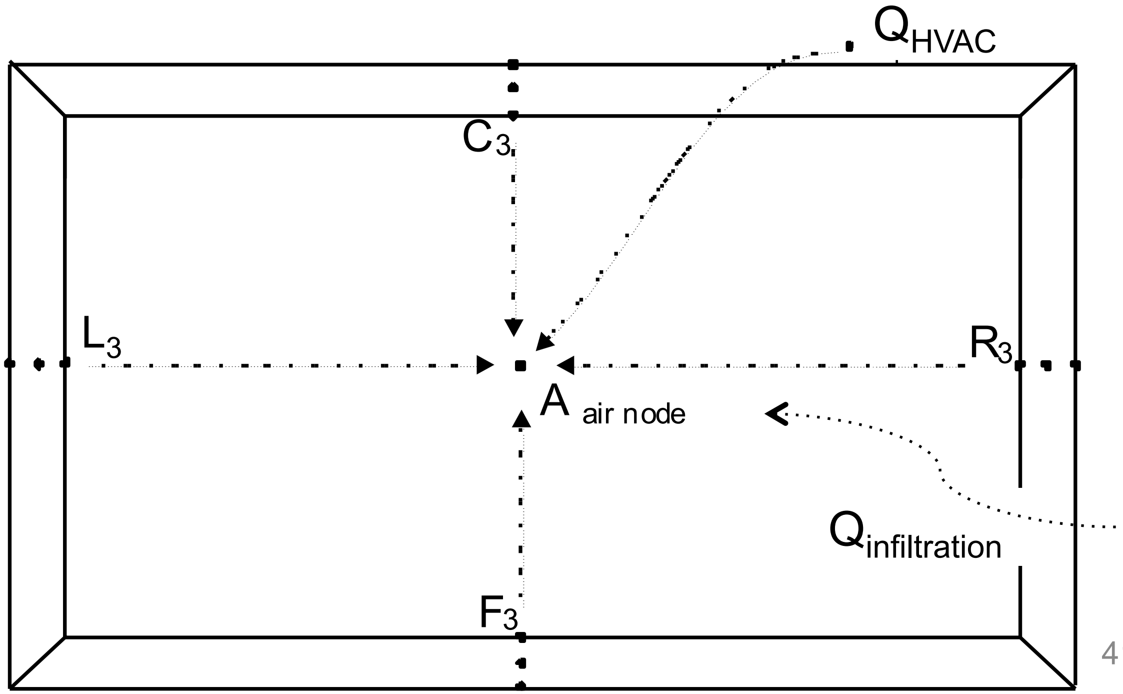


Bringing all modes (and nodes) together

- To get the impact on indoor air temperature (and close the system of equations)
  - Write an energy balance on the indoor air node
  - Air impacted directly only by convection (bulk and/or surface)

$$(V_{room}\rho_{air}c_{p,air})\frac{\partial T_{air}}{\partial \tau} = \sum_{i=1}^{n} h_i A_i \left(T_{i,surf} - T_{air,in}\right) + \dot{m}c_p\left(T_{out} - T_{air,in}\right) + Q_{HVAC}$$

In plain English:
The change in indoor air temperature is equal to the sum of convection from each interior surface plus outdoor air delivery (by infiltration or dedicated outdoor air supply), plus the bulk convective heat transfer delivered by the HVAC system



Bringing all modes (and **nodes**) together

- For now, and most of this class, we will stick to the previous steady-state heat transfer assumptions, where the energy balance on an exterior surface is:

Solar gain	$+\alpha I_{total}$
Surface-sky radiation	$+F_{surface-sky}\varepsilon_{surface}\varepsilon_{sky}\sigma(T_{sky}^4 - T_{surface}^4)$
Surface-ground radiation	$+F_{surface-ground}\varepsilon_{surface}\varepsilon_{ground}\sigma(T_{ground}^4 - T_{surface}^4)$
Convection on external wall	$+h_{conv}(T_{air,out} - T_{surface})$
Conduction through wall	$= U(T_{surface} - T_{surface,in})$

We will now spend some time working on this term

3. CONDUCTION THROUGH ENVELOPE ASSEMBLIES

Conduction through envelope assemblies

- Building enclosures typically include several layers
  1. Sometimes those layers are made of uniform materials
     - Relatively easy to deal with
  2. Often those layers are divided into different materials
     - With different widths, conductivities, and resistances
     - More complicated to deal with
  3. Often involves more transient (time-varying) heat flow
     - Even more complicated to deal with!
  4. Often those layers involve more than 1-D heat flow
     - ***Even more*** complicated to deal with!!

These increase in complexity

- We will learn about layers for #1, then complex layers for #2, then a heat storage term for #3, and finally using software for #4

Conduction through multiple layers

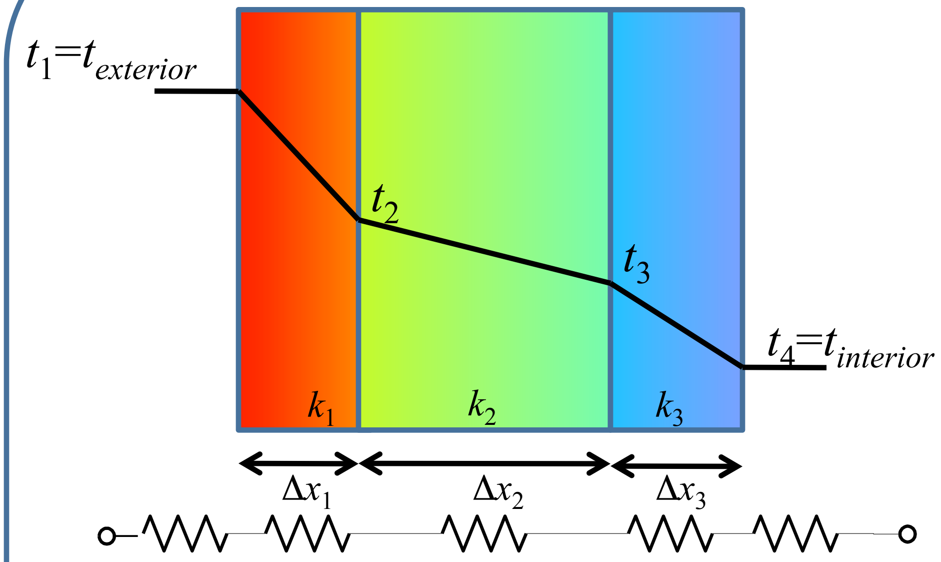
- Just as in electrical circuits, the overall thermal resistance of a series of elements (layers) can be expressed as the sum of the resistances of each layer
  - Do not forget the interior and exterior convective resistances!

- By continuity of energy we can write:

$$q = \frac{t_1 - t_2}{R_1} = \frac{t_2 - t_3}{R_2} = \frac{t_3 - t_4}{R_3}$$

so

$$q = \frac{t_1 - t_4}{R_{total}} \quad \text{where} \quad R_{total} = R_o + R_1 + R_2 + R_3 + R_i$$



$$R_{exterior} = 0.029 \quad R_1 = \frac{\Delta x_1}{k_1} \quad R_2 = \frac{\Delta x_2}{k_2} \quad R_3 = \frac{\Delta x_3}{k_3} \quad R_{interior} = 0.12$$

Typical "film" values in $[(m^2K)/W]$

Can only add resistances (R), not conductances (C or U)

Simple conduction through multiple layers

- Calculate the R-value of an enclosure assembly

Steps:

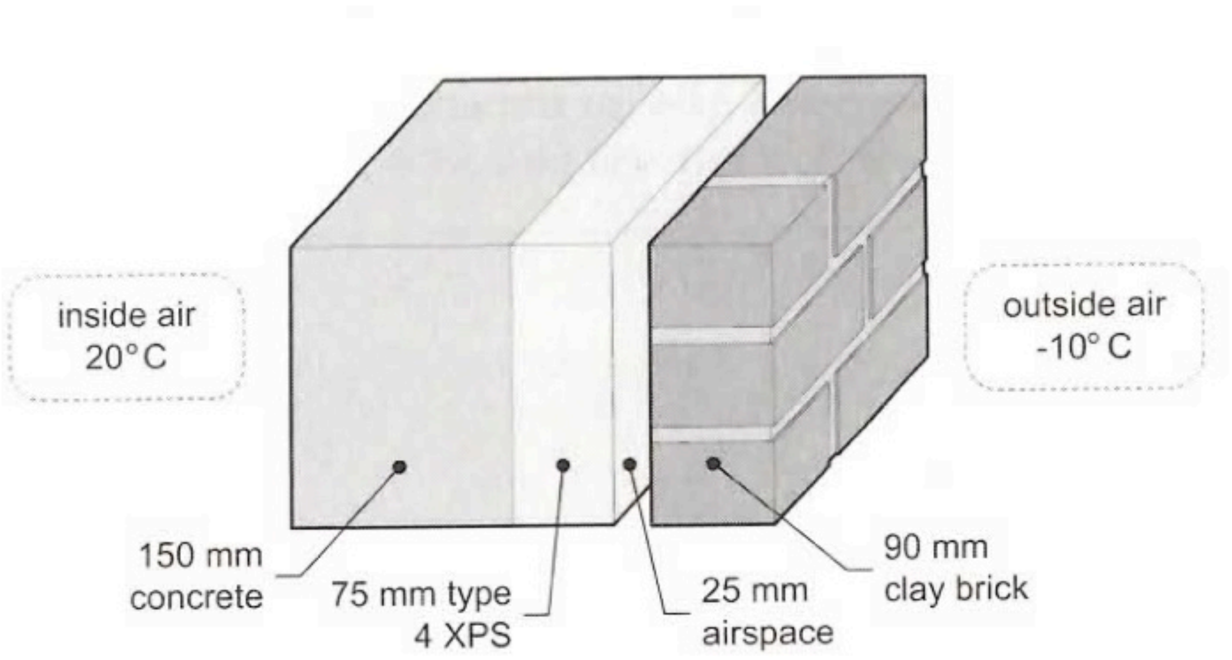
1. List each material in the assembly
    - And its conductivity and thickness
2. Calculate conductance of each layer
    - C (or U) = k/L
3. Calculate thermal resistance of each layer
    - R = 1/C
4. Sum the individual thermal resistances to get R_{total}

Conduction through multiple layers

- Example problem 3.5

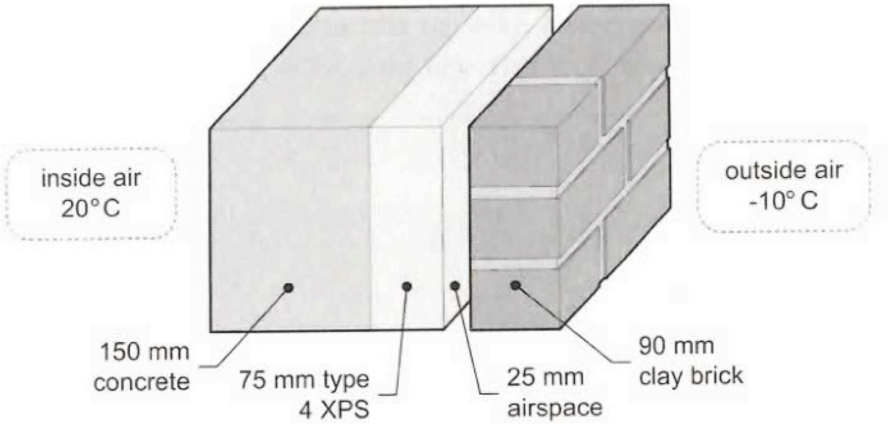
  A) Calculate the total thermal resistance, R_{total}, and overall heat transfer coefficient, U_{total}, of the wall shown below

  B) Calculate temperature at each material surface



Conduction through multiple layers

- Refer to ASHRAE 2005 HOF Ch. 25 for data



Layer material	Conductivity, k W/mK	Thickness m	Conductance W/m^{2}K	Resistance m^{2}K/W

A note on R-values of air cavities (air spaces)

- ASHRAE has measured the combined convective + radiative R-values for thin cavities of various orientations and depths with various effective emissivities, ε_{eff}
- These are the best data to use for air spaces in assemblies
  - If you do not know that the material in the cavity is reflective or “low-e”, just assume that both walls of the cavity have ε=0.9 for each surface, so that when combined, ε_{eff}=0.82

ASHRAE HOF 2005, Chapter 25 (small cavities)

R-values for different air gap characteristics

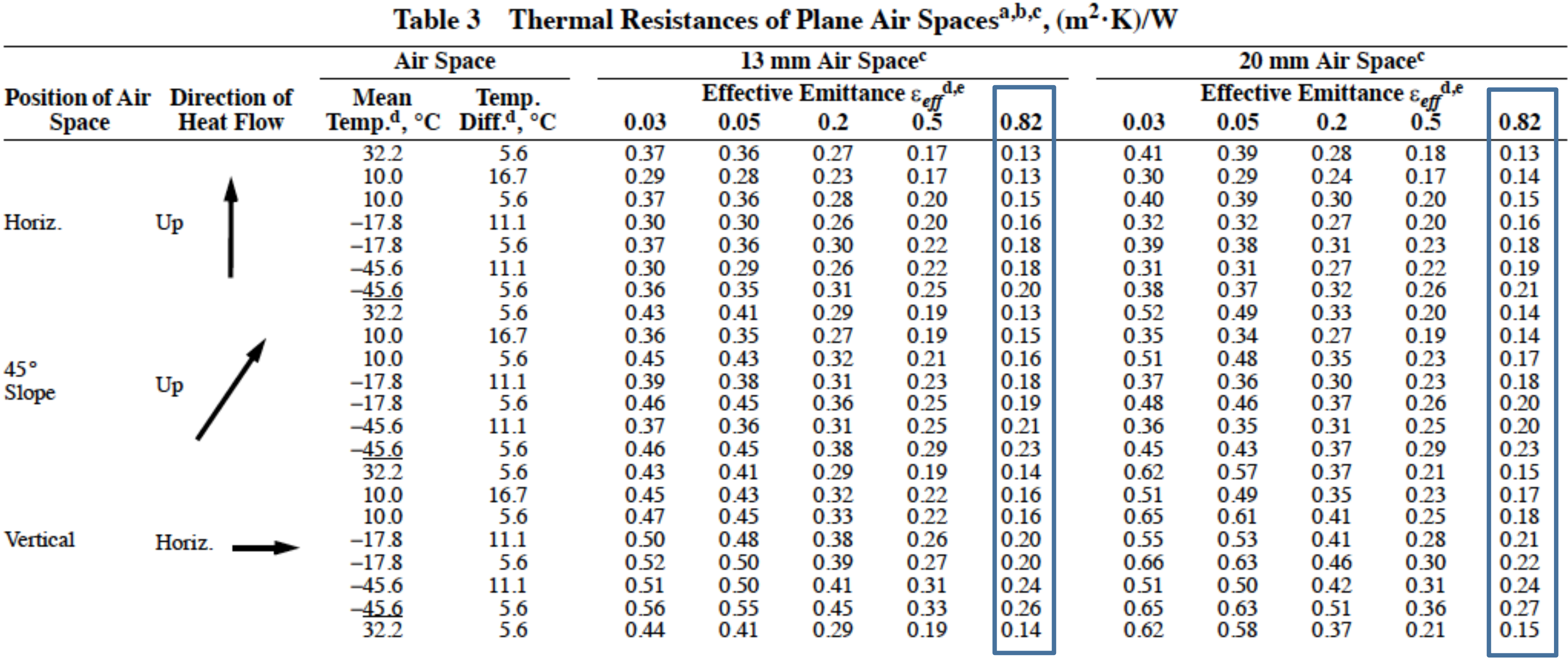


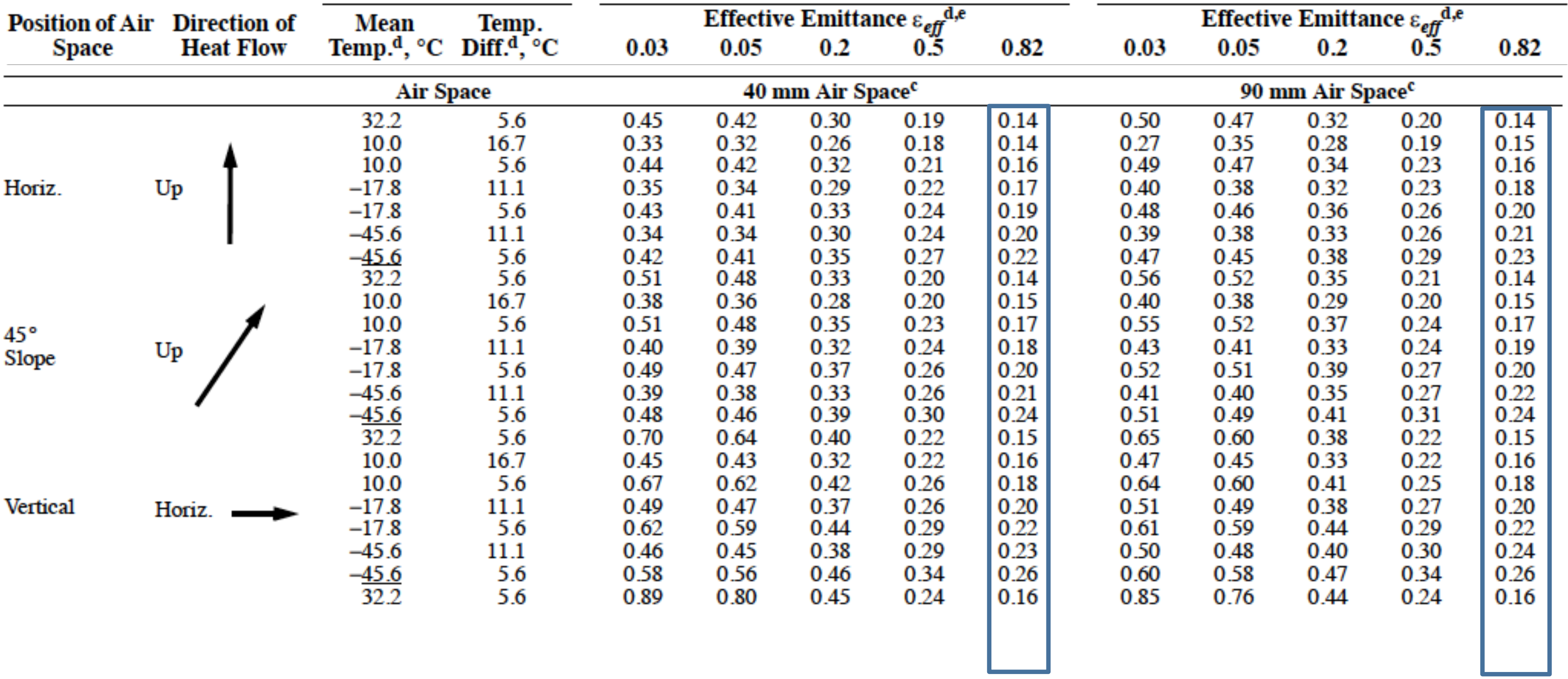
Table 3 Thermal Resistances of Plane Air Spaces[a,b,c], $(m^2 \cdot K)/W$

Position of Air Space	Direction of Heat Flow	Air Space		13 mm Air Space[c]					20 mm Air Space[c]				
		Mean Temp.[d], °C	Temp. Diff.[d], °C	Effective Emittance ε_{eff}[d,e]					Effective Emittance ε_{eff}[d,e]				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
Horiz.	Up ↑	32.2	5.6	0.37	0.36	0.27	0.17	0.13	0.41	0.39	0.28	0.18	0.13
		10.0	16.7	0.29	0.28	0.23	0.17	0.13	0.30	0.29	0.24	0.17	0.14
		10.0	5.6	0.37	0.36	0.28	0.20	0.15	0.40	0.39	0.30	0.20	0.15
		−17.8	11.1	0.30	0.30	0.26	0.20	0.16	0.32	0.32	0.27	0.20	0.16
		−17.8	5.6	0.37	0.36	0.30	0.22	0.18	0.39	0.38	0.31	0.23	0.18
		−45.6	11.1	0.30	0.29	0.26	0.22	0.18	0.31	0.31	0.27	0.22	0.19
		−45.6	5.6	0.36	0.35	0.31	0.25	0.20	0.38	0.37	0.32	0.26	0.21
45° Slope	Up ↗	32.2	5.6	0.43	0.41	0.29	0.19	0.13	0.52	0.49	0.33	0.20	0.14
		10.0	16.7	0.36	0.35	0.27	0.19	0.15	0.35	0.34	0.27	0.19	0.14
		10.0	5.6	0.45	0.43	0.32	0.21	0.16	0.51	0.48	0.35	0.23	0.17
		−17.8	11.1	0.39	0.38	0.31	0.23	0.18	0.37	0.36	0.30	0.23	0.18
		−17.8	5.6	0.46	0.45	0.36	0.25	0.19	0.48	0.46	0.37	0.26	0.20
		−45.6	11.1	0.37	0.36	0.31	0.25	0.21	0.36	0.35	0.31	0.25	0.20
		−45.6	5.6	0.46	0.45	0.38	0.29	0.23	0.45	0.43	0.37	0.29	0.23
Vertical	Horiz. →	32.2	5.6	0.43	0.41	0.29	0.19	0.14	0.62	0.57	0.37	0.21	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.51	0.49	0.35	0.23	0.17
		10.0	5.6	0.47	0.45	0.33	0.22	0.16	0.65	0.61	0.41	0.25	0.18
		−17.8	11.1	0.50	0.48	0.38	0.26	0.20	0.55	0.53	0.41	0.28	0.21
		−17.8	5.6	0.52	0.50	0.39	0.27	0.20	0.66	0.63	0.46	0.30	0.22
		−45.6	11.1	0.51	0.50	0.41	0.31	0.24	0.51	0.50	0.42	0.31	0.24
		−45.6	5.6	0.56	0.55	0.45	0.33	0.26	0.65	0.63	0.51	0.36	0.27
		32.2	5.6	0.44	0.41	0.29	0.19	0.14	0.62	0.58	0.37	0.21	0.15

Usually we use values from the $\varepsilon_{eff} = 0.82$ column

ASHRAE HOF 2005, Chapter 25 (larger cavities)

R-values for different air gap characteristics



Position of Air Space	Direction of Heat Flow	Mean Temp.^{d}, °C	Temp. Diff.^{d}, °C	Effective Emittance $\varepsilon_{eff}^{d,e}$					Effective Emittance $\varepsilon_{eff}^{d,e}$				
				0.03	0.05	0.2	0.5	0.82	0.03	0.05	0.2	0.5	0.82
		Air Space		40 mm Air Space^c					90 mm Air Space^c				
Horiz.	Up ↑	32.2	5.6	0.45	0.42	0.30	0.19	0.14	0.50	0.47	0.32	0.20	0.14
		10.0	16.7	0.33	0.32	0.26	0.18	0.14	0.27	0.35	0.28	0.19	0.15
		10.0	5.6	0.44	0.42	0.32	0.21	0.16	0.49	0.47	0.34	0.23	0.16
		–17.8	11.1	0.35	0.34	0.29	0.22	0.17	0.40	0.38	0.32	0.23	0.18
		–17.8	5.6	0.43	0.41	0.33	0.24	0.19	0.48	0.46	0.36	0.26	0.20
		–45.6	11.1	0.34	0.34	0.30	0.24	0.20	0.39	0.38	0.33	0.26	0.21
		–45.6	5.6	0.42	0.41	0.35	0.27	0.22	0.47	0.45	0.38	0.29	0.23
45° Slope	Up ↗	32.2	5.6	0.51	0.48	0.33	0.20	0.14	0.56	0.52	0.35	0.21	0.14
		10.0	16.7	0.38	0.36	0.28	0.20	0.15	0.40	0.38	0.29	0.20	0.15
		10.0	5.6	0.51	0.48	0.35	0.23	0.17	0.55	0.52	0.37	0.24	0.17
		–17.8	11.1	0.40	0.39	0.32	0.24	0.18	0.43	0.41	0.33	0.24	0.19
		–17.8	5.6	0.49	0.47	0.37	0.26	0.20	0.52	0.51	0.39	0.27	0.20
		–45.6	11.1	0.39	0.38	0.33	0.26	0.21	0.41	0.40	0.35	0.27	0.22
		–45.6	5.6	0.48	0.46	0.39	0.30	0.24	0.51	0.49	0.41	0.31	0.24
Vertical	Horiz. →	32.2	5.6	0.70	0.64	0.40	0.22	0.15	0.65	0.60	0.38	0.22	0.15
		10.0	16.7	0.45	0.43	0.32	0.22	0.16	0.47	0.45	0.33	0.22	0.16
		10.0	5.6	0.67	0.62	0.42	0.26	0.18	0.64	0.60	0.41	0.25	0.18
		–17.8	11.1	0.49	0.47	0.37	0.26	0.20	0.51	0.49	0.38	0.27	0.20
		–17.8	5.6	0.62	0.59	0.44	0.29	0.22	0.61	0.59	0.44	0.29	0.22
		–45.6	11.1	0.46	0.45	0.38	0.29	0.23	0.50	0.48	0.40	0.30	0.24
		–45.6	5.6	0.58	0.56	0.46	0.34	0.26	0.60	0.58	0.47	0.34	0.26
		32.2	5.6	0.89	0.80	0.45	0.24	0.16	0.85	0.76	0.44	0.24	0.16

Usually we use values from the $\varepsilon_{eff} = 0.82$ column

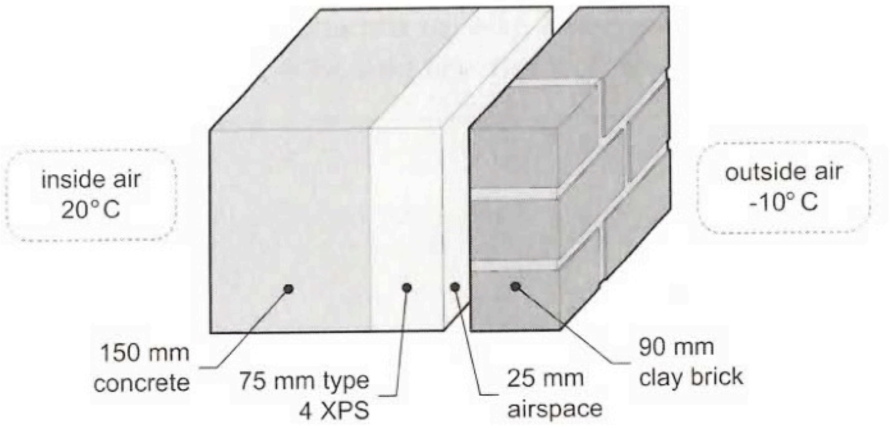
Note on R Value of deeper cavities

- The R-value of cavities stops increasing much at 3 inches (75 mm) depth
  - Beyond 3 inches (75 mm), they are dominated by radiation heat transfer, and convection starts to play a role
    - You double skin façade project groups will work with this!
  - For a deep cavity, either compute R with more advanced methods or use the 3 inch (75 mm) value

- Do **NOT** take the R value of a 1 inch (25 mm) cavity and multiply by the thickness of the cavity for thick cavities
  - If you did that, you would guess that an 8 foot attic would have an R value of about 100 BTU/(hr ft^2 F), which is a factor of 20 too high!

Conduction through multiple layers

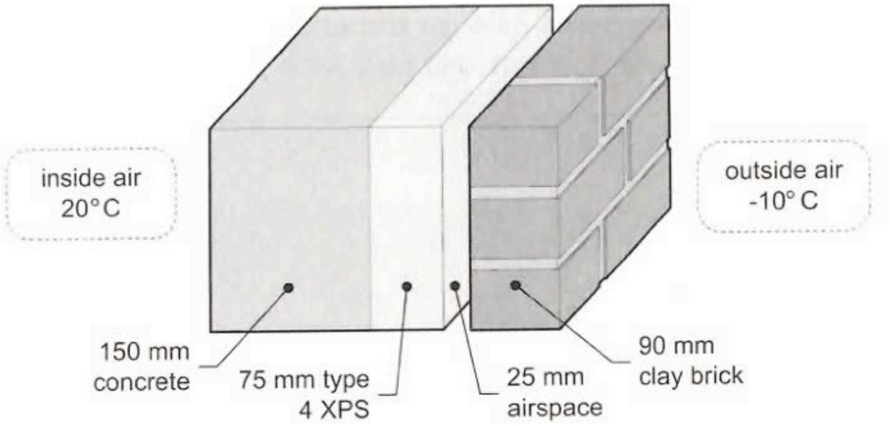
- Refer to ASHRAE 2005 HOF Ch. 25 for data



Layer material	**Conductivity, k** W/mK	**Thickness** m	**Conductance** W/m^{2}K	**Resistance** m^{2}K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17

Conduction through multiple layers

- Refer to ASHRAE 2005 HOF Ch. 25 for data

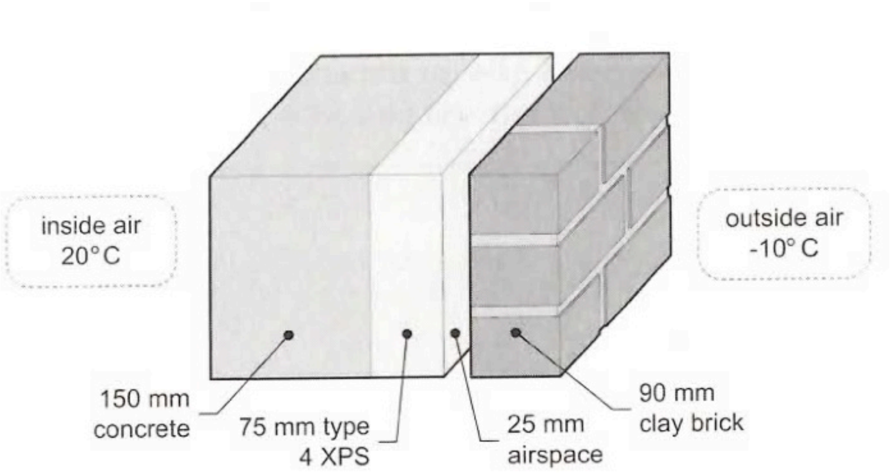


Layer material	Conductivity, k W/mK	Thickness m	Conductance W/m^2K	Resistance m^2K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			R_{total} (m^2K/W)	3.04
			U_{total} (W/m^2K) = 1/R	0.33

Conduction through multiple layers

- U_{total} = 0.33 W/m^{2}K

- Calculate steady-state heat flow through the enclosure



- q = UΔt
- q = (0.33 W/m^{2}K)*(t_{inside} - $t_{outside}$)
- q = (0.33 W/m^{2}K)*(20 – (-10)) = 10 W/m^2
  - From inside to outside

Conduction through multiple layers

- Calculating the temperature gradient through an enclosure of *i* materials

$$\Delta T_i = \frac{t_{internal} - t_{external}}{\sum_{i=0}^{n} R_i} R_i$$

Layer	Conductivity, k W/mK	Thickness m	Conductance W/m²K	Resistance m²K/W
Interior film	n/a	n/a	8.3	0.121
Concrete	1.8	0.15	12	0.083
Type 4 XPS	0.029	0.075	0.4	2.564
Air space	n/a	0.025	n/a	0.17
Brick	1.3	0.09	14.4	0.069
Exterior film	n/a	n/a	34	0.029
			R_{total} (m²K/W)	3.04
			U_{total} (W/m²K)	0.33

Conduction through multiple layers

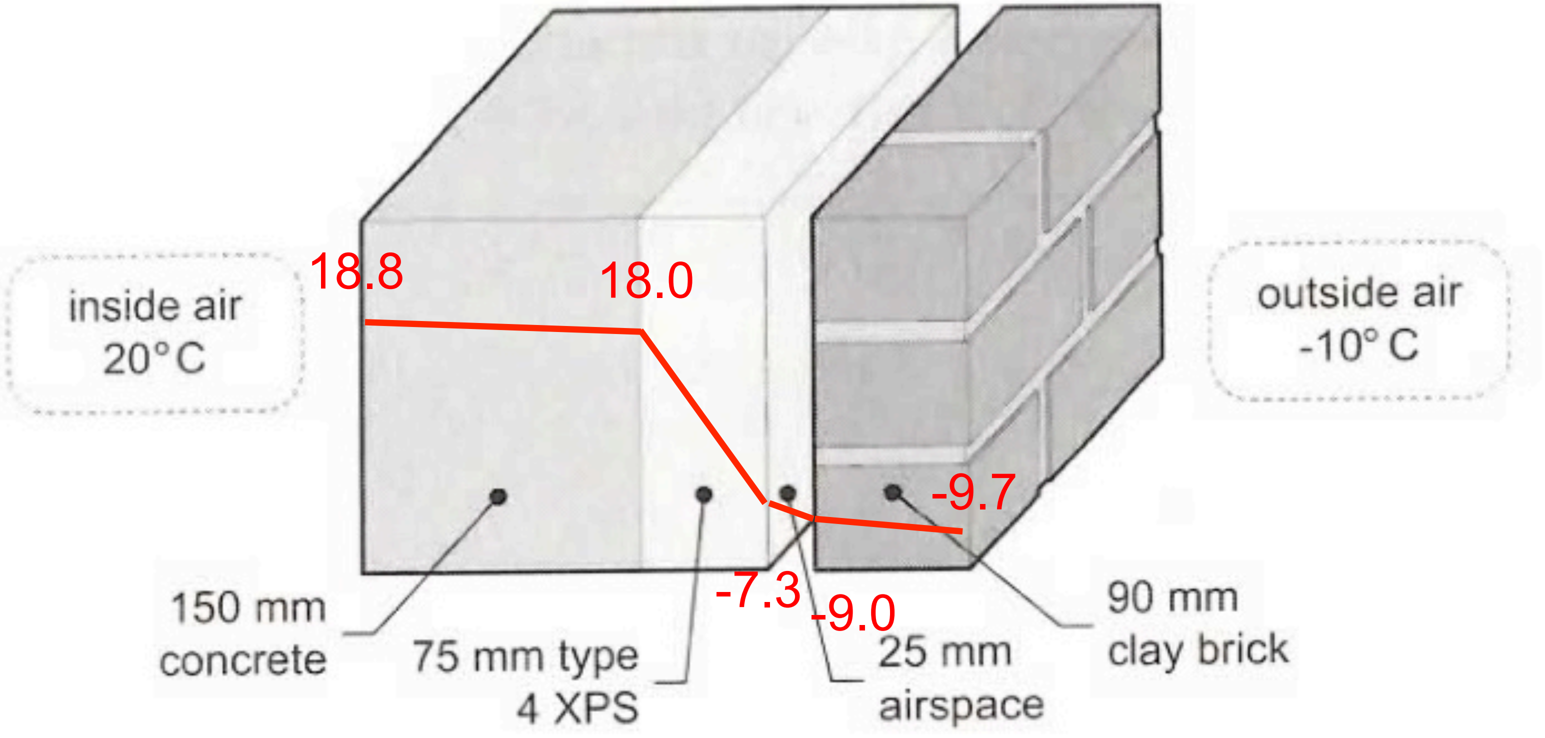
- Calculating the temperature gradient through an enclosure

$$\Delta T_i = \frac{t_{internal} - t_{external}}{\sum_{i=0}^{n} R_i} R_i$$

Layer	Conductivity, k W/mK	Thickness m	Conductance W/m²K	Resistance m²K/W	ΔT_i °C	t_i °C
Interior film	n/a	n/a	8.3	0.121		20
					1.2	
Concrete	1.8	0.15	12	0.083		18.8
					0.8	
Type 4 XPS	0.029	0.075	0.4	2.564		18.0
					25.3	
Air space	n/a	0.025	n/a	0.17		-7.3
					1.7	
Brick	1.3	0.09	14.4	0.069		-9.0
					0.7	
Exterior film	n/a	n/a	34	0.029		-9.7
					0.3	
			R_{total} (m²K/W)	3.04	Exterior t:	-10
			U_{total} (W/m²K)	0.33		

Conduction through multiple layers

- Calculating the temperature gradient through an enclosure



Total heat transfer through multiple layers

- Continue to use the electrical resistance analogy

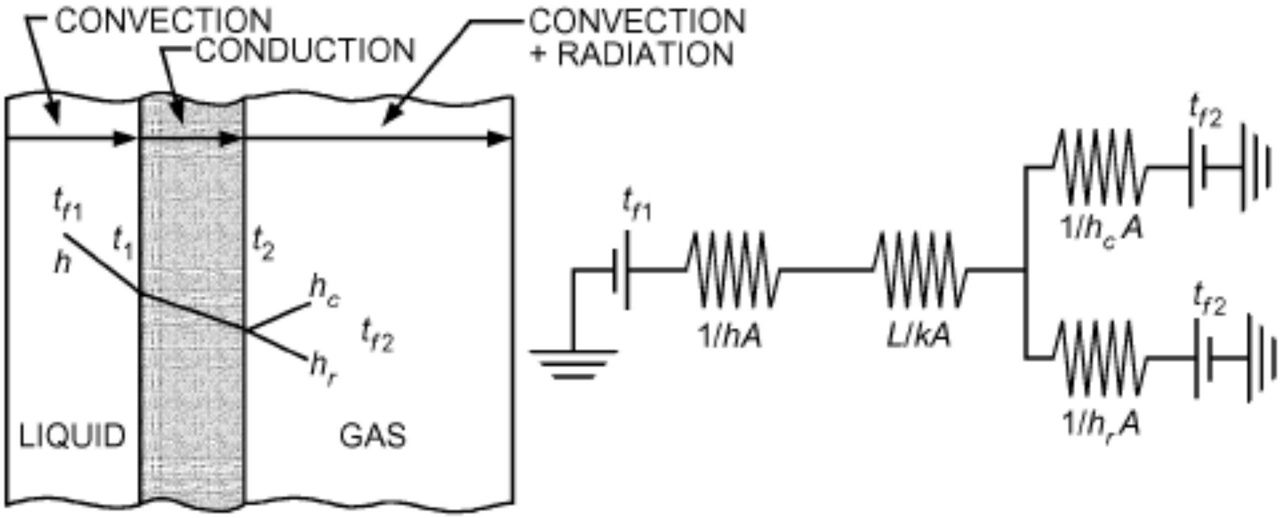


Fig. 1 Thermal Circuit

Limitations to resistance summation rule

The summation rule for finding R_{total} has several limitations:

- Only works for **layers**
- Layers must be **same area**
- Layers must be **uniform thickness**
- Layers must have **constant material properties**
  - This is the **biggest** limitation

What do we do with more realistic constructions?

Parallel path or ISO thermal equivalents

ASHRAE zone methods

4. CONDUCTION IN MORE COMPLICATED ENCLOSURE ASSEMBLIES

Thermal networks

- We have learned how to combine layers to get an overall thermal resistance (or U value) for an assembly made of homogenous layers
  - But we often have to find R (or U) of a more complicated assembly
- This more complicated analysis is best done using thermal networks
  - We've already touched on networks a bit

- References:
  - ISO 6946 Building components and building elements thermal resistance and thermal transmittance calculation method
  - ASHRAE HOF

Developing a thermal network

1. Identify the layers of the assembly
2. Identify all elements in the layers with differing thermal conductivity
3. Find the R value for each element
4. Draw a resistor for each element
   - Don’t forget the internal and external convection resistance!
5. Set the resistance to the R value divided by the fractional cross sectional area of the element
6. Connect resistors assuming isothermal or parallel path conditions

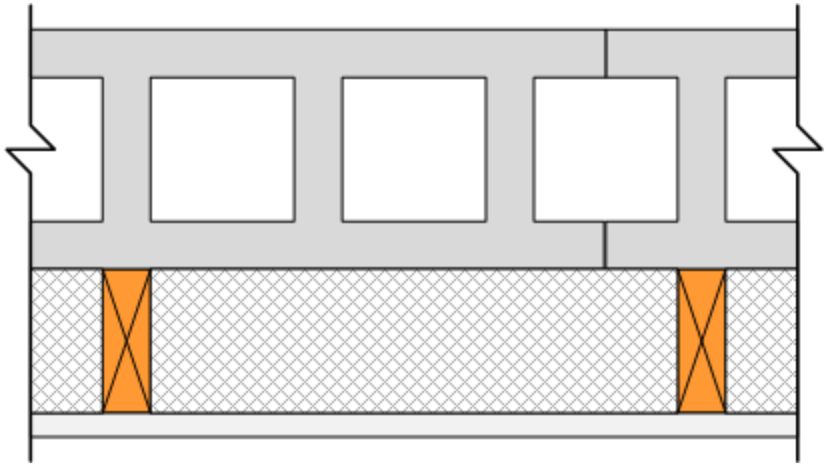
Isothermal vs. Parallel Path?

- In the isothermal assumption, the temperature at any layer interface is assumed to be constant, even if the layer is more than one material
  - This means that there is a network node that corresponds to the interface of each layer
- In the parallel path assumption, the heat transfer is assumed to be only normal to planes
  - This means that the network is several parallel branches

- This is probably best illustrated by example

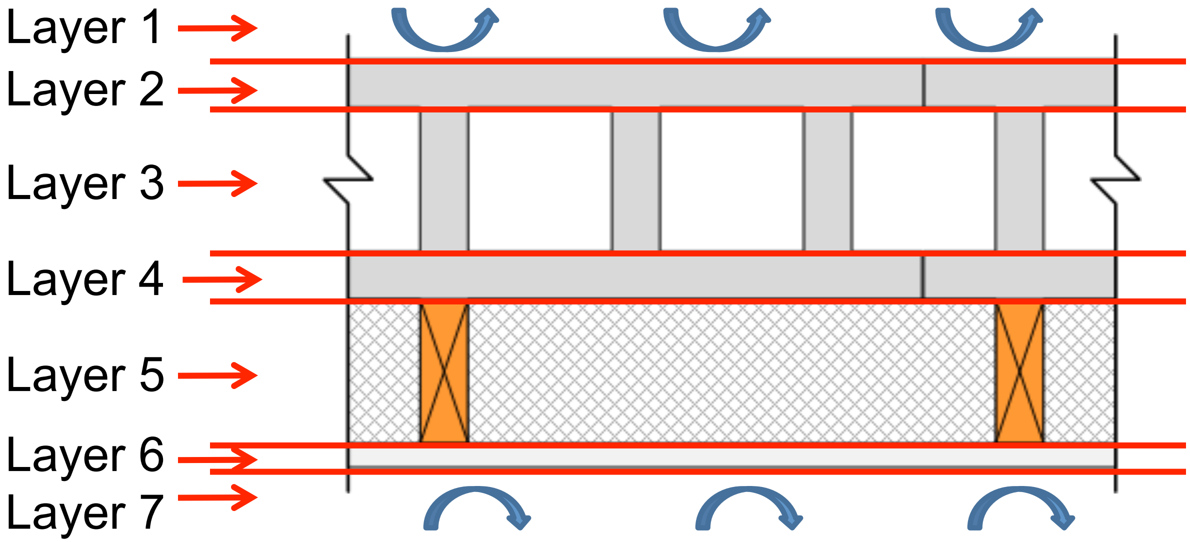
Isothermal/parallel path example

- Consider a CMU Block wall with an attached stud wall
  - A section of the wall is shown below in plan view
  - Draw the isothermal and parallel path thermal networks



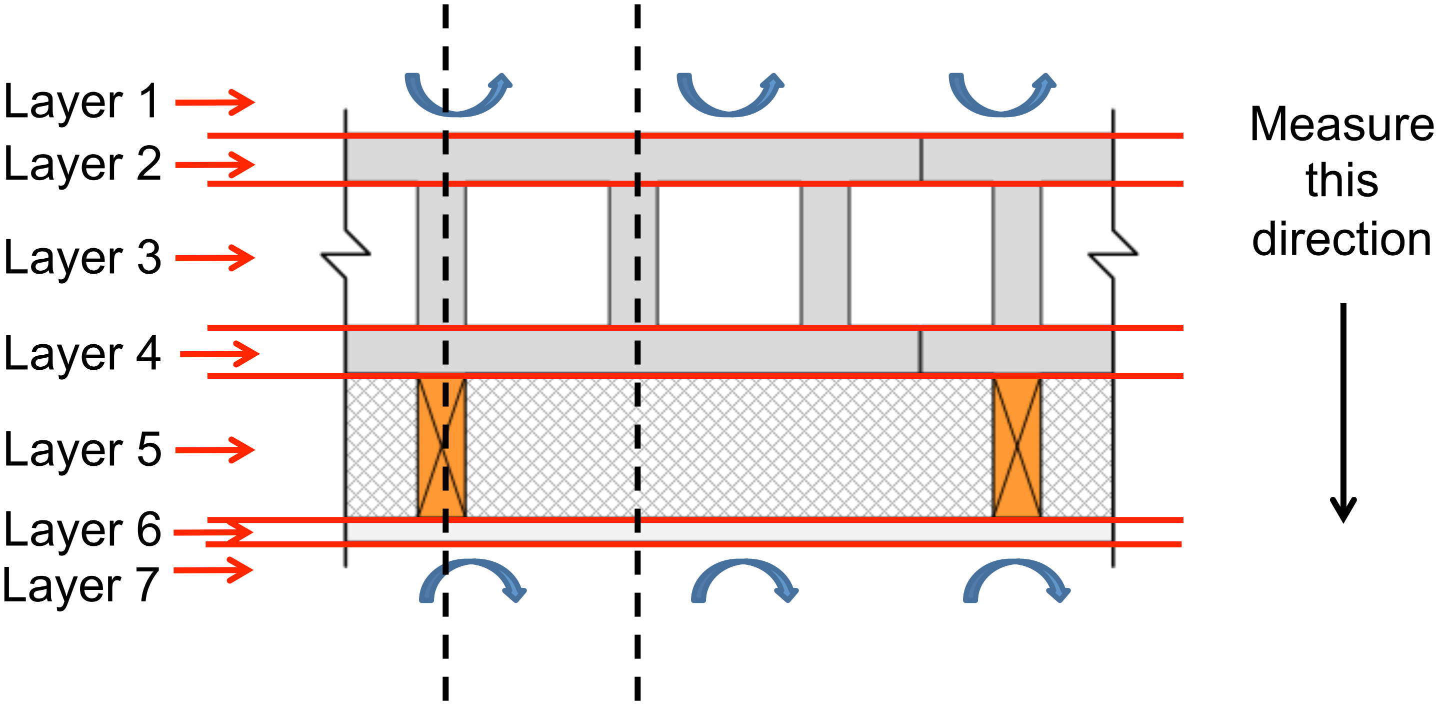
Isothermal/parallel path example

1. **Identify the layers**
   - This example has seven layers
   - Five are within the assembly
     - The other two are internal and external convection resistances



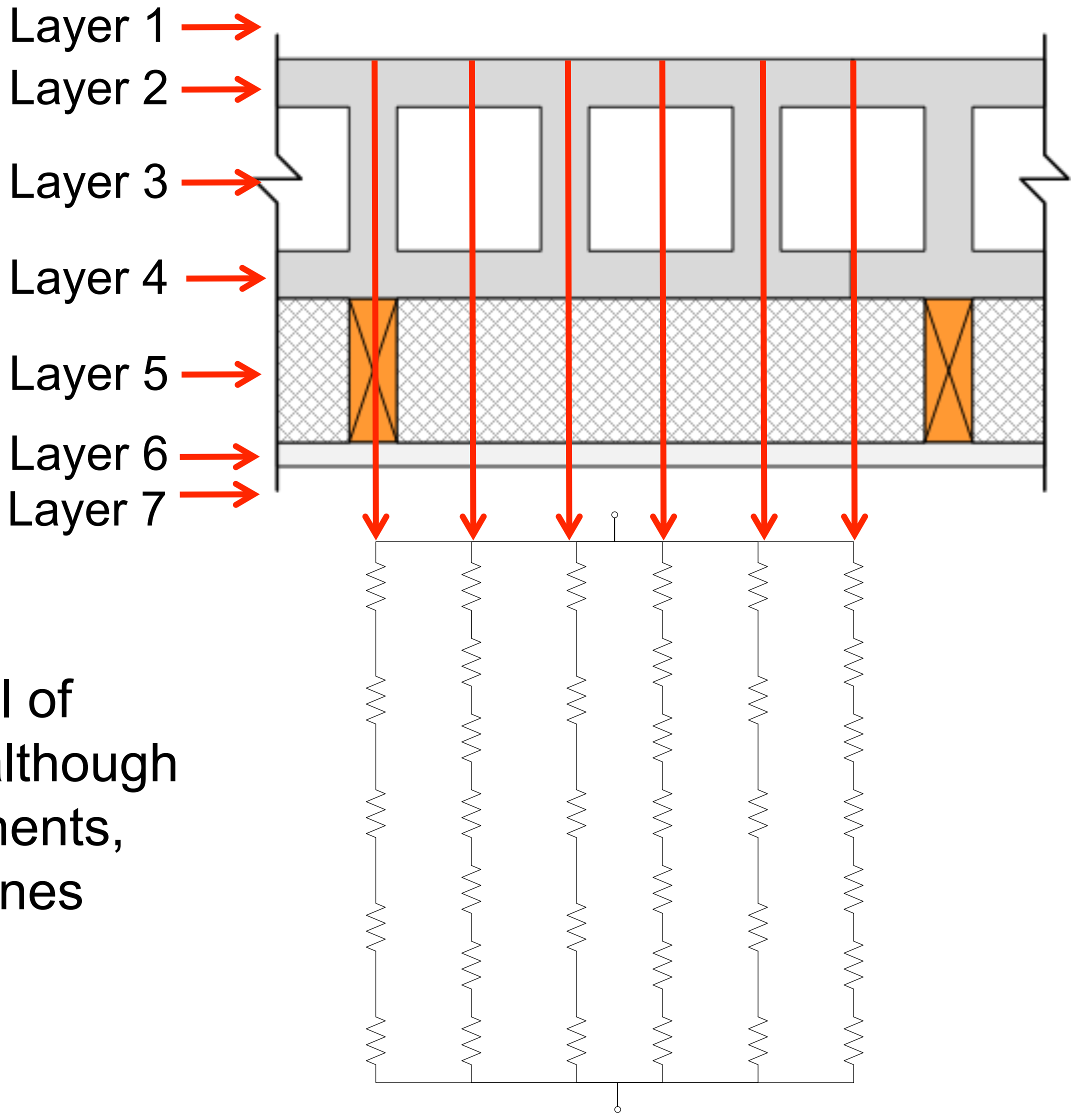
2. Identify the elements and 3. Find R-value for each element

- Layer 1 and 7 have 1 element (convection)
- Layer 2 and 4 have 1 element (1.5 inches of concrete)
- Layer 6 has 1 element (0.5 inches of gypsum wallboard)
- Layer 3 has 2 elements (3.5 inches air cavity and 3.5 inches concrete)
- Layer 5 has 2 elements (3.5 inches insulation and 3.5 inches wood stud)



Parallel Path method

- Identify the different paths and draw them in parallel



Layer 1

Layer 2

Layer 3

Layer 4

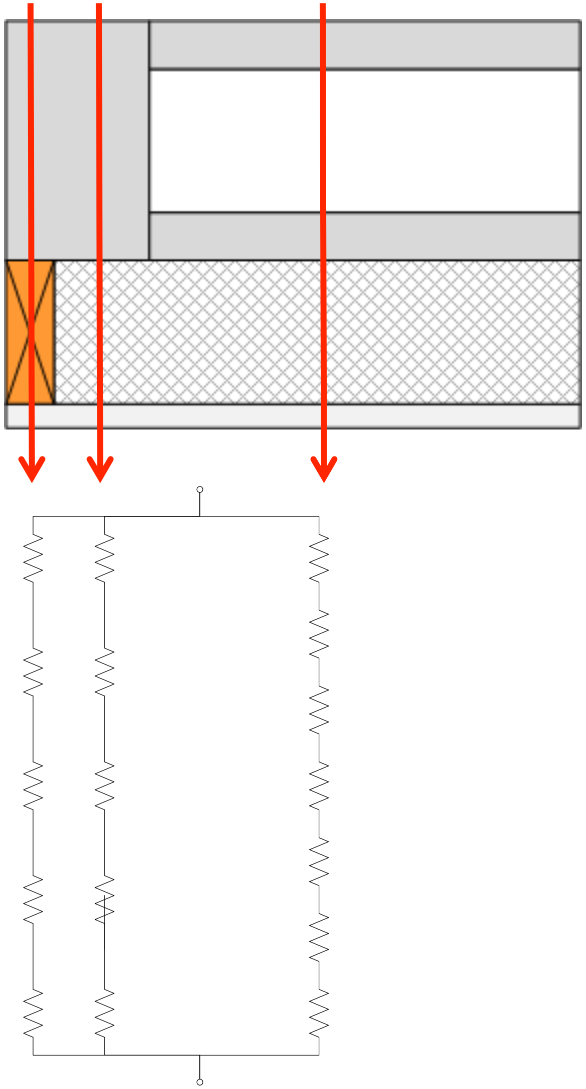
Layer 5

Layer 6

Layer 7

Note: In this example, several of these paths are identical so although there are many more components, there are only a few unique ones

Alternate Parallel Path

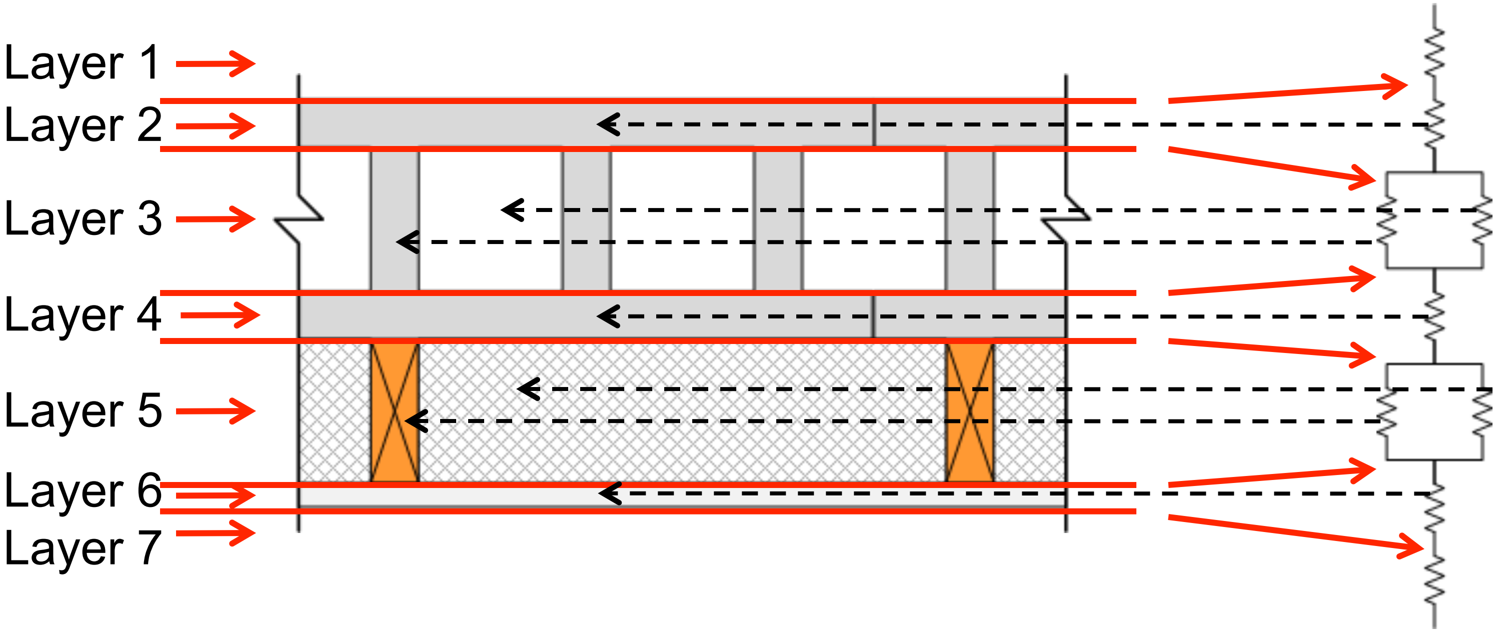


- We could combine like elements in each layer to reduce the number of paths to analyze without changing the answer for either isothermal or parallel path
  - This parallel network only has three paths, but will have the same temperatures at each interface location as the previous network

- Note the difference in the number of resistances between each path – varies according to # of elements/layers involved in each path

Isothermal method

- To apply the isothermal method, we put a node at each layer interface and add a resistor for each element in the layer



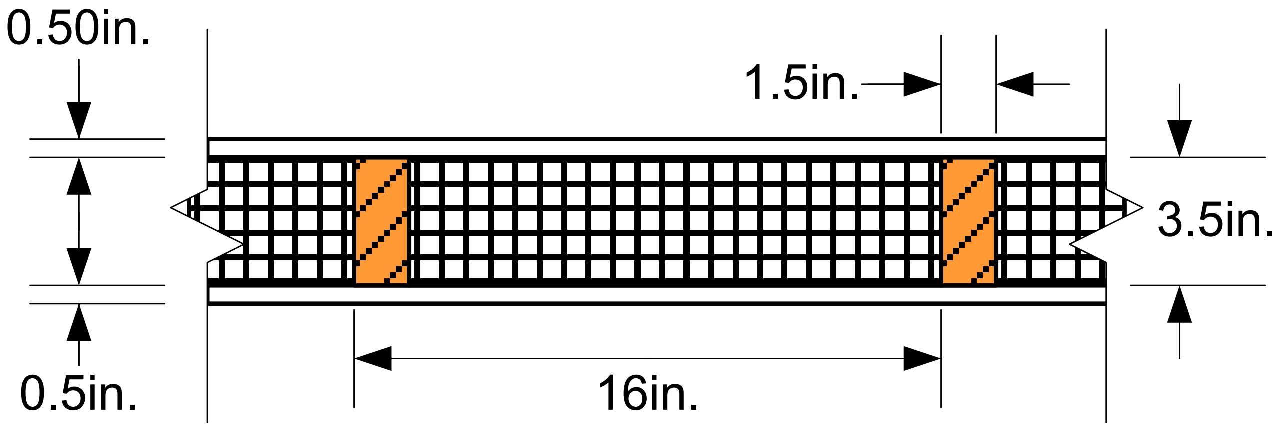
Assigning resistance values

- The resistance value for the network elements will be the R-value of the element represented **divided by the fractional area, *f*,** of each element
  - Fractional area is the fraction of the entire cross section that the element takes up
  - This must be a number between 0 and 1

$$R_{network} = \frac{\mathrm{R}_{element}}{f}$$

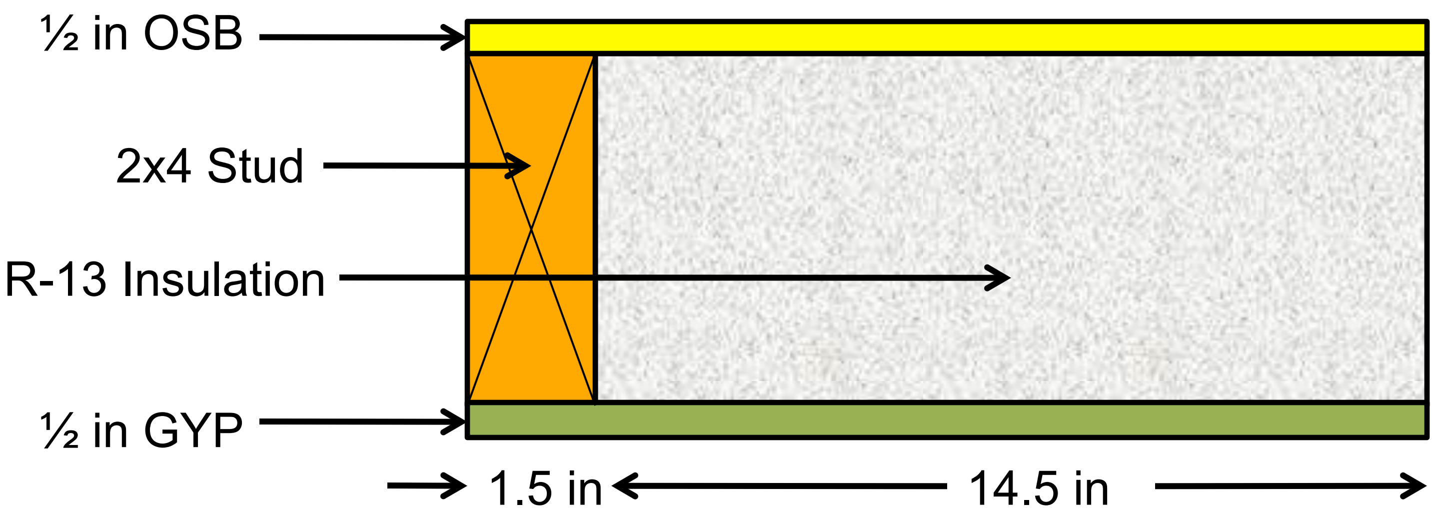
Example 3.6

- A wood frame wall has 2x4 studs spaced 24 inches OC (“on center”) with R-13 insulation in the cavity
- The interior wall is 0.5 inches of gypsum wallboard and the outside wall is 0.5 inches of OSB sheathing
- Draw the isothermal and parallel path networks for this wall
  - Assume winter outdoor conditions



Example 3.6 continued

- Let’s redraw the wall construction to make it easier to identify the layers
  - We can show only the unique part of the wall
  - Note: This drawing is not to scale



Example 3.6 continued

Identify the layers and elements:

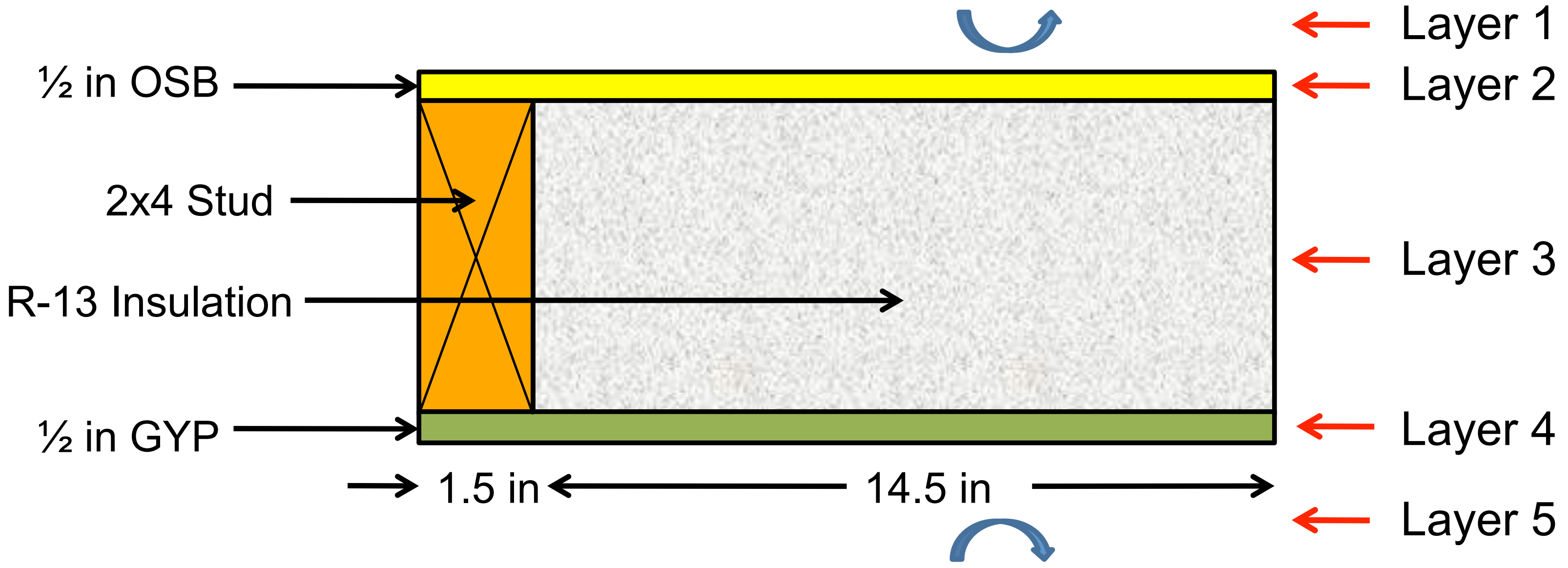
Layer 1: Exterior convection (winter conditions)

Layer 2: 0.5 inches of OSB

Layer 3: 3.5 inches of wood stud and 3.5 inches R-13 insulation

Layer 4: 0.5 inches of gypsum wallboard

Layer 5: Interior convection



Example 3.6 continued

Look-up R values for each element:

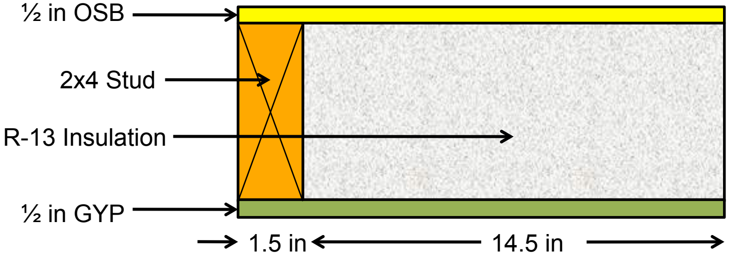
- Outdoor winter convection: R_e = 0.03 (m^2K)/W
- ½ in. OSB, R_{osb} = 0.12 (m^2K)/W
- R-13 insulation: R_{ins} = 2.29 (m^2K)/W
- 2x4 (3.5 in. thick) wood stud: R_{2x4} = 0.96 (m^2K)/W
- ½ in. gypsum: R_{gyp} = 0.079 (m^2K)/W
- Indoor convection: R_i = 0.12 (m^2K)/W

Example 3.6 continued

Find fractional areas of each element

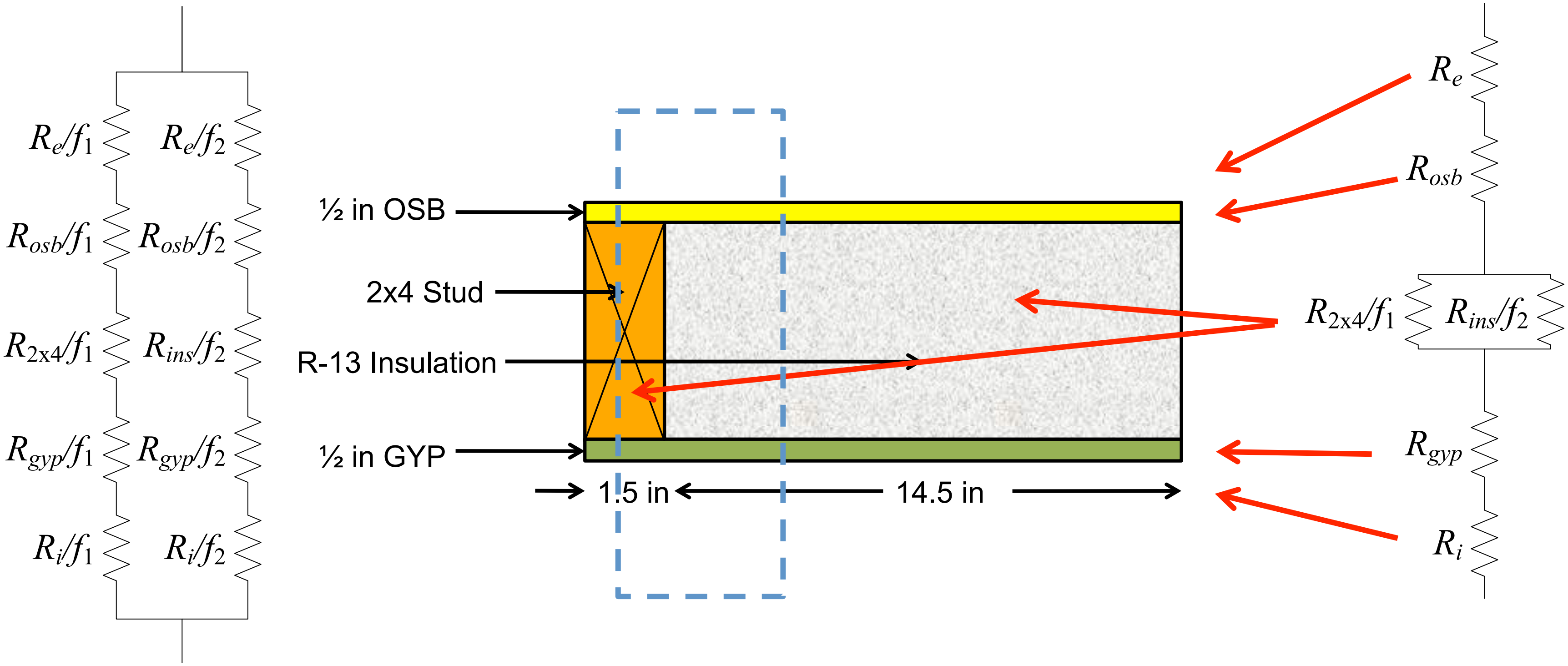
- The full width of the assembly is 16 inches
- All elements are full height of the wall, so the fractional width = fractional area

- Layers 1, 2, 4 and 5 are all 16 inches
  - $f = 1.0$
- 2x4 stud is 1.5 inches
  - $f_1 = 1.5/16 = 0.094$
- R-13 insulation is 14.5 inches
  - $f_2 = 14.5/16 = 0.906$



Example 3.6 continued

- Draw the thermal networks

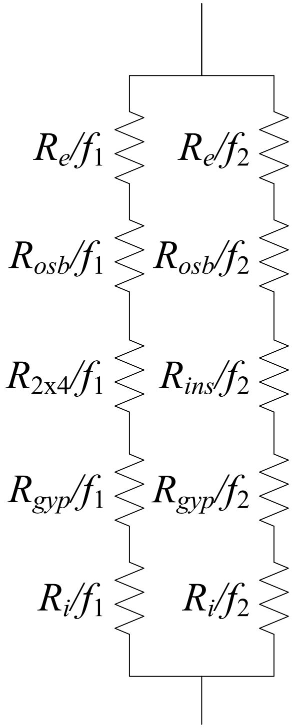


Isothermal

Parallel path

Example 3.6 continued

Parallel path network resistor values



$$\frac{R_{ext}}{f_1} = \frac{0.03}{0.094} = 0.32\,\tfrac{m^2K}{W}, \qquad \frac{R_{ext}}{f_2} = \frac{0.03}{0.906} = 0.03\,\tfrac{m^2K}{W}$$

$$\frac{R_{int}}{f_1} = \frac{0.12}{0.094} = 1.27\,\tfrac{m^2K}{W}, \qquad \frac{R_{int}}{f_2} = \frac{0.12}{0.906} = 0.13\,\tfrac{m^2K}{W}$$

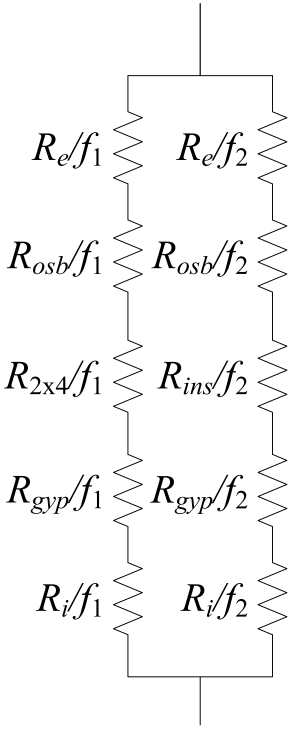
$$\frac{R_{osb}}{f_1} = \frac{0.12}{0.094} = 1.27\,\tfrac{m^2K}{W}, \qquad \frac{R_{osb}}{f_2} = \frac{0.12}{0.906} = 0.13\,\tfrac{m^2K}{W}$$

$$\frac{R_{gyp}}{f_1} = \frac{0.08}{0.094} = 0.84\,\tfrac{m^2K}{W}, \qquad \frac{R_{gyp}}{f_2} = \frac{0.08}{0.906} = 0.09\,\tfrac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25\,\tfrac{m^2K}{W}, \qquad \frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53\,\tfrac{m^2K}{W}$$

Example 3.6 continued

Parallel path network resistor values



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_1 = R_{ext}\big/f_1 + R_{osb}\big/f_1 + R_{gyp}\big/f_1 + R_{int}\big/f_1 + R_{2x4}\big/f_1$$

$$R_2 = R_{ext}\big/f_2 + R_{osb}\big/f_2 + R_{gyp}\big/f_2 + R_{int}\big/f_2 + R_{insulation}\big/f_2$$

$$R_1 = 0.32 + 1.27 + 10.25 + 1.27 + 0.84 = 13.96$$

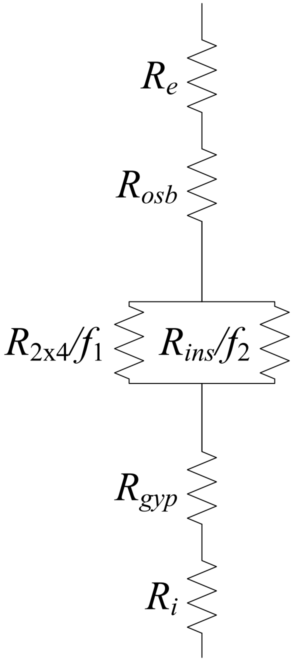
$$R_2 = 0.03 + 0.13 + 2.53 + 0.13 + 0.09 = 2.91$$

$$\frac{1}{R_{total}} = \frac{1}{13.96} + \frac{1}{2.91} = 0.415$$

$$R_{total} = 2.41 = \text{R-}13.68\ \text{(IP)}$$

Example 3.6 continued

We can now find the network resistor values for the **isothermal** network



$$R_{ext} = \frac{0.03}{1.0} = 0.03\,\tfrac{m^2K}{W} \qquad R_{int} = \frac{0.12}{1.0} = 0.12\,\tfrac{m^2K}{W}$$

$$R_{osb} = \frac{0.12}{1.0} = 0.12\,\tfrac{m^2K}{W} \qquad R_{gyp} = \frac{0.08}{1.0} = 0.08\,\tfrac{m^2K}{W}$$

$$\frac{R_{2x4}}{f_1} = \frac{0.96}{0.094} = 10.25\,\tfrac{m^2K}{W} \qquad \frac{R_{ins}}{f_2} = \frac{2.29}{0.906} = 2.53\,\tfrac{m^2K}{W}$$

$$R_{total} = R_e + R_{osb} + \frac{1}{\dfrac{1}{R_{2x4}/f_1} + \dfrac{1}{R_{insulation}/f_2}} + R_{gyp} + R_i$$

$$R_{total} = 0.03 + 0.12 + \frac{1}{1/10.25 + 1/2.53} + 0.08 + 0.12 = 2.38 = \text{R-}13.49\text{ (IP)}$$

Example 3.6 continued

- Looking at the overall R value networks we can also see the dominant path for heat transfer
  - It is the path of lowest resistance

- In this case, we have about 5x more heat transfer through the insulation than through the wood stud
  - Even though insulation has much higher resistance, the width of the wood stud is very small relative to the insulation

Utility of thermal networks

By developing the full thermal network and then combining elements, we can better see the heat transfer paths

In particular:

- We can identify thermal bridges more easily
  - Areas of particularly low resistance

- We can identify the relative contribution of the elements to heat transfer more easily

- We can use nodal analysis techniques to find the temperature everywhere quickly and easily